Fourier series-based optimization of LED angular intensity profiles for displays and backlighting

Andrea Beauchamp Supervisor: Adrian Kitai December 2019 Engineering Physics MaSC Thesis Defense

Abstract:

A method using a Fourier series is demonstrated to optimize an LED array for local dimming applications in liquid crystal display backlighting. The same optimization method is also suitable for LED displays in which the Moiré effect must be suppressed during photography with a minimum loss of spatial resolution. Initially, the angular intensity profile of a Lambertian LED is modelled when backlighting a Lambertian rear projection screen and compared to experimental data. An array of optimized LEDs and the resulting screen intensity pattern is then derived such that an intensity distribution with an intensity deviation of less than 2% is achieved. The angular intensity profile of the LED is modified using adjustable Fourier coefficients optimized according to an algorithm. The algorithm is designed to achieve an illuminated screen area of maximum size for a bounded LED backlight array to appear uniform in intensity to an observer. This Fourier series approach provides an elegant method to optimize the intensity profile of LED backlight arrays without the use of ray tracing. A lens was designed in order to provide this optimized intensity profile as well as created and tested.

Table of Contents

Chapter 1: Introduction

- 1.1 LCD Displays and full array backlighting
- 1.2 Local Dimming
- 1.3 LED Displays and the Moiré Effect
- 1.4 Lenses
- **1.5** Rear projection Application
- 1.6 Objectives

Chapter 2: Theory

- 2.1 LED Intensity Profiles
- **2.2** Screen interaction
- 2.3 Observing Intensity
- 2.4 Factors that Affect Image
- 2.5 Fourier Representation
- **2.6** Finding Fourier Coefficients

Chapter 3: Methodology

- 3.1 Initial Theory Validation
- 3.2 Matlab Modelling and Flow charts
- **3.3** Lens design

Chapter 4: Results and Discussion

- 4.1 Experimental Results and theory validation
- 4.2 Fourier Coefficients and optimized intensity profile
- 4.3 Analysis of Uniformity and Degree of Edge Effects
- 4.4 Analyzing Bloom
- 4.5 Lens Design
 - 4.5.1.1 Intensity collection
 - 4.5.1.2 Lambertian Lens
 - 4.5.1.3 Optimized Intensity Lens
 - **4.5.1.4** ray diagram and improvements
- 4.6 Testing Lens

Chapter 5: Conclusion

Chapter 6: References

1. Introduction

1.1: LCD Displays and full array backlighting

LEDs are currently used in a wide variety of display and lighting applications due to their low cost and high brightness. Such applications include the transportation sector with traffic lights and headlights and displays on appliances such as microwave and oven timers. Due to low energy consumption, low maintenance and small size, LEDs are very common for use in indicators and signs such as message and advertising signs on highways or large LED displays in stadiums. LEDs are furthermore used in destination displays for airports and train stations.

LEDs are also used in lighting as a better alternative to the incandescent and fluorescent light bulbs due to their high efficiency, also leading to a more sustainable outcome. LEDs can now be seen in streetlights on poles and parking garages, as well as general interior lighting. They are also used in cellphone cameras for flash due to their small size.

The third area of application is for data communication and other such signalling. LEDs can send data through fiber optic cables and infrared LEDs can be used for remote controls and such niche applications as in movie theatres to send sound to audience members with assisted listening devices.

Another area for application of LEDs is in machine vision systems, as bright and consistent illumination is required. For instance, a certain type of barcode scanner called a charge-coupled device (CCD) scanner uses hundreds of small light sensors to determine the intensity of light reflected at it to measure the ambient light of the barcode. [1]

A final area of application that will be mentioned is in biological detection. Ultra-violet LEDs are incorporated into light induced fluorescence sensors used for biological agent detection. [2]

One major application of interest in this paper is of LEDs within the backlight of a liquid crystal display (LCD). As time has passed backlights have been getting smaller and thinner while the displays, they illuminate have been getting larger. Improving the light guiding of backlight units and optimizing light extraction efficiency allows for smaller and thinner LCDs for handheld devices such as laptops. [3]

The backlight often comprises an array of LEDs that, in combination with a diffusing sheet, is used to provide a light source with a uniform intensity output over the LCD area. This is known as full array backlighting as opposed to edge-lit LCD backlighting. Edge-lit backlighting is as it sounds: the LEDs are placed around the edges of the LCD screen and the light is spread out across the back of the panel using light guides. This method allows for a thinner display that uses less power. However, the issue with edge lit LCD backlighting is the inconsistency with the spread of light across the screen and possibly visibly brighter areas around the edge of the screen when a dark image is used on the LCD. Full array backlighting has the option to

implement a technique known as local dimming which is much more difficult to implement into edge lit backlights. This technique will be discussed in more detail in the next section.

The LCD dominates the display industry today due to its superior lifetime, low cost, high resolution and high peak brightness. [4] The achievement of uniform full array backlighting is dependent on parameters of the LED array including LED spacing, system depth and LED angular light distribution. As the spacing between LEDs is increased, less LEDs can be used. The effect of each LED on the overall intensity of the backlight becomes more prominent as the spacing is increased, meaning that if the spacing is increased too much, the output intensity on the screen will not be uniform and an observer would be able to see 'polka dots' of light where the LEDs are located. Similarly, the depth of the system can create this effect. If the screen is too close to the backlight, the light from the LEDs is not able to spread out enough to create a uniform light, and the distance between LEDs appears larger. If the screen is further away, the resulting light is more uniform, but the system would be quite thick. Ideally the system should be as thin as possible while maintaining a uniform light emission. The angular distribution of each LED in the backlight is another important parameter. The wider the angular distribution, the wider the spread of light, allowing for an easier way of creating uniform illumination. However, there is also a need in local dimming for limiting that angular spread of light from the LEDs. Such as selectively boosting different areas of the backlight to boost the output intensity by over 100% to improve contrast and local brightness, while saving power. [5]

1.2: Local Dimming

Local dimming is a method used to improve LCD contrast ratio, by only powering specific LEDs to illuminate selected areas of an image while leaving the rest of the LEDs unpowered. This allows for both very dark and very bright portions of an image and a substantial improvement in contrast ratio. It is a critical enabler of high dynamic range LCDs. Using local dimming to only power specific LEDs that are required for an image instead of the entire backlight array also improves the power efficiency of the LCD. [6] High dynamic range refers to technology that improves the range of colour and contrast in an image

A variety of local dimming strategies including edge-lit LCD backlighting and full array backlighting have been developed. [7-10] In "Edge-lit LCD backlight unit for 2D local dimming" by Yoon et al, local dimming has been applied to an edge-lit backlight. [7] First the light is guided from the edges by way of a light guiding plate that is patterned with inverse trapezoidal microstructures. Yoon aimed to accomplish 2D local dimming, so they divided up the backlight into "illuminating blocks" where each block is edge lit by LEDs. Multiple blocks make up the entire backlight.

In [9] a high dynamic range liquid crystal display with pixel level local dimming was proposed. This was accomplished by creating a pixelated LCD dimming panel to control what light came from the backlight and continued onto the 'master' LCD panel. Using this dual panel system they found that the contrast ratio was improved but the moiré effect became a big issue

as well as pixel misalignment, decreased optical efficiency, increased power consumption and excessive thickness. The moiré effect will be further discussed in chapter 4.

In [10], the authors attempted to use a new linear programming algorithm with a full array backlight to attempt local dimming. Methods tested included the "average "method, where the brightness of the LED is set to an average value of brightness for the area, the "maximum" method, where the LED intensity is set to the maximum pixel value for the area, and the "square root" method where the intensity is chosen by the quadratic mean pixel values of specific areas to light the LEDs. [10] After comparing edge-lit local dimming, local dimming using full array backlighting is well known to be a preferred solution.

LCD display technology is rapidly developing to satisfy High Dynamic Range (HDR) requirements. Mini-LEDs and micro LEDs are being studied for LCD backlighting due to their ability to provide many more local dimming zones. [11] Full array backlight local dimming-type LCD displays require the ability for portions of the LED array to be powered while adjacent LEDs are not powered.

Local dimming can improve the contrast and power consumption of the backlight; however, local dimming does introduce a few issues. One major issue is one called the haloeffect, which describes light spreading beyond the lit area of an image due to the backlight. For example, if an image of a night sky with a moon was used, local dimming method would propose only the LEDs responsible for lighting the area of the moon would be on, and all other LEDs would be off. When this happens, there is a possibility of having light spill over from the moon and into the sky around it, creating a "halo" of light around this moon. This halo-effect that is well known in local dimming LCDs can be minimized by ensuring minimum bloom or unwanted light spreading between powered and non powered portions within the LED backlight. [11] At the same time, the LED array should produce spatially uniform light over the desired portion of the LED array that is to be powered.

Assessments have been done on the power saving of local dimming compared to backlights that do not use local dimming. Optimizations to backlighting have been created based on the results of such assessments. One of these algorithms is described in [12] and includes application specific variables relevant to local area dimming. In contrast, the methods described throughout this thesis are more broadly applicable and rely on Fourier analysis as well as a novel algorithm.

1.3: LED Displays and the Moiré Effect

Up till now the term LCD has been used which refers to a display using a liquid crystal layer as the active medium that holds the image. Both LED displays and LCDs use LEDs as the backlight for the display, however the LCD has the liquid crystal layer that controls the pixels in the image while in an LED display, each LED represents a pixel.

Another key application for LED arrays in display technology that can benefit from our approach is in camera-ready LED displays. These LED displays, well known in the digital billboard industry, are increasingly used in TV studio and movie set applications. A challenge in these applications is the Moiré effect. During photography of an LED display, unwanted patterning appears in the camera image due to the grid of camera pixels interacting with the grid of LED pixels. [13] The current solution to mitigate the Moiré effect is to place a diffusing screen in front of the LED display to produce more uniform light emission over the LED display surface; however, this results in a drop in effective display resolution. A method that would mitigate the Moiré effect while also causing minimal degradation of the resolution of the image would be valuable for camera-ready LED displays. Once again, a diffused LED array capable of substantially uniform light emission when a region of one or more LEDs is powered while an adjacent region of LEDs is not powered is required, such that minimum unwanted light spreading occurs. Excessive light spreading limits effective display resolution.

