# NOVEL ELECTROMAGNETIC OPTIMIZATION TECHNIQUES, INCLUDING SPACE MAPPING

J.W. Bandler

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J.W. Bandler

Optimization Systems Associates Inc.
P.O. Box 8083, Dundas, Ontario
Canada L9H 5E7



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**Background**

assume that $X_{os}$ (optimization space) and $X_{em}$ (EM space) have the same dimensionality, i.e.,

$$x_{os} \in \mathbb{R}^n \quad \text{and} \quad x_{em} \in \mathbb{R}^n,$$

but may not represent the same parameters

the $X_{os}$-space model can be comprised of empirical models, or an efficient coarse-grid EM model

the $X_{em}$-space model is typically a fine-grid EM model but, ultimately, can represent actual hardware prototypes

we assume that the $X_{os}$-space model responses, $R_{os}(x_{os})$, are much faster to calculate but less accurate than the $X_{em}$-space model responses, $R_{em}(x_{em})$

we initially perform optimization in $X_{os}$ to obtain the optimal design $x_{os}^*$, for instance in the minimax sense

subsequently, apply SM to find the mapped solution $\bar{x}_{em}$ in $X_{em}$ to reproduce the optimal performance predicted by the empirical model

**The Concept of Space Mapping**
(*Bandler, Biernacki, Chen, Grobelny and Hemmers, 1994*)

our aim is to find an appropriate mapping, $\boldsymbol{P}$, from the $X_{em}$-space to the $X_{os}$-space, i.e.,

$$\boldsymbol{x}_{os} = \boldsymbol{P}(\boldsymbol{x}_{em})$$

such that

$$\boldsymbol{R}_{os}(\boldsymbol{P}(\boldsymbol{x}_{em})) \approx \boldsymbol{R}_{em}(\boldsymbol{x}_{em})$$

we assume that such a mapping exists and is one-to-one within some local modeling region encompassing our SM solution

once the mapping is established, the SM solution is

$$\bar{\boldsymbol{x}}_{em} = \boldsymbol{P}^{-1}(\boldsymbol{x}_{os}^{*})$$

## Original Space Mapping Method

the mapping is established through an iterative process

to obtain the initial approximation to the mapping, $P^{(0)}$, we perform EM analyses at a preselected set of base points in $X_{em}$ around the starting point

as the first base point we may select the starting point, i.e.,

$$x_{em}^{(1)} = x_{os}^{*}$$

assuming $x_{em}$ and $x_{os}$ represent the same physical parameters, followed by additional base points chosen by perturbation as

$$x_{em}^{(i)} = x_{em}^{(1)} + \Delta x_{em}^{(i-1)}, \quad i = 2, 3, \ldots, m$$

this is followed by parameter extraction optimization in $X_{os}$ to obtain the set of corresponding base points $x_{os}^{(i)}$ according to

$$\underset{x_{os}^{(i)}}{\text{minimize}} \; \| R_{os}(x_{os}^{(i)}) - R_{em}(x_{em}^{(i)}) \|$$

for $i = 1, 2, \ldots, m$, where $\| \cdot \|$ indicates a suitable norm

## Original Space Mapping Method (continued)

at the $j$th iteration, both sets may be expanded to contain $m_j$ points which are used to establish the updated mapping $P^{(j)}$

the current approximation $P^{(j)}$ is used to estimate $\bar{x}_{em}$ as

$$x_{em}^{(m_j+1)} = P^{(j)^{-1}}(x_{os}^*)$$

the process continues until the termination condition

$$\| R_{os}(x_{os}^*) - R_{em}(x_{em}^{(m_j+1)}) \| \leq \epsilon$$

is satisfied, where $\epsilon$ is a small positive constant, then $P^{(j)}$ is our desired $P$

if not, the set of base points in $X_{em}$ is augmented by $x_{em}^{(m_j+1)}$ and correspondingly, $x_{os}^{(m_j+1)}$ determined by parameter extraction augments the set of base points in $X_{os}$

upon termination, we set $\bar{x}_{em} = x_{em}^{(m_j+1)} = P^{(j)^{-1}}(x_{os}^*)$ as the SM solution

