A NOVEL APPROACH TO STATISTICAL MODELING USING CUMULATIVE PROBABILITY DISTRIBUTION FITTING

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Abstract

We present a novel approach to statistical modeling. The statistical model is directly extracted by fitting the cumulative probability distributions (CPDs) of the model responses to those of the measured data. This new technique is based on a solid mathematical foundation and, therefore, should prove more reliable and robust than the existing methods. The approach is illustrated by statistical MESFET modeling based on a physics-oriented model which combines the modified Khatibzadeh and Trew model and the Ladbrooke model (KTL). The approach is compared with the established parameter extraction/postprocessing approach (PEP) in the context of yield verification.

Introduction

random variations in the manufacturing environment result in complicated distributions and correlations of device responses

statistical modeling is a prerequisite for statistical analysis and yield optimization

device model types for statistical modeling:

equivalent circuit models (ECMs)
physics-based models (PBMs)
data bases

indirect statistical modeling

parameter extraction/postprocessing (PEP)

we present a novel approach: direct statistical modeling

cumulative probability distribution (CPD) fitting

Comparison of Statistical Modeling Methods

indirect statistical modeling

optimization is applied to extract parameters of individual devices

the parameter statistics are obtained by postprocessing the resulting models

optimization variables are the parameter values of individual devices

statistical model may not be accurate even though the individual device fitting is excellent

direct statistical modeling

optimization is applied to fit the statistics (e.g., CPD) of model responses to those of the measured data

the parameter statistics are obtained directly from statistical fitting

optimization variables are parameter statistics such as mean values and standard deviations

based on a solid mathematical foundation, more reliable and robust

Definition of CPD and Matching Error

CPD (cumulative probability distribution)

the CPD of a sample of data $S = [X_1 X_2 \ldots X_n]^T$ is defined as

$$C(x) = \frac{n_x}{n}$$

where n_x is the number of data points in S which are smaller than or equal to x

CPD matching error

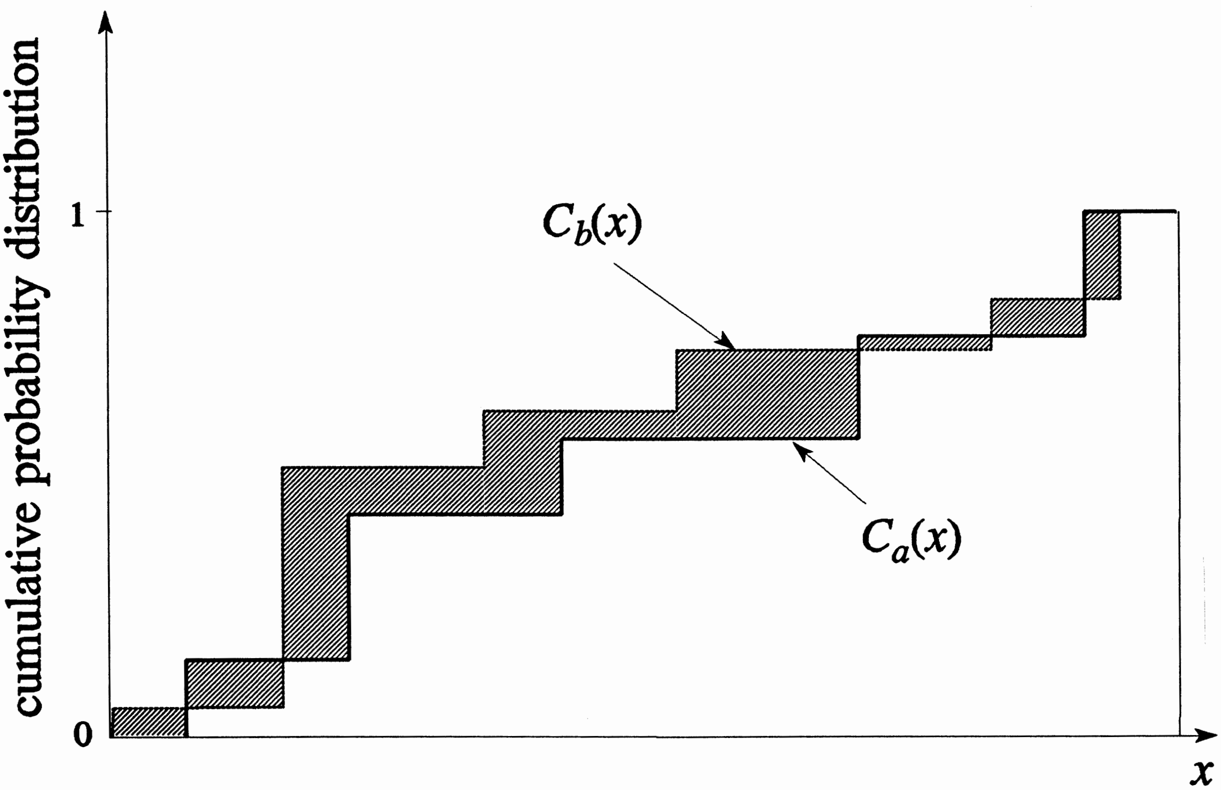
the matching error between two CPDs $C_a(x)$ and $C_b(x)$ is defined as

