

**STATISTICAL DESIGN/YIELD OPTIMIZATION
OF NONLINEAR MICROWAVE CIRCUITS**

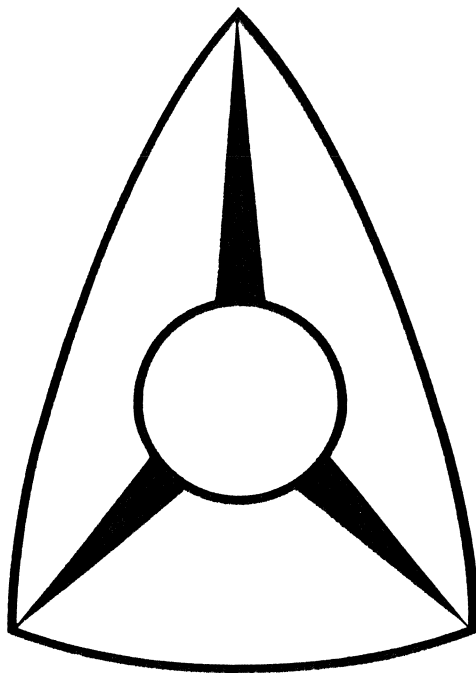
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STATISTICAL DESIGN/YIELD OPTIMIZATION OF NONLINEAR MICROWAVE CIRCUITS

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Introduction

advances in GaAs MMIC technology demand the next generation CAE software

efficient algorithms for large-scale optimization of nonlinear circuits are crucial

cost reduction is a primary goal

yield-driven optimization methodology is essential

statistical representation of physical, process and geometrical parameters must be integral to CAE



Increasing Sophistication of Design Methodology

deterministic

performance-driven design

fixed tolerance worst-case design

variable tolerance worst-case design

statistical

fixed tolerance yield-driven design

correlated tolerances

variable tolerance cost-driven design



Nonlinear Circuit Simulation

most analog microwave circuits operate under steady-state conditions

harmonic balance has become the major simulation tool for nonlinear circuits

popular harmonic balance software:

- typical algorithms inhibit fast optimization

- yield-driven design may be prohibitively CPU intensive

- inaccurate, slow sensitivity/gradient evaluation

OSA's research and development successfully addresses these issues



Yield Optimization

high yield is essential for large volume production of MMICs

useful CAE must account for manufacturing tolerances, model uncertainties, variations of process parameters, environmental uncertainties, etc.

yield optimization is computationally intensive

optimization is iterative

each iteration requires simulation of many statistically related circuits

simulation of each nonlinear circuit is iterative

yield-driven design demands powerful, robust, fast optimizers and effective statistical representation of devices and subcircuits



Manufacturing Yield

manufacturing yield is simply the ratio

$$Y = N_{\text{pass}}/N_t$$

where

N_{pass} is the number of circuit outcomes satisfying the design specifications

N_t is the total number of circuit outcomes

Yield Optimization Problem Formulation

centering problem with fixed tolerances

$$\begin{array}{l} \text{maximize } Y \\ \mathbf{x}^0 \end{array}$$

where \mathbf{x}^0 denotes the nominal design parameter vector



Pioneering Work on Statistical Design

circuit design exploiting parameter tolerances
(Karafin 1971, Pinel and Roberts 1972)

worst-case design with optimized tolerances
(Bandler 1972-1974)

optimal centering, tolerancing and tuning
(Bandler 1976)

integrated approach to microwave design with
parameter tolerances, model uncertainties,
mismatches and reference plane uncertainties
(Bandler 1976)



Major Approaches to Statistical Design

simplicial approximation (Director and Hachtel 1977)

updated approximations and cuts (Bandler and Abdel-Malek 1978)

outer-approximation (Polak and Sangiovanni-Vincentelli 1979)

arbitrary and discrete distributions (Abdel-Malek and Bandler 1980)

statistical exploration (Soin and Spence 1980)

stochastic approximation with Monte Carlo based optimization (Styblinski and Ruszczynski 1980)

parametric sampling (Singhal and Pinel 1981)