The spatial extent of light spreading that occurs in either a local dimming-type LCD backlight or in a camera-ready LED display depends on the degree of diffusion of the diffusing sheet placed in front of the LCD array. With insufficient diffusion, it is not possible to produce acceptably uniform illumination for the application at hand. If diffusion is increased, then spatial resolution decreases and blooming increases.

Increasing the spatial density of LEDs will lead to an improvement in spatial light uniformity and a reduction in light spreading since a lower degree of diffusion is then required, however cost constraints limit the extent to which this approach may be applied. There is, however, another way to optimize the LED array to minimize the requirement for diffusion: The angular emission profile of each LED within the LED array can be optimized to allow for a diffuser having a minimum degree of diffusion to enable substantially uniform light emission. This will then lead to a reduction in light spreading or bloom.

1.4: Lenses

A common way to alter LED angular emission profiles is to place lenses over the LEDs. Many lens types are known. Freeform lenses, multiple curvature lenses, and double freeform lenses are well developed and may be manufactured cost effectively using plastic molding. [14-16]

There are two types of freeform surfaces. The first is a single freeform surface (SFS) where only once side of the lens is freeform, while the other is standard. The second type is called a double freeform surface (DSF) which means both surfaces of the lens are freeform.



Fig.1. Structures of freeform lens a) SFS lens, b) DFS lens [15]

The light pattern of an LED generally is a circular spot. In some cases the need for a rectangular light pattern occurs. [14] In order to accomplish this a lens with a single freeform surface was created. The first incident surface is spherical with the LED source at the center of said sphere, and the exit surface is the freeform surface. Using this set up, coordinates can be established, and the rays from the incident surface traced, using partial differential equations to form the curve of the lens.



Fig.2. Coordinate diagram of solving a lens [14]

A multicurvature lens is used to improve the uniformity of illumination of LEDs. [15] The goal here was to use energy redistribution by controlling the direction of incident light to improve uniformity compared to traditional light sources. Using and optimizing a multicurvature lens for this application helped improve the Lambertian radiation profile of the LED. This type of lens might not be suited for backlight applications but rather the industry of LED lighting.

Using a double freeform surface lens designed using the edge-ray principle to create an ultra-compact rotationally symmetric lens was attempted in [16]. The double freeform surface lens allows light to be taken in on one side, redistributed, then passed through the second surface to be redistributed again. The figure below describes the ray tracing method used, how the incident rays interact with the first surface at different points, and then continue to the second surface, and shows what the output location of these rays would be. This is a very

common method when designing lenses to be placed over LEDs in order to improve illumination uniformity.



Fig.3. A) Principle of luminance Engineering. Light source with dimension D has a projection D_{ϑ} in the ϑ direction. B) ray tracing diagram to achieve profile of lens.[16]

Much like this method in [16], ray mapping was also used in [17] to be used in creating ultra-thin backlights. An example of such ray tracing is described in the figure below. This figure is more simplified than the one above but shows the method more clearly.



Construction schematic of the contour curve for DF

Fig.4. Coordinate diagram to find lens profile. [15]

Other methods to design freeform lenses include the differential equation method, the tailoring method, the simultaneous multiple surfaces method (SMS), the parameter optimization method, and supporting paraboloids method. [17-19]

A new ODE (Ordinary differential equation) method was proposed for the design of a total internal reflection (TIR) lens. [18] This lens would have a double freeform surface and be used to increase luminous flux at small beam spread angles. Using conditions such as Minimizing Fresnel loss, illumination models, Snell's Law of ray propagation, and constraints on the incident angle of a ray on the light-exiting surface of the lens, differential equations were created. This lens is shown in the figure below.



Fig.5. Schematic of a TIR LED lens, and propagation of rays through the system [18]

1.4: Rear projection Application

Liquid Crystal Displays (LC Displays) and LED displays are widespread, however they have limitations. LC Displays are size-limited since they are fabricated on continuous glass panels. LED displays are not limited in size, but they have resolution limitations due to the cost involved as LED density increases.

In standard colour LCDs, there exists a white backlight and to be able to control the color and brightness of the pixels, a liquid crystal (LC) light modulator with a colour filter array is used. This is where the size limitation appears. The LC modulator is made of glass, so is limited by the size of the glass. "Both plasma and liquid crystal displays rely on glass sheets and are not readily available in sizes above approximately 100 inches. Making much larger units is a challenge due to glass processing and transportation issues." [20]



Fig.6. Example of Standard Rear Projection showing that it is not a flat panel, and the bezel gaps. [21]



Fig.7. Schematic of Rear Projection

The main disadvantage with these displays is "the inability to economically produce large size, high resolution displays having no visible bezel gaps". [20] "Bezel" refers to the outside frame of the LC unit. A current solution to this problem is to create multiple display units and place them side by side and use a technique called edge blending. This technique uses multiple projectors to overlap the images on the screens. The disadvantage of this solution is that it is not a flat panel nor is it an appropriate solution for certain wall mounted displays which require a thin form factor. [20] Thin form factor describes the depth of the display.



Fig.8. Edge blending technique in rear projetion. [22]

In order to make displays at any size, one approach is to use LEDs as backlights to form an array of projectors that are made up of LC light modulators, much like making finger puppets on a wall using a flashlight. The result is an edge-blended projector array of very small projectors. This approach was described in [23]. The display size would be dependent on the number of modules used. The light emanating from each module would overlap with each other to form an image on a screen.[23]

Something similar was done in [24]. Although, here, the method of using edge blending to eliminate bezel gaps was the focus. The method to eliminate bezel gaps was to tilt the LEDs at the edge of each modulator in such a way as to make sure every area of the screen was illuminated. [24] This method has been shown to achieve a nice thin display due to having a small distance between screen and LC modulators.

Using arrays of LEDs behind each LC modulator lowers the number of LEDs compared to an LED display as well as the cost. However, there is an image artifact trade off with this method as mentioned previously. Rear projection screens are sensitive to the direction that the light is directed onto the screen. This results in viewing angle issues from the other side of the screen. The image will not be the same at each viewing angle due to inappropriate brightness levels. The viewer will notice the boundaries between adjacent LC light modulators. This method also requires more power, lack of screen contrast, contrast limitations, and degeneration of light modulators due to the brightness of the backlight LEDs. [20]

In order to improve these displays, the idea is to use multiple LC light modulators, each with arrays of LEDs to project a clear image onto a screen, while showing no gaps between the

LC light modulators , reducing power consumption, increasing screen contrast, and decreasing the degradation of light modulators. [20]

LED and LC displays will enable ultra-high resolution and arbitrarily large displays which are desired to replace printed advertising displays as well as for specialty applications such as ultra-high resolution map displays and navigation displays.

The seamless tiling solution being targeted uses LC units. The challenge is to achieve high image quality and to eliminate any visible seams or joins between these LC units. Normally bezel gaps exist between LC units but these must be eliminated. The approach of interest uses an array of small size backlight LEDs behind a desired number of LC units to illuminate a single, continuous screen. The picture on the screen would be produced by controlling the image information on the LC units as shown in Fig.9. The screen would eliminate the gaps between LC units. The final image would be produced by overlapping screen images produced by the array of LEDs. Due to the divergence of the light from each LED the screen image could be continuous even at the join between LC units. Colour would be obtained through the use of white LEDs and colour LCs.



Fig.9. New Rear projection diagram with backlight LEDs and LFA LEDs [20]

This alternative rear projection method solves a lot of issues with current displays as mentioned previously and could benefit from my research with altering the output intensity profiles of the LEDs acting as the backlight.

1.6: Objectives

The purpose of this paper is to investigate details of modified LED angular emission profiles, to study the impact of these profiles on light emission uniformity from an array of such modified LEDs, and more specifically to minimize light spreading or bloom. For the first time, an approach has been undertaken in which modified LED angular profiles are modeled using

Fourier series in conjunction with a minimizing algorithm. This is shown to be a powerful tool to optimize LED array design and to minimize the degree of diffusion required.

Modelling and optimizing backlight uniformity commonly includes the Liquid Crystal layer as well as the backlight and the diffusing layer. [25] The method presented in my research is a more general model that will be applied to a basic setup of an LED backlight and a Lambertian screen in order to explain and illustrate our method in the most simplistic way. The method models an area of intensity on a screen from an LED array using a Lambertian intensity distribution of an LED. After modeling the Lambertian distribution using a Fourier series, the model is extended to allow for modification of the intensity distribution, and its application to all LEDs in the array such as producing a substantially uniform screen intensity distribution with a local deviation of less than 2%.

In order to control bloom, a minimization algorithm minimizes light extending beyond a boundary established around a zone of illuminated LEDs. The method introduced in the upcoming chapters aims to improve local dimming. Implementing this method could also increase the distance between LEDs, thus reducing cost while still maintaining uniformity.

Light intensity angular emission profiles resulting from this model could subsequently be implemented using commercially available LEDs combined with low cost molded plastic lenses. In this paper, conventional LEDs spaced 5cm apart will be studied to illustrate the modelling and optimization methods. It is trivial to change the scale to suit smaller LED pitches.

2. Theory

2.1: LED Intensity Profiles

Firstly, the intensity profile of an LED must be considered. An LED intensity profile is represented graphically as intensity versus angle θ between the direction of emitted light and the LED optical axis. For a lambertian LED, the luminous intensity varies as the cosine of this angle. In general, most LEDs will have a Lambertian radiation pattern. This LED intensity is represented by Eq. (1).

$$I_{LED} = I_0 \cos\theta \tag{1}$$

The intensity profile of a substantially Lambertian light source is depicted graphically in Fig.10. This figure combines two ways of graphically representing the intensity profile of a Lambertian LED. The left side is depicted as a circle in polar coordinates, while the right hand side is depicted as a cosine wave. Each of these depictions is symmetric about 0, hence why this representation is possible. The intensity of an LED as a function of theta should appear as a circle as depicted on the left side of Fig.10. The intensity is the vector that exists from 0 to the edge of the circle and travels around the circle as a function of theta. The value of the intensity vector as it travels around the circle should reveal a cosine wave when plotted against theta as depicted in Fig.10.



Fig.10. Intensity profile of a Lambertian source, using both Polar coordinates (Left), and Cartesian coordinates (Right).

