OPTIMAL CENSORING SCHEMES FOR NON-PARAMETRIC INTERVALS

OPTIMAL PROGRESSIVE TYPE-II CENSORING SCHEMES FOR NON-PARAMETRIC CONFIDENCE INTERVALS OF QUANTILES

By

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ABSTRACT

In this work, optimal censoring schemes are investigated for the non-parametric confidence intervals of population quantiles under progressive Type-II right censoring. The proposed inference can be universally applied to any probability distributions for continuous random variables. By using the interval mass as an optimality criterion, the optimization process is also independent of the actual observed values from a sample as long as the initial sample size n and the number of observations m are pre-determined. This study is based on the fact that each (uncensored) order statistic observed from progressive Type-II censoring can be represented as a mixture of underlying ordinary order statistics with exactly known weights [11, 12]. Using several sample sizes combined with various degrees of censoring, the results of the optimization are tabulated here for a wide range of quantiles with selected levels of significance (*i.e.*, $\alpha = 0.01, 0.05, 0.10$). With the optimality criterion under consideration, the efficiencies of the worst progressive Type-II censoring scheme and ordinary Type-II censoring scheme are also examined in comparison with the best censoring scheme obtained for a given quantile with fixed n and m.

KEY WORDS : **confidence interval, interval mass, mixture representation, non-parametric inference, optimal censoring scheme, order statistic, progressive Type-II censoring, quantile**

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Chapter 1

INTRODUCTION

In order to set a warranty period of a new product or even to compare alternative manufacturing designs, the estimation of quantiles is routinely performed in the reliability and lifetime analysis. If one applies a parametric procedure to estimate a quantile, an important presumption underlying the method is that the model fits the data well. Unless this is verified in the very first stage of analysis, the inferential results may lose the power considerably and lead the analyst to a severely distorted conclusion. One way to overcome this obstacle is to apply a non-parametric procedure which does not specify the model structure a priori so that the results of inference are virtually free from the hazard of the model violation. Furthermore, for financial and temporal reasons, censoring is usually unavoidable in practice of a reliability experiment and one special form of intentional censoring to look at is progressive Type-II right censoring. Thus, the primary focus of this project is to review the procedure to construct an exact non-parametric confidence interval for a quantile of interest under progressive Type-II right censoring and to numerically investigate the associated problem of selecting the optimal censoring schemes using the expected interval mass as an optimality criterion.

During the past decades, the problem of optimal scheduling and optimal censoring has received much attention in different areas of the reliability literature. Balakrishnan and Aggarwala [3] have addressed this problem in general and investigated it using the trace and determinant functions based on the variance-covariance matrix of BLUEs[1] as optimality criteria for several continuous parametric distributions including exponential, normal, extreme-value, and log-normal. Recently, Gouno, Sen, and Balakrishnan [10] tackled the selection problem of optimal stress change points for a multiple step-stress model when the available data are progressively Type-I censored. In this project, we look at the case of progressive Type-II censoring in a non-parametric setting and the work is based on the results derived by Guilbaud [11, 12] on the mixture representation of order statistics from a continuous parent distribution.

Here is the outline of each chapter. Chapter 2 provides a brief overview on non-parametric interval estimation and how to construct it for a given quantile of interest in the complete sample case. In Chapter 3, the concept of censoring is explored, in particular, progressive Type-II right censoring and its mathematical formulation. Chapter 4 then reviews the key results of Guilbaud [11, 12] on the mixture representation of order statistics from progressive Type-II censoring and how to derive a confidence interval for a given quantile non-parametrically. In Chapter 5, the issue of finding optimal censoring schemes is discussed and the results of the numerical study

[1]best linear unbiased estimators

are tabulated for a range of popular quantiles. Finally, a summary of the results from Chapter 5 along with some suggestions for the future research is provided in Chapter 6.

Chapter 2

NON-PARAMETRIC CONFIDENCE INTERVALS FOR QUANTILES

Interval estimation for an unknown quantity of interest is an important problem in statistical inference along with hypothesis testing for a decision making process. Unlike a point estimate alone, an interval estimate conveys an indication of reliability and precision of the estimation with a desired level of confidence. A confidence interval provides a range of values within which the true but unknown value of a quantity of interest may lie, and the confidence is gained by constructing such intervals using the same method over and over again [9]. In order to describe the concept of a confidence interval mathematically, let $\hat{\theta}_L$ and $\hat{\theta}_U$ be functions of a sample $X_1, X_2, \ldots, X_n$ such that

$$1 - \alpha = Pr[\hat{\theta}_L \leq \theta \leq \hat{\theta}_U] \tag{2.1}$$

holds where $1 - \alpha$ is referred to as the confidence coefficient and θ is the unknown quantity of interest one wants to estimate. Then, the interval $[\hat{\theta}_L, \hat{\theta}_U]$ is called a two-sided confidence interval for θ with $100(1 - \alpha)\%$ level of confidence, and $\hat{\theta}_L$ and $\hat{\theta}_U$ are called the lower and upper confidence limits, respectively. One can also construct one-sided confidence intervals in a similar manner. Note that both endpoints of the confidence interval are random variables as they are functions of random variables. Thus, the length and location of the interval are random in nature and no one can be sure whether the unknown quantity θ will actually fall within the confidence limits estimated from a given sample. The objective is then to generate an interval which is as narrow as possible but still includes θ with an acceptable level of probability.

For a parametric model, the unknown quantities of interest for the point or interval estimations are usually location, scale, and shape parameters as they summarize the distributional information of a parametric model and characterize its behaviour. For an instance, a confidence interval for μ, the population mean of a normal distribution can be easily formulated using a pivotal quantity method depending on whether σ, the population standard deviation is known or not. For other probability distributions, however, the method of constructing confidence intervals may not be straightforward (*e.g.*, [2]) and one may have to employ a variety of different techniques to obtain the approximate results (*e.g.*, the bootstrap methods).

Another issue concerned with the interval estimation for any types of unknown quantities solely based on a parametric model is the validity of the distribution model from the very first stage of analysis. Parametric statistics has earned considerable popularity and wide applicability with its robustness and power to analyze data using distribution models indexed by few parameters. Despite the popularity of the parametric approaches, however, the validity of all the related inferential procedures and results intrinsically relies on the validity of the model being assumed. Therefore, the power of any parametric techniques associated with a particular parametric model is at stake when the model assumption is clearly violated. On the other hand, non-parametric statistics does not depend on the choice of a model and with minimal assumptions, its properties and inferential techniques are virtually distribution-free or model-free. Hence, if one doubts about the adequacy of a parametric model and there are no suitable ways to assess the model validity, non-parametric methods work as an alternative.

Here we illustrate a simple non-parametric procedure to construct the confidence intervals for quantiles based on complete observations from a sample (*viz.*, no censoring). Quantiles of the underlying population distribution are often the quantities one is interested in estimating as they give an insight into the way the population is distributed. Some well-known special quantiles are median, quartiles, centiles, percentiles, etc. These quantiles are used to group the population being studied or to quote some important characteristics of the population. For example, median and quartiles are used to draw the box-and-whisker plots and to describe a data set instead of sample mean and sample standard deviation when the data in hand exhibit a highly skewed distribution. For a model checking purpose, the normal quantile plots are commonly used to assess graphically whether it is safe to assume that the data set in question comes from a normal distribution. In a clinical trial or a medical study, quantiles are frequently used to summarize the survival data, and in a reliability experiment or a lifetime analysis, quantiles are also used to evaluate the performance of a testing unit.

Now, in order to construct a non-parametric confidence interval for a given quantile from a continuous distribution $F(t) = Pr[X \le t]$, let ξ_p be the quantile that satisfies

$$Pr[X \le \xi_p] = F(\xi_p) = p, \qquad 0 < p < 1, \tag{2.2}$$

where X is a random variable whose cdf is $F(t)$. Then, suppose that a sample of size n is taken from this population and ordered so that $X_1 < X_2 < \ldots < X_n$ denote such order statistics from the sample. The probability of the interval $(-\infty, X_j]$ covering ξ_p is then given by the lower tail binomial probability,

$$\begin{aligned} Pr[X_j > \xi_p] &= Pr[X_j \ge \xi_p] \\ &= \sum_{k=0}^{j-1} \binom{n}{k} p^k (1-p)^{n-k} = b_j, \end{aligned} \tag{2.3}$$

for each $j = 1, 2, \ldots, n$. Using (2.3), for integers r and s which satisfy $1 \le r < s \le n$, the probability of $X_r \le \xi_p \le X_s$ is determined by

$$\begin{aligned} Pr[X_r \le \xi_p \le X_s] &= Pr[X_s \ge \xi_p] - Pr[X_r > \xi_p] = b_s - b_r \\ &= \sum_{k=r}^{s-1} \binom{n}{k} p^k (1-p)^{n-k}. \end{aligned} \tag{2.4}$$

For a selected confidence coefficient $1 - \alpha$, the values of r and s can be searched so that $Pr[X_r \leq \xi_p \leq X_s] \geq 1 - \alpha$ holds. Substituting the observed values x_r and x_s for X_r and X_s, the two-sided $100(1 - \alpha)\%$ confidence interval for ξ_p is $[x_r, x_s]$. It is remarked that it is not always possible to find r and s to give the conventional 0.90, 0.95, 0.99 values for $1 - \alpha$. Especially with small sample sizes, it gets more difficult to produce a confidence interval non-parametrically with a high level of confidence. Nevertheless, the confidence interval constructed as above is free of any particular forms of the probability distribution $F(t)$ as long as the underlying parent distribution is continuous.

Chapter 3

PROGRESSIVE TYPE-II CENSORING

For any statistical analyses, a complete collection of data is the most favorable scenario prior to the actual analysis step as the inference made is considered relatively resistant to the uncertainty. In reality, however, statistical analysts and practitioners frequently encounter situations where the data are not all observable. Then, it is questionable whether comparable inference based on the incomplete sample can be devised to the complete sample case. One type of such incomplete data which arises commonly in practice is censored data. Censored data are observed when the experiments involving lifetimes of testing units have to be terminated earlier than scheduled. For the reasons of cost reduction and time constraint, intentional censoring is unavoidable in practice, especially for a reliability study and a survival analysis. The two traditional forms of censoring which have been studied extensively in the past are Type-I and Type-II censoring. Type-I censoring occurs when the experiment is terminated at a prefixed time T, independent of the failure times. Within this, Type-I right censoring implies that no failures would be observed beyond this time T. While Type-I censoring specifies the time of termination, conventional Type-II censoring restricts the number of failures to be observed. As such, in Type-II right censoring, there would be a prefixed number m so that the experiment is terminated at the time of the mth failure and all the remaining units are removed from the experiment.

In addition, a more general censoring, named progressive censoring, has recently been developed. The concept of progressive censoring was first introduced in 1956 by Herd at Iowa State College in his Ph.D. thesis entitled *Estimation of the parameters of a population from a multi-censored sample* [3]. The subject was further developed in 1963 by A. C. Cohen [6] and it has received much attention for research since then. The importance of progressive censoring lies in its efficient exploitation of the available resources compared to the traditional sampling. Withdrawn unfailed testing units are typically used in other experiments in the same or at a different facility [10]. Progressive censoring can also be either Type-I or Type-II, and in fact, it includes both the conventional Type-I and Type-II censorings as special cases. Progressive Type-I right censored samples are observed when a pre-specified number or proportion of unfailed units are continuously removed during the experiment at each pre-determined time point. Similarly, progressive Type-II right censored samples arise when a pre-specified number of unfailed units are continuously withdrawn from the experiment at each failure time observed until the pre-determined number of units have failed.

Among various types of censoring, it is the interest in this project to study progressive censoring, in particular, progressive Type-II right censoring as it attains more tractable and interesting mathematical properties. Now, let us spell out how progressive Type-II censoring proceeds in terms of the underlying ordinary order statistics. Consider a life-testing experiment involving n experimental units and m failures are to be observed with $2 \leq m \leq n$. Besides, let $\mathbf{R} = (R_1, R_2, \ldots, R_m)$ be the planned progressive censoring scheme of one's choice which satisfies $R_i \geq 0$ for $i = 1, 2, \ldots, m$ and

$$\sum_{i=1}^{m} R_i + m = n. \tag{3.1}$$

The failure times of the testing units since time zero can be viewed as a random sample of size n from a random variable X with cdf $F(t)$, and the corresponding order statistics of the successive failure times are denoted by $X_1 < X_2 < \cdots < X_n$. As the parent distribution $F(t)$ is assumed to be continuous, $X_1, X_2, \ldots, X_n$ are distinct with probability 1. Nevertheless, $X_1, X_2, \ldots, X_n$ may not be all observable because of progressive Type-II censoring outlined as follows.

Right after observing the first failure at time $Y_1 (\equiv X_1)$, R_1 of the $N_1 = n - 1$ surviving units are selected at random (without replacement) and removed from the experiment so that only $n_1 = N_1 - R_1$ units remain under observation. Immediately after the second failure at time Y_2, R_2 of the $N_2 = n_1 - 1$ surviving units are selected at random (without replacement) and removed so that only $n_2 = N_2 - R_2$ units remain under observation, and so on. Hence, for $i = 1, 2, \ldots, m - 1$, $N_i = n_{i-1} - 1$ denotes the number of units surviving beyond Y_i just before R_i units are removed and $n_i = N_i - R_i$ denotes the number of units remaining beyond Y_i right after R_i

units are removed, with $n_0 = n$. It is also defined that $N_m = n_{m-1} - 1 = R_m$ and $n_m = 0$ since all N_m units left after the mth failure at time Y_m are withdrawn from the experiment if there are any. Therefore, all N_i's and n_i's are determined by the censoring scheme **R**. The progressive Type-II censoring just described leads to m observable uncensored order statistics $Y_1 < Y_2 < \cdots < Y_m$ that are available for inference. As mentioned before, the conventional Type-II right censoring is a special case with $R_1 = R_2 = \cdots = R_{m-1} = 0$ and $R_m = n - m > 0$, whereas the complete sample case (*viz.*, no censoring) corresponds to the case with $m = n$ and $R_1 = R_2 = \cdots = R_m = 0$.

Chapter 4

CONFIDENCE INTERVALS USING MIXTURE REPRESENTATIONS

4.1 MIXTURE WEIGHTS OF UNCENSORED ORDER STATISTICS

Recently, Guilbaud [11, 12] derived how each observed uncensored order statistic $Y_1 < Y_2 < \cdots < Y_m$ can be represented as a mixture of underlying ordinary order statistics $X_1 < X_2 < \cdots < X_n$ under progressive Type-II right censoring. In order to summarize the results, let $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_m)^T$ and $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$. Also, let $\hat{w}_{i,j}$ be the indicator of the event $Y_i = X_j$, which equals 1 if the event occurs and 0 otherwise. Hence, $\hat{w}_{i,j}$'s simply tell which of the order statistics $X_1 < X_2 < \cdots < X_n$

is selected as one of $Y_1 < Y_2 < \cdots < Y_m$. Then, the $m \times n$ random matrix $\hat{W} = (\hat{w}_{i,j})$ composed of the indicators holds the relationship of

$$\mathbf{Y} = \hat{W}\mathbf{X}, \tag{4.1}$$

with $\hat{w}_{1,1} = 1$ since $Y_1 = X_1$ by definition. Obviously, $\hat{W}$ has a structural property of unit row sums. Moreover, for $2 \leq i \leq m$, $\hat{w}_{i,j} = 0$ for $j < i$ and the column numbers of 1's are in the strictly increasing order.

