**A UNIFIED FRAMEWORK FOR**
**HARMONIC BALANCE SIMULATION AND**
**SENSITIVITY ANALYSIS**

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# A UNIFIED FRAMEWORK FOR HARMONIC BALANCE SIMULATION AND SENSITIVITY ANALYSIS

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*Unified Theoretical Framework*

under this framework the following features are unified

simulation and sensitivity analysis

linear and nonlinear circuits

hierarchical and nonhierarchical approaches

voltage and current excitations

open and short circuit terminations

*Theoretical Breakthroughs*

harmonic balance technique expanded from simulation to adjoint sensitivity analysis

hierarchical approach generalized to permit upward and downward analysis in both the original and adjoint networks

*Impact on the Next Generation Circuit CAD*

important features of linear simulators, e.g., Super-Compact

- syntax-oriented hierarchical approach
- design optimization
- statistical analysis
- yield maximization

these features now applicable to nonlinear circuits

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*Gradient Optimizers for Circuit CAD*

fast convergence

gradient from perturbation

Broyden update formulas

gradients from adjoint network approach

*Perturbation Approach to Gradient Evaluation*

simulate the circuit to obtain response function **f**

for i=1, 2, ..., n

perturb the ith variable

simulate the ith perturbed circuit

approximate the derivative of **f** w.r.t. the ith variable by finite differences

restore the ith variable

*Features*

n + 1 circuit simulations required

for nonlinear circuits, the starting point for each circuit simulation can be the previous solution; this reduces the number of iterations in nonlinear simulation

*Adjoint Network Approach to Gradient Evaluation*

for linear circuits

simulate the original circuit to obtain branch voltages $V_b$

simulate the adjoint network to obtain adjoint branch voltages, $\hat{V}_b$

compute sensitivity expressions at element level $G_b$

response sensitivity is typically the algebraic operation of $V_b$, $\hat{V}_b$ and $G_b$

we have extended this approach to nonlinear circuits

*Features*

extremely fast as compared with the perturbation approach

cost of solving the adjoint network is almost free compared with the cost of simulating the original circuit

adjoint network simulation only solves a set of linear equations

original network simulation solves nonlinear equations using iterative methods

all relevant matrices for the adjoint network equations are already preprocessed, i.e., inverse or LU factors are available and reusable

*Notation*

$\mathbf{V}(k)$ contains <u>external voltages</u> of a linear subcircuit at harmonic k

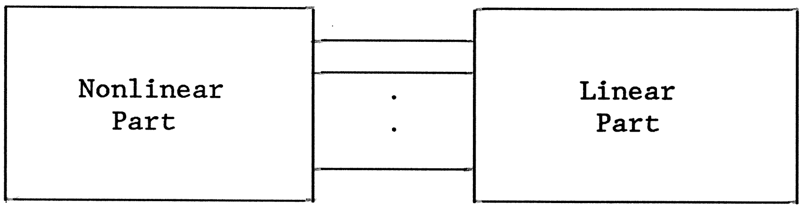
$\mathbf{V}_t(k)$ contains <u>both internal and external voltages</u> of a linear subcircuit at harmonic k

$\overline{\mathbf{V}}$ contains real and imaginary parts of $\mathbf{V}(k)$ for all harmonics

$\wedge$ denotes adjoint quantities, e.g., $\hat{\mathbf{V}}(k)$

current vectors $\mathbf{I}(k)$, $\mathbf{I}_t(k)$, $\overline{\mathbf{I}}$ and $\hat{\mathbf{I}}(k)$ similarly defined

*Harmonic Balance Simulation*




time domain ⟵ Fourier Transform ⟶ frequency domain

harmonic balance equation

$$\overline{\mathbf{F}}(\overline{\mathbf{V}}) \triangleq \overline{\mathbf{I}}_{NL}(\overline{\mathbf{V}}) + \overline{\mathbf{I}}_{L}(\overline{\mathbf{V}}) = 0$$

where $\overline{\mathbf{I}}_{NL}(\overline{\mathbf{V}})$ and $\overline{\mathbf{I}}_{L}(\overline{\mathbf{V}})$ represent currents from the linear and the nonlinear parts, respectively

*Sensitivity Analysis*

sensitivity of output response $V_{out}$ w.r.t. design variable x

$$\frac{\partial V_{out}}{\partial x}$$

essential for gradient optimizers

*Hierarchical Analysis*

UPWARD analysis is to obtain the overall circuit matrix, e.g., Y matrix, for the linear part