To prove this relationship, it is needed to consider Fig.11 which depicts the circle from Fig.10 with an amplitude of $I_0=1$. The intensity vector I is labeled in red. Wherever the intensity

vector traces around the edge of the circle (point F in this diagram) will make a 90° angle with the top of the circle to make a triangle. This triangle consists of the intensity vector I as well as the blue line. From trigonometry we know that if we trace a line from point f to the center of the circle to represent the radius of the circle, this makes two triangles. Each of these triangles will have 2 sides the same length (the radius of the circle). Knowing that the 2 sides of each triangle will always be the same it is perceived that 2 angles will be the same as well. Using the example in figure 3, this means that angles

$$\angle A = \angle B, \angle C = \angle D$$

Angles **A** +**D**+**B**+**C** need to add up to 180 degrees in order to obey the law of cosines, stating all angles in a triangle add up to 180deg.



Fig.11. Trigonometry breakdown I=cosϑ

Knowing that point f will always make a 90° angle with the top of the circle no matter where the intensity vector points and assuming the diameter of the circle to be 1, it can be seen from triangle **ADF** that the intensity vector **I** is equal to the cosine of θ , where θ is depicted in Fig.11.

This is true as I travels for any point around the circle, as seen in Fig.12.

Subbing in,

Therefore,



Fig.12. Example 2: The same principles can be seen where the intensity vector forms a 90° angle with the top of the circle, and so $I=cos(\vartheta)$ using this triangle.

2.2: Screen interaction

An ideal rear projection screen would normally be Lambertian in nature. The screen situated in front of an LED array, if Lambertian, should have ideal diffuse transmittance, in that it takes all incoming light from the LEDs and transmits the light through the screen and diffuses it equally in all directions. The behavior of a Lambertian screen is depicted in Fig.13A. A non Lambertian screen would have directional behaviour as shown in Fig.13B. It is assumed that the rear projection screen is Lambertian.



Fig.13. Lambertian surface A: Ideal Diffuse transmittance, B: transmittance with directional component (non Lambertian)

Assuming a single Lambertian LED illuminates a Lambertian rear projection screen, the screen light intensity as a function of θ as shown in Fig.14 will be described by Eq.(2) and Fig.15.



Fig.14. Diagram depicting intensity on a screen due to an LED as a function of ϑ



Fig.15. Expected screen intensity versus angle θ from Eq.(2). Both LED and screen are assumed Lambertian, and θ is defined in Fig.13.

Justification for the fourth power or $\cos^4 \theta$ dependence in Eq.(2) will now be reviewed. The first cosine term comes from the LED, which is shown in Eq.(1).

Two cosine terms arise from the increasing distance between the LED and the screen as a function of angle. Brightness is inversely proportional to the square of the distance as shown in Fig 16.

$$I_0 \propto \frac{1}{D^2}, I \propto \frac{1}{H^2}, I = I_0 (\frac{D}{H})^2$$

Since
$$\frac{D}{H} = \cos \theta$$
, therefore $I = I_0 \cos^2 \theta$.



Fig.16. Relationship between I_o and I due to LED- screen distance

The fourth cosine term arises from the amount of light that appears on the screen due to the surface area subtended by a differential angular range of $d\theta$. As depicted in Fig.17 dx shows the size of the spot where light will arrive, and this is increased as the magnitude of the angle θ gets larger.

$$dy = dx \cos \theta$$



Fig.17. Subtended screen area versus ϑ

The intensity output of an LED on a screen will initially be analyzed for screen output described by Eq.(2). This will be considered a baseline for modifications.

2.3: Observing Intensity

When an array of LEDs illuminates a rear projection screen, an observer of the screen sees an image that depends on the distance between the screen and the LED array, the spatial density of LEDs, and the angular intensity profile of the light from each LED. When considering local dimming or moiré elimination, the spread of light from each LED on the screen needs to be as limited as possible.

LED arrays may result in non-uniform intensity patterns on the rear projection screen when the LED array is positioned too close to the screen and/or the spatial density of the LED array is low. Examples of this phenomenon is displayed in Fig.18.B.



Figure 18: (A) Image requirements. (B) Actual image with visible LED backlight patterns. [26]

Metrics to assess the illumination uniformity based on the human visual system perception have been developed. [26] Sparrow's Criterion is described as the resolution limit when attempting to resolve the joint intensity map of two separate disks that are close together but have equal intensity. The two disks are considered separate once there is an intensity value located spatial between them, that has a minimum value different from either peak.

In [27] a polynomial equation was used to represent intensity of an LED array and Sparrows Criterion was used to achieve uniform illuminance in the central region, while a second and third method were used for the non central regions. In [28] Sparrow's Criterion was compared with a Contrast Sensitivity Function (CSF) as a way to evaluate an LED array lighting system.

CSF is a way to describe how well the human visual system is able to detect patterns. It measures how well one can differentiate between finer and finer increments between dark and light areas. Due to the nature of Sparrow's Criterion, which does not include the human visual system, authors of [28] chose to use CSF as the criterion to create a uniform lighting condition

for LED arrays. Similarly, the method chosen to go further needs to include how the human visual system responds to the patters. Human visual perception is a key component in determining uniformity. If luminance intensity deviations within luminous intensity patterns are less than approximately 2%, then the illuminated area will appear substantially uniform in intensity to the average observer. [29] If the luminous intensity patterns are greater than 2%, the illuminated area may appear similar to any of the examples in Fig.18.B.

An example of strong LED illumination patterns on a screen can be seen in Fig.19.



Fig.19. LED array pattern on a screen due to an LED backlight located 3cm behind the screen. LED pitch is 4.96cm and each LED is a Lambertian emitter.

The percent deviation in Fig.19 is approximately 37%. Looking at the figure below it can be seen that as this deviation decreases and approaches a value less than 2%, the image appears more uniformly white.







Fig.20. Approaching uniformity by lowering deviation between maximum and minimum intensities

2.4: Factors that Affect Image

The modelling depicted in this paper is based on a square LED array having 5cm LED pitch. The chosen goal in this paper is to achieve optimum results with a height of 3cm between the screen and the LED array since these values match to a physical LED array that we used to validate our modelling. These values may be scaled as desired to suit other geometries. Modelling will be restricted to angular emission profiles that are symmetrical about the optical axis of a given LED.

Examining two LEDs placed side by side with the LED spacing and screen height mentioned above, the resulting intensity profile can be seen in Fig.21. The resulting percentage deviation is approximately 35.8% which is much higher than 2% and therefore two bright spots would be perceived on the screen. The relationship between screen height and percentage deviation for multiple LED spacing options is analyzed in Fig.22. It can be seen for an LED spacing of 5cm the screen height would need to be 5cm to produce a percent deviation of 2% or less when only 2 LEDs are considered. It is clear, that for LED spacing of 5cm and screen height of 3cm as an

example, the intensity profile of the LED must be modified to achieve a uniform screen appearance.



Fig.21. Overlap of 2 adjacent LEDs and the resulting intensity profile (green) that an observer would see on a screen due to 2 LEDs. The difference between the two peaks and the valley between them is 34.28% of the intensity of the peak. x is defined in Fig.17





2.5: Fourier Representation

The angular intensity profile of an LED will now be represented using a Fourier series.

$$F(\alpha) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(\frac{n\pi\alpha}{p}) + b_n \sin(\frac{n\pi\alpha}{p}) \right]$$
(3)

F(x) is defined over the interval [-p, p], where $p=\pi/2$ and Fourier coefficients a_n and b_n are given by

$$a_n = \frac{1}{p} \int_{-p}^{p} f(\alpha) \cos(\frac{n\pi\alpha}{p}) d\alpha$$
(4)

$$b_n = \frac{1}{p} \int_{-p}^{p} f(\alpha) \sin(\frac{n\pi\alpha}{p}) d\alpha$$
(5)

In order to represent Eq.(2) using the Fourier series of Eq.3, the required coefficients found using Eq.(4) and Eq.(5) are $a_0=3/4$, $a_1=1/2$, $a_2=1/8$, $a_3=0$ and $b_0=b_1=...=0$. This results in a Fourier series

$$I = \cos^4 \theta = \frac{3}{8} + \frac{1}{2}\cos(2\theta) + \frac{1}{8}\cos(4\theta)$$
 (6)

The Fourier coefficients can now be modified in order to modify the shape of the LED angular intensity profile. Extra terms can be added to the Fourier series to refine the angular dependence of the LED illumination profile. This general equation with 5 coefficients is

$$I_{optimized} = a_0 + a_1 \cos(2\theta) + a_2 \cos(4\theta) + a_3 \cos(6\theta) + a_4 \cos(8\theta) + \dots$$
(7)

Coefficients may now be sought that result in an LED intensity profile that, when applied to the array of LEDs, will allow for uniform screen illumination that stays within a 2% deviation.

2.6: Finding Fourier Coefficients

A second optimization requirement is now introduced. Whereas uniform screen illumination may be achieved using very wide-angle lensed LEDs, our goal is also to minimize bloom. To this end, we invoke the use of a finite or bounded LED array. This finite LED array will result in limited spread of light beyond the bounded array. The Fourier representation of Eq.(7) which has a limited number of terms cannot properly represent wide angle illumination commonly found in low cost full array backlit LCD displays. Since wide angle illumination is not wanted, the limited number of terms (5 coefficients) is acceptable and it allows us to effectively optimize small angle luminance dependence.

A constraint is therefore introduced by means of a square "window of uniformity". This window is defined by the area on the screen illuminated by the finite LED array providing screen intensity deviation less than +/- 1%. Our algorithm seeks to maximize the size of the window.

This window of uniformity is optimized to be as large as possible and thereby ideally close to the size of the bounded LED array. The window can only be optimized by changing the Fourier coefficients. An example of a finite LED array and a resulting window of illumination are shown in Fig.23.



Fig.23. Window uniformity is defined by screen area over which uniformity may be achieved (red) in this example overlaying an 8x8 LED array. Outside of the window, screen illumination will drop off rapidly and deviates by more than +/- 1%.

Matlab code has been created to optimize the Fourier coefficients in Eq.(7).to maximize the window of uniformity. Given an estimate of the starting coefficients for the Fourier expression, the code will then create an array and attempt to optimize the window according to the parameters by adjusting the coefficients. This code will be described in the methodology section.

