DEVICE STATISTICAL MODELING AND VERIFICATION

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ABSTRACT

Optimization Systems Associates Inc. (OSA) pioneered and introduced the commercial microwave CAD community to statistical modeling and yield-driven design. Following almost a decade of algorithm developments we now present novel, sophisticated techniques for statistical modeling of active devices: (1) indirect, based on multi-device parameter extraction and statistical postprocessing of the resulting sample of models, and (2) direct, where the statistics of the model parameters are determined through matching the statistical distribution of the model to that of the data. Statistical models thus created reflect the random variations of the device parameters and are suitable for Monte Carlo simulations and yield-driven design of microwave circuits containing such uncertain devices.

INTRODUCTION

In integrated circuit manufacturing, fabricated circuits and devices exhibit parameter values deviating randomly from their nominal (or designed) values. These random variations result in statistical spreads of circuit responses and directly affect production yield and cost. Since all the active and passive components are fabricated on a common semi-insulating substrate, postproduction tuning is restricted and device replacement is not feasible. Therefore, yield and cost analysis and optimization are becoming widely accepted as indispensable ingredients of circuit CAD methodology.

Statistical modeling is a prerequisite for yield and cost analysis, and consequently optimization. In statistical modeling we determine a device model whose parameters are described as random variables. The distribution of the model responses due to random variations of the model parameters must reflect the actual distribution of device responses. The latter is characterized by multi-device measurement data (measurements taken on a number of supposedly identical devices). Therefore, statistical modeling is a process of matching the statistical distribution of the model to that of the measurement data. The advanced statistical modeling methods described here are based on OSA's pioneering work [1-6].

In this paper we present two techniques for statistical modeling of active devices. The first, a two-stage, indirect method is based on multi-device parameter extraction followed by statistical postprocessing of the resulting sample of models. In the second, direct, technique the statistics of the model parameters are determined through direct matching of statistical distributions of the model and of the data. Matching of either cumulative probability distribution (CPD) functions or histograms can be employed [1,2]. The two techniques for statistical modeling have been implemented in OSA's HarPE [1], a powerful parameter extraction and device characterization CAD software system. An example of various graphical plots generated by HarPE is shown in Fig. 1, including DC I-V curves, the Smith charts and polar plots of S parameters, histograms and a Monte Carlo sweep.

INDIRECT STATISTICAL MODELING

The indirect statistical modeling technique consists of two stages. In the first stage each of the devices represented by the multi-device measurement data is modeled individually in a deterministic fashion. Parameter extraction optimization is invoked for each of the outcomes to fit the simulated responses to the corresponding measurement data set. Such parameter extraction is carried out for all data sets in the multi-device measurements and results in as many device models as the number of data sets in the measurement data. They form a sample of device models, with different parameter values for different outcome models. Statistical postprocessing of those different parameter values leads to a single, consolidated model whose parameter values are described by the means, standard deviations and the correlation matrix. For non-Gaussian distributions, a discrete distribution function (DDF) approximation to the marginal distributions [6] is used to enhance model accuracy. Fig. 2 illustrates the two stages of indirect statistical modeling.

DIRECT STATISTICAL MODELING

In direct statistical modeling, the model parameter statistics are obtained directly instead of from postprocessing a set of individually extracted models. The whole multi-device measurement data is utilized simultaneously in a statistical fashion by generating the distribution of the measured device responses. The statistics of the model parameters are determined by fitting the distributions (CPDs or histograms) of the model responses to those of the measurement data. To obtain the distributions of the model responses we employ Monte Carlo simulation. Model parameters are randomly generated according to the parameter distribution and the resulting distribution of model responses is found. The fitting is carried out in a single optimization. Direct statistical modeling using CPD fitting is depicted in Fig. 3.

The optimization variables include the parameter statistics, for example, the mean values and standard deviations in the case of normal distributions, or the nominal values and tolerances in the case of uniform distributions (other types of distributions can also be applied). The initial parameter statistics and distributions need to be assumed at the starting point. At the solution we obtain the parameter statistics leading to the best match of the corresponding CPDs or histograms.