DOWNWARD analysis is to obtain responses for the linear part at individual components, e.g., voltage or power for an element

TOP LEVEL analysis is to solve the harmonic balance equations for nonlinear networks

TOP LEVEL analysis is to solve the terminated circuit for linear networks

*Top Level Simulation of Nonlinear Circuits*

harmonic balance equation

$\overline{\mathbf{F}}(\overline{\mathbf{V}}) = \mathbf{0}$

Newton update

$\overline{\mathbf{V}}_{new} = \overline{\mathbf{V}}_{old} - \overline{\mathbf{J}}^{-1}\overline{\mathbf{F}}(\overline{\mathbf{V}}_{old})$

$\overline{\mathbf{J}}$ is the Jacobian matrix

the Newton solution provides the top level voltages V(k)

*Downward Simulation of the Original Linear Network*

consider a typical subcircuit

internal and external voltages $V_t(k)$ of the subcircuit can be computed from its external voltages $V(k)$ by

$$A(k) \begin{bmatrix} V_t(k) \\ I(k) \end{bmatrix} = \begin{bmatrix} 0 \\ V(k) \end{bmatrix}$$

$A(k)$ is the modified nodal admittance matrix of the subcircuit

$I(k)$ contains currents into the subcircuit from its external ports

$V_t(k)$, along with $I(k)$, is a solution to the subcircuit

*Implementation*

suppose subcircuits C1, C2 and C3 are directly connected to the subcircuit under consideration from below

then solution $V_t(k)$ contains internal and external voltages of the subcircuit under consideration

$V_t(k)$ also provides external voltages of the lower level subcircuits C1, C2, C3

the equation is used iteratively down the hierarchy until all desired voltages are found

*Top Level Adjoint Simulation for Nonlinear Networks*

suppose the second entry of $\bar{V}$ is selected as the output voltage

$\bar{V}_{out} = [\ 0\ 1\ 0\ 0\ ..\ 0\ ]\bar{V}$

the corresponding adjoint system is

$$\bar{J}^T\hat{\bar{V}} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ . \\ . \\ 0 \\ 0 \end{bmatrix}$$

in the adjoint system, the RHS is the output voltage selection vector whose second entry is 1 and all other entries are 0

the matrix $\bar{J}$ is the Jacobian matrix previously used in the Newton iteration

the LU factors of $\bar{J}$ available and reusable

the solution gives the top level external adjoint voltages $\hat{V}(k)$

*Adjoint Voltages when Output Port is Suppressed*

disconnect nonlinear part

solve linear part for individual harmonics

$\hat{\mathbf{V}}_L$ contains adjoint voltages at the external ports of the linear subnetwork

adjoint excitations of the overall circuit

$$\bar{\mathbf{J}}^T \hat{\mathbf{V}} = \bar{\mathbf{Y}}^T \hat{\mathbf{V}}_L$$

*Downward Simulation of the Adjoint Linear Network*

consider a typical subcircuit

internal and external adjoint voltages $\hat{V}_t(k)$ of the subcircuit can be computed from its external adjoint voltages $\hat{V}(k)$ by

$$A^T(k) \begin{bmatrix} \hat{V}_t(k) \\ -\hat{I}(k) \end{bmatrix} = \begin{bmatrix} 0 \\ -\hat{V}(k) \end{bmatrix}$$

matrix A(k) is the modified nodal admittance matrix previously used in downward simulation of the original circuit

$\hat{I}(k)$ contains adjoint currents into the subnetwork from its external ports

$\hat{V}_t(k)$, along with $\hat{I}(k)$, is a solution to the adjoint subcircuit

*Implementation*

solution $\hat{V}_t(k)$ contains internal and external adjoint voltages of the subnetwork

$\hat{V}_t(k)$ also provides external adjoint voltages for downward analysis

the LU factors of A(k) are available and reusable

*Sensitivity Expressions*

variable x belongs to branch b

$$\frac{\partial \overline{V}_{out}}{\partial x} = \begin{cases} -\sum_k \mathrm{Real}\,[\hat{V}_b(k) V_b^*(k) G_b^*(k)] & \text{(a)} \\ -\sum_k \mathrm{Real}\,[\hat{V}_b(k) G_b^*(k)] & \text{(b)} \\ -\sum_k \mathrm{Imag}\,[\hat{V}_b(k) G_b^*(k)] & \text{(c)} \end{cases}$$

