# PRACTICAL, HIGH SPEED GRADIENT COMPUTATION FOR HARMONIC BALANCE SIMULATORS

OSA-89-MT-27-V

October 17, 1989

Presented at the 1989 MTT-S Symposium, Long Beach, CA, June 13-15, 1989

# PRACTICAL, HIGH SPEED GRADIENT COMPUTATION FOR HARMONIC BALANCE SIMULATORS

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## *Introduction*

resurgence of interest in harmonic balance (HB) simulation

extensive number crunching required by available HB simulators inhibits fast optimization of nonlinear circuits

inaccurate sensitivity and gradient calculations

## *Improving* HB *Efficiency and Robustness*

*Kundert and Sangiovanni-Vincentelli* and *Rizzoli et al.* suggested exact Jacobian approach

*Rizzoli et al.* combined both optimization and solving nonlinear equations - disadvantage - incompatibility with established yield optimization formulations

*Bandler, Zhang and Biernacki* developed the exact adjoint sensitivity technique (EAST) for the HB method

## *Our Research*

implementation aspects of the adjoint sensitivity approach

impact on general purpose CAD programs

*Summary*

FAST - the future adjoint sensitivity technique

powerful computational concept

combines efficiency of exact adjoint sensitivities with simplicity of conventional perturbations

concept carries over to practically implementable Jacobians for fast harmonic balance simulation

our results promise high speed gradient evaluation essential for yield optimization of nonlinear MMIC circuits by general purpose software

FAST - *The Future Adjoint Sensitivity Technique*

performs adjoint sensitivities

retains most of the efficiency and accuracy of the EAST (exact adjoint sensitivity technique)

accommodates the simplicity of the conventional perturbation method, which we name PAST (perturbation approximate sensitivity technique)

directly applicable to both the node/port formulation and the state variable formulation

suitable for implementation within the framework of general purpose software

extends to the computation of Jacobians for high speed HB simulation

the features of FAST and its Jacobian extensions are illustrated by a mixer example and a frequency doubler example

*Notation and Definitions*

the overall nonlinear circuit is divided into linear and nonlinear parts

| | |
|---|---|
| $\mathbf{v}(t)$ and $\mathbf{i}(t)$ | voltage and current waveforms at the linear-nonlinear connection ports |
| $\mathbf{V}(k)$ or $\mathbf{I}(k)$ | kth harmonic of voltage and current spectrum |

bar denotes the split real and imaginary parts of complex quantities

| | |
|---|---|
| $\bar{\mathbf{V}}$ or $\bar{\mathbf{I}}$ | real vectors containing the real and the imaginary parts of $\mathbf{V}(k)$ or $\mathbf{I}(k)$ for all harmonics k, $k = 0, 1, \ldots, H$ |

hat distinguishes quantities of the adjoint system

| | |
|---|---|
| $\hat{\bar{\mathbf{V}}}$ | real vector containing the real and the imaginary parts of adjoint voltages for all harmonics k, $k = 0, 1, \ldots, H$ |

*Basics of the* HB *Technique*

suppose the harmonic balance equation is

$$\bar{\mathbf{F}}\,(\bar{\mathbf{V}}) = \mathbf{0}$$

a simple Newton's update for the equation is

$$\bar{\mathbf{V}}_{\text{new}} = \bar{\mathbf{V}}_{\text{old}} - \bar{\mathbf{J}}^{-1}\,\bar{\mathbf{F}}\,(\bar{\mathbf{V}}_{\text{old}})$$

where $\bar{\mathbf{J}}$ is the Jacobian matrix

*Gradient Analysis Using* FAST

suppose the output voltage $\bar{V}_{out}$ can be computed from $\bar{\mathbf{V}}$ as

$$\bar{V}_{out} = \bar{\mathbf{e}}^T \bar{\mathbf{V}}$$

the approximate sensitivity of output voltage $\bar{V}_{out}$ w.r.t. variable $\phi$ can be computed as

$$\partial \bar{V}_{out} / \partial \phi = - \hat{\bar{\mathbf{V}}}^T \bar{\mathbf{F}} ( \phi + \Delta \phi , \bar{\mathbf{V}}_{solution}) / \Delta \phi$$

where

$\bar{\mathbf{V}}_{solution}$ solution of the harmonic balance equations

$\hat{\bar{\mathbf{V}}}$ adjoint voltages obtained from solving the linear equations:

$$\bar{\mathbf{J}}^T \hat{\bar{\mathbf{V}}} = \bar{\mathbf{e}}$$

the adjoint linear equations are easy to solve since the LU factors of the Jacobian matrix are available from solving the harmonic balance equations