COMBINING THE INDIRECT AND DIRECT APPROACHES

Each of the two approaches can be applied alternatively to carry out complete statistical modeling independently. However, each has its own advantages and disadvantages.

Indirect statistical modeling is straightforward and easy to use. No initial statistics need to be guessed at. However, it relies on the uniqueness of the parameter extraction process and, therefore, the resulting statistical models may not reflect the actual distribution of measurement data, even if the fit of the simulated responses to the corresponding measurements for individual device models is excellent. Direct statistical modeling, on the other hand, is based on a solid mathematical foundation and, therefore, should prove more reliable and robust than the indirect method. However, the initial parameter statistics need to be assigned and the parameter distribution types guessed. It may be justified to assume normal distributions in the case of physical or process parameters, but it may not be correct for equivalent circuit models. All of that may affect the solution. Also, a good starting point is important to reduce the computation time and assure successful optimization.

A practical approach is to combine the two methods. We use the indirect method first to obtain an initial statistical model and then apply the direct method to improve the model accuracy. For efficiency, the initial modeling may be carried out with a small number of devices.

STATISTICAL MODEL VERIFICATION

Two practical methods for model verification are statistical comparison [4,5] and yield verification [2]. The former compares the statistics of the model responses generated by Monte Carlo simulation with the statistics of the measurement data. The latter checks the consistency between yield predicted by statistical models and the yield estimated by the actual device data.

Visual comparisons of distributions may also be used for effective and quick model validation. HarPE provides useful graphical displays where the histograms or CPDs of the model responses and the corresponding data including the mean values and standard deviations can be shown in the same diagram.

STATISTICAL MODELING OF GaAs MESFETs USING A PHYSICS-ORIENTED MODEL

Consider statistical modeling of a GaAs MESFET using a physics-oriented model which we call the KTL (Khatibzadeh-Trew-Ladbrooke) model. The KTL model combines the advantages of the Khatibzadeh and Trew model [7] and the small-signal Ladbrooke model [8] while overcoming their respective shortcomings. Its attractive statistical properties have already been presented in [2,3].

The KTL small-signal equivalent circuit follows the Ladbrooke model as shown in Fig. 4. The model includes the intrinsic FET parameters, L, Z, a, N_d, V_{b0}, v_{sat}, μ_0, ε, L_{G0}, a_0, r_{01}, r_{02}, r_{03}, and the linear extrinsic elements, L_g, R_g, L_d, R_d, L_s, R_s, G_{ds}, C_{ds}, C_{ge}, C_{de}, where L is the gate length, Z the gate width, a the channel thickness, N_d the doping density, V_{b0} the zero-bias barrier potential, v_{sat} the saturation value of electron drift velocity, μ_0 the low-field mobility of GaAs, ε

the dielectric constant, L_{G0} the inductance from gate bond wires and pads, a_0 the proportionality coefficient, and r_{01}, r_{02} and r_{03} the fitting coefficients [1,3,5].

The bias-dependent small-signal parameters, namely, g_m, C_{gs}, C_{gd}, R_i, L_g, r_0 and τ, as shown in Fig. 4, are derived using the modified Ladbrooke formulae once the DC operating point is obtained using the Khatibzadeh and Trew model [3].

A sample of GaAs MESFET data which is obtained by aligning wafer measurements to a consistent bias condition [3] is used for statistical modeling. There are 35 data sets (devices) containing the small-signal S parameters measured at the frequencies from 1 to 21 GHz with a 2 GHz step under the bias condition of V_{gs} = -0.7 V and V_{ds} = 5 V.

We first use multi-device parameter extraction and statistical postprocessing based on 15 devices to obtain the initial parameter statistics including the mean values, standard deviations and the correlation matrix. Then, the initial model is optimized using direct approach of CPD fitting. We consider 16 statistical parameters with normal distributions. This results in 32 optimization variables, namely all the means and standard deviations. The statistical KTL model parameter values after optimization are listed in Table I. The CPDs of the real part of S_{21} (RS21) at 11 GHz from the data and from the statistical KTL model before and after optimization are shown in Fig. 5. We can see that after optimization the CPD matching between the data and the KTL model is significantly improved. The histograms of the imaginary part of S_{21} (IS21) at 11 GHz from the data and from the statistical KTL model before and after optimization are shown in Fig. 6. We can see that after optimization the histogram matching between the data and the KTL model is also improved.