$V_b(k)$ and $\hat{V}_b(k)$ are voltages of branch b at harmonic k

$G_b(k)$ is the sensitivity expression at the element level

*Examples of* $G_b(k)$

for a linear resistor $G_b(k)=1$

for a nonlinear resistor described by $i(t)=i(v(t), x)$

$G_b(k)$=[kth Fourier coefficient of $\partial i/\partial x$]

*Formula vs. Identity of Variables*

equation (a) is used if x belongs to the linear part of the circuit

equation (b) is used if x belongs to nonlinear resistive elements, nonlinear voltage controlled current sources or the real part of a complex driving source

equation (c) is used if x belongs to nonlinear capacitive elements or the imaginary part of a complex driving source

SENSITIVITY EXPRESSIONS AT THE ELEMENT LEVEL

| Type of Element* | | Expression for $G_b(k)$ | Applicable Equation |
|---|---|---|---|
| linear conductor | G | 1 | (a) |
| linear resistor | R | $-1/R^2$ | (a) |
| linear capacitor | C | $j\omega_k$ | (a) |
| linear inductor | L | $-1/(j\omega_k L^2)$ | (a) |
| nonlinear VCCS or nonlinear resistor described by $i = i(\mathbf{v}(t),x)$ | | [kth Fourier coefficient of $\partial i/\partial x$] | (b) |
| nonlinear capacitor described by $q = q(\mathbf{v}(t),x)$ | | $\omega_k$ [kth Fourier coefficient of $\partial q/\partial x$] | (c) |
| current driving source | | 1 | (b) or (c)$^+$ |
| voltage driving source | | 1/(source impedance) | (b) or (c)$^+$ |

* the element is located in branch b and contains the variable x.

\+ (b) is used if x is the real part of the driving source.
(c) is used if x is the imaginary part of the driving source.

$\omega_k$ is the kth harmonic frequency used in the harmonic equation.

*FET Mixer Example of Camacho-Penalosa and Aitchison (1987)*

compute exact sensitivities of the conversion gain w.r.t.
26 variables

- all parameters in the linear and nonlinear parts
- DC bias, LO power, RF power,
- IF, LO and RF terminations

results in excellent agreement with those from perturbation

CPU time for simulation only is 22 seconds on a VAX 8600

CPU time for sensitivity computation

- our approach 3.7 seconds
- perturbation approach 240 seconds

GRADIENTS OF MIXER CONVERSION GAIN

| Variable x | Gradient Expression |
|---|---|
| RF power | $c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}\} - 1$ |
| $R_g(f_{RF})$ | $c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}\} + c/(2R_g(f_{RF}))$ |
| $R_d(f_{IF})$ | $c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}$ |
| | $- 1/(R_d(f_{IF}) + jX_d(f_{IF}))\} + c/(2R_d(f_{IF}))$ |
| $X_d(f_{IF})$ | $c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}$ |
| | $- j/(R_d(f_{IF}) + jX_d(f_{IF}))\}$ |
| any parameter other than above | $c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}\}$ |

$c = 20/\ell n10$

R and X represent the real and the imaginary parts of the impedance terminations, respectively. Subscripts g and d represent the gate and the drain terminations, respectively.

NUMERICAL VERIFICATION OF SENSITIVITIES OF THE MIXER

| Location of Variables | Variable | Exact Sensitivity | Numerical Sensitivity | Difference ( % ) |
|---|---|---|---|---|
| linear | $C_{ds}$ | 2.23080 | 2.23042 | 0.02 |
| subnetwork | $C_{gd}$ | −29.44595 | −29.44659 | 0.00 |
| | $C_{de}$ | 0.00000 | 0.00000 | 0.03 |
| | $R_g$ | 3.17234 | 3.17214 | 0.01 |
| | $R_d$ | 6.42682 | 6.42751 | 0.01 |
| | $R_s$ | 11.50766 | 11.50805 | 0.00 |
| | $R_{de}$ | −0.02396 | −0.02412 | 0.66 |
| | $L_g$ | −0.50245 | −0.50346 | 0.20 |
| | $L_d$ | −0.20664 | −0.20679 | 0.07 |
| | $L_s$ | 1.15334 | 1.15333 | 0.00 |
| nonlinear | $C_{gs0}$ | −6.17770 | −6.17786 | 0.00 |
| subnetwork* | $\tau$ | 0.49428 | 0.49414 | 0.03 |
| | $V_\phi$ | −20.85730 | −20.85758 | 0.00 |
| | $V_{p0}$ | −26.48210 | −26.48041 | 0.01 |
| | $V_{dss}$ | 0.01064 | 0.01028 | 3.33 |
| | $I_{dsp}$ | 9.93696 | 9.93680 | 0.00 |
| bias and | $V_{GS}$ | −31.62080 | −31.62423 | 0.01 |
| driving | $V_{DS}$ | −2.17821 | −2.17823 | 0.00 |
| sources | $P_{LO}$ | 2.76412 | 2.76412 | 0.00 |
| | $P_{RF}$ | −0.05401 | −0.05392 | 0.16 |

