A UNIFIED FRAMEWORK FOR HARMONIC BALANCE SIMULATION AND SENSITIVITY ANALYSIS

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A UNIFIED FRAMEWORK FOR HARMONIC BALANCE SIMULATION AND SENSITIVITY ANALYSIS

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Unified Theoretical Framework

simulation and sensitivity analysis

linear and nonlinear circuits

hierarchical and nonhierarchical approaches

voltage and current excitations

open and short circuit terminations

Theoretical Breakthroughs

harmonic balance technique expanded from simulation to adjoint sensitivity analysis

hierarchical approach generalized to permit upward and downward analysis in both the original and adjoint networks

Impact on the Next Generation Circuit CAD

important features for linear CAD

- syntax-oriented hierarchical approach
- design optimization
- statistical analysis
- yield maximization

these features now applicable to nonlinear circuits

Notation

$\mathbf{V}(k)$ contains external voltages of a linear subcircuit at harmonic k

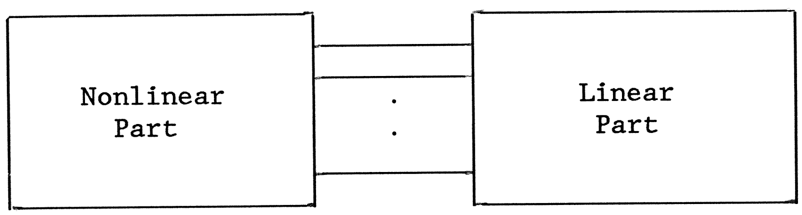
$\mathbf{V}_t(k)$ contains both internal and external voltages of a linear subcircuit at harmonic k

$\overline{\mathbf{V}}$ contains real and imaginary parts of $\mathbf{V}(k)$ for all harmonics

$\wedge$ denotes adjoint quantities, e.g., $\hat{\mathbf{V}}(k)$

current vectors $\mathbf{I}(k)$, $\mathbf{I}_t(k)$, $\overline{\mathbf{I}}$ and $\hat{\mathbf{I}}(k)$ similarly defined

Harmonic Balance Simulation



time domain ⟵ Fourier Transform ⟶ frequency domain

harmonic balance equation

$$\overline{\mathbf{F}}(\overline{\mathbf{V}}) \triangleq \overline{\mathbf{I}}_{NL}(\overline{\mathbf{V}}) + \overline{\mathbf{I}}_{L}(\overline{\mathbf{V}}) = 0$$