CONCLUSIONS

Statistical fluctuations in the manufacturing process cause variations in device parameters values, and consequently in device performance. The ultimate purpose of statistical modeling is to characterize devices for accurate yield and cost analysis and optimization.

We have presented two approaches to statistical modeling: indirect and direct. By combining the two methods we can obtain accurate statistical device models. This has been demonstrated on the example of statistical modeling of a GaAs MESFET using the KTL model. Model verification has been illustrated by comparing the distributions (CPDs and histograms) of the model responses and those of the data.

It should be pointed out that measurement errors may significantly affect accuracy of the resulting statistical model. If the measurement data contains some wild points (e.g., due to faulty devices) they may severely degrade the resulting model and should be removed. A robust approach using the Huber function has been proposed to automatically handle such errors [9].

Our advanced algorithms together with a number of state-of-the-art optimizers including ℓ_1, ℓ_2 (the least squares) and Huber, and aided by useful statistical displays implemented in HarPE, provide an efficient and user-friendly environment for statistical modeling. The extracted statistical models can be used, for example, in OSA90/hope [10] for yield-driven and cost-driven circuit design. A new exciting development of invoking HarPE directly as a child process from within OSA90/hope provides a consolidated modeling/simulation/optimization software system for microwave circuit design and further enhances this CAD environment.

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TABLE I

THE OPTIMIZED KTL MODEL PARAMETER STATISTICS

Parameter	Mean	σ(%)	Parameter	Mean	σ(%)
L(μm)	0.4685	3.57	C_{ds}(pF)	0.0547	1.58
a(μm)	0.1308	5.19	C_{ge}(pF)	0.0807	5.92
N_d(m^{-3})	2.3x10^{23}	3.25	C_{de}(pF)	0.0098	6.22
v_{sat}(m/s)	10.5x10^4	2.27	C_x(pF)	2.4231	4.03
μ_0(m^2/Vns)	6.5x10^{-10}	2.16	Z(μm)	300	*
L_{G0}(nH)	0.0396	10.9	ε	12.9	*
R_d(Ω)	1.2867	4.32	V_{b0}(V)	0.6	*
R_s(Ω)	3.9119	1.91	r_{01}(Ω/V^2)	0.35	*
R_g(Ω)	8.1718	0.77	r_{02}(V)	7.0	*
L_d(nH)	0.0659	5.74	r_{03}(Ω)	2003	*
L_s(nH)	0.0409	5.49	a_0	1.0	*
G_{ds}(1/Ω)	3.9x10^{-3}	1.78			

σ denotes standard deviation.

* Assumed fixed (non-statistical) parameters.