Two parameters are used for seeking an optimized condition. The first parameter is the deviation parameter defined by the percentage difference between the maximum and minimum intensities within the window, representing the flatness of the screen intensity profile. A square window is created based on the area that respects this threshold value. The second parameter is the side length of the square window which we seek to maximize. Eq.(8) defines R, a merit function, to be the ratio between the deviation parameter and the window size. R is to be minimized in order to find a maximum window size while keeping an appropriately small deviation parameter within the threshold.

$$R = \frac{DeviationParameter}{WindowSize}$$
(8)

The merit function does not explicitly include a variable to address bloom, but rather, by finding a balance between intensity uniformity and area size, bloom is inherently addressed. This occurs because, by seeking to maximize the area of uniformity from a finite LED array size, localized LED levelling is optimized, and the spatial extent of bloom is thereby automatically minimized.

Matlab code now uses the minimization of R to find the coefficients in Eq.(7) by a successive approximation algorithm.

Once this ideal intensity profile is found, a lens can be created to be placed over the LED to give it this improved intensity profile.

3. Methodology

3.1: Initial Theory Validation: Model Single LED radiation Pattern

The process being applied is as follows

- i) use Matlab to first model the ideal output of an LED on a screen using an expected function of $\cos^4\theta$. This is assuming a Lambertian screen.
- ii) Create first a 2D model as a function of both the distance away from center of LED, and of theta (the angle between the point straight above the LED on the screen, and the maximum distance away on the screen).
- iii) create a 3d model of this radiation pattern in Matlab as well. Need to make sure this matches with the theory. The Matlab approach for this will be described in section 3.2.
- iv) Measure single LED radiation Pattern: LED will be attached to a moving platform with screen set above the LED and a photometer situated above that. The LED will move in 1 direction by a consistent distance. At each distance a new intensity measurement will be taken. This gives an intensity measurement at each distance away from the center of the LED, giving data for an intensity profile. The data to be recorded is the intensity in cd/m², the distance the LED moves at each interval, the number of intervals. The height that the screen is located above the LED is also measured, as this height has a great impact on the intensity values recorded. Measurements will be taken with multiple screens in order to find the most Lambertian screen to use for the remainder of the project.
- v) Compare results of model to experimental results: Answer questions such as: How different is the model compared to the actual? Why is the experiment different? Does this difference lie within the expected difference? How does the screen height affect the radiation pattern?

The initial experiments consist of testing a XQ-E high intensity LED from Cree by measuring the intensity profile on a screen and to compare it to the expected Lambertian intensity profile. In order to measure the intensity profile and compare to a Lambertian profile, a Lambertian screen is necessary. Therefore 2 different screens will be tested for the Lambertian result. The first screen will be referenced as the Vutec "square" screen, as it came in a square shape and is more glass like, the second screen is an Aeroview 70 (Stewart Filmscreen Inc) screen and is a membrane like material, so when comparing will be referenced as "membrane" screen. In order to use this "membrane" screen, a frame was necessary to keep the screen material taught. To create this frame, steel rods will be used and pinned at each corner so that once the membrane screen is clamped to the steel rods on each side, the frame can be expanded in size to stretch the screen. Both screens are shown below in Fig.24.



Fig.24. Vutec non-Lambertian screen (left) Aeroview 70 membrane screen (right)



Fig.25. Frame for membrane screen

First the circuit will be described. The current provided to the LED will be 0.1A, as the maximum drive current allowed is 1A. Choose a 20 Ω series-connected resistor to limit the current given a 5v power supply.

$$R = \frac{V_{cc} - V_{LED}}{I_{LED}} = \frac{5v - 3v}{0.1A} = 20\Omega$$



Fig.26. LED circuit diagram

The LED is situated on a board and is placed in a vice that is movable in the x direction. The screen is set on top of the LED at a distance of 3cm for both screens.

The vice is able to move in 1 direction via a crank. Knowing the total distance the vice can move in the 1 dimension and the number of cycles needed to move the crank in order to achieve this distance one can calculate the distance traveled by the LED in one cycle of the crank. This distance was found to be 3.22mm. Knowing the distance traveled each cycle as well as the distance between the screen and the LED, the angle of light can be found. See Fig.27.

$\Theta = \arctan(\frac{x}{v})$

where y is the distance between the LED and the screen, and x is the horizontal distance between the photometer and the LED. What is not shown is that the LED is on a movable mount that allows it to move in the x direction relative to the photometer allowing the X value to increase. The photometer was centered along the axis of the brightest point before any movement of the LED.



Fig.27. LED and photometer set up. (Distances between LED/Screen/photometer not accurate)

The photometer is placed not directly over the LED for the starting position but off to one side so that as the LED is moved the photometer will pass directly over the LED. This was done so that the peak intensity is clear in the measurements as well as a tail end, given the limited movement of the photometer preventing it from full covering the range of LED illumination.

Using the intensity found when $\theta=0$ for both screens we can find the theoretical intensity output $I=I_0\cos^4(\theta)$. This output will be plotted alongside the results for each screen. The polar plot of this will be used as well for comparison between the two.

vi) Measure Single LED Radiation Pattern with Camera: Use a camera to take pictures of the LED array, and import this picture into Matlab. Knowing the number of pixels, and dimensions of the actual space in the picture, we can attribute intensity values to specific distances and use this to create an intensity profile. The photometer results will be necessary to calibrate the intensity results of the camera, as the camera outputs pixel values in the range 0-255. The camera is needed to acquire results of a 6x6 LED array, as the moving platform used for the single LED array has a restriction on the distance it can move and so will not allow the photometer to get full results of the array.

Along side the photometer a Teledyne Dalsa Genie TS camera is used to gather information in all directions. The results from the camera will be compared to the photometer to confirm accuracy in the results. Once the camera results are confirmed, it will then be used going forward when analysing the intensity output of an array of LEDs.

The camera takes a picture and outputs this to the computer. In order to extract intensity values the pixel value from each pixel is taken. These pixel values have a possible range from 0 to 255, where 0 is no light, and 255 is the maximum amount of light. These pixel values are extracted in a matrix format in Matlab where the picture can be recreated with just pixel values in order to get the intensity profile in all directions. In order to compare the camera results with the photometer results, since the photometer results are in one dimension, the maximum brightness point is found in the camera results and one row of data is taken along this maximum intensity point. This should represent the same data as the photometer. Both these sets of data will be normalized as the photometer data points are in cd/m^2, while the camera data points are in pixel values. The flow chart for Matlab code will be shown in section 3.2.



Fig.28. Camera photograph of LED through a screen. The red lines depict the row and column of pixels where the maximum intensity is located.



Fig.29. Showing that the row and column of maximum intensity is interchangeable

A picture of one LED in use is taken with the camera first to compare with the photometer output. Using the maximum intensity from the photometer, this can be used to

calibrate a scale for the camera, since the camera uses pixel values. Converting the maximum intensity in pixel value to CD/m² a conversion formula is created. Generally, the scaling factor would be equal to the max intensity of the photometer/max intensity of the camera, all multiplied by the exposure value used, where the exposure value was given in 1/1000 μ sec. When taking repeated measurements using the same settings or measuring at different points, if the conversion formula predicts the correct result each time, then the scale is linear and this formula can be used for converting pixel values to cd/m² for all future measurements.

Once a single LED is measured and the camera values are calibrated using 1 row of pixels, a contour plot of the intensity can be displayed in 3 dimensions using Matlab.

- vii) Exposure settings: The first step is to take multiple pictures of the LED array using different exposure settings. Then we need to figure out where the exposure limit for saturation occurs, and decide which exposure settings to use for optimal results.
- viii) We also need to check if the camera uses gamma correction because this would alter the results. Gamma correction is when the camera accounts for different changes in light to mimic the human eye. For our experiment the camera needs to behave more like a photometer. In order to check for this phenomenon, use a single LED and take pictures with different input voltages, if when plotting intensity against input current, the relationship is linear, then there is no gamma correction. The result of this has been shown here in the method section as to show that no other method to account for a gamma correction in the camera needs to be added.



Fig.30. Gamma curve for the camera. Linear relationship

Knowing that a gamma correction is not needed, the camera can continue to be used as before.

ix) Measure LED array radiation pattern: Use the Dalsa camera method that was used for the single LED. The LED array will need to be created via soldering, and checked to make sure all LEDs have a cohesive intensity output, meaning there is no singular dim or bright LED that stands out from the rest. The LED array will have the same circuitry as the singular LED, but they will all be connected in parallel.

3.2: Matlab Modelling and Flow charts



Fig.31. Camera data acquisition code

The data from the camera is stored as a matrix the size of the pixel size of the photograph, filled with intensity values ranging from 0-255. Here, 0 is complete lack of light, and 255 is the maximum intensity. The "imread" function reads this file and stores the data in Matlab as "A" to be used. From here the "find" function is used to find the x and y location of maximum intensity and what that intensity value is. This location is given by pixel/matric values as they are both the same. Knowing the X point of max intensity called Xm, similar with Ym. A row of data can now be extrapolated that includes the peak intensity. As one goes across this row for all values of y, the intensity value should behave as the theory suggests. Therefore, intensity profile from the camera can be expressed as the values from "A" located at (Xm,:). This could be done with a column as well instead of a row. Using the location as the x axis this can be plotted as a function of pixel. To be converted into intensity as a function of theta, the x axis can be modified. Every picture taken has a ruler that extends from the top of the picture to the bottom, so that each pixel can be converted to cm scale, knowing the length of the picture in cm, and the number of pixels. Knowing the pixel size and the height to the camera, simple trigonometry is used to find the angle for each data point. To plot this in 3D, use meshgrid or contour3 function.

The intensity profile of a Lambertian LED on a Lambertian screen needs to be modeled in 2D. This is simply achieved by plotting $\cos^4\theta$. This intensity profile of an LED is to be considered rotationally symmetric and therefore to plot this in 3D, this one vector of data needs to be converted into a matrix where the data is symmetric about the origin and is depicted as a function.

To convert 2D intensity data into a matrix that is representative of a rotationally symmetric intensity profile the flow chart below is used.



Fig.32. Code plan to create 3D intensity data for LED

First input data such as the height of the screen relative to the LED, and the Fourier coefficients need to be stated. These Fourier coefficients are the coefficients that, when used in our Fourier equation, give the Lambertian intensity profile on a screen ($\cos^4\theta$). The data can be either

plotted as a function of θ , or as a function of x where x=htan θ , and the cosine terms in the Fourier equation is modified to account for this.