Sensitivity Analysis

sensitivity of output response V_{out} w.r.t. design variable x

$$\frac{\partial V_{out}}{\partial x}$$

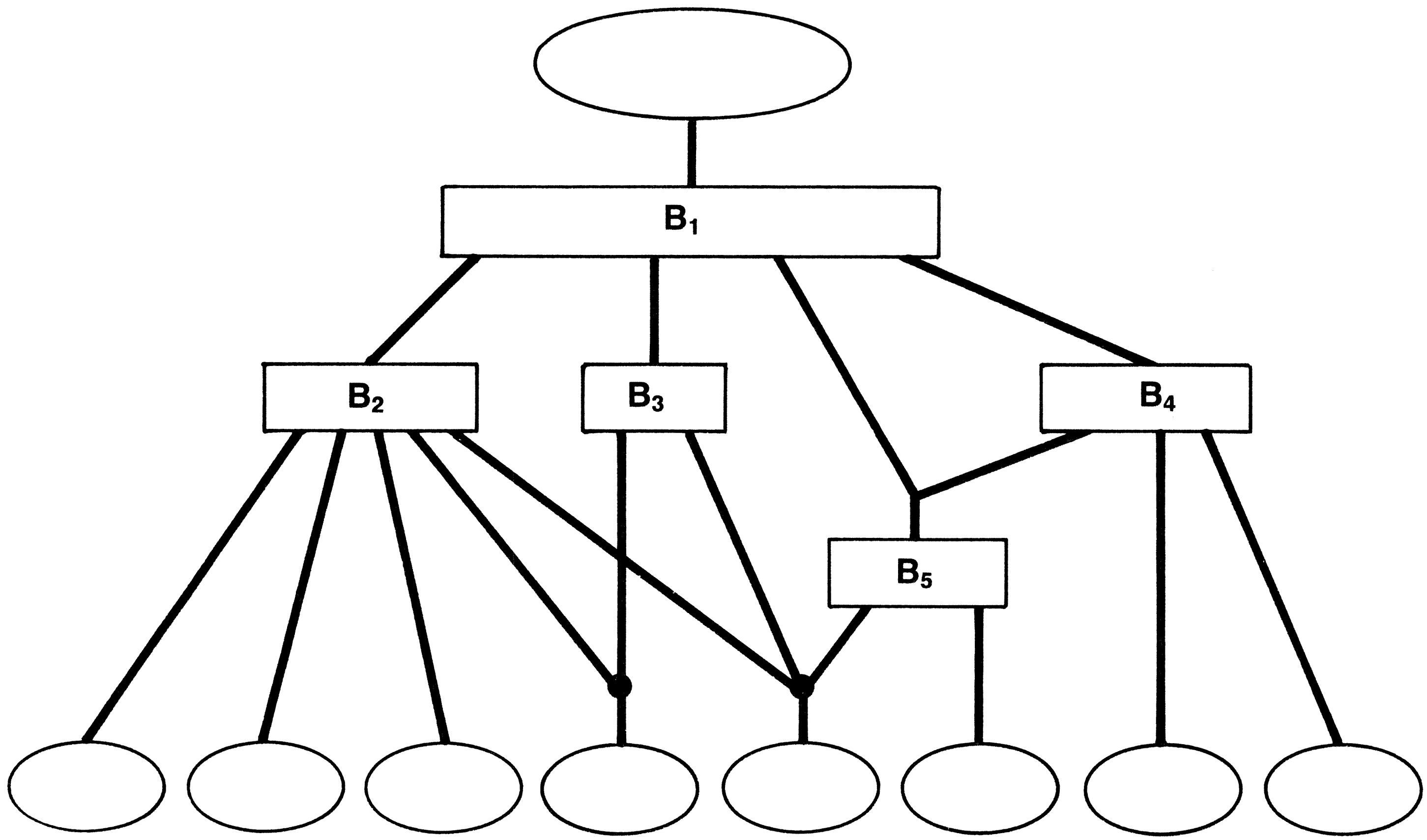
essential for gradient optimizers

Hierarchical Analysis

UPWARD analysis to obtain the overall circuit matrix, e.g., Y matrix

DOWNWARD analysis to obtain responses at individual components, e.g., voltage or power for an element

TOP LEVEL analysis to solve harmonic balance equation for nonlinear networks or to solve the terminated circuit for linear networks



B1
B2
B3
B4
B5

Top Level Simulation of Nonlinear Circuits

harmonic balance equation

$\overline{\mathbf{F}}(\overline{\mathbf{V}}) = \mathbf{0}$

Newton update

$\overline{\mathbf{V}}_{new} = \overline{\mathbf{V}}_{old} - \overline{\mathbf{J}}^{-1}\overline{\mathbf{F}}(\overline{\mathbf{V}}_{old})$

$\overline{\mathbf{J}}$ is the Jacobian matrix

the Newton solution provides the top
level external voltages $\mathbf{V}(k)$

Downward Simulation of the Original Linear Network

internal and external voltages $\mathbf{V}_t(k)$ of a subcircuit can be computed from its external voltages $\mathbf{V}(k)$ by

$$\mathbf{A}(k) \begin{bmatrix} \mathbf{V}_t(k) \\ \mathbf{I}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{V}(k) \end{bmatrix}$$

$\mathbf{A}(k)$ is the modified nodal admittance matrix of the subcircuit

$\mathbf{I}(k)$ contains currents into the subcircuit from its external ports

the solution $\mathbf{V}_t(k)$ provides external voltages for downward analysis

Top Level Adjoint Simulation for Nonlinear Networks

let $\overline{V}_{out} = [\ 0\ 1\ 0\ 0\ ..\ 0\ ]\overline{\mathbf{V}}$

corresponding adjoint system

$$\overline{\mathbf{J}}^T \hat{\overline{\mathbf{V}}} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

the solution gives the top level external adjoint voltages $\hat{V}(k)$

LU factors of $\overline{\mathbf{J}}$ available and reusable

Downward Simulation of the Adjoint Linear Network

internal and external adjoint voltages $\hat{V}_t(k)$ of a subcircuit can be computed from its external adjoint voltages $\hat{V}(k)$ by

$$A^T(k) \begin{bmatrix} \hat{V}_t(k) \\ -\hat{I}(k) \end{bmatrix} = \begin{bmatrix} 0 \\ -\hat{V}(k) \end{bmatrix}$$

the solution $\hat{V}_t(k)$ provides the external adjoint voltages for downward analysis

