NONLINEAR DEVICE CHARACTERIZATION
AND STATISTICAL MODELING

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OSA-96-MT-5-V

March 18, 1996

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presented at

WORKSHOP ON NONLINEAR CAD
1996 IEEE MTT-S Int. Microwave Symposium, San Francisco, CA, June 17, 1996

Introduction

simulation of linear/nonlinear circuits requires accurate linear/nonlinear device models

both deterministic and statistical models are needed to address increasing sophistication of design methodology

deterministic

performance-driven design
cost functions
variable tolerance worst-case design

statistical

fixed tolerance yield-driven design
correlated tolerances
variable tolerance cost-driven design

CAD goal: first-pass success design

examples for this presentation were produced by OSA's software system HarPE™

Overview

nonlinear device characterization

- device modeling
- parameter extraction
- ℓ_1 and Huber data fitting
- Space Mapping™ model alignment

statistical modeling

- multi-device parameter extraction
- ℓ_2 and Huber statistical postprocessing
- direct CPD fitting
- model verification

Device Models

local vs. global models

equivalent circuit models (ECMs)

high computational efficiency

interpolation models

physics-based models (PBMs)

relate the circuit elements to the device physics based on the simplified analytical solution of device equations

slower but, in general, more accurate than ECMs

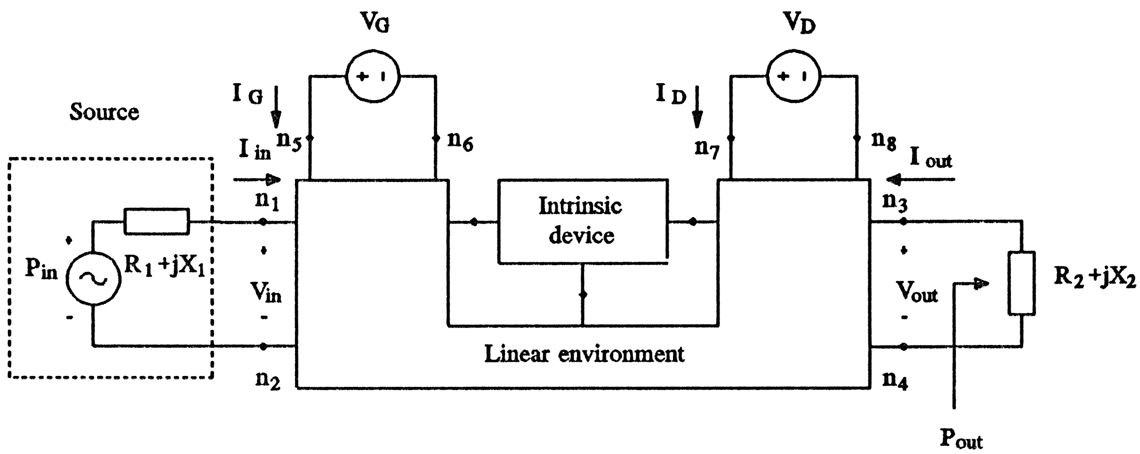
physical models (PMs)

based on the numerical solution of fundamental device equations

the most accurate but computationally intensive

both PBMs and PMs are capable of performance prediction, permitting device optimization

Measurement/Simulation Setup for Parameter Extraction



Sequential Parameter Extraction

conventional methods for parameter extraction are based on DC and small-signal measurement data fitting

model parameters are determined from DC, cold and hot device measurements in a sequential manner

the already extracted parameters are fixed when identifying the remaining parameters

specific measurements, approaches, deembeding formulas, etc., are highly model dependent

Integrated DC/Small-Signal Parameter Extraction
(*Bandler et al., 1988*)

taking into account the relationship between the DC and small-signal parameters

combining DC and small-signal data into one optimization problem using multi-bias and multi-frequency measurements

substantial improvement of uniqueness and reliability

Multi-Bias *S*-Parameter Data for Parameter Extraction

```
PARAMETER VG = 0  VD = 4
FORMAT   FREQ(GHZ)      MS11    PS11    MS21    PS21   ...
         2.0            0.9546  -46.72  4.0405  145.54  ...
         3.0            0.9392  -66.98  3.6149  129.27  ...
          ...

PARAMETER VG = -1.74  VD = 4
FORMAT   FREQ(GHZ)      MS11    PS11    MS21    PS21   ...
         2.0            0.9585  -36.75  3.1389  150.53  ...
          ...

PARAMETER VG = -3.1  VD = 4
FORMAT   FREQ(GHZ)      MS11    PS11    MS21    PS21   ...
         2.0            0.9614  -32.46  2.5494  152.25  ...
          ...

FORMAT     VG    VD    ID
           0.0   4.0   0.177
          -1.74  4.0   0.092
          -3.10  4.0   0.037
```

