NOVEL SPACE MAPPING OPTIMIZATION TECHNIQUE FOR ELECTROMAGNETIC DESIGN

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Background

assume that X_{os} (optimization space) and X_{em} (EM space) have the same dimensionality, i.e.,

$$x_{os} \in \mathbb{R}^n \quad \text{and} \quad x_{em} \in \mathbb{R}^n,$$

but may not represent the same parameters

the X_{os}-space model can be comprised of empirical models, or an efficient coarse-grid EM model

the X_{em}-space model is typically a fine-grid EM model but, ultimately, can represent actual hardware prototypes

we assume that the X_{os}-space model responses, $R_{os}(x_{os})$, are much faster to calculate but less accurate than the X_{em}-space model responses, $R_{em}(x_{em})$

we initially perform optimization in X_{os} to obtain the optimal design x_{os}^*, for instance in the minimax sense

subsequently, apply SM to find the mapped solution $\bar{x}_{em}$ in X_{em} to reproduce the optimal performance predicted by the empirical model

The Concept of Space Mapping
(*Bandler, Biernacki, Chen, Grobelny and Hemmers, 1994*)

our aim is to find an appropriate mapping, $\boldsymbol{P}$, from the X_{em}-space to the X_{os}-space, i.e.,

$$\boldsymbol{x}_{os} = \boldsymbol{P}(\boldsymbol{x}_{em})$$

such that

$$\boldsymbol{R}_{os}(\boldsymbol{P}(\boldsymbol{x}_{em})) \approx \boldsymbol{R}_{em}(\boldsymbol{x}_{em})$$

we assume that such a mapping exists and is one-to-one within some local modeling region encompassing our SM solution

once the mapping is established, the SM solution is

$$\bar{\boldsymbol{x}}_{em} = \boldsymbol{P}^{-1}(\boldsymbol{x}_{os}^{*})$$

Original Space Mapping Method

the mapping is established through an iterative process

to obtain the initial approximation to the mapping, $P^{(0)}$, we perform EM analyses at a preselected set of base points in X_{em} around the starting point

as the first base point we may select the starting point, i.e.,

$$x_{em}^{(1)} = x_{os}^{*}$$

assuming x_{em} and x_{os} represent the same physical parameters, followed by additional base points chosen by perturbation as

$$x_{em}^{(i)} = x_{em}^{(1)} + \Delta x_{em}^{(i-1)}, \quad i = 2, 3, ..., m$$

this is followed by parameter extraction optimization in X_{os} to obtain the set of corresponding base points $x_{os}^{(i)}$ according to

$$\underset{x_{os}^{(i)}}{\text{minimize}} \; \| R_{os}(x_{os}^{(i)}) - R_{em}(x_{em}^{(i)}) \|$$

for $i = 1, 2, ..., m$, where $\| \cdot \|$ indicates a suitable norm

Original Space Mapping Method (continued)

at the jth iteration, both sets may be expanded to contain m_j points which are used to establish the updated mapping $P^{(j)}$

the current approximation $P^{(j)}$ is used to estimate $\bar{x}_{em}$ as

$$x_{em}^{(m_j+1)} = P^{(j)^{-1}}(x_{os}^*)$$

the process continues until the termination condition

$$\| R_{os}(x_{os}^*) - R_{em}(x_{em}^{(m_j+1)}) \| \leq \epsilon$$

is satisfied, where ϵ is a small positive constant, then $P^{(j)}$ is our desired P

if not, the set of base points in X_{em} is augmented by $x_{em}^{(m_j+1)}$ and correspondingly, $x_{os}^{(m_j+1)}$ determined by parameter extraction augments the set of base points in X_{os}

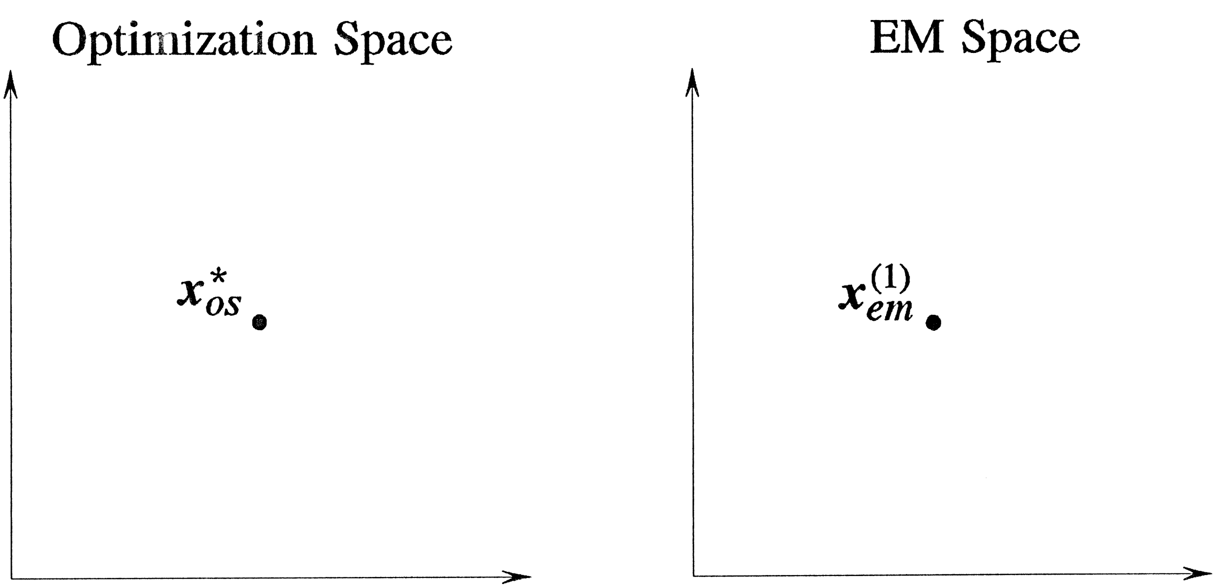
upon termination, we set $\bar{x}_{em} = x_{em}^{(m_j+1)} = P^{(j)^{-1}}(x_{os}^*)$ as the SM solution