$$e_{ab} = \int_{-\infty}^{\infty} D_{ab}(x)\, dx$$

where $D_{ab}(x)$ is the distance between $C_a(x)$ and $C_b(x)$ at the point x calculated as

$$D_{ab}(x) = | C_a(x) - C_b(x) |$$

CPD and Matching Error



$C_a(x)$ and $C_b(x)$ are two cumulative probability distributions

CPD matching error is indicated by the shaded area

Formulation of CPD Fitting for Statistical Modeling

n_r measured responses for n_{mo} manufactured outcomes

$$S_i = [X_{i1}\, X_{i2} \ldots X_{in_{mo}}]^T, \quad i = 1, 2, \ldots, n_r$$

n_{so} Monte Carlo outcomes simulated from the model

$$R_i(\phi) = [R_{i1}(\phi)\, R_{i2}(\phi) \ldots R_{in_{so}}(\phi)]^T$$

where $\phi = [\phi_1\, \phi_2 \ldots \phi_{n_\phi}]^T$ is the set of optimization variables such as the mean values and standard deviations

CPD matching error

$$e(\phi) = [e_1(\phi)\; e_2(\phi)\; \ldots\; e_{n_r}(\phi)]^T$$

where $e_i(\phi)$ is the CPD matching error between S_i and R_i

optimization problem of CPD fitting for statistical modeling

$$\underset{\phi}{minimize} \quad U(\phi) \triangleq H[e(\phi)]$$

where $H[e(\phi)]$ is a norm of $e(\phi)$ (e.g., ℓ_1, ℓ_2 or Huber norm)

Statistical Modeling of GaAs MESFET

the combined Khatibzadeh and Trew/Ladbrooke model (KTL)

alignment of Plessey data to a consistent bias condition for statistical modeling

35 individual devices containing the *S* parameters from 1 GHz to 21 GHz with 2 GHz step under the bias condition of V_{gs} = -0.7 V and V_{ds} = 5 V

16 statistical parameters assuming normal distributions

32 optimization variables, namely the mean values and standard deviations of all 16 statistical parameters

initial values for the means and standard deviations were estimated from PEP based on 15 devices

