COMBINED DISCRETE/NORMAL STATISTICAL MODELING OF MICROWAVE DEVICES

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OSA

COMBINED DISCRETE/NORMAL STATISTICAL

MODELING OF MICROWAVE DEVICES

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Outline

statistical characterization of the equivalent circuit parameters for microwave FET devices

statistics derived from the S-parameters measured for a sample of devices

parameter extraction and statistical estimation

combined discrete/normal approach

confidence levels and confidence intervals

size of the sample

Introduction

variations of process and geometrical parameters in device manufacturing result in complex statistical behaviour of devices

statistical modeling provides tools to generate device random outcome responses that reflect the actual distribution of the responses

growing demand for reliable statistical models

needed for accurate yield estimation

needed in yield optimization

Equivalent Circuit Statistical Models

equivalent model parameters are needed as the input data to popular microwave CAD software

model parameters are strongly correlated

multidimensional distributions

an approach based on measurements of the S-parameters for a sample of finished devices - *Purviance, Criss and Monteith* (1988)

Parameter Extraction

each set of measured S-parameters corresponding to one FET outcome is converted to the corresponding parameters of the equivalent circuit

parameter extraction procedure leading to a reliable and unique solution must be applied

this step provides a sample of equivalent circuit models

Statistical Estimation

the statistics of the equivalent circuit parameters are examined

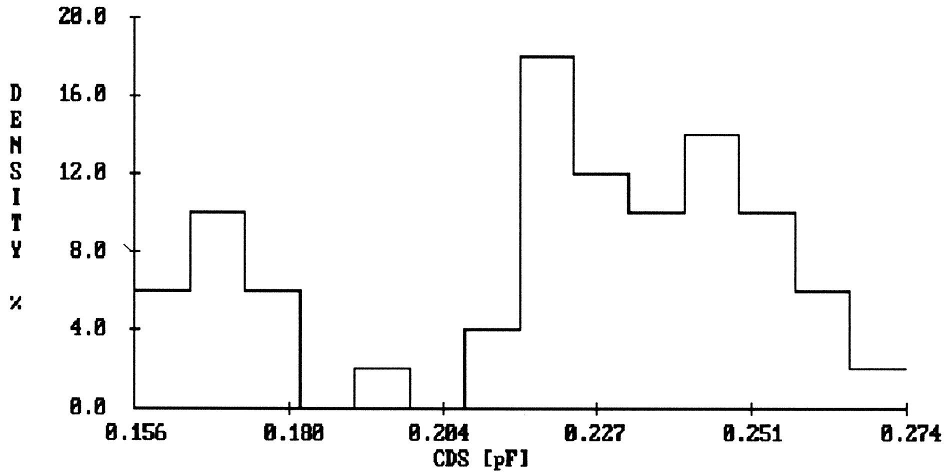
estimates of the means, standard deviations and correlation coefficients are calculated

Multidimensional Normal Distribution

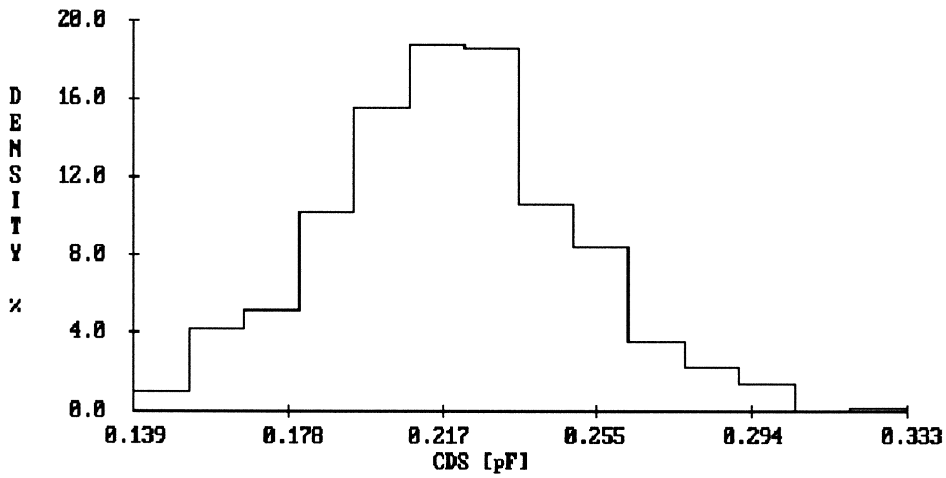
mean values, standard deviations and correlation coefficients are sufficient to describe pdf

easy to generate outcomes

assumption of multidimensional normal distribution is often made in practice for simplicity



