COMBINED DISCRETE/NORMAL STATISTICAL MODELING OF MICROWAVE DEVICES

OSA-89-EM-11-R

March 2, 1989

Submitted to the 1989 European Microwave Conference (Wembley, England, September 1989)

COMBINED DISCRETE/NORMAL STATISTICAL MODELING OF MICROWAVE DEVICES

J.W. Bandler, R.M. Biernacki, S.H. Chen, J.F. Loman,
M.L. Renault and Q.J. Zhang

Optimization Systems Associates Inc.
P.O. Box 8083
Dundas, Ontario, Canada L9H 5E7

Abstract

This paper deals with statistical characterization of the equivalent circuit parameters for microwave FET devices. The statistics are derived from the S-parameters measured for a sample of devices. The use of a multidimensional normal distribution seems justified for most parameters. For parameters exhibiting sample distributions substantially different from normal we propose a combined discrete/normal approach that can preserve the means, standard deviations, correlations and marginal distributions derived from the sample. This provides enhanced accuracy of the model while retaining the simplicity of the normal distribution. The problem of the size of the sample is also addressed in terms of confidence levels and confidence intervals.

SUMMARY

Introduction

As the microwave industry advances into MMIC technology there is a growing demand for reliable statistical models. Statistical models are needed for accurate yield estimation and in yield optimization. Variations of process and geometrical parameters in device manufacturing result in complex statistical behaviour of devices. The device responses not only vary from one device to another but also are strongly correlated. The purpose of statistical modeling is to provide tools to generate device random outcome responses that reflect the actual distribution of the responses [1].

Probably the most appropriate approach to statistical modeling should be based on process parameters monitoring [2] and on inter-process measurements. At this level, distributions are typically normal and the variables are independent. Such foundry data and its relations to device responses, however, may not be available to circuit designers.

Recently, Purviance et al. [3] investigated an alternative approach based on measurements of the S-parameters for a sample of finished devices. Statistical properties of the model parameters are derived from such measurements. The choice of the equivalent model parameters comes from typical requirements for the input data to popular microwave CAD software such as Touchstone or Super-Compact. Our work follows this approach.

Parameter Extraction and Statistical Estimation

In the first step of the approach discussed in this paper each set of measured data (the S-parameters) corresponding to one FET outcome has to be converted to the corresponding parameters of the equivalent circuit. A parameter extraction procedure leading to a reliable and unique solution must be applied. This step provides a sample of equivalent circuit models.

In the second step the statistics of the equivalent circuit parameters are examined and estimates of the means (μ), standard deviations (σ) and correlation coefficients (ρ) between different parameters of the model are calculated.

Multidimensional Normal Distribution vs. Discretized Distribution Function

The estimated statistical parameters μ, σ and ρ are sufficient to describe the multidimensional normal distribution. If it were the case the process of statistical modeling would be finished. These parameters could then be used to simulate statistical outcomes during yield analysis or optimization. This assumption is often made. Its verification can be done through examining the histograms of the model parameters.

In the case of an arbitrary distribution, an attempt can be made to approximate the joint probability density function (pdf) based on the data available. A discretized pdf is a practical solution [4]. The limitation of such an approach comes from a limited size of the sample (leading to a poor approximation) as well as from the exponential growth of the number of cells with the number of variables.

Combined Discrete/Normal Approach

In our experiment we processed S-parameter data for 50 simulated FETs derived from actual measurement data. Our Robust Model Parameter Extraction program RoMPE [5] was used to identify the equivalent model parameters to create a sample of 50 equivalent circuit models. The Materka-Kacprzak model was used [6]. From these 50 models, we estimated the values of μ, σ and ρ listed in Table I. Examining the histograms we noticed that for most of the model parameters a normal distribution could be assumed. The only exception was the parameter C_{DS}. The corresponding histogram is shown in Fig. 1. (A similar result was obtained in [3].)

To illustrate the consequence of assuming a multidimensional normal distribution we generated 50 outcomes using the values μ, σ and ρ listed Table I and a multidimensional normal random number generator. The regenerated histogram for C_{DS} is shown in Fig. 2. It is obviously a very poor match to the original histogram of Fig. 1.