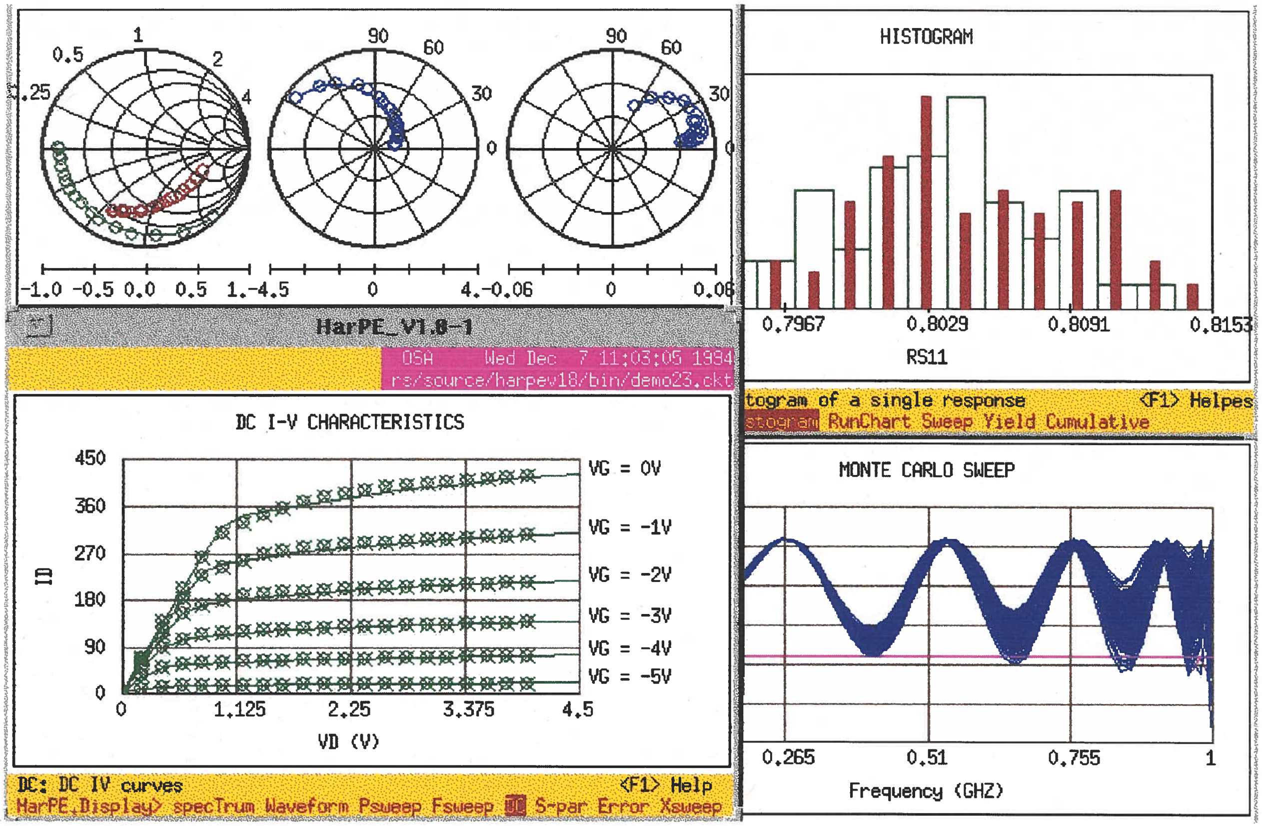


Fig. 1 Various device responses featured by HarPE.

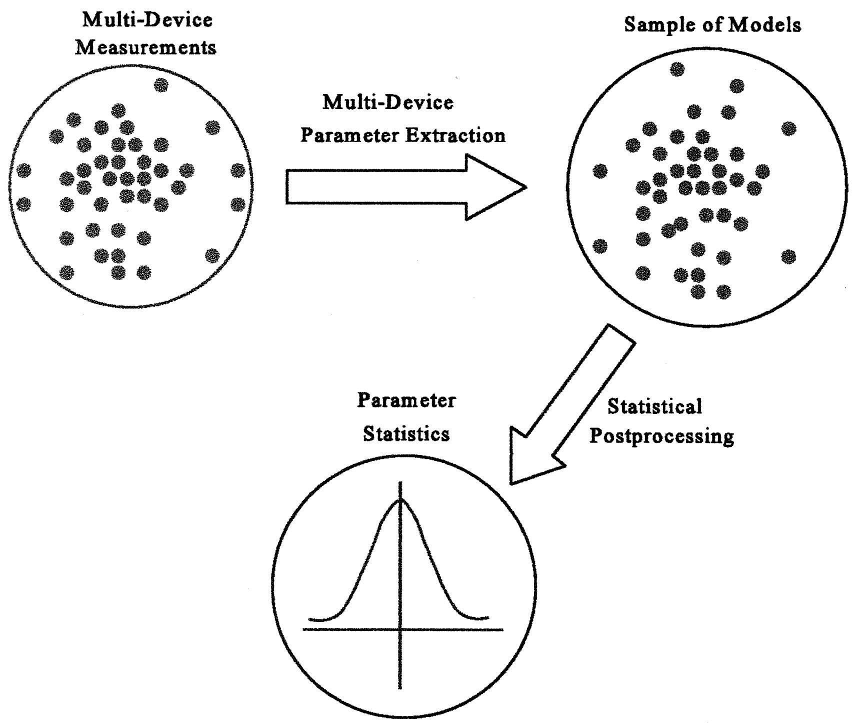


Fig. 2 Illustration of indirect statistical modeling.