**Aggressive Approach to Space Mapping**
(*Bandler, Biernacki, Chen, Hemmers and Madsen, 1995*)

at the SM solution, $R_{em}(x_{em}^{(M)})$ will closely match $R_{os}(x_{os}^{*})$,

$$\| R_{os}(x_{os}^{*}) - R_{em}(x_{em}^{(M)}) \| \leq \epsilon$$

where $M$ is the number of iterations needed to converge to an SM solution

hence, after an additional parameter extraction optimization in $X_{os}$, the resulting point

$$x_{os}^{(M)} = P(x_{em}^{(M)})$$

approaches the point $x_{os}^{*}$ (optimal solution in $X_{os}$), or

$$\| x_{os}^{(M)} - x_{os}^{*} \| \leq \eta \quad \text{as} \quad j \to M$$

where $\eta$ is a small positive constant

by setting $\eta$ to 0, we consider the set of $n$ nonlinear equations

$$f(x_{em}) = \mathbf{0}$$

of the form

$$f(x_{em}) = P(x_{em}) - x_{os}^{*}$$

where $x_{os}^{*}$ is a given vector

## Aggressive Space Mapping - Quasi-Newton Iteration

let $x_{em}^{(j)}$ be the $j$th approximation to the solution and $f^{(j)}$ written for $f(x_{em}^{(j)})$

the next iterate is found by a quasi-Newton iteration

$$x_{em}^{(j+1)} = x_{em}^{(j)} + h^{(j)}$$

by solving the linear system

$$B^{(j)} h^{(j)} = -f^{(j)}$$

$B^{(j)}$ is an approximation to the Jacobian matrix

$$J(x_{em}^{(j)}) = \left. \left( \frac{\partial f^T(x_{em})}{\partial x_{em}} \right)^T \right|_{x_{em} = x_{em}^{(j)}}$$

in our implementation, $B^{(1)}$ is set to the identity matrix

the approximation to the Jacobian matrix is updated by the classic Broyden formula (*Broyden, 1965*)

$$B^{(j+1)} = B^{(j)} + \frac{f(x_{em}^{(j)} + h^{(j)}) - f(x_{em}^{(j)}) - B^{(j)} h^{(j)}}{h^{(j)T} h^{(j)}} h^{(j)T}$$

## Aggressive Space Mapping - Implementation

begin with a point, $\boldsymbol{x}_{os}^* \triangleq arg\,min\,\{H(\boldsymbol{x}_{os})\}$, representing the optimal design in $X_{os}$ where $H(\boldsymbol{x}_{os})$ is some appropriate objective function

*Step* 0. initialize $\boldsymbol{x}_{em}^{(1)} = \boldsymbol{x}_{os}^*$, $\boldsymbol{B}^{(1)} = \mathbf{1}$, $\boldsymbol{f}^{(1)} = \boldsymbol{P}(\boldsymbol{x}_{em}^{(1)}) - \boldsymbol{x}_{os}^*$, $j = 1$; stop if $\|\boldsymbol{f}^{(1)}\| \leq \eta$

*Step* 1. solve $\boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$

*Step* 2. set $\boldsymbol{x}_{em}^{(j+1)} = \boldsymbol{x}_{em}^{(j)} + \boldsymbol{h}^{(j)}$

*Step* 3. evaluate $\boldsymbol{P}(\boldsymbol{x}_{em}^{(j+1)})$

*Step* 4. compute $\boldsymbol{f}^{(j+1)} = \boldsymbol{P}(\boldsymbol{x}_{em}^{(j+1)}) - \boldsymbol{x}_{os}^*$; if $\|\boldsymbol{f}^{(j+1)}\| \leq \eta$, stop

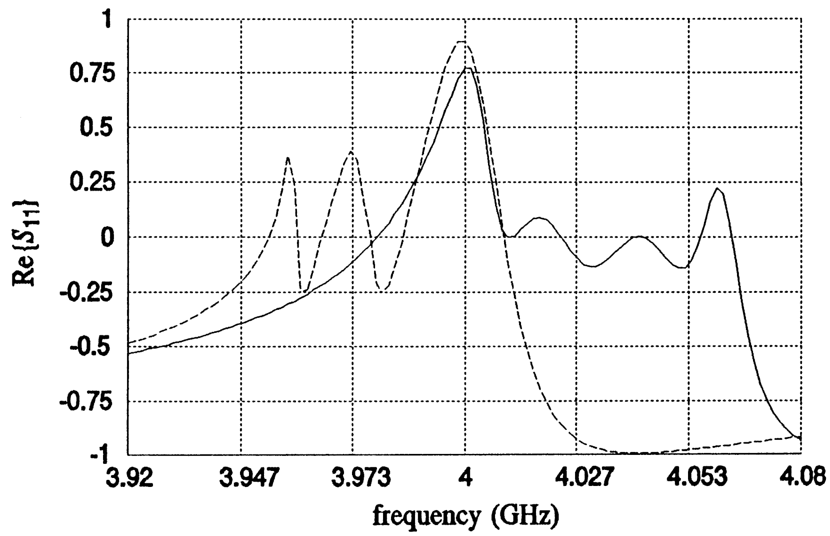
*Step* 5. update $\boldsymbol{B}^{(j)}$ to $\boldsymbol{B}^{(j+1)}$