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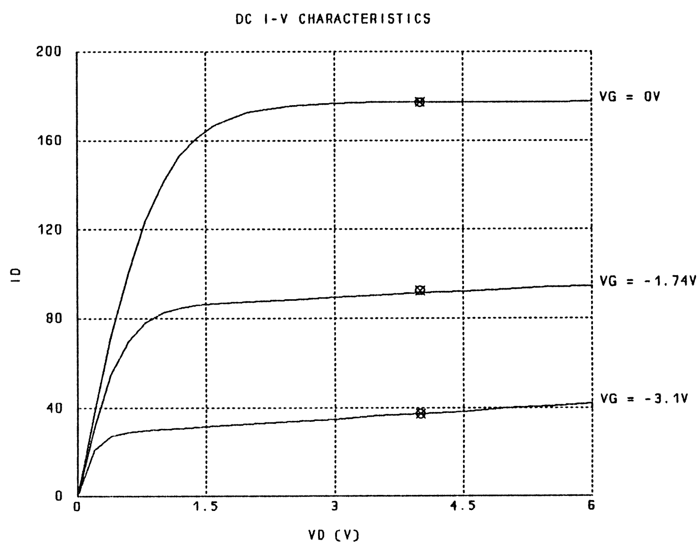
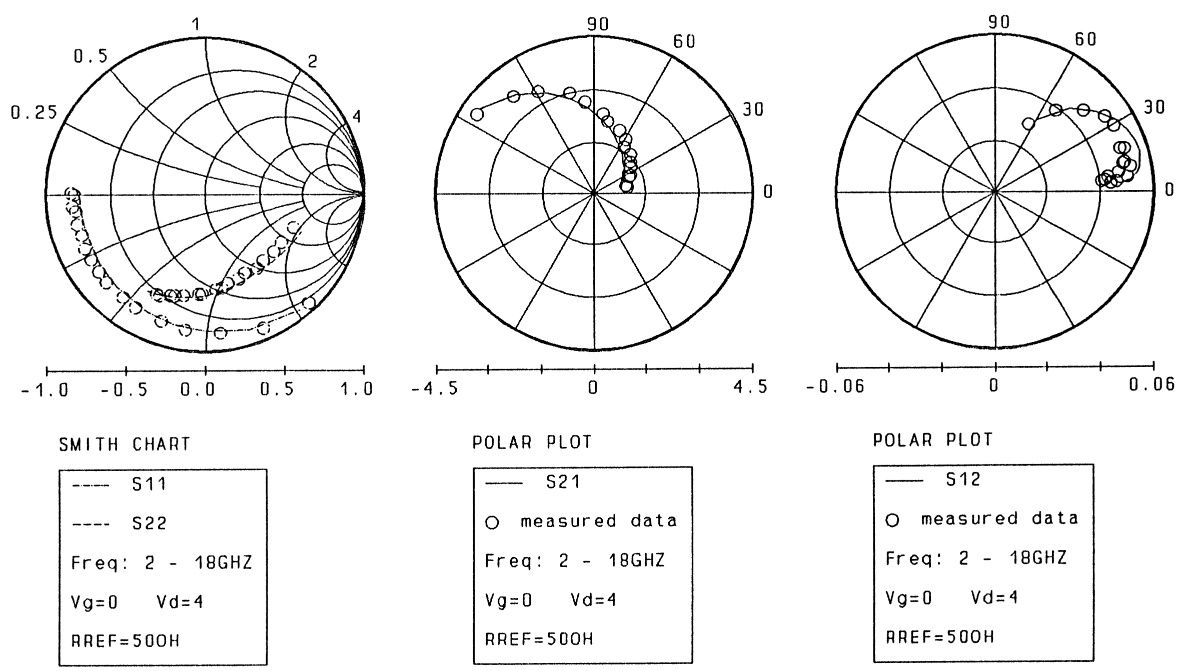
DC and *S*-Parameter Match After Optimization





model used: modified Materka and Kacprzak

Large-Signal Device Parameter Extraction Using Harmonics: Spectrum Data Fitting
(*Bandler et al., 1989*)

device is excited under practical (large-signal) working conditions

spectrum measurements are taken at different bias, input power and fundamental frequency combinations

parameters are extracted by optimizing the model response to match the spectrum measurements

harmonic balance simulation technique for nonlinear circuit simulation in the frequency domain is used

nonlinear adjoint sensitivity analysis for gradient computation of nonlinear circuit responses (FAST)

the first true nonlinear large-signal device model parameter extraction approach

extended to large-signal waveform data fitting (*Werthof, van Raay and Kompa, 1993*)

Harmonic Data for Parameter Extraction

(Texas Instruments, 1989)

PARAMETER VG = -0.372 VD = 2 FREQ = 6GHZ

FORMAT PIN(DBM)	POUT1(DBM)	POUT2(DBM)	POUT3(DBM)	ID0(MA)
+10.0	+15.1	+2.4	-5.7	38.9
+5.0	+13.0	-5.2	-11.9	42.3
0.0	+9.6	-19.5	-27.3	44.3
-5.0	+4.9	-32.4	-45.6	44.7
-10.0	0.0	-42.7	-60.1	44.9
-15.0	-5.2	-52.8	-99.9	45.1