In our new approach we combine a discrete distribution with a multidimensional normal distribution. The discrete distribution is an approximation to the marginal distribution of the parameter of interest and as its pdf we simply take the histogram. The corresponding ogive, i.e., the integral

$$f(x) = \int_{-\infty}^{x} \mathrm{pdf}(y)\, dy \tag{1}$$

is an approximation to the (cumulative) marginal distribution function. We assume that the model parameters are described by a multidimensional random variable X which in turn can be expressed as a function of another multidimensional random variable Y such that

1. $X_i = g_i(Y_i)$,
2. X_i and Y_i have the same means and standard deviations,
3. Y has a multidimensional normal distribution described by μ, σ and ρ.

The analytical form of g_i does not have to be known. Under these assumptions a very simple scheme for generating simulated outcomes can be devised. A multidimensional normal random number generator is used to generate outcomes of Y, and then the outcomes of X_i are obtained from Y_i using the one-dimensional mapping through the discrete distribution function (1) and the corresponding normal distribution function. Fig. 3 shows the two functions for C_{DS}.

Applying the combined approach to our data we generated 50 simulated outcomes and used them to regenerate μ, σ and ρ and the histograms. Fig. 4 shows the regenerated histogram for C_{DS}. This result is a much better match to the original distribution of Fig. 1. Another feature of our approach is its ability to preserve the mean values, standard deviations and correlation coefficients. Table II lists the regenerated values of μ, σ and ρ. They agree well with the original values of Table I, that is, they all are well inside the confidence intervals corresponding to 90% confidence level.

Statistical Confidence and Sample Size

In the full paper we will also discuss the problem of deciding on a particular statistical sample size as a trade-off between measurement cost and statistical accuracy. The optimal sample size corresponds to the knee points of confidence curves. Additional samples will not significantly reduce the confidence interval. A detailed investigation shows that under the assumption of a normal distribution a reasonable sample size is about 20 - 50.

Conclusions

This paper proposes a novel heuristic technique which combines a flexible discrete distribution with the multi-dimensional normal distribution. Our approach successfully handles arbitrary distributions and at the same time preserves the μ, σ and ρ values. A FET statistical modeling example with a 10-dimensional distribution is presented. A bi-modal distribution is successfully recovered. This paper also describes the confidence of statistical modeling and suggests a sample size for efficient statistical data processing.

References

[1] J.W. Bandler and S.H. Chen, "Circuit optimization: the state of the art", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, 1988, pp. 424-443.

[2] S. Liu and K. Singhal, "A statistical model for MOSFETs", *IEEE Int. Conf. on Computer-Aided Design* (Santa Clara, CA), 1985, pp. 78-80.

[3] J. Purviance, D. Criss and D. Monteith, "FET model statistics and their effects on design centering and yield prediction for microwave amplifiers", *IEEE Int. Microwave Symp. Digest* (New York, NY), 1988, pp. 315-318.

[4] H.L. Abdel-Malek and J.W. Bandler, "Yield optimization for arbitrary statistical distributions: Part II-implementation", *IEEE Trans. Circuits and Systems*, vol. CAS-27, 1980, pp. 253-262.

[5] *RoMPE Users Manual*, Optimization Systems Associates Inc., Dundas, Ontario, Canada L9H 5E7, 1988.

[6] A. Materka and T. Kacprzak, "Computer calculation of large-signal GaAs FET amplifier characteristics", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, 1985, pp. 129-135.

TABLE I

ORIGINAL MEANS, STANDARD DEVIATIONS AND SELECTED CORRELATION COEFFICIENTS FOR MATERKA-KACPRZAK MODEL

Model Parameter	Mean Value μ		Standard Deviation σ (%)	Selected Correlation Coefficients ρ		
				G_{DS}	R_I	R_S
R_G	2.4291	ohm	14	-.44	-.83	.33
R_D	.9703	ohm	19	.18	.25	.27
G_{DS}	5.0558	mS	12	1.00	.17	-.42
R_I	.8114	ohm	38	.17	1.00	-.34
R_S	1.0110	ohm	12	-.42	-.34	1.00
C_{GS}	.6187	pF	6	-.10	.05	.03
C_{DG}	.0276	pF	15	.68	.04	-.43
C_{DS}	.2206	pF	14	.12	.07	-.12
g_m	.0611	S	10	.58	.19	-.13
τ	3.6697	psec	11	-.67	-.09	.02