NUMERICAL VERIFICATION OF SENSITIVITIES OF THE MIXER (continued)

| Location of Variables | Variable | Exact Sensitivity | Numerical Sensitivity | Difference ( % ) |
|---|---|---|---|---|
| termina-tions | $R_g(f_{LO})$ | 0.06671 | 0.06657 | 0.22 |
| | $X_g(f_{LO})$ | 0.37855 | 0.37854 | 0.00 |
| | $R_g(f_{RF})$ | 0.78812 | 0.78798 | 0.02 |
| | $X_g(f_{RF})$ | 0.45120 | 0.45119 | 0.00 |
| | $R_d(f_{IF})$ | 0.71451 | 0.71436 | 0.02 |
| | $X_d(f_{IF})$ | 0.10886 | 0.10871 | 0.14 |

* Nonlinear elements are characterized by

$$C_{gs}(v_1) = C_{gs0} / \sqrt{1-v_1/V_\phi},$$

$$R_i(v_1)C_{gs}(v_1) = \tau$$

and the function for $i_m(v_1, v_2)$ is shown in the I-V curve of the FET device, whose mathematical expression is consistent with Camacho-Penalosa and Aitchison (1987). $V_\phi$, $V_{p0}$, $V_{dss}$ and $I_{dsp}$ are parameters in the function $i_m(v_1, v_2)$.

*Comparison with the Perturbation Method*

perturbation method

- simulate the original nonlinear circuit
- perturb all variables and resimulate for each perturbation

our method

- simulate the original circuit
- solve the adjoint equations once

features of our method

- adjoint simulation noniterative
- exact gradient
- computation speed fast

*Conclusions*

unified theory for

- simulation and sensitivity analysis
- linear and nonlinear circuits
- hierarchical and nonhierarchical

since nonlinear simulation is costly, the adjoint sensitivity approach is very significant

our hierarchical approach permits

- voltages anywhere in the original and adjoint networks
- variables anywhere in the entire circuit

a key for the coming generation of microwave CAD software

*Application to Nonlinear Parameter Extraction*

operate the device under large signal AC conditions

perform measurements at DC, fundamental as well as higher harmonics

optimize all parameters such that the computed large signal responses match the measured responses at DC, fundamental and all higher harmonics simultaneously

powerful $l_1$ gradient optimizer should be used

the error function and its derivatives w.r.t. optimization variables are provided by appropriate simulation and sensitivity analysis

*Simulation and Sensitivity Analysis for Nonlinear Parameter Extraction*

formulate the harmonic balance equations

solve harmonic balance equations, retaining LU factors of the Jacobian matrix at the solution

compute error functions for optimization

define the vector for selecting accessible measurement ports, e.g., drain port

solve the adjoint system using the voltage selection vector as the RHS and the transposed Jacobian as the adjoint network matrix

use downward simulation to obtain all necessary branch voltages (both original and adjoint)