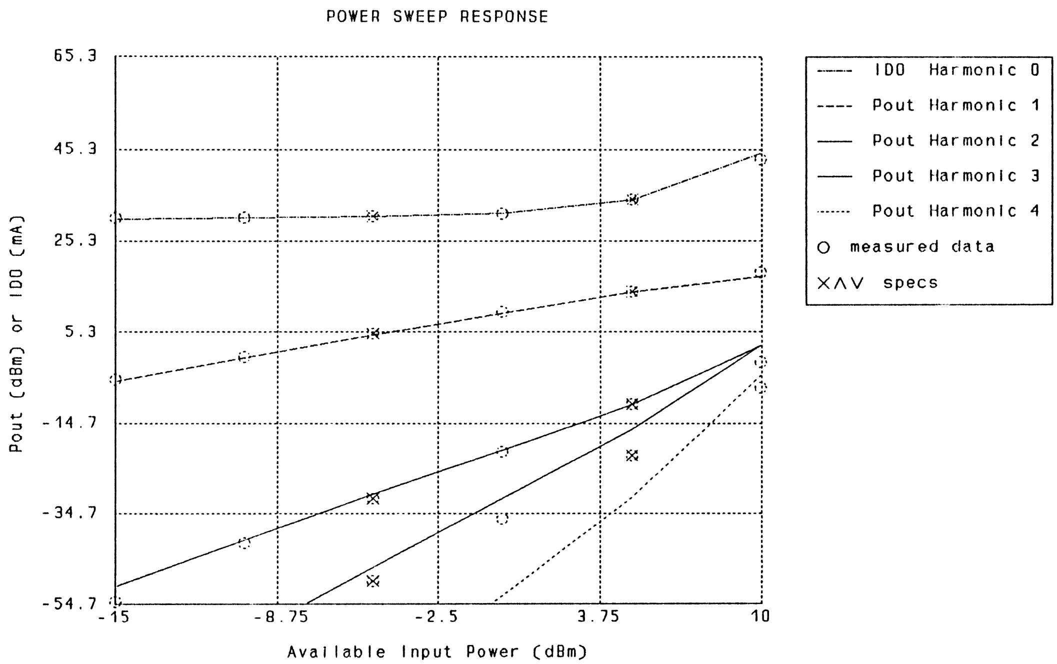
PARAMETER VG = -0.673 VD = 4 FREQ = 6GHZ

FORMAT PIN(DBM)	POUT1(DBM)	POUT2(DBM)	POUT3(DBM)	ID0(MA)
+10.0	+18.1	-1.5	-7.3	42.8
+5.0	+13.9	-10.7	-22.1	34.0
0.0	+9.5	-21.2	-36.1	31.0
-5.0	+4.6	-31.5	-49.9	30.2
-10.0	-0.3	-41.4	-62.1	30.0
-15.0	-5.5	-54.4	-99.9	30.0

PARAMETER VG = -1.073 VD = 6 FREQ = 6GHZ

FORMAT PIN(DBM)	POUT1(DBM)	POUT2(DBM)	POUT3(DBM)	ID0(MA)
+10.0	+16.1	+1.9	-10.2	31.2
+5.0	+11.7	-5.8	-20.6	21.3
0.0	+7.3	-14.8	-33.5	17.1
-5.0	+2.4	-24.6	-47.8	15.6
-10.0	-2.6	-34.4	-61.1	15.1
-15.0	-7.8	-46.9	-99.9	15.0

Power Spectrum Match After Optimization



model used: Curtice symmetrical cubic formulas

implemented as a user-defined model

The ℓ_1 Norm

$$\sum_{j=1}^{m} |f_j(\boldsymbol{\phi})|$$

f_j represent error functions

The Huber Function (*Huber, 1981*)

$$\rho_k(f) = \begin{cases} f^2/2 & \text{if } |f| \leq k \\ k|f| - k^2/2 & \text{if } |f| > k \end{cases}$$

