



OSA

**STATISTICAL PHYSICS-BASED
MODELING AND YIELD-DRIVEN DESIGN
OF MICROWAVE CIRCUITS**

OSA-92-OS-4-V

April 28, 1992

Optimization Systems Associates Inc.

Dundas, Ontario, Canada

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Quadratic Modeling

to reduce the number of actual time-consuming circuit simulations

suitable for any application where a large number of expensive simulations is required

multidimensional quadratics

maximally flat interpolation technique: applying the least-squares constraint to the second order term coefficients provides unique solution

fixed pattern of base points leads to extremely efficient modeling



Gradient Quadratic Modeling

to take advantage of available gradient information - in OSA90/hope and in HarPE available through the *FAST* technique

quadratic modeling applied to responses and their gradients simultaneously

especially suitable for gradient-based yield-driven design

a low-pass filter and an MMIC amplifier design illustrate the merits



Implementation of Gradient Quadratic Modeling

types of variables

n_{DS} designable variables with statistics

n_D designable variables without statistics

n_S non-designable variables with statistics

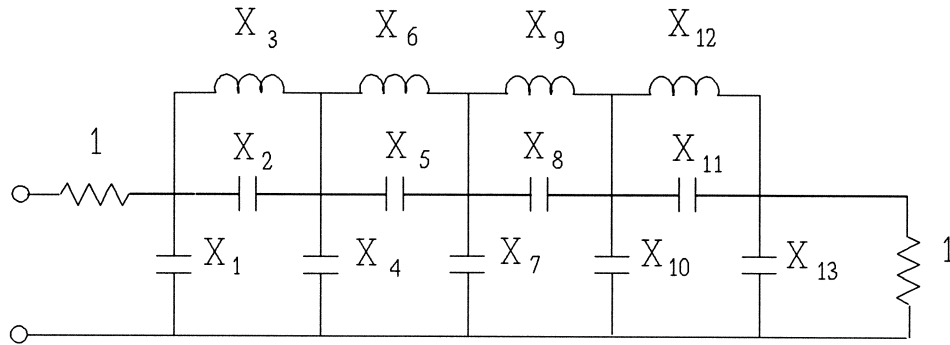
use the same set of $2(n_{DS} + n_S) + 1$ base points for both responses and gradients

build quadratic models for all responses and gradients at each iteration

evaluate the models for all statistical outcomes



13-Element Low-Pass Filter



Specifications

insertion loss less than 0.4dB at 21 angular frequencies from 0.25 to 1 and greater than 49dB at 7 frequencies from 1.05 to 1.115

Design Parameters

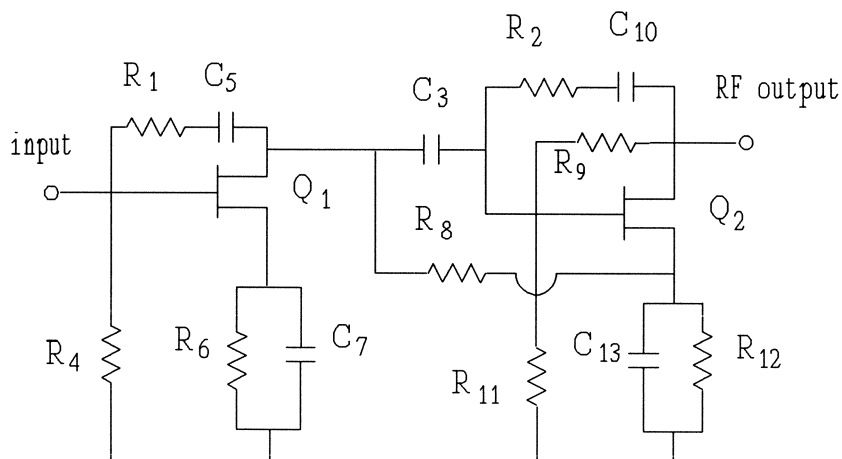
13 elements, normal distribution with 0.5% standard deviation

Yield

33.4% at the initial minimax solution is increased to 76%



Two-Stage GaAs MMIC Feedback Amplifier



FET Model

small-signal physics-based Ladbroke model

Specifications

small-signal gain: between 7dB and 9dB

VSWR at the input port: less than 2

VSWR at the output port: less than 2.2

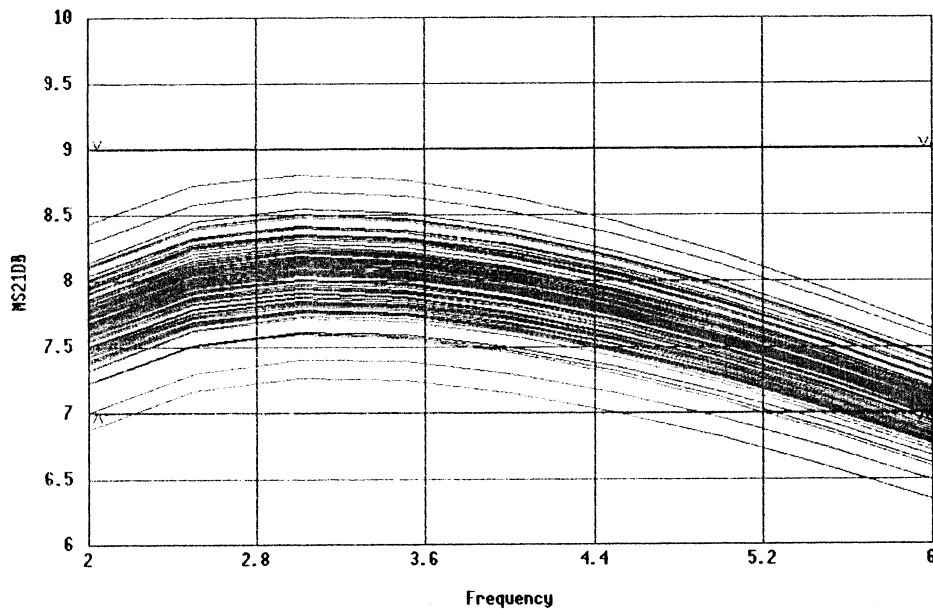
Design Parameters

R_1 , R_2 , and C_3

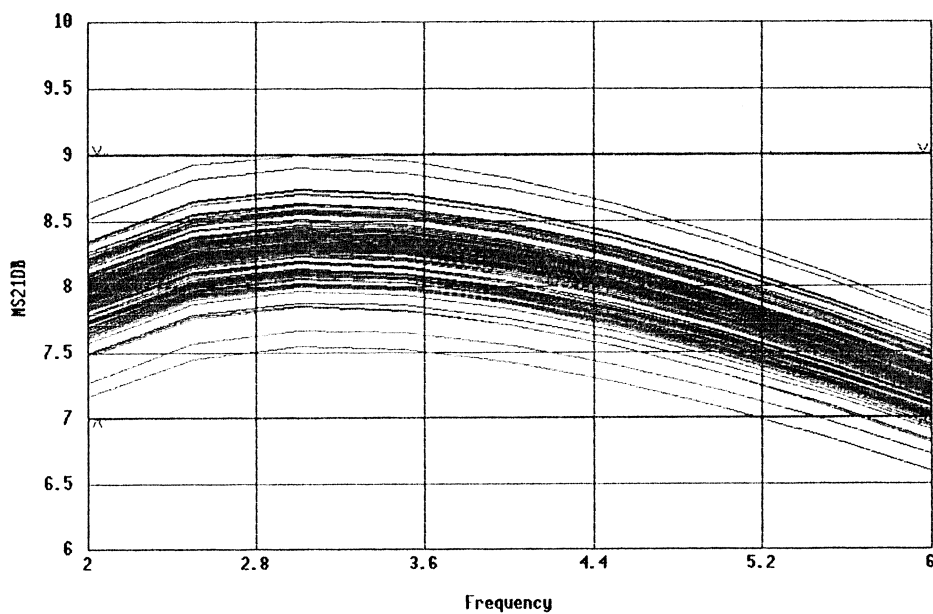


Responses of Two-Stage GaAs MMIC Feedback Amplifier

at the starting point (minimax solution) yield is 32.1%



at the solution yield is increased to 77.8%





Physics-Based Design and Yield Optimization of MMICs

the ability to predict and enhance production yield is critical for the continued success of MMIC technology

physics-based models (PBMs) for active and passive devices dealing directly with the lowest level of fabrication/technological parameters are essential for the next generation of microwave CAD