Using the idea of sequential and independent simple random sampling without replacement, $\hat{W}$ can be further decomposed into a product of m matrices that are mutually independent of each other and of $\mathbf{X}$. It is expressed by

$$\hat{W} = K^{(m)}\hat{K}^{(m-1)} \cdots \hat{K}^{(1)}, \tag{4.2}$$

where $\hat{K}^{(i)}$ is a $(i + n_i) \times (i + N_i)$ matrix defined as

$$\hat{K}^{(i)} = \left(\begin{array}{c|c} I^{(i)} & 0_1^{(i)} \\ \hline 0_2^{(i)} & \hat{H}^{(i)} \end{array} \right), \tag{4.3}$$

for $i = 1, 2, \ldots, m-1$, and $K^{(m)}$ is a non-random $m \times (m + N_m)$ matrix given by

$$K^{(m)} = \begin{cases} \left(I^{(m)} | 0_1^{(m)} \right), & \text{if } N_m > 0, \\ I^{(m)}, & \text{if } N_m = 0. \end{cases} \tag{4.4}$$

Here, $I^{(i)}$ is a $i \times i$ identity matrix and $\hat{H}^{(i)} = (\hat{h}_{r,s}^{(i)})$ is the $n_i \times N_i$ matrix of indicators $\hat{h}_{r,s}^{(i)}$ of the events $a_r^{(i)} = b_s^{(i)}$, with $a_1^{(i)} < \cdots < a_{n_i}^{(i)}$ and $b_1^{(i)} < \cdots < b_{N_i}^{(i)}$ denoting the order statistics remaining beyond Y_i immediately after/before withdrawing R_i surviving units. $0_1^{(i)}$ and $0_2^{(i)}$ are simply conformable zero matrices. Clearly, the distribution of $\hat{H}^{(i)}$ is determined by n_i and N_i, and the expectation of each $\hat{h}_{r,s}^{(i)}$ is given by

$$\begin{aligned} h_{r,s}^{(i)} &= Pr[a_r^{(i)} = b_s^{(i)}] \\ &= \begin{cases} \binom{s-1}{r-1}\binom{N_i-s}{n_i-r} \Big/ \binom{N_i}{n_i}, & \text{if } r \le s \le r+R_i, \\ 0, & \text{otherwise ,} \end{cases} \end{aligned} \tag{4.5}$$

for $1 \le r \le n_i$ and $1 \le s \le N_i$. As a result, the expectation of $\hat{H}^{(i)}$ equals $H^{(i)} = (h_{r,s}^{(i)})$ whose elements are defined by (4.5). Structurally, $H^{(i)}$ is a band matrix with positive elements only if $r \le s \le r + R_i$ and 0 otherwise. For each row, the sum is exactly 1, and in the complete sample case (*viz.*, no censoring), $H^{(i)}$ becomes an identity matrix as $n = m$ and $R_1 = R_2 = \cdots = 0$.

Now, from the independence of the factors in (4.2), it easily follows that the expectation of $\hat{W}$ equals

$$W = K^{(m)} K^{(m-1)} \cdots K^{(1)}, \tag{4.6}$$

where $K^{(i)} = E[\hat{K}^{(i)}]$ for $i = 1, 2, \ldots, m-1$ and it is the right member of (4.3) with $\hat{H}^{(i)}$ replaced by $H^{(i)}$. The properties of $W = (w_{i,j})$ are such that each row sum is 1 and the number of positive elements in the successive rows form a non-decreasing sequence. Consequently, one can see that for each $i = 2, 3, \ldots, m$, $(\hat{w}_{i,1}, \hat{w}_{i,2}, \ldots, \hat{w}_{i,n})$ has multinomial distribution with parameters 1 and $(w_{i,1}, w_{i,2}, \ldots, w_{i,n})$. Given the independence of $\hat{W}$ and $\mathbf{X}$, the mixture representation (4.1) thus holds with the weights $w_{i,j} = Pr[Y_i = X_j] = Pr[\hat{w}_{i,j} = 1] = E[\hat{w}_{i,j}]$ for $1 \leq i \leq m$ and $1 \leq j \leq n$, and the mixture weights are given by the non-random matrix $W = E[\hat{W}]$. It is also important to note that W is completely determined by n, m, and $\mathbf{R} = (R_1, R_2, \ldots, R_m)$, and the computation of all the elements of W can be carried out via an efficient recursive relation described in Guilbaud [11].

4.2 CONFIDENCE INTERVALS UNDER PROGRESSIVE TYPE-II CENSORING

In Chapter 2, a procedure to construct a non-parametric confidence interval for a quantile ξ_p was illustrated in the complete sample case. Using the mixture representation given in (4.1), a non-parametric confidence interval for ξ_p under progressive Type-II right censoring can be constructed in a similar way as described in [11, 12]. Suppose that ξ_p is a given p-quantile that satisfies (2.2) and it is uniquely defined through $\xi_p = F^{-1}(p)$ for any continuous distribution function $F(t)$. It is then our interest here to estimate its two-sided confidence interval non-parametrically under progressive Type-II right censoring. Before proceeding, let $\mathbf{b} = (b_j)$ be the $n \times 1$ matrix whose elements are defined by (2.3) and let $\mathbf{a} = (a_j)$ be the $m \times 1$ matrix whose elements are defined in terms of $W = (w_{i,j})$ through

$$\mathbf{a} = W\mathbf{b}. \tag{4.7}$$

The vector $\mathbf{a}$ is simply a collection of the probabilities of covering ξ_p by each Y_r since

$$\begin{aligned} Pr[Y_r \geq \xi_p] &= Pr[Y_r > \xi_p] \\ &= \sum_{j=1}^{n} w_{r,j} Pr[X_j \geq \xi_p] \\ &= \sum_{j=1}^{n} w_{r,j} b_j = a_r, \end{aligned} \tag{4.8}$$

for $r = 1, 2, \ldots, m$. It is clear that $a_1 \leq a_2 \leq \cdots \leq a_m$ as $Y_1 < Y_2 < \cdots < Y_m$ with probability 1, and in the special case with no censoring, W becomes a $n \times n$ identity matrix, yielding $\mathbf{a} = \mathbf{b}$. Now, suppose that r and s are some integers satisfying $1 \leq r < s \leq m$. The coverage probability of the interval estimator $[Y_r, Y_s]$ for ξ_p can then be easily expressed in terms of the elements of $\mathbf{a}$ through

$$Pr[Y_r \leq \xi_p \leq Y_s] = Pr[Y_s \geq \xi_p] - Pr[Y_r > \xi_p] = a_s - a_r. \tag{4.9}$$

Provided $\mathbf{a}$ has been evaluated via (4.7), the integers r and s can be determined so that the confidence level of this interval is at least a specified value $1 - \alpha$ (*i.e.*, $a_s - a_r \geq 1 - \alpha$). It is recognized here that $\mathbf{a}$ is such that this is feasible. It is also remarked that the confidence interval constructed as described above does not depend on a functional form of $F(t)$ and thus, it is a distribution-free property as long as the underlying parent distribution is continuous.

Since the coverage probability (4.8) is established by the matrix operation of

(4.7) and it is the final key ingredient to accomplish our goal of finding the interval estimator for ξ_p under progressive Type-II censoring, it is worth introducing other efficient way to estimate (4.8) without going through direct computation of the matrix W. Using the expression of the explicit density function of Y_r given in [4], one can easily calculate (4.8) and this is particularly useful when one wants to estimate only one or few elements of $\mathbf{a}$ and to avoid intensive and costly computation of the whole matrix W. Adopting the usual conventions that $\prod_{i=1}^{0} u_i \equiv 1$ and $\sum_{i=1}^{0} u_i \equiv 0$, the density function of Y_r is given by

$$f_{r:m:n}(y_r) = c\sum_{i=0}^{r-1} c_{i,r-1}(R_1+1, R_2+1, \ldots, R_{r-1}+1) f(y_r)\Big(1-F(y_r)\Big)^{R_i''-1}, \quad (4.10)$$

$-\infty < y_r < \infty$ for $r = 1, 2, \ldots, m$, where

$$c = n(n-R_1-1)\cdots(n-R_1-\ldots-R_{r-1}-r+1), \quad (4.11)$$

$$R_i'' = n - \sum_{j=1}^{r-i-1}(R_j+1), \quad (4.12)$$

and $f(t)$ denotes the corresponding pdf of an absolutely continuous distribution function $F(t)$ such that $f(t) = \frac{d}{dt}F(t)$. In addition, $c_{i,r-1}(R_1+1, R_2+1, \ldots, R_{r-1}+1)$ is obtained through a function

$$c_{i,q}(\mathbf{u}_q) = \frac{(-1)^i}{\left(\prod_{j=1}^{i}\sum_{k=q-i+1}^{q-i+j} u_k\right)\left(\prod_{j=1}^{q-i}\sum_{k=j}^{q-i} u_k\right)}, \quad (4.13)$$

defined for any real vector $\mathbf{u}_q = (u_1, u_2, \ldots, u_q)$ of length $q \geq 1$. Integrating the density function, the corresponding distribution function or the cdf of Y_r is given by

$$\begin{aligned} F_{r:m:n}(y_r) &= \int_{-\infty}^{y_r} f_{r:m:n}(y)\, dy \\ &= c\sum_{i=0}^{r-1} c_{i,r-1}(R_1+1, R_2+1, \ldots, R_{r-1}+1) R_i''^{-1} \\ &\qquad \times \left(1 - \left(1 - F(y_r)\right)^{R_i''}\right), \end{aligned} \tag{4.14}$$

$-\infty < y_r < \infty$ for $r = 1, 2, \ldots, m$. Using the complementary event, the coverage probability (4.8) can be explicitly formulated as

$$\begin{aligned} Pr[Y_r \geq \xi_p] &= Pr[Y_r > \xi_p] \\ &= 1 - Pr[Y_r \leq \xi_p] = 1 - F_{r:m:n}(\xi_p) \\ &= 1 - c\sum_{i=0}^{r-1} c_{i,r-1}(R_1+1, R_2+1, \ldots, R_{r-1}+1) R_i''^{-1} \\ &\qquad \times \left(1 - (1-p)^{R_i''}\right), \end{aligned} \tag{4.15}$$

and the last step results from the definition of ξ_p stated in (2.2).

Chapter 5

OPTIMAL PROGRESSIVE CENSORING SCHEMES

5.1 OPTIMALITY CRITERION AND OPTIMAL SCHEMES

In the previous chapter, the exact non-parametric interval estimator for any given quantile ξ_p has been derived when a particular progressive Type-II censoring scheme is to be applied. Then, as pointed out in [3], some natural questions that arise are: "how can a practitioner decide on which censoring scheme to be used out of numerous censoring schemes?" "Is the decision made strictly on the basis of convenience, or can one select a censoring scheme which makes the most sense within some statistical settings?" From a practical point of view, the question of choosing the optimal values for $\mathbf{R} = (R_1, R_2, \ldots, R_m)$ is certainly an indispensable one and it has to be addressed when one designs a progressive Type-II censoring experiment as there are astronomical figures of distinct censoring schemes even for a moderate size of n and m.

Before selecting the optimal censoring scheme, one must first devise an optimality criterion or an objective function to be optimized [3]. Consequently, the meaning of the optimal censoring scheme is restricted to the criterion of one's choice. In the case of non-parametric interval estimation for a quantile ξ_p with n and m fixed, a simple optimization with respect to the choice of $\mathbf{R} = (R_1, R_2, \ldots, R_m)$ is to select $\mathbf{R}$ which enables to find r and s in (4.9) that satisfy $a_s - a_r \approx 1 - \alpha$ and $a_r \approx 1 - a_s$. This constraint is equivalent to find r and s satisfying $a_r \approx \alpha/2$ and $a_s \approx 1 - \alpha/2$ in order to yield a symmetric confidence interval when it is possible.

In the circumstance without any censoring, a reasonable objective function with respect to the choice of r and s is the index difference $s - r$ corresponding to the relevant interval $[X_r, X_s]$ with the level of confidence at least $1 - \alpha$. To minimize this function is to minimize the expected probability mass $F(X_s) - F(X_r)$ of the underlying distribution within the interval [7]. This clearly reflects the purpose of the interval estimation, which is to produce the shortest interval with a desired confidence level. Under progressive Type-II right censoring, it follows from (4.1) that $Y_i = \sum_{j=1}^{n} \hat{w}_{i,j} X_j$ and the expectation of $F(Y_i) = \sum_{j=1}^{n} \hat{w}_{i,j} F(X_j)$ equals

$$e_i = \sum_{j=1}^{n} w_{i,j} \frac{j}{n+1}, \tag{5.1}$$

for $i = 1, 2, \ldots, m$. As noted in [11], it is convenient to realize that the $m \times 1$ matrix $\mathbf{e} = (e_i)$ whose elements are defined by (5.1) can be computed by $(n+1)^{-1} W J$ with

$J = (1, 2, \ldots, n)^T$. Also, notice that (5.1) depends on the elements of the weight matrix W. Even if W is not available, (5.1) can still be computed easily using the equivalent expression derived from the probability integral transformation under progressive Type-II censoring and the generalization of Malmquist's transformation results through the independent ratios from beta distribution [3]. The expression is rather explicit and it is given by

$$e_i = 1 - \prod_{j=m-i+1}^{m} \frac{\delta_j}{\delta_j + 1}, \tag{5.2}$$

where

$$\delta_j = j + \sum_{k=m-j+1}^{m} R_k, \tag{5.3}$$

for $i = 1, 2, \ldots, m$. Therefore, the expectation of the probability mass within the interval $[Y_r, Y_s]$ is equal to $e_s - e_r$ for $1 \leq r < s \leq m$. Then, for given n and m, the optimal progressive Type-II censoring scheme for the non-parametric interval estimator of ξ_p is obtained by minimizing the expected interval mass

$$M(\mathbf{R}) = \min_{(r,s) \in \mathcal{S}} \{e_s - e_r\}, \tag{5.4}$$

where $\mathcal{S}$ is a subset of binary Cartesian product of positive integers (r, s) such that $1 \leq r < s \leq m$ and $a_s - a_r \geq 1 - \alpha$ under given conditions. If $\mathcal{S}$ is an empty set for some $\mathbf{R}$'s, $M(\mathbf{R})$ is simply defined to be 1. It is remarked that the objective function $M(\cdot)$ and the index set $\mathcal{S}$ are both depending on the choices of n, m, $\mathbf{R}$, and α for a given quantile ξ_p, but $M(\mathbf{R})$ is minimized with respect to every possible progressive censoring scheme $\mathbf{R} = (R_1, R_2, \ldots, R_m)$ under n, m, α all fixed in advance along with ξ_p. It provides flexibility to the practitioners as the number of units to be put on the test and the number of complete failure times to be observed are both to be determined by a test designer based on the availability of units and experimental facilities. If one or both of these are to be determined in the planning stage, one may also use the tables presented in the next section to decide upon the values of n and m which are feasible given an agreeable value of the objective function.

In this finite sample case, to minimize (5.4) with all the other values given, one may list each and every possible choice of censoring schemes and the corresponding values of the objective function. After determining the best value, the value which minimizes the expected interval mass $M(\cdot)$, or a certain region of satisfactory values from this list, one can pick out either the best censoring scheme or one which gives a value very close to the best but may be practically more convenient (*i.e.*, a suboptimal censoring scheme). As mentioned in [3], for fixed n and m, "the increase in efficiencies by employing the optimal progressive Type-II censoring scheme is often substantial compared to the case of conventional Type-II censoring scheme frequently used by practitioners". For the purpose of comparing different censoring schemes with the selected objective function, a sensible definition of the efficiency of a censoring scheme A, $\mathbf{R}_A$ with respect to a censoring scheme B, $\mathbf{R}_B$ can be given in percentage by

$$Efficiency(\mathbf{R}_A, \mathbf{R}_B) = \frac{M(\mathbf{R}_A)}{M(\mathbf{R}_B)} \times 100\%. \tag{5.5}$$

This is simply a ratio of the interval masses and if one is interested in searching for a region containing a number of satisfactory censoring schemes, such a region can be defined in terms of efficiencies [3]. For instance, an experimenter may be pleased with any censoring schemes which are at least 95% as efficient as the optimal (best) scheme. Then, he can choose the censoring scheme out of these satisfactory ones which is the most practicable or convenient to the experimental setting. Hence, without complicating the calculations, practicality is also built into the objective function by employing a *two-stage* approach to optimization [3]. Moreover, one can see that for any fixed values of n and m, the efficiency of the conventional Type-II censoring scheme with respect to the optimal censoring scheme would be at most 100%. Therefore, "there is no loss in efficiency over the conventional censoring scheme if the optimal scheme is to be used" [3].

5.2 NUMERICAL STUDY

Up to now, how to construct a non-parametric confidence interval for a given quantile has been reviewed as well as how to determine the associated optimal progressive Type-II censoring scheme. The weight matrix W can be computed through (4.4) to (4.6) as described in Section 4.1 and the elements (2.3) of $\mathbf{b}$ can be evaluated by the binomial distribution function. Then, an appropriate confidence interval can be searched for a given quantile using the vector $\mathbf{a}$ which can be easily estimated from (4.7). Moreover, the vector $\mathbf{e}$ can be calculated with the procedure just described in the previous section in order to explore the optimal censoring schemes.