$k > 0$ is a threshold separating "large" and "small" errors

the definition of ρ_k ensures a smooth transition at k

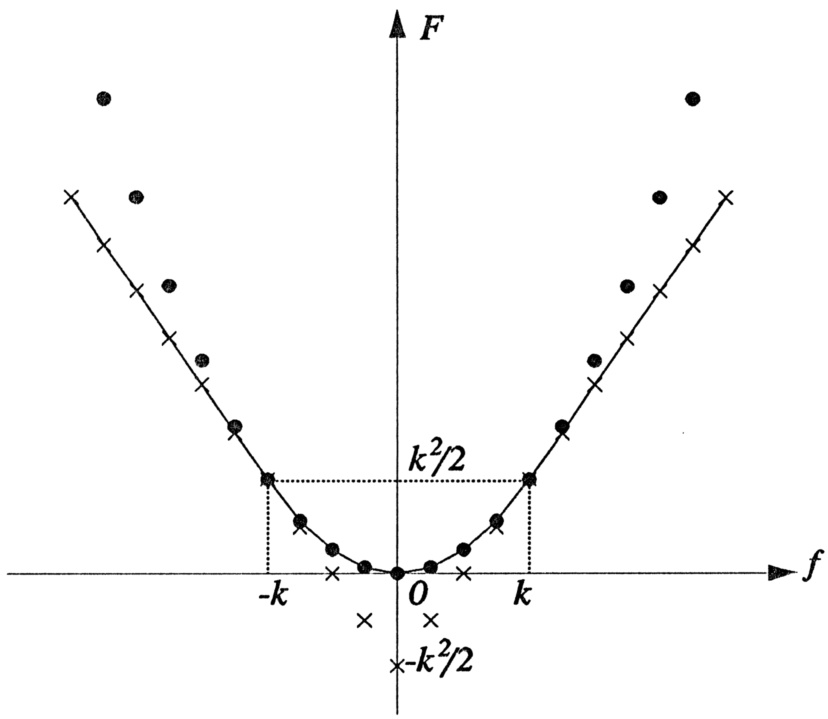
The Huber Norm

$$\sum_{j=1}^{m} \rho_k(f_j(\boldsymbol{\phi}))$$

a hybrid of the ℓ_2 and the ℓ_1 norms

Huber Function as a Hybrid of ℓ_1 and ℓ_2

the Huber, ℓ_1 and ℓ_2 objective functions in the one-dimensional case

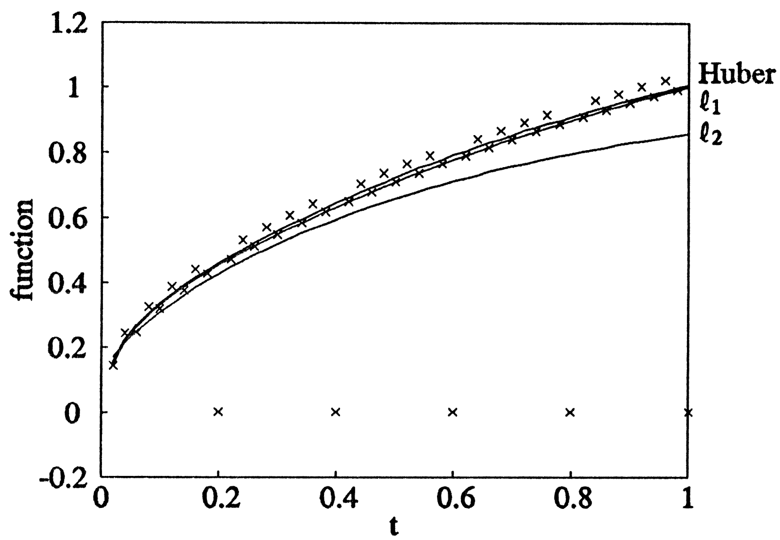


the large errors are treated in the ℓ_1 sense and the small errors are measured in terms of least squares

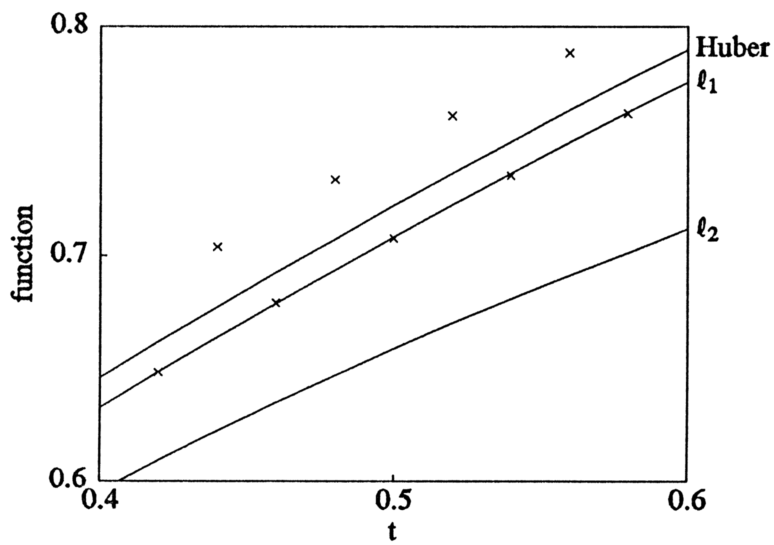
by selecting k we can control the proportion of errors treated in the ℓ_1 or ℓ_2 sense

ℓ_1, ℓ_2 and Huber Data Fitting

ℓ_1, ℓ_2 and Huber solutions for data fitting in the presence of large and small errors