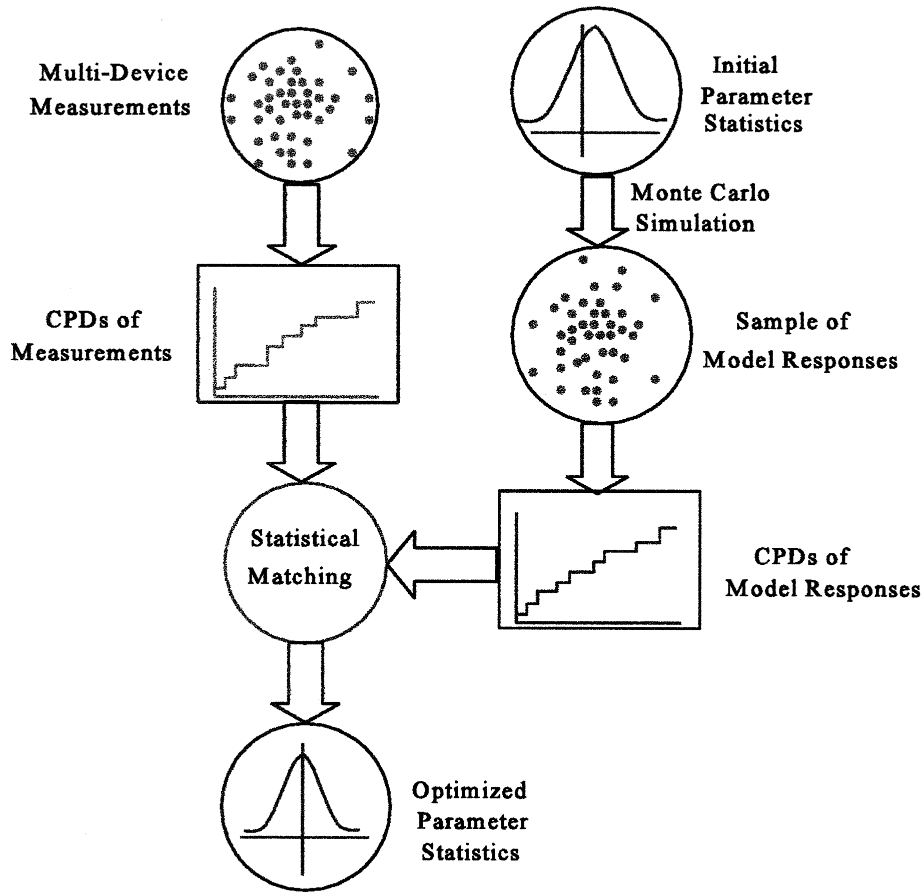


Fig. 3 Direct statistical modeling using CPD matching.