For illustration and reference for practitioners, a numerical study has been conducted with some selected values of parameters and the results are tabulated in the appendix. The implementation was done using R and FORTRAN programming languages, and the codes are also provided in the appendix. In the original computation performed, the choices of the sample size n ranged from 10 to 100 with an increment of 5 and the pre-determined number of failure observations m ranged from 2 to n with a unit increment for each choice of n. In addition, the choices of m were restricted in such a way that the total number of available censoring schemes $\binom{n-1}{m-1}$ does not exceed 3.0×10^6 in order to keep the computational time and space manageable. To examine a variety of quantiles, p of ξ_p was selected from 0.05 to 0.95 with an increment of 0.05. Besides, to yield the conventional 99%, 95%, and 90% confidence intervals, α was chosen to be 0.01, 0.05, and 0.10, respectively.

Due to a limitation on space, the tables presented in the appendix represent only a portion of the complete results and it is intended to provide the general behaviour of this discrete optimization. Each table is dedicated to a specific quantile with some fixed n and m, and it lists the best progressive censoring scheme and the worst censoring scheme along with conventional Type-II censoring scheme for comparison. The best censoring scheme is simply the one which minimizes the objective function $M(\mathbf{R})$ given in (5.4) and the worst censoring scheme is, on the other hand, the one that maximizes

$$\bar{M}(\mathbf{R}) = \max_{(r,s)\in\mathcal{S}}\{e_s - e_r\}, \tag{5.6}$$

whose definition is analogous to (5.4). For some censoring schemes given in the tables, the meaning of $0 \star k$ with some positive integer k is to repeat zero k times. Hence, it simply denotes a zero vector of size k embedded in the censoring scheme $\mathbf{R}$. The meaning of $1 \star k$ can be interpreted in a similar way and they are given to dramatically simplify the notation of $\mathbf{R}$. In each table, count denotes the total number of distinct progressive censoring schemes which can produce at least one interval with the confidence level greater than or equal to the nominal level $1 - \alpha$. In other words, it is the number of censoring schemes for which $\mathcal{S}$ in (5.4) is a non-empty set. Since the non-parametric confidence interval depends on the order or the index of the uncensored observations, the confidence interval is given in the form of $[r, s]$ from the relevant interval $[Y_r, Y_s]$. In each table, the actual level of confidence for each interval estimator is also provided together with its expected probability mass. Additionally, efficiency was calculated using (5.5) with respect to the best censoring scheme found within fixed values of n, m, p, and α. In the cases where a certain censoring scheme can not yield any intervals with the desired level of confidence, the efficiency is simply noted as not available.

Chapter 6

CONCLUSION

Based on the results of the numerical study presented in Section 5.2 and the appendix, a number of comments can be made on some interesting observations. First of all, it is noted that with certain choices of n and m, the available progressive censoring schemes which can generate confidence intervals for a given quantile with a desired level of confidence can be scarce (*e.g.*, Table A.1 and Table A.2). Moreover, the higher the required level of confidence is, the fewer the choices of censoring schemes which can produce such intervals are. One can also observe that for fixed n, m, and α, the actual level of confidence increases initially and then decreases as p increases. Consequently, when the quantile of interest is too small or too large, there may not be any censoring schemes to generate a confidence interval with a selected level of confidence unless n and m are both reasonably large.

Then, how large should m be compared to n? It was found that the size of m related to n is also a significant factor to boost up the available censoring schemes which can yield confidence intervals with a desired level. By examining tables, it can be seen that if m is too small compared to n or too close to n for fixed ξ_p and α, the number of censoring schemes which can yield confidence intervals with a desired level decreases. This in turn reduces the number of suboptimal censoring scheme choices whose efficiencies are close to that of the best scheme. To the surprise, this finding also implies that the case without any censoring actually performs worse than the case with an appropriate censoring in constructing a non-parametric confidence interval for a quantile. Therefore, the censoring proportion $\frac{n-m}{n}$ is an important factor to consider at the planning and designing stage of a progressive Type-II censoring experiment if one wants to construct a confidence interval with a desired coverage probability.

Another general observation related to n and m is that raising n with m fixed enables to find the optimal censoring schemes for small quantiles like $\xi_{0.05}$ and $\xi_{0.10}$ within a limited range. On the other hand, increasing m with fixed n enables to find the optimal censoring schemes for a large range of quantiles of interest. Although it is not shown in the tables, it was observed that the maximum level of confidence could be achieved by a censoring scheme $\mathbf{R} = (n-m, 0\star(m-1))$ in any cases where none of the censoring schemes examined could produce an interval with at least the nominal level. Thus, withdrawing every unit immediately after observing the first failure time is the best in a sense that the interval $[Y_1, Y_m]$ attains the highest confidence level. Nevertheless, it turned out to be also the worst censoring scheme in some situations (*e.g.*, Table A.37) when one could locate the optimal censoring scheme. This is because $[Y_1, Y_m]$ bears the highest probability mass within the interval.

Although the tables compiled in the appendix can be used for reference for statisticians and other practitioners who are planning and designing a progressive Type-II censoring experiment, it is rather difficult to see whether there is a universal pattern or a mathematical trend of the best censoring schemes or the worst censoring schemes. On the contrary, it can be clearly pointed out that if one randomly chooses a progressive censoring scheme for an experiment in which a large censoring takes place, it will be less likely to obtain a desired non-parametric confidence interval for the quantity of interest (*e.g.*, Table A.28). Hence, the thorough planning of an experiment is important regarding the inferential matter. Other interesting observation to point out is that in a few cases, the best censoring scheme coincides with the conventional Type-II censoring scheme. For example, Table A.22 shows that for $n = 25$ and $m = 20$, the conventional Type-II censoring scheme turns out to be the best censoring scheme to construct a 90% confidence interval for $\xi_{0.35}$. This result is however valid only with the objective function under consideration and it may not be true for other types of optimality criteria. In most cases, the ordinary Type-II censoring schemes could not even produce a single confidence interval with an acceptable level and the efficiency gained by the optimal censoring scheme was proven to be indeed substantial compared to the conventional Type-II censoring scheme as well as to the worst censoring scheme.

After reviewing the up-to-date literature on the subjects of non-parametric interval estimation and progressive censoring, it becomes obvious that one of the main virtues of the non-parametric interval estimation is its distribution-free property and it makes the method applicable as a supplement to any continuous distribution families [9]. Certain drawbacks do exist, however. It may not be always possible to construct a confidence interval with a desired coverage level, particularly with a small sample size, and even if one can, the actual level of confidence will be separated in the neighborhood of the nominal level. With no exception, this intrinsic problem also occurs in the complete sample case (*i.e.*, no censoring). One should therefore carefully select the values of n, m, and α in the designing stage so that they will certainly lead to find the corresponding optimal censoring scheme or a nearly optimal censoring scheme for a quantile of interest. For the future research, it is desired to develop and investigate several different types of non-parametric optimality criteria other than the expected interval mass considered in this project so that the properties of different objective functions and their best uses can be analyzed and compared. Another possibility of the future research is to conduct the optimization under several well-known parametric distributions (*e.g.*, exponential, Weibull, log-normal, gamma, etc.) and to compare the results with the non-parametric results obtained in this project in order to assess how robust the non-parametric method can be.

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Appendix A

Results of Numerical Study

The optimization results of the numerical studies conducted in Section 5.2 are tabulated in this section.

Table A.1: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.45}$ with $n = 10$ and $m = 7$

(total 84 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI[2]	Actual CL[3]	CI Mass	Efficiency
0.01	2	Best PC[4]	(0, 3, 0⋆5)	[1, 7]	0.9918	0.7727	100.00 %
		Worst PC	(3, 0⋆6)	[1, 7]	0.9929	0.7792	99.17 %
		Type-II	(0⋆6, 3)	[1, 7]	0.8955	0.5455	NA[5]
0.05	74	Best PC	(0, 2, 0⋆4, 1)	[2, 7]	0.9536	0.5844	100.00 %
		Worst PC	(2, 1, 0⋆5)	[1, 7]	0.9887	0.7765	75.26 %
		Type-II	(0⋆6, 3)	[1, 7]	0.8955	0.5455	NA
0.10	83	Best PC	(0⋆4, 1, 0, 2)	[2, 7]	0.9046	0.4909	100.00 %
		Worst PC	(0, 0, 1, 2, 0⋆3)	[2, 7]	0.9601	0.6623	74.12 %
		Type-II	(0⋆6, 3)	[1, 7]	0.8955	0.5455	NA

[2] confidence interval

[3] confidence level

[4] progressive censoring

[5] not available

Table A.2: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 15$ and $m = 6$ (total 2002 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	2	Best PC	(0, 9, 0⋆4)	[1, 6]	0.9928	0.7625	100.00 %
		Worst PC	(9, 0⋆5)	[1, 6]	0.9953	0.7812	97.60 %
		Type-II	(0⋆5, 9)	[1, 6]	0.5627	0.3125	NA
0.05	111	Best PC	(0, 8, 0⋆3, 1)	[2, 6]	0.9643	0.5833	100.00 %
		Worst PC	(8, 1, 0⋆4)	[1, 6]	0.9888	0.7768	75.10 %
		Type-II	(0⋆5, 9)	[1, 6]	0.5627	0.3125	NA
0.10	650	Best PC	(0, 6, 0, 0, 1, 2)	[2, 6]	0.9036	0.4648	100.00 %
		Worst PC	(7, 1, 1, 0⋆3)	[1, 6]	0.9785	0.7666	60.64 %
		Type-II	(0⋆5, 9)	[1, 6]	0.5627	0.3125	NA

Table A.3: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 15$ and $m = 10$ (total 2002 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1997	Best PC	(0⋆5, 1, 0⋆3, 4)	[1, 10]	0.9903	0.5903	100.00 %
		Worst PC	(0, 0, 1, 0, 1, 1, 2, 0⋆3)	[1, 10]	0.9974	0.8085	73.01 %
		Type-II	(0⋆9, 5)	[1, 10]	0.9860	0.5625	NA
0.05	2002	Best PC	(0⋆6, 1, 0, 0, 4)	[2, 9]	0.9511	0.4531	100.00 %
		Worst PC	(4, 0⋆5, 1, 0⋆3)	[1, 8]	0.9855	0.6179	73.33 %
		Type-II	(0⋆9, 5)	[2, 10]	0.9734	0.5000	90.62 %
0.10	2002	Best PC	(0⋆7, 1, 0, 4)	[3, 9]	0.9003	0.3839	100.00 %
		Worst PC	(0, 2, 0, 0, 2, 1, 0⋆4)	[2, 8]	0.9602	0.5375	71.43 %
		Type-II	(0⋆9, 5)	[3, 10]	0.9258	0.4375	87.76 %

Table A.4: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 15$ and $m = 10$ (total 2002 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	212	Best PC	(4, 0⋆8, 1)	[2, 10]	0.9904	0.6818	100.00 %
		Worst PC	(1⋆5, 0⋆5)	[1, 10]	0.9901	0.8328	81.87 %
		Type-II	(0⋆9, 5)	[1, 10]	0.8491	0.5625	NA
0.05	1831	Best PC	(0⋆3, 3, 0⋆5, 2)	[4, 10]	0.9524	0.5000	100.00 %
		Worst PC	(0⋆4, 1, 0, 0, 1, 3, 0)	[1, 10]	0.9501	0.7370	67.84 %
		Type-II	(0⋆9, 5)	[1, 10]	0.8491	0.5625	NA
0.10	1992	Best PC	(0, 0, 1, 0⋆4, 1, 0, 3)	[4, 10]	0.9004	0.4288	100.00 %
		Worst PC	(0⋆3, 2, 0, 0, 2, 1, 0, 0)	[4, 10]	0.9609	0.6100	70.30 %
		Type-II	(0⋆9, 5)	[1, 10]	0.8491	0.5625	NA

Table A.5: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 20$ and $m = 6$ (total 11628 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	26	Best PC	(0,13, 0⋆3, 1)	[1, 6]	0.9915	0.6508	100.00 %
		Worst PC	(14, 0⋆5)	[1, 6]	0.9963	0.7937	82.00 %
		Type-II	(0⋆5,14)	[1, 6]	0.6140	0.2381	NA
0.05	1302	Best PC	(9, 0⋆3, 1, 4)	[1, 6]	0.9523	0.4473	100.00 %
		Worst PC	(10, 4, 0⋆4)	[1, 6]	0.9853	0.7810	57.28 %
		Type-II	(0⋆5,14)	[1, 6]	0.6140	0.2381	NA
0.10	5888	Best PC	(0, 9, 0⋆3, 5)	[2, 6]	0.9097	0.3619	100.00 %
		Worst PC	(7, 5, 2, 0⋆3)	[1, 6]	0.9711	0.7640	47.37 %
		Type-II	(0⋆5,14)	[1, 6]	0.6140	0.2381	NA

Table A.6: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 20$ and $m = 10$ (total 92378 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	92207	Best PC	(0, 1, 0, 1, 0⋆5, 8)	[1, 10]	0.9900	0.4698	100.00 %
		Worst PC	(0⋆4, 1, 3, 6, 0⋆3)	[1, 10]	0.9960	0.7908	59.42 %
		Type-II	(0⋆9,10)	[1, 10]	0.9830	0.4286	NA
0.05	92378	Best PC	(0, 0, 1, 0, 1, 0⋆4, 8)	[2, 9]	0.9501	0.3645	100.00 %
		Worst PC	(4, 0, 0, 1, 4, 1, 0⋆4)	[1, 8]	0.9865	0.5876	62.04 %
		Type-II	(0⋆9,10)	[2, 10]	0.9618	0.3810	95.69 %
0.10	92378	Best PC	(0, 0, 1, 0⋆3, 2, 0, 0, 7)	[2, 8]	0.9001	0.3089	100.00 %
		Worst PC	(0⋆6, 1, 9, 0, 0)	[2, 9]	0.9635	0.4945	62.46 %
		Type-II	(0⋆9,10)	[2, 9]	0.9348	0.3333	92.67 %

Table A.7: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 20$ and $m = 10$ (total 92378 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	36115	Best PC	(0, 6, 0⋆5, 1, 0, 3)	[2, 10]	0.9903	0.5800	100.00 %
		Worst PC	(6, 0, 0, 2, 2, 0⋆5)	[1, 10]	0.9982	0.8415	68.92 %
		Type-II	(0⋆9,10)	[1, 10]	0.8780	0.4286	NA
0.05	91386	Best PC	(0⋆3, 4, 0⋆5, 6)	[3, 10]	0.9514	0.4212	100.00 %
		Worst PC	(0⋆3, 2, 2, 2, 4, 0⋆3)	[3, 10]	0.9773	0.7032	59.90 %
		Type-II	(0⋆9,10)	[1, 10]	0.8780	0.4286	NA
0.10	92369	Best PC	(0⋆3, 3, 0⋆5, 7)	[4, 10]	0.9025	0.3469	100.00 %
		Worst PC	(1, 2, 0⋆5, 2, 5, 0)	[3, 10]	0.9572	0.5992	57.90 %
		Type-II	(0⋆9,10)	[1, 10]	0.8780	0.4286	NA

Table A.8: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 20$ and $m = 10$ (total 92378 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	32	Best PC	(0⋆4,10, 0⋆5)	[4, 10]	0.9922	0.6825	100.00 %
		Worst PC	(8, 1, 1, 0⋆7)	[1, 10]	0.9905	0.8542	79.91 %
		Type-II	(0⋆9,10)	[1, 10]	0.4119	0.4286	NA
0.05	3550	Best PC	(0, 0, 8, 0⋆6, 2)	[4, 10]	0.9507	0.5143	100.00 %
		Worst PC	(4, 1, 1, 2, 1, 0, 1, 0⋆3)	[1, 10]	0.9500	0.8317	61.83 %
		Type-II	(0⋆9,10)	[1, 10]	0.4119	0.4286	NA
0.10	27873	Best PC	(7, 0⋆8, 3)	[4, 10]	0.9143	0.4396	100.00 %
		Worst PC	(0, 1, 0, 3, 1, 1, 4, 0⋆3)	[1, 10]	0.9000	0.8024	54.78 %
		Type-II	(0⋆9,10)	[1, 10]	0.4119	0.4286	NA