The next step is to take the Intensity data from V to create two vectors. The first we will call r, and it is equal to V. The second vector will be called r', and is equal to V'. We now have the same data in a vertical vector and a horizontal vector. Taking these vectors and lining them up along the x and y axis in a 2D plane, the location within the vector can be treated as a location along the axis, and the value at each point will be the intensity value that will give the 3^{rd} dimension. This can be visually depicted using a simple example vector [2,1,0,1,2] below.





Looking at this figure, the grid can be treated as locations within the matrix. The scale along the outside shows how the 2 vectors are crossed over at the origin as this is where the rotation is to occur. To create a matrix that holds the intensity data, all the empty spots in the matrix need to be populated. R_i will be new values along the diagonal calculated using the function in Matlab called bsxfun. This vector is depicted by the green ellipse in the figure below.


Fig.34.Visual depiction of rotating a vector to fill in missing matrix values

For this vector example, data is only known up to the edge of the red outline. Drawing a concentric circle with the corresponding radius of the limit of our data knowledge, it can be seen that the squares filled in with $\sqrt{5}$ is not actually known if we rotate the r vector. Only the area within the large blue circle is known if the vector is rotated. Using the interpolation function stated in the flow chart above, a full matrix can be created comprised of the rotated vector where the vector is the 2D intensity profile.

x) Model LED array radiation Pattern: Further the model by adding other LED intensity profiles to both the 3D model and the 2D model. The 3D model of an array will be created by creating a separate matrix for each LED, then convolving them together into 1 big matrix, making sure to attribute for the correct distance between the LEDs.

A zero matrix is created that is large enough to represent a 20x20 array of these LEDs considering the distance between them. The "img" matrix created previously that shows the

intensity values for an LED is placed within this zero matrix such that the first row of values lines up with the first row in the zero matrix, and such that the first columns line up as well. Example plotted at (1:f,1:f) where f is the length of the zero matrix. Another zero matrix is created and populated with the "img" matrix, except this time the "img" matrix is shifted in the x direction by the distance between LEDs. This was done for the location of each LED, shifting in the x direction by the distance between LEDs each time to create a row, and then also shifted by the same distance in the y direction to create all other rows. This results in 20 matrixes of the same size each with intensity values for an LED at a specific location. These matrixes can now all be added together to create one matrix that corresponds to the intensity values of the full 20x20 LED array.



Fig.35. Setup simulation function

Optimize Fourier coefficients to achieve optimal LED radiation pattern: using Matlab one can adjust the coefficients until a Fourier series is found. When side by-side LED's are considered, in order not to see a difference in intensity there needs to be a restricted percentage difference of the maximum intensity, and the value between the 2 LEDs of less than 2%. This new Fourier series needs to allow for such a percentage difference less than 2%.

Percent deviation= $2\% = \frac{Maximum Intensity - Minimum Intensity}{maximum intensity} * 100$



Fig.36. Function to be minimized includes the setup simulation function as well as the fitness function

Once this code is created, the optimized coefficients are found by minimizing this function displayed in Fig.36, or more specifically minimizing the out puts R. This minimizing is done using a matlab function that already exists called fmin. The parameters for this function, are what we are looking to optimize which are the guess coefficients. This minimizing function takes these guess input coefficients and alters them until the R value is minimized. Minimizing R means decreaseing the deviation between intensity peaks and valleys, as well as maximixing the area over which this occurs. Optimized coefficients are then outputed and can be used as

inputs into the setup simulation function in Fig.35 to show the intensity of the array in 3D and compare to the original intensity of the array.

xi) Model LED Array Radiation Pattern using Optimized Fourier Series: Using Matlab again, much like when modelling the LED array radiation pattern, except this time use the modified Fourier equation in place of each LED. This would model how an LED array, with lenses on each LED that provide this new radiation pattern, would behave.

In order to analyze the spatial deviation in the intensity, the data will be cut into smaller sections. These sections will be square in size with length corresponding to 5cm. This length is chosen as the LEDs are spaced 4.96 cm apart. This will allow each section to include the peak intensity value located directly where the LED is as well as whatever minimum intensity value exists between the LEDs horizontally/vertically or diagonally. Within each section the difference between peak and valley is found and compared as a percentage of the peak intensity. Repeating this for spatial sections across the entire array, a topographical representation of spatial intensity deviation can be created and plotted. An example code for this section is shown below.

1	length=18;			
2	for i=1:length			
3	a1(i)=i*5-5; a2(i)=i*5;			
4	x_10=find(x1>0); y_10=find(x1>a1(i));			
5	x_15=find(x1<5); y_15=find(x1 <a2(i));< td=""></a2(i));<>			
6				
7	xa=x_10(1,1); xb=max(x_15); xc=max(y_15); xd=y_10(1,1);			
8	x_min=x1(xa):(x1(xb)-x1(xa))/(xb-xa):x1(xb);			
9	y_min=y1(xd):(y1(xc)-y1(xd))/(xc-xd):y1(xc);			
10				
11	newsumm=summ(x_10(1,1):max(x_15),y_10(1,1):max(y_15));			
12	[MM,II] = max(newsumm(:));			
13	<pre>[I_rowsum, I_colsum] = ind2sub(size(newsumm),II); %location of max intensity circle</pre>			
14	minnn=min(newsumm); minfinal=min(minnn);			
15	percentdeviation3d(i)=((MM-minfinal)/MM)*100;			
16				
17	end			

Line 1 specifies a length of 18, as this is how many sections there will be, as there are 20 LEDs. Line 3 specifies where each section will end in the x and y axis. Line 4 and 5 are creating the vectors that represent the x and y axis of the first column of sections. X1 was a previous variable that denoted the x axis of the entire 20x20 array, since the x and y axis for this array is the same, x1 was used interchangably for the y axis as well. Line 7 is creating variables that represent the max x and y value for each section. Lines 8 and 9 are scaling the x and y axis to have the correct amount of points. Row 12 finds the maximum intensity within this section, line 13, outputs the row and column within the matrix where this maximum intensity exists. Line 14 finds the minimum intensity value in this area and line 15 outputs the difference between these intensities compared to peak intenisty. This code is then repeated for each column of sections until the entire array has been analyzed. These intensity deviation values are then plotted in a contour map. Because each of these deviation values represents a small area, these values are plotted at the center location of each section. Visual example of sectioning is shown below in Fig.37.



Fig.37. Sectioning of LED array spatially for analization of intensity deviation

Once this new Fourier series is found and optimized in order to give the led array an intensity pattern that is acceptable, Opticslab software will be used and compared to first-principles theory and Fourier analysis. Actual near and mid-field Opticslab results will be contrasted and compared to first principles results.

3.3: Lens design

Designing the lens to give the optimized intensity profile is the next step. Opticslab software will be used to create an aspheric lens with refractive index of 1.5. With optics lab a point source will be selected as the LED light source. However since the point source in optics lab does not behave as a Lambertian source, adjustments need to be made to give the source a Lambertian intensity profile.

The intensity data from optics lab will need to be extracted for use in the Matlab code to test the results of the lens. While designing the lens, the output intensity profile needs to constantly be compared and contrasted with the optimized intensity profile. Optics lab will simulate a source with a specified number of light rays that will shine through a lens and onto an observational screen. The intensity profile on this screen should be equal to $\cos^4\theta$ with no lens, and equal to the optimized intensity profile with a lens. The data on this observational screen is what is extracted to a text file. How the observational screen works is that it is cut into 127 sections in the x and y direction to create a grid. As the screen size is increased, the sizes of these sections are increased, not the number of sections. The intensity gathered in one section is represented as one data point. Knowing the size of the screen and how many sections exist, the size of each section can be calculated.

The format of the textfile is shown in an example below.

```
Optics Lab Ray Trace Data
Screen 1 Data:
Intensity Data:
FOV (mm):
                 197421.
Power (%):
                 19.6477710512
Peak Power (W/sq mm):
                        4.33987607321e-010
                 Intensity(RGB)
X coor
        Y coor
-64
        -64
                  0
                      0
                          0
                      0
                          0
-64
        -63
                  0
-64
        -62
                  0
                      0
                          0
-64
        -61
                  0
                      0
                          0
-64
        -60
                  0
                      0
                          0
        -59
                      0
-64
                  0
                          0
-64
        -58
                  0
                      0
                          0
-64
        -57
                  0
                      0
                          0
```

Fig.38. Snippet of the textfile that is create from Opticslab

For every x coordinate, the y coordinate ranges from -64 to +63, and the x coordinates have the same range. Again, note that these coordinates stay the same regardless of the size of the observation screen and the difference between these coordinates needs to be scaled. Also note that the first 13 lines are not important to the data we want. The intensity shows up as 3 columns, one for red, one for green, and the last for blue. This is because there are different colour rays that can be used. For colours such as purple, the red and blue intensity columns are filled in, both equally. Intensity data need only be extracted from one colour column as long as it is a populated one. For example, if the colour is purple, we can extract data from either the red or blue column but not the green. In this snapshot of the textfile, the intensity values are 0 as this is near the edge of the screen and so no light is appearing.

To import the textfile into Matlab the function dlmread("filename") is used and assigned to a variable called "m". Now, we want to alter this textfile to only include the coordinates and intensity data. To accomplish this a blank matrix will be created called "txtfile". The first column of this new matrix is then populated with the x coordinate values skipping the first 13 lines of text. This is done for column 2, populated with the y coordinate, then again for the intensity value of the red column. The colour of source rays was chosen arbitrarily. Now this new matrix called textfile is a more accessible version of the data. This data now needs to be extracted into x and y vectors so that it can be plotted, because at it is currently, we can not extract column 1 and call that the x axis, same with column 2 and the y axis. What we do is to use the find function. In this "txtfile" matrix we find where in column 1, the value is -64. As this happens multiple times, we call this vector location 1, as it is filled with the location in the 1st column of the matrix where x=-64. This is done for every x value from -64 to +63.

A second matrix is created called "screenoutput". This matrix will only be populated by intensity values. The matrix will represent the screen output, the location within the matrix corresponding to the location on the screen. This is where the location values from the "txt" file matrix will be used. To populate the first row of this matrix where x=-64, use screenoutput(1,1:128)=txtfile(loc1,3). This example of code is saying that at row one in "screenoutput" for columns 1 to 128 (corresponding to y values of -64 to +63) we populate it with the intensity value in column 3 whenever x=-64 (loc1).

