ROBUSTIZING CIRCUIT OPTIMIZATION USING HUBER FUNCTIONS

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Abstract

We introduce a novel approach to "robustizing" circuit optimization using Huber functions: both two-sided and one-sided. We compare Huber optimization with ℓ_1, ℓ_2 and minimax methods in the presence of faults, large and small measurement errors, bad starting points and statistical uncertainties. We demonstrate FET statistical modeling, multiplexer optimization, analog fault location and data fitting.

Introduction

designers are concerned with the robustness of numerical optimization techniques

engineering data is frequently contaminated by model/measurement/statistical errors

the classical least-squares method is vulnerable to gross errors: a few wild data points can alter the least-squares solution significantly

the ℓ_1 method is robust against gross errors, but it can be undesirably biased when the data contains many small variations

the Huber optimization is more robust than ℓ_2 w.r.t. large errors, and smoother, less biased than ℓ_1

the benefits of this novel approach are demonstrated by FET statistical modeling, analog fault location and data fitting

the one-sided Huber function can be more effective and efficient than minimax in overcoming a bad starting point for large-scale optimization problems

Huber Functions

$$\rho_k(f) = \begin{cases} f^2/2 & \text{if } |f| \le k \\ k|f| - k^2/2 & \text{if } |f| > k \end{cases}$$

where k is a positive constant

the Huber function ρ_k is a hybrid of the least-squares (ℓ_2) (when $|f| \le k$) and the ℓ_1 (when $|f| > k$)

if k is sufficiently large, the Huber function becomes the ℓ_2 function; if k approaches zero, ρ_k approaches the ℓ_1 function

Huber Optimization

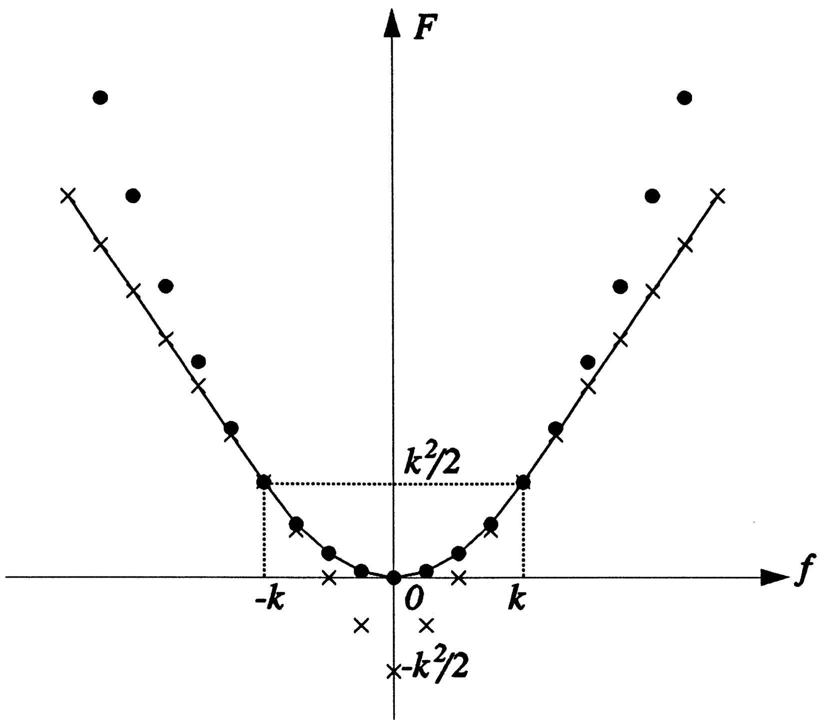
$$\underset{x}{\text{minimize}} \quad F(x) \triangleq \sum_{j=1}^{m} \rho_k(f_j(x))$$