Illustration of Space Mapping Optimization

Step 0

find the optimal design x_{os}^{*} in Optimization Space

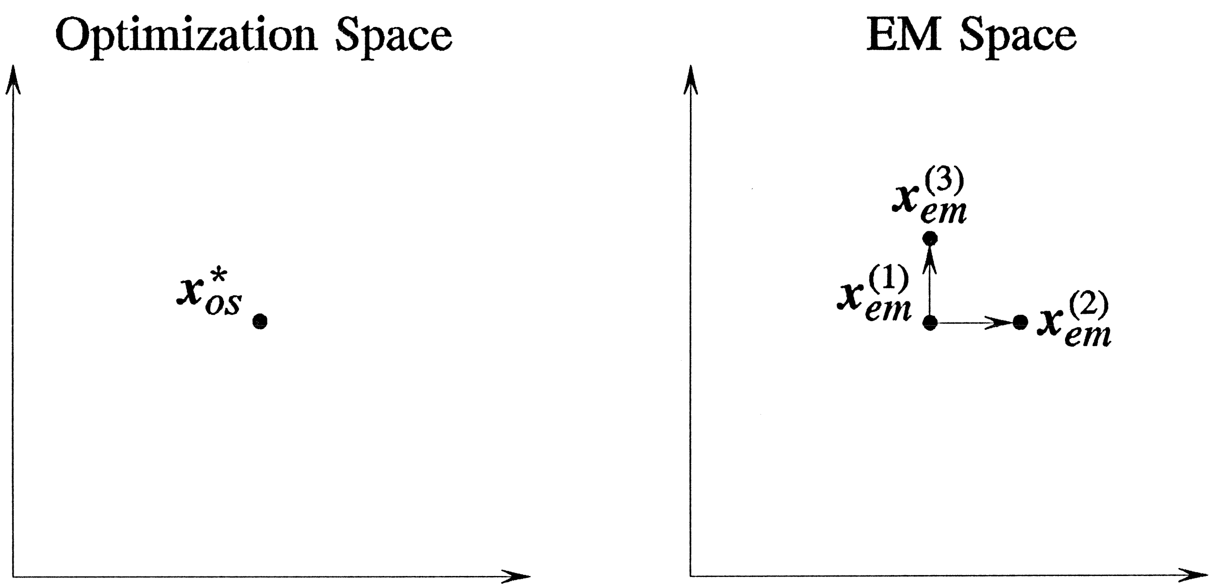
Step 1



set $x_{em}^{(1)} = x_{os}^{*}$ assuming x_{em} and x_{os} represent the same physical parameters