*Comparison of* FAST *with* PAST

consider 10 design variables in a nonlinear circuit

PAST analysis

> the circuit has to be perturbed 10 times
> 10 nonlinear circuits have to be solved

FAST analysis

> the circuit has to be perturbed 10 times
> no nonlinear circuits have to be solved
> only 10 errors need to be evaluated

the best possible situation for PAST is that all 10 simulations use the same Jacobian and all converge in one iteration

FAST always requires less computation than that of the best possible situation of PAST since error evaluation takes a fraction of time needed for simulation

FAST is more accurate than PAST

*Comparison of* FAST *with* EAST

EAST

commonly accepted as the most powerful tool
a need to keep track of variable locations
expensive to implement

FAST

no need to keep track of variable locations
only the output port has to be identified
implementable in general CAD programs

FAST *Analysis of a FET Mixer*

the mixer circuit

| | |
|---|---|
| LO frequency | $f_{LO}$ = 11 GHz |
| RF frequency | $f_{RF}$ = 12 GHz |
| IF frequency | $f_{IF}$ = 1 GHz |
| DC bias voltages | $V_{GS}$ = -0.9, $V_{DS}$ = 3.0 |
| LO power | $P_{LO}$ = 8 dBm |
| RF power | $P_{RF}$ = -15 dBm |
| conversion gain | 6.9 dB |

compute sensitivities of the conversion gain w.r.t. all 26 variables

all parameters in the linear part
all parameters in the nonlinear part
DC bias, LO power, RF power
IF, LO and RF terminations

excellent agreement between sensitivities computed using FAST, PAST and EAST

CPU times on VAX 8600

| | |
|---|---|
| circuit simulation | 22 seconds |
| FAST sensitivity analysis | 10.7 seconds |
| EAST sensitivity analysis | 3.7 seconds |
| PAST sensitivity analysis | 240 seconds |