PBMs allow to include physical parameters as design variables

the advantages of PBMs over traditional equivalent circuit models (ECMs) are particularly clear in statistical modeling

yield optimization of a three stage X-band amplifier illustrates the merits



Equivalent Circuit Models (ECMs)

equivalent circuit models representing active devices

statistical properties assigned to the parameters of ECMs

difficult to represent actual statistical properties of the devices

high computational efficiency



Physics-Based Models (PBMs)

using PBMs for both active devices and passive components

statistical properties assigned to physical parameters

reflecting actual statistical properties of the devices

more computationally intensive than ECMs



PBMs for Passive Devices

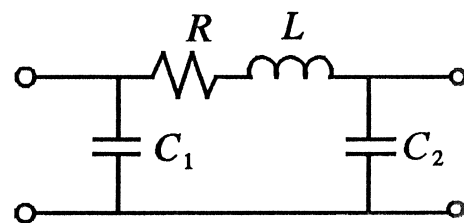
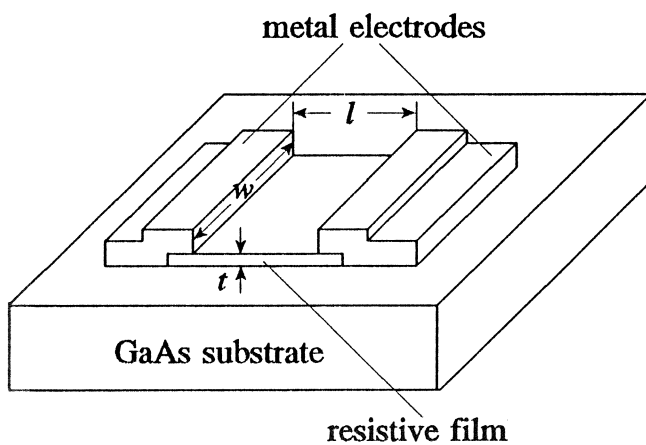
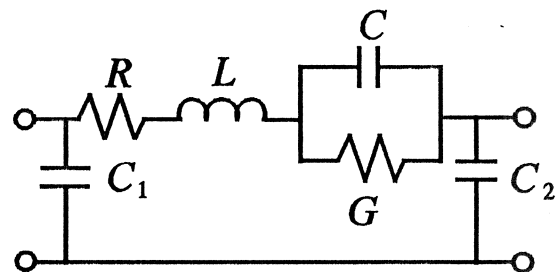
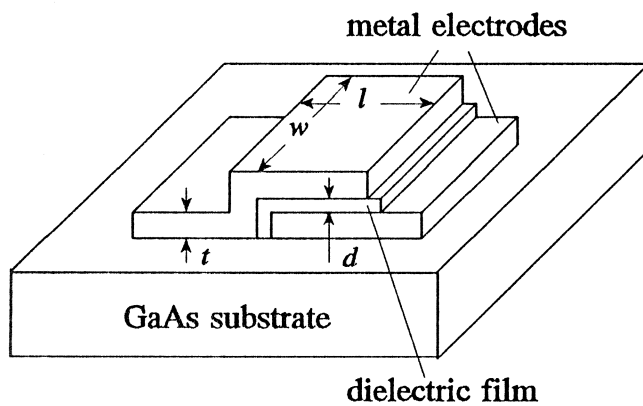
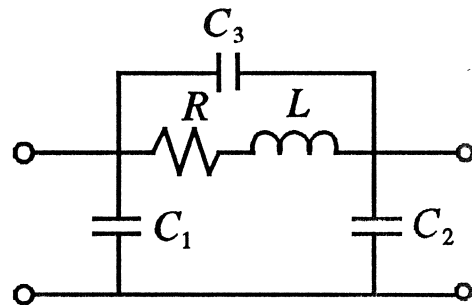
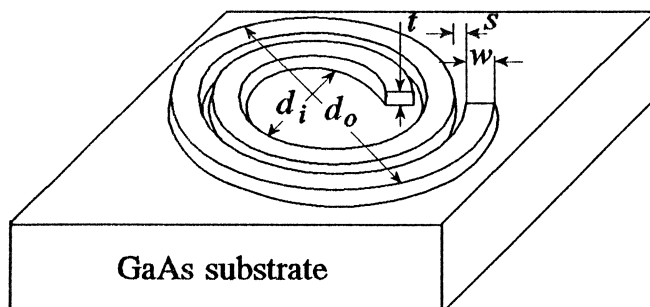
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the entries of \mathbf{Y} are calculated from equivalent circuit components

the expressions of the equivalent circuit components are derived from (simplified) device physics

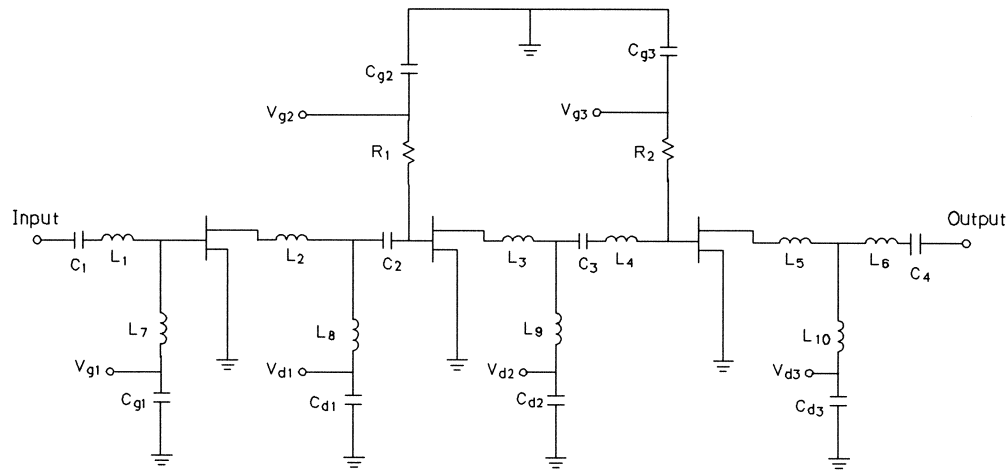


Examples of PBMs for Passive Devices





Yield Optimization of a Three Stage X-band Amplifier using OSA90/hope



Specifications

8GHz - 12GHz: gain between 12dB and 16dB, VSWR < 2
in the stopband: gain < 2dB

Variables

37 statistical variables with correlations
16 design variables



Assumed Distributions for Statistical Variables for the Three Stage X-band Amplifier

Variable	Mean Value	Std Dev	Variable	Mean Value	Std Dev
$N_d(\text{cm}^{-3})$	$2.0 \cdot 10^{17}$	7.0	$d(\text{ m})$	0.1	4.0
$L_G(\text{ m})$	1.0	3.5	$S_{C1}(\text{ m}^2)$	532.7	3.5
$A_G(\text{ m})$	0.24	3.5	$S_{C2}(\text{ m}^2)$	2278.9	3.5
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The doping density N_d , gate length L_G , channel thickness A_G and gate width W_G of the three MESFETs have the same distribution. The conductor width W_L and spacing S_L of the 10 spiral inductors L_1, L_2, \dots, L_{10} have the same distribution. d is the thickness of the dielectric film for all MIM capacitors. S_{Ci} is the area of the metal plate of MIM capacitor C_i .



Predicted and Verified Yield for Different Specifications

Specification	Before Yield Optimization		After Yield Optimization	
	Predicted Yield (%)	Verified Yield (%)	Predicted Yield (%)	Verified Yield (%)
Spec. 1	17.5	15.7	67	57.9
Spec. 2	21	20	83	75.7
Spec. 3	44	37.1	98	93.6