LU factors of A(k) available and reusable

Sensitivity Expressions

variable x belongs to branch b

$$
\frac{\partial \overline{V}_{out}}{\partial x} = \begin{cases} -\sum_k \text{Real}\,[\hat{V}_b(k) V_b^*(k) G_b^*(k)] & \text{(a)} \\ -\sum_k \text{Real}\,[\hat{V}_b(k) G_b^*(k)] & \text{(b)} \\ -\sum_k \text{Imag}\,[\hat{V}_b(k) G_b^*(k)] & \text{(c)} \end{cases}
$$

$V_b(k)$ and $\hat{V}_b(k)$ are voltages of branch b at harmonic k

$G_b(k)$ is an element sensitivity expression

Examples

for a linear resistor $G_b(k)=1$

for a nonlinear resistor described
by $i(t)=i(v(t), x)$
$G_b(k)$=[kth Fourier coefficient of $\partial i/\partial x$]

FET Mixer Example of Camacho-Penalosa and Aitchison (1987)

compute exact sensitivities of the conversion gain w.r.t.
26 variables

all parameters in the linear and nonlinear parts
DC bias, LO power, RF power,
IF, LO and RF terminations

results in excellent agreement with those from perturbation

CPU time for simulation only is 22 seconds on a VAX 8600

CPU time for sensitivity computation

our approach 3.7 seconds
perturbation approach 240 seconds

Comparison with the Perturbation Method

perturbation method

simulate the original nonlinear circuit
perturb all variables and resimulate for each perturbation

our method

simulate the original circuit
solve the adjoint equations once

features of our method

adjoint simulation noniterative
exact gradient
significant saving of CPU times

Conclusions

unified theory for

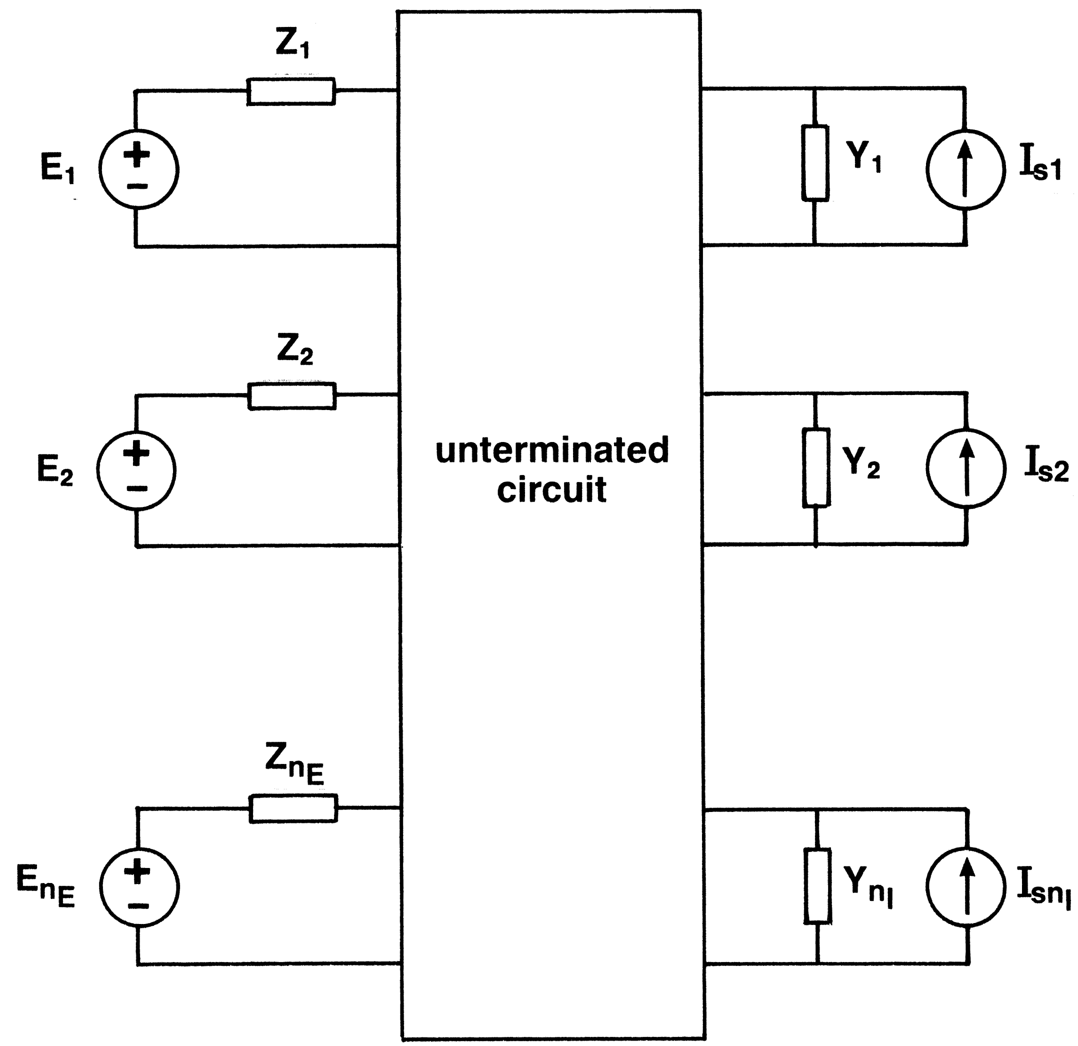
simulation and sensitivity analysis
linear and nonlinear circuits
hierarchical and nonhierarchical