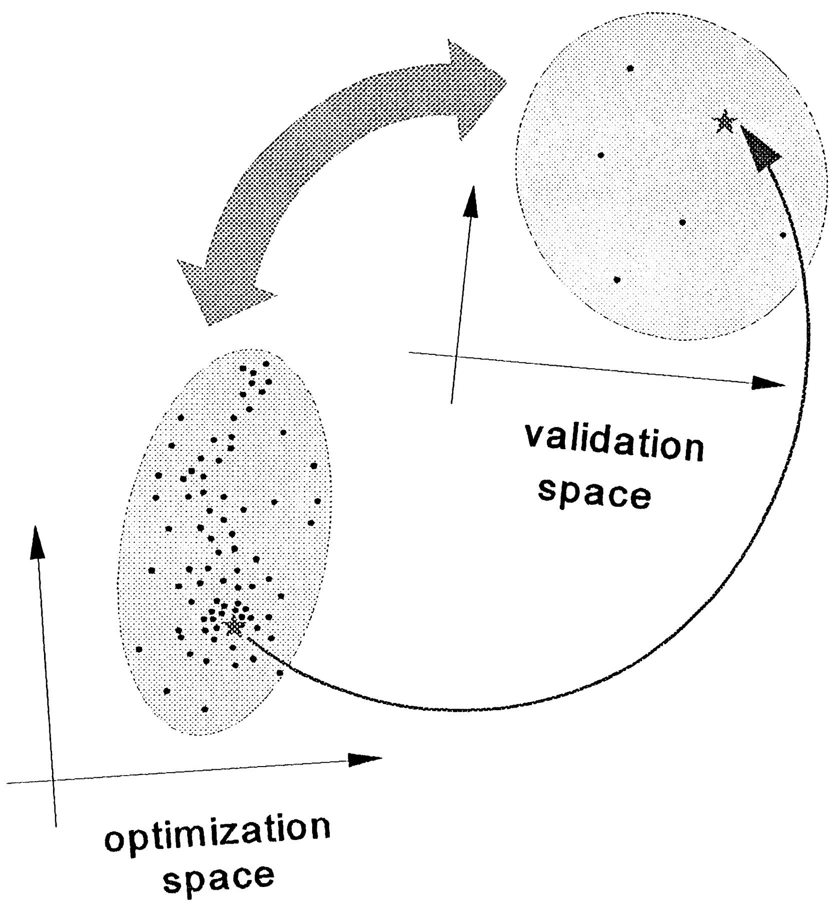
enlarged view



Space Mapping for Physical Models

using PBMs for fast optimization

using PMs for accurate validation



Statistical Device Modeling

random variations in the manufacturing environment result in complicated distributions and correlations of device responses

statistical modeling is a prerequisite for statistical analysis and yield optimization (design centering)

device model types for statistical modeling

- equivalent circuit models
- physics-based and physical models
- measurement databases