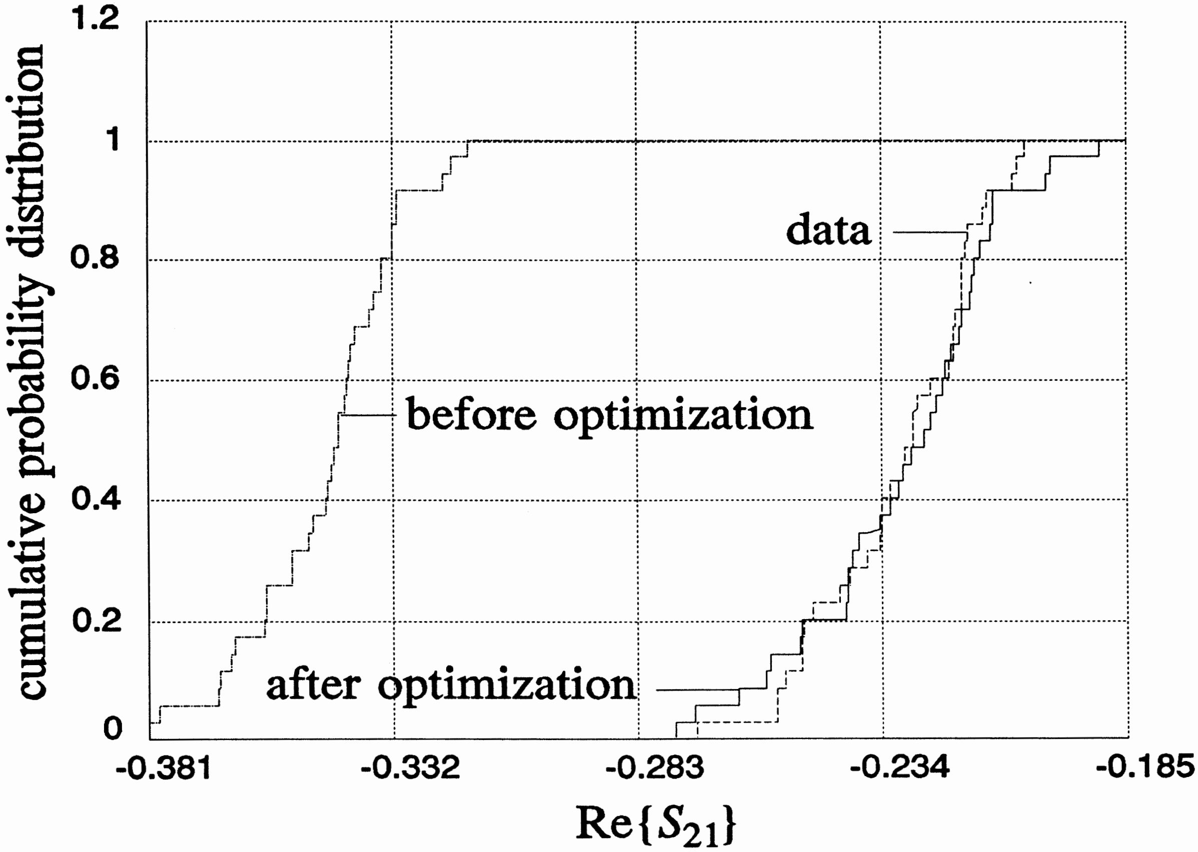
the correlation matrix obtained by PEP was assumed to be valid for the model and was kept fixed during CPD fitting

TABLE I
CPD OPTIMIZED KTL MODEL PARAMETERS

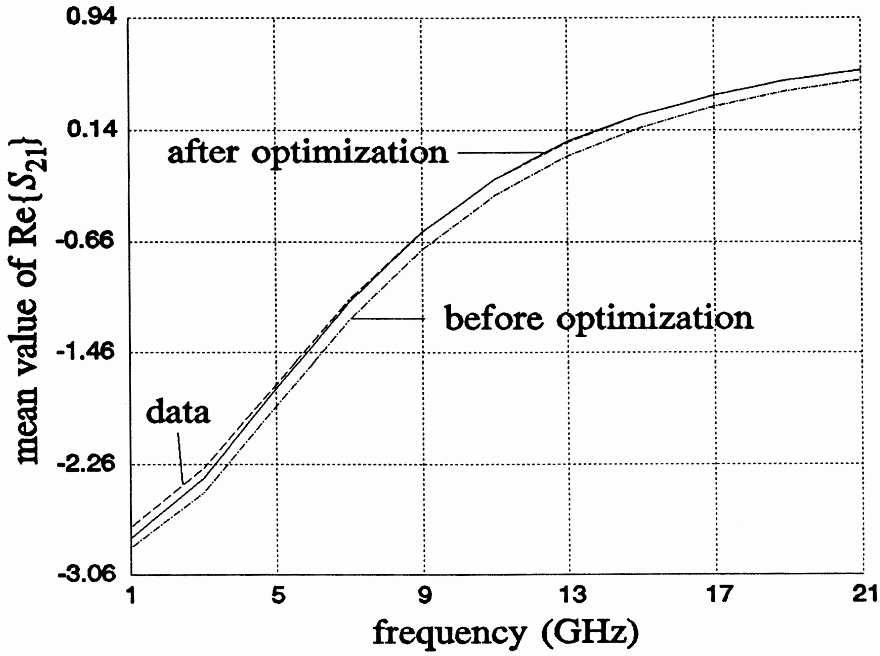
Parameter	Mean	σ(%)	Parameter	Mean	σ(%)
L(μm)	0.4685	3.57	C_{ds}(pF)	0.0547	1.58
a(μm)	0.1308	5.19	C_{ge}(pF)	0.0807	5.92
N_d(m^{-3})	2.3×10^{23}	3.25	C_{de}(pF)	0.0098	6.22
v_{sat}(m/s)	10.5×10^{4}	2.27	C_x(pF)	2.4231	4.03
μ_0(m^2/Vns)	6.5×10^{-10}	2.16	Z(μm)	300	*
L_{G0}(nH)	0.0396	10.9	ε	12.9	*
R_d(Ω)	1.2867	4.32	V_{b0}(V)	0.6	*
R_s(Ω)	3.9119	1.91	r_{01}(Ω/V^2)	0.35	*
R_g(Ω)	8.1718	0.77	r_{02}(Ω)	2003	*
L_d(nH)	0.0659	5.74	r_{03}(Ω)	7.0	*
L_s(nH)	0.0409	5.49	a_0	1.0	*
G_{ds}(1/Ω)	3.9×10^{-3}	1.78			