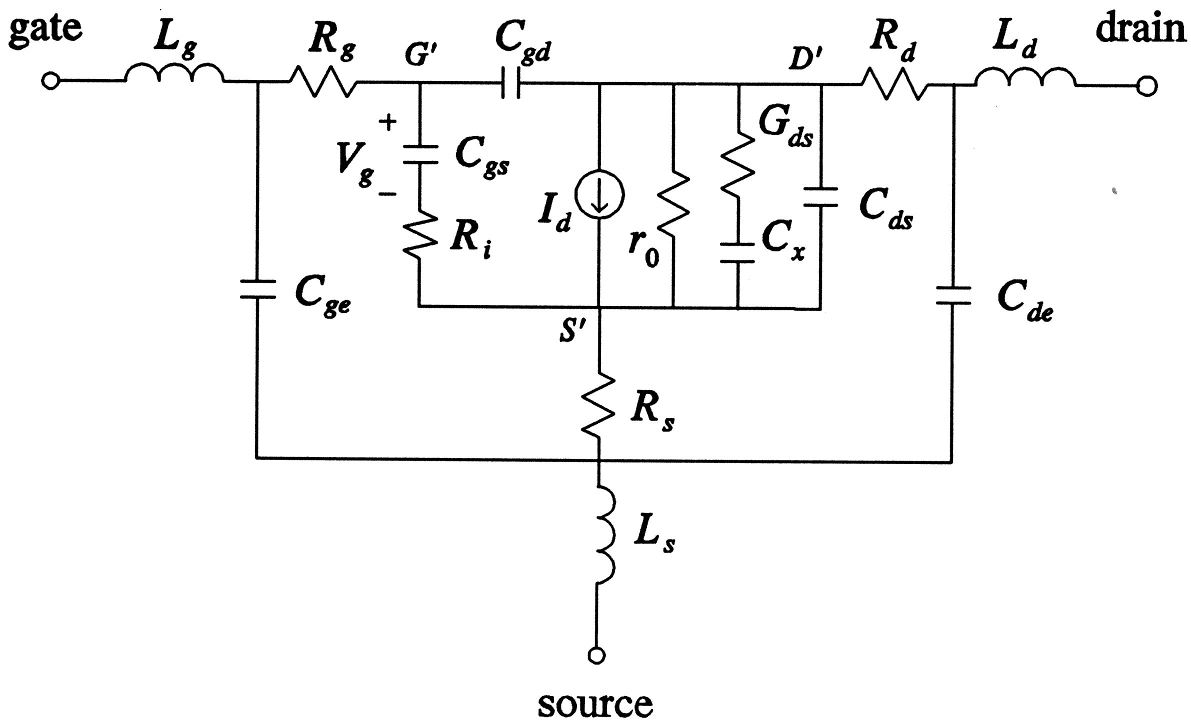
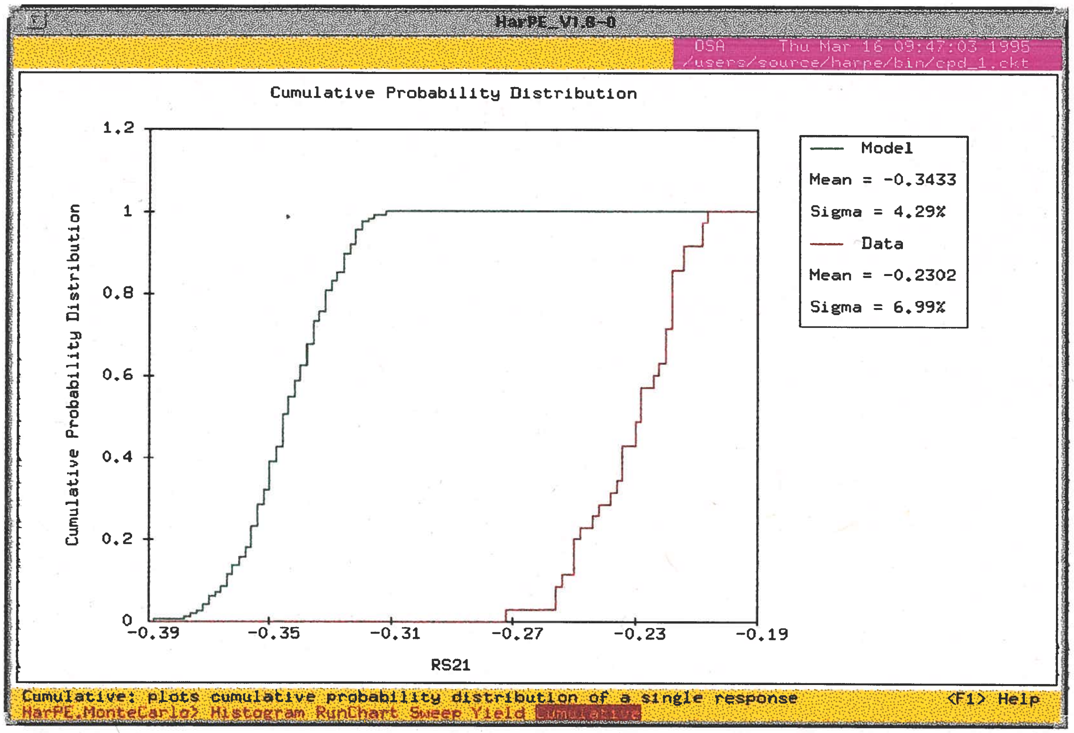
