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OF NONLINEAR MICROWAVE CIRCUITS

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John W. Bandler, Radoslaw M. Biernacki, Shao Hua Chen and Qi-jun Zhang
Optimization Systems Associates Inc.
P.O. Box 8083, Dundas, Ontario
Canada L9H 5E7

Abstract

We address cost-effective, yield-driven design of nonlinear circuits such as (monolithic) microwave integrated circuits, (M)MICs, integrating statistical variations with physical/process/geometrical parameters. Powerful, novel techniques for intensive number crunching complemented by flexible, state-of-the-art software architectures dedicated to microwave integrated circuit design methodology are essential.

Effective yield optimization requires meaningful statistical representation of devices and subcircuits. The statistics arise from unknown mismatches at various reference planes, environmental fluctuations, random variations of basic physical, geometrical and process parameters, etc. Two important ways of addressing statistical modeling, namely, the equivalent circuit based approach and the physical/geometrical/process based approach, are discussed.

Yield optimization of nonlinear microwave circuits is a challenging task facing microwave CAD researchers today. The pioneering work in statistical design carried out over the past two decades is reviewed. Recent advances in yield optimization of nonlinear microwave circuits are emphasized. We also examine a unified theory for frequency domain simulation and sensitivity analysis of linear and nonlinear circuits involving the elegant theoretical concept EAST (exact adjoint sensitivity technique).

An expedient implementation combining the efficiency of EAST with the simplicity of traditional perturbation has realized the breakthrough FAST™ (feasible adjoint sensitivity technique). FAST, particularly suitable for implementation in general purpose microwave CAD programs, is a key to the power of a new software system HarPE™. The foregoing concepts and their relevance to modern (M)MIC/CAD are explained.

The world's most powerful harmonic balance optimizer and the first commercially available statistical modeling system dedicated to the microwave domain is featured in HarPE Version 1.4+S. Currently, HarPE solves device-oriented simulation, characterization and design optimization problems. The incorporation of the physics based model of Khatibzadeh and Trew is discussed. Statistical modeling and Monte Carlo analyses relevant to nonlinear device modeling are also addressed.

INTRODUCTION

The rapid progress in GaAs MMIC technology has been a driving force in the development of next generation microwave CAE software. A crucial feature in such software is the requirement for efficient algorithms for large-scale optimization of nonlinear circuits. The primary goals for circuit design is cost reduction. Yield-driven optimization methodology must be fully available. Statistical representation of physical, process and geometrical parameters must be integral to CAE of (M)MICs.

Such requirements resulted in the increasing sophistication of design methodology in computer-aided engineering over the past two decades. Two major design categories have been developed: deterministic and statistical. In the deterministic category, the conventional performance-driven design is complemented by the more advanced fixed tolerance and variable tolerance worst-case design. Progress in the statistical approach resulted in design methodology ranging from fixed tolerance yield-driven design to variable tolerance cost-driven design with correlated tolerances.

In this paper, we provide a brief overview of statistical modeling and design. In particular, we address statistical design techniques for microwave circuits operating under large-signal periodic steady state conditions. Efficient nonlinear circuit simulation and optimization techniques using powerful adjoint sensitivity analysis is emphasized. Physical/geometrical based statistical characterization of devices is discussed. A complete treatment of device, chip and wafer statistics is achieved by a hierarchical statistical modeling approach. The world's first commercially available statistical modeling system dedicated to the microwave domain is described.

YIELD OPTIMIZATION: A BRIEF REVIEW

Massive effort has been initiated into the production of MMIC chips. In such production, devices are designed and manufactured in batches rather than individually. Small random variations in the manufacturing process typically result in some "bad" devices. The yield, defined as the ratio of the number of "good" devices to the total number of devices, directly affects the cost of manufacturing. Consequently, the need for yield optimization has become more and more pressing. This need requires CAE systems to account for manufacturing tolerances, model uncertainties, variations of process parameters, environmental uncertainties, etc.

Unfortunately, yield optimization of nonlinear circuits is extremely computationally intensive. It involves three numerical procedures: optimization, statistical analysis, and nonlinear circuit simulation. The computational effort in each of the three procedures is nontrivial and is multiplied sequentially: optimization is iterative, each iteration of optimization requires simulation of many statistically related circuits, and simulation of each nonlinear circuit is iterative. Therefore, yield-driven design demands efficient algorithms for circuit simulations, powerful, robust, fast optimizers and effective statistical representation of devices and subcircuits.