compute (adjoint) sensitivity at element level

compute circuit response sensitivity using our novel formulas

compute gradient vector for optimization

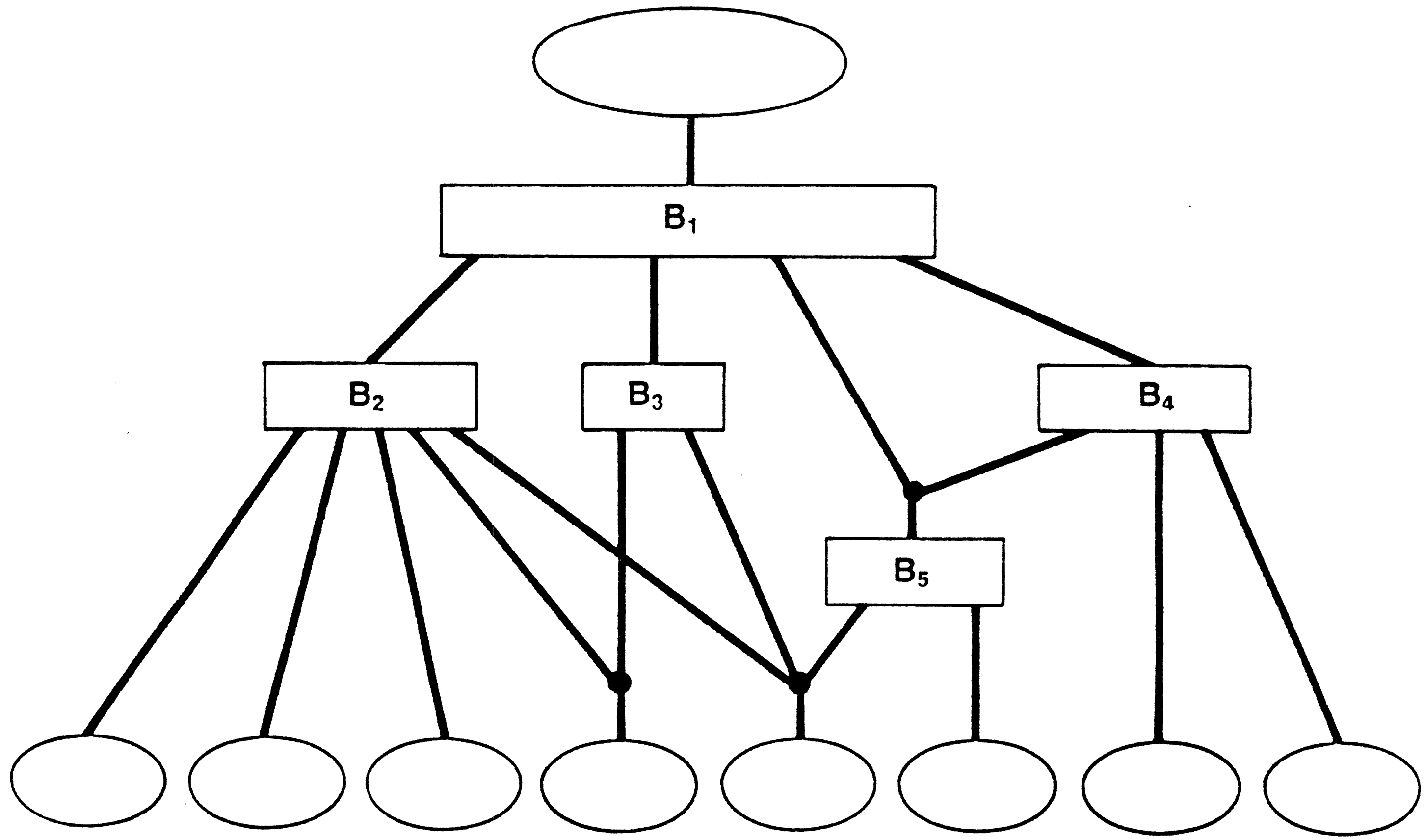
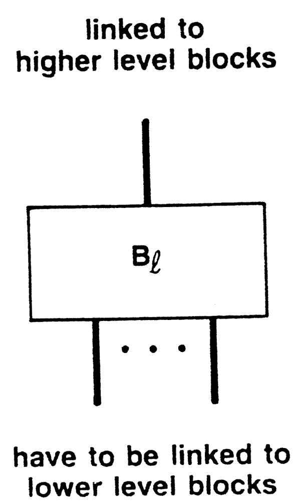
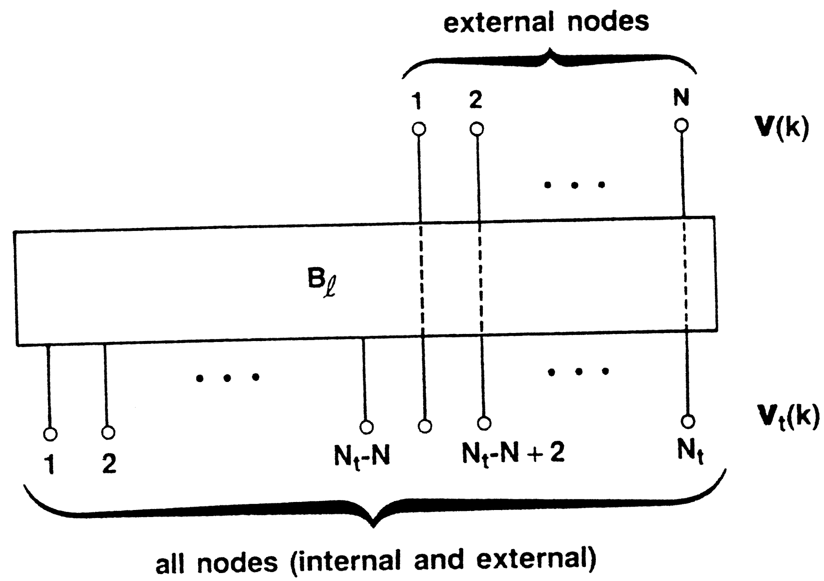