X and y axis are created using a vector of -64 to +63 for each, then this "screenoutput" matrix can be 3D plotted displaying intensity versus location on the screen. To make this plot to show the intensity as a function of angle or cm, the locations on the screen can be converted based on the size of the screen, and the distance the screen is from the source or lens.

An aspherical lens will be used as it has the most parameters that are alterable. Using a standard concave or convex lens may not be adequate, aspherical gives more room to play. The parameters for an aspherical lens in optics lab is given below in Fig.39.



Fig.39. Aspherical lens parameters

Parameters that will be used are the diameter, the center thickness and the x location which is the distance between the lens and the source. The refractive index will be set at 1.5. On the right half of Fig.39, how the surfaces behave are shown. Both right and left surface of the lens as well as the edges of the lens will be set to refract light. On the bottom right the major alterations can occur. The options are to change the radial parameters of the left and right surfaces, having toroids on either surface or adding in x and y components on either surface.

Clicking on the left radial coefficient button, another set of parameters can be shown. This is displayed in Fig.40.



Fig.40. Left surface radial parameters

To alter the left surface, the radius of curvature can be increase or decreased, and can be made concave or convex by specifying a positive or negative value. The Schwarzchild constant can be altered as well. If it is 0, the radial shape of the surface is spherical, and the rest of the options for the Schwarzchild constant are listed on the bottom left of Fig.40. Polynomial coefficients can be entered as well, but these introduce a lot of complexity that is not needed.

In order to test various lens designs, only one parameter is analyzed at a time, changing the values, a trend in the effect on the intensity profile may be seen.

Due to expense and time limits, only one lens will be made. The lens will be created in Autodesk inventor. The sketch and file will be given to the machine shop at McMaster where the lens is created according to the design. The lens will be made out of acrylic plastic (PMMA), and then will be optically polished.

To test the lens, it will be placed over one single LED, and intensity measurements will be taken exactly as previously without the lens, using the Teledyne Dalsa Genie TS camera. The results from this will then be compared to the model of the optimized intensity profile to see how accurately the lens alters the intensity profile.

4. Results

4.1: Experimental Results and theory validation

A Cree LED (Xlamp XQ-E) in conjunction with an almost Lambertian screen (Stewart Filmscreen Aeroview 70) were used to test and validate modelling. The screen was situated 3 cm above the LED. The spot intensity at the screen was measured with a Minolta photometer while the LED was moved in 1 dimension to acquire light from limited LED angles between -76.2 and 37.1 degrees. The angular limitation is due to physical limitations of the movement system. The intensity profile in Fig.41 was obtained. In order to further validate these results, a Teledyne Dalsa Genie TS camera was used to obtain a luminance distribution of the screen for all angles. This is also presented in Fig.41. Good agreement between measured and expected Lambertian profiles is clear suggesting a Lambertian model is sufficient to represent an experimental system. Other screens were also tested, but they did not have a sufficiently Lambertian response and were therefore not used. The Vutec "glass" screen described in the methodology chapters compared to the Aeroview 70 "membrane" screen used, are compared in Fig.42. Non Lambertian screens introduce viewing angle effects and are therefore not suitable unless additional viewing angle considerations are modelled.



Fig.41. Intensity profile of XQ-E high Intensity LED on a screen; Photometer results (green), Theory of Eq.(2) (blue), Camera results (red)



Fig.42. Intensity profile comparing Lambertian screen(red) and a non-Lambertian screen (light blue)

A test on how the exposure rate of the camera affects the intensity profile of the picture was carried out by altering the exposure rates and comparing them on the same plot. This is shown in Fig.43.



Fig.43. Intensity profile using multiple exposure rates.

Based on these results an exposure that maximizes the scale, while not flattening out at the top was chosen. If the exposure is too high the sensitivity with high intensity light will decrease, if the exposure is too low, the sensitivity with low intensities will decrease. We need to have an exposure that maximizes sensitivity in both areas. More specifically, any exposure rate between 2000µsec-10000µs was found to be acceptable.

Using these LEDs and the Aeroview 70 screen, an LED backlight array situated behind the screen was constructed and analysed. The LED array was 3cm behind the screen and the LED array had pitch of 5cm. The resulting luminance pattern is shown in Fig.44A. The LED pattern is quite discernible. A model based on the single LED intensity profile of Eq.(2) is shown in Fig.44B which comprises a superposition of luminance profiles of every LED in the array.



Fig.44. A) Actual LED array, B) LED array model, both viewed after a screen.

4.2 Fourier Coefficients and Optimized Intensity Profile

In Section 23.5 a model was developed that would find a set of coefficients that, when applied to Eq.(7), would create an intensity profile for one LED that would be optimized to create a uniform intensity profile on a screen for a finite LED array. The LED array used to model results is a 20x20 array with pitch of 5cm and a distance of 3cm between the LED array and the screen. Inserting these parameters into the model, the optimized coefficients found via the successive approximation algorithm are shown in Table 1. These coefficients, when applied to Eq.(7), provide the ideal intensity equation for each individual LED. These values were chosen to model an experimental setup that exists in our lab, however the dimensions and results may readily be scaled

Table 1. Fourier coefficients determined via Matlab algorithm						
A ₀	A ₁	A ₂	A ₃	A ₄		
0.9735	1.0156	-0.091	-0.816	0.0544		

Using the coefficients found in Table 1 and applying them to Eq.(7), the optimized LED angular luminance profile is displayed in Fig.45. Comparing the original Lambertian distribution(blue) to the new optimized function(red), it can be seen that the angular spread of light has been increased for smaller angles.



Fig.45. Lambertian Intensity profile of $\cos^4 \vartheta$ (blue) compared to the new optimized intensity profile (red)

An LED array in which each LED is modified to have this optimized angular intensity profile is now simulated. In Fig.46 a full 20x20 array is modeled with the original Lambertian distribution. In Fig.47 the same 20x20 array is modeled with the new distribution. Clearly, the modified angular distribution results in a substantial improvement in uniformity.



Fig.46. (Left) 20x20 LED array with Lambertian LED distribution. Fig.47. (right) 20x20 LED array using Fourier coefficients from Table 1 for each LED

4.3: Analysis of Uniformity and Degree of Edge Effects

The results displayed in the previous section appear very promising to the naked eye, but they must be analyzed further. The percentage deviation that is being aimed for is +/-1%, over as much of the area of the array as possible. A topographical map of the percent deviation of the intensity was created and is shown in Fig.48.



Fig.48. Topographical percent deviation of the uniformity of the screen in Fig.47

Fig.48 shows details of areas where the percent deviation in luminance increases above the threshold limit of 2% of the max intensity. The edge of the window of uniformity can be seen at locations in x and y from 10-80 cm. There are some small regions within this window, where the percent deviation slightly rises above 2%, however it does not reach more than 2.1%. The asymmetry is due to a propagation of a small rounding error.

The size of this window of uniformity needs to be compared to the size of the array. A visual aid is presented in Fig.49 by superimposing both the LED array and the window. There is a difference between the window of uniformity (yellow) and the edge of the LED array (pink). The difference between these two edges is approximately 10cm in width on every side. Based on the spacing between LEDs used in this model, the area between the edge of the LED array and the window of uniformity would correspond to 2 extra rows/columns of LEDs. Knowing this, if the size of the required display is known, there would need to be a buffer of 2 rows/columns of

LEDs around the display area to allow for edge effects that the viewer would be exposed to. This effect could be scaled depending on the spacing of the LEDs.



Fig.49. Window of uniformity (yellow) edges of the LED array (pink)

Analysing this edge area, the percent deviation increases to 5% halfway between the edge of the window of uniformity and the edge of the LED array, and it reaches a maximum of approximately 15% at the edge of the array. Human vision is known to disregard small deviations near a boundary and therefore this increase in deviation would not be very noticeable to the average observer. Edge effects comprise a complex area of study for human vision detection.

4.4 Analyzing Bloom

Fig.50 describes how the overall shape of the intensity profile of a 20x20 LED array changes as the R parameter described in Section 2.5 is increased. As R is increased a more prominent halo effect is displayed as well as a decrease in uniformity.

When R=0.03 the window of uniformity is approximately 70cm^2 . For R=0.14 the window of uniformity is approximately 14cm^2 both with an intensity deviation of 2%. For R=0.2 the intensity deviation never reached 2% but instead resulted in a window size of 20cm^2 for a deviation of 4%. In order to change R, the coefficients were modified, producing a new window size where the percent deviation was minimum and constant.



Fig.50. Visually analyzing how increasing R affects bloom. The first row depicts a cross section of the intensity profile of a 20x20 LED array. The bottom row shows a top down view as an observer would see on a Lambertian screen.

R does not specifically have a bloom parameter and therefore there is no direct connection between bloom and R. The merit function R attempts to maximize uniformity and also to maximize the size of a substantially uniform area. However, based on the visual results of Fig.50 there is a correlation between R and bloom, as was predicted in Section 2.6.

A substantially uniform intensity profile has been modeled using an optimized intensity distribution that must be applied to all LEDs in an array. In order to reduce this approach to practice the required intensity profile must be achieved using optics applied to each LED. A lens or other optics would be designed that provides the intensity profile discovered in Fig.45 and represented by Eq.(7). Applying one lens per LED in an array would allow for a uniform array equal to that of the model presented in Section 4.3.

This method provides optimized illumination uniformity, and also optimizes local dimming. Because this method does not require a broad expanse and overlap of light from the LEDs, turning off particular LEDs would continue to allow for substantially uniform intensity where the remaining LEDs are powered. As introduced previously in Section 4.3, the light spread from each LED is small, allowing for accurate local dimming and minimizing bloom. This method of designing a uniform array would also minimize the moiré effect while minimizing unnecessary bloom and a corresponding loss of display resolution in camera-ready LED displays.

The use of a Fourier series to represent the Lambertian distributions of an LED and screen represent a powerful approach to optimizing the LED angular intensity profile. The superposition of light from a resulting LED array may be used to achieve optimum properties of the individual LED.

Comparing the results of this Fourier method to that of the polynomial method described previously in [27], the optimized intensity profile has a wider peak centered at an emitting angle of 0° and dropping off at +/- 80° . The polynomial method resulted in peak light emission at 60° - 70° off axis with less light emitted at smaller angles. Each method provided drastically different shapes while providing high uniformity, yet the polynomial method did not address the issue of bloom and would introduce substantial bloom compared to our results.