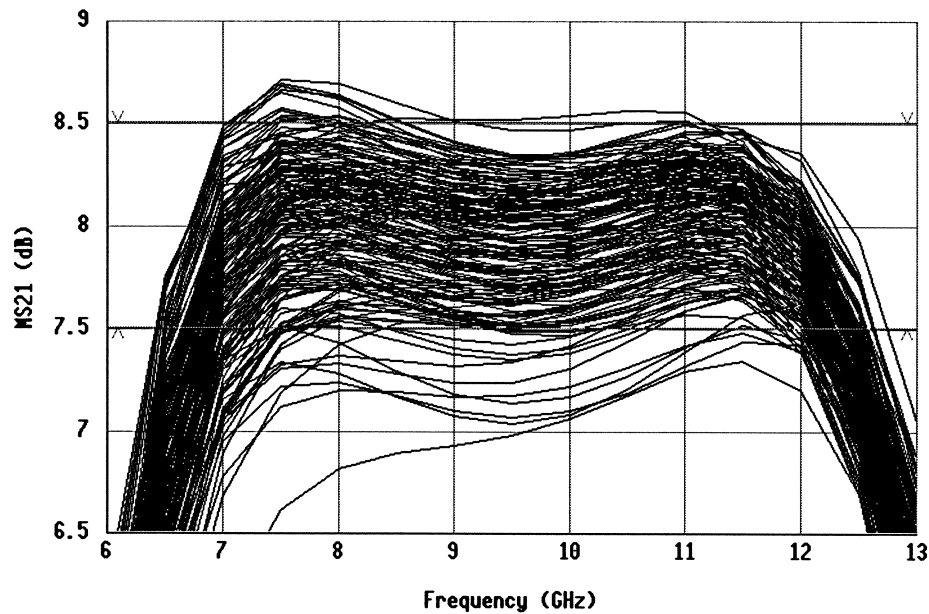
Spec. 1: $7.5\text{dB} < S_{21} < 8.5\text{dB}$, $ S_{11} < 0.5$, $ S_{22} < 0.5$
Spec. 2: $6.5\text{dB} < S_{21} < 7.5\text{dB}$, $ S_{11} < 0.5$, $ S_{22} < 0.5$
Spec. 3: $6.0\text{dB} < S_{21} < 8.0\text{dB}$, $ S_{11} < 0.5$, $ S_{22} < 0.5$

200 Monte Carlo outcomes are used for predicted yield, 140 for verified yield.

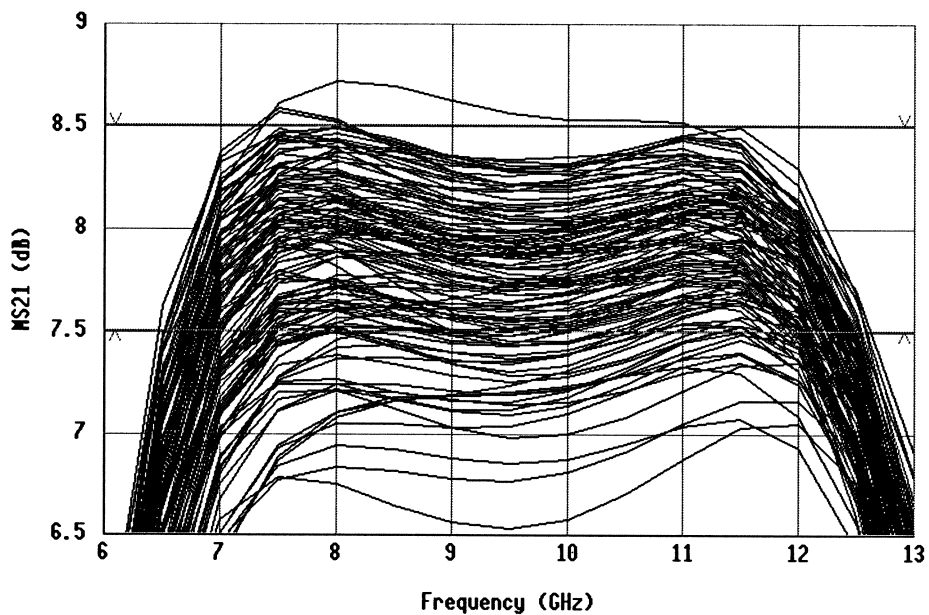


Yield Comparison

Monte Carlo simulation using the new MESFET model



Monte Carlo simulation using device data





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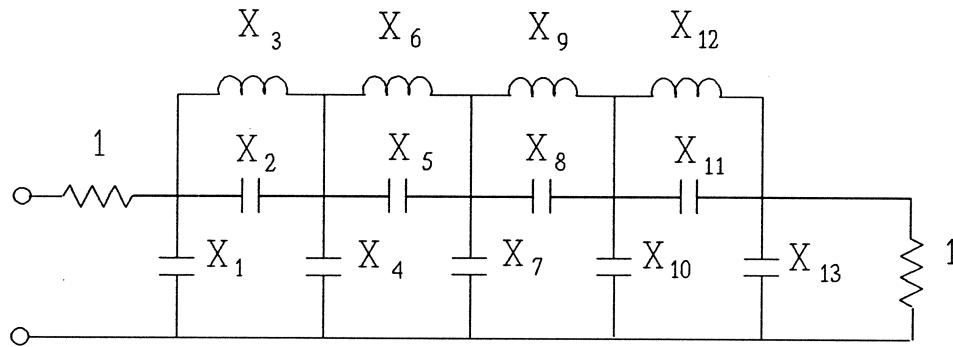
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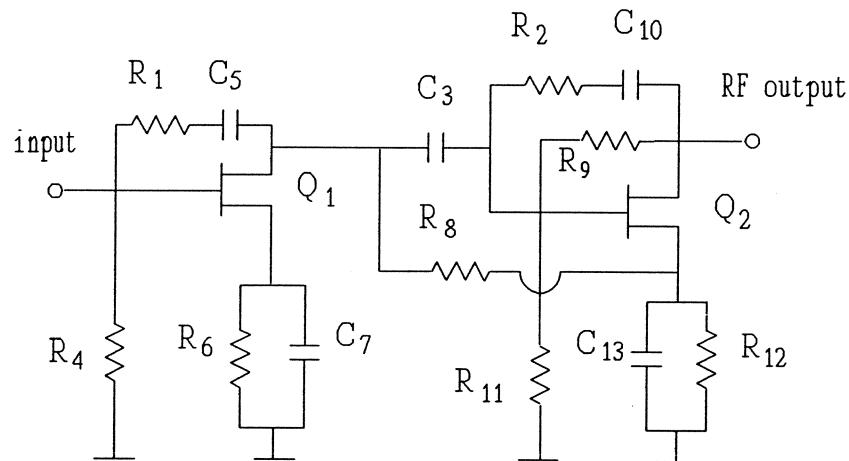
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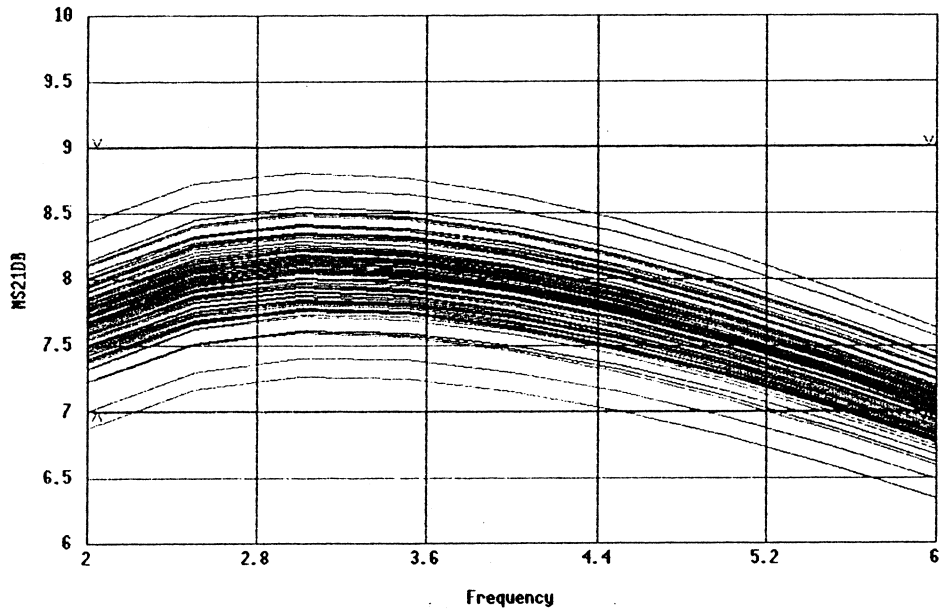
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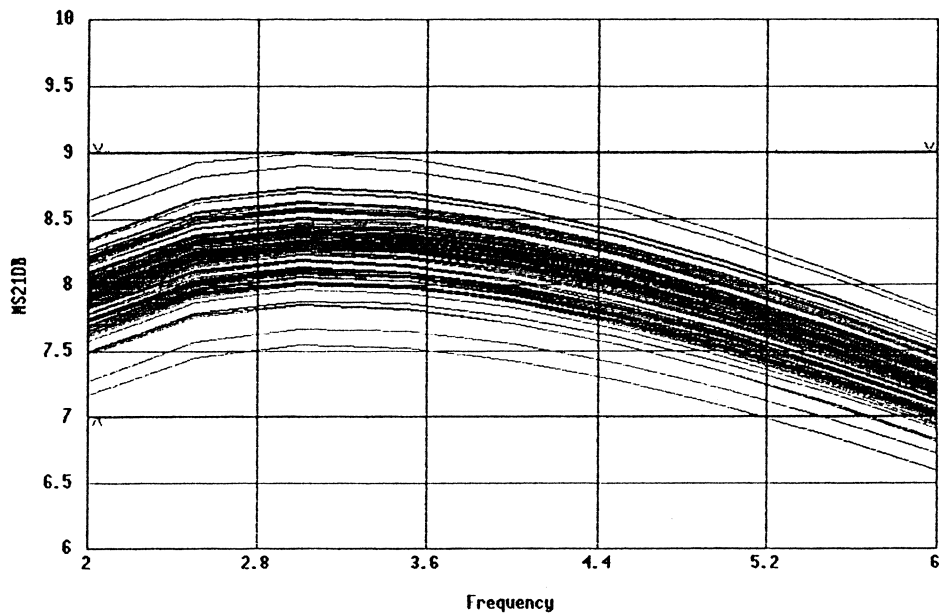