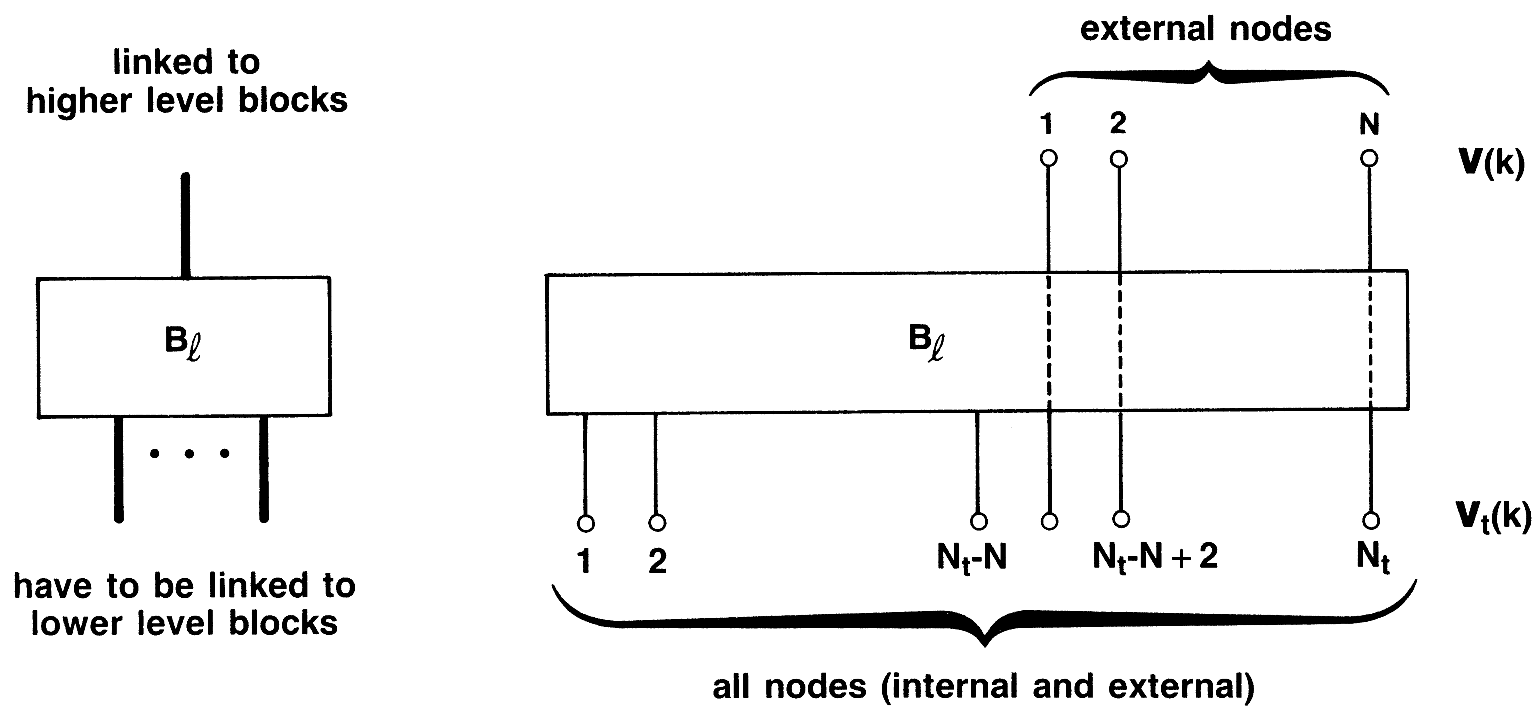
since nonlinear simulation is costly, the adjoint sensitivity approach is very significant