Original histogram of C_{DS} obtained from a sample of 50 FETs



Regenerated histogram of C_{DS} using multidimensional normal distribution

Arbitrary Multidimensional Distributions

approximation to the joint pdf

discretized pdf is a practical solution

limitations:

limited size of the sample (poor approximation)

exponential growth of the number of cells with the number of variables

FET Statistical Modeling Example

S-parameters for 50 FETs derived from measurement data

our parameter extraction system was used to uniquely identify the equivalent model parameters

the Materka-Kaćprzak model was used

sample of 50 equivalent circuit models was obtained

from these 50 models we estimated the means, standard deviations and correlation coefficients

ORIGINAL MEANS, STANDARD DEVIATIONS AND SELECTED CORRELATION COEFFICIENTS

Model Parameter	Mean Value μ		Standard Deviation σ (%)	Selected Correlation Coefficients ρ		
				G_{DS}	R_I	R_S
R_G	2.4291	ohm	14	-.44	-.83	.33
R_D	.9703	ohm	19	.18	.25	.27
G_{DS}	5.0558	mS	12	1.00	.17	-.42
R_I	.8114	ohm	38	.17	1.00	-.34
R_S	1.0110	ohm	12	-.42	-.34	1.00
C_{GS}	.6187	pF	6	-.10	.05	.03
C_{DG}	.0276	pF	15	.68	.04	-.43
C_{DS}	.2206	pF	14	.12	.07	-.12
g_m	.0611	S	10	.58	.19	-.13
τ	3.6697	psec	11	-.67	-.09	.02

Our Approach

for most of the model parameters a normal distribution could be assumed

some parameters exhibit distributions substantially different from normal

we combine a discrete distribution with a multidimensional normal distribution

Assumptions

model parameters are described by a multidimensional random variable **X**

X is a function of multidimensional normal random variable **Y**

$X_i = g_i(Y_i)$

X_i and Y_i have the same means and standard deviations

Implementation

the analytical form of g_i does not have to be known

multidimensional normal random number generator is used to generate outcomes of Y

the outcomes of X_i are obtained from Y_i using one-dimensional mapping

discrete distribution function and the corresponding normal distribution function are used

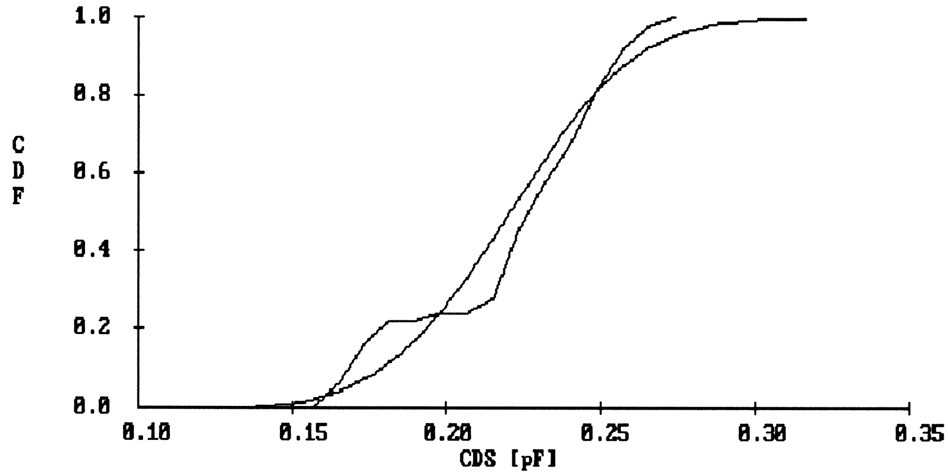
Discrete Distribution Function

approximation to the marginal distribution of the parameter of interest

as its pdf we simply take the histogram

as its cumulative marginal distribution function we simply take the integral

$$f(x) = \int_{-\infty}^{x} \mathrm{pdf}(y)\, dy$$



Cumulative distribution functions of the discrete and normal distributions

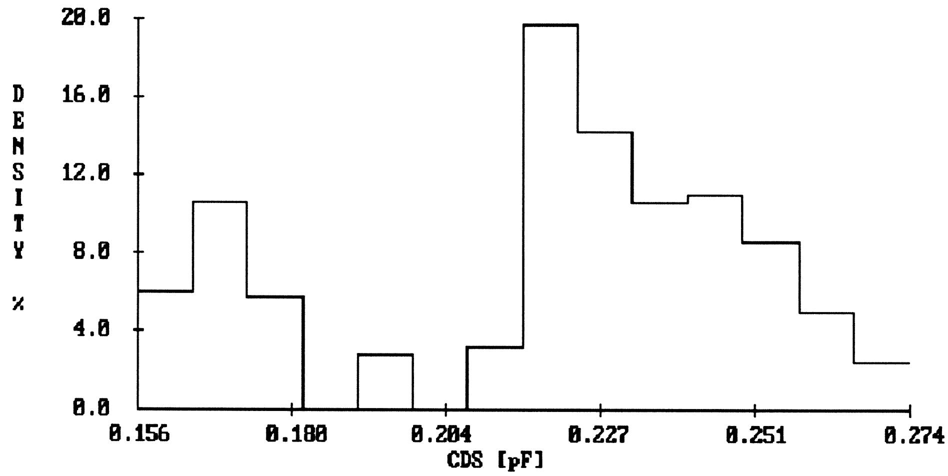
Results

the combined approach was used

500 simulated outcomes were used to regenerate means, standard deviations, correlation coefficients and histograms