where

$x = [x_1\, x_2 \ldots x_n]^T$ is the set of variables

$f_j \;\; j = 1, 2, \ldots, m$, are the error functions

Huber Function as a Hybrid ℓ_1 / ℓ_2



the Huber, ℓ_1 and ℓ_2 objective functions in the one-dimensional case

the strikes represent the discrete points on the ℓ_1 curve

the dots represent the discrete points on the ℓ_2 curve

the continuous curve indicates the Huber objective function

Comparison in Data Fitting

approximating $\sqrt{t}$ by a rational function for $0 \le t \le 1$

large errors deliberately introduced at 5 of the sample points and small variations added to the remaining data

the ℓ_1, ℓ_2 and Huber approximations are compared

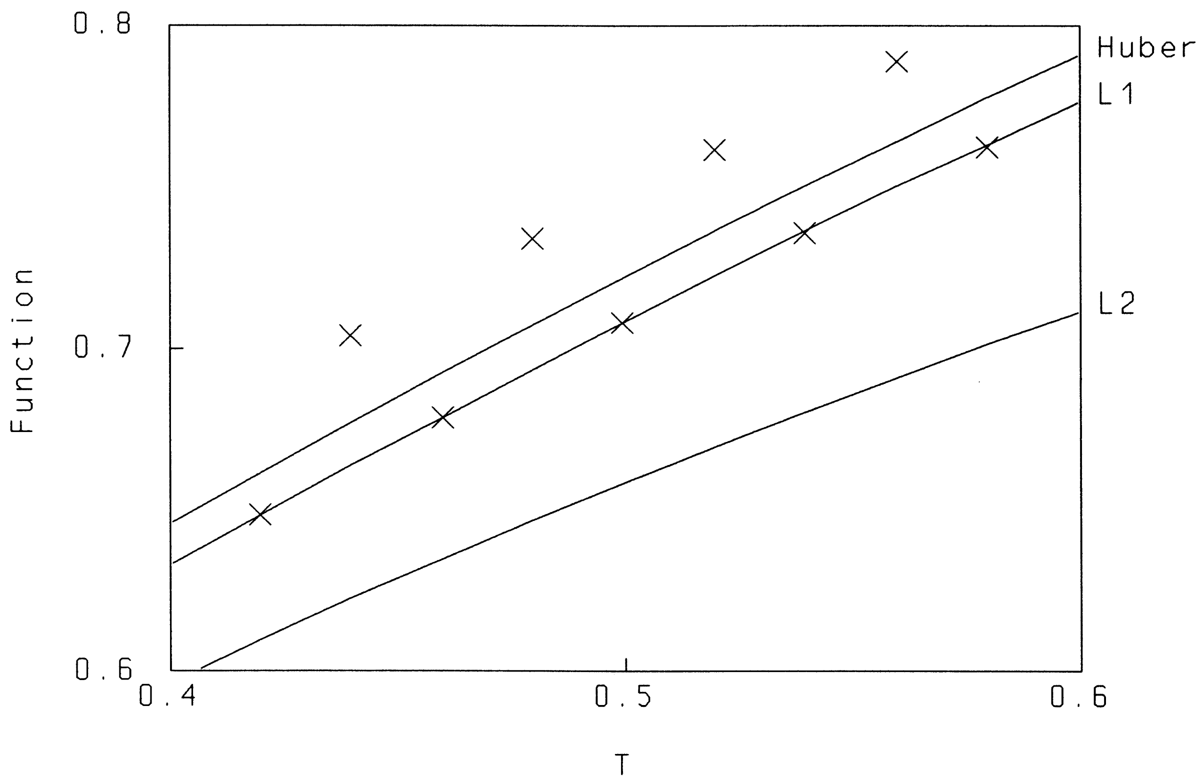
ℓ_2 solution suffers significantly from the presence of gross errors

ℓ_1 solution is dictated by a subset of residual functions which have zero values at the solution and all the nonzero residuals are treated as large errors

the Huber solution provides a flexible combination of the robustness of the ℓ_1 and the unbiasedness of the ℓ_2

the Huber solution is equivalent to an ℓ_2 solution with the gross errors reduced to the threshold value k

ℓ_1, ℓ_2 and Huber Data Fitting



ℓ_1, ℓ_2 and Huber solutions for data fitting in the presence of large and small errors: an enlarged view