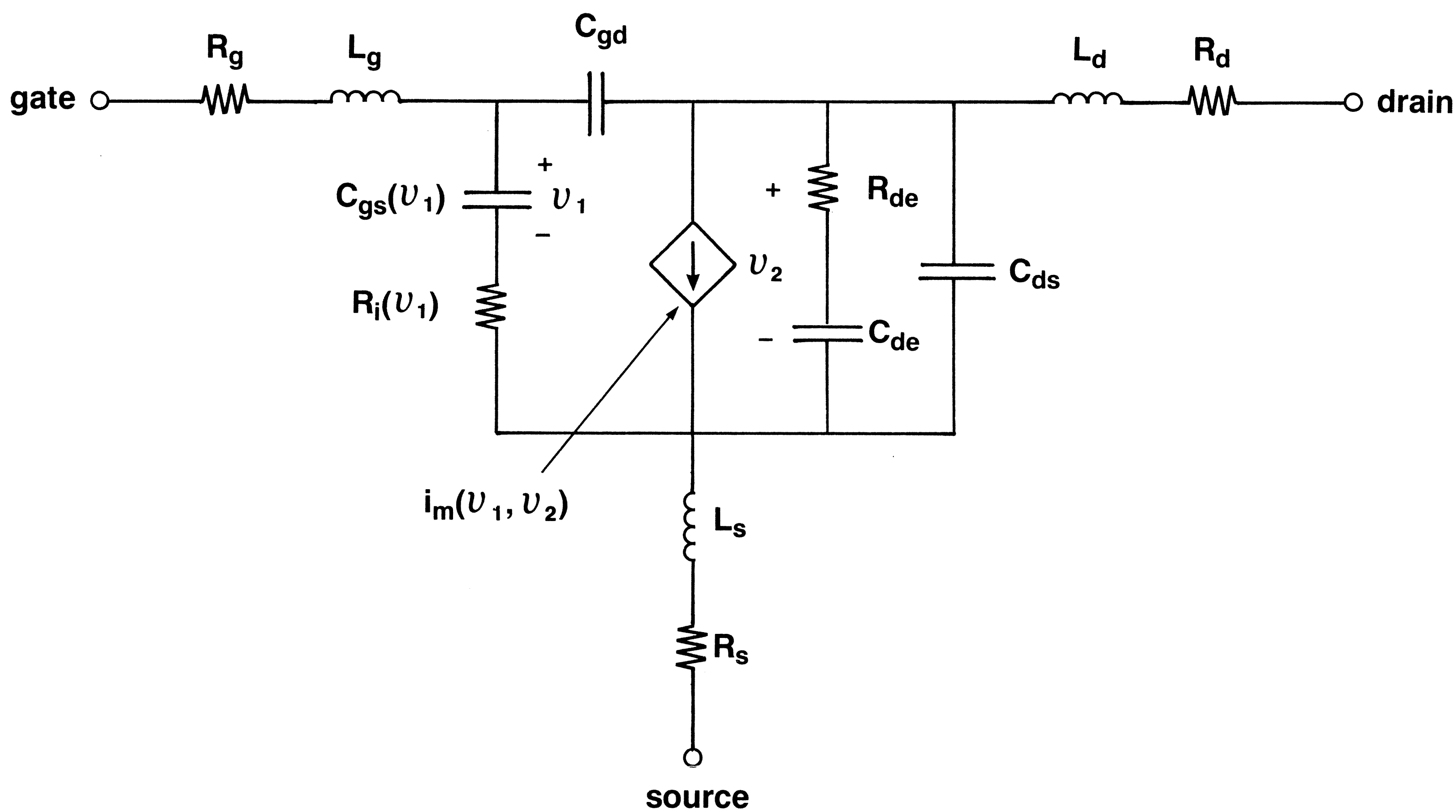
our hierarchical approach permits

voltages anywhere in the original and adjoint networks
variables anywhere in the entire circuit

a key for the coming generation of microwave CAD software


Z1
Z2
ZnE
E1
E2
EnE
unterminated
circuit
Y1
Y2
YnI
Is1
Is2
IsnI


linked to
higher level blocks
B_ℓ
have to be linked to
lower level blocks
external nodes
1
2
N
$\mathbf{V}(k)$
B_ℓ
1
2
N_t-N
N_t-N + 2
N_t
$\mathbf{V}_t(k)$
all nodes (internal and external)



R_g
L_g
C_{gd}
L_d
R_d
gate
drain
+
$C_{gs}(\upsilon_1)$
υ_1
−
$R_i(\upsilon_1)$
υ_2
+
R_{de}
C_{ds}
−
C_{de}
$i_m(\upsilon_1, \upsilon_2)$
L_s
R_s
source