Fig. 1 An arbitrary circuit hierarchy. Each thick line represents a group of nodes. Each rectangular box represents a connection block for a subcircuit. Each bottom circular box represents a circuit element and the top circular box represents the sources and loads.




(a)




(b)

Fig. 2 A typical subcircuit connection block: (a) as seen from Fig. 1, (b) detailed representation of all the nodes of the subnetwork. Nodes at the top (bottom) of the rectangular box are the external (external and internal) nodes of the subnetwork.

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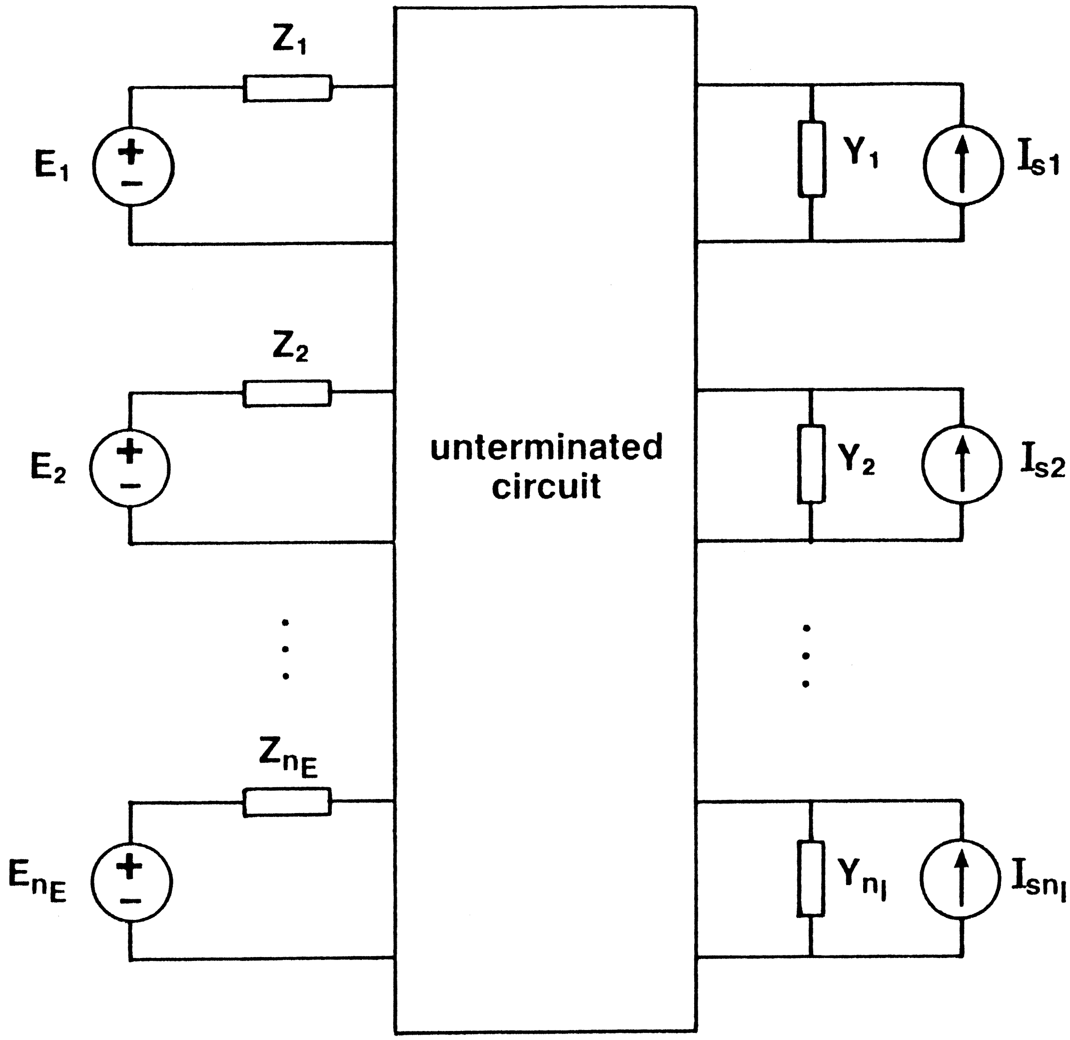