Illustration of Space Mapping Optimization

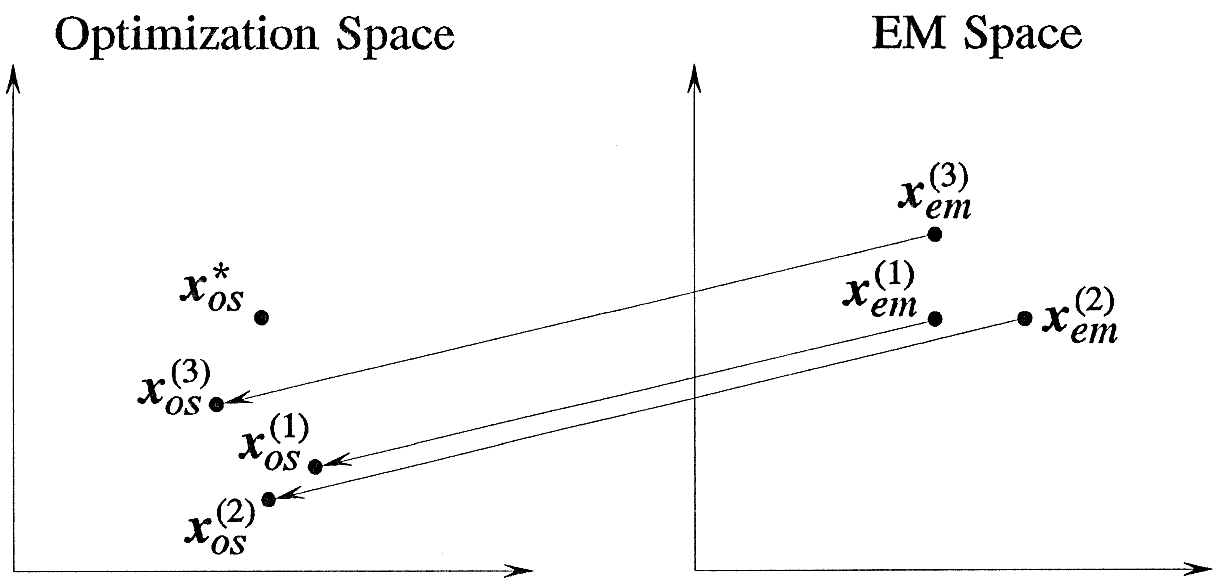
Step 2



generate additional base points around $x_{em}^{(1)}$

Illustration of Space Mapping Optimization

Step 3

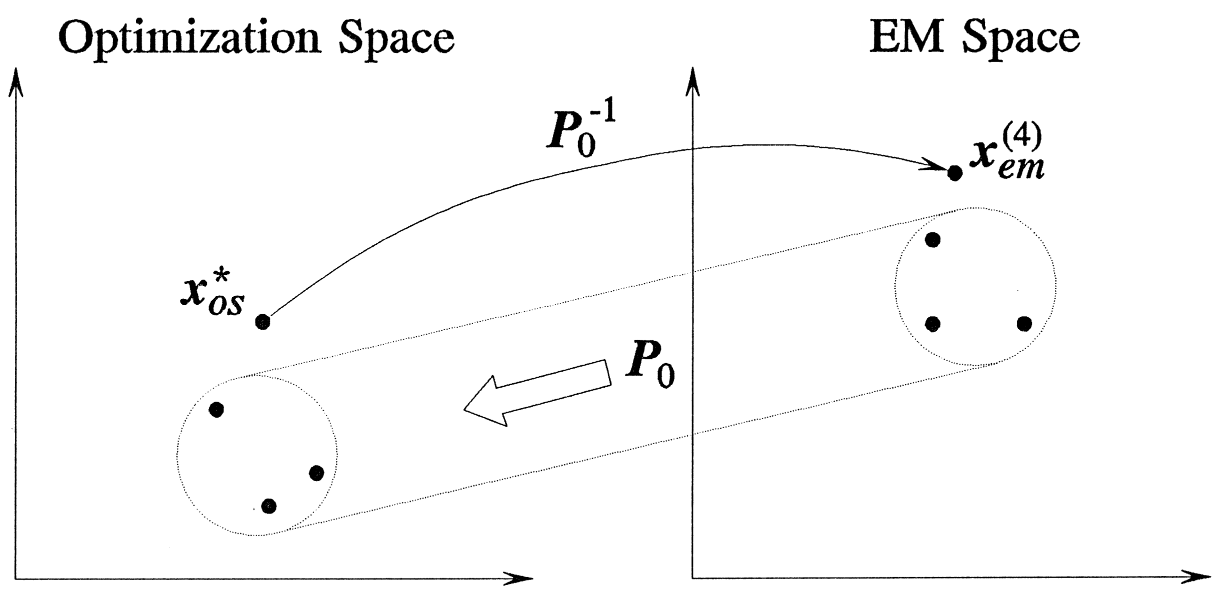


perform X_{os}-space model parameter extraction for each of the base points to match the EM and OS responses