statistical models are determined from multi-device measurements

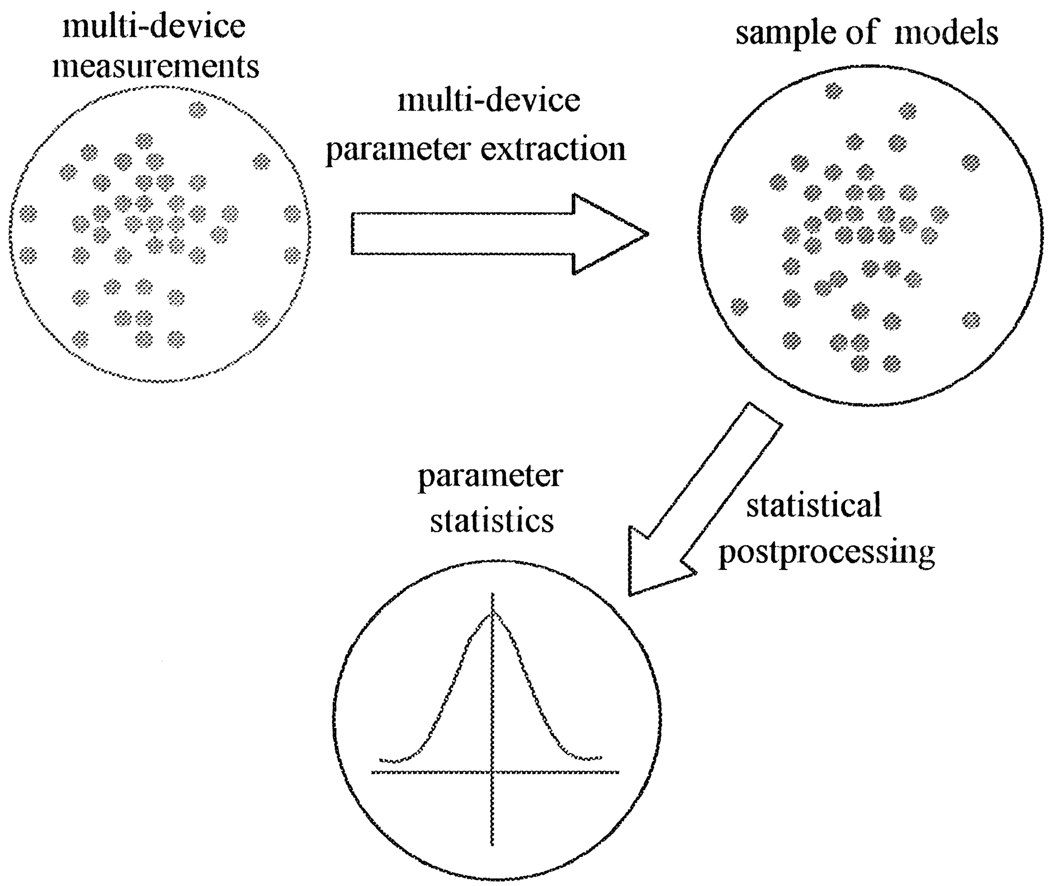
indirect statistical modeling

- parameter extraction/postprocessing

direct statistical modeling

- cumulative probability distribution (CPD) fitting
- histogram fitting

Indirect Statistical Modeling



first, we extract model parameters for individual devices

then the sample of model parameters is postprocessed to estimate the statistics

Sample of Device Models

SAMPLE
FORMAT

INDEX	T(PS)	GM	C1(PF)	CDG(PF)	GDS
1	3.63999	0.050866	0.0514826	0.0490503	0.00754406
2	3.55698	0.0486995	0.045284	0.0485504	0.00666363
3	3.58932	0.0497634	0.0479207	0.0483223	0.00730487
4	3.62658	0.0489517	0.0481039	0.0498116	0.00665159
...					
100	3.46184	0.049053	0.0452641	0.0464511	0.00727283

END

Consolidated Statistical Model

T: 3.50406PS {Normal Sigma=2.69% Correlation=CORMAT[1]
DDF= 5 5 6 14 18 10 16 12 7 7}

GM: 0.0490743 {Normal Sigma=2.28% Correlation=CORMAT[2]
DDF= 1 3 13 15 17 14 12 18 5 2}

Statistical Estimation

the error functions to estimate mean values

$$f_j(\bar{\phi}) = \bar{\phi} - \phi^j$$

the error functions to estimate standard deviations

$$f_j(V_\phi) = V_\phi - (\phi^j - \bar{\phi})^2$$

where

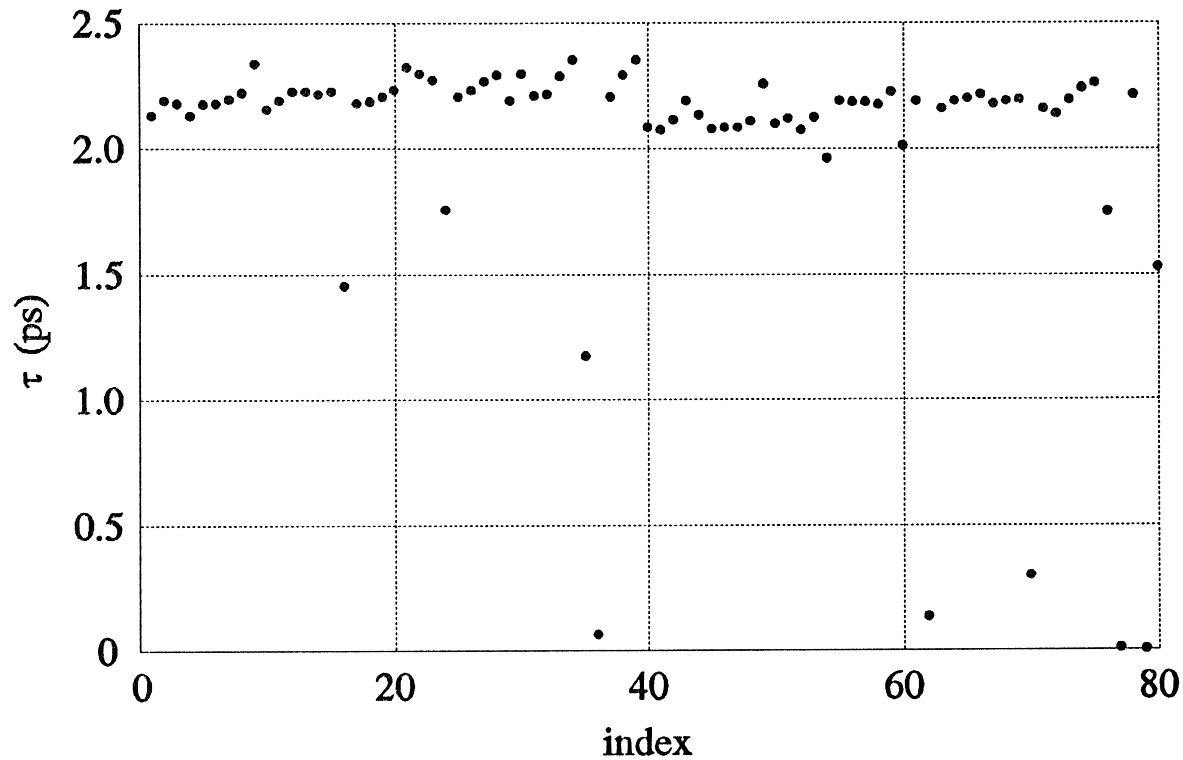
ϕ^j	the extracted value of a parameter of the jth device
j	1, 2, ..., N
N	the total number of devices
V_ϕ	the estimated variance from which we can calculate the standard deviation σ_ϕ

we normally apply least-squares estimators

wild points severely degrade the least-squares estimates

the Huber function can be used as an automatic robust statistical estimator in place of least-squares estimators