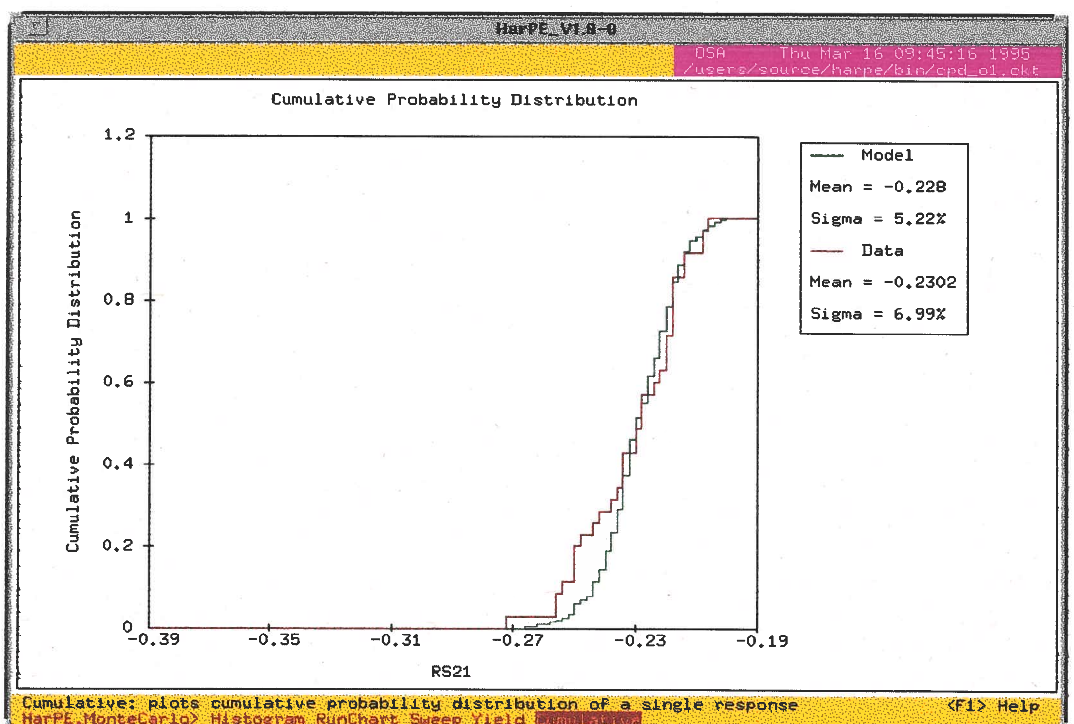