Process/Geometrical Approach to Statistical Design

developed for VLSI statistical design with
process/geometrical parameters

identification of significant parameters in MOS VLSI
(Yang, Hocevar, Cox, Machala and Chatterjee 1986)

software tools for IC yield optimization with respect to
manufacturing process parameters (e.g., diffusion time and
temperature) and layout mask dimensions (Styblinski and
Opalski 1986)

worst-case analysis with respect to a set of statistically
independent process disturbances (Nassif, Strojwas and
Director 1986)



Recent Explorations in Statistical Design

yield gradient estimation for truncated probability density functions (Styblinski 1986)

theoretical relations between parametric and catastrophic yield (Maly, Strojwas and Director 1986)

profit-based statistical design (Riley and Sangiovanni-Vincentelli 1986)

maximum income approach (Opalski and Styblinski 1986)

regression model w.r.t. critical system parameters (Yu, Kang, Hajj and Trick 1987)

gradient based optimization exploiting transient sensitivity (Hocevar, Cox and Yang 1988)



Recent OSA Contributions to Statistical Design

world's first commercial, microwave oriented
yield-driven design (OSA for Super-Compact 1987)

a novel one-sided ℓ_1 formulation of design centering
(Bandler and Chen 1988)

efficient quadratic modeling and large scale yield
optimization (Bandler, Biernacki, Chen, Renault, Song and
Zhang 1988)

yield optimization of nonlinear microwave circuits
(Bandler, Zhang, Song and Biernacki 1989)

FAST and IGAT gradient based yield optimization
(Bandler, Zhang, Song and Biernacki 1990)



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Typical statistical distributions



Yield interpretation in the parameter space



Envelope diagram for the magnitude of S_{21}
for centered design of 11 element LC filter



Yield Optimization of Nonlinear Microwave Circuits

pioneered by OSA

comprehensive treatment of yield optimization of nonlinear microwave circuits with statistically characterized devices

one-sided χ^2 circuit centering with gradient approximations

efficient harmonic balance simulation with exact Jacobians

statistical representation of nonlinear devices:

multidimensional statistical distributions of the intrinsic device and parasitic parameters

yield enhanced from 25% to 61% for a frequency doubler design having 34 statistically toleranced parameters



Yield Optimization of the FET Frequency Doubler

design specifications

lower specification of 2.5 dB on conversion gain

lower specification of 19 dB on spectral purity

optimization variables

input inductance L_1

microstrip lengths l_1 and l_2

bias voltages V_{GB} and V_{DB}

driving power level P_{IN}



Yield Optimization of the FET Frequency Doubler (cont'd)

tolerances assumed

uniform distributions with 3% tolerances for 6
optimization variables

uniform distributions with 5% tolerances for 6 other
elements

the large-signal FET model

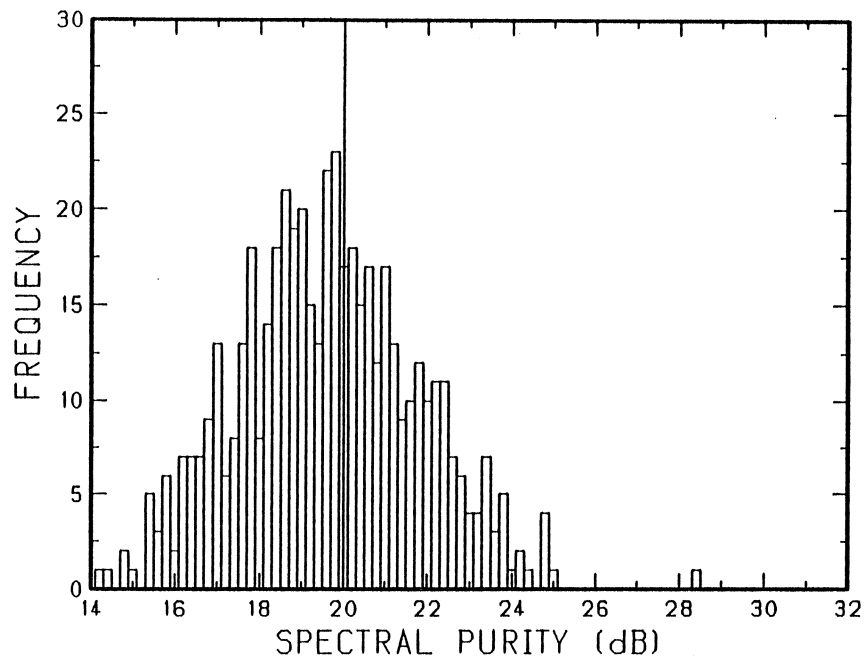
modified Materka and Kacprzak model, normal
distributions and correlations assumed for 22
parameters