Data Containing Wild Points



run chart of the extracted FET time-delay τ; a few abnormal values in the data due to faulty devices and/or gross measurement errors

in our work using the ℓ_2 estimator these wild points had to be manually excluded

applying Huber estimators to the same data we obtain similar results but without excluding any points

Statistical Modeling Using Huber Estimator

ESTIMATED STATISTICS OF SELECTED FET PARAMETERS

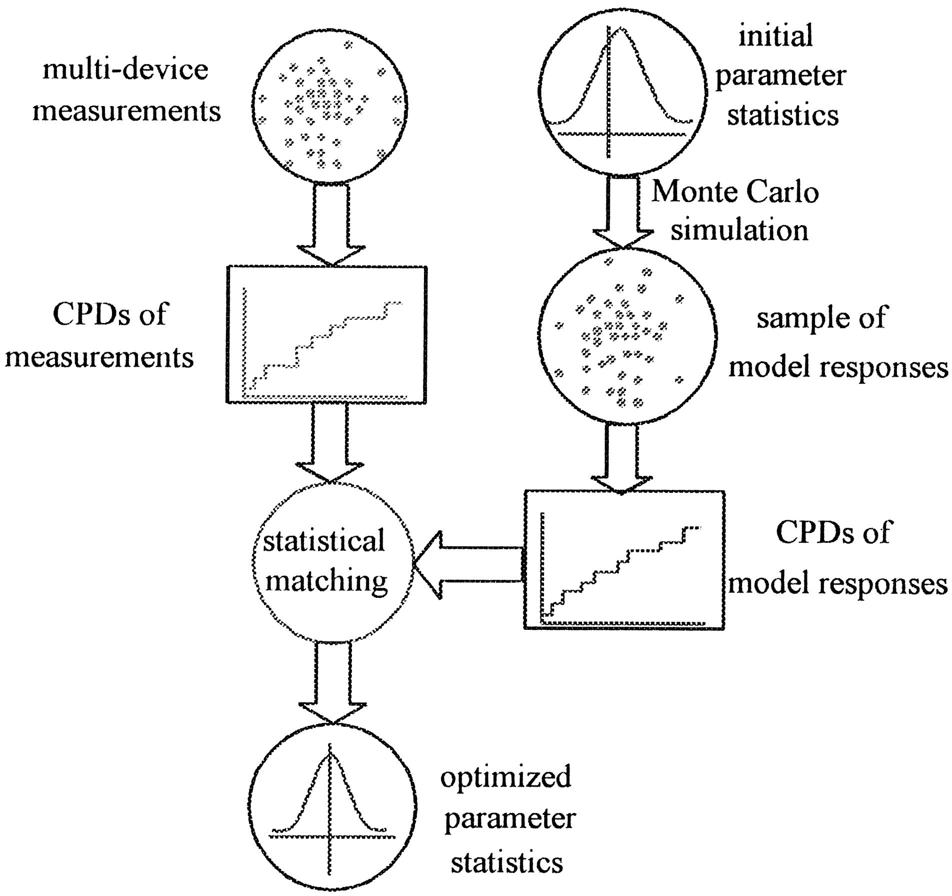
Parameter	$\bar{\phi}(\ell_2)$	$\bar{\phi}(H)$	$\bar{\phi}(\ell_2^*)$	$\sigma_\phi(\ell_2)$	$\sigma_\phi(H)$	$\sigma_\phi(\ell_2^*)$
L_G(nH)	0.04387	0.03464	0.03429	94.6%	21.8%	17.4%
G_{DS}(1/KΩ)	1.840	1.820	1.839	28.6%	6.3%	4.9%
I_{DSS}(mA)	47.36	47.53	47.85	14.0%	12.7%	11.3%
τ(ps)	2.018	2.154	2.187	26.3%	5.8%	3.4%
C_{10}(pF)	0.3618	0.3658	0.3696	8.2%	4.6%	3.5%
K_1	1.2328	1.231	1.233	15.5%	10.8%	8.7%

$\bar{\phi}$ denotes the mean and σ_ϕ the standard deviation
H denotes Huber estimates
ℓ_2^* denotes ℓ_2 estimates after 11 abnormal data sets are manually excluded