Review of the Literature

In the late 1960s, Bandler [1] provided a review of optimization methods directed at performance-driven design. Early work in statistical design, formally exploiting parameter tolerances in circuit optimization, began at the end of that decade. Many fundamental concepts and techniques were developed during the 1970s. Karafin's paper is one of the earliest [2]. Pinel and Roberts [3] addressed the problem of design with tolerances. Bandler addressed cost-driven worst-case design with optimized tolerances [4-6], optimal centering and tolerance assignment integrated with tuning at the design stage [7], and an integrated approach to microwave design with parameter tolerances, model uncertainties, mismatches and reference plane uncertainties [8].

A later development is optimal tuning and alignment at the production stage [9].

Statistical design has been an important theoretical topic in VLSI design for a long time. Both circuit theoreticians and design engineers have contributed substantially. Director and Hachtel [10] developed a simplicial approximation approach to design centering. Bandler and Abdel-Malek [11] treated optimal centering, tolerancing, and yield determination via updated approximations and cuts. Polak and Sangiovanni-Vincentelli [12] used an outer-approximation. Yield optimization for arbitrary statistical distributions was presented by Abdel-Malek and Bandler [13].

Various approximation schemes were developed, such as the statistical exploration of Soin and Spence [14]. Styblinski and Ruszczynski [15] proposed the use of the stochastic approximation as a powerful, Monte Carlo based optimization technique. An important contribution by Singhal and Pinel [16] is the parametric sampling approach which is based on the concept of importance sampling. Downs, Cook and Rogers [17] estimated the yield for large circuits using a partitioning approach.

A special issue of the IEEE Transactions on Computer-Aided Design on Statistical Design of VLSI Circuits edited by Strojwas and Sangiovanni-Vincentelli was published in January 1986 [18]. Advanced approaches to statistical design of large-scale integrated circuits were presented. Yang, Hocevar, Cox, Machala and Chatterjee [19] identified significant process/geometrical parameters in active devices and presented an integrated approach for MOS VLSI statistical circuit design. An efficient sampling method is given by Stein [20]. Yield gradient estimation for truncated probability density functions was described by Styblinski [21]. Spoto, Coston and Hernandez [22] described a system for parametric statistical characterization and design of integrated circuits. Styblinski and Opalski [23] described algorithms and software tools for IC yield optimization based on fundamental fabrication parameters. Nassif, Strojwas and Director [24] presented an approach for realistic estimation of variations in device and circuit performance and performed worst-case analysis in terms of a set of statistically independent process disturbances. Maly, Strojwas and Director [25] bridged the gap between the traditional concepts of parametric and catastrophic yield. An interesting expansion of the statistical design concept to profit-based design methodology was introduced by Riley and Sangiovanni-Vincentelli [26].

Statistical design continues to be an important area of research in circuit design. Another extension of statistical design is the maximum income approach used by Opalski and Styblinski [27]. Aoki, Masuda, Shimada and Sato [28] approached the statistical centering problem from a design of experiment concept. Yu, Kang, Hajj and Trick [29] used a regression model to approximate system performance in terms of critical system parameters. Hocevar, Cox and Yang [30] used powerful gradient based optimizers including quasi-Newton and minimax optimizations and exploiting transient sensitivity information.

In 1987, the world's first commercial, microwave oriented yield-driven design features were introduced into Super-Compact [31] by Optimization Systems Associates Inc. [32]. Bandler and Chen [33] introduced a novel one-sided ℓ_1 formulation of the design centering problem. Bandler, Biernacki, Chen, Renault, Song and Zhang [34] and Biernacki, Bandler, Song and Zhang [35] used efficient quadratic modeling and presented the largest example of yield optimization of microwave circuits to that date.

The first formal demonstration of yield optimization of nonlinear microwave circuits with statistically characterized devices was presented by Bandler, Zhang, Song and Biernacki [36].

The most recent advance is the implementation of efficient gradient evaluation techniques. Implementation of IGAT (integrated gradient approximation technique) and FAST (feasible adjoint sensitivity technique) in gradient based yield optimization of nonlinear circuits has been described by Bandler, Zhang, Song and Biernacki [37]. Presently, pioneering work in yield-driven design with dynamically integrated physics based device models and statistical modeling is being carried out within the new software system HarPE Version 1.4+S [38].