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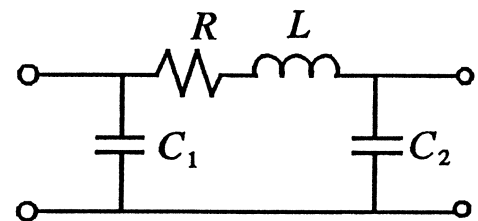
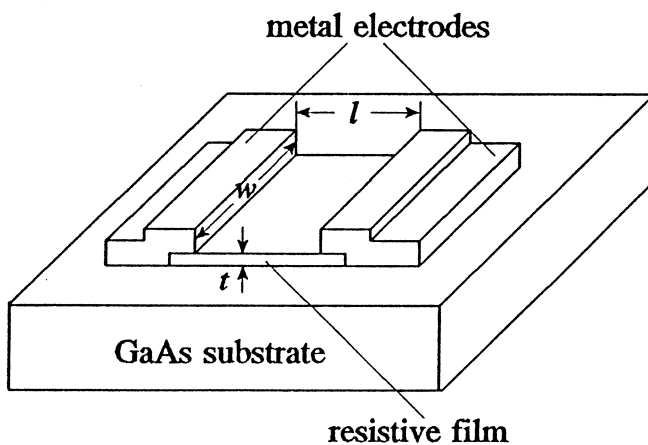
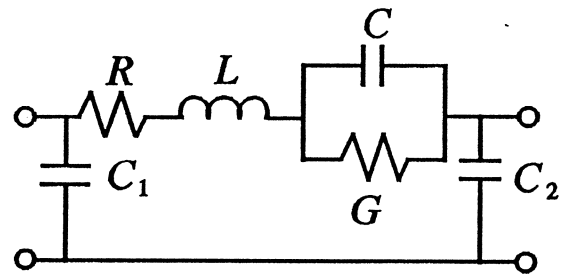
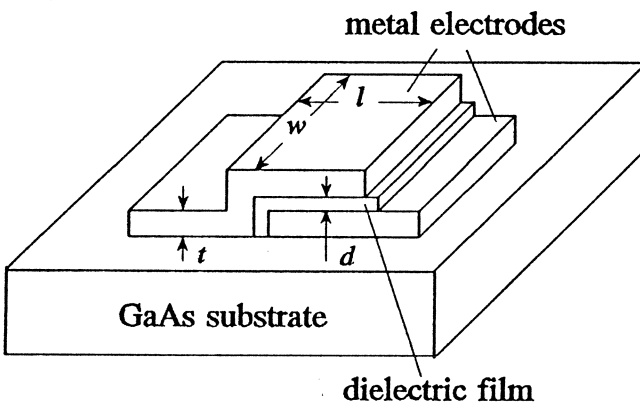
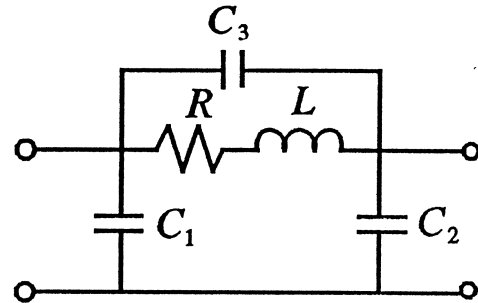
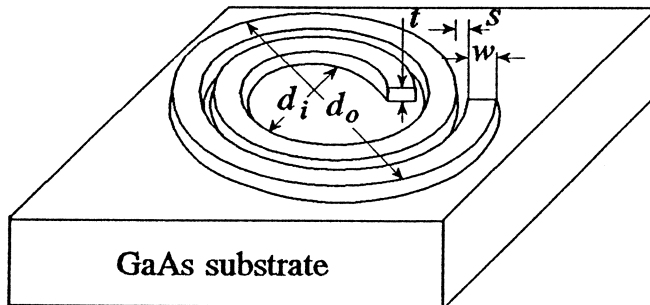
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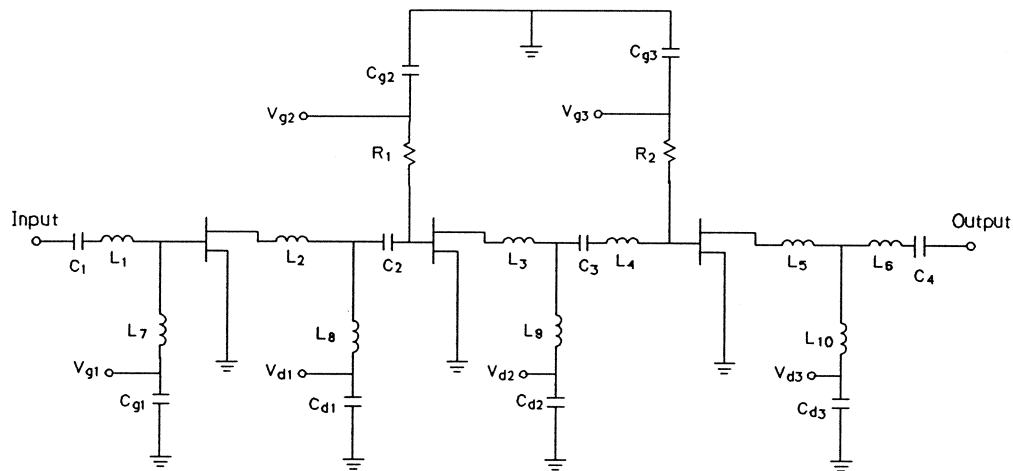


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ASSUMED PARAMETER CORRELATIONS FOR THE THREE MESFETS

	A _{G1}	L _{G1}	W _{G1}	N _{d1}	A _{G2}	L _{G2}	W _{G2}	N _{d2}	A _{G3}	L _{G3}	W _{G3}	N _{d3}
A _{G1}	1.00	0.00	0.00	-0.25	0.80	0.00	0.00	-0.20	0.78	0.00	0.00	-0.10
L _{G1}	0.00	1.00	0.00	-0.10	0.00	0.80	0.00	-0.05	0.00	0.78	0.00	-0.05
W _{G1}	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.00	0.00	0.00	0.78	0.00
N _{d1}	-0.25	-0.10	0.00	1.00	-0.20	-0.05	0.00	0.80	-0.15	-0.05	0.00	0.78
A _{G2}	0.80	0.00	0.00	-0.20	1.00	0.00	0.00	-0.25	0.80	0.00	0.00	-0.20
L _{G2}	0.00	0.80	0.00	-0.05	0.00	1.00	0.00	-0.10	0.00	0.80	0.00	-0.10
W _{G2}	0.00	0.00	0.80	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.00
N _{d2}	-0.20	-0.05	0.00	0.80	-0.25	-0.10	0.00	1.00	-0.20	-0.05	0.00	0.80
A _{G3}	0.78	0.00	0.00	-0.15	0.80	0.00	0.00	-0.20	1.00	0.00	0.00	-0.25
L _{G3}	0.00	0.78	0.00	-0.05	0.00	0.80	0.00	-0.05	0.00	1.00	0.00	-0.10
W _{G3}	0.00	0.00	0.78	0.00	0.00	0.00	0.80	0.00	0.00	0.00	1.00	0.00
N _{d3}	-0.10	-0.05	0.00	0.78	-0.20	-0.10	0.00	0.80	-0.25	-0.10	0.00	1.00