mean values, standard deviations and correlations based on
information given by *Purviance et al.* (1988)

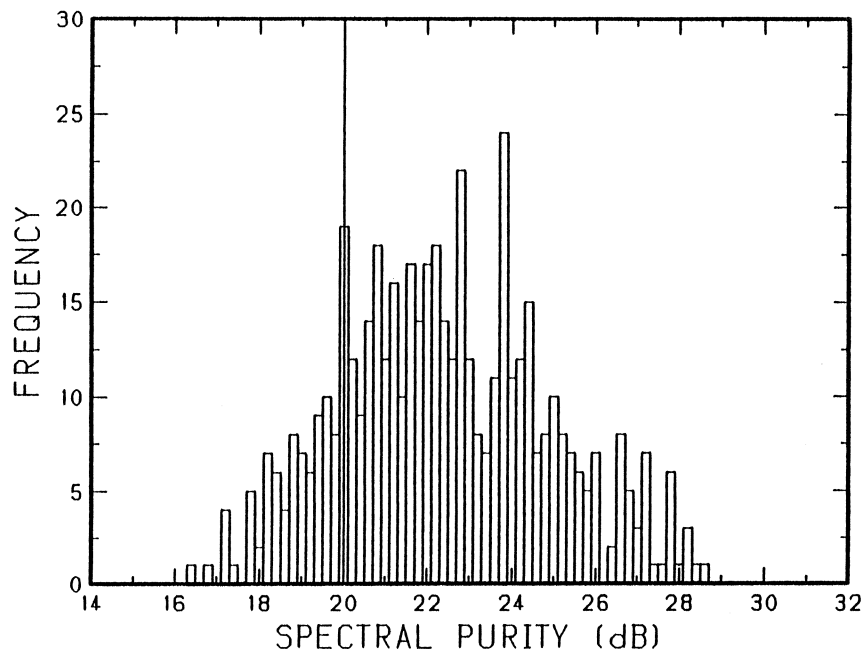


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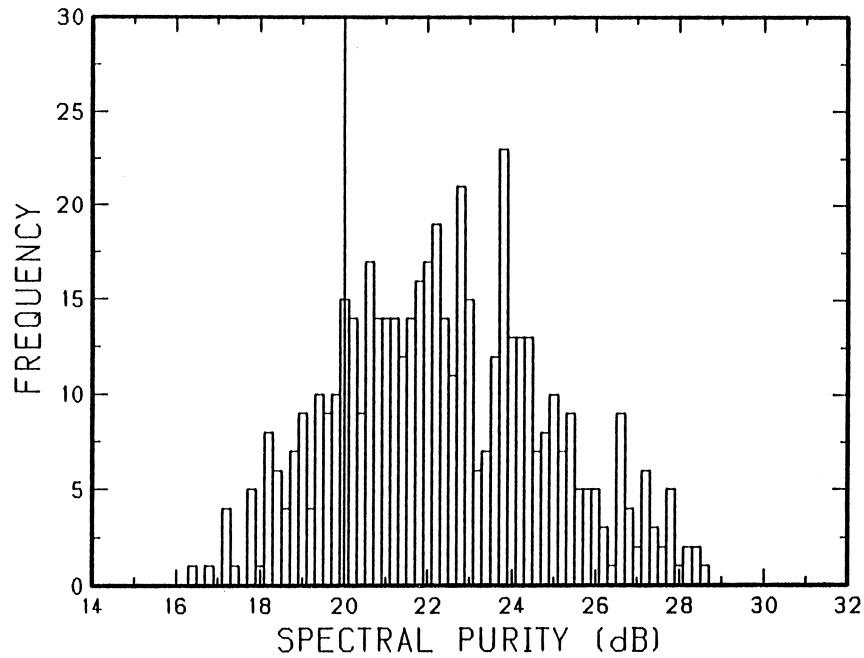
FET frequency doubler