This method could be extended to non Lambertian LEDs, as the Fourier series is powerful. Due to the nature of most LEDs being all or nearly Lambertian, only a Lambertian LED was modeled here. Attention was restricted to a Lambertian screen due to the assumption of zero viewing angle effects. Viewing angle independence was maintained.

4.5: Lens Design

4.5.1 Intensity collection

In optics lab the source is classified as a point source. The number of rays coming from the source is customizable, in a 1D or 2D setup with a wide range on the number of rays one can use. In order to collect as much data while not overpowering the computer this software is run on, 1000 rays are used. For intensity data collection 2D ray setup is used, while for ray diagrams, only 1D is used, as this makes it simpler to trace the rays to see how lens shape affect the direction.

When analyzing the 2D data collected from the observational screen in optics lab it was apparent that the 2D rays tend to be skewed along the x and y axis. This was apparent for even perfectly spherical lenses. An example is shown in Fig.51. 1D rays will generally be along the x-axis. For the 2D rays it is expected that the source rays behave with rotational symmetry. This was discovered when analyzing how the source behaves. The source in Opticslab is not a Lambertian LED source, but rather a point source. Knowing that a point source on a screen, the intensity would behave as $I=\cos^3\theta$, as Lambertian light source adds an extra $\cos\theta$. This was discussed in Chapter 2: Theory.



Fig.51. Observational screen diagram with datapoints skewed in x and y

In order to account for this skewing done by the screen the intensity at each radius from the center is averaged. A visual depiction of these radii is located in Fig.52. As the observational screen is already separated into a grid, which can be defined as a matrix of intensity values, where the location in the matrix is the location on the grid, it makes sense to use this geometry to our advantage. For a radius of 1, the values located 1 spot away from the center point in all directions (making a square shape to approximate a circle) are all added together and divided by the number of points. This allows us to have an average intensity value at a radius of 1 away from the center. This is done for each radius until the edge of the screen. As the number of points on the screen ranges from -64 to +63 in the x and y direction, this allows up to a radius of r=63. (The data is uneven and therefore rows and columns -64 will be excluded)



Fig.52. Observational screen depicting concentric squares with radius R upon where intensity values are averaged

This averaged intensity as a function of radius from the center will be plotted against an x axis that is given by x=xscreen*a, where xscreen is a vector of values that range from -63 to +63, and a is the size of the screen divided by the length of xscreen. This converts the location on the screen into an actual location in cm. This method was confirmed to have worked when the intensity of the Point source in Opticslab on the screen was found to follow the expected theory of I=cos³ θ . Actual 3D Intensity data showing this skew effect in the x and y directions is shown below in Fig.53.



Fig.53. 3D intensity plot of point source on a screen from Optics lab where the skew in x and y is shown in the lower dark blue values.

4.5.2 Lambertian Lens

The source is known to be non Lambertian in Opticslab. Before a lens can be designed, the light source needs to be modified to produce a Lambertian intensity profile. This was done by introducing a very small plano-concave lens in front of the point source. This lens was made to be so small that when compared to the observational screen, it was part of the point source. Therefore, when comparing the screen to the source and lens, the screen sees the lens and point source as one. The parameters of this lens were found using the equation

$$\frac{1}{f} = (n-1)\left[\frac{1}{R1} - \frac{1}{R2}\right]$$

Where f is the focal length parameter, n is the refractive index, and R1 and R2 are the radius of curvature of each surface. The parameters of the plano-concave lens in optics lab are given in Fig.54. As this lens is not to be created, but just used to modify the source in Opticslab, a high refractive index is allowed. Even though a high refractive index limits the amount of light transmitted through the lens, if this loss is considered when analyzing the loss of light from the main lens that will be designed then this is acceptable.

Plano Concave Lens	×	
OK	Delete Cancel	
50.	Diameter (mm)	
-15.	Focal Length (mm)	
4.	X Location (mm)	
0.	Y Location (mm)	
0.	Z Location (mm)	
0.	Rotation (deg)	
0.	Z Rotation (deg)	
5.	Center Thickness (mm)	
Trans Coef	(enter 0 for auto update)	
	Glass Type	
View Index n	= 2.4	
Group	= 3.42 = 4 0	
0		
Radius (mm):	To Enter Radii:	
-45.	Use a Simple Lens instead	

Fig.54. Parameters for Plano-concave lens used to create a Lambertian source in Opticslab

This Lambertian lens is shown below. In Fig.55. The source is using 1D light rays. There are 33 rays emitted from the source, and 17 that are transmitted through the lens.



Fig.55. Ray diagram of source with plano-concave lens to create a Lambertian light source. Point source is located inside the opening of the lens on the left of the figure.

This lens combined with the source displays an intensity output on the observational screen that matches the Lambertian intensity profile.

4.5.3 Optimized Intensity Lens

To design a lens in optics lab that gives an intensity profile that matches the optimized intensity profile that was created using the Matlab model, was very difficult. The parameters were given in Chapter 3: Methodology. Each parameter was varied and analyzed, along with the ray diagrams until one design was created that provided as close of an intensity profile as possible. This lens consisted of a refractive index of 1.5, with refracting edges on all surfaces as the material to be used is acrylic. The design of the lens is shown in Fig.56. The left surface is flat and includes a spherical cut-out in the center with a radius of 11.7mm. The sphere is exactly a hemisphere. Similarly, on the exiting surface there is a cut out of a hemisphere with a slightly smaller radius of 11.28mm. The center thickness between the spheres is 4.6mm and the diameter is 167.1mm.



Fig.56. AutoCAD depiction of left side of lens.

The output intensity profile on the observational screen in Opticslab using this lens gives the profile depicted in Fig.57. As each lens parameter would affect the fit of the peak of the intensity profile while very few parameters would affect the intensity at more wide angles, a perfect fit is difficult. There are limitations to optics lab that will be covered at the end of this chapter as well as methods to improve the fit of this profile using an alternative software that did not have these limitations.



Fig.57. Intensity profile of optimized lens compared to the optimized Fourier intensity profile.

The intensity due to the optimized lens in optics lab is a close fit to the optimized Fourier intensity profile, however, differences can be seen, mainly at wide angles and a slight double peak occurs with the Opticslab model. Other lenses created fit with more accurate peaks, or more accurate on lower ends of the intensity profile, however when tested in an array, did not have a low Intensity deviation. However, when the optimized lens was applied to our lens array model in Matlab, the topographical percent deviation was the most promising with a uniform 7% intensity deviation. This map is shown in Fig.58.



Fig.58. Topographical intensity deviation of an array with the Opticslab lens data applied to each LED in the array. 7% intensity deviation is seen for the majority of the array up to the edges.

How the intensity of the array would appear to the human eye is depicted in Fig.59. As the intensity deviation is 7% and not 2% or less, we still can see a polka dot like effect. However, it is not as drastic as it would be without the lens as in Fig.46.



Fig.59. LED array with Opticslab optimized Lens, 7% intensity deviation

4.5.4 ray diagram and improvements

Looking at the ray diagram of the lens in Opticslab the light loss due to the lens can be calculated. Using a Light source with 33 1D rays, we know from previously that the Lambertian lens only allows 17 rays to be transmitted. Therefore, the number of rays entering the optimized lens is 17. The number of rays that make it to the observational screen is 11. Therefore, 35% of light entering the lens does not reach the screen and is therefore lost. A ray diagram of the optimized lens with 1000 1D source rays is depicted in Fig.60. More rays are used in order to show more information on how the lens behaves and where exactly the light is going. Note that this only includes rays in 1 dimension, and so a lot of data in the other dimension is lost, as rays can be angled into the screen at varying degrees. The outline of the spherical cut-outs of the lens can also be seen in Fig.60.



Fig.60. Ray diagram in Opticslab of the optimized lens.

The dotted vertical line along the right side of this diagram is the observational screen. This figure is zoomed in very far, but the observational screen extends vertically enough to encompass all the light rays emitted by the source and lens. Looking at Fig.60, the importance of the diameter of the lens can be seen. As the edges of the lens refract light, many rays are heading towards the observational screen and create an influx of intensity at these points. Making the lens longer in terms of diameter allows us to increase the intensity profile at larger angles, to fill out that bottom area of the intensity profile. If the lens diameter is decreased more of the light is directed along the mid section of the intensity profile or even closer towards the peak to create a large double peak. (if the diameter was small enough) The purpose of the spherical section on the left surface is to collect the light an disperse it outwards, while the spherical cut out on the right surface is to direct a portion of the light towards the peak in the intensity profile. Changing the thickness of the lens also affected the shape of the intensity of the peak.

The gaps in rays emitting from the right surface at the edges of the sphere, are due to the angle at which the rays hit the spherical surface. Towards the edge of the spherical surface the rays are almost going straight past which creates an overshadow area due to the sphere itself. This is shown as a diagram in Fig.61. On the observational screen, the size of these blank areas was calculated and found to be smaller than the size of 1 observational point. The observational screen works as a grid system, described in section 4.5.1. The intensity within one section of that grid is summed up and given in one data point for the area. The size of this blank space is much smaller than the size of one grid section. Therefore, there is no noticeable missing intensity on the intensity profile.



Fig.61. Example diagram of sphere creating an area devoid of rays

In order to improve the lens, we need to look at modifying the direction of the rays at specific points. To change the angle at which the output ray leaves the surface without altering the direction of the input ray, the curvature of the surface can be altered. In Opticslab there are limitations on the number of curved surfaces that can exist on one surface. For example, on the right exiting surface, because a spherical cut-out already exists, the rest of the surface can not be curved and is therefore flat. The edges of the lens are not optimizable other than defined as to whether they are absorbing, reflecting or refracting edges. If a curvature could be applied to the edges of the lens, more light could be directed towards the screen instead of in the opposite direction as in Fig.60. In order to make the diameter of the lens smaller, as currently it is too large to be considered a solution to place on an LED in the array described in this paper as the spacing between each LED is 4.96cm, and the diameter of the lens is much larger than this. Directing the direction of the rays that are emitted from the edges of the lens would allow one to keep the rays going to the location they are going now but being emitted at a different angle due to curvature of the edges. Considering the critical angle formula for a material with a

refractive index of 1.5 transitioning into air with a refractive index of 1. The equation and resulting critical angle are listed below.

$$\Theta_{c} = \sin^{-1}(\frac{n_{2}}{n_{1}})$$
$$\Theta_{c} = 41.8^{\circ}$$

If the incident angle of the ray is less than the critical angle, the light will be transmitted, shown in Fig.62, while if the incident angle is greater than the critical angle, total internal reflection will occur as in Fig.63. With total internal reflection, the light is not lost but can be transmitted out through a different surface. Examples of this critical angle and its effect on light rays are shown in Fig.30 and Fig.31.



