

**EFFICIENT LARGE-SIGNAL FET PARAMETER  
EXTRACTION USING HARMONICS**

OSA-88-MT-28-R

December 9, 1988

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Full Length

Topic Area: Computer Aided Design

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*Abstract*

We present a novel approach to large-signal nonlinear parameter extraction of GaAs MESFET devices measured under harmonic conditions. Powerful nonlinear adjoint-based optimization simultaneously processes multi-bias, multi-power-input, multi-fundamental-frequency excitations and multi-harmonic measurements to uniquely reveal the parameters of the intrinsic FET. One test successfully processed 111 error functions of 20 model parameters.

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## SUMMARY

### *Introduction*

An accurate nonlinear large-signal FET model is critical to nonlinear microwave CAD. Various approaches to FET modeling have been proposed, e.g., [1]-[4]. The dominant nonlinear bias-dependent current source, namely, the drain-to-source current source, in these models is commonly determined by fitting static or dynamic DC I-V characteristics only [1, 2, 4, 5, 6], or by matching DC characteristics and small-signal S-parameters simultaneously [3]. Other nonlinear elements in the model are either determined by applying special DC biases so as to determine the parameters of the gate-to-source nonlinear current source in the Materka and Kacprzak model [2], or by using small-signal S-parameters.

The FET models obtained by these methods may provide accurate results under DC and/or small-signal conditions. They may not, however, be accurate enough for high-frequency large-signal applications [7], since they are determined under small-signal conditions and then used to predict the behaviour for large-signal operation.

For the first time, a truly nonlinear large-signal FET parameter extraction procedure is proposed which utilizes spectrum measurements, including DC bias information and power output at different harmonics under practical working conditions [8]. The harmonic balance method [9] is employed for fast nonlinear frequency domain simulation in conjunction with  $\ell_1$  and  $\ell_2$  optimization for extracting the parameters of the nonlinear elements in the large-signal FET model.

Numerical experiments have shown that all the parameters can be uniquely identified under actual high-frequency large-signal working conditions, demonstrating the importance of higher harmonics in large-signal parameter extraction. In addition, powerful nonlinear adjoint analysis for sensitivity computation [10] has been

implemented with attendant advantages in computation time.

### *Formulation*

Nonlinear intrinsic FET parameters are to be determined using large signal data. All linear FET model elements such as parasitics are extracted using small-signal data.

The FET and its measurement environment are shown in Fig. 1, where  $Y_{in}$  and  $Y_{out}$  are input and output 2-ports,  $Y_g$  and  $Y_d$  are gate and drain bias 2-ports, respectively. We apply a large signal power input  $P_{in}$  to the circuit. DC voltages and output power  $P_{out}$  at several harmonics [8] are measured.

In addition to the multi-bias, multi-frequency concept we pioneered for small-signal parameter extraction, we allow the circuit to be excited at several input power levels. Various combinations of bias points, fundamental frequencies and input levels result in a variety of measurement information needed for parameter extraction. Assume for the  $j$ th bias-input-frequency combination,  $j=1, 2, \dots, M$ , the measurement is

$$S_j = [S_j(0) \ S_j(\omega_1) \ S_j(\omega_2) \ \dots \ S_j(\omega_H)]^T, \quad (1)$$

where  $S_j(0)$  is the DC component of the measurement, and  $S_j(\omega_k)$ ,  $k=1, \dots, H$ , are the  $k$ th harmonic components. Correspondingly, the model response  $F_j(\phi)$  can be expressed as

$$F_j(\phi) = [F_j(\phi, 0) \ F_j(\phi, \omega_1) \ F_j(\phi, \omega_2) \ \dots \ F_j(\phi, \omega_H)]^T, \quad (2)$$

where  $\phi$  stands for the parameters of the model to be determined.

The parameter extraction optimization problem can then be formulated as

$$\min_{\phi} \sum_{j=1}^M (w_{jdc} |F_j(\phi, 0) - S_j(0)|^p + \sum_{k=1}^H w_{jk} |F_j(\phi, \omega_k) - S_j(\omega_k)|^p), \quad (3)$$

where  $w_{jdc}$  and  $w_{jk}$  are weighting factors, and  $p=1$  or  $2$  corresponds to  $\ell_1$  or  $\ell_2$  optimization, respectively.

Model responses  $F_j(\phi)$  are computed using the harmonic balance method [9]. The powerful nonlinear adjoint sensitivity analysis [10] is implemented to provide gradients for optimization. This significantly accelerates the optimization and makes our parameter extraction approach computationally practical.