Table A.9: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 20$ and $m = 16$ (total 3876 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	3876	Best PC	(0, 1, 0, 1, 0⋆11, 2)	[1, 10]	0.9900	0.4698	100.00 %
		Worst PC	(0⋆5, 1, 1, 2, 0⋆8)	[1, 11]	0.9955	0.5471	85.89 %
		Type-II	(0⋆15, 4)	[1, 11]	0.9929	0.4762	98.67 %
0.05	3876	Best PC	(0, 0, 1, 0, 1, 0⋆10, 2)	[2, 9]	0.9501	0.3645	100.00 %
		Worst PC	(2, 0⋆6, 2, 0⋆8)	[1, 9]	0.9789	0.4350	83.80 %
		Type-II	(0⋆15, 4)	[2, 10]	0.9618	0.3810	95.69 %
0.10	3876	Best PC	(0, 0, 1, 0⋆3, 2, 0⋆8, 1)	[2, 8]	0.9001	0.3089	100.00 %
		Worst PC	(0, 4, 0⋆14)	[2, 8]	0.9410	0.3619	85.35 %
		Type-II	(0⋆15, 4)	[2, 9]	0.9348	0.3333	92.67 %

Table A.10: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 20$ and $m = 16$ (total 3876 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	3876	Best PC	(0⋆15, 4)	[2, 13]	0.9919	0.5238	100.00 %
		Worst PC	(0⋆7, 1, 1, 0, 2, 0⋆5)	[2, 13]	0.9959	0.6021	86.99 %
		Type-II	(0⋆15, 4)	[2, 13]	0.9919	0.5238	100.00 %
0.05	3876	Best PC	(0, 0, 1, 0⋆12, 3)	[3, 11]	0.9505	0.4034	100.00 %
		Worst PC	(0, 4, 0⋆14)	[3, 11]	0.9693	0.4825	83.59 %
		Type-II	(0⋆15, 4)	[3, 12]	0.9683	0.4286	94.12 %
0.10	3876	Best PC	(0⋆15, 4)	[4, 11]	0.9025	0.3333	100.00 %
		Worst PC	(0, 1, 2, 1, 0⋆12)	[3, 10]	0.9491	0.4251	78.42 %
		Type-II	(0⋆15, 4)	[4, 11]	0.9025	0.3333	100.00 %

Table A.11: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 20$ and $m = 16$ (total 3876 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	3876	Best PC	(0⋆12, 1, 0, 0, 3)	[5, 16]	0.9902	0.5442	100.00 %
		Worst PC	(0, 0, 1, 1, 0⋆5, 1, 1, 0⋆5)	[4, 15]	0.9967	0.6655	81.77 %
		Type-II	(0⋆15, 4)	[4, 16]	0.9928	0.5714	95.24 %
0.05	3876	Best PC	(0⋆6, 1, 0⋆3, 1, 0⋆4, 2)	[6, 14]	0.9503	0.4258	100.00 %
		Worst PC	(0, 1, 1, 0⋆8, 1, 1, 0⋆3)	[5, 14]	0.9761	0.5140	82.84 %
		Type-II	(0⋆15, 4)	[6, 15]	0.9586	0.4286	99.36 %
0.10	3876	Best PC	(0⋆8, 1, 0⋆6, 3)	[7, 14]	0.9015	0.3550	100.00 %
		Worst PC	(0⋆5, 2, 0, 2, 0⋆8)	[6, 13]	0.9484	0.4457	79.65 %
		Type-II	(0⋆15, 4)	[6, 14]	0.9216	0.3810	93.18 %

Table A.12: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 20$ and $m = 16$ (total 3876 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1614	Best PC	(0⋆9, 4, 0⋆6)	[8, 16]	0.9903	0.5442	100.00 %
		Worst PC	(0⋆6, 2, 0⋆3, 1, 1, 0⋆4)	[3, 16]	0.9900	0.7810	69.69 %
		Type-II	(0⋆15, 4)	[1, 16]	0.8818	0.7143	NA
0.05	3823	Best PC	(0⋆12, 3, 0, 0, 1)	[9, 16]	0.9507	0.4190	100.00 %
		Worst PC	(0⋆10, 1, 0, 0, 1⋆3)	[4, 16]	0.9500	0.6402	65.45 %
		Type-II	(0⋆15, 4)	[1, 16]	0.8818	0.7143	NA
0.10	3874	Best PC	(0⋆10, 2, 0⋆4, 2)	[10, 16]	0.9036	0.3452	100.00 %
		Worst PC	(0, 0, 1, 0⋆8, 1, 0, 2, 0, 0)	[9, 16]	0.9489	0.4586	75.28 %
		Type-II	(0⋆15, 4)	[1, 16]	0.8818	0.7143	NA

Table A.13: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 25$ and $m = 5$ (total 10626 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	3	Best PC	(0,20, 0⋆3)	[1, 5]	0.9918	0.7308	100.00 %
		Worst PC	(20, 0⋆4)	[1, 5]	0.9967	0.7692	95.00 %
		Type-II	(0⋆4,20)	[1, 5]	0.2130	0.1538	NA
0.05	49	Best PC	(17, 0⋆3, 3)	[1, 5]	0.9507	0.4808	100.00 %
		Worst PC	(19, 1, 0⋆3)	[1, 5]	0.9902	0.7612	63.16 %
		Type-II	(0⋆4,20)	[1, 5]	0.2130	0.1538	NA
0.10	177	Best PC	(0,17, 0, 0, 3)	[2, 5]	0.9028	0.3956	100.00 %
		Worst PC	(18, 2, 0⋆3)	[1, 5]	0.9795	0.7555	52.36 %
		Type-II	(0⋆4,20)	[1, 5]	0.2130	0.1538	NA

Table A.14: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 25$ and $m = 10$ (total 1307504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1061107	Best PC	(6, 0⋆5, 1, 0, 0, 8)	[1, 10]	0.9902	0.4681	100.00 %
		Worst PC	(2, 1, 3, 4, 3, 2, 0⋆4)	[1, 10]	0.9975	0.8293	56.45 %
		Type-II	(0⋆9,15)	[1, 10]	0.9279	0.3462	NA
0.05	1307377	Best PC	(0, 2, 0⋆6, 1,12)	[2, 10]	0.9505	0.3387	100.00 %
		Worst PC	(0⋆4, 1, 2, 5, 7, 0, 0)	[3, 10]	0.9584	0.6658	50.87 %
		Type-II	(0⋆9,15)	[1, 10]	0.9279	0.3462	NA
0.10	1307504	Best PC	(0⋆7, 1, 0,14)	[3, 10]	0.9015	0.2738	100.00 %
		Worst PC	(0⋆6, 2, 6, 7, 0)	[3, 10]	0.9479	0.5751	47.60 %
		Type-II	(0⋆9,15)	[2, 10]	0.9216	0.3077	88.97 %

Table A.15: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 25$ and $m = 10$ (total 1307504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	11179	Best PC	(0, 0,12, 0⋆6, 3)	[3, 10]	0.9908	0.5629	100.00 %
		Worst PC	(11, 1, 1, 2, 0⋆6)	[1, 10]	0.9976	0.8563	65.74 %
		Type-II	(0⋆9,15)	[1, 10]	0.6303	0.3462	NA
0.05	630981	Best PC	(0⋆3,10, 0⋆5, 5)	[4, 10]	0.9557	0.4231	100.00 %
		Worst PC	(0, 1, 0, 1, 3, 4, 5, 1, 0, 0)	[1, 10]	0.9502	0.7796	54.27 %
		Type-II	(0⋆9,15)	[1, 10]	0.6303	0.3462	NA
0.10	1165349	Best PC	(0⋆5,10, 0⋆3, 5)	[5, 10]	0.9020	0.3462	100.00 %
		Worst PC	(0⋆5, 2, 4, 7, 2, 0)	[1, 10]	0.9000	0.6933	49.93 %
		Type-II	(0⋆9,15)	[1, 10]	0.6303	0.3462	NA

Table A.16: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 25$ and $m = 10$ (total 1307504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	21	Best PC	(0⋆4,15, 0⋆5)	[5, 10]	0.9903	0.6731	100.00 %
		Worst PC	(0,15, 0⋆8)	[2, 10]	0.9979	0.8205	82.03 %
		Type-II	(0⋆9,15)	[1, 10]	0.1148	0.3462	NA
0.05	766	Best PC	(0⋆6,15, 0⋆3)	[7, 10]	0.9577	0.5481	100.00 %
		Worst PC	(11, 0, 1⋆3, 0, 0, 1, 0, 0)	[1, 10]	0.9502	0.8347	65.66 %
		Type-II	(0⋆9,15)	[1, 10]	0.1148	0.3462	NA
0.10	7122	Best PC	(0, 0,13, 0⋆6, 2)	[5, 10]	0.9016	0.4423	100.00 %
		Worst PC	(8, 0, 3, 2, 0, 2, 0⋆4)	[1, 10]	0.9000	0.8406	52.62 %
		Type-II	(0⋆9,15)	[1, 10]	0.1148	0.3462	NA

Table A.17: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 25$ and $m = 15$ (total 1961256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1961256	Best PC	(0⋆9, 1, 0⋆4, 9)	[2, 13]	0.9901	0.4308	100.00 %
		Worst PC	(1, 0, 1, 2, 3, 3, 0⋆9)	[1, 11]	0.9972	0.5935	72.59 %
		Type-II	(0⋆14,10)	[2, 14]	0.9921	0.4615	93.33 %
0.05	1961256	Best PC	(0, 0, 1, 1, 0⋆10, 8)	[3, 11]	0.9501	0.3358	100.00 %
		Worst PC	(0, 6, 4, 0⋆12)	[2, 9]	0.9781	0.4536	74.01 %
		Type-II	(0⋆14,10)	[3, 12]	0.9572	0.3462	96.99 %
0.10	1961256	Best PC	(0⋆7, 1, 0⋆6, 9)	[3, 10]	0.9015	0.2738	100.00 %
		Worst PC	(1, 9, 0⋆13)	[2, 8]	0.9577	0.3949	69.32 %
		Type-II	(0⋆14,10)	[3, 11]	0.9382	0.3077	88.97 %

Table A.18: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 25$ and $m = 15$ (total 1961256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1961256	Best PC	(0⋆11, 1, 0, 0, 9)	[3, 15]	0.9900	0.4704	100.00 %
		Worst PC	(0⋆10, 1, 3, 5, 1, 0)	[3, 15]	0.9964	0.7038	66.84 %
		Type-II	(0⋆14,10)	[2, 15]	0.9904	0.5000	94.08 %
0.05	1961256	Best PC	(0⋆5, 1, 0⋆6, 1, 0, 8)	[5, 14]	0.9503	0.3660	100.00 %
		Worst PC	(1, 3, 4, 2, 0⋆11)	[3, 11]	0.9787	0.5350	68.42 %
		Type-II	(0⋆14,10)	[4, 14]	0.9649	0.3846	95.17 %
0.10	1961256	Best PC	(0⋆14,10)	[5, 13]	0.9075	0.3077	100.00 %
		Worst PC	(4, 3, 3, 0⋆12)	[4, 11]	0.9286	0.4641	66.30 %
		Type-II	(0⋆14,10)	[5, 13]	0.9075	0.3077	100.00 %

Table A.19: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 25$ and $m = 15$ (total 1961256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	571742	Best PC	(0⋆5, 9, 0⋆8, 1)	[6, 14]	0.9903	0.5594	100.00 %
		Worst PC	(0, 1, 0, 1, 2, 0, 1⋆3, 2, 0, 1, 0⋆3)	[1, 15]	0.9900	0.8649	64.68 %
		Type-II	(0⋆14,10)	[1, 15]	0.7878	0.5385	NA
0.05	1908500	Best PC	(0⋆7, 6, 0⋆6, 4)	[8, 15]	0.9506	0.4038	100.00 %
		Worst PC	(1, 0⋆7, 1, 0, 1, 0, 4, 3, 0)	[1, 15]	0.9500	0.7686	52.54 %
		Type-II	(0⋆14,10)	[1, 15]	0.7878	0.5385	NA
0.10	1959458	Best PC	(1, 0⋆6, 3, 0⋆6, 6)	[8, 15]	0.9010	0.3405	100.00 %
		Worst PC	(0⋆3, 1, 0⋆6, 1, 1, 0, 3, 4)	[2, 15]	0.9000	0.5680	59.95 %
		Type-II	(0⋆14,10)	[1, 15]	0.7878	0.5385	NA

Table A.20: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 25$ and $m = 15$ (total 1961256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	43	Best PC	(0⋆4,10, 0⋆10)	[7, 15]	0.9909	0.5874	100.00 %
		Worst PC	(0⋆3, 9, 1, 0⋆10)	[5, 15]	0.9905	0.7101	82.73 %
		Type-II	(0⋆14,10)	[1, 15]	0.2288	0.5385	NA
0.05	14507	Best PC	(0, 0, 9, 0⋆11, 1)	[8, 15]	0.9509	0.4423	100.00 %
		Worst PC	(2, 1, 3, 0, 2, 2, 0⋆9)	[3, 15]	0.9500	0.8046	54.97 %
		Type-II	(0⋆14,10)	[1, 15]	0.2288	0.5385	NA
0.10	262889	Best PC	(0⋆3, 8, 0⋆10, 2)	[9, 15]	0.9009	0.3626	100.00 %
		Worst PC	(1⋆3, 0, 0, 4, 0, 0, 1, 0, 0, 2, 0⋆3)	[2, 15]	0.9000	0.8256	43.92 %
		Type-II	(0⋆14,10)	[1, 15]	0.2288	0.5385	NA

Table A.21: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 25$ and $m = 20$ (total 42504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	42504	Best PC	(0⋆9, 1, 0⋆9, 4)	[2, 13]	0.9901	0.4308	100.00 %
		Worst PC	(1, 0⋆3, 2, 1, 1, 0⋆13)	[1, 12]	0.9968	0.5055	85.22 %
		Type-II	(0⋆19, 5)	[2, 14]	0.9921	0.4615	93.33 %
0.05	42504	Best PC	(0, 0, 1, 1, 0⋆15, 3)	[3, 11]	0.9501	0.3358	100.00 %
		Worst PC	(0, 5, 0⋆18)	[2, 10]	0.9754	0.3887	86.39 %
		Type-II	(0⋆19, 5)	[3, 12]	0.9572	0.3462	96.99 %
0.10	42504	Best PC	(0⋆7, 1, 0⋆11, 4)	[3, 10]	0.9015	0.2738	100.00 %
		Worst PC	(0, 5, 0⋆18)	[3, 10]	0.9307	0.3401	80.50 %
		Type-II	(0⋆19, 5)	[3, 11]	0.9382	0.3077	88.97 %

Table A.22: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 25$ and $m = 20$ (total 42504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	42504	Best PC	(0⋆11, 1, 0⋆7, 4)	[3, 15]	0.9900	0.4704	100.00 %
		Worst PC	(0⋆8, 2, 1⋆3, 0⋆8)	[3, 15]	0.9955	0.5432	86.60 %
		Type-II	(0⋆19, 5)	[3, 16]	0.9949	0.5000	94.08 %
0.05	42504	Best PC	(0⋆5, 1, 0⋆6, 1, 0⋆6, 3)	[5, 14]	0.9503	0.3660	100.00 %
		Worst PC	(1, 2, 2, 0⋆17)	[4, 13]	0.9684	0.4388	83.42 %
		Type-II	(0⋆19, 5)	[4, 14]	0.9649	0.3846	95.17 %
0.10	42504	Best PC	(0⋆19, 5)	[5, 13]	0.9075	0.3077	100.00 %
		Worst PC	(2, 1, 1, 0⋆3, 1, 0⋆13)	[3, 11]	0.9144	0.3820	80.55 %
		Type-II	(0⋆19, 5)	[5, 13]	0.9075	0.3077	100.00 %