Huber Estimator for Statistical Modeling

use the Huber function as an automated robust estimator for FET statistical modeling

model parameters are extracted from the measurements of 80 FETs using HarPE™ and then postprocessed to estimate the parameter statistics

to estimate the mean we define the error functions as

$$f_j(\bar{\phi}) = \bar{\phi} - \phi^j, \quad j = 1, 2, ..., N$$

where ϕ^j is the extracted parameter value for the jth device and N is the total number of devices

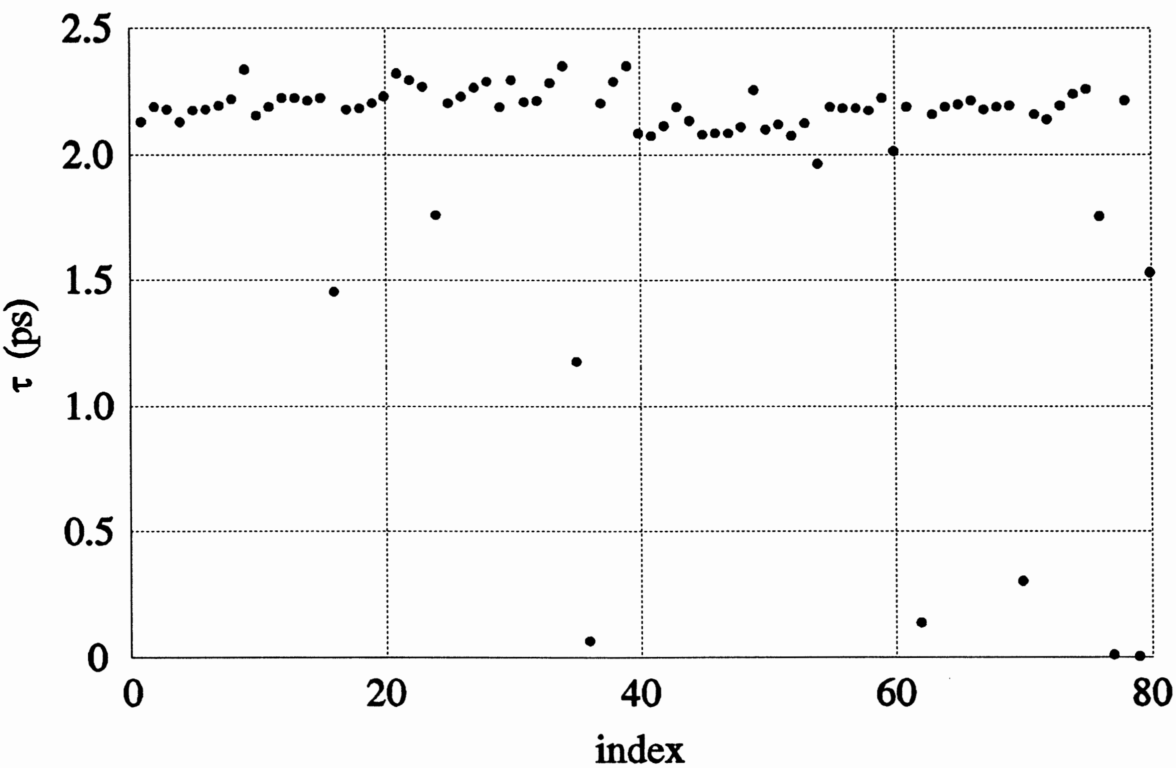
choose threshold k for the Huber function according to the normal spread of the parameter values (e.g., $k = 0.25$ for τ)

to estimate the variance we define the error functions as

$$f_j(V_\phi) = V_\phi - (\phi^j - \bar{\phi})^2, \quad j = 1, 2, ..., N$$

where V_ϕ denotes the estimated variance

Data Containing Wild Points



run chart of the extracted FET time-delay τ

a few abnormal values in the data due to faulty devices and/or gross measurement errors

manual deletion has been adopted to exclude abnormal data points in earlier work using ℓ_2 estimator