An automatic weight assignment algorithm has been developed, improving robustness and enhancing convergence speed. Also, by converting measured powers to voltages, we achieve a well-conditioned optimization problem.

#### *The FET Model Used in Our Experiment*

In our numerical experiments, we used the Microwave Harmonica [11] modified Materka and Kacprzak FET model as the intrinsic FET, as shown in Fig.

2. The nonlinear elements of the model are described by [11]

$$\begin{aligned}
i_D &= F[v_G(t - \tau), v_D(t)] (1 + S_S \frac{v_D}{I_{DSS}}), \\
F(v_G, v_D) &= I_{DSS} \left[ 1 - \frac{v_G}{V_{p0} + \gamma v_D} \right]^{(E + K_E v_G)} \cdot \tanh \left[ \frac{S_1 v_D}{I_{DSS}(1 - K_G v_G)} \right], \\
i_G &= I_{G0} [\exp(\alpha_G v_G) - 1], \\
i_B &= I_{B0} \exp[\alpha_B(v_D - v_1 - V_{BC})], \\
\left\{ \begin{array}{l} R_1 = R_{10}(1 - K_R v_G), \\ R_1 = 0, \quad \text{if } K_R v_G \geq 1, \end{array} \right. & \tag{4} \\
\left\{ \begin{array}{l} C_1 = C_{10}(1 - K_1 v_G)^{-1/2} + C_{1S}, \\ C_1 = C_{10}\sqrt{5} + C_{1S}, \quad \text{if } K_1 v_G \geq 0.8, \end{array} \right. \\
\text{and} \\
\left\{ \begin{array}{l} C_F = C_{F0}[1 - K_F(v_1 - v_D)]^{-1/2}, \\ C_F = C_{F0}\sqrt{5}, \quad \text{if } K_F(v_1 - v_D) \geq 0.8, \end{array} \right.
\end{aligned}$$

where  $I_{DSS}$ ,  $V_{P0}$ ,  $\gamma$ ,  $E$ ,  $K_E$ ,  $S_1$ ,  $K_G$ ,  $\tau$ ,  $S_s$ ,  $I_{G0}$ ,  $\alpha_G$ ,  $I_{B0}$ ,  $\alpha_B$ ,  $V_{BC}$ ,  $R_{10}$ ,  $K_R$ ,  $C_{10}$ ,  $K_1$ ,  $C_{1S}$ ,  $C_{F0}$ , and  $K_F$  are the parameters to be determined. Since only one of  $I_{B0}$  and  $V_{BC}$  is independent, we fix  $V_{BC}$  and let

$$\phi = [I_{DSS} \ V_{P0} \ \gamma \ E \ K_E \ S_1 \ K_G \ \tau \ S_s \ I_{G0} \ \alpha_G \ I_{B0} \ \alpha_B \ R_{10} \ K_R \ C_{10} \ K_1 \ C_{1S} \ C_{F0} \ K_F]^T. \quad (5)$$

The parasitics of the FET are illustrated in Fig. 1 and their values are listed in Table I [3].

*Test 1: Robustness of the Parameter Extraction Approach*

Assume that the solution of the model is [3]

$$\phi = [0.1888 \ -4.3453 \ -0.3958 \ 2 \ 0 \ 0.0972 \ -0.1678 \ 3.654 \ 0 \ 0.5 \times 10^{-9} \ 20 \ 0.5 \times 10^{-9} \ 1 \ 4.4302 \ 0 \ 0.6137 \ 0.7686 \ 0 \ 0.0416 \ 0]^T. \quad (6)$$

The circuit is simulated at three bias points:  $(V_{GB}=0, V_{DB}=5)$ ,  $(V_{GB}=-1.5, V_{DB}=5)$  and  $(V_{GB}=-3, V_{DB}=5)$ .  $P_{in}=5$  and 10dBm are applied at  $(V_{GB}=0, V_{DB}=5)$ , and  $P_{in}=5$ , 10 and 15dBm at the other two bias points, respectively. At each bias point two fundamental frequencies (1 and 2GHz) are used, respectively. There are 16 bias-input-frequency combinations in total. Six harmonics are considered in the harmonic balance simulation. The output power at the first 3 harmonics are measured and used in the objective function, i.e.,  $H=3$  in (3). Therefore we have 20 variables and 64 error functions.