a set of OS points corresponding to the EM base points is established

Illustration of Space Mapping Optimization

Step 4

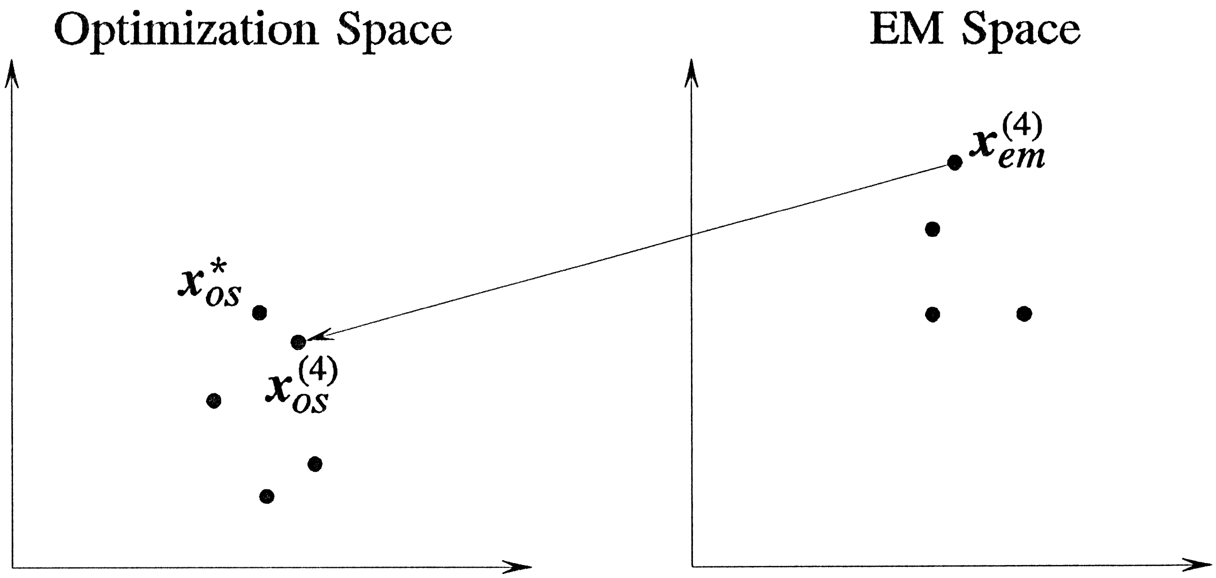


determine the initial mapping P_0

use the inverse mapping to obtain $x_{em}^{(4)}$

Illustration of Space Mapping Optimization

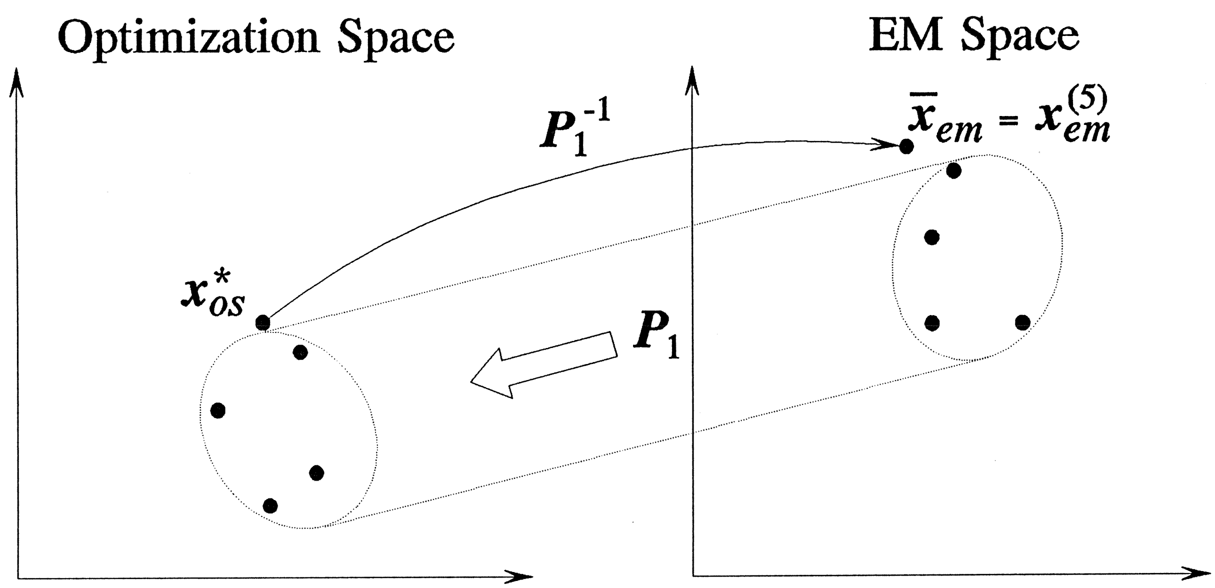
Step 5



perform X_{os}-space model parameter extraction to obtain $x_{os}^{(4)}$

Illustration of Space Mapping Optimization

Step 6



use the additional pair of points to update the mapping to P_1

apply the updated inverse mapping to obtain $x_{em}^{(5)}$

if $\| R_{os}(x_{os}^*) - R_{em}(x_{em}^{(5)}) \| \leq \epsilon$ then $\bar{x}_{em} = x_{em}^{(5)}$ is considered as the SM solution

Aggressive Approach to Space Mapping
(*Bandler, Biernacki, Chen, Hemmers and Madsen, 1995*)

at the SM solution, $R_{em}(x_{em}^{(M)})$ will closely match $R_{os}(x_{os}^{*})$,

$$\| R_{os}(x_{os}^{*}) - R_{em}(x_{em}^{(M)}) \| \leq \epsilon$$

where M is the number of iterations needed to converge to an SM solution

hence, after an additional parameter extraction optimization in X_{os}, the resulting point

$$x_{os}^{(M)} = P(x_{em}^{(M)})$$

approaches the point x_{os}^{*} (optimal solution in X_{os}), or

$$\| x_{os}^{(M)} - x_{os}^{*} \| \leq \eta \quad \text{as} \quad j \to M$$

where η is a small positive constant

by setting η to 0, we consider the set of n nonlinear equations

$$f(x_{em}) = \mathbf{0}$$

of the form

$$f(x_{em}) = P(x_{em}) - x_{os}^{*}$$

where x_{os}^{*} is a given vector

Aggressive Space Mapping - Quasi-Newton Iteration

let $\boldsymbol{x}_{em}^{(j)}$ be the jth approximation to the solution and $\boldsymbol{f}^{(j)}$ written for $\boldsymbol{f}(\boldsymbol{x}_{em}^{(j)})$

the next iterate is found by a quasi-Newton iteration