Table A.23: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 25$ and $m = 20$ (total 42504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	42504	Best PC	(0⋆19, 5)	[7, 20]	0.9906	0.5000	100.00 %
		Worst PC	(0, 1, 1, 2, 1, 0⋆15)	[6, 18]	0.9924	0.5956	83.94 %
		Type-II	(0⋆19, 5)	[7, 20]	0.9906	0.5000	100.00 %
0.05	42504	Best PC	(0⋆10, 2, 0⋆8, 3)	[8, 17]	0.9501	0.3817	100.00 %
		Worst PC	(0⋆4, 1, 0, 3, 1, 0⋆12)	[7, 16]	0.9749	0.4660	81.90 %
		Type-II	(0⋆19, 5)	[8, 18]	0.9567	0.3846	99.23 %
0.10	42504	Best PC	(0⋆13, 1, 0⋆5, 4)	[9, 17]	0.9010	0.3182	100.00 %
		Worst PC	(0, 1, 0, 1⋆4, 0⋆13)	[8, 16]	0.9288	0.4051	78.54 %
		Type-II	(0⋆19, 5)	[8, 17]	0.9245	0.3462	91.92 %

Table A.24: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 25$ and $m = 20$ (total 42504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	36505	Best PC	(1, 0⋆9, 3, 0⋆8, 1)	[10, 20]	0.9902	0.4990	100.00 %
		Worst PC	(0⋆7, 1, 0, 0, 1, 0⋆3, 1, 0, 1, 0, 1, 0)	[4, 20]	0.9900	0.7573	65.89 %
		Type-II	(0⋆19, 5)	[1, 20]	0.9174	0.7308	NA
0.05	42485	Best PC	(0⋆15, 3, 0⋆3, 2)	[12, 20]	0.9516	0.3736	100.00 %
		Worst PC	(0⋆8, 2, 0⋆7, 1, 2, 0, 0)	[11, 20]	0.9764	0.4819	77.53 %
		Type-II	(0⋆19, 5)	[1, 20]	0.9174	0.7308	NA
0.10	42504	Best PC	(0⋆13, 1, 0, 1, 0⋆3, 3)	[13, 20]	0.9011	0.3112	100.00 %
		Worst PC	(0⋆7, 1, 0⋆8, 1, 0, 3, 0)	[12, 20]	0.9458	0.4131	75.34 %
		Type-II	(0⋆19, 5)	[11, 20]	0.9081	0.3462	89.90 %

Table A.25: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.75}$ with $n = 25$ and $m = 20$ (total 42504 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	113	Best PC	(0⋆6, 5, 0⋆13)	[11, 20]	0.9906	0.4698	100.00 %
		Worst PC	(0, 3, 1, 0, 1, 0⋆15)	[7, 20]	0.9900	0.6391	73.51 %
		Type-II	(0⋆19, 5)	[1, 20]	0.6217	0.7308	NA
0.05	26247	Best PC	(0⋆14, 5, 0⋆5)	[15, 20]	0.9521	0.3526	100.00 %
		Worst PC	(1, 1, 0⋆4, 1, 0⋆9, 1, 0, 1, 0)	[7, 20]	0.9500	0.6319	55.79 %
		Type-II	(0⋆19, 5)	[1, 20]	0.6217	0.7308	NA
0.10	39544	Best PC	(0⋆8, 4, 0⋆10, 1)	[14, 20]	0.9030	0.3018	100.00 %
		Worst PC	(0⋆10, 2, 0⋆5, 1, 0, 2, 0)	[8, 20]	0.9000	0.5888	51.26 %
		Type-II	(0⋆19, 5)	[1, 20]	0.6217	0.7308	NA

Table A.26: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 30$ and $m = 8$ (total 1560780 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	145800	Best PC	(13, 0⋆4, 1, 0, 8)	[1, 8]	0.9900	0.4088	100.00 %
		Worst PC	(11,11, 0⋆6)	[1, 8]	0.9920	0.8368	48.86 %
		Type-II	(0⋆7,22)	[1, 8]	0.9226	0.2258	NA
0.05	1559645	Best PC	(3, 0⋆6,19)	[1, 8]	0.9502	0.2509	100.00 %
		Worst PC	(0, 0, 1, 5, 7, 9, 0, 0)	[1, 8]	0.9847	0.7129	35.20 %
		Type-II	(0⋆7,22)	[1, 8]	0.9226	0.2258	NA
0.10	1560780	Best PC	(0, 0, 2, 0, 0, 1, 0,19)	[2, 8]	0.9002	0.2091	100.00 %
		Worst PC	(0⋆4, 3,10, 9, 0)	[2, 8]	0.9351	0.5678	36.83 %
		Type-II	(0⋆7,22)	[1, 8]	0.9226	0.2258	92.61 %

Table A.27: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 30$ and $m = 8$ (total 1560780 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	3078	Best PC	(0,18, 0⋆5, 4)	[2, 8]	0.9903	0.5103	100.00 %
		Worst PC	(18, 4, 0⋆6)	[1, 8]	0.9979	0.8410	60.67 %
		Type-II	(0⋆7,22)	[1, 8]	0.5141	0.2258	NA
0.05	179308	Best PC	(0, 0,16, 0⋆4, 6)	[3, 8]	0.9542	0.3763	100.00 %
		Worst PC	(1,14, 7, 0⋆5)	[2, 8]	0.9855	0.7898	47.65 %
		Type-II	(0⋆7,22)	[1, 8]	0.5141	0.2258	NA
0.10	799191	Best PC	(0, 0,13, 0⋆3, 1, 8)	[3, 8]	0.9012	0.3071	100.00 %
		Worst PC	(7, 8, 0, 2, 5, 0⋆3)	[2, 8]	0.9634	0.7471	41.10 %
		Type-II	(0⋆7,22)	[1, 8]	0.5141	0.2258	NA

Table A.28: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 30$ and $m = 8$ (total 1560780 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	59	Best PC	(0, 0,21, 0⋆4, 1)	[3, 8]	0.9906	0.6452	100.00 %
		Worst PC	(21, 1, 0⋆6)	[1, 8]	0.9980	0.8449	76.36 %
		Type-II	(0⋆7,22)	[1, 8]	0.1238	0.2258	NA
0.05	1592	Best PC	(0⋆3,20, 0⋆3, 2)	[4, 8]	0.9540	0.4977	100.00 %
		Worst PC	(14, 8, 0⋆6)	[1, 8]	0.9516	0.8381	59.38 %
		Type-II	(0⋆7,22)	[1, 8]	0.1238	0.2258	NA
0.10	9863	Best PC	(0⋆4,20, 0, 0, 2)	[5, 8]	0.9067	0.4194	100.00 %
		Worst PC	(9, 9, 3, 1, 0⋆4)	[1, 8]	0.9000	0.8241	50.89 %
		Type-II	(0⋆7,22)	[1, 8]	0.1238	0.2258	NA

Table A.29: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 30$ and $m = 25$ (total 118755 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	118755	Best PC	(0⋆3, 1, 0⋆20, 4)	[1, 11]	0.9900	0.3313	100.00 %
		Worst PC	(5, 0⋆24)	[1, 11]	0.9918	0.3871	85.58 %
		Type-II	(0⋆24, 5)	[1, 12]	0.9916	0.3548	93.36 %
0.05	118755	Best PC	(3, 0⋆23, 2)	[1, 8]	0.9502	0.2509	100.00 %
		Worst PC	(0, 0, 1, 2, 2, 0⋆20)	[1, 9]	0.9783	0.2930	85.64 %
		Type-II	(0⋆24, 5)	[1, 9]	0.9646	0.2581	97.22 %
0.10	118755	Best PC	(0, 0, 2, 0, 0, 1, 0⋆18, 2)	[2, 8]	0.9002	0.2091	100.00 %
		Worst PC	(3, 0⋆5, 1, 0⋆17, 1)	[1, 8]	0.9514	0.2527	82.76 %
		Type-II	(0⋆24, 5)	[2, 9]	0.9242	0.2258	92.61 %

Table A.30: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 30$ and $m = 25$ (total 118755 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
		Best PC	(0⋆12, 1, 0⋆11, 4)	[2, 14]	0.9902	0.3890	100.00 %
0.01	118755	Worst PC	(2, 0⋆6, 1, 2, 0⋆16)	[1, 13]	0.9924	0.4405	88.31 %
		Type-II	(0⋆24, 5)	[2, 15]	0.9953	0.4194	92.76 %
		Best PC	(0⋆3, 1, 0⋆5, 1, 0⋆14, 3)	[3, 12]	0.9502	0.3038	100.00 %
0.05	118755	Worst PC	(0, 5, 0⋆23)	[3, 12]	0.9697	0.3508	86.59 %
		Type-II	(0⋆24, 5)	[4, 14]	0.9544	0.3226	94.17 %
		Best PC	(0⋆3, 2, 0, 0, 2, 0⋆17, 1)	[4, 11]	0.9003	0.2578	100.00 %
0.10	118755	Worst PC	(0, 5, 0⋆23)	[4, 12]	0.9201	0.3118	82.68 %
		Type-II	(0⋆24, 5)	[4, 12]	0.9119	0.2581	99.90 %

Table A.31: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 30$ and $m = 25$ (total 118755 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
		Best PC	(0⋆3, 1, 0⋆11, 1, 0⋆8, 3)	[4, 17]	0.9901	0.4381	100.00 %
0.01	118755	Worst PC	(3, 2, 0⋆23)	[4, 17]	0.9927	0.5048	86.78 %
		Type-II	(0⋆24, 5)	[5, 19]	0.9910	0.4516	97.00 %
		Best PC	(0⋆13, 1, 1, 0⋆9, 3)	[6, 16]	0.9500	0.3291	100.00 %
0.05	118755	Worst PC	(0, 1, 1, 3, 0⋆21)	[4, 14]	0.9607	0.3943	83.46 %
		Type-II	(0⋆24, 5)	[6, 17]	0.9644	0.3548	92.74 %
		Best PC	(0⋆6, 2, 0⋆6, 1, 0⋆10, 2)	[7, 15]	0.9008	0.2840	100.00 %
0.10	118755	Worst PC	(2, 1, 0⋆10, 1, 0⋆11, 1)	[5, 14]	0.9289	0.3256	87.24 %
		Type-II	(0⋆24, 5)	[7, 16]	0.9113	0.2903	97.84 %

Table A.32: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 30$ and $m = 25$ (total 118755 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	118755	Best PC	(0⋆19, 1, 0⋆4, 4)	[8, 22]	0.9904	0.4581	100.00 %
		Worst PC	(0⋆4, 1, 2, 2, 0⋆18)	[8, 21]	0.9933	0.5259	87.11 %
		Type-II	(0⋆24, 5)	[8, 23]	0.9948	0.4839	94.67 %
0.05	118755	Best PC	(0⋆11, 1, 0, 1, 0⋆10, 3)	[10, 20]	0.9500	0.3505	100.00 %
		Worst PC	(0⋆5, 1, 0, 0, 3, 1, 0⋆15)	[9, 19]	0.9721	0.4141	84.66 %
		Type-II	(0⋆24, 5)	[10, 21]	0.9572	0.3548	98.79 %
0.10	118755	Best PC	(0⋆24, 5)	[11, 20]	0.9013	0.2903	100.00 %
		Worst PC	(0, 1⋆3, 0, 0, 1, 1, 0⋆17)	[8, 17]	0.9066	0.3605	80.53 %
		Type-II	(0⋆24, 5)	[11, 20]	0.9013	0.2903	100.00 %

Table A.33: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 30$ and $m = 25$ (total 118755 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	118678	Best PC	(0⋆12, 2, 0⋆11, 3)	[13, 25]	0.9900	0.4355	100.00 %
		Worst PC	(0⋆18, 1, 0, 1, 0⋆3, 3)	[9, 25]	0.9901	0.5513	78.99 %
		Type-II	(0⋆24, 5)	[1, 25]	0.9767	0.7742	NA
0.05	118755	Best PC	(0⋆22, 1, 0, 4)	[15, 25]	0.9512	0.3318	100.00 %
		Worst PC	(0⋆9, 1, 0⋆8, 1, 0, 1, 2, 0⋆3)	[14, 24]	0.9727	0.4142	80.11 %
		Type-II	(0⋆24, 5)	[14, 25]	0.9644	0.3548	93.51 %
0.10	118755	Best PC	(0⋆15, 1, 0⋆6, 1, 0, 3)	[16, 24]	0.9000	0.2823	100.00 %
		Worst PC	(0⋆8, 1, 0, 0, 1, 1, 2, 0⋆11)	[15, 23]	0.9259	0.3562	79.23 %
		Type-II	(0⋆24, 5)	[16, 25]	0.9116	0.2903	97.22 %

Table A.34: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.75}$ with $n = 30$ and $m = 25$ (total 118755 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	44002	Best PC	(0⋆4, 4, 0⋆19, 1)	[14, 25]	0.9901	0.4194	100.00 %
		Worst PC	(0⋆7, 1, 0⋆4, 2, 0⋆3, 1, 0, 1, 0⋆6)	[9, 25]	0.9900	0.6602	63.52 %
		Type-II	(0⋆24, 5)	[1, 25]	0.7974	0.7742	NA
0.05	116290	Best PC	(0⋆19, 4, 0⋆4, 1)	[18, 25]	0.9521	0.3180	100.00 %
		Worst PC	(0⋆8, 1, 0⋆11, 1, 1, 0, 2, 0)	[10, 25]	0.9500	0.5872	54.15 %
		Type-II	(0⋆24, 5)	[1, 25]	0.7974	0.7742	NA
0.10	118642	Best PC	(2, 0⋆16, 1, 0⋆6, 2)	[18, 25]	0.9002	0.2661	100.00 %
		Worst PC	(0⋆15, 1, 0⋆6, 1, 1, 2)	[13, 25]	0.9000	0.4294	61.97 %
		Type-II	(0⋆24, 5)	[1, 25]	0.7974	0.7742	NA

Table A.35: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 35$ and $m = 7$ (total 1344904 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	12819	Best PC	(20, 0⋆5, 8)	[1, 7]	0.9905	0.3889	100.00 %
		Worst PC	(22, 6, 0⋆5)	[1, 7]	0.9957	0.8226	47.27 %
		Type-II	(0⋆6,28)	[1, 7]	0.7315	0.1667	NA
0.05	736131	Best PC	(13, 0⋆4, 1,14)	[1, 7]	0.9505	0.2679	100.00 %
		Worst PC	(7,10,10, 1, 0⋆3)	[1, 7]	0.9858	0.7884	33.98 %
		Type-II	(0⋆6,28)	[1, 7]	0.7315	0.1667	NA
0.10	1254051	Best PC	(0,12, 0⋆4,16)	[2, 7]	0.9032	0.2146	100.00 %
		Worst PC	(0, 4, 9,13, 2, 0, 0)	[2, 7]	0.9522	0.7035	30.51 %
		Type-II	(0⋆6,28)	[1, 7]	0.7315	0.1667	NA

Table A.36: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 35$ and $m = 7$ (total 1344904 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	193	Best PC	(25, 0⋆5, 3)	[1, 7]	0.9932	0.5833	100.00 %
		Worst PC	(26, 2, 0⋆5)	[1, 7]	0.9979	0.8282	70.43 %
		Type-II	(0⋆6,28)	[1, 7]	0.1919	0.1667	NA
0.05	4063	Best PC	(0, 0,24, 0, 0, 1, 3)	[3, 7]	0.9513	0.4278	100.00 %
		Worst PC	(23, 5, 0⋆5)	[1, 7]	0.9882	0.8237	51.93 %
		Type-II	(0⋆6,28)	[1, 7]	0.1919	0.1667	NA
0.10	22868	Best PC	(21, 0⋆5, 7)	[2, 7]	0.9051	0.3472	100.00 %
		Worst PC	(11, 9, 8, 0⋆4)	[1, 7]	0.9021	0.7992	43.45 %
		Type-II	(0⋆6,28)	[1, 7]	0.1919	0.1667	NA

Table A.37: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 35$ and $m = 30$ (total 278256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	278256	Best PC	(1, 0, 1, 0⋆3, 1, 0⋆22, 2)	[1, 11]	0.9900	0.2979	100.00 %
		Worst PC	(0⋆5, 1, 2, 2, 0⋆22)	[1, 12]	0.9947	0.3331	89.42 %
		Type-II	(0⋆29, 5)	[1, 12]	0.9929	0.3056	97.48 %
0.05	278256	Best PC	(0⋆3, 1, 0⋆25, 4)	[2, 10]	0.9500	0.2276	100.00 %
		Worst PC	(5, 0⋆29)	[2, 10]	0.9534	0.2593	87.79 %
		Type-II	(0⋆29, 5)	[2, 11]	0.9647	0.2500	91.04 %
0.10	278256	Best PC	(0⋆29, 5)	[2, 9]	0.9068	0.1944	100.00 %
		Worst PC	(5, 0⋆29)	[2, 9]	0.9338	0.2269	85.71 %
		Type-II	(0⋆29, 5)	[2, 9]	0.9068	0.1944	100.00 %