L, Z, a	gate length, gate width, channel thickness
N_d, V_{b0}	doping density, zero-bias barrier potential
v_{sat}	saturation electron drift velocity
μ_0, ε	low-field mobility, dielectric constant
L_{G0}	inductance from gate bond wires and pads
a_0, r_{01}, r_{02}, r_{03}	fitting coefficients
R_d, R_s, R_g, L_d, L_s, G_{ds}, C_{ds}, C_{ge}, C_{de}, C_x	extrinsic elements
σ	standard deviation
*	fixed (non-statistical) parameters

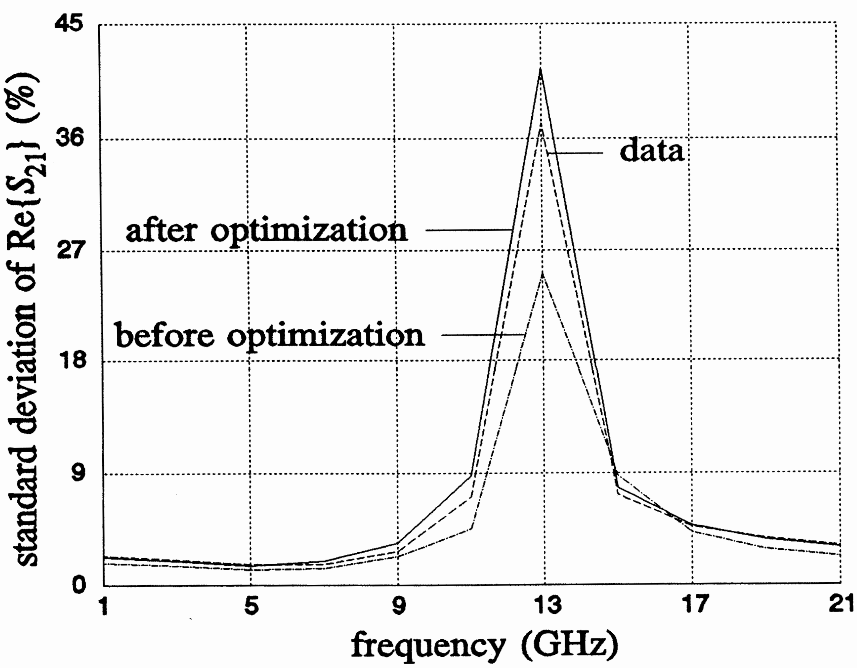
CPDs of Re{S_{21}} at 11 GHz from Data and KTL