Histogram of the spectral purity for 500
frequency doubler outcomes - before optimization



Histogram of the spectral purity for 500 frequency doubler outcomes - after optimization using IGAT



Histogram of the spectral purity for 500 frequency doubler outcomes - after optimization using FAST



Efficient Harmonic Balance Optimization

traditional perturbation (PAST): inaccurate, time-consuming Jacobian and gradient calculations

Kundert and Sangiovanni-Vincentelli and *Rizzoli et al.* suggested exact Jacobian approach

Rizzoli et al. combined both optimization and solving nonlinear equations - disadvantage - incompatibility with established yield optimization formulations

Bandler, Zhang and Biernacki (1988) developed the exact adjoint sensitivity technique (EAST) for harmonic balance

Bandler, Zhang and Biernacki (1989) developed the feasible adjoint sensitivity technique (FASTTM) for harmonic balance



FASTTM

a breakthrough in circuit theory

an expedient implementation of the EAST concept

unmatched speed and accuracy over perturbation

implementable in general purpose CAE architectures

combines efficiency of exact adjoint sensitivities with
simplicity of conventional perturbation

concept applicable to Jacobian evaluation for fast harmonic
balance simulation

the basis of the world's most powerful harmonic balance
optimizer (featured in HarPE)



Statistical Modeling

variations of process and geometrical parameters in device manufacturing result in complex statistical behaviour of devices

complicated distributions and correlations of device responses or equivalent model parameters

statistical modeling provides tools to generate device random outcome responses that reflect the actual distribution of the responses

growing demand for reliable statistical models

needed for accurate yield estimation

needed in yield optimization



Increasing Complexity of Device Statistics

statistically independent variables

multidimensional distributions - correlated variables

measurement errors superimposed

multi-level distributions



Equivalent Circuit Statistical Models

equivalent model parameters are needed as the input data to popular microwave CAD software

model parameters are strongly correlated

multidimensional distributions

an approach based on measurements of the S-parameters for a sample of finished devices - *Purviance, Criss and Monteith* (1988)



Physics Based Models

models based on semiconductor device physics equations

two approaches to solving device equations

analytical models where device equations are analytically simplified or solved under certain assumptions (e.g., quasi-static assumption)

numerical models where numerical techniques are used to directly solve device equations

analytical models evaluate faster than numerical ones



Advantages of Physics Based Models

true device design: optimization to adjust physical, geometrical and process variables before device fabrication

overall circuit performance reflects geometrical dimensions, material parameters, doping profile, channel thickness, etc.

essential for meaningful statistical analysis and yield optimization



Parameter Extraction in Statistical Modeling

each set of measurements corresponding to one device outcome is converted to the corresponding parameters of the model

parameter extraction procedure leading to a reliable and unique solution must be applied

this step provides a sample of models

Statistical Estimation

the statistics of the model parameters are examined

estimates of the means, standard deviations and correlation coefficients are calculated