Fig. 4 The small-signal equivalent circuit of the KTL model, where $I_d = g_m V_g e^{-j\omega\tau}$.

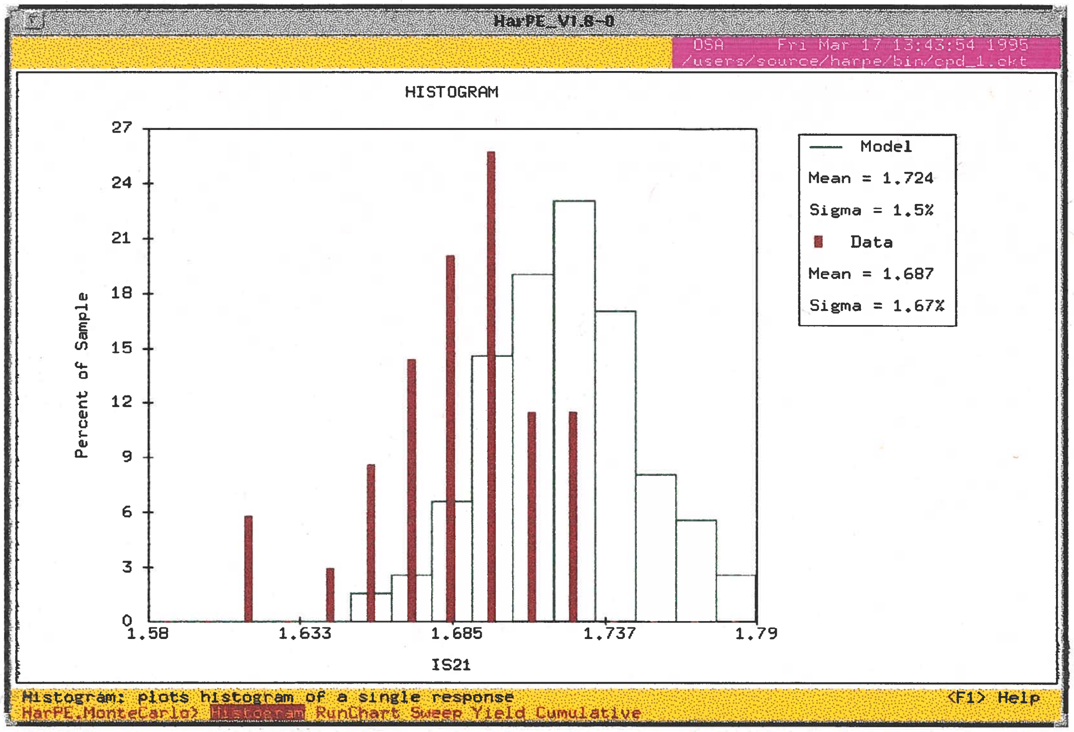


(a)

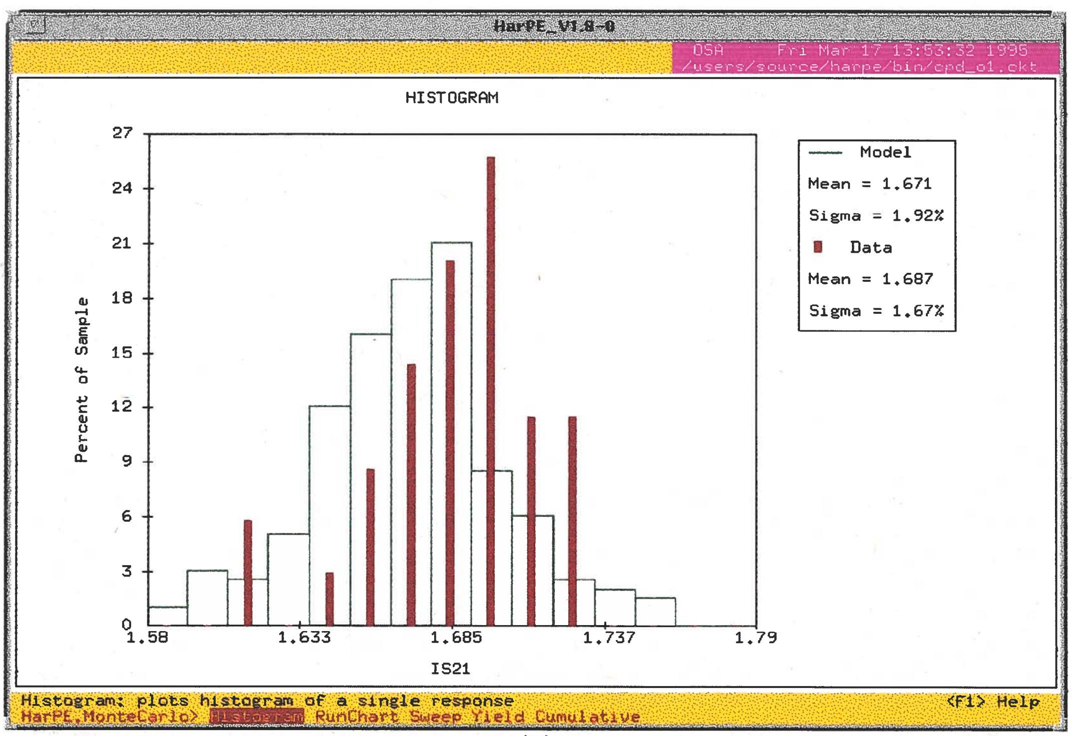


(b)

Fig. 5 CPDs of the simulated RS21 (the real part of S_{21}) and the corresponding data at 11 GHz, (a) before optimization and (b) after optimization.



(a)



(b)

Fig. 6 Histograms of the simulated IS21 (the imaginary part of S_{21}) and the corresponding data at 11 GHz, (a) before optimization and (b) after optimization.