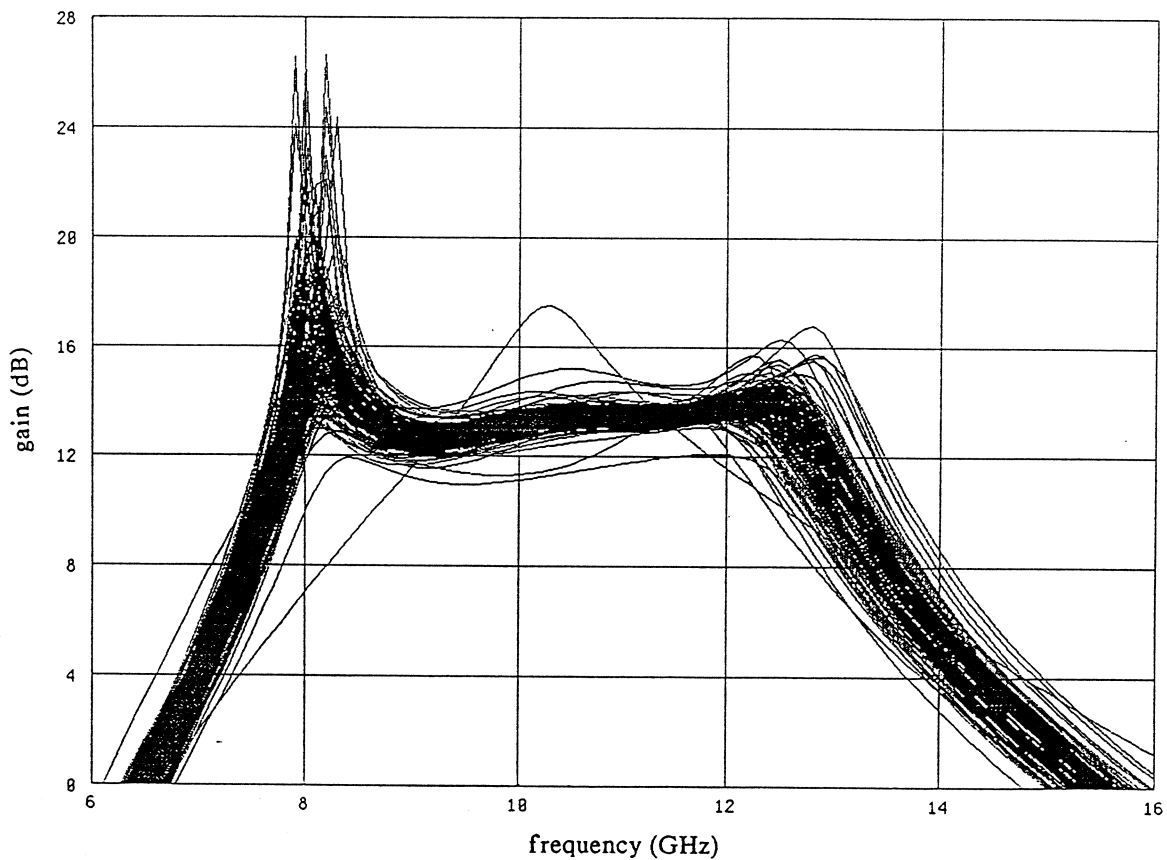
Design Variables for Yield Optimization of the Three Stage X-band Amplifier

Design Variable	Before Opt.	After Opt.	Design Variable	Before Opt.	After Opt.
A_G (m)	0.24	0.243	n_{L3}	2.33	2.04
N_d (cm ⁻³)	$2.0 \cdot 10^{17}$	$2.03 \cdot 10^{17}$	n_{L4}	2.29	2.34
S_{C1} (m ²)	532.7	552.2	n_{L5}	2.32	2.39
S_{C2} (m ²)	2278.9	1910.2	n_{L6}	1.84	2.08
S_{C3} (m ²)	583.1	554.2	n_{L7}	1.49	1.50
S_{C4} (m ²)	468.7	477.2	n_{L8}	2.65	2.82
n_{L1}	2.88	2.79	n_{L9}	2.43	2.48
n_{L2}	3.98	4.11	n_{L10}	3.27	3.35

n_{Li} is the number of turns of the spiral inductor L_i .