Table A.38: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 35$ and $m = 30$ (total 278256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	278256	Best PC	(0⋆29, 5)	[3, 16]	0.9905	0.3611	100.00 %
		Worst PC	(4, 1, 0⋆28)	[1, 14]	0.9906	0.4207	85.84 %
		Type-II	(0⋆29, 5)	[3, 16]	0.9905	0.3611	100.00 %
0.05	278256	Best PC	(0⋆29, 5)	[4, 14]	0.9501	0.2778	100.00 %
		Worst PC	(2, 1, 2, 0⋆27)	[3, 13]	0.9681	0.3258	85.25 %
		Type-II	(0⋆29, 5)	[4, 14]	0.9501	0.2778	100.00 %
0.10	278256	Best PC	(0⋆4, 1, 0, 0, 1, 0⋆21, 3)	[5, 13]	0.9004	0.2351	100.00 %
		Worst PC	(3, 0⋆7, 1, 0⋆20, 1)	[3, 12]	0.9162	0.2774	84.77 %
		Type-II	(0⋆29, 5)	[5, 14]	0.9227	0.2500	94.06 %

Table A.39: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 35$ and $m = 30$ (total 278256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	278256	Best PC	(0⋆5, 1, 0⋆23, 4)	[6, 20]	0.9901	0.4023	100.00 %
		Worst PC	(2, 0, 2, 1, 0⋆26)	[5, 19]	0.9940	0.4572	87.99 %
		Type-II	(0⋆29, 5)	[6, 21]	0.9920	0.4167	96.55 %
0.05	278256	Best PC	(0⋆16, 1, 0⋆12, 4)	[7, 18]	0.9505	0.3071	100.00 %
		Worst PC	(1, 0, 0, 1, 3, 0⋆25)	[7, 18]	0.9610	0.3625	84.71 %
		Type-II	(0⋆29, 5)	[8, 20]	0.9520	0.3333	92.13 %
0.10	278256	Best PC	(0⋆8, 1, 0⋆20, 4)	[8, 17]	0.9005	0.2585	100.00 %
		Worst PC	(0⋆6, 5, 0⋆23)	[8, 17]	0.9268	0.3021	85.59 %
		Type-II	(0⋆29, 5)	[8, 18]	0.9244	0.2778	93.08 %

Table A.40: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 35$ and $m = 30$ (total 278256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	278256	Best PC	(0⋆22, 1, 1, 0⋆5, 3)	[10, 25]	0.9900	0.4243	100.00 %
		Worst PC	(0, 1⋆3, 0⋆11, 1, 0, 0, 1, 0⋆11)	[8, 23]	0.9918	0.4826	87.92 %
		Type-II	(0⋆29, 5)	[11, 27]	0.9907	0.4444	95.47 %
0.05	278256	Best PC	(0⋆11, 1, 0⋆7, 1, 0⋆9, 3)	[12, 23]	0.9502	0.3251	100.00 %
		Worst PC	(0⋆9, 5, 0⋆20)	[12, 23]	0.9602	0.3783	85.92 %
		Type-II	(0⋆29, 5)	[12, 24]	0.9590	0.3333	97.52 %
0.10	278256	Best PC	(0⋆12, 2, 0⋆16, 3)	[13, 22]	0.9007	0.2738	100.00 %
		Worst PC	(3, 1, 0⋆12, 1, 0⋆15)	[10, 20]	0.9168	0.3238	84.57 %
		Type-II	(0⋆29, 5)	[13, 23]	0.9105	0.2778	98.57 %

Table A.41: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 35$ and $m = 30$

(total 278256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	278256	Best PC	(0⋆25, 1, 0⋆3, 4)	[16, 30]	0.9900	0.4012	100.00 %
		Worst PC	(0⋆9, 1, 0⋆13, 1, 2, 1, 0⋆4)	[15, 29]	0.9952	0.4797	83.64 %
		Type-II	(0⋆29, 5)	[15, 30]	0.9920	0.4167	96.30 %
0.05	278256	Best PC	(0⋆27, 1, 0, 4)	[18, 29]	0.9510	0.3095	100.00 %
		Worst PC	(0⋆3, 1, 0, 1⋆3, 0, 0, 1, 0⋆19)	[16, 27]	0.9608	0.3710	83.42 %
		Type-II	(0⋆29, 5)	[18, 30]	0.9606	0.3333	92.86 %
0.10	278256	Best PC	(0⋆22, 1, 0⋆6, 4)	[19, 28]	0.9003	0.2616	100.00 %
		Worst PC	(0⋆9, 1, 0, 1, 0, 1, 0, 1, 0, 0, 1, 0⋆11)	[16, 25]	0.9198	0.3209	81.51 %
		Type-II	(0⋆29, 5)	[19, 29]	0.9149	0.2778	94.17 %

Table A.42: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.75}$ with $n = 35$ and $m = 30$ (total 278256 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	254874	Best PC	(0⋆7, 1, 0⋆9, 3, 0⋆11, 1)	[19, 30]	0.9901	0.3848	100.00 %
		Worst PC	(0⋆5, 1, 0, 1, 0⋆16, 1, 0, 0, 1, 1, 0)	[12, 30]	0.9900	0.5936	64.83 %
		Type-II	(0⋆29, 5)	[1, 30]	0.9024	0.8056	NA
0.05	278195	Best PC	(0⋆20, 2, 0⋆8, 3)	[21, 30]	0.9505	0.2885	100.00 %
		Worst PC	(0⋆24, 1, 0, 0, 1, 1, 2)	[16, 30]	0.9500	0.4219	68.38 %
		Type-II	(0⋆29, 5)	[1, 30]	0.9024	0.8056	NA
0.10	278256	Best PC	(0⋆13, 1, 0⋆14, 1, 3)	[22, 30]	0.9002	0.2386	100.00 %
		Worst PC	(0⋆24, 1⋆6)	[22, 30]	0.9391	0.3137	76.07 %
		Type-II	(0⋆29, 5)	[19, 30]	0.9002	0.3056	78.10 %

Table A.43: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 40$ and $m = 6$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	626	Best PC	(29, 0⋆4, 5)	[1, 6]	0.9927	0.4435	100.00 %
		Worst PC	(31, 3, 0⋆4)	[1, 6]	0.9973	0.8022	55.28 %
		Type-II	(0⋆5,34)	[1, 6]	0.4310	0.1220	NA
0.05	15801	Best PC	(23, 0, 0, 1, 0,10)	[1, 6]	0.9506	0.2958	100.00 %
		Worst PC	(25, 9, 0⋆4)	[1, 6]	0.9867	0.7935	37.27 %
		Type-II	(0⋆5,34)	[1, 6]	0.4310	0.1220	NA
0.10	93276	Best PC	(0,23, 0⋆3,11)	[2, 6]	0.9085	0.2378	100.00 %
		Worst PC	(19,14, 1, 0⋆3)	[1, 6]	0.9712	0.7820	30.41 %
		Type-II	(0⋆5,34)	[1, 6]	0.4310	0.1220	NA

Table A.44: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 40$ and $m = 6$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	28	Best PC	(32, 0⋆4, 2)	[1, 6]	0.9906	0.6098	100.00 %
		Worst PC	(34, 0⋆5)	[1, 6]	0.9993	0.8130	75.00 %
		Type-II	(0⋆5,34)	[1, 6]	0.0433	0.1220	NA
0.05	288	Best PC	(0,31, 0⋆3, 3)	[2, 6]	0.9575	0.4756	100.00 %
		Worst PC	(31, 3, 0⋆4)	[1, 6]	0.9849	0.8022	59.29 %
		Type-II	(0⋆5,34)	[1, 6]	0.0433	0.1220	NA
0.10	1072	Best PC	(0, 0,31, 0, 0, 3)	[3, 6]	0.9076	0.3972	100.00 %
		Worst PC	(30, 4, 0⋆4)	[1, 6]	0.9747	0.8000	49.65 %
		Type-II	(0⋆5,34)	[1, 6]	0.0433	0.1220	NA

Table A.45: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 40$ and $m = 35$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	575757	Best PC	(0, 0, 2, 0⋆31, 3)	[1, 12]	0.9900	0.2805	100.00 %
		Worst PC	(0⋆5, 1, 0, 1⋆4, 0⋆24)	[1, 13]	0.9958	0.3105	90.32 %
		Type-II	(0⋆34, 5)	[1, 13]	0.9942	0.2927	95.83 %
0.05	575757	Best PC	(0, 4, 0⋆5, 1, 0⋆27)	[2, 10]	0.9502	0.2194	100.00 %
		Worst PC	(4, 0⋆3, 1, 0⋆30)	[1, 10]	0.9646	0.2483	88.36 %
		Type-II	(0⋆34, 5)	[2, 11]	0.9580	0.2195	99.93 %
0.10	575757	Best PC	(0⋆3, 2, 0, 1, 0⋆28, 2)	[3, 10]	0.9001	0.1823	100.00 %
		Worst PC	(4, 0⋆33, 1)	[3, 11]	0.9174	0.2168	84.09 %
		Type-II	(0⋆34, 5)	[3, 11]	0.9215	0.1951	93.44 %

Table A.46: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 40$ and $m = 35$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	575757	Best PC	(0⋆34, 5)	[4, 18]	0.9907	0.3415	100.00 %
		Worst PC	(1, 2, 0⋆3, 1, 1, 0⋆28)	[2, 16]	0.9913	0.3867	88.31 %
		Type-II	(0⋆34, 5)	[4, 18]	0.9907	0.3415	100.00 %
0.05	575757	Best PC	(0⋆5, 3, 0⋆28, 2)	[5, 15]	0.9500	0.2645	100.00 %
		Worst PC	(3, 1, 1, 0⋆32)	[4, 15]	0.9723	0.3074	86.05 %
		Type-II	(0⋆34, 5)	[5, 16]	0.9577	0.2683	98.58 %
0.10	575757	Best PC	(0⋆34, 5)	[6, 15]	0.9023	0.2195	100.00 %
		Worst PC	(0⋆4, 5, 0⋆30)	[6, 15]	0.9257	0.2549	86.11 %
		Type-II	(0⋆34, 5)	[6, 15]	0.9023	0.2195	100.00 %

Table A.47: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 40$ and $m = 35$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	575757	Best PC	(0⋆8, 1, 0⋆25, 4)	[7, 22]	0.9900	0.3761	100.00 %
		Worst PC	(0⋆4, 5, 0⋆30)	[7, 22]	0.9914	0.4249	88.52 %
		Type-II	(0⋆34, 5)	[7, 23]	0.9926	0.3902	96.37 %
0.05	575757	Best PC	(0⋆34, 5)	[9, 21]	0.9525	0.2927	100.00 %
		Worst PC	(3, 0, 1, 0, 1, 0⋆30)	[7, 19]	0.9699	0.3362	87.05 %
		Type-II	(0⋆34, 5)	[9, 21]	0.9525	0.2927	100.00 %
0.10	575757	Best PC	(0⋆17, 1, 0⋆16, 4)	[9, 19]	0.9014	0.2450	100.00 %
		Worst PC	(0, 0, 1⋆4, 0, 1, 0⋆27)	[9, 19]	0.9254	0.2838	86.33 %
		Type-II	(0⋆34, 5)	[10, 21]	0.9183	0.2683	91.32 %

Table A.48: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 40$ and $m = 35$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	575757	Best PC	(0⋆21, 1, 0⋆12, 4)	[12, 28]	0.9902	0.3984	100.00 %
		Worst PC	(2, 1, 1, 0, 1, 0⋆30)	[11, 27]	0.9927	0.4486	88.80 %
		Type-II	(0⋆34, 5)	[12, 29]	0.9936	0.4146	96.08 %
0.05	575757	Best PC	(0⋆13, 1, 0⋆20, 4)	[14, 26]	0.9502	0.3039	100.00 %
		Worst PC	(0⋆8, 1, 0, 2, 2, 0⋆23)	[13, 25]	0.9700	0.3515	86.46 %
		Type-II	(0⋆34, 5)	[15, 28]	0.9514	0.3171	95.86 %
0.10	575757	Best PC	(0⋆15, 1, 0⋆6, 1, 0⋆11, 3)	[15, 25]	0.9002	0.2562	100.00 %
		Worst PC	(0⋆11, 1, 4, 0⋆22)	[14, 24]	0.9332	0.2965	86.40 %
		Type-II	(0⋆34, 5)	[16, 27]	0.9038	0.2683	95.50 %

Table A.49: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 40$ and $m = 35$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	575757	Best PC	(0⋆23, 1, 0⋆10, 4)	[19, 34]	0.9900	0.3811	100.00 %
		Worst PC	(0⋆17, 1, 0⋆10, 2, 2, 0⋆5)	[18, 33]	0.9949	0.4363	87.34 %
		Type-II	(0⋆34, 5)	[19, 35]	0.9912	0.3902	97.66 %
0.05	575757	Best PC	(0⋆34, 5)	[21, 33]	0.9513	0.2927	100.00 %
		Worst PC	(0⋆14, 1, 0, 1, 2, 1, 0⋆16)	[20, 31]	0.9624	0.3414	85.74 %
		Type-II	(0⋆34, 5)	[21, 33]	0.9513	0.2927	100.00 %
0.10	575757	Best PC	(0⋆29, 1, 0⋆4, 4)	[21, 31]	0.9027	0.2463	100.00 %
		Worst PC	(0⋆5, 1, 0⋆3, 1, 1, 0, 1, 1, 0⋆21)	[20, 30]	0.9205	0.2933	83.98 %
		Type-II	(0⋆34, 5)	[22, 33]	0.9177	0.2683	91.82 %

Table A.50: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.75}$ with $n = 40$ and $m = 35$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	574293	Best PC	(0⋆24, 3, 0⋆9, 2)	[23, 35]	0.9901	0.3490	100.00 %
		Worst PC	(0⋆20, 1, 0⋆10, 1, 0, 3, 0)	[16, 35]	0.9900	0.5364	65.06 %
		Type-II	(0⋆34, 5)	[1, 35]	0.9567	0.8293	NA
0.05	575757	Best PC	(0⋆26, 1, 0⋆6, 1, 3)	[25, 35]	0.9503	0.2642	100.00 %
		Worst PC	(0⋆8, 1, 0⋆23, 1, 3, 0)	[24, 35]	0.9718	0.3294	80.20 %
		Type-II	(0⋆34, 5)	[23, 35]	0.9521	0.2927	90.26 %
0.10	575757	Best PC	(0⋆34, 5)	[26, 35]	0.9023	0.2195	100.00 %
		Worst PC	(0⋆33, 5, 0)	[26, 35]	0.9347	0.2805	78.26 %
		Type-II	(0⋆34, 5)	[26, 35]	0.9023	0.2195	100.00 %

Table A.51: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.85}$ with $n = 40$ and $m = 35$ (total 575757 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	3682	Best PC	(0⋆14, 5, 0⋆20)	[25, 35]	0.9902	0.3020	100.00 %
		Worst PC	(0⋆6, 3, 0, 0, 1, 0⋆5, 1, 0⋆19)	[18, 35]	0.9900	0.4952	60.98 %
		Type-II	(0⋆34, 5)	[1, 35]	0.5675	0.8293	NA
0.05	443366	Best PC	(0⋆13, 4, 0⋆20, 1)	[27, 35]	0.9500	0.2291	100.00 %
		Worst PC	(0⋆18, 1, 1, 0⋆5, 1, 1, 0⋆4, 1, 0⋆3)	[20, 35]	0.9500	0.4710	48.64 %
		Type-II	(0⋆34, 5)	[1, 35]	0.5675	0.8293	NA
0.10	557320	Best PC	(0, 4, 0⋆32, 1)	[28, 35]	0.9007	0.1902	100.00 %
		Worst PC	(0, 0, 1, 0⋆26, 1, 0, 0, 3, 0, 0)	[21, 35]	0.9000	0.4203	45.27 %
		Type-II	(0⋆34, 5)	[1, 35]	0.5675	0.8293	NA

