ROBUST MODEL PARAMETER EXTRACTION USING LARGE-SCALE OPTIMIZATION CONCEPTS

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Abstract

A robust approach to model parameter extraction is presented. This approach utilizes multi-bias measurements and dc device characteristics. Novel automatic decomposition concepts for large-scale optimization detect possible model topology deficiencies. Powerful ℓ_1 optimization is employed with adjoint analyses for both dc and ac sensitivities.

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SUMMARY

Introduction

Model parameter extraction, i.e., the determination of equivalent circuit parameters from dc, rf and microwave measurements on devices (such as FETs), is of fundamental importance to microwave circuit designers. This paper describes a robust approach which substantially expands the multi-circuit algorithm introduced in [1] and [2], and exploits the automatic decomposition concepts for large-scale optimization proposed in [3].

Conventionally, we seek a set of model parameters which minimizes the difference between the model responses and the measurements. To alleviate indeterminacy as well as for simplicity, techniques have been implemented (e.g., [4], [5]) which separate the dc, low frequency and high frequency measurements and divide the model parameters into corresponding subsets. This defines a set of subproblems to be solved <u>sequentially</u>. However, such a sequentially decoupled solution may not be reliable: a parameter determined solely from dc measurements may not be suitable for the purpose of microwave simulation.

Bandler et al. [1], [2] have recently proposed to simultaneously process multiple sets of S-parameter measurements made under different biasing conditions. From these measurements, multiple sets of model parameters are identified. The authors showed that the uniqueness of the solution may be improved by constraining model parameters that are insensitive to bias as common variables. However, their classification of the model parameters as either <u>completely</u> bias-independent or <u>arbitrarily</u> bias-dependent is rather simplistic. Our new approach employs the dc characteristics of the device to constrain the bias-dependent parameters in order to reduce the degrees of freedom in modeling and improve the unique determinacy of the problem.

Bandler and Zhang [3] have proposed the automatic construction of a decomposition dictionary to reveal interdependency between model responses and model parameters. We exploit this approach to examine a sequence of models of increasingly more complex topologies. We start the modeling process with a basic topology, subsequently adding elements consistent with the dictionary requirements to achieve a better match between the model responses and the measurements.

ℓ_1 optimization is highly favored for device modeling [1]. We have integrated a powerful ℓ_1 algorithm [6] into our new approach. To provide gradients, efficiently, adjoint analyses are applied to obtaining both dc and ac sensitivities.

Multi-circuit Formulation and dc Constraints

Let $\boldsymbol{\phi}^k$ be the set of model parameters and $\mathbf{v}^k$ be biasing parameters, where superscript k is used to indicate different bias points. We represent in general the functional dependency of $\boldsymbol{\phi}^k$ on $\mathbf{v}^k$ by $\boldsymbol{\phi}^k = \mathbf{f}(\mathbf{v}^k)$. Then ϕ_i being a common variable implies a completely known functional dependency as

$$\phi_i^k = \phi_i, \quad \text{for all k.} \tag{1}$$

Otherwise, ϕ_i^k, k=1, ..., K, are treated as totally unrelated variables (where K is the total number of different bias points).

In our new approach, we use index sets I_a, I_b and I_c to allow for a more flexible classification of the model parameters according to our knowledge of the functional dependency $\phi_i = f_i(\mathbf{v})$.

For $i \in I_a$, ϕ_i is taken as independent of the bias, i.e., $f_i(\mathbf{v})$ has the simplest form as given by (1). For each i we need to identify only ϕ_i.

For $i \in I_b$, suppose we know only the form of $f_i(\mathbf{v})$. We write

$$\phi_i^k = f_i(\boldsymbol{\alpha}, \mathbf{v}_k), \quad \text{for } i \in I_b, \tag{2}$$

where $\boldsymbol{\alpha}$ is a set of unknown coefficients to be determined through the modeling process. In fact, (2) describes constraints imposed on the multiple sets of model parameters $\boldsymbol{\phi}^k$. These constraints may be derived from physical characteristics of the device such as the dc equations. They may also include mathematical expressions, such as polynomials, based on engineering experience that a parameter varies 'slightly' or 'moderately' with the bias. The number of optimization variables in this group, namely the number of coefficients in $\boldsymbol{\alpha}$, is independent of the number of different bias points. In contrast, if we treat each ϕ_i^k as a separate variable then the total number of variables would grow in proportion to the number of bias points. In other words, the constraints introduced by (2) have reduced the degrees of freedom so that the modeling problem may be better determined and, consequently, the uniqueness of the solution may be further strengthened. Furthermore, the resulting coefficients $\boldsymbol{\alpha}$ provide a model for the functional dependency of $\boldsymbol{\phi}$ on the bias, which is useful for dc and large-signal simulation of the device.

For $i \in I_c$, we assume nothing about $f_i(\mathbf{v})$ and therefore in this group for every different bias ϕ_i^k has to be a separate variable.

A FET Example

We use the same measurement data and small-signal equivalent circuit model of a FET as those considered by Bandler et al. [2], [7]. The small-signal equivalent circuit model has 11 parameters, namely

$$\{R_g, R_d, L_s, \tau, G_{ds}, R_i, R_s, C_{gs}, C_{dg}, C_{ds}, g_m\}.$$

Measurements are made under three different biasing conditions. We organize the data into groups, according to bias, dc measurements, low frequencies (from 2GHZ to 9GHZ) and high frequencies (from 10GHZ to 18GHZ), and different scattering parameters (S_{11}, S_{12}, S_{21} and S_{22}). A sensitivity dictionary [3] is constructed to reveal the interdependency between different data groups and model parameters. We take four of the model parameters as bias-independent, i.e., they are considered as common variables:

$$\{R_g,\ R_d,\ L_s,\ \tau\}. \tag{3}$$

The remaining seven model parameters are considered as bias-dependent. Three of them, namely G_{ds}, C_{gs} and g_m, are also constrained by the dc characteristics given by Materka and Kacprzak [5]:

$$
\begin{aligned}
i_r &= I_{sr}\,[\exp(\alpha_{sr} v_{dg}) - 1] \\
i_f &= I_s\,[\exp(\alpha_s v_g) - 1] \\
i_d &= I_{dss}(1 - v_g/V_p)^2 \tanh(\alpha v_d/(v_g - V_p)) \\
V_p &= V_{po} + \gamma v_d \\
C_{gs} &= C_{go}(1 - v_g/V_{bi})^{-\frac{1}{2}},\ \ \text{for } v_g < 0.8V_{bi}.
\end{aligned} \tag{4}
$$

The ten coefficients in these constraints, namely,

$$\{I_s,\ \alpha_s,\ I_{sr},\ \alpha_{sr},\ I_{dss},\ \alpha,\ V_{po},\ \gamma,\ C_{go},\ V_{bi}\} \tag{5}$$

are included in the set of optimization variables. At each iteration of the modeling process, we solve the dc circuit equations for the bias-dependent dc voltages and currents, and then the values of G_{ds}, C_{gs} and g_m are determined from (4). In other words, the extraction of G_{ds}, C_{gs} and g_m utilizes both the dc and ac measurements. They are the type of model parameters we have defined in (2). In the previous work by Bandler et al. [2], [7], the dependency