$$\boldsymbol{x}_{em}^{(j+1)} = \boldsymbol{x}_{em}^{(j)} + \boldsymbol{h}^{(j)}$$

by solving the linear system

$$\boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$$

$\boldsymbol{B}^{(j)}$ is an approximation to the Jacobian matrix

$$\boldsymbol{J}(\boldsymbol{x}_{em}^{(j)}) = \left.\left(\frac{\partial \boldsymbol{f}^T(\boldsymbol{x}_{em})}{\partial \boldsymbol{x}_{em}}\right)^T\right|_{\boldsymbol{x}_{em} = \boldsymbol{x}_{em}^{(j)}}$$

in our implementation, $\boldsymbol{B}^{(1)}$ is set to the identity matrix

the approximation to the Jacobian matrix is updated by the classic Broyden formula (*Broyden, 1965*)

$$\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \frac{\boldsymbol{f}(\boldsymbol{x}_{em}^{(j)} + \boldsymbol{h}^{(j)}) - \boldsymbol{f}(\boldsymbol{x}_{em}^{(j)}) - \boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)}}{\boldsymbol{h}^{(j)T}\boldsymbol{h}^{(j)}}\boldsymbol{h}^{(j)T}$$

Aggressive Space Mapping - Implementation

begin with a point, $\boldsymbol{x}_{os}^{*} \triangleq \arg\min \{H(\boldsymbol{x}_{os})\}$, representing the optimal design in X_{os} where $H(\boldsymbol{x}_{os})$ is some appropriate objective function

Step 0. initialize $\boldsymbol{x}_{em}^{(1)} = \boldsymbol{x}_{os}^{*}$, $\boldsymbol{B}^{(1)} = \mathbf{1}$, $\boldsymbol{f}^{(1)} = \boldsymbol{P}(\boldsymbol{x}_{em}^{(1)}) - \boldsymbol{x}_{os}^{*}$, $j = 1$; stop if $\|\boldsymbol{f}^{(1)}\| \leq \eta$

Step 1. solve $\boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$

Step 2. set $\boldsymbol{x}_{em}^{(j+1)} = \boldsymbol{x}_{em}^{(j)} + \boldsymbol{h}^{(j)}$

Step 3. evaluate $\boldsymbol{P}(\boldsymbol{x}_{em}^{(j+1)})$

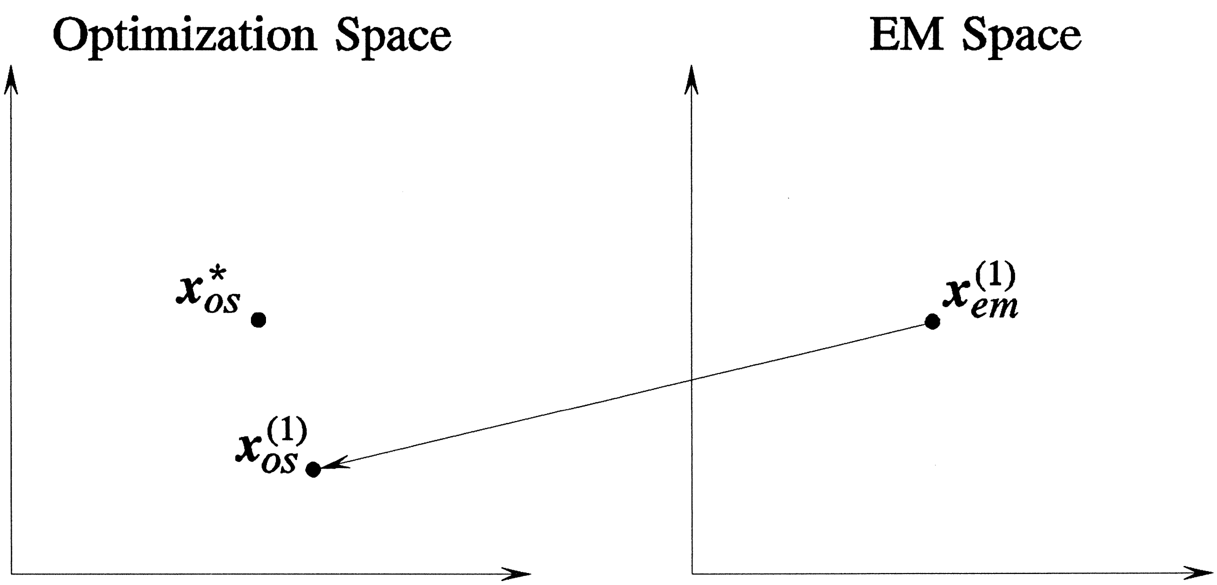
Step 4. compute $\boldsymbol{f}^{(j+1)} = \boldsymbol{P}(\boldsymbol{x}_{em}^{(j+1)}) - \boldsymbol{x}_{os}^{*}$; if $\|\boldsymbol{f}^{(j+1)}\| \leq \eta$, stop