To examine uniqueness of the solution we uniformly perturbed the solution in (6) by 20 to 40 percent, and re-optimized with the  $\ell_1$  norm, i.e.,  $p=1$  in (3). Several starting points were tested and all of them converged to the known solution exactly. This verifies the strong identifiability induced by the higher harmonics.

Table II demonstrates the significance of the nonlinear adjoint sensitivity analysis, where we compare the CPU execution time for two different starting points with and without nonlinear adjoint analysis for gradient computation. With the

adjoint analysis, the program runs approximately 10 times faster than that without adjoint analysis.

*Test 2: Parameter Extraction with Measurement Errors*

In this test, we added 10% normally distributed random noise to the simulated measurements used in Test 1. We used the same bias-input-frequency combinations and the same starting points as those in Test 1. We applied  $\ell_2$  optimization, i.e.,  $p=2$  in (3). Measurements at the first 4 harmonics are used, i.e.,  $H=4$  in (3). Any signal below  $-35\text{dBm}$  was discarded. After optimization, all points converged to virtually one solution quite close to (6) except for  $I_{B0}$  and  $\alpha_B$  because of their relatively low sensitivities to the response functions. Still,  $I_{B0}$  and  $\alpha_B$  converged to their respective order.

*Test 3: Fitting to the Curtice Model*

Here we use set of data generated by the Curtice model [4,11]. The circuit is similar to that of Fig. 1 except that the intrinsic FET is replaced by the intrinsic part of the Curtice model. Some of the parameters of the Curtice model are taken from Fig. 13 of [4]. See Table III.

We selected 32 bias-input-frequency combinations, as shown in Table IV. The first 3 harmonics were assumed as measurement data. Any signal below  $-30\text{dBm}$  was discarded. There were 111 error functions in total.

$$\begin{aligned} \ell_2 \text{ optimization was applied to extract the model parameters, resulting in} \\ \phi = [0.05208 \ -1.267 \ -0.08774 \ 1.269 \ -0.3224 \ 0.07312 \ -0.6482 \ 5.322 \\ 4.462 \times 10^{-5} \ 8.782 \times 10^{-9} \ 34.04 \ 5.96 \times 10^{-12} \ 4.245 \ 0.03610 \ 9.892 \times 10^{-3} \ 1.066 \\ 1.531 \ 0.03141 \ 1.321 \ 1.638]^T. \end{aligned}$$

Fig. 3 illustrates the modeling results at a bias point other than those considered in the optimization. Excellent agreement is observed.

As for Test 1, parameters at the solution were perturbed uniformly by 20



to 40 percent and re-optimized. Of six starting points, four converged to the same solution with little variances in  $R_{10}$  and  $K_R$ . The other two converged to different points with different final objective function values.

Fig. 4 shows the characteristics of drain-to-source nonlinear current sources of the Curtice model and the modified Materka and Kacprzak model, and again we have reached an excellent match. Notice that only 6 bias points are used in the optimization which is even less than the total number of parameters for this current source. However, since we modeled under actual large-signal conditions, employing higher harmonic measurements, a much larger range of information has been covered than single individual points on the DC I-V curve can provide.

### *Conclusions*

In this paper an accurate and truly nonlinear large-signal parameter extraction approach has been presented, where not only DC bias and fundamental frequency, but also higher harmonic responses have been used. Such information effectively reflects the nonlinearities of the model. The harmonic balance method for nonlinear circuit simulation, adjoint analysis for nonlinear circuit sensitivity calculation and optimization methods have been applied. Numerical results demonstrate that the method can uniquely and efficiently determine the parameters of the nonlinear elements of the GaAs MESFET model under actual large-signal operating conditions.

Consideration of the parameter extraction problem under two-tone measurements is planned.

### *Acknowledgements*

The authors thank Dr. R.A. Pucel of Raytheon Company, Lexington, MA, for suggesting Test 3. They thank Dr. A.M. Pavio of Texas Instruments, Dallas, TX, for his interest and willingness to prepare experimental data to test our algorithm.