Gain of the Three Stage X-band Amplifier Before Yield Optimization

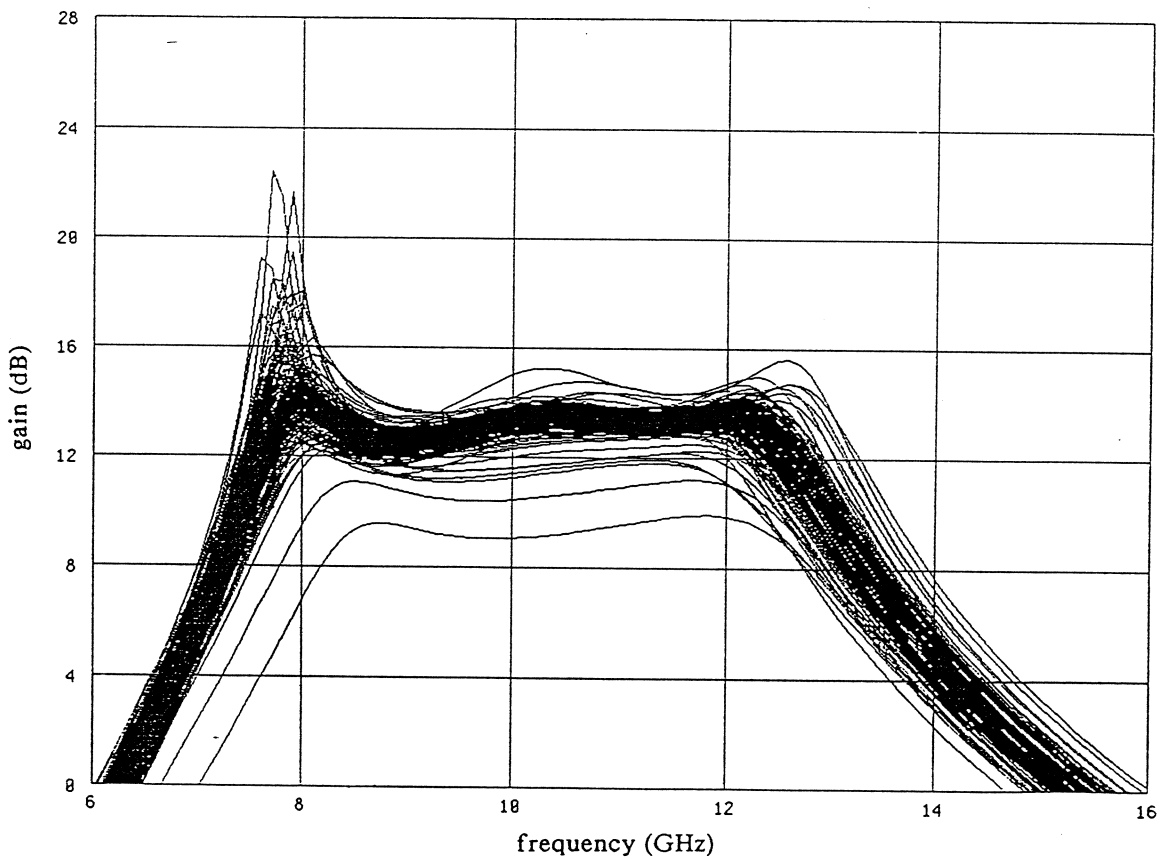


200 outcomes

initial yield is 47.5%



Gain of the Three Stage X-band Amplifier After Yield Optimization



200 outcomes

yield is increased to 78.5%



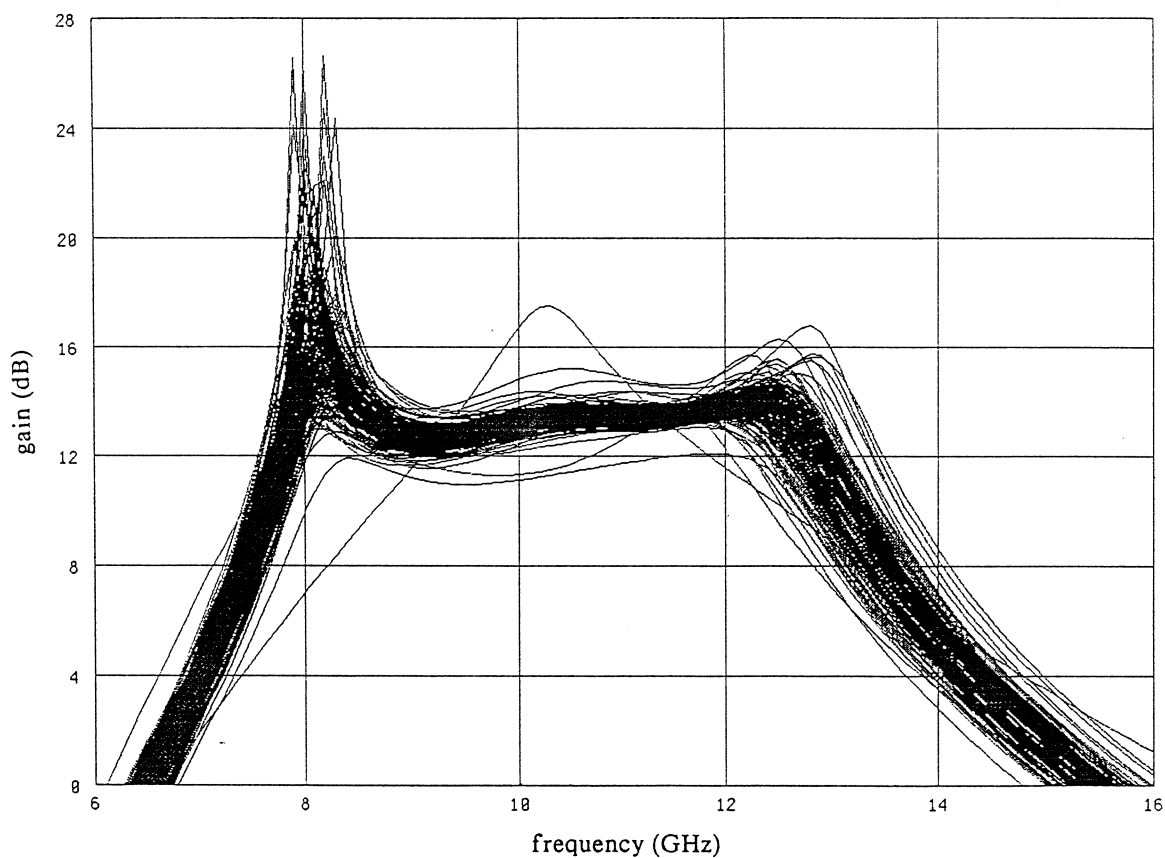
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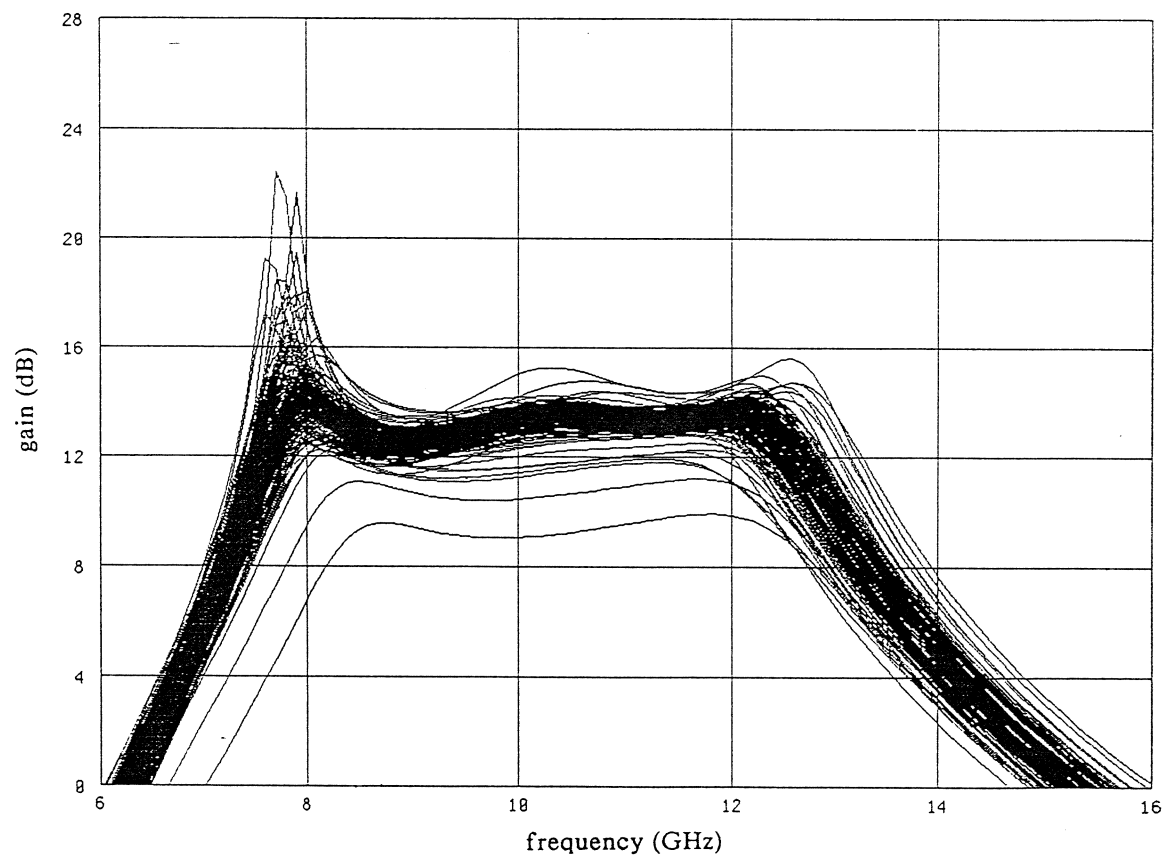


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Statistical Modeling of GaAs MESFETs

statistical modeling is a prerequisite for yield-driven and cost-driven circuit design

statistical modeling may be approached at

- equivalent circuit parameter level

- measurement level

- physical and material parameter level

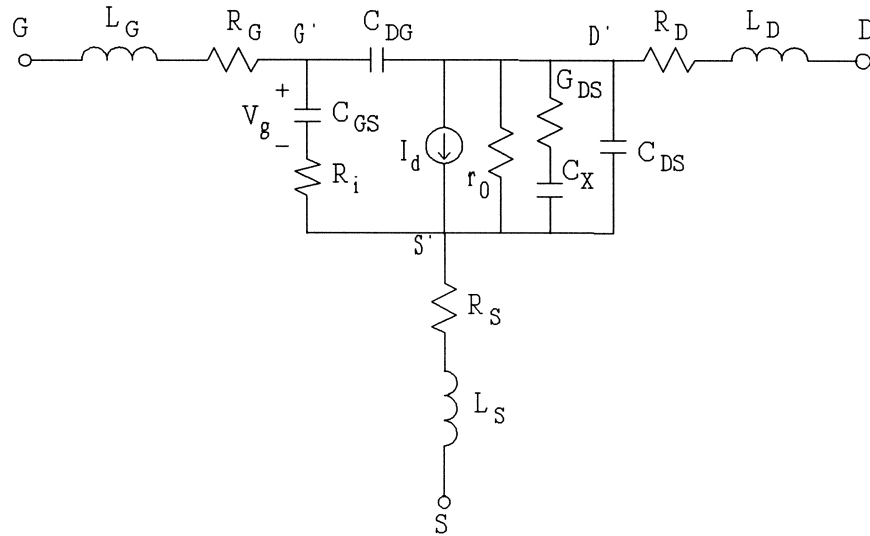
the equivalent circuit model can accurately fit the data from which the model parameters are extracted, because it has fewer constraints and more variables than the physical model

the model based on physical parameters can preserve the statistical characteristics of the actual device

for yield-driven and cost-driven circuit design, physics-based statistical models are more accurate



FET Model Equivalent Circuit



the Materka and Kacprzak nonlinear FET model

nonlinear intrinsic FET parameters

$\{I_{DSS}, V_{p0}, \gamma, E, K_E, \tau, S_S, R_{10}, K_R, C_{10}, C_{1S}, K_1, C_{F0}, K_F\}$

linear extrinsic parameters

$\{L_G, R_G, R_D, L_D, R_S, L_S, G_{DS}, C_{DS}\}$



The Ladbroke Physics-Based MESFET Model

the model is in equivalent circuit form

the elements of the equivalent circuit are expressed in terms of physical and geometrical parameters

g_m , τ , r_0 , C_{GS} , C_{DG} , R_i , R_D , R_S , and L_G are functions of the physical parameters and bias conditions

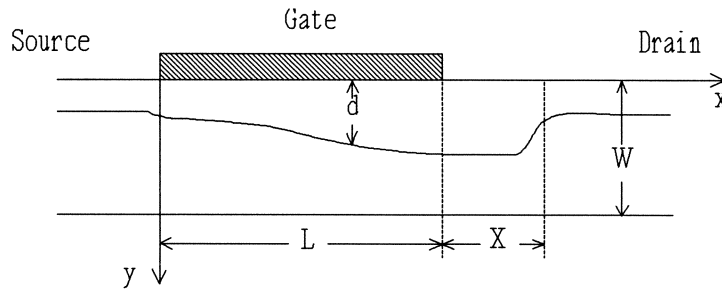
R_G , L_D , L_S , G_{DS} and C_{DS} are assumed to be linear components

the model parameters of the Ladbroke MESFET model to be extracted are

$$\{L_{g0}, W, N, v_{sat}, V_{B0}, a_0, r_{01}, r_{02}, r_{03}, L_{G0}, R_G, L_D, L_S, G_{DS}, C_{DS}\}$$