Step 5. update $\boldsymbol{B}^{(j)}$ to $\boldsymbol{B}^{(j+1)}$

Step 6. set $j = j + 1$; go to *Step* 1

Illustration of Aggressive Space Mapping Optimization

Step 2



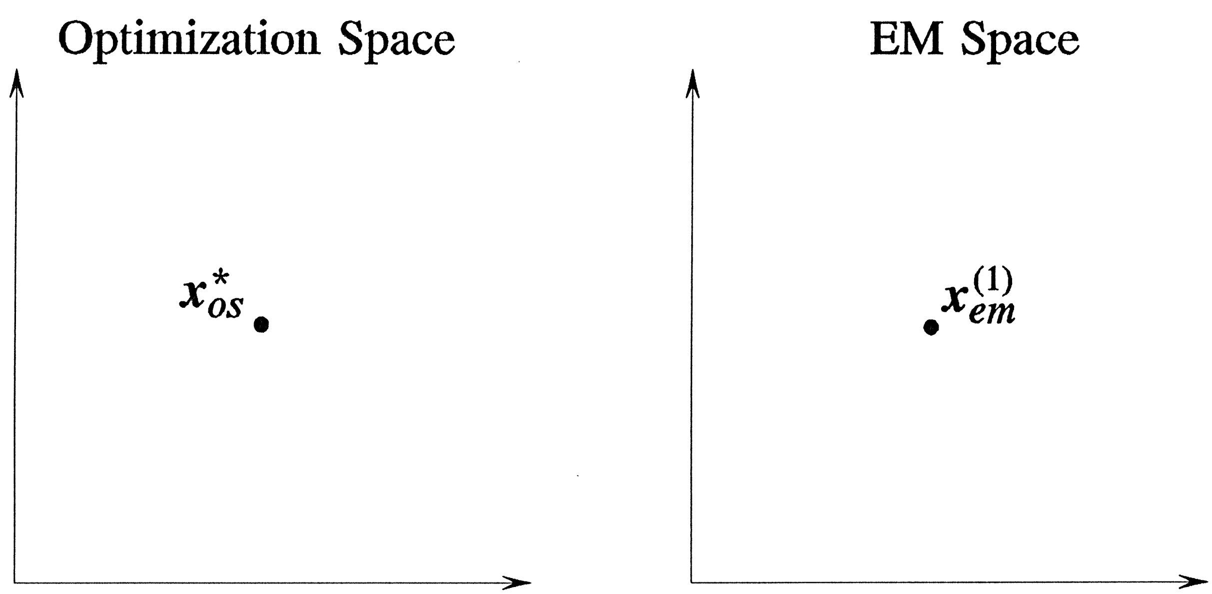
perform X_{os}-space model parameter extraction

Illustration of Aggressive Space Mapping Optimization

Step 0

find the optimal design x_{os}^* in Optimization Space

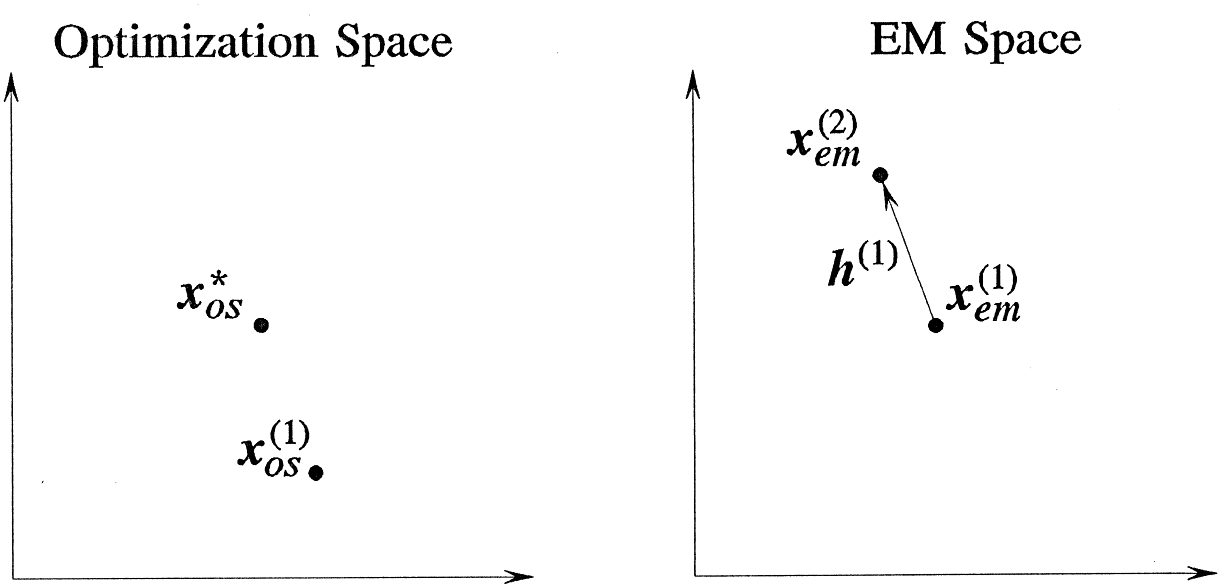
Step 1



set $x_{em}^{(1)} = x_{os}^*$ assuming x_{em} and x_{os} represent the same physical parameters