a good match to the original distribution was obtained

our approach preserves the mean values, standard deviations and correlation coefficients



Regenerated histogram of C_{DS} using combined discrete/normal approach

RECOVERED MEANS, STANDARD DEVIATIONS
AND SELECTED CORRELATION COEFFICIENTS

Model Parameter	Mean Value μ		Standard Deviation σ (%)	Selected Correlation Coefficients ρ		
				G_{DS}	R_I	R_S
R_G	2.4274	ohm	14	-.43	-.83	.31
R_D	.9632	ohm	19	.23	.25	.24
G_{DS}	5.0483	mS	12	1.00	.19	-.43
R_I	.8053	ohm	38	.19	1.00	-.34
R_S	1.0036	ohm	12	-.43	-.34	1.00
C_{GS}	.6169	pF	6	-.11	.06	.01
C_{DG}	.0277	pF	15	.63	.05	-.38
C_{DS}	.2192	pF	14	.14	.08	-.08
g_m	.0608	S	10	.54	.22	-.14
τ	3.6633	psec	11	-.66	-.12	.02

Confidence Level

probability of p falling into the confidence interval

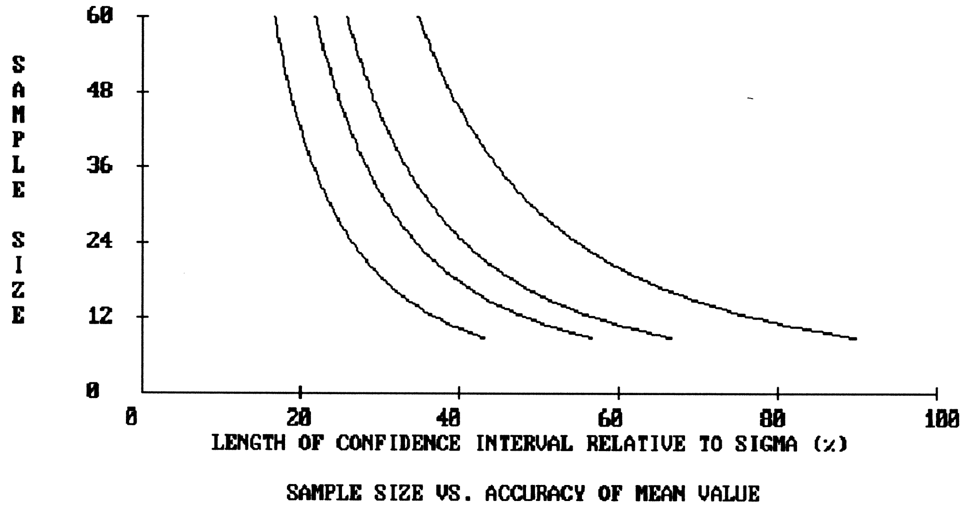
$$P - \Delta_{\text{lower}} \leq p \leq P + \Delta_{\text{upper}}$$

where

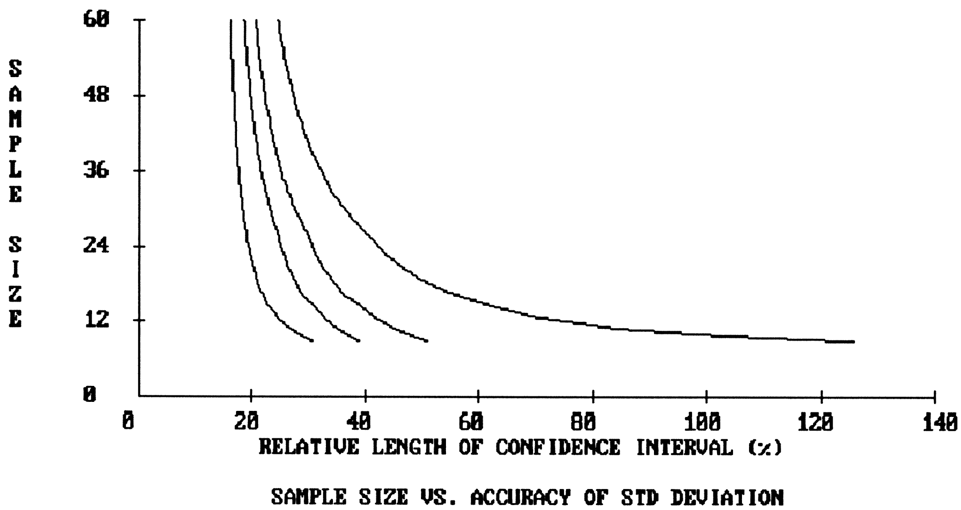
Δ_{lower} and Δ_{upper} define the confidence interval