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TABLE I  
LINEAR PARASITIC PARAMETER VALUES OF  
THE FET MODEL (FIG. 1) USED IN ALL NUMERICAL TESTS

Parameter	Value	Unit
$R_g$	0.0119	$\Omega$
$L_g$	0.1257	nH
$R_s$	0.3740	$\Omega$
$L_s$	0.0107	nH
$R_d$	0.0006	$\Omega$
$L_d$	0.0719	nH
$C_{ds}$	0.1927	pF
$R_{de}$	440	$\Omega$
$C_{de}$	1.5	pF

TABLE II  
CPU TIME WITH AND WITHOUT NONLINEAR  
ADJOINT GRADIENT CALCULATIONS IN TEST 1

	Without Adjoint Ana.*		With Adjoint Ana.	
	CPU** (sec.)	Obj. fun. value	CPU** (sec.)	Obj. fun. value
Starting point 1	1800	$1.158 \times 10^{-3}$	230	$1.013 \times 10^{-3}$
	2600	$2.235 \times 10^{-4}$	260	$1.734 \times 10^{-4}$
Starting point 2	2600	$1.366 \times 10^{-3}$	200	$1.115 \times 10^{-3}$
	2900	$5.479 \times 10^{-4}$	220	$4.894 \times 10^{-4}$

\* The gradient is estimated by perturbations at every other iteration.

\*\* Fortran program run on a VAX 8650.

TABLE III  
PARAMETERS OF THE CURTICE MODEL USED IN TEST 3

Parameter	Value	Unit
$\beta_2$	0.04062	1/V
$A_0$	0.05185	A
$A_1$	0.04036	A/V
$A_2$	-0.009478	A/V <sup>2</sup>
$A_3$	-0.009058	A/V <sup>3</sup>
$\gamma$	1.608	1/V
$V_{DS0}$	4.0	V
$I_S$	$1.05 \times 10^{-9}$	A
$N$	1.0	-
$C_{GS0}$	1.1	pF
$C_{GD0}$	1.25	pF
$F_C$	0.5	-
$G_{MIN}$	0.0	1/ $\Omega$
$V_{BI}$	0.7	V
$V_{BR}$	20	V
$\tau$	5.0	ps
see [4] and [11]		

TABLE IV  
INPUT LEVELS USED WITH DIFFERENT FUNDAMENTAL  
FREQUENCIES AND DIFFERENT BIASES IN TEST 3

$(V_{GB}, V_{DB})$	$P_{in}$ (dBm)			
	$f_1=0.5\text{GHZ}$	$f_1=1.0\text{GHZ}$	$f_1=1.5\text{GHZ}$	$f_1=2.0\text{GHZ}$
$(-0.3, 3)$	0, 4	0, 4	0, 4	0, 4
$(-0.3, 7)$	0, 4	0, 4	0, 4	0, 4
$(-1.0, 3)$	0	0	0	0
$(-1.0, 7)$	0	0, 4	0, 4	0
$(-0.5, 3)$	--	8	8	--
$(-0.5, 7)$	8	8	8	8
$f_1$ denotes the fundamental frequency				

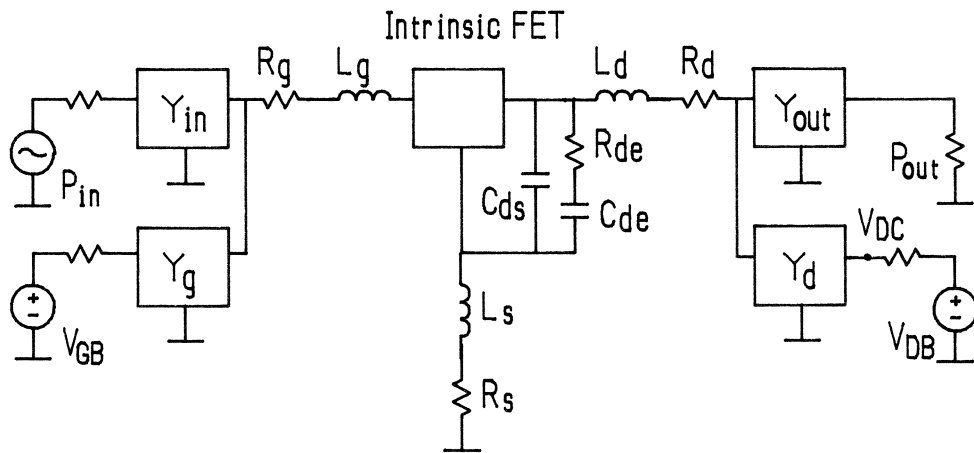


Fig. 1 Circuit setup for large-signal multi-harmonic FET measurement.