Illustration of Aggressive Space Mapping Optimization

Step 3



initialize Jacobian approximation $B^{(1)} = \mathbf{1}$

obtain $x^{(2)}_{em}$ by solving

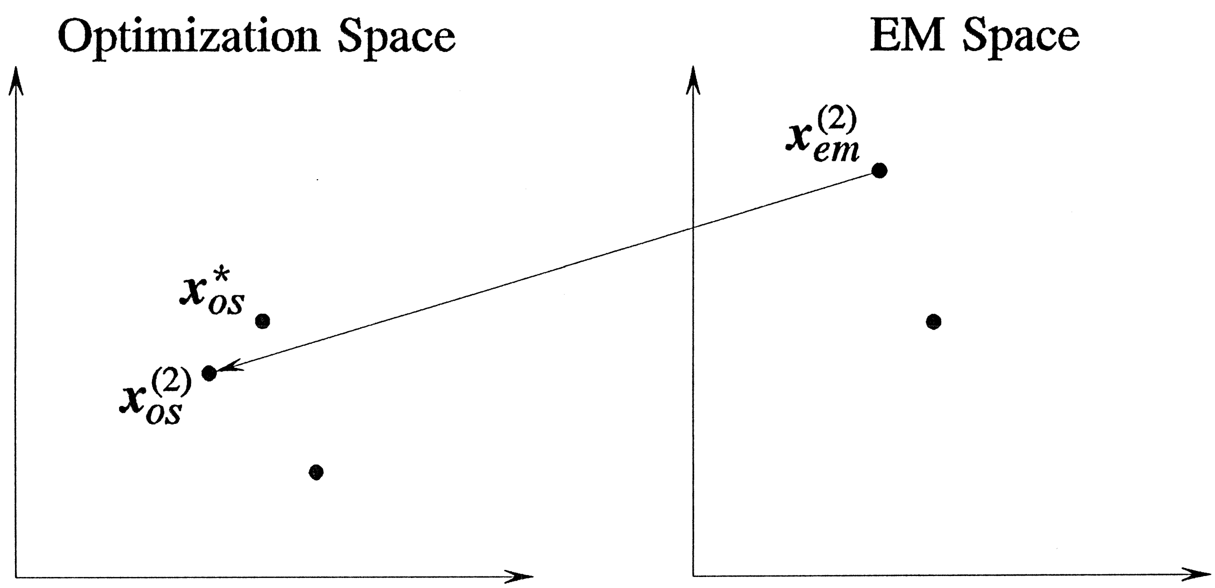
$$B^{(1)} h^{(1)} = -f^{(1)}$$

where

$$f^{(1)} = x^{(1)}_{os} - x^*_{os}$$

Illustration of Aggressive Space Mapping Optimization

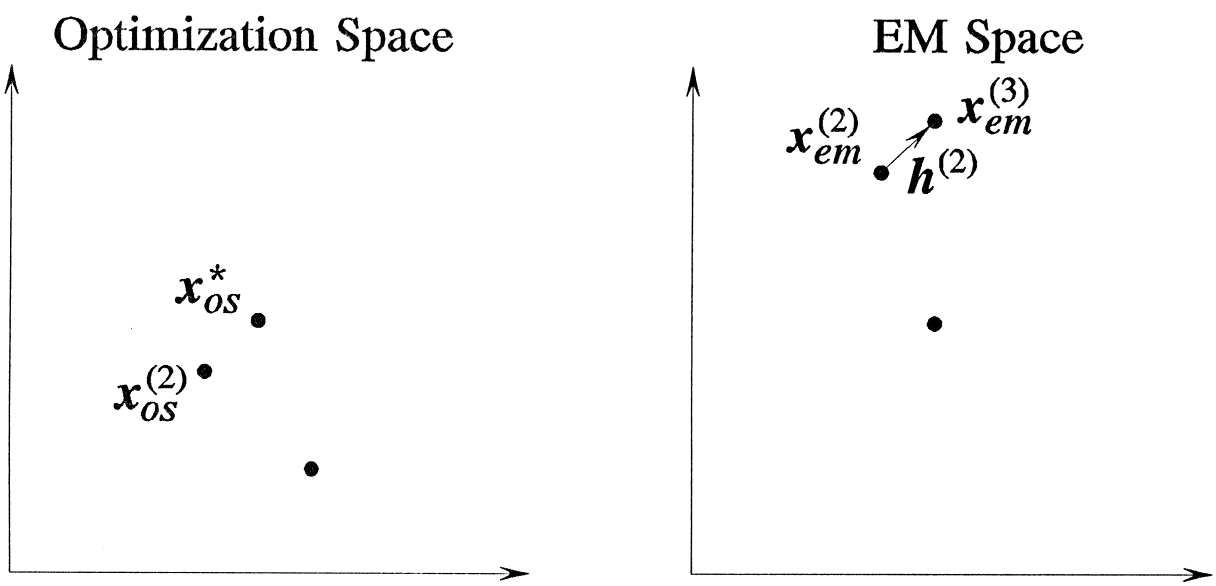
Step 4



perform X_{os}-space model parameter extraction

Illustration of Aggressive Space Mapping Optimization

Step 5



update Jacobian approximation from $B^{(1)}$ to $B^{(2)}$

obtain $x_{em}^{(3)}$ by solving

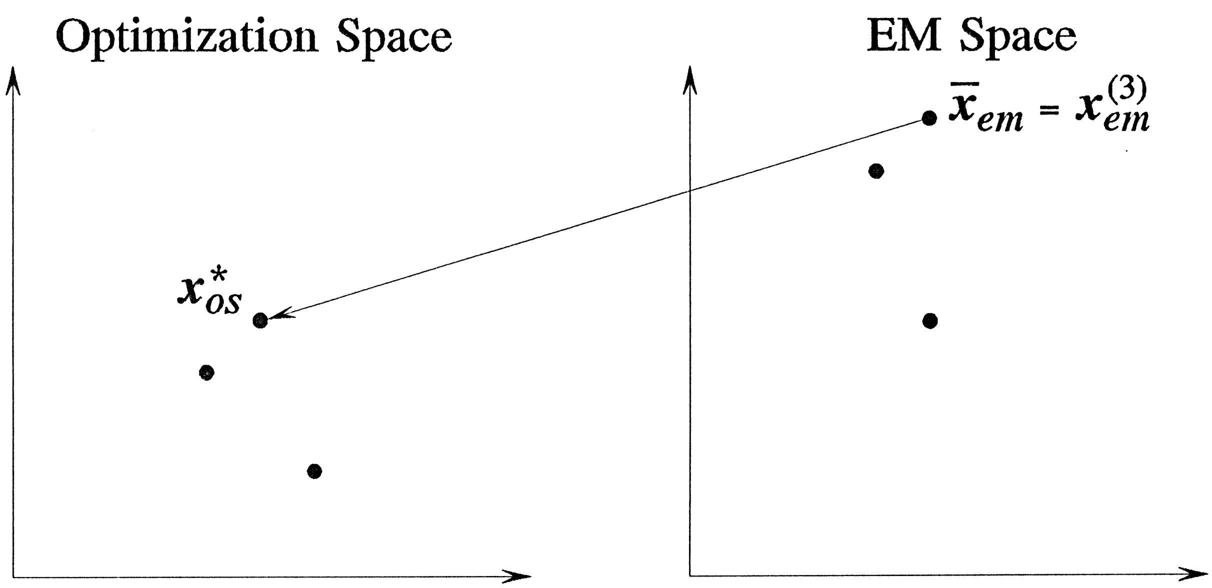
$$B^{(2)} h^{(2)} = -f^{(2)}$$

where

$$f^{(2)} = x_{os}^{(2)} - x_{os}^{*}$$

Illustration of Aggressive Space Mapping Optimization

Step 6



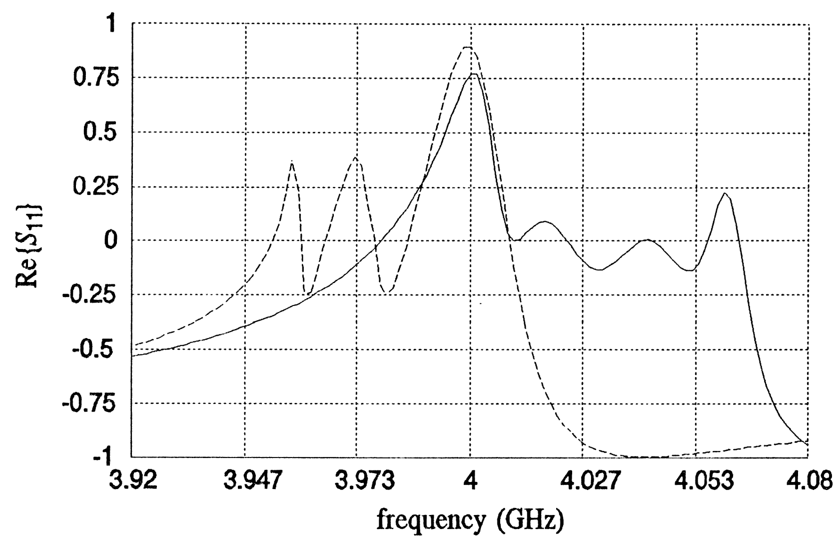
perform X_{os}-space model parameter extraction

if $\| x_{os}^{(3)} - x_{os}^{*} \| \leq \epsilon$ then $\bar{x}_{em} = x_{em}^{(3)}$ is considered as the SM solution

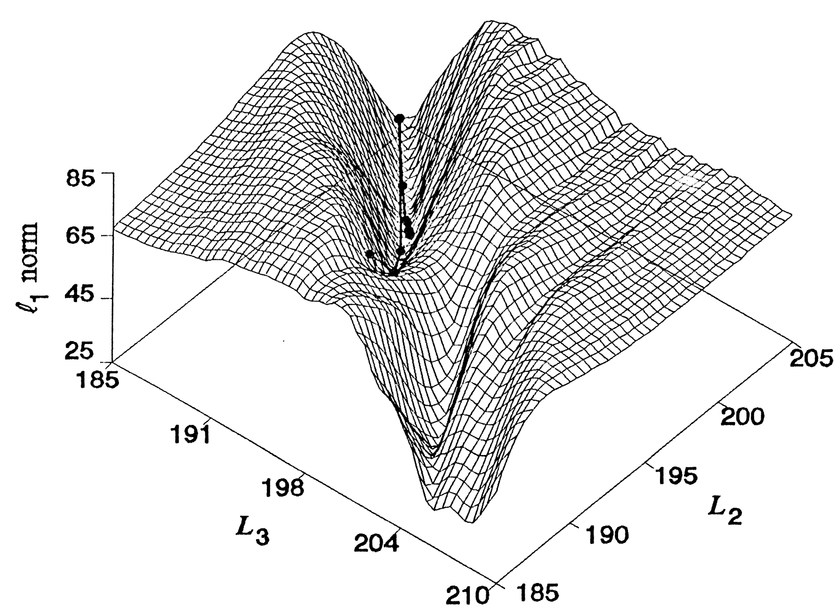
Frequency Space Mapping for Parameter Extraction

parameter extraction can be a serious challenge, especially at the starting point, if the model responses are misaligned

Re$\{S_{11}\}$ using OSA90/hope (—) and ***em*** (---) at $\boldsymbol{x}_{os}^*$



straightforward optimization from such a starting point can lead to a local minimum