Introduction to HarPE™

device-oriented nonlinear/linear simulation,
characterization and optimization

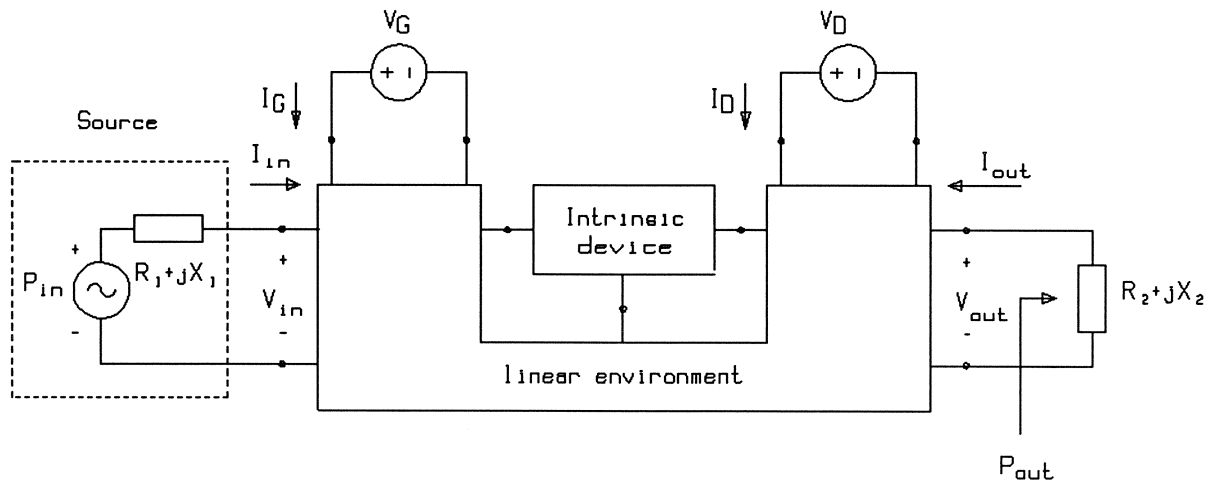
the world's first system for exploitation of harmonic data
for complete and accurate device characterization

nonlinear adjoint sensitivity analysis for gradient
optimization of nonlinear circuits (FAST™)

provides individualized models usable either in HarPE or
in other CAD software

writing in-house modeling programs is unnecessary

statistical modeling from multi-device measurements, and
Monte Carlo analysis



Schematic representation of a single-device circuit



HarPE Implementation of Physics Based Model

an analytical large-signal model based on the work of Khatibzadeh and Trew (1988)

model parameters include gate length, width, channel thickness, doping density, critical electric field, saturation velocity, built-in potential, low-field mobility, high-field diffusion coefficient

applicable to both small- and large-signal analysis

dynamically integrated into harmonic balance simulation

velocity - electric field curve is optimizable



Optimization of Physics Based Model

HarPE can optimize all parameters in the physics based model

parameter extraction allows determination of model parameters which can neither be accurately measured nor analytically derived, e.g., doping density, velocity - electric field curve

in parameter extraction excellent agreement was achieved using S-parameter measurements at 3 bias points

in design optimization of an amplifier an improvement of 12% power added efficiency was achieved using the physics based model parameters