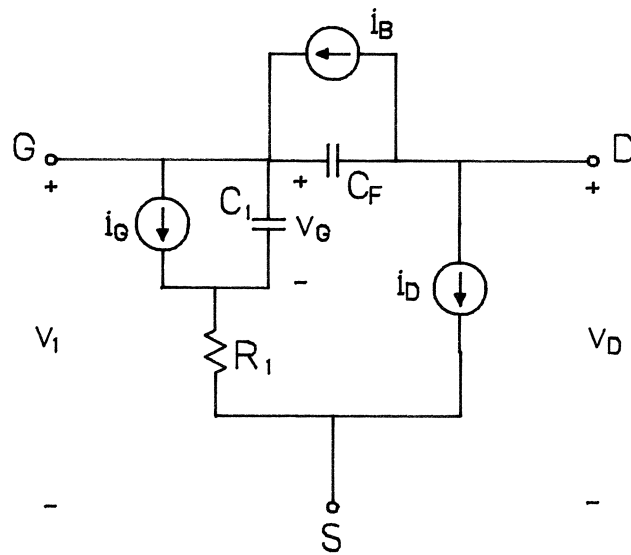
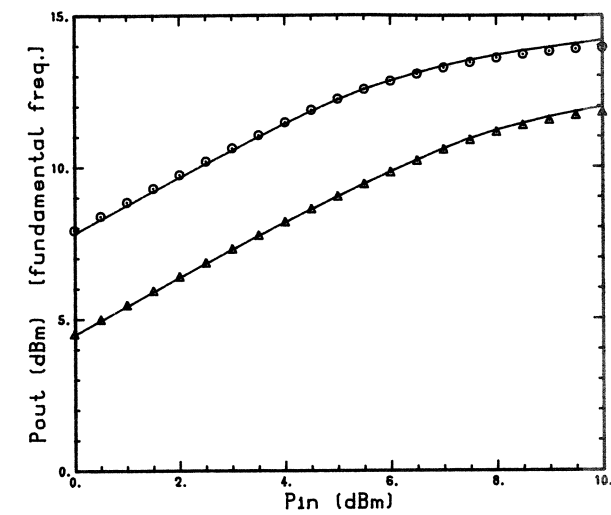
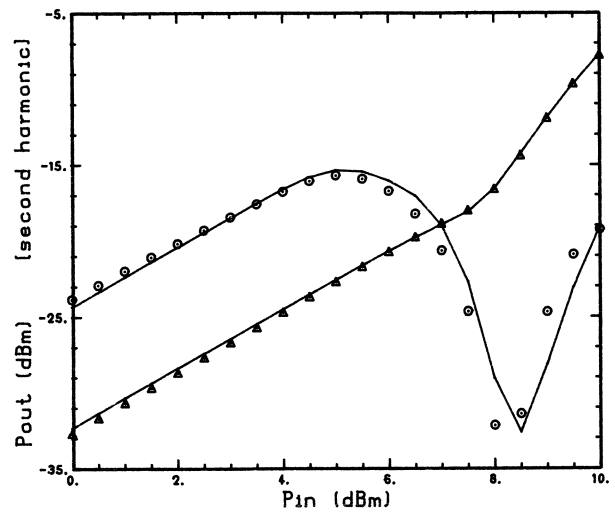


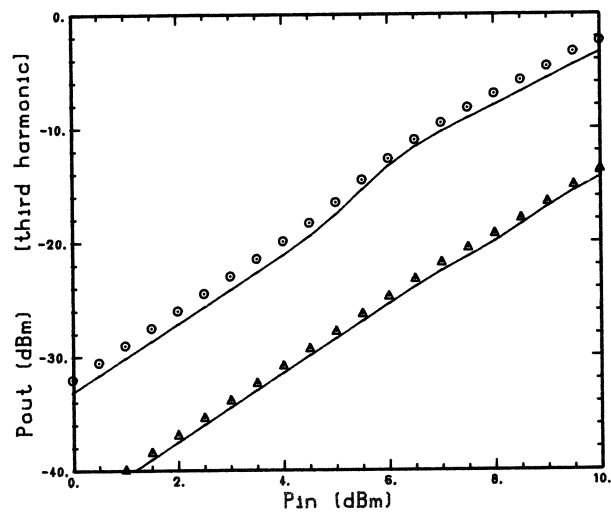
Fig. 2 Intrinsic part of the modified Materka and Kacprzak FET model.