Statistical Modeling Using Huber Estimator

ESTIMATED STATISTICS OF SELECTED FET PARAMETERS

Parameter	$\bar{\phi}(\ell_2)$	$\bar{\phi}(H)$	$\bar{\phi}(\ell_2^*)$	$\sigma_\phi(\ell_2)$	$\sigma_\phi(H)$	$\sigma_\phi(\ell_2^*)$
L_G(nH)	0.04387	0.03464	0.03429	94.6%	21.8%	17.4%
G_{DS}(1/KΩ)	1.840	1.820	1.839	28.6%	6.3%	4.9%
I_{DSS}(mA)	47.36	47.53	47.85	14.0%	12.7%	11.3%
τ(ps)	2.018	2.154	2.187	26.3%	5.8%	3.4%
C_{10}(pF)	0.3618	0.3658	0.3696	8.2%	4.6%	3.5%
K_1	1.2328	1.231	1.233	15.5%	10.8%	8.7%

$\bar{\phi}$ denotes the mean and σ_ϕ the standard deviation
H denotes Huber estimates
ℓ_2^* denotes ℓ_2 estimates after 11 abnormal data sets are manually excluded

ℓ_2 and Huber estimates of the statistics for selected model parameters

Huber estimator does not require manual manipulation of the data and is more appropriate when there are data points which cannot be clearly classified as abnormal

Analog Diagnosis Using Huber Optimization

penalty function approach

$$\underset{x}{\text{minimize}} \sum_{j=1}^{n+K} \rho_k(f_j(x))$$

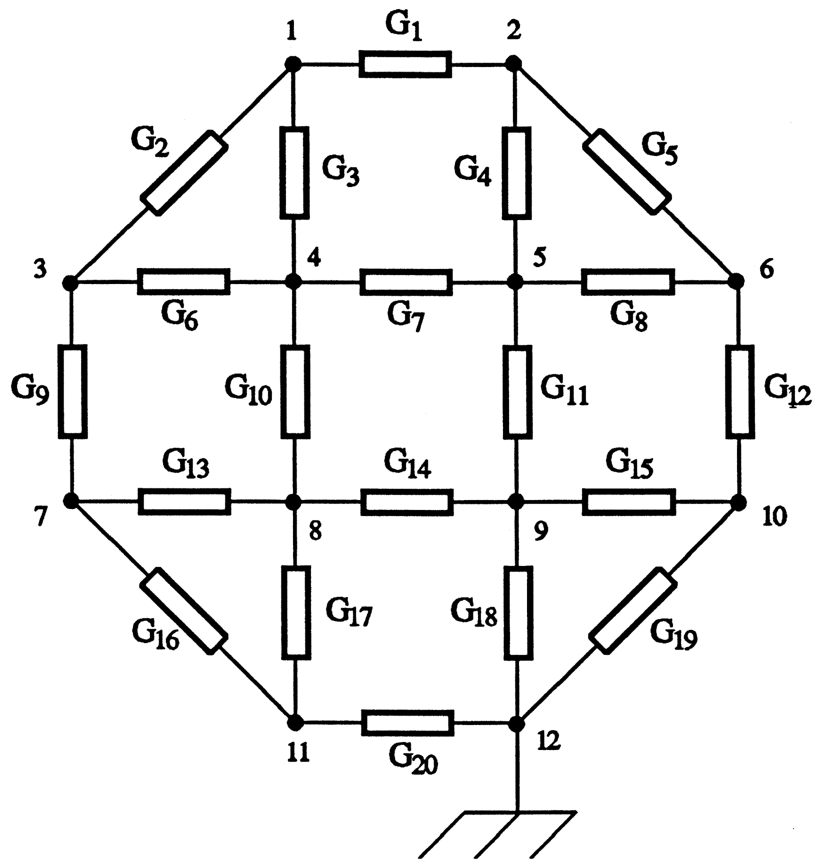
where

$$f_i(x) = \Delta x_i / x_i^0, \quad i = 1, 2, ..., n$$

$$f_{n+i}(x) = \beta_i (V_i^c - V_i^m), \quad i = 1, 2, ..., K$$

β_i	appropriate multipliers for the penalty terms
$x \equiv [x_1 \, x_2 \, ... \, x_n]^T$	vector of circuit parameters
x^0	nominal values
$\Delta x \equiv x - x^0$	deviations from the nominal
$V_1^m, ..., V_K^m$	K measurements
$V_1^c, ..., V_K^c$	calculated circuit responses