TABLE II

RECOVERED MEANS, STANDARD DEVIATIONS AND SELECTED CORRELATION COEFFICIENTS FOR MATERKA-KACPRZAK MODEL

Model Parameter	Mean Value μ		Standard Deviation σ (%)	Selected Correlation Coefficients ρ		
				G_{DS}	R_I	R_S
R_G	2.4001	ohm	14	-.55	-.79	.40
R_D	.9425	ohm	19	.10	.17	.07
G_{DS}	4.9755	mS	12	1.00	.23	-.54
R_I	.8386	ohm	34	.23	1.00	-.35
R_S	1.0358	ohm	12	-.54	-.35	1.00
C_{GS}	.6222	pF	6	-.07	.13	.08
C_{DG}	.0282	pF	15	.68	.11	-.25
C_{DS}	.2129	pF	15	.10	.02	-.28
g_m	.0617	S	11	.51	.29	-.05
τ	3.6551	psec	11	-.65	-.16	.11

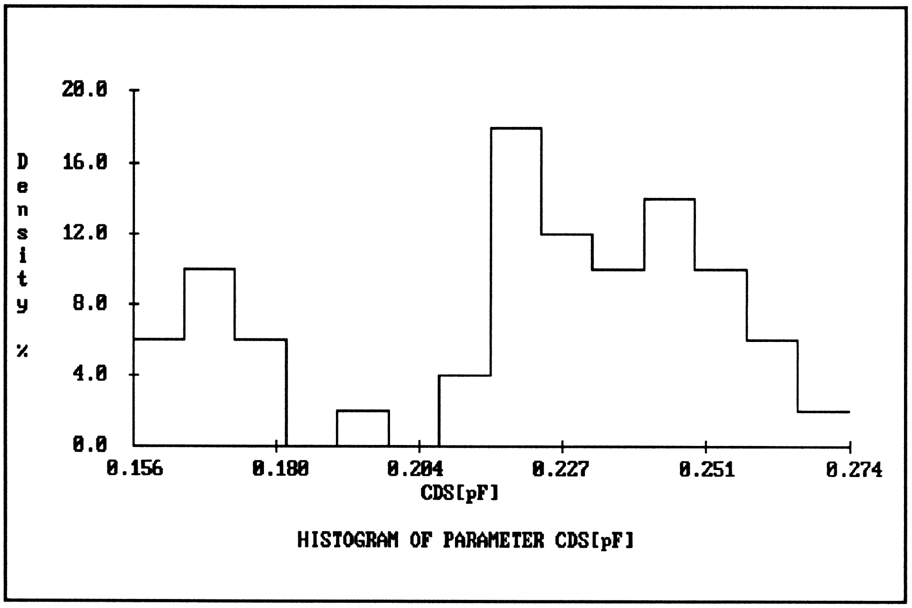

Fig. 1. Original histogram of C_{DS} obtained from a sample of 50 FETs.

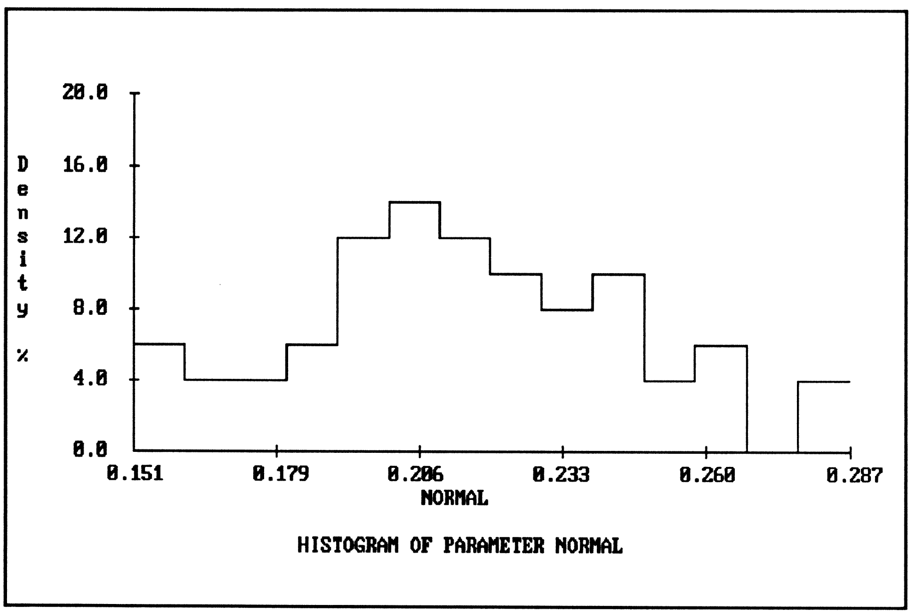

Fig. 2. Histogram of C_{DS} regenerated from a simulation based on the multidimensional normal distribution assumption. This is a poor match to Fig. 1.