Circuit Optimization with Physical Models

three stage power amplifier circuit

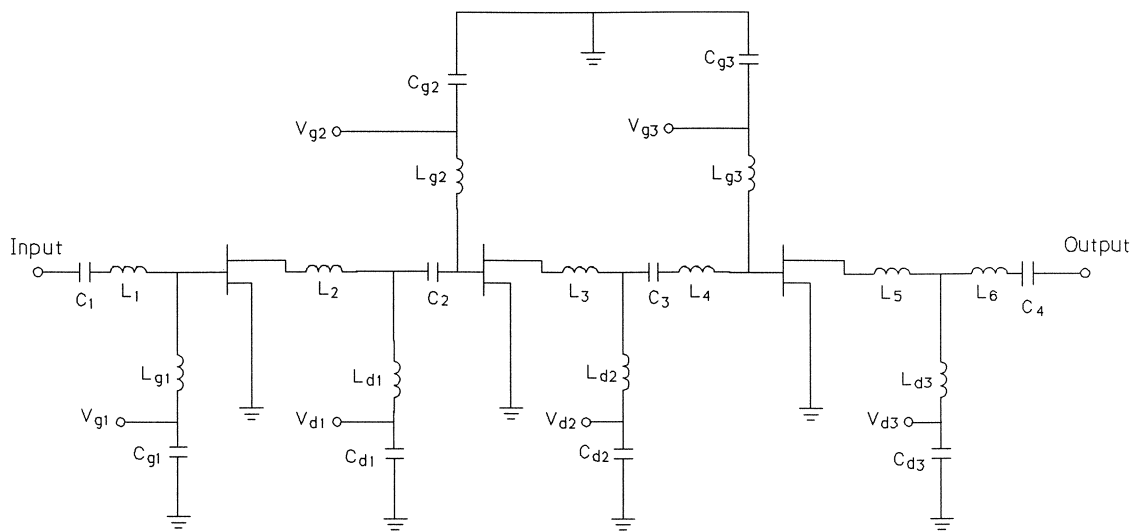
specifications are imposed on gain and power added efficiency

variables in the linear subcircuit

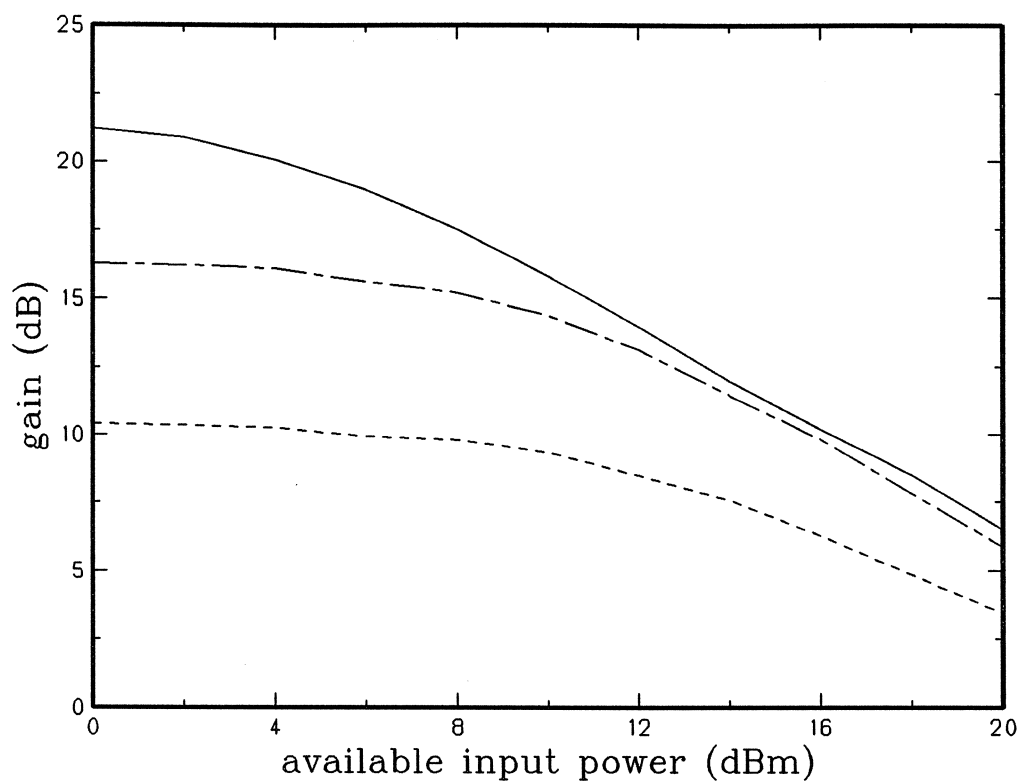
$$C_1, C_2, C_3, C_4, L_1, L_2, L_3, L_4, L_5, L_6$$