Analog Diagnosis Using Huber Optimization



resistive mesh circuit

only external nodes are available for excitation and measurements

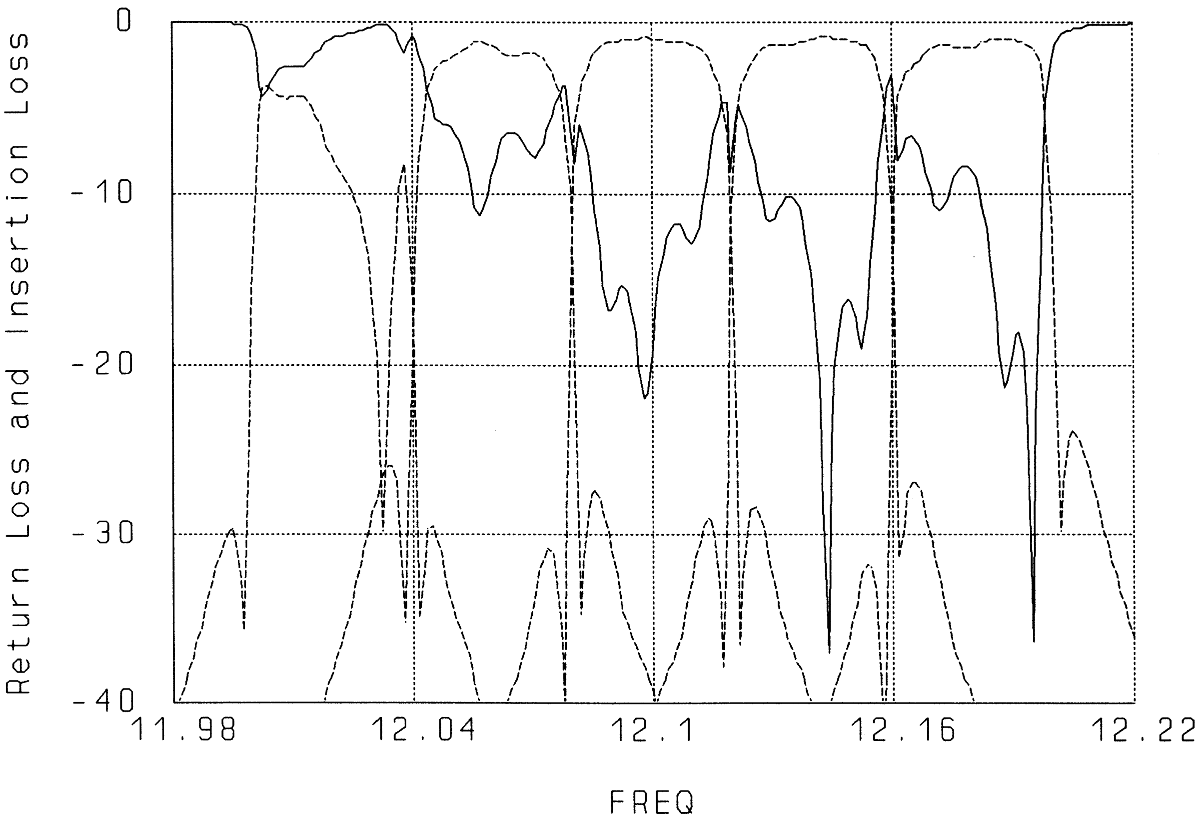
Huber Solution of Analog Diagnosis Problem