Table A.52: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 45$ and $m = 6$ (total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	618	Best PC	(33, 1, 0⋆3, 5)	[1, 6]	0.9901	0.4402	100.00 %
		Worst PC	(36, 3, 0⋆4)	[1, 6]	0.9980	0.8043	54.73 %
		Type-II	(0⋆5,39)	[1, 6]	0.3135	0.1087	NA
0.05	11417	Best PC	(0,31, 0⋆3, 8)	[2, 6]	0.9516	0.2943	100.00 %
		Worst PC	(31, 8, 0⋆4)	[1, 6]	0.9884	0.7966	36.95 %
		Type-II	(0⋆5,39)	[1, 6]	0.3135	0.1087	NA
0.10	61302	Best PC	(0,28, 0⋆3,11)	[2, 6]	0.9073	0.2391	100.00 %
		Worst PC	(26,13, 0⋆4)	[1, 6]	0.9718	0.7929	30.16 %
		Type-II	(0⋆5,39)	[1, 6]	0.3135	0.1087	NA

Table A.53: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 45$ and $m = 6$ (total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	28	Best PC	(37, 0⋆4, 2)	[1, 6]	0.9903	0.6114	100.00 %
		Worst PC	(39, 0⋆5)	[1, 6]	0.9993	0.8152	75.00 %
		Type-II	(0⋆5,39)	[1, 6]	0.0179	0.1087	NA
0.05	247	Best PC	(0,36, 0⋆3, 3)	[2, 6]	0.9551	0.4783	100.00 %
		Worst PC	(37, 2, 0⋆4)	[1, 6]	0.9919	0.8071	59.26 %
		Type-II	(0⋆5,39)	[1, 6]	0.0179	0.1087	NA
0.10	939	Best PC	(0, 0,36, 0, 0, 3)	[3, 6]	0.9011	0.4006	100.00 %
		Worst PC	(35, 4, 0⋆4)	[1, 6]	0.9727	0.8022	49.94 %
		Type-II	(0⋆5,39)	[1, 6]	0.0179	0.1087	NA

Table A.54: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 45$ and $m = 40$

(total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1086008	Best PC	(0, 0, 1, 0⋆36, 4)	[2, 14]	0.9900	0.2666	100.00 %
		Worst PC	(5, 0⋆39)	[1, 13]	0.9947	0.2935	90.83 %
		Type-II	(0⋆39, 5)	[2, 15]	0.9923	0.2826	94.32 %
0.05	1086008	Best PC	(0, 0, 1, 1, 0⋆35, 3)	[3, 12]	0.9501	0.2048	100.00 %
		Worst PC	(3, 0⋆8, 1, 0⋆29, 1)	[1, 11]	0.9568	0.2336	87.64 %
		Type-II	(0⋆39, 5)	[3, 13]	0.9608	0.2174	94.19 %
0.10	1086008	Best PC	(0⋆39, 5)	[3, 11]	0.9083	0.1739	100.00 %
		Worst PC	(0, 5, 0⋆38)	[3, 11]	0.9325	0.1962	88.64 %
		Type-II	(0⋆39, 5)	[3, 11]	0.9083	0.1739	100.00 %

Table A.55: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 45$ and $m = 40$ (total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1086008	Best PC	(0⋆39, 5)	[5, 20]	0.9906	0.3261	100.00 %
		Worst PC	(0, 1, 1, 0⋆6, 1⋆3, 0⋆28)	[3, 18]	0.9915	0.3576	91.18 %
		Type-II	(0⋆39, 5)	[5, 20]	0.9906	0.3261	100.00 %
0.05	1086008	Best PC	(0⋆5, 1, 0⋆7, 1, 0⋆25, 3)	[6, 17]	0.9501	0.2475	100.00 %
		Worst PC	(2, 0⋆38, 3)	[6, 18]	0.9639	0.2730	90.66 %
		Type-II	(0⋆39, 5)	[6, 18]	0.9630	0.2609	94.87 %
0.10	1086008	Best PC	(0⋆8, 3, 0⋆30, 2)	[7, 16]	0.9001	0.2091	100.00 %
		Worst PC	(0, 1⋆4, 0⋆7, 1, 0⋆27)	[5, 15]	0.9171	0.2417	86.49 %
		Type-II	(0⋆39, 5)	[7, 17]	0.9159	0.2174	96.18 %

Table A.56: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 45$ and $m = 40$ (total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1086008	Best PC	(0⋆7, 1, 0⋆31, 4)	[8, 24]	0.9900	0.3572	100.00 %
		Worst PC	(0, 0, 1, 0, 1, 2, 1, 0⋆33)	[8, 24]	0.9927	0.3969	90.01 %
		Type-II	(0⋆39, 5)	[8, 25]	0.9929	0.3696	96.66 %
0.05	1086008	Best PC	(0⋆12, 2, 0⋆26, 3)	[10, 22]	0.9501	0.2735	100.00 %
		Worst PC	(0, 1, 0, 0, 1, 0, 1, 0⋆12, 1, 0⋆19, 1)	[8, 21]	0.9587	0.3059	89.41 %
		Type-II	(0⋆39, 5)	[10, 23]	0.9589	0.2826	96.77 %
0.10	1086008	Best PC	(0⋆12, 2, 0⋆5, 1, 0⋆20, 2)	[11, 21]	0.9002	0.2305	100.00 %
		Worst PC	(0⋆3, 1⋆3, 0⋆10, 1, 0⋆22, 1)	[9, 20]	0.9190	0.2608	88.38 %
		Type-II	(0⋆39, 5)	[11, 22]	0.9149	0.2391	96.41 %

Table A.57: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 45$ and $m = 40$ (total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1086008	Best PC	(0⋆24, 1, 0⋆14, 4)	[14, 31]	0.9901	0.3761	100.00 %
		Worst PC	(1⋆3, 0, 1, 0, 1, 0⋆33)	[13, 30]	0.9928	0.4193	89.69 %
		Type-II	(0⋆39, 5)	[15, 33]	0.9907	0.3913	96.11 %
0.05	1086008	Best PC	(0⋆26, 2, 0⋆12, 3)	[16, 29]	0.9501	0.2877	100.00 %
		Worst PC	(0⋆7, 1⋆4, 0⋆5, 1, 0⋆23)	[14, 27]	0.9595	0.3277	87.81 %
		Type-II	(0⋆39, 5)	[17, 31]	0.9557	0.3043	94.54 %
0.10	1086008	Best PC	(0⋆24, 1, 0⋆14, 4)	[17, 28]	0.9009	0.2424	100.00 %
		Worst PC	(0⋆6, 1, 0⋆3, 1, 0, 1⋆3, 0⋆25)	[17, 28]	0.9193	0.2810	86.25 %
		Type-II	(0⋆39, 5)	[16, 28]	0.9146	0.2609	92.92 %

Table A.58: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 45$ and $m = 40$ (total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1086008	Best PC	(0⋆10, 1, 0⋆28, 4)	[21, 37]	0.9901	0.3581	100.00 %
		Worst PC	(0⋆9, 1, 2, 1, 0, 1, 0⋆26)	[19, 35]	0.9945	0.4073	87.90 %
		Type-II	(0⋆39, 5)	[22, 39]	0.9901	0.3696	96.89 %
0.05	1086008	Best PC	(0⋆12, 1, 0⋆17, 1, 0⋆8, 3)	[23, 35]	0.9500	0.2759	100.00 %
		Worst PC	(0⋆15, 1, 1, 0, 1⋆3, 0⋆19)	[23, 35]	0.9554	0.3200	86.22 %
		Type-II	(0⋆39, 5)	[24, 37]	0.9526	0.2826	97.63 %
0.10	1086008	Best PC	(0, 0, 1, 0⋆22, 1, 0⋆13, 3)	[24, 34]	0.9001	0.2325	100.00 %
		Worst PC	(0⋆18, 1, 1, 0, 3, 0⋆18)	[24, 34]	0.9216	0.2723	85.37 %
		Type-II	(0⋆39, 5)	[24, 35]	0.9149	0.2391	97.21 %

Table A.59: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.75}$ with $n = 45$ and $m = 40$ (total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1085986	Best PC	(1, 0⋆27, 1, 0⋆10, 3)	[26, 40]	0.9900	0.3276	100.00 %
		Worst PC	(0⋆29, 1, 0⋆9, 4)	[22, 40]	0.9900	0.4058	80.72 %
		Type-II	(0⋆39, 5)	[1, 40]	0.9821	0.8478	NA
0.05	1086008	Best PC	(0⋆19, 1, 0⋆19, 4)	[28, 39]	0.9501	0.2487	100.00 %
		Worst PC	(0⋆34, 2, 2, 1, 0⋆3)	[26, 38]	0.9545	0.3019	82.37 %
		Type-II	(0⋆39, 5)	[28, 40]	0.9630	0.2609	95.33 %
0.10	1086008	Best PC	(0⋆3, 1, 0⋆27, 1, 0⋆7, 3)	[29, 38]	0.9000	0.2116	100.00 %
		Worst PC	(0⋆22, 1, 1, 0, 1, 1, 0, 1, 0⋆11)	[27, 36]	0.9203	0.2583	81.91 %
		Type-II	(0⋆39, 5)	[30, 40]	0.9068	0.2174	97.32 %

Table A.60: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.85}$ with $n = 45$ and $m = 40$

(total 1086008 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	264034	Best PC	(0⋆11, 5, 0⋆28)	[29, 40]	0.9903	0.2804	100.00 %
		Worst PC	(0⋆6, 1, 0, 1, 0⋆9, 1, 0, 1, 0⋆9, 1, 0⋆9)	[21, 40]	0.9900	0.4984	56.26 %
		Type-II	(0⋆39, 5)	[1, 40]	0.6858	0.8478	NA
0.05	1040086	Best PC	(4, 0⋆38, 1)	[31, 40]	0.9544	0.2147	100.00 %
		Worst PC	(0⋆24, 1, 0⋆3, 2, 0⋆8, 1, 1, 0)	[24, 40]	0.9500	0.4293	50.02 %
		Type-II	(0⋆39, 5)	[1, 40]	0.6858	0.8478	NA
0.10	1082886	Best PC	(0⋆8, 1, 0⋆24, 3, 0⋆5, 1)	[34, 40]	0.9001	0.1843	100.00 %
		Worst PC	(0⋆20, 1, 0⋆3, 1, 0⋆12, 1, 0, 2)	[25, 40]	0.9000	0.3671	50.21 %
		Type-II	(0⋆39, 5)	[1, 40]	0.6858	0.8478	NA

Table A.61: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 50$ and $m = 6$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	605	Best PC	(38, 1, 0⋆3, 5)	[1, 6]	0.9900	0.4412	100.00 %
		Worst PC	(41, 3, 0⋆4)	[1, 6]	0.9983	0.8061	54.73 %
		Type-II	(0⋆5,44)	[1, 6]	0.2191	0.0980	NA
0.05	9090	Best PC	(0,36, 0⋆3, 8)	[2, 6]	0.9513	0.2956	100.00 %
		Worst PC	(36, 8, 0⋆4)	[1, 6]	0.9873	0.7983	37.03 %
		Type-II	(0⋆5,44)	[1, 6]	0.2191	0.0980	NA
0.10	42269	Best PC	(0,33, 0⋆3,11)	[2, 6]	0.9039	0.2402	100.00 %
		Worst PC	(32,12, 0⋆4)	[1, 6]	0.9711	0.7952	30.21 %
		Type-II	(0⋆5,44)	[1, 6]	0.2191	0.0980	NA

Table A.62: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 50$ and $m = 6$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	28	Best PC	(42, 0⋆4, 2)	[1, 6]	0.9900	0.6127	100.00 %
		Worst PC	(44, 0⋆5)	[1, 6]	0.9993	0.8170	75.00 %
		Type-II	(0⋆5,44)	[1, 6]	0.0070	0.0980	NA
0.05	215	Best PC	(0,41, 0⋆3, 3)	[2, 6]	0.9530	0.4804	100.00 %
		Worst PC	(42, 2, 0⋆4)	[1, 6]	0.9915	0.8088	59.39 %
		Type-II	(0⋆5,44)	[1, 6]	0.0070	0.0980	NA
0.10	825	Best PC	(38, 0⋆4, 6)	[1, 6]	0.9069	0.4085	100.00 %
		Worst PC	(40, 4, 0⋆4)	[1, 6]	0.9710	0.8039	50.81 %
		Type-II	(0⋆5,44)	[1, 6]	0.0070	0.0980	NA

Table A.63: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.10}$ with $n = 50$ and $m = 45$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1906884	Best PC	(4, 0⋆5, 1, 0⋆38)	[1, 11]	0.9901	0.2153	100.00 %
		Worst PC	(1, 1, 3, 0⋆42)	[1, 12]	0.9934	0.2372	90.78 %
		Type-II	(0⋆44, 5)	[1, 12]	0.9916	0.2157	99.83 %
0.05	1906884	Best PC	(2, 0⋆3, 1, 0⋆39, 2)	[1, 9]	0.9500	0.1653	100.00 %
		Worst PC	(0, 0, 1, 1, 0, 2, 1, 0⋆38)	[1, 10]	0.9775	0.1877	88.06 %
		Type-II	(0⋆44, 5)	[2, 11]	0.9569	0.1765	93.67 %
0.10	1906884	Best PC	(0⋆44, 5)	[2, 9]	0.9083	0.1373	100.00 %
		Worst PC	(5, 0⋆44)	[2, 9]	0.9231	0.1525	90.00 %
		Type-II	(0⋆44, 5)	[2, 9]	0.9083	0.1373	100.00 %

Table A.64: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.15}$ with $n = 50$ and $m = 45$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1906884	Best PC	(0, 3, 0⋆5, 1, 0⋆4, 1, 0⋆32)	[2, 14]	0.9900	0.2545	100.00 %
		Worst PC	(4, 0⋆10, 1, 0⋆33)	[1, 14]	0.9936	0.2783	91.44 %
		Type-II	(0⋆44, 5)	[2, 15]	0.9918	0.2549	99.84 %
0.05	1906884	Best PC	(0⋆44, 5)	[3, 13]	0.9558	0.1961	100.00 %
		Worst PC	(0, 5, 0⋆43)	[3, 13]	0.9677	0.2184	89.80 %
		Type-II	(0⋆44, 5)	[3, 13]	0.9558	0.1961	100.00 %
0.10	1906884	Best PC	(0⋆4, 1, 0⋆3, 1, 0, 2, 0⋆33, 1)	[4, 12]	0.9000	0.1626	100.00 %
		Worst PC	(3, 0⋆7, 1, 0⋆35, 1)	[2, 11]	0.9116	0.1888	86.09 %
		Type-II	(0⋆44, 5)	[4, 13]	0.9239	0.1765	92.12 %

Table A.65: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.25}$ with $n = 50$ and $m = 45$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1906884	Best PC	(0⋆6, 3, 0⋆37, 2)	[5, 20]	0.9901	0.3128	100.00 %
		Worst PC	(0⋆10, 1⋆5, 0⋆30)	[4, 20]	0.9916	0.3341	93.61 %
		Type-II	(0⋆44, 5)	[5, 21]	0.9916	0.3137	99.70 %
0.05	1906884	Best PC	(0⋆44, 5)	[7, 19]	0.9519	0.2353	100.00 %
		Worst PC	(0, 0, 1, 0, 1, 3, 0⋆39)	[7, 19]	0.9635	0.2642	89.06 %
		Type-II	(0⋆44, 5)	[7, 19]	0.9519	0.2353	100.00 %
0.10	1906884	Best PC	(0⋆16, 1, 0⋆27, 4)	[8, 18]	0.9004	0.1967	100.00 %
		Worst PC	(0⋆6, 5, 0⋆38)	[8, 18]	0.9219	0.2212	88.90 %
		Type-II	(0⋆44, 5)	[8, 19]	0.9260	0.2157	91.18 %