*Step* 6. set $j = j + 1$; go to *Step* 1

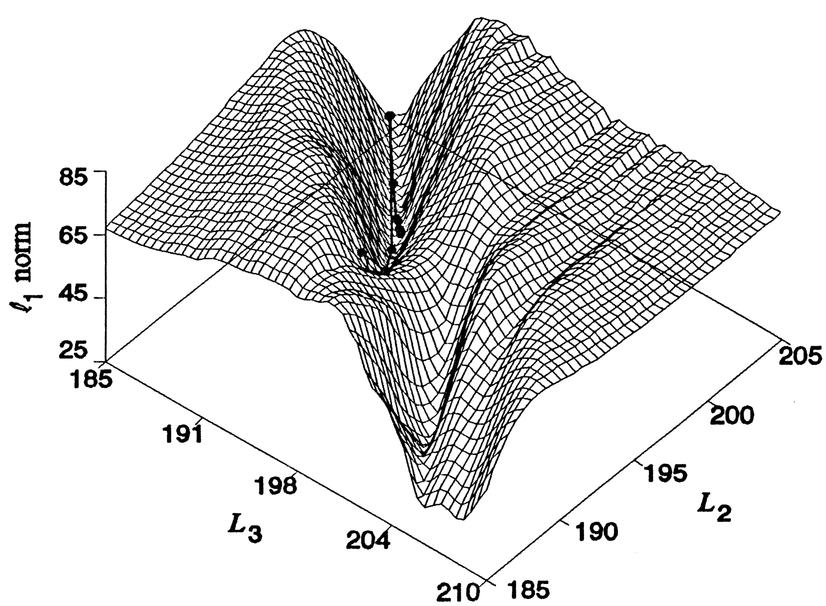
## Frequency Space Mapping for Parameter Extraction

parameter extraction can be a serious challenge, especially at the starting point, if the model responses are misaligned

Re$\{S_{11}\}$ using OSA90/hope (—) and *em* (---) at $x_{os}^*$




straightforward optimization from such a starting point can lead to a local minimum



## Frequency Space Mapping - Mapping and Alignment

to better condition the parameter extraction subproblem

first, we align $R_{os}$ and $R_{em}$ along the frequency axis using

$$\omega_{os} = P_{\omega}(\omega)$$

this frequency space mapping can be as simple as

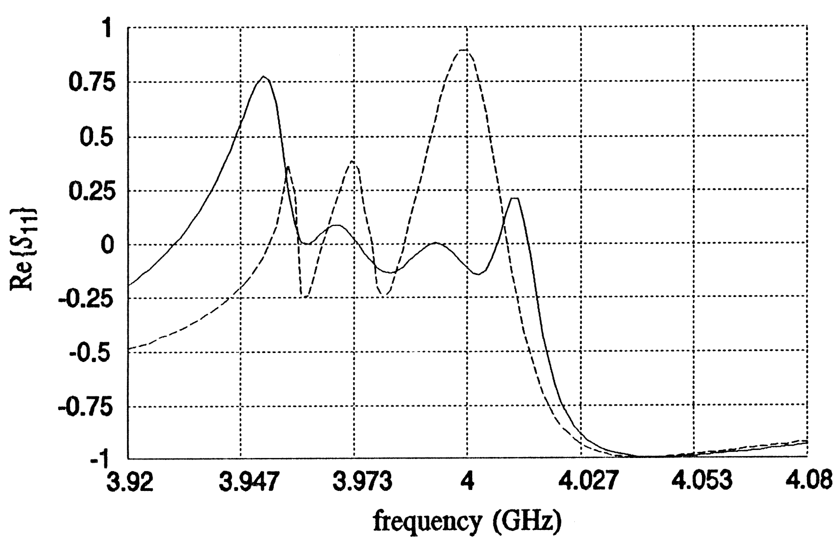
$$\omega_{os} = \sigma\,\omega + \delta$$

at the starting point, we determine $\sigma_0$ and $\delta_0$ by

$$\underset{\sigma_0,\ \delta_0}{\text{minimize}} \ \| R_{os}(x_{os}, \sigma_0, \delta_0) - R_{em}(x_{em}) \|$$

where $x_{os}$ and $x_{em}$ are fixed and $x_{os} = x_{em}$

resulting alignment between OSA90/hope (—) and *em* (---):