Fig. 3 A representation of a terminated subnetwork. Both current and voltage sources can be accommodated. The overall port sequence is such that ports 1, 2, ..., $n_E$ correspond to voltage sources and ports $n_E+1$, $n_E+2$, ..., $n_E+n_I$ correspond to current sources. The total number of ports is N, i.e., $N = n_E+n_I$.

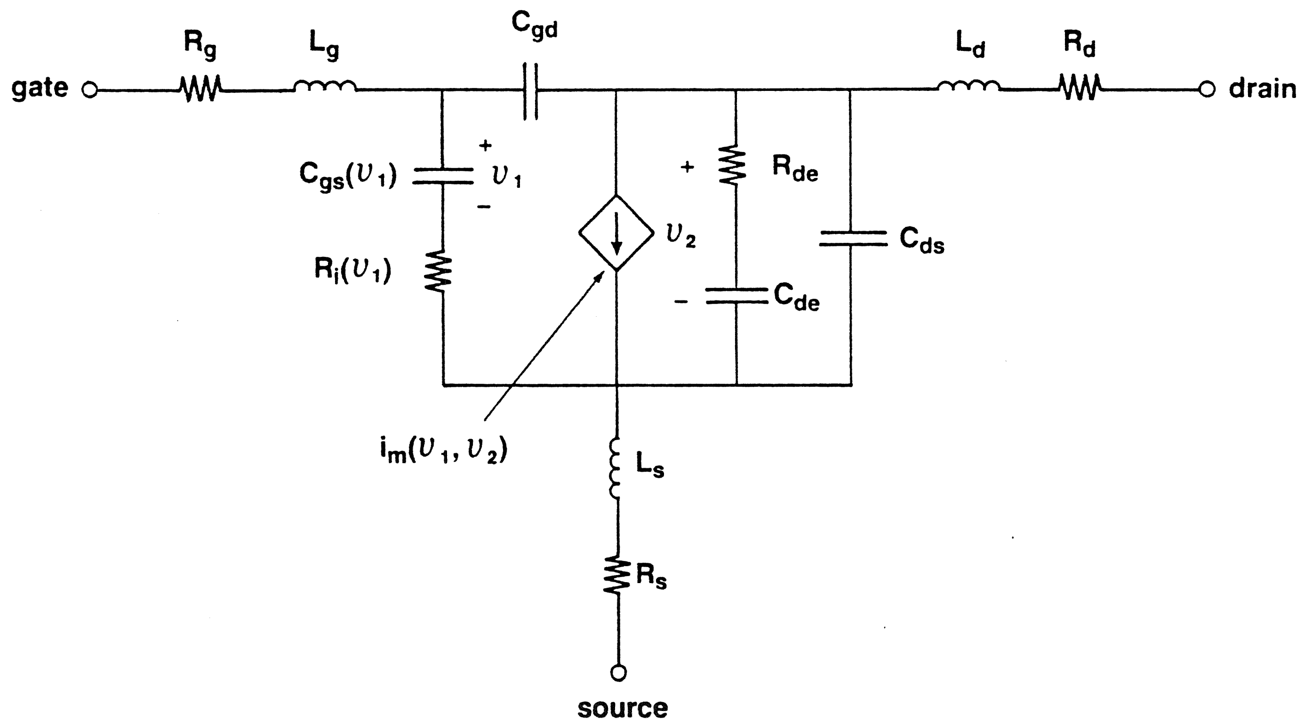


Fig.4 A large signal MESFET model. All parameters are consistent with Camacho-Penalosa and Aitchison (1987).