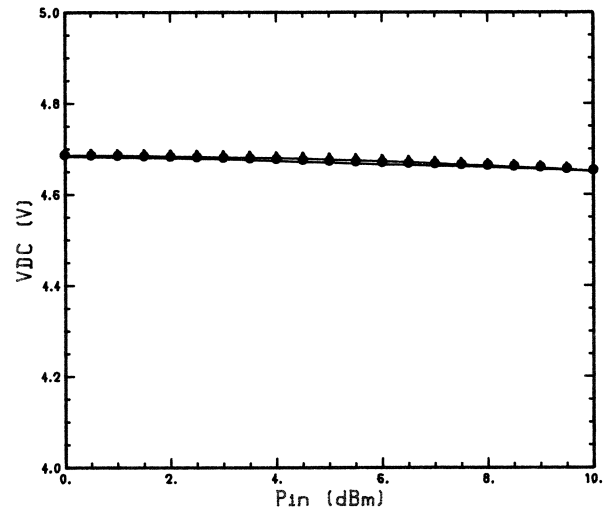
(a)



(b)



(c)



(d)

Fig. 3 Agreement between the (Materka) model response and the simulated measurements (using the Curtice model) at  $V_{GB}=-0.5$  and  $V_{DB}=5$  in Test 3. Solid lines represent the (Materka) computed model response. Circles denote the simulated measurements at fundamental frequency 0.5GHz and triangles the simulated measurements at fundamental frequency 1.0GHz. (a) Fundamental component, (b) second harmonic component, (c) third harmonic component, and (d) DC component.

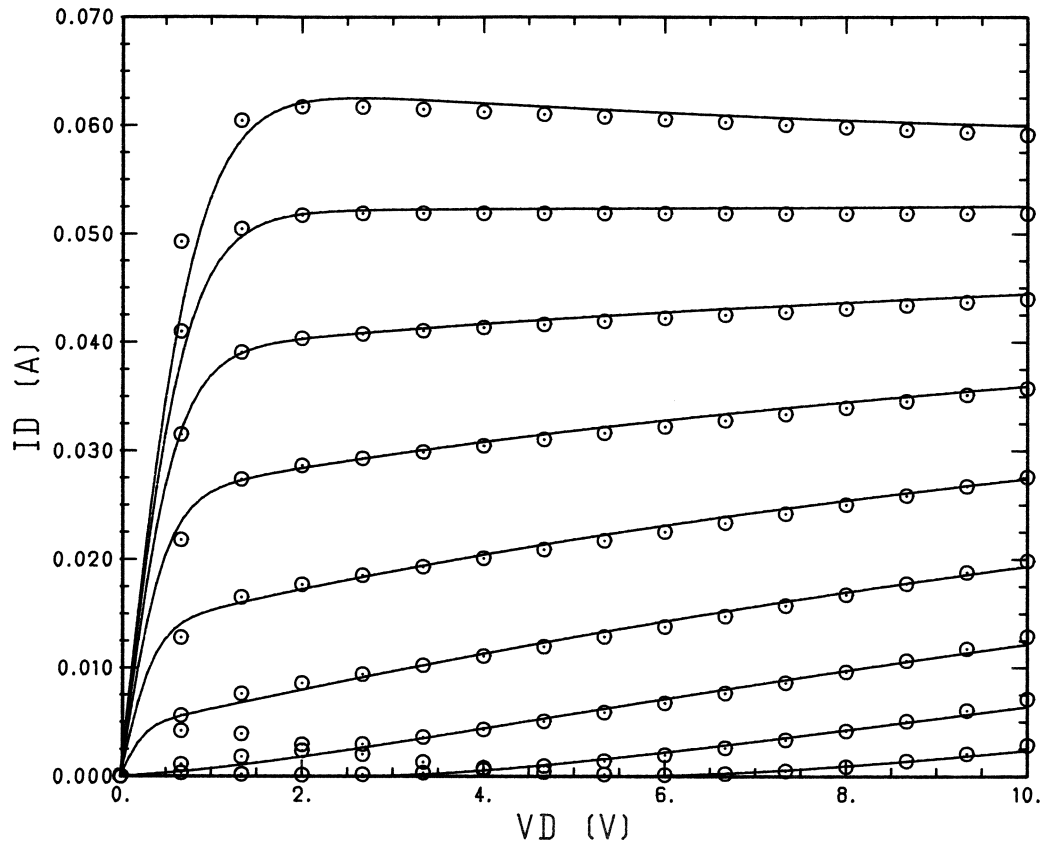


Fig. 4 Agreement between the DC characteristics of the modified Materka and Kacprzak model and the simulated measurements (from the Curtice model) in Test 3.  $V_G$  is from  $-1.75\text{V}$  to  $0.25\text{V}$  in steps of  $0.25\text{V}$ , and  $V_D$  is from 0 to  $10\text{V}$ . (Curtice uses  $V_{in}$  and  $V_{out}$ , respectively.) Solid lines represent the (Materka) model, and the circles represent the measurements.