Fig.62. Transmittance occurs when $\vartheta < \vartheta_c$



Fig.63. Total internal reflection occurs when $\vartheta > \vartheta_c$

As the angle of incident rays in the case of our lens is not variable, how the angle can be influenced is by adding a curvature to the surface. By either making the incident angle larger or smaller, one can choose where transmittance will occur and where internal reflection is the better choice.

4.5.6. Testing Lens

The optimized lens was manufactured in the McMaster machine shop according to AutoCAD design and specifications. Then lens was placed over the same LED used for the initial theory validation and a picture was taken with the Teledyne Dalsa camera above the Aeroview 70 screen.

Before the lens was used, the spherical surface was polished with sandpaper starting at a grade of 600 up to 4000. The lens was also created in 3 parts to make the machining process simple. The first part was the front of the lens, including 1 spherical cut out. The second part was a center disk with no spherical cut outs, and the third piece was the backside of the lens, with the second spherical cut out. The lens parts were not bonded together but placed atop one another. The expected light loss at each interface is approximately 7%. There should be no altercation to the direction of the light rays as the material and refractive index is the same for each piece. The Intensity results are depicted as a function of angle in degrees plotted against the desired Fourier model in Fig.64.



Fig.64.Intensity profile given by the optimized lens compared to the desired Fourier model.

Looking at Fig.64, it can be seen that the intensity profile given by the optimized lens does not match the desired Fourier model. It is clear that the we were able to widen out the intensity profile at larger angles, however not able to widen out the peak of the intensity profile. This is also shown in Fig.65. when comparing the lens to no lens, the peak is very similar, just large angle intensity is increase. Looking at the picture taken in Fig.66, there is a defined circle. The intensity emitted in this circle is due to the spherical cut-outs in the lens. This was found by taking a piece of paper and sliding it over the lens until the shadow interacted with this ring of light. This shows that the spherical cut outs may not be accurate to the mode. The intensity profile was also compared to the lens model in Fig.67.



Fig.65. Intensity with the experimental lens vs without any lens



Fig.66. Camera picture of LED shining through Aeroview 70 screen



Fig.67. Intensity profile of optimized lens compared to the lens model as well as the Fourier model as a function of cm.

Reasons for the discrepancy in the model and the experimental is likely due to a combination of factors. The main reason that the intensity profile of the lens is sharper and thinner than the model is due to the location of the lens relative to the LED. In the model, the LED source is located approximately 1mm into the spherical cutout of the lens. The exact distance this LED is located within the lens is hard to achieve and is likely not as accurate as it should be. A second note, the sensitivity of the model dimensions is higher than the order of magnitude the machine shop is able to give in regards to sizing the lens, more specifically in regards to the spherical cut-out. The dimensions are given in Table.2.

	Dimensions	Model	Actual		
First part of lens	diameter	2.256cm	2.306cm		
(bottom piece)	depth	1.128cm	1.118cm		
3 rd part of lens (top	diameter	2.342cm	2.360cm		
	depth	1.171cm	1.232cm		

Table.2 Lens dimensions of model vs actual LED

5. Conclusion

A new method for modelling and creating a uniform intensity distribution for a rear projection screen illuminated by an LED array has been proposed and demonstrated. The method is a very general approach that provides an elegant solution without the use of ray tracing. The Lambertian intensity distribution of an LED has been modeled and expressed using a Fourier series. The coefficients of this Fourier series expression were then modified using an algorithm that optimized both the spatial uniformity of the screen intensity and the screen area, described as a window of uniformity, over which this uniformity corresponded to less than a +/-1% luminance deviation. The optimized Fourier coefficients may be applied to individual LEDs in an array to create a window of uniformity that approaches the area of the LED array. The resulting window of uniformity was further analyzed and found to retain adequate uniformity, and to be sufficient in size to allow for improved local dimming. Due to the large spacing between LEDs, this method is shown to lead to a low cost approach to LCD backlighting, as well as an effective approach for LED displays where photography is required.

A lens was designed using Optics lab software to obtain the optimized intensity profile. Due to software restrictions a lens was designed that did not fully agree with the model and gave a uniform intensity deviation of 7% instead of <2%. This deviation is still a considerable improvement over the existing 37% without any optimization. Methods on how to improve the lens design to match the model has been described. This optimized lens was created and tested; the results did not match the model. Reasons for this mismatch have been discussed but is primarily due to the inability to raise the LED into an opening in the lens.
6. References

- Alex A. Goldberg, "Barcode Scanners and how they actually work" GA International Labtag. (2019)
- 2. Cabalo, Jerry et al. <u>"Overview of the TAC-BIO detector"</u>. *Optically Based Biological and Chemical Detection for Defence IV*. International Society for Optics and Photonics. **7116**: 71160D. (2008)
- 3. D. DeAgazio, "Thin is In: LED Backlight Units Are Becoming Thinner and Brighter to Meet Consumer-Product Requirements," SID (2), 16-20 (2009)
- 4. H. Chen, J. Lee, B. Lin, S. Chen and S. Wu, "Liquid crystal display and organic light-emitting diode display: present status and future perspectives," Light-Sci Appl. **25**(2), 16-20 (2009)
- 5. P. de Greef and H.G. Hulze, "Adaptive Dimming and Boosting Backlight for LCD-TV Systems," SID **38**(1), 1332-1335 (2007)
- 6. H. Chen, J. Sung, T. Ha, Y. Park and C. Hong, "Backlight Local Dimming Algorithm for High Contrast LCD-TV," in Proc. ASID, New Delhi, India, Oct. 2006, pp. 168–171.
- 7. G. Yoon, S. Bae, Y. Boklee, and J. Yoon, "Edge-lit LCD backlight unit for 2D local Dimming," Opt. Express **26**(16), (2018)
- 8. T. Jung, M. Albrecht, C. Xu, and A. Karrenbauer, "P-56: Application of the SS C Local Dimming Algorithm for an Edge-Lit TV," SID **41**(1), 1450-1453 (2010)
- 9. H. Chen, R. Zhu, M. Li, S. Lee and S. Wu, "Pixel-by-pixel local dimming for high dynamic-range liquid crystal displays," Opt. Express **25**(3), (2017)
- X. Shu, X. Wu, S. Forchhammer, "Optimal local dimming for LED backlit LCD displays via linear programming," Proc. SPIE 8305, Visual Information Processing and Communication III, 83050K (2012)
- 11. G. Tan, Y. Huang, M. Li, S. Lee, and S. Wu, "High dynamic range liquid crystal displays with a mini-LED backlight," Opt. Express **26**(13), (2018)
- 12. N. Burini, E. Nadernejad, J. Korhonen, S. Forchhammer, X. Wu, "Modeling Power-Constrained Optimal Backlight Dimming for Color Displays," JDT **9**(8), 656-665 (2013)
- 13. K. Lin, N. Liao, D. Zhao, S. Dong, Y. Li, "Analysis of Moiré minimization in color LED flat-panel display," Proc. SPIE 9618, 2015
- 14. Y. Shi, B. Li, M. Zhao, Y. Zhou, and D. Zhang, "The design of LED rectangular uniform illumination lens system," Optik **144**, 251-256 (2017)
- A. J. Whang, S. Chao, C. Chen, Y. Chen, H. Hsiao, and X. Hu, "High Uniform Illumination of Light-Emitting Diodes Lighting with Applying the Multiple-Curvature Lens," Opt. Rev. 18(2), 218-223 (2011)
- 16. S. Hu, K. Du, T. Mei, L. Wan, and N.Zhu, "Ultra-compact LED lens with double freeform surfaces for uniform illumination," Opt. Express **23**(16), (2015)
- 17. H. Wu, X. Zhang, and P. Ge, "Double freeform surfaces lens design for LED uniform illumination with high distance–height ratio," Opt Laser Technol. **73**, 166-172 (2015)
- J. S. Yang, J. Park, B. O, S. Park, and S. G. Lee, "Design Method for a Total Internal Reflection LED Lens with Double Freeform Surfaces for Narrow and Uniform Illumination," JOSK 20(5), 614-622 (2016)
- 19. Z. Zhao, H. Zhang, H. Zheng, and S. Liu, "New reversing freeform lens design method for LED uniform illumination with extended source and near field," Opt Commun. **410**, 123-129 (2018)
- 20. A. Kitai, "Seamless Ultrathin Rear Projection Display: SID (2017). Patent US 9,829,744 B2.
- 21. "Square Rear Projection Film" (2018) [WEB] https://www.indiamart.com/proddetail/rearprojection-film-19088085612.html

- 22. W. Kozuki "Edge Overlap: Simplify multi-projector edge blending" (2018) [WEB] https://www.matrox.com/graphics/en/products/graphics_cards/m_series/powerdesk/edge_ove rlap/
- 23. Whitehead et al. (2018). US 0284677 US Patent Application US2008/0284677A1 publication date Nov. 20, 2008.
- 24. Kitai, (2012). US 0236,217 B2. U.S. Provisional Application No. 61/240,412 "LIGHT EMITTING DIODE ILLUMINATED DISPLAY" filed on Sep. 8, 2009
- 25. J. Korhonen, N. Burini, S. Forchhammer, J. M. Pedersen, "Modeling LCD displays with local backlight dimming for image quality assessment," Proc. SPIE 7866, (2011)
- 26. I. Moreno, "Illumination uniformity assessment based on human vision," Opt. Lett. 35(23) (2010)
- 27. K. wang, D. wu, Z. Qin, F. Chen, X. Luo, and S. Liu, "New reversing design method for LED uniform illumination," Opt.Express **19**(104), A830-A840 (2011)
- Z. Quin, C. Ji, K. Wang, and S. Liu, "Analysis of light emitting diode array lighting system based on human vision: normal and abnormal uniformity condition," Opt.Express 20(21) 23927-23943(2012)
- 29. A. Rose, "The Sensitivity Performance of the Human Eye on an Absolute Scale," JOSA **38**(2) 196-208 (1948)