# TABLE I

# NUMERICAL VERIFICATION OF FAST FOR THE MIXER EXAMPLE

| Variable | Sensitivity from FAST | Sensitivity from EAST | Sensitivity from PAST | Difference between FAST and EAST (%) | Difference between FAST and PAST (%) |
|---|---|---|---|---|---|
| linear subnetwork | | | | | |
| $C_{ds}$ | -24.28082 | -24.28081 | -24.03669 | 0.00 | 1.01 |
| $C_{gd}$ | -32.16238 | -32.16237 | -32.33670 | 0.00 | -0.54 |
| $C_{de}$ | $-8.8\times10^{-13}$ | $1.7\times10^{-13}$ | 0 | 120.21 | 100.00 |
| $R_g$ | 10.00754 | 10.00756 | 9.89609 | -0.00 | 1.11 |
| $R_d$ | 11.71325 | 11.71327 | 11.71338 | -0.00 | -0.00 |
| $R_s$ | -4.98829 | -4.98827 | -4.98861 | 0.00 | -0.01 |
| $R_{de}$ | -0.07171 | -0.07171 | -0.07115 | 0.00 | 0.79 |
| $L_g$ | -0.30238 | -0.30238 | -0.30054 | 0.00 | 0.61 |
| $L_d$ | -0.87824 | -0.87824 | -0.87247 | 0.00 | 0.66 |
| $L_s$ | -0.33527 | -0.33527 | -0.33191 | 0.00 | 1.00 |
| nonlinear subnetwork | | | | | |
| $C_{gs0}$ | -5.43110 | -5.43110 | -5.38265 | 0.00 | 0.89 |
| $\tau$ | 1.52983 | 1.52984 | 1.56057 | -0.00 | -2.01 |
| $V_\phi$ | -20.84224 | -20.84223 | -20.84308 | -0.00 | -0.00 |
| $V_{p0}$ | -14.62206 | -14.62206 | -14.62469 | 0.00 | -0.02 |
| $V_{dss}$ | 0.30209 | 0.30209 | 0.30210 | 0.00 | -0.00 |
| $I_{dsp}$ | 9.39335 | 9.39335 | 9.39338 | -0.00 | -0.00 |
| bias and driving sources | | | | | |
| $V_{GS}$ | -4.94402 | -4.94402 | -4.94271 | -0.00 | 0.03 |
| $V_{DS}$ | -0.67424 | -0.67424 | -0.67429 | 0.00 | -0.01 |
| $P_{LO}$ | 2.02886 | 2.02885 | 2.02882 | 0.00 | 0.00 |
| $P_{RF}$ | -0.09073 | -0.09072 | -0.09077 | 0.01 | -0.05 |
| terminations | | | | | |
| $R_g(f_{LO})$ | 8.83598 | 8.83596 | 8.76244 | 0.00 | 0.83 |
| $X_g(f_{LO})$ | 2.20500 | 2.20496 | 2.16567 | 0.00 | 1.78 |
| $R_g(f_{RF})$ | 0.71282 | 0.71281 | 0.70568 | 0.00 | 1.00 |
| $X_g(f_{RF})$ | 0.46410 | 0.46409 | 0.45702 | 0.00 | 1.53 |
| $R_d(f_{IF})$ | 0.65950 | 0.65950 | 0.65272 | -0.00 | 1.03 |
| $X_d(f_{IF})$ | 0.09024 | 0.09024 | 0.09207 | -0.00 | -2.02 |

*Simple and Efficient Computation of Jacobian*

exact Jacobian for HB simulation is available but very expensive to implement

*Kundert and Sangiovanni-Vincentelli* (1986)
*Rizzoli et. al.* (1986)

the perturbation (or incremental) approach is typically used in practice but is slow

FAST concept extends to Jacobian calculation by

computing time domain derivatives at the device level using perturbations

converting these derivatives to the frequency domain by a Fourier transform

assembling the resulting Fourier coefficients into the Jacobian matrix

*Approximate Jacobian for a Frequency Doubler*

the doubler circuit

| | |
|---|---|
| input frequency | 5 GHz |
| output frequency | 10 GHz |

four harmonics were considered

the Jacobians were computed using our FAST approach and the conventional perturbation approach

the numerical results from the two approaches agreed very well

the corresponding CPU times on MicroVAX II

| | |
|---|---|
| FAST | 0.89 second |
| perturbations | 5.3 seconds |

*Conclusions*

FAST is an expedient tool for gradient calculation in the HB environment

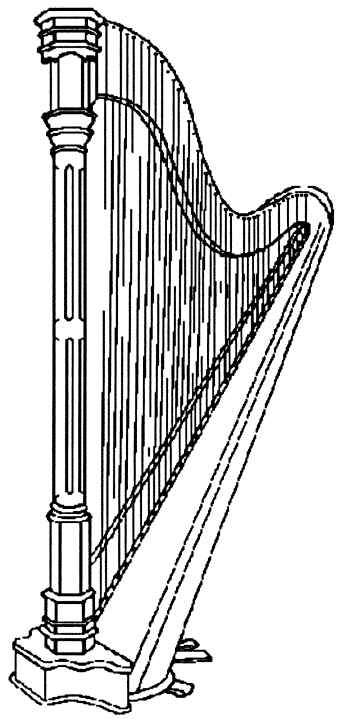
advantages of FAST over PAST are its unmatched speed and accuracy

advantage of FAST over EAST is its implementational simplicity

FAST is directly compatible with established formulations of yield optimization

FAST is particularly suitable for implementation in general purpose microwave CAD software

FAST provides a key to the new generation of yield optimizers for MMICs



the FAST technique is fully implemented in a new program HarPE™ for harmonic balance driven FET model simulation and parameter extraction

visit OSA booth #1714 for demonstration of HarPE™