FAULT LOCATION OF THE RESISTIVE MESH CIRCUIT

Element	Nominal Value	Actual Value	Percentage Deviation		
			Actual	ℓ_1	Huber
G_1	1.0	0.98	-2.0	0.00	-0.11
G_2	1.0	0.50	-50.0*	-48.89	-47.28
G_3	1.0	1.04	4.0	0.00	-2.46
G_4	1.0	0.97	-3.0	0.00	-1.18
G_5	1.0	0.95	-5.0	-2.70	-3.16
G_6	1.0	0.99	-1.0	0.00	-0.06
G_7	1.0	1.02	2.0	0.00	-0.19
G_8	1.0	1.05	5.0	0.00	-0.41
G_9	1.0	1.02	2.0	2.41	3.75
G_{10}	1.0	0.98	-2.0	0.00	0.39
G_{11}	1.0	1.04	4.0	0.00	-0.37
G_{12}	1.0	1.01	1.0	2.73	1.32
G_{13}	1.0	0.99	-1.0	0.00	-0.26
G_{14}	1.0	0.98	-2.0	0.00	-0.50
G_{15}	1.0	1.02	2.0	0.00	-0.05
G_{16}	1.0	0.96	-4.0	-3.36	-2.67
G_{17}	1.0	1.02	2.0	0.00	-0.61
G_{18}	1.0	0.50	-50.0*	-50.09	-47.33
G_{19}	1.0	0.98	-2.0	-1.41	-3.81
G_{20}	1.0	0.96	-4.0	-4.40	-4.72