Mean Values of Re{S_{21}} vs. Frequency from Data and KTL

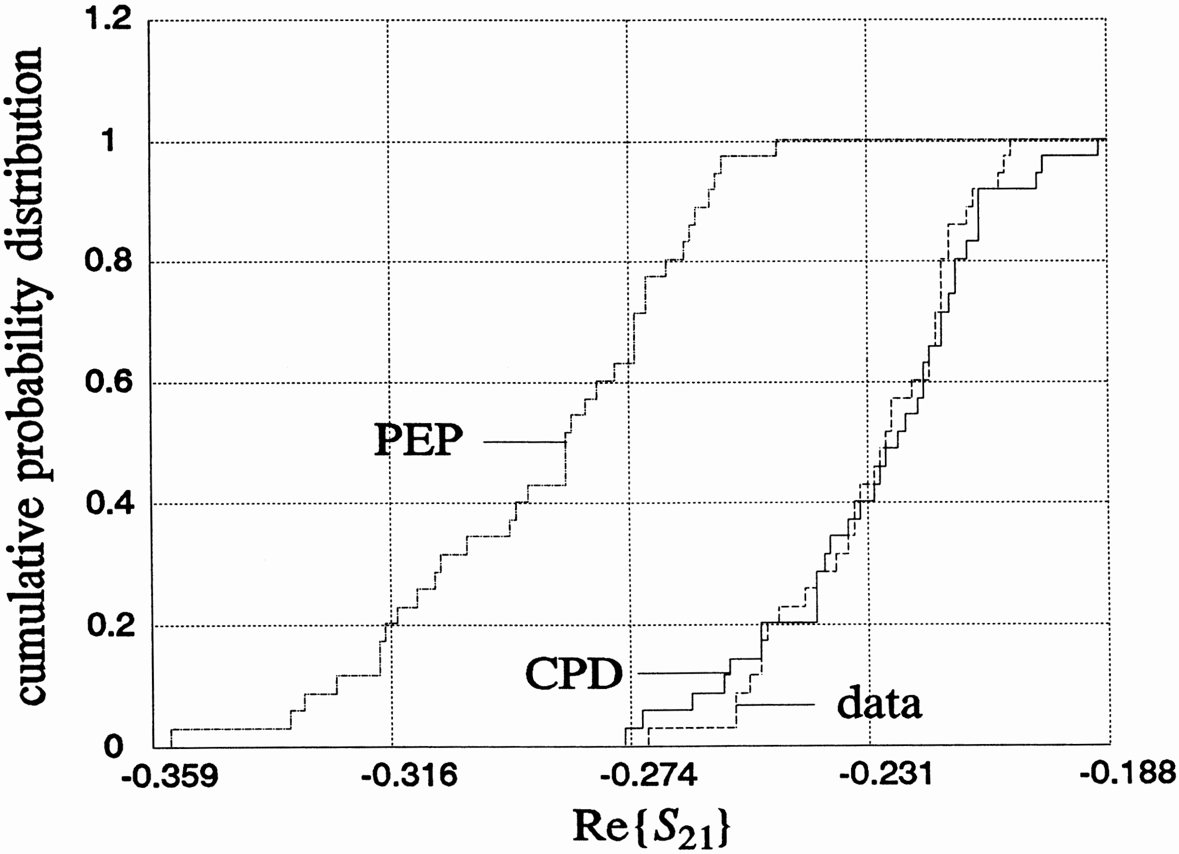


Standard Dev. of Re{S_{21}} vs. Frequency from Data and KTL

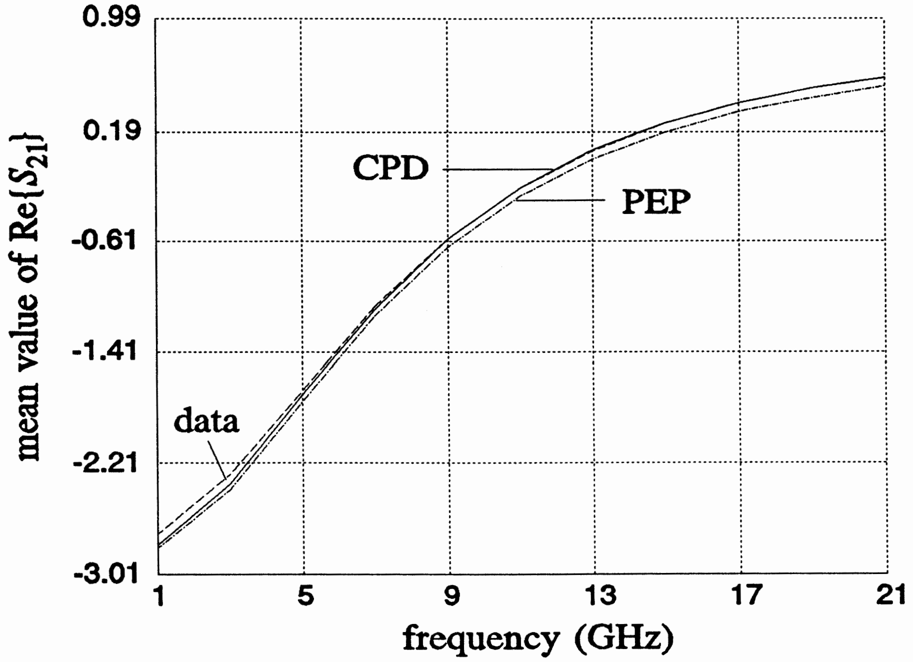


Comparison between CPD Method and PEP Method

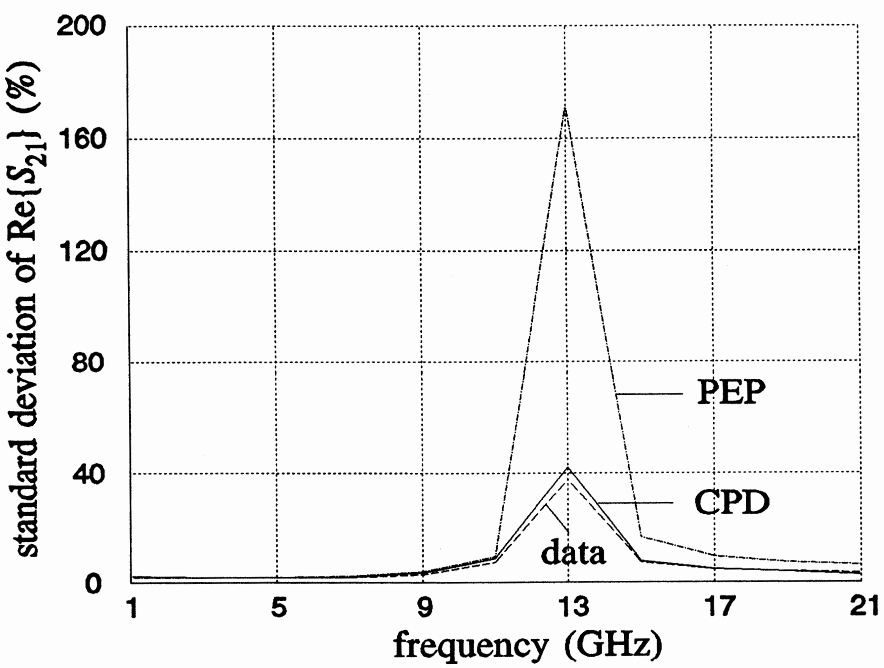
CPDs of Re{S_{21}} at 11 GHz from CPD and PEP



Mean Values of Re{S_{21}} vs. Frequency from CPD and PEP



Standard Dev. of Re{S_{21}} vs. Frequency from CPD and PEP



Yield Verification

ultimate goal of statistical modeling is to provide accurate statistical models for yield optimization

verification of statistical model by comparing the yield estimations by the model and data

yield estimation using Monte Carlo simulation

yield optimization of a small-signal broadband amplifier (passband: 8 GHz - 12 GHz) using OSA90/hope

yield optimization was performed using the two statistical KTL models (CPD and PEP) w.r.t. three different specifications

the yields predicted by both models are in good agreement for every specification

TABLE II
YIELD PREDICTED BY THE KTL MODELS AND VERIFIED BY DATA

Spec.	Before Optimization CPD (%)	Before Optimization PEP (%)	Before Optimization Data (%)	After Optimization CPD (%)	After Optimization PEP (%)	After Optimization Data (%)
Sp. 1	22	26	28.6	71	69.5	77.6