Frequency Space Mapping - Mapping and Alignment

to better condition the parameter extraction subproblem

first, we align R_{os} and R_{em} along the frequency axis using

$$\omega_{os} = P_{\omega}(\omega)$$

this frequency space mapping can be as simple as

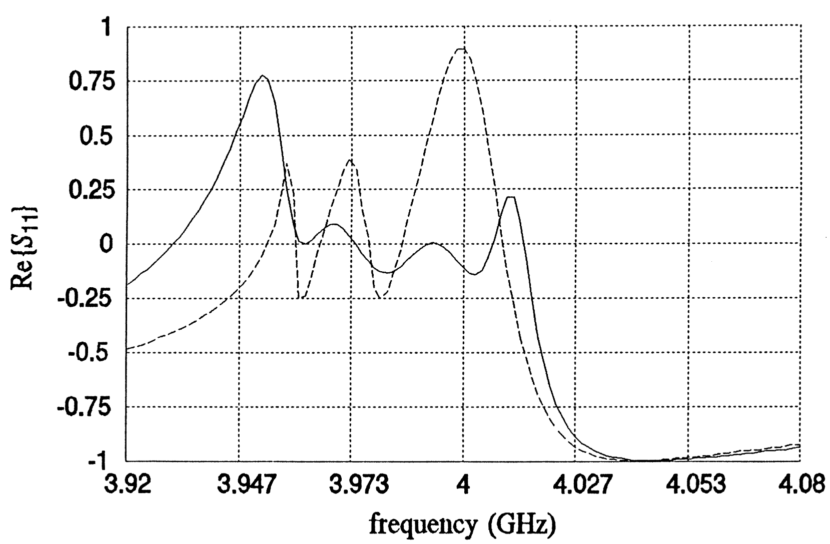
$$\omega_{os} = \sigma\,\omega + \delta$$

at the starting point, we determine σ_0 and δ_0 by

$$\underset{\sigma_0,\,\delta_0}{\text{minimize}}\ \| R_{os}(x_{os}, \sigma_0, \delta_0) - R_{em}(x_{em}) \|$$

where x_{os} and x_{em} are fixed and $x_{os} = x_{em}$

resulting alignment between OSA90/hope (—) and *em* (---):



Frequency Space Mapping:
Sequential FSM (SFSM) Algorithm

we perform a sequence of optimizations to gradually achieve the identity Frequency Space Mapping

we optimize x_{os} to match R_{os} and R_{em}:

$$\underset{x_{os}^{(j)}}{\text{minimize}} \; \| R_{os}(x_{os}^{(j)}, \sigma^{(j)}, \delta^{(j)}) - R_{em}(x_{em}) \|$$

the values $\sigma^{(j)}$ and $\delta^{(j)}$ are updated according to

$$\sigma^{(j)} = 1 + (\sigma_o - 1)\frac{(K - j)}{K}$$

and

$$\delta^{(j)} = \delta_o \frac{(K - j)}{K},$$

respectively, for $j = 0, 1, ..., K$

K determines the number of steps in the sequence

larger values of K increase the probability of success in the parameter extraction subproblem at the expense of longer optimization time

Frequency Space Mapping:
Exact Penalty Function (EPF) Algorithm

we perform only one optimization to achieve the identity Frequency Space Mapping and optimize $\boldsymbol{x}_{os}$ to match $\boldsymbol{R}_{os}$ to $\boldsymbol{R}_{em}$

the ℓ_1 norm version of the EPF formulation is given by

$$\underset{x_{os},\, \sigma,\, \delta}{\text{minimize}} \quad \{\| R_{os}(x_{os}, \sigma, \delta) - R_{em}(x_{em}) \|_1 + \alpha_1 | \sigma - 1 | + \alpha_2 | \delta | \}$$

the minimax version is given by

$$\underset{x_{os},\, \sigma,\, \delta}{\text{minimize}} \left\{ \max_{1 \le i \le 4} \left[U(x_{os}, \sigma, \delta),\; U(x_{os}, \sigma, \delta) - \alpha_i g_i \right] \right\}$$

where

$$U(x_{os}, \sigma, \delta) = \| R_{os}(x_{os}, \sigma, \delta) - R_{em}(x_{em}) \|$$

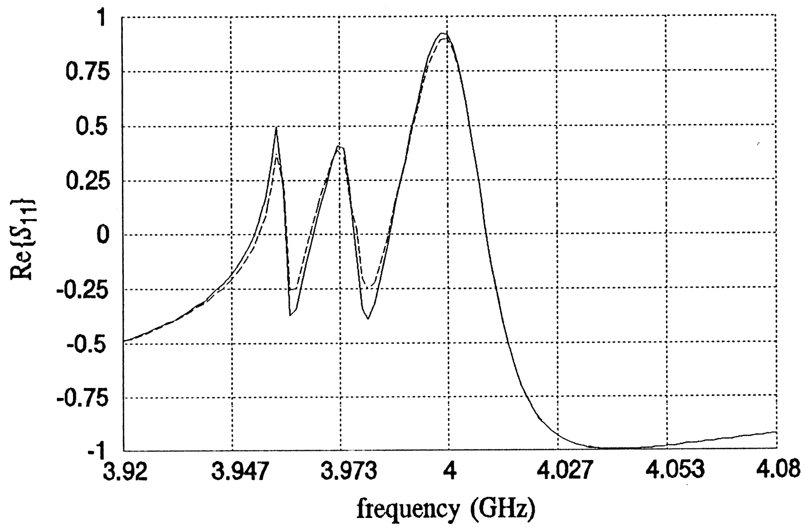
and

$$g(\sigma, \delta) = \begin{bmatrix} \sigma - 1 \\ 1 - \sigma \\ \delta \\ -\delta \end{bmatrix}$$

in both EPF formulations, α_i are kept fixed and must be sufficiently large to obtain the identity mapping and hence the solution to the parameter extraction problem

Frequency Space Mapping - Results

Re$\{S_{11}\}$ using OSA90/hope (—) and *em* (---)



resulting match after applying the FSM algorithm