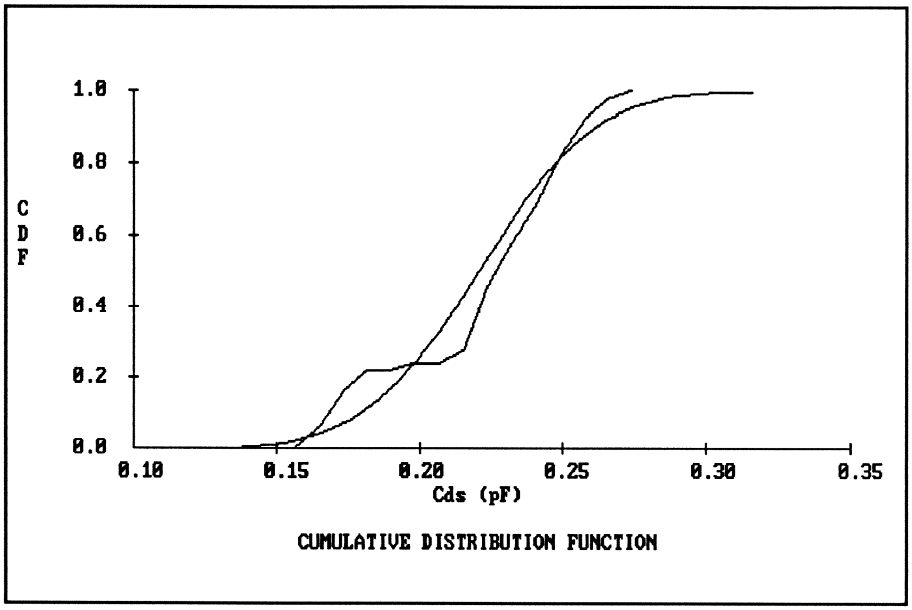

Fig. 3. Distribution functions of the discrete and normal distributions used for mapping the outcomes.

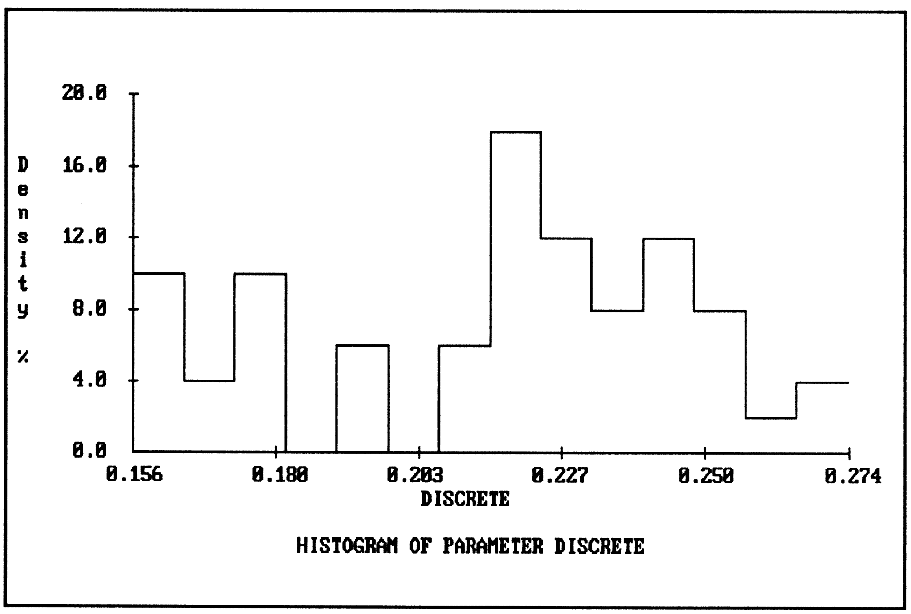

Fig. 4. Histogram of C_{DS} regenerated from a simulation based on the combined discrete/normal approach. This is a reasonable match to Fig. 1.