Table A.66: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.35}$ with $n = 50$ and $m = 45$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1906884	Best PC	(1, 0⋆21, 1, 0⋆21, 3)	[9, 26]	0.9900	0.3424	100.00 %
		Worst PC	(0, 1, 0, 1⋆3, 0, 1, 0⋆37)	[9, 26]	0.9931	0.3742	91.51 %
		Type-II	(0⋆44, 5)	[10, 28]	0.9914	0.3529	97.03 %
0.05	1906884	Best PC	(0⋆5, 1, 0⋆38, 4)	[11, 24]	0.9500	0.2607	100.00 %
		Worst PC	(0⋆6, 1, 0, 2, 2, 0⋆35)	[11, 24]	0.9650	0.2894	90.07 %
		Type-II	(0⋆44, 5)	[12, 26]	0.9557	0.2745	94.97 %
0.10	1906884	Best PC	(0⋆19, 1, 1, 0⋆23, 3)	[12, 23]	0.9001	0.2191	100.00 %
		Worst PC	(0⋆9, 2, 3, 0⋆34)	[13, 24]	0.9014	0.2462	89.00 %
		Type-II	(0⋆44, 5)	[13, 25]	0.9132	0.2353	93.12 %

Table A.67: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.50}$ with $n = 50$ and $m = 45$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1906884	Best PC	(0⋆31, 2, 0⋆12, 3)	[16, 34]	0.9900	0.3576	100.00 %
		Worst PC	(1, 1, 0, 1, 1, 0, 0, 1, 0⋆37)	[15, 33]	0.9927	0.3953	90.46 %
		Type-II	(0⋆44, 5)	[17, 36]	0.9910	0.3725	95.98 %
0.05	1906884	Best PC	(0⋆44, 5)	[18, 32]	0.9511	0.2745	100.00 %
		Worst PC	(0, 1, 0, 1, 2, 1, 0⋆39)	[17, 31]	0.9620	0.3076	89.24 %
		Type-II	(0⋆44, 5)	[18, 32]	0.9511	0.2745	100.00 %
0.10	1906884	Best PC	(0⋆19, 1, 1, 0⋆8, 1, 0⋆14, 2)	[20, 31]	0.9001	0.2313	100.00 %
		Worst PC	(1, 2, 0, 1, 1, 0⋆40)	[18, 30]	0.9210	0.2626	88.07 %
		Type-II	(0⋆44, 5)	[20, 32]	0.9081	0.2353	98.29 %

Table A.68: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.65}$ with $n = 50$ and $m = 45$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1906884	Best PC	(0⋆36, 1, 0⋆7, 4)	[24, 41]	0.9901	0.3394	100.00 %
		Worst PC	(0⋆4, 1, 0⋆5, 1⋆3, 0⋆12, 1, 0⋆19)	[21, 38]	0.9919	0.3831	88.60 %
		Type-II	(0⋆44, 5)	[23, 41]	0.9914	0.3529	96.15 %
0.05	1906884	Best PC	(0⋆4, 1, 0⋆39, 4)	[26, 39]	0.9500	0.2606	100.00 %
		Worst PC	(0⋆13, 1⋆4, 0, 0, 1, 0⋆25)	[23, 36]	0.9524	0.2984	87.31 %
		Type-II	(0⋆44, 5)	[27, 41]	0.9537	0.2745	94.92 %
0.10	1906884	Best PC	(0⋆34, 1, 0⋆9, 4)	[27, 38]	0.9006	0.2196	100.00 %
		Worst PC	(0⋆13, 1, 1, 0, 0, 1, 0, 1, 1, 0⋆24)	[26, 37]	0.9240	0.2543	86.35 %
		Type-II	(0⋆44, 5)	[26, 38]	0.9132	0.2353	93.33 %

Table A.69: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.75}$ with $n = 50$ and $m = 45$ (total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1906884	Best PC	(0⋆33, 1, 0⋆10, 4)	[30, 45]	0.9901	0.3076	100.00 %
		Worst PC	(0⋆19, 1, 0⋆23, 4, 0)	[29, 45]	0.9951	0.3647	84.34 %
		Type-II	(0⋆44, 5)	[29, 45]	0.9903	0.3137	98.05 %
0.05	1906884	Best PC	(0⋆44, 5)	[32, 44]	0.9519	0.2353	100.00 %
		Worst PC	(0⋆28, 1, 2, 0, 0, 1, 1, 0⋆11)	[30, 41]	0.9606	0.2784	84.53 %
		Type-II	(0⋆44, 5)	[32, 44]	0.9519	0.2353	100.00 %
0.10	1906884	Best PC	(0⋆41, 1, 0, 0, 4)	[33, 43]	0.9029	0.1985	100.00 %
		Worst PC	(0⋆21, 1, 1, 0, 1, 1, 0⋆10, 1, 0⋆8)	[30, 40]	0.9198	0.2382	83.34 %
		Type-II	(0⋆44, 5)	[33, 44]	0.9255	0.2157	92.05 %

Table A.70: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.85}$ with $n = 50$ and $m = 45$

(total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme (**R**)	CI	Actual CL	CI Mass	Efficiency
0.01	1259139	Best PC	(0⋆36, 5, 0⋆8)	[36, 45]	0.9903	0.2636	100.00 %
		Worst PC	(0⋆8, 1, 0⋆6, 1, 0⋆6, 1, 0⋆15, 1, 1, 0⋆5)	[23, 45]	0.9900	0.5098	51.71 %
		Type-II	(0⋆44, 5)	[1, 45]	0.7806	0.8627	NA
0.05	1898085	Best PC	(0⋆18, 3, 0⋆23, 1, 0, 1)	[36, 45]	0.9500	0.2055	100.00 %
		Worst PC	(0⋆22, 1, 0⋆11, 1, 0⋆8, 3, 0)	[29, 45]	0.9500	0.3725	55.17 %
		Type-II	(0⋆44, 5)	[1, 45]	0.7806	0.8627	NA
0.10	1906588	Best PC	(2, 0⋆42, 1, 2)	[37, 45]	0.9016	0.1685	100.00 %
		Worst PC	(0⋆24, 1, 0⋆17, 1, 1, 2)	[32, 45]	0.9002	0.2778	60.65 %
		Type-II	(0⋆44, 5)	[1, 45]	0.7806	0.8627	NA

Table A.71: Optimal PC Schemes for Non-Parametric CIs of $\xi_{0.90}$ with $n = 50$ and $m = 45$

(total 1906884 progressive Type-II censoring schemes examined)

α	Count	Category	Censoring Scheme ($\mathbf{R}$)	CI	Actual CL	CI Mass	Efficiency
0.01	10	Best PC	(5, 0⋆44)	[33, 45]	0.9909	0.2614	100.00 %
		Worst PC	(0⋆9, 5, 0⋆35)	[32, 45]	0.9901	0.2903	90.06 %
		Type-II	(0⋆44, 5)	[1, 45]	0.3839	0.8627	NA
0.05	830821	Best PC	(5, 0⋆44)	[37, 45]	0.9596	0.1743	100.00 %
		Worst PC	(0⋆8, 1, 0⋆17, 2, 0⋆8, 2, 0⋆9)	[29, 45]	0.9500	0.3916	44.51 %
		Type-II	(0⋆44, 5)	[1, 45]	0.3839	0.8627	NA
0.10	1627876	Best PC	(4, 0⋆43, 1)	[38, 45]	0.9157	0.1492	100.00 %
		Worst PC	(0⋆8, 1, 0⋆10, 1, 1, 0⋆18, 1, 0⋆3, 1, 0)	[30, 45]	0.9000	0.3509	42.51 %
		Type-II	(0⋆44, 5)	[1, 45]	0.3839	0.8627	NA

Appendix B

Partial R Codes for Numerical Study

The programs written in R are provided here for conducting the numerical studies in Section 5.2 to search for the optimal progressive Type-II censoring schemes. For an illustration purpose, the values of $n = 15$ and $m = 8$ are used throughout the programs.

```
n <- 15             # total sample size
m <-  8             # number of observations to be made

p   <- seq(0.05, 0.95, by=0.05)
alpha <- c(0.01, 0.05, 0.10)

time1 <- Sys.time()
```

```
Rs <- NULL
for (R1 in 0:(n-m))
for (R2 in 0:(n-m-R1))
for (R3 in 0:(n-m-R1-R2))
for (R4 in 0:(n-m-R1-R2-R3))
for (R5 in 0:(n-m-R1-R2-R3-R4))
for (R6 in 0:(n-m-R1-R2-R3-R4-R5))
for (R7 in 0:(n-m-R1-R2-R3-R4-R5-R6)) {
     R8 <-    n-m-R1-R2-R3-R4-R5-R6-R7
     Rs <- rbind(Rs, c(R1, R2, R3, R4, R5, R6, R7, R8)) }

time2 <- Sys.time()
rtime <- time2 - time1; rtime




time1 <- Sys.time()

matW  <- function(n, m, R) {
   W  <- c(1, rep(0, n-1))
   T  <- diag(rep(1, n-1))
   Ni <- n - 1

   for (i in 1:(m-1)) {
      ni <- Ni - R[i]

      H  <- matrix(0, nrow=ni, ncol=Ni)

      for (r in 1:ni)
```

```
      for (s in r:(r + R[i]))
         H[r,s] <- choose(s-1, r-1) * choose(Ni-s, ni-r) / choose(Ni, ni)

    HT <- H %*% T

    W  <- rbind(W, c(rep(0, i), HT[1,]))
    if ((i != (m-1)) | (R[m] != 0)) T <- HT[2:ni, 2:(n-i)]
    Ni <- ni - 1 }
  W }

b <- NULL
for (j in 1:length(p))
  b <- cbind(b, pbinom(0:(n-1), n, p[j]))

arr <- NULL
for (indx in 1:(dim(Rs)[1])) {
  R <- Rs[indx,]

  W <- matW(n, m, R)
  a <- W %*% b

  as <- 1:m + cumsum(rev(R))
  al <- as / (as + 1)
  EU <- 1 - cumprod(rev(al))     # equivalently, EU <- W %*% 1:n / (n+1)

  rows <- NA
  for (i in 1:length(p))
    for (j in 1:length(alpha))
      rows <- cbind(rows, p=p[i], opt(a[,i], alpha[j], m, EU))
```

```
   arr <- rbind(arr, rows) }
   arr <- arr[,-1]

time2 <- Sys.time()
ctime <- time2 - time1; ctime
```

Appendix C

Partial FORTRAN Codes for Numerical Study

The programs written in FORTRAN are given here for conducting the numerical studies in Section 5.2 to search for the optimal progressive Type-II censoring schemes. Again, for illustration, the values of $n = 15$ and $m = 8$ are used throughout the programs.

```
PROGRAM MAIN1R

INTEGER    N, M
PARAMETER (N = 15, M = 8)
INTEGER    R1, R2, R3, R4, R5, R6, R7, R8

INTEGER    OUT
REAL       ETIME, ELAPSED(2)          ! user time and system time

OUT = 10
OPEN (OUT, FILE='N15M08R.txt', STATUS='NEW')
```

```
DO R1 = 0, (N-M)
DO R2 = 0, (N-M-R1)
DO R3 = 0, (N-M-R1-R2)
DO R4 = 0, (N-M-R1-R2-R3)
DO R5 = 0, (N-M-R1-R2-R3-R4)
DO R6 = 0, (N-M-R1-R2-R3-R4-R5)
DO R7 = 0, (N-M-R1-R2-R3-R4-R5-R6)
   R8 =     N-M-R1-R2-R3-R4-R5-R6-R7
   WRITE (OUT,*) R1, R2, R3, R4, R5, R6, R7, R8
ENDDO
ENDDO
ENDDO
ENDDO
ENDDO
ENDDO
ENDDO

CLOSE (OUT)
WRITE (*,*) 'RTIME=', ETIME(ELAPSED)

STOP
END


SUBROUTINE BCALC (N, LENP, P, B)

INTEGER          N, LENP
DOUBLE PRECISION P(LENP), B(N,LENP)

INTEGER          I, J
DOUBLE PRECISION PBINOM
EXTERNAL         PBINOM

DO I = 1, N
   DO J = 1, LENP
      B(I,J) = PBINOM(I-1, N, P(J))
```

```
         ENDDO
      ENDDO

      RETURN
      END



      SUBROUTINE MATW (N, M, R, W)

      INTEGER          N, M, R(M)
      DOUBLE PRECISION W(M,N)

      INTEGER          I, J, NBIG, NSMALL, Q, S
      DOUBLE PRECISION H(N-1,N-1), T(N-1,N-1), HT(N-1,N-1)
      DOUBLE PRECISION CHOOSE
      EXTERNAL         CHOOSE, IDMAT, ZEROMAT, MATMUL, SUBMAT

      W(1,1) = 1.0D0

      DO I = 2, (N-1)
         W(1,I) = 0.0D0
      ENDDO

      CALL IDMAT (N-1, T)

      NBIG = N - 1

      DO I = 1, (M-1)
         NSMALL = NBIG - R(I)

         CALL ZEROMAT (NSMALL, NBIG, H, N-1)

         DO Q = 1, NSMALL
            DO S = Q, (Q + R(I))
               H(Q,S) = CHOOSE(S-1, Q-1) * CHOOSE(NBIG-S, NSMALL-Q)
     &                                  / CHOOSE(NBIG,   NSMALL)
            ENDDO
```

```
      ENDDO

      CALL MATMUL (NSMALL, NBIG, N-I, H, T, HT, N-1, N-1)

      DO J = 1, I
         W(I+1,J) = 0.0D0
      ENDDO

      DO J = 1, (N-I)
         W(I+1,I+J) = HT(1,J)
      ENDDO

      IF ((I .NE. (M-1)) .OR. (R(M) .NE. 0)) THEN
         CALL SUBMAT (2, NSMALL, 2, N-I, HT, T, N-1)
      ENDIF

      NBIG = NBIG - 1
   ENDDO

   RETURN
   END



   SUBROUTINE EUCALC (M, R, EU)

   INTEGER          M, R(M)
   DOUBLE PRECISION EU(M)

   INTEGER          AS, I
   DOUBLE PRECISION AL(M), CUMPR

   AS = 0

   DO I = 1, M
      AS = AS + 1 + R(M-I+1)
      AL(I) = DBLE(AS) / DBLE(AS + 1)
   ENDDO
```

```
      CUMPR = 1.0D0

      DO I = 1, M
         CUMPR = CUMPR * AL(M-I+1)
         EU(I) = 1.0D0 - CUMPR
      ENDDO

      RETURN
      END



      PROGRAM MAIN2C

      INTEGER          N, M
      PARAMETER       (N = 15, M = 8)
      INTEGER          R(M)

      INTEGER          LENP, LENA, LENPA
      PARAMETER       (LENP = 19, LENA = 3, LENPA = LENP * LENA)
      DOUBLE PRECISION P(LENP), ALPHA(LENA)

      INTEGER          I, J, K, L, U
      DOUBLE PRECISION W(M,N), B(N,LENP), A(M,LENP), EU(M), CL, MASS
      LOGICAL          EXIST
      EXTERNAL         FILEOC, BCALC, MATW, MATMUL, EUCALC, OPT

      INTEGER          IN, OUT(LENPA)
      REAL             ETIME, ELAPSED(2)        ! user time and system time

      DATA P     /0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35,
     &            0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70,
     &            0.75, 0.80, 0.85, 0.90, 0.95/,
     &     ALPHA /0.01, 0.05, 0.10/

      CALL FILEOC (IN, OUT, LENPA, .TRUE.)
```

```
      K = 1

      DO I = 1, LENP
         DO J = 1, LENA
            WRITE (OUT(K),'(2F6.2/)') P(I), ALPHA(J)

            K = K + 1
         ENDDO
      ENDDO

      CALL BCALC (N, LENP, P, B)

      DO
         READ (IN,*,END=100) R

         CALL MATW (N, M, R, W)
         CALL MATMUL (M, N, LENP, W, B, A, M, N)
         CALL EUCALC (M, R, EU)

         K = 1

         DO I = 1, LENP
            DO J = 1, LENA
               CALL OPT (A(1,I), ALPHA(J), M, EU, EXIST, L, U, CL, MASS)
               WRITE (OUT(K),*)                R, EXIST, L, U, CL, MASS

               K = K + 1
            ENDDO
         ENDDO
      ENDDO

  100 CALL FILEOC (IN, OUT, LENPA, .FALSE.)
      WRITE (*,*) 'CTIME=', ETIME(ELAPSED)

      STOP
      END
```