p represents the true value of a statistical parameter

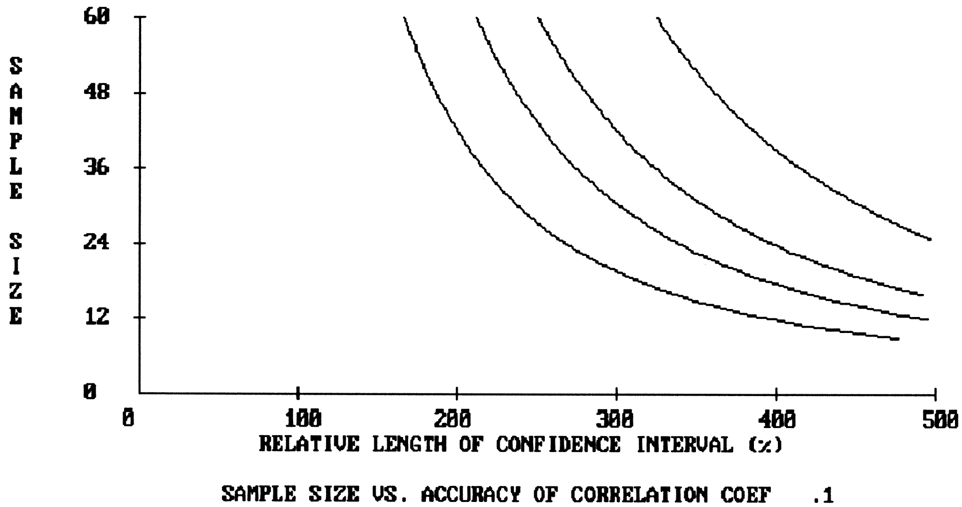
P represents the estimate of p from the statistical sample



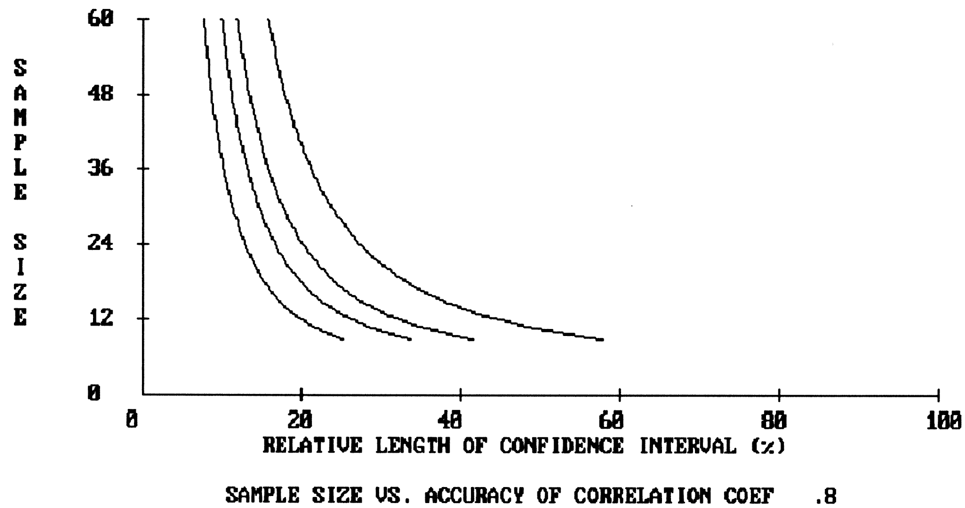
Sample size vs. confidence interval of the mean value



Sample size vs. confidence interval of the standard deviation



Sample size vs. confidence interval of a correlation coefficient



Sample size vs. confidence interval of a correlation coefficient

Sample Size

result of trade-off between measurement cost and statistical accuracy

decision on the size of a statistical sample can be based on relationships between the sample sizes and the confidence intervals

optimal sample sizes correspond to the knee points on the confidence curves

investigation shows that, under the assumption of a normal distribution, a reasonable sample size is between 20 and 50

Conclusions

a novel heuristic technique: flexible discrete distribution combined with multidimensional normal distribution

enhanced accuracy of the model

retains simplicity of the normal distribution

successfully handles arbitrary distributions

preserves the means, standard deviations and correlation coefficients