ℓ_2 and Huber estimates of the statistics for selected model parameters

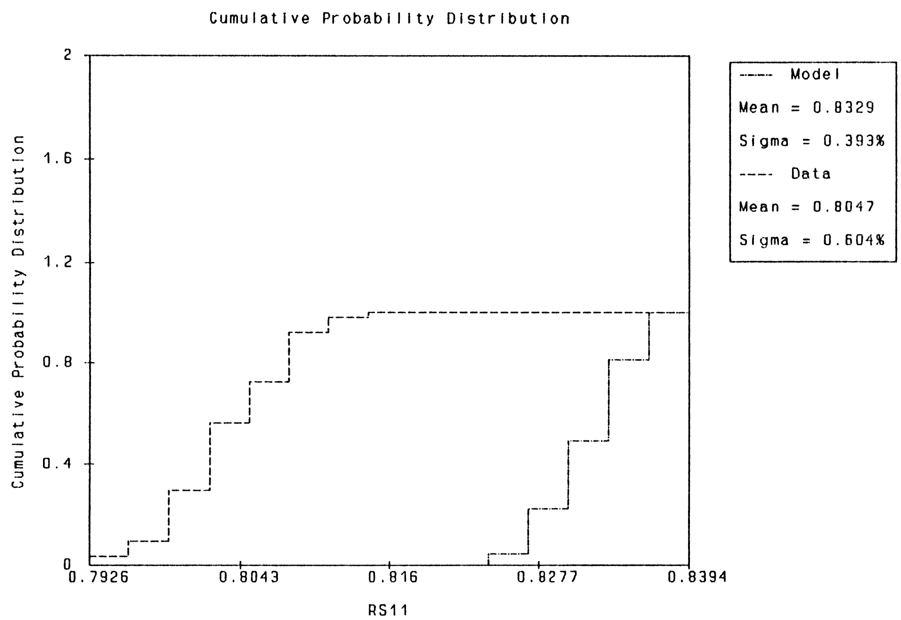
Huber estimator does not require manual manipulation of the data and is more appropriate when there are data points which cannot be clearly classified as abnormal

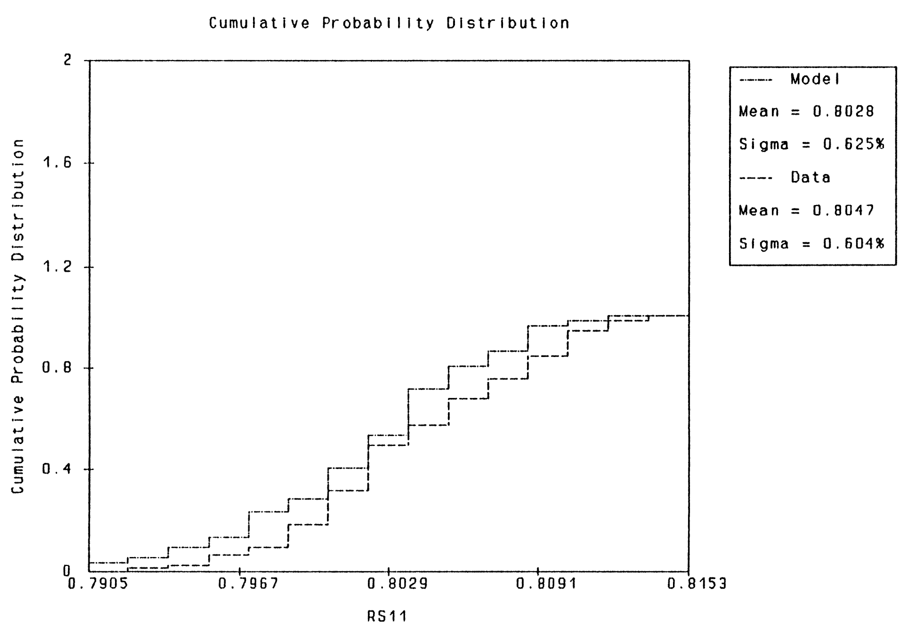
Direct Statistical Modeling



parameter statistics are determined directly by optimization

CPD Match Before and After Optimization





Data Alignment and Model Verification

data alignment

- the measurement conditions may vary for different device outcomes

- statistical modeling requires identical measurement conditions for all device outcomes

- measurement data may need to be preprocessed and aligned for statistical modeling

statistical model verification

- comparing the statistics of the model responses generated by Monte Carlo simulation with the statistics of the measurement data

- checking consistency between the yield predicted by the statistical model and the yield estimated from the measurement data

Concluding Remarks

deficiencies in parameter extraction techniques may include nonuniqueness, wild solution values

nonlinear device characterization needs to address the intended operation of the device

nondestructive device measurements and corresponding techniques must address characterization of difficult to model phenomena

the Huber approach is worth to be promoted for both parameter extraction and robust statistical modeling

Space Mapping technique promises practicality of exploiting physical models in circuit-level CAD

uniqueness of parameter extraction in indirect statistical modeling must be carefully monitored

direct statistical modeling needs to be extended to handle nonstandard distributions and correlations