Sp. 2	30	38.5	37.1	76.5	78.5	90.9
Sp. 3	64.5	67.5	76.7	98.5	93.5	99.5

Sp. 1: 7.5 dB $< |S_{21}| <$ 8.5 dB, $|S_{11}| < 0.5$, $|S_{22}| < 0.5$.

Sp. 2: 6.5 dB $< |S_{21}| <$ 7.5 dB, $|S_{11}| < 0.5$, $|S_{22}| < 0.5$.

Sp. 3: 6.0 dB $< |S_{21}| <$ 8.0 dB, $|S_{11}| < 0.5$, $|S_{22}| < 0.5$.

200 outcomes are used for yield prediction by the statistical KTL model, 210 for yield verification using the device data.

Conclusions

a novel approach to statistical modeling using cumulative probability distribution fitting

parameter statistics such as mean values and standard deviations are directly optimized

our new approach avoids parameter extraction of individual devices and, therefore, is not affected by possible pitfalls of the parameter extraction process

the new CPD technique is based on a solid mathematical foundation and, therefore, should prove more reliable and robust

in principle the proposed method can be applied with any statistical distributions

we have extended the new technique to statistical modeling using histogram fitting

our investigations set the stage for further research:

- application of Huber optimization to statistical fitting
- optimization of correlation coefficients
- statistical modeling using yield matching

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