* Faults

Five-channel 12 GHz Waveguide Manifold Multiplexer



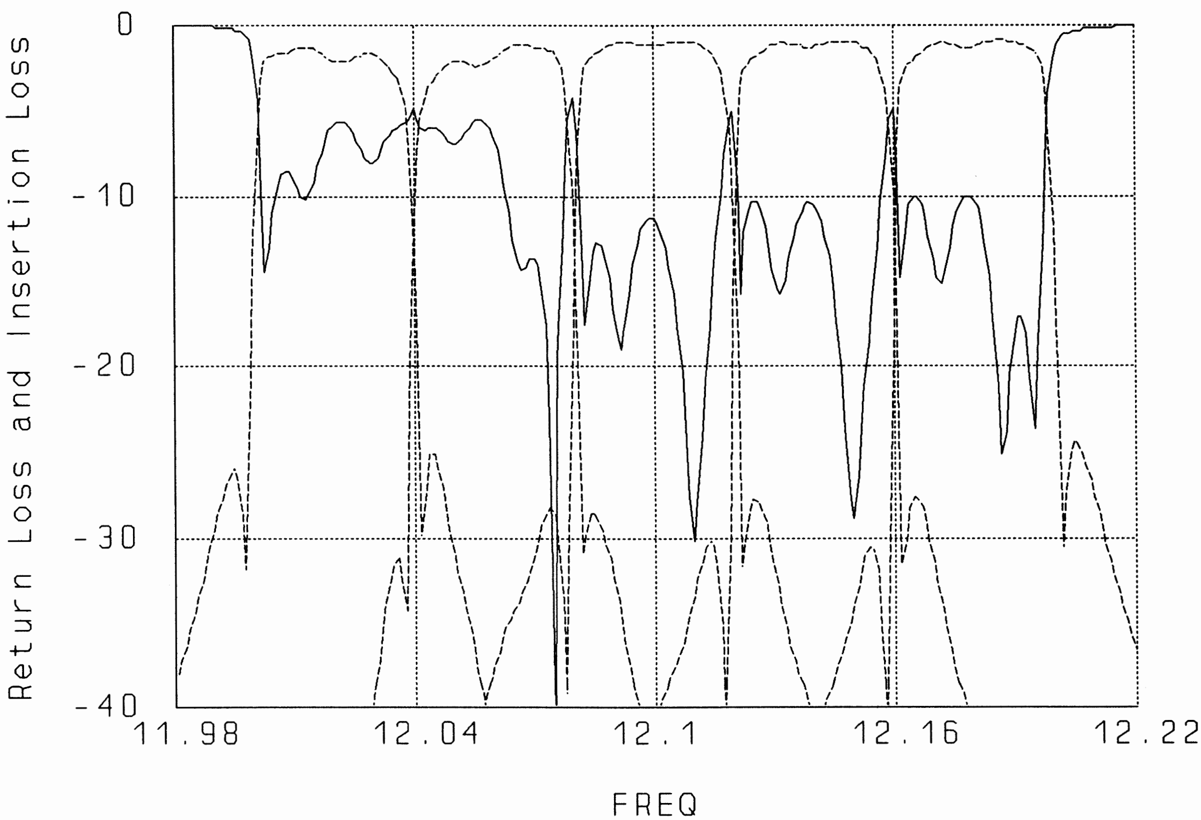
responses before optimization

(_____) common port return loss

(------) individual channel insertion losses

75 optimizable variables

Partial Minimax Optimization of the 5-Channel Multiplexer

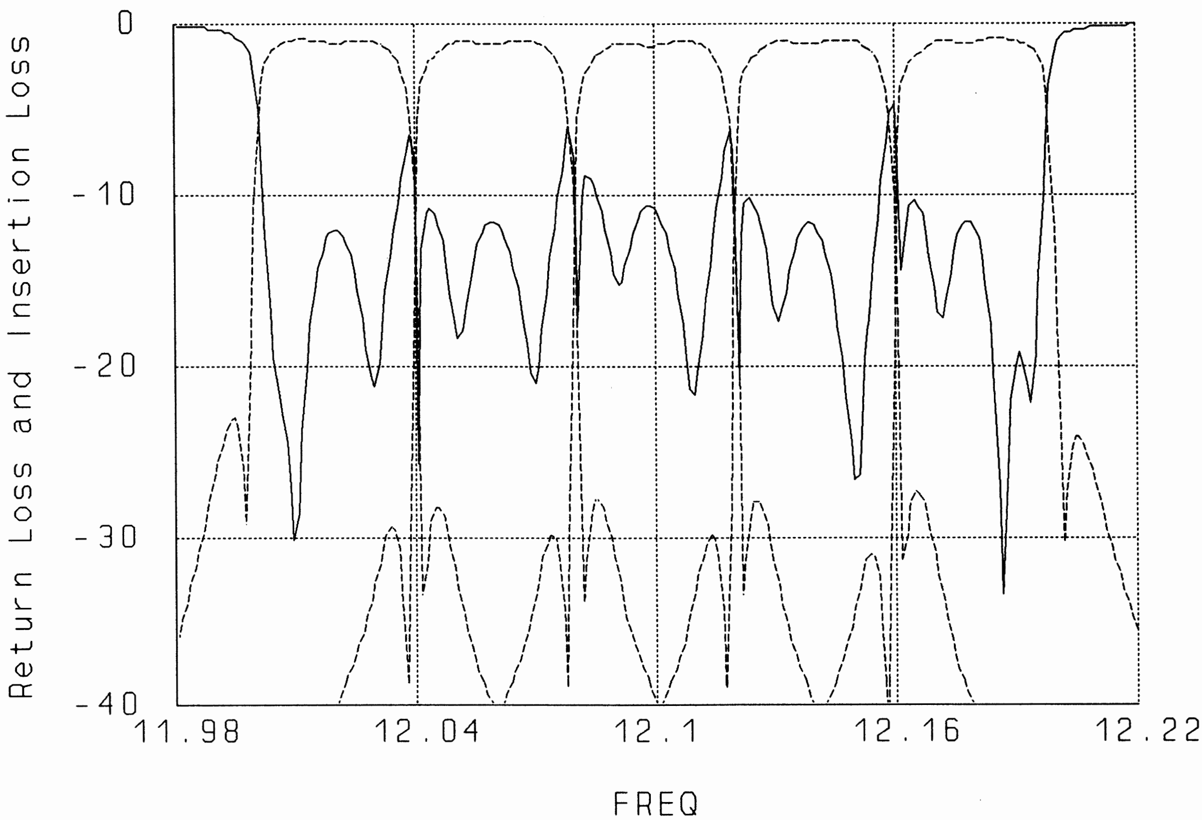


multiplexer responses after minimax optimization with 10 variables: spacings and channel input transformer ratios