**Frequency Space Mapping:**
**Sequential FSM (SFSM) Algorithm**

we perform a sequence of optimizations to gradually achieve the identity Frequency Space Mapping

we optimize $x_{os}$ to match $R_{os}$ and $R_{em}$:

$$\underset{x_{os}^{(j)}}{\text{minimize}} \ \| R_{os}(x_{os}^{(j)}, \sigma^{(j)}, \delta^{(j)}) - R_{em}(x_{em}) \|$$

the values $\sigma^{(j)}$ and $\delta^{(j)}$ are updated according to

$$\sigma^{(j)} = 1 + (\sigma_0 - 1)\frac{(K - j)}{K}$$

and

$$\delta^{(j)} = \delta_0 \frac{(K - j)}{K},$$

respectively, for $j = 0, 1, ..., K$

$K$ determines the number of steps in the sequence

larger values of $K$ increase the probability of success in the parameter extraction subproblem at the expense of longer optimization time

**Frequency Space Mapping:**
**Exact Penalty Function (EPF) Algorithm**

we perform only one optimization to achieve the identity Frequency Space Mapping and optimize $x_{os}$ to match $R_{os}$ to $R_{em}$

the $\ell_1$ norm version of the EPF formulation is given by

$$\underset{x_{os},\, \sigma,\, \delta}{\text{minimize}} \quad \{ \| R_{os}(x_{os}, \sigma, \delta) - R_{em}(x_{em}) \|_1 + \alpha_1 | \sigma - 1 | + \alpha_2 | \delta | \}$$

the minimax version is given by

$$\underset{x_{os},\, \sigma,\, \delta}{\text{minimize}} \left\{ \max_{1 \le i \le 4} \left[ U(x_{os}, \sigma, \delta),\ U(x_{os}, \sigma, \delta) - \alpha_i g_i \right] \right\}$$

where

$$U(x_{os}, \sigma, \delta) = \| R_{os}(x_{os}, \sigma, \delta) - R_{em}(x_{em}) \|$$

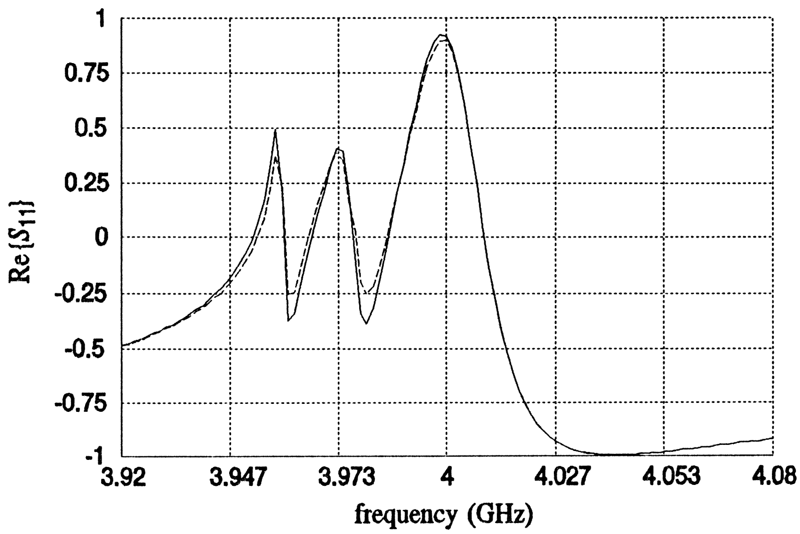
and

$$g(\sigma, \delta) = \begin{bmatrix} \sigma - 1 \\ 1 - \sigma \\ \delta \\ -\delta \end{bmatrix}$$

in both EPF formulations, $\alpha_i$ are kept fixed and must be sufficiently large to obtain the identity mapping and hence the solution to the parameter extraction problem

## Frequency Space Mapping - Results

Re$\{S_{11}\}$ using OSA90/hope (—) and *em* (---)




resulting match after applying the FSM algorithm