variables in the FET

$$L, a, W, N_d$$

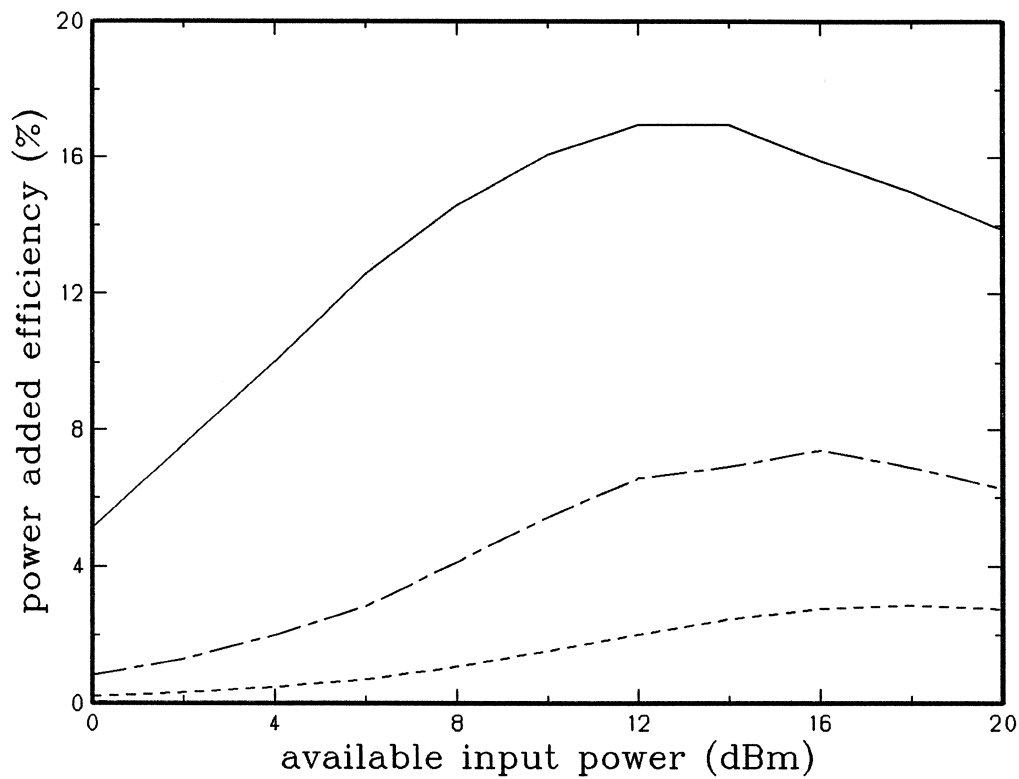


**Three stage power amplifier
(Kermarrec and Rumelhard 1988)**



Gain versus input power of the amplifier

- before optimization
- - - after optimizing only linear elements
- after optimizing both linear elements and FET parameters



Power added efficiency versus input power of the amplifier

- before optimization
- - - after optimizing only linear elements
- after optimizing both linear elements and FET parameters



Statistical Modeling

histogram of the small-signal transconductance

a small-signal equivalent circuit FET statistical model

model extracted from S-parameter measurements of 100 devices



Correlations in Statistical Models

correlation between the FET gate length and the channel delay time

a small-signal statistical equivalent circuit model extracted from 100 sets of synthetic S-parameter data generated by the physics-based model with variations on the gate length, width, etc.



Statistical Response

Monte Carlo analysis of an LC filter

200 outcomes

before yield optimization



Yield Optimization (coming soon)

statistical response of an LC filter after yield optimization

Monte Carlo analysis with 200 outcomes

yield increases from 54.5% to 78.5%



Statistical Response

Monte Carlo analysis of a FET frequency doubler

run chart of spectrum purity for 500 outcomes

before yield optimization



Yield Optimization

run chart of spectrum purity of the FET frequency doubler
after yield optimization

Monte Carlo analysis with 500 outcomes

yield increases from 32.4% to 78.6%