worst-case errors cannot be further reduced with only 10 variables by minimax

hardly improved upon the starting point

Partial Huber Optimization of the 5-Channel Multiplexer

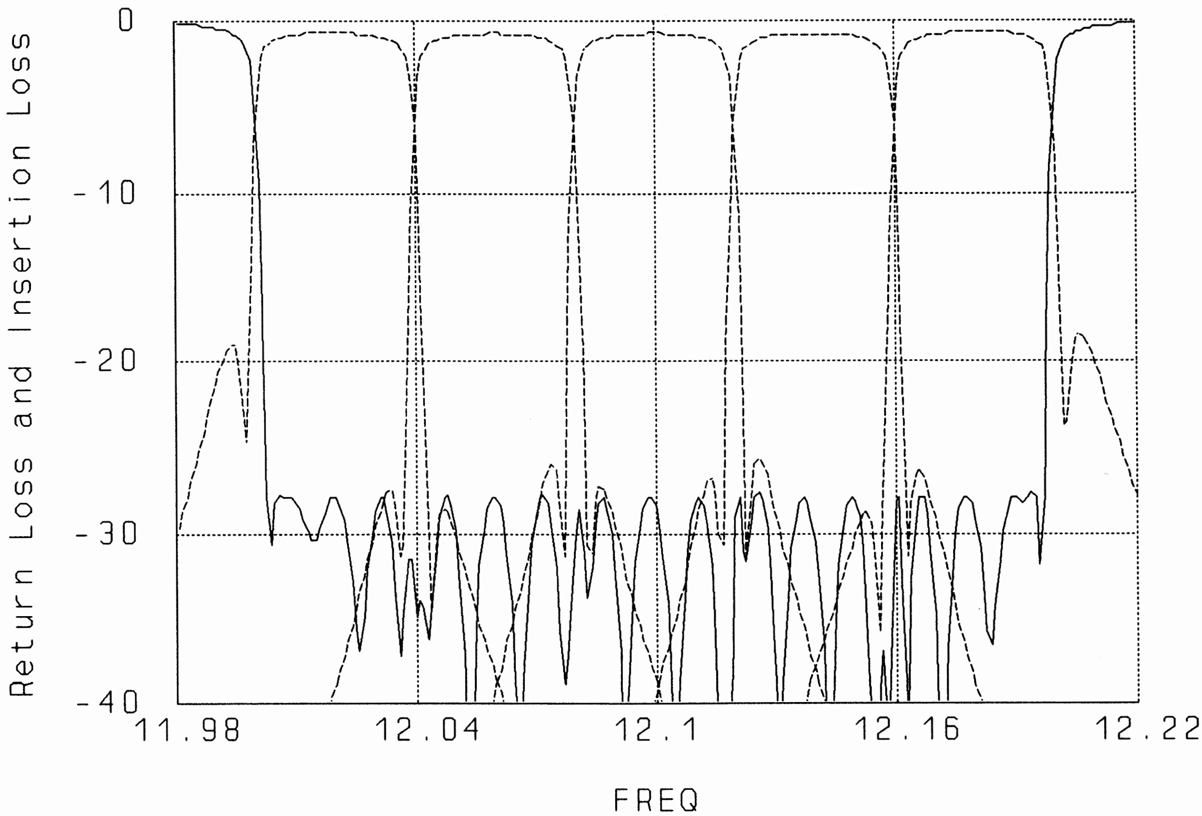


multiplexer responses after one-sided Huber optimization with 10 variables

significantly better than minimax solution with the same 10 variables

provides a good starting point for full-scale minimax optimization

Full-scale Minimax Optimization of the 5-Channel Multiplexer



multiplexer responses after minimax optimization with the full set of 75 variables

Conclusions

we have introduced the concept and applications of the Huber method to circuit CAD

the novel Huber concept is consistent with practical engineering intuition

the Huber method will have a far-reaching and profound impact on modeling, design, design validation, fault diagnosis and statistical processing of circuits and devices

we have presented strong evidence in a number of application areas