The Ladbroke Physics-Based MESFET Model



the elements of the equivalent circuit are expressed in terms of physical and geometrical parameters

for example

$$g_m = \epsilon v_{sat} Z_G / d$$

$$C_{DG} = 2 \epsilon Z_G / (1 + 2X/L_{g0})$$

$$L_G = \mu_0 d Z_G / (m^2 L_{g0}) + L_{G0}$$

the equivalent depletion depth d , the voltage dependent space-charge layer extension X and the channel current are obtained as

$$d = [2 \epsilon (-V_{G'S'} + V_{B0}) / (qN)]^{0.5}$$

$$X = a_0 \{2 \epsilon / [qN(-V_{G'S'} + V_{B0})]\}^{0.5} (V_{D'G'} + V_{B0})$$

$$I_{CH} = qNv_{sat}(W-d)Z_G$$



Statistical Modeling Procedure

measurement data

four-finger 0.5 μm GaAs MESFET measurement data
from Plessey Research Caswell

69 individual devices from two wafers

each device is measured under 3 bias conditions and
at frequencies from 1GHz to 21GHz with 0.4GHz step

modeling procedure

extract model parameters for each individual device

postprocess to obtain the statistics of the model

the form of the statistical model

nominal values (mean values)

tolerances (standard deviations)

discrete distribution functions (DDFs)

correlation matrix



Verifications of Statistical FET Device Modeling

statistical models must be able to predict the statistical behaviour of actual devices

the model responses and the actual device responses must be statistically consistent

we calculate S parameters of the Ladbroke and the Materka and Kacprzak models through Monte Carlo simulations at one bias point and one frequency point

the statistics of the simulation results are compared with the statistics of the measurements

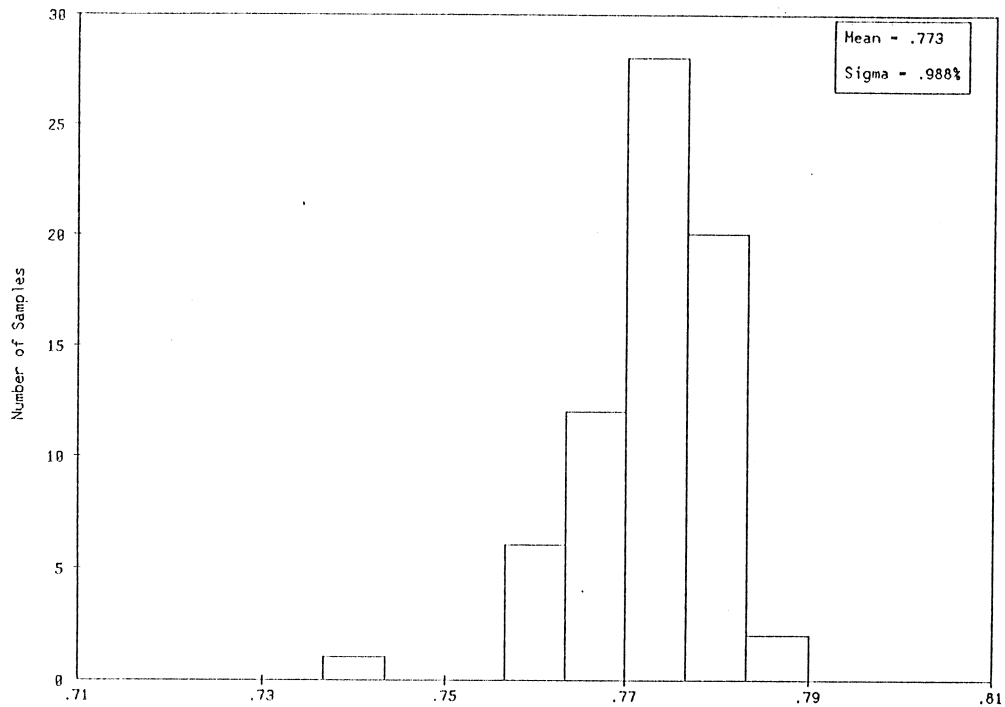


**MEAN VALUES AND STANDARD DEVIATIONS OF MEASURED
AND SIMULATED S PARAMETERS AT 11GHZ**

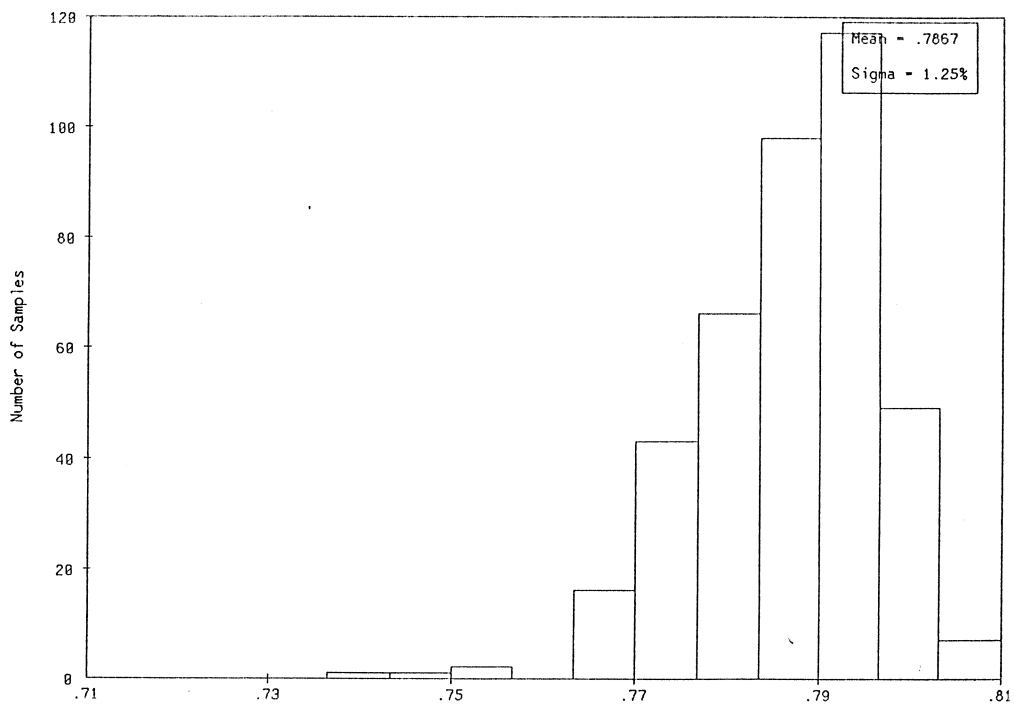
	Measured		Simulated			
			Ladbroke		Materka	
	Mean	Dev.(%)	Mean	Dev.(%)	Mean	Dev.(%)
$ S_{11} $.7730	.988	.7856	.764	.7725	1.74
$\angle S_{11}$	-114.3	1.36	-119.3	1.10	-114.9	1.63
$ S_{21} $	1.919	.802	1.679	1.34	1.933	15.2
$\angle S_{21}$	93.35	.856	94.06	.835	93.43	.860
$ S_{12} $.0765	3.77	.07542	3.68	.07564	5.07
$\angle S_{12}$	34.00	2.51	31.98	2.33	33.72	2.14
$ S_{22} $	0.5957	1.48	.5838	1.54	.5935	4.19
$\angle S_{22}$	-38.69	2.10	-36.86	1.42	-37.85	3.31