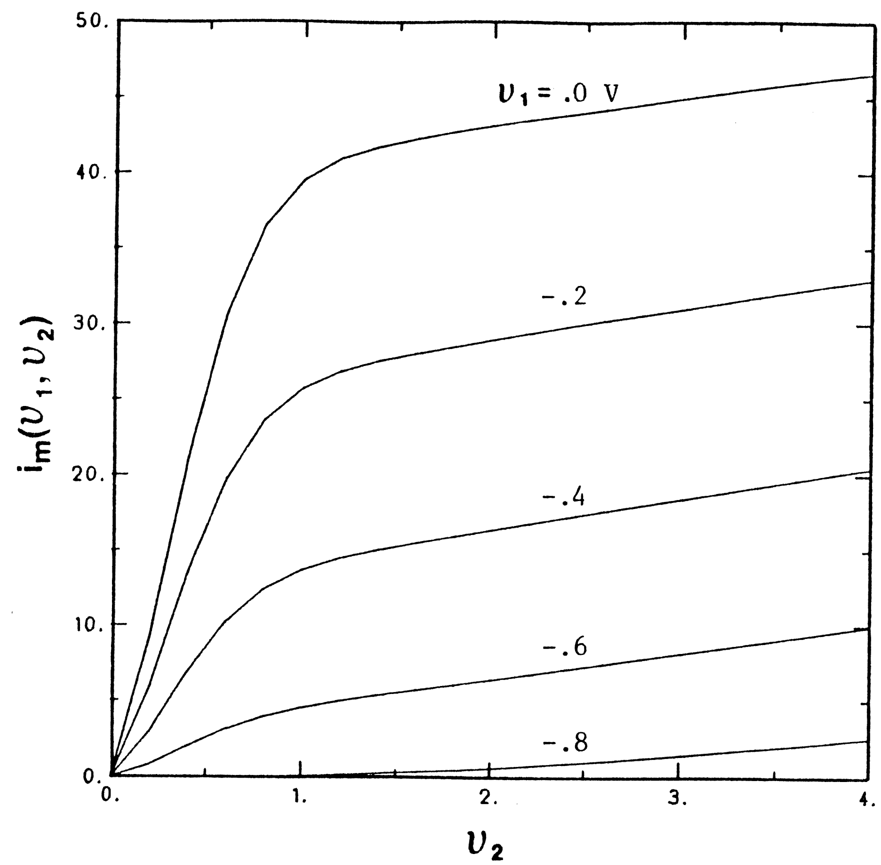


Fig. 5 The DC characteristics of the MESFET model.

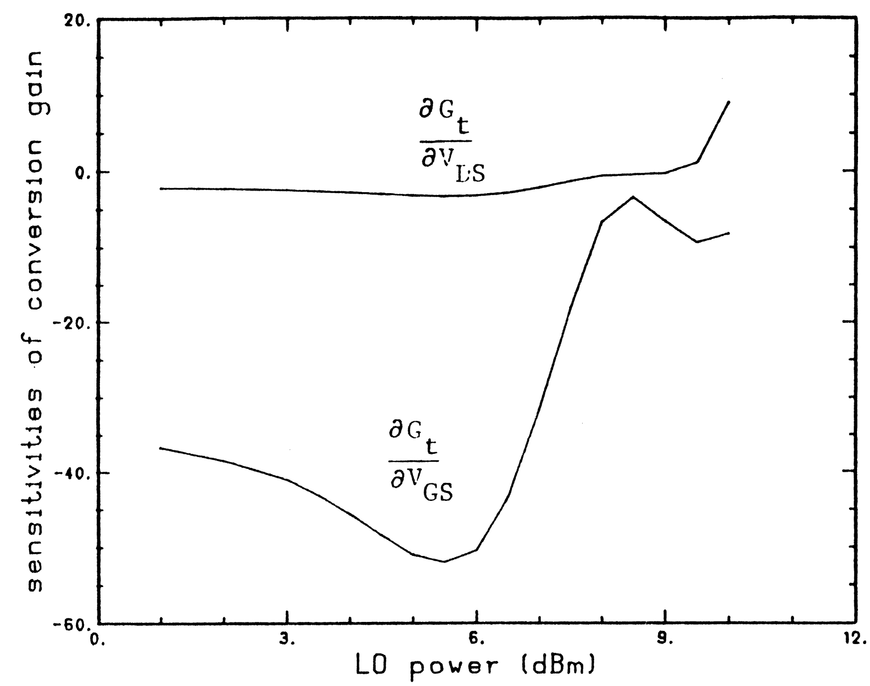


Fig. 6 Sensitivities of conversion gain w.r.t. bias voltages as functions of LO power.