Systematic reviews of statistical design have been given by Bandler and Chen [33], Wehrhahn and Spence [39], Strojwas [40] and Soin and Spence [41].

Advanced Topics for Statistical Design

Statistical design of large-scale circuits is one of the most challenging tasks in CAD. Bandler, Biernacki, Chen, Renault, Song and Zhang [34] performed design centering for a 5-channel multiplexer circuit. A fast and dedicated branched cascaded circuit simulator described by Bandler, Daijavad and Zhang [42] was used. For general circuits, efficient approaches include sparse matrix techniques (e.g., Reid [43], Duff [44] and Kundert [45]), decomposition (e.g., Salama, Starzyk and Bandler [46] and Bandler and Zhang [47, 48]), approximation (e.g., Biernacki, Bandler, Song and Zhang [35] and Tan, Pan, Ku and Shey [49]) and/or supercomputers (e.g., Bandler, Biernacki, Chen, Renault, Song and Zhang [34] and Rizzoli, Ferlito and Neri [50]).

Approximation techniques have been instrumental in solving computationally intensive problems. Approximations can be made at the circuit response level or device/element level. Quadratic approximation (e.g., Biernacki and Styblinski [51] and Low and Director [52]) and regression models (e.g., Yu, Kang, Hajj and Trick [29]) are examples of circuit response approximation. An important vehicle for device approximation is the black box approach (e.g., Murray-Lasso [53]). Burns, Newton and Pederson [54] and Jain, Agnew and Nakhla [55] used a table lookup approach to approximate active devices. The table lookup approach is also used by Jansen Microwave to speed up simulation and optimization in the field theory based MMIC CAD package LINMIC+ [56]. Lewis [57] used a 2^N tree concept to select simulation samples for model building in which more samples were automatically selected in highly nonlinear regions. Barby, Vlach and Singhal [58] used polynomial splines. Meijer [59] made recent progress in the table lookup approach by integrating into the table lookup the basic knowledge of device characteristics.

An elegant formulation of the design problem into optimization is a prerequisite for meaningful and efficient circuit design. Examples of efficient formulations for circuit optimization can be found in Nye, Riley, Sangiovanni-Vincentelli and Tits [60] and Bandler and Zhang [61].

YIELD OPTIMIZATION OF NONLINEAR MICROWAVE CIRCUITS

Bandler, Zhang, Song and Biernacki [36] addressed the challenge of yield optimization of nonlinear microwave circuits operating in the steady-state under large-signal periodic excitations. Yield-driven design was formulated as a one-sided ℓ_1 optimization problem. The nonlinear circuit simulation was carried out using the harmonic balance method with exact Jacobians. Multidimensional statistical distributions of the intrinsic and parasitic parameters of FETs were handled.

Nonlinear Circuit Simulation

Most analog microwave circuits operate under steady-state conditions. Harmonic balance [62-65] has become the major simulation tool for nonlinear circuits. In harmonic balance, the overall nonlinear circuit is divided into linear and nonlinear parts. The linear part is computed in the frequency domain while the nonlinear part is computed in the time domain. The Fourier transform is used to communicate between the two domains. The harmonic balance equations are nonlinear and are solved by an iterative procedure.

Algorithms used presently in popular harmonic balance software inhibit fast optimization. Due to their inaccurate and slow sensitivity/gradient evaluation, yield-driven design may be prohibitively CPU intensive. This problem is addressed by the recent development of a unified framework for harmonic balance simulation and sensitivity analysis.

Efficient Harmonic Balance Optimization

Traditional microwave CAD software uses perturbation (PAST - perturbation approximate sensitivity technique) to calculate Jacobians and gradients. This approach is very simple to implement but inaccurate and time-consuming. Kundert and Sangiovanni-Vincentelli [63] and Rizzoli and Neri [64] suggested an exact Jacobian approach. Rizzoli, Lipparini and Marazzi [65] combined both optimization and solving nonlinear equations. A serious disadvantage, however, is the incompatibility with established yield optimization formulations.