Histogram of $|S_{11}|$ at 11GHz from Measurement

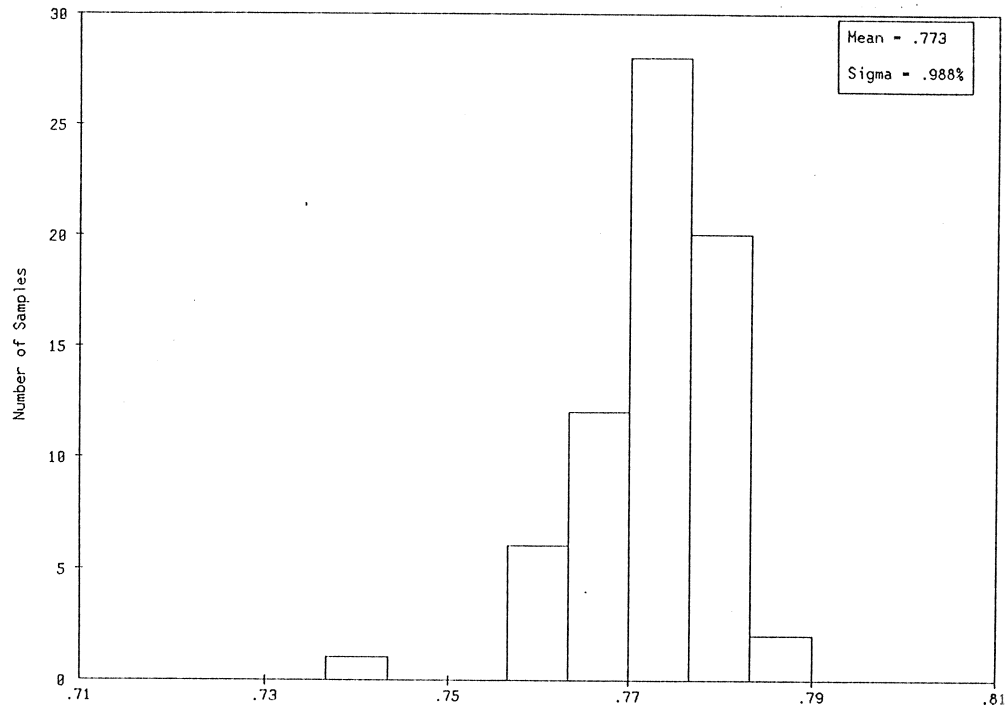


Histogram of $|S_{11}|$ at 11GHz from the Ladbroke Model

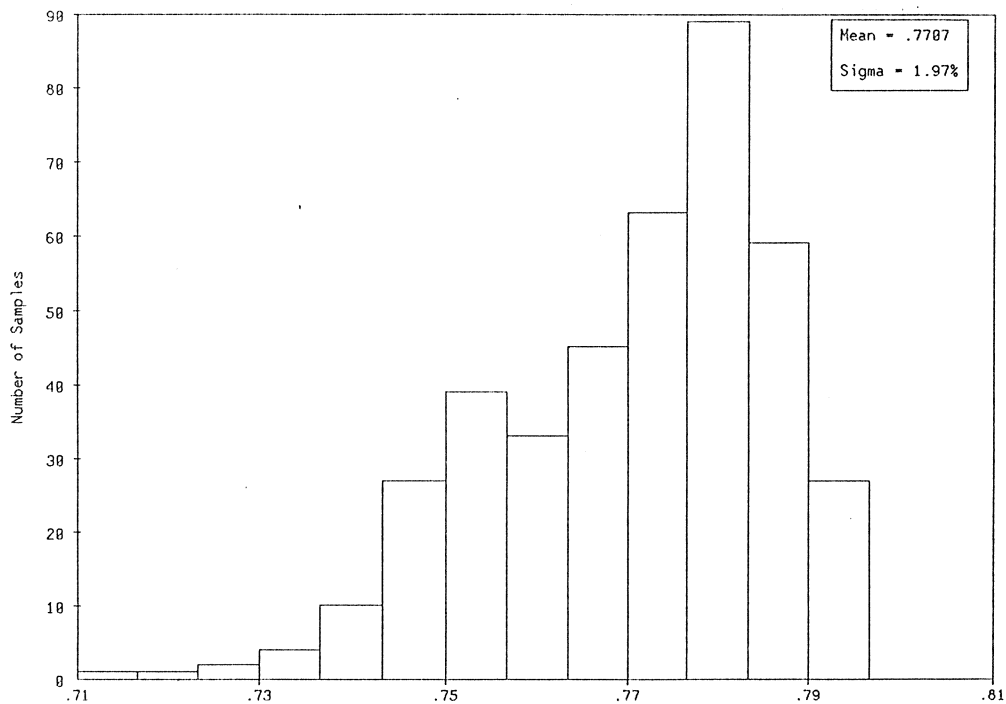




Histogram of $|S_{11}|$ at 11GHz from Measurement



Histogram of $|S_{11}|$ at 11GHz from the Materka Model





Physics-Based Small-Signal Statistical FET Model

the small-signal Ladbroke model provides more reliable estimates of device statistics than normal equivalent circuit models

the Ladbroke model does not have DC simulation capability

the physics-based Khatibzadeh and Trew model provides good DC simulation but is inaccurate for small-signal simulation

we combine the DC Khatibzadeh and Trew characterization and the small-signal Ladbroke formulas to form a new physics-based small-signal FET model

the new model is being tested within the statistical environment of HarPE and OSA90/hope

the new model is particularly suitable for bias-dependent small-signal FET statistical modeling



Predictable Yield-Driven Circuit Optimization

combined Khatibzadeh and Trew/Ladbroke model

the model is characterized from device measurements of
0.5 μm GaAs MESFET provided by Plessey Research
Caswell

design of a small-signal broadband amplifier is investigated
using OSA90/hope

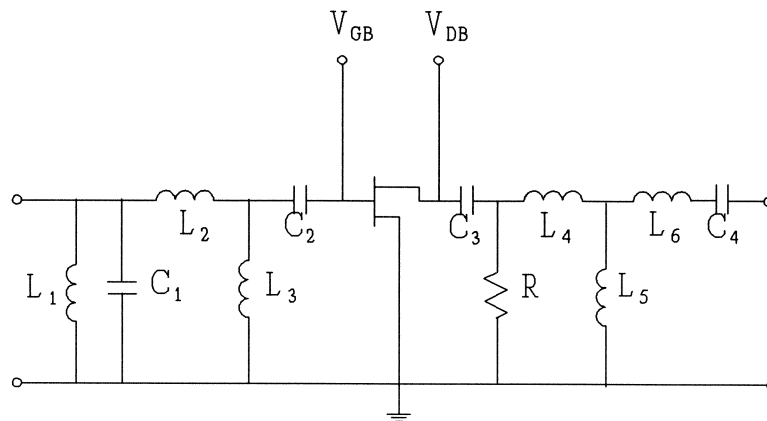
(1) nominal design

(2) yield optimization (using 100 outcomes)

yield estimates by Monte Carlo analyses before and after
yield optimization are 17.5% and 67%



Small-Signal Broadband Amplifier



specifications

$$|S_{21}| = 8\text{dB} \pm 0.5\text{dB}$$

$$|S_{11}| \leq 0.5$$

$$|S_{22}| \leq 0.5$$

design variables are the matching network elements

$$L_1, L_2, L_3, L_4, L_5, L_6, C_1, C_2, C_3, C_4, R$$

ASSUMED PARAMETER CORRELATIONS FOR THE THREE MESFETS

	A _{G1}	L _{G1}	W _{G1}	N _{d1}	A _{G2}	L _{G2}	W _{G2}	N _{d2}	A _{G3}	L _{G3}	W _{G3}	N _{d3}
A _{G1}	1.00	0.00	0.00	-0.25	0.80	0.00	0.00	-0.20	0.78	0.00	0.00	-0.10
L _{G1}	0.00	1.00	0.00	-0.10	0.00	0.80	0.00	-0.05	0.00	0.78	0.00	-0.05
W _{G1}	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.00	0.00	0.00	0.78	0.00
N _{d1}	-0.25	-0.10	0.00	1.00	-0.20	-0.05	0.00	0.80	-0.15	-0.05	0.00	0.78
A _{G2}	0.80	0.00	0.00	-0.20	1.00	0.00	0.00	-0.25	0.80	0.00	0.00	-0.20
L _{G2}	0.00	0.80	0.00	-0.05	0.00	1.00	0.00	-0.10	0.00	0.80	0.00	-0.10
W _{G2}	0.00	0.00	0.80	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.00
N _{d2}	-0.20	-0.05	0.00	0.80	-0.25	-0.10	0.00	1.00	-0.20	-0.05	0.00	0.80
A _{G3}	0.78	0.00	0.00	-0.15	0.80	0.00	0.00	-0.20	1.00	0.00	0.00	-0.25
L _{G3}	0.00	0.78	0.00	-0.05	0.00	0.80	0.00	-0.05	0.00	1.00	0.00	-0.10
W _{G3}	0.00	0.00	0.78	0.00	0.00	0.00	0.80	0.00	0.00	0.00	1.00	0.00
N _{d3}	-0.10	-0.05	0.00	0.78	-0.20	-0.10	0.00	0.80	-0.25	-0.10	0.00	1.00