Bandler, Zhang and Biernacki developed the exact adjoint sensitivity technique (EAST) [66, 67] and the feasible adjoint sensitivity technique (FAST) [68], both within the harmonic balance environment. These powerful sensitivity analysis techniques have been applied to multi-device harmonic balance parameter extraction optimization [69, 70] and yield optimization of nonlinear microwave circuits [37].

Sensitivity Analysis

EAST is theoretically the most efficient method for sensitivity analysis [67]. The sensitivities of the output response w.r.t. many variables can be obtained from two sets of voltages, i.e., the original and the adjoint voltages. The adjoint equations are linear and are byproducts of solving the original nonlinear circuit. No network perturbation is necessary. Only one original circuit and one adjoint network need to be solved regardless of how many variables there are.

FAST combines the efficiency of EAST with the simplicity of PAST. It is an expedient implementation of the EAST concept. It provides unmatched speed and accuracy over perturbation and is implementable in general purpose CAE architectures. FAST is the basis of the world's most powerful harmonic balance optimizer featured in HarPE [38].

The integrated gradient approximation technique (IGAT) developed by Bandler, Chen, Daijavad and Madsen [71] is another powerful approach. IGAT utilizes the Broyden formula [72] with special iterations of Powell to update the approximate gradients. It is particularly suitable for optimization since the function evaluations in each optimization iteration can be reused to construct an approximate gradient vector. IGAT is slower than FAST but faster than PAST. IGAT is the best choice when modification of the circuit simulator is not possible.

Yield Optimization of Nonlinear Circuits

Yield-driven design of nonlinear circuits can be formulated as a one-sided ℓ_1 optimization problem. For each circuit outcome, we calculate a set of weighted errors between simulated circuit responses and specifications. A generalized ℓ_p function of those error functions is formulated. Only the circuit outcomes that violate specifications are included in the final ℓ_1 function, i.e., the one-sided ℓ_1 function [33]. Optimization will tend to minimize the number of circuits that violate specifications. Consequently, the yield is increased. This formulation permits the use of powerful gradient based optimizers. Therefore, harmonic balance simulation with exact Jacobians and efficient sensitivity analysis can be exploited, making the difficult task of nonlinear circuit yield optimization tractable. We have integrated harmonic balance based IGAT and FAST with one-sided ℓ_1 yield optimization. IGAT and FAST are compared with PAST on the one extreme and the theoretical EAST on the other. FAST, linking state-of-the-art optimization and efficient harmonic balance simulation, is the key to making our approach to nonlinear microwave circuit design the most powerful available.

Yield Optimization of a FET Frequency Doubler

A FET frequency doubler consisting of a common-source FET with a lumped input matching network and a microstrip output matching and filter section is considered for yield optimization [37]. The fundamental frequency is 5GHz. Six optimization variables include an input inductance, two microstrip lengths, two bias voltages and the driving power level. The specifications for the doubler include a 2.5 dB lower specification on the conversion gain and a 20 dB lower specification on the spectral purity.

The large-signal FET statistical model includes an intrinsic large-signal FET model, statistical distributions and correlations of parameters. The multidimensional normal distribution is assumed for all FET intrinsic and extrinsic parameters. The means, standard deviations and correlations between FET parameters are assumed [37]. Uniform distributions with fixed tolerances of 3% are assumed for all the six optimization variables. The random number generator used is capable of generating outcomes from the independent and multidimensional correlated normal distributions and from uniform distributions.

The starting point for yield optimization is the solution of the minimax nominal design w.r.t. the same specifications, using the same six design variables. At this point, the estimated yield based on 500 outcomes is 39.6%.

We conducted two designs using IGAT and FAST gradient calculation in the same environment. Fifty outcomes were used during optimization. Both approaches increased the yield to 71%, based on 500 outcomes at the solutions. The CPU time using IGAT and FAST are 37 and 20 minutes, respectively on a Multiflow Trace 14/300. The speed of the Multiflow Trace 14/300 is about 10 MFLOPS.

STATISTICAL MODELING

As the microwave industry advances into MMIC technology, the capability of reliable statistical modeling becomes a prerequisite for accurate yield and cost analysis and optimization. Variations of process and geometrical parameters in manufacturing result in complex statistical behaviour of devices. The device responses not only vary from one device to another but are also strongly correlated. The aim of statistical modeling is to provide tools for generating device random outcomes which reproduce the actual distribution of the responses [73].

Dutton, Divekar, Gonzalez, Hansen and Antoniadis [74] and Divekar, Dutton and McCalla [75] studied the correlation of fabrication process and electrical device parameter variations and experimented on bipolar junction transistors. Styblinski [76] used factor analysis to describe resistor correlations for monolithic integrated circuits. Rankin [77] described fundamental concepts in statistical modeling for integrated circuits. Freeman [78] presented statistical techniques for calibrating simulation models of analog circuits and devices. Liu and Singhal [79] described a combined equivalent circuit/physical modeling scheme using empirical formulas. Cox, Yang, Mahant-Shetti and Chatterjee [80] identified critical physical/geometrical parameters for MOS VLSI circuit statistical description. Herr and Barnes [81] investigated CMOS VLSI statistical modeling. Spanos and Director [82] used a hierarchical approach to relate the process disturbances to circuit responses.

Microwave FET statistical modeling was approached by Purviance, Criss and Monteith [83] using equivalent circuit models, by Purviance, Petzold and Potratz [84] using principal component analysis at the S-parameter level, and by Purviance, Meehan and Collins [85] using a data base approach. Bandler, Biernacki, Chen, Loman, Renault and Zhang [73] combined discrete and normal distributions to characterize the irregular distributions and at the same time preserve statistical correlations.

Hierarchical Statistical Modeling

A statistical model can be described by a statistical distribution at the device response level, the equivalent model parameter level and/or the basic process/geometrical parameter level. A complete model includes the overall effect of device statistics, chip statistics and wafter statistics. Inter-device correlations originate from chip and wafer statistics. Inter-chip correlations originate from wafer statistics.

Equivalent Circuit Based Statistical Modeling

In the equivalent circuit based statistical modeling approach, the model is described by the statistical distributions of equivalent circuit parameters [73, 83]. The advantage of this approach is that the equivalent circuit models are generally available in popular microwave CAD software and model evaluation is generally fast. However, the distribution functions may not be simple. The model parameters are strongly correlated resulting in complicated and multidimensional distributions. Also, the number of statistical variables may be large.

Physics/Geometrical/Process Based Statistical Modeling

Probably the most appropriate approach to statistical modeling should be based on process parameter monitoring and on inter-process measurements. In this approach, we use physical models that are based on semiconductor device physics equations. The statistical properties are described directly for process/geometrical parameters such as FET gate length, gate width, doping density, etc. The statistical distributions at this level are generally simpler than that at the equivalent circuit level. A small number of statistically independent variables can be used. However, the model evaluation is slower than that of the equivalent circuit model. It also requires that the physical and empirical formulas be made available in advanced or specialized CAD software.

A physics based model is necessary for realistic device design: to optimize physical, geometrical and process variables that are truly adjustable in device fabrication. We will then be able to relate the circuit performance to geometrical dimensions, material parameters, doping profiles, channel thicknesses, etc. The integration of physics models with iterative optimization has been presented by Bandler, Zhang and Cai [86]. It demonstrated the essence of meaningful statistical analysis and yield optimization.

Parameter Extraction in Statistical Modeling

Statistical modeling requires measurements made on a large number of devices. For each device, a set of model parameters is extracted from the corresponding measurements. From the measurements of all the devices, a sample of models is obtained. It is essential to the success of this step to have a parameter extraction procedure which produces efficient, consistent and unique solutions. The statistics of the extracted sample of model parameters can be examined. Estimates of the means, standard deviations and correlation coefficients can be calculated.

Multidimensional Distributions

The statistics of model parameters can be described by a normal or arbitrary multidimensional distribution. The assumption of a multidimensional normal distribution is often made in practice for simplicity. The normal distributions can be completely described by mean values, standard deviations and correlation coefficients. Using this distribution it is easy to generate outcomes.

Arbitrary multidimensional distributions can be used to describe complicated statistics. In this case, an approximation to the joint pdf is usually made. A practical attempt is to use a discretized pdf [13]. The accuracy of this approach is heavily dependent upon the size of the samples. When the number of statistical variables increases, the memory requirement for the distribution grows exponentially.

NEW CAD SYSTEM FOR STATISTICAL MODELING

Advanced features of statistical modeling and analysis are available in the new CAD system HarPE Version 1.4+S [38]. HarPE is a state-of-the-art software system for device modeling, simulation and optimization. It is the world's first system for exploitation of harmonic data for complete and accurate device characterization. And now it is the first commercially available statistical modeling system. It implements the nonlinear adjoint sensitivity analysis for gradient optimization of nonlinear circuits (FAST) with state-of-the-art ℓ_1, least-squares and minimax optimizers. An analytically unified DC/small-signal/large-signal circuit simulation [87, 88] ensures a truly seamless nonlinear/linear simulation and optimization.

HarPE's statistical features include multi-device modeling, postprocessing of statistical data, Monte Carlo analysis and physics based device modeling.

Multi-Device Modeling

Suitable measurements (large-signal power spectra, small-signal S-parameters and/or DC IV data) are collected from a sample of devices. The size of the sample is determined by a trade-off between measurement cost and statistical estimation accuracy. Typically, a sample of 100 to 300 devices is used. A theoretical study by Bandler, Biernacki, Chen, Loman, Renault and Zhang [73] on the relationship between statistical confidence level and sample size has indicated that, if the distribution is indeed normal, sufficient confidence can be achieved from a sample size as small as 20 to 50. Measurements on multiple devices may be automated by some systems, such as Cascade Microtech's MicroCAT™ Test Executive system which is supported by HarPE.

From the measured data, multiple device models are extracted through repeated optimization. Execution times vary according to the model used, the sample size, and the amount of data per device. Our test examples show a typical run time for 100 devices, with S-parameter measurements at two bias points and five frequencies per bias, ranging from 45 minutes on an Apollo DN3500 to 15 minutes on a Sun SPARCstation 1.

Postprocessing

The multi-device modeling produces a sample of extracted model parameters. From this sample the program can generate histograms showing parameter spreads and scatter diagrams showing parameter correlations. The sample of parameters can be postprocessed to generate a consolidated statistical model, which is then back annotated to produce a HarPE circuit file immediately suitable for Monte Carlo analysis.

Parameter correlations are automatically estimated and included in the multidimensional normal distribution model. For parameters exhibiting substantially non-Gaussian distributions, the histograms can be used directly as discrete approximations of the marginal density functions. This combined discrete/normal approach as described by Bandler, Biernacki, Chen, Loman, Renault and Zhang [73] preserves the means, standard deviations, correlations and marginal distributions derived from the sample. It provides enhanced accuracy of the model while retaining the simplicity of the normal distribution.

Monte Carlo Analysis

Monte Carlo analysis is a practical means of studying the statistical behaviour of devices and circuits. The Monte Carlo analysis option in HarPE includes large-signal, DC and small-signal simulations. Uniform, normal, exponential and lognormal distributions with absolute or relative tolerances can be defined in Monte Carlo analysis. Histograms, run charts and sweep diagrams are displayed. Yield can be computed from specifications on spectra, S-parameters, DC currents and arbitrary user-defined functions.

Physics Based FET Model

The statistical features in HarPE can be used with built-in or user-created device models. The built-in models include the Materka and Kacprzak model [89], the Curtice and Ettenberg model [90], the Raytheon (Statz, Newman, Smith, Pucel and Haus) model [91], the Khatibzadeh and Trew model [92], the Gummel-Poon bipolar transistor model, and the diode model.

HarPE is the first commercial software to offer a physics based model, i.e., the Khatibzadeh and Trew model [92], together with statistical modeling and analysis capabilities to microwave engineers. The physics based model parameters include gate length, gate width and channel thickness. Uniform doping and arbitrary doping profiles are accommodated. Physics based models enjoy a distinctive advantage in statistical modeling and analysis. Because such models directly relate to physical, geometrical and process parameters, their statistical distributions are generally simpler than that at the equivalent circuit level. Therefore, physics based statistical models should be more accurate and reliable.

CONCLUSIONS

This paper offers a critical review of statistical design and modeling. We examined a number of important issues including yield optimization, efficient harmonic balance simulation, sensitivity analysis and optimization of nonlinear microwave circuits, statistical sampling, hierarchical statistical modeling, different modeling approaches, model approximation and CAD software aspects. Theoretical unification and a coherent implementation of such a large variety of aspects is a driving force in the development of the next generation of microwave CAD software systems. In this aspect an efficient software architecture is of significant importance [93, 94]. Breakthrough design technologies are required before effectively solving statistical design of large-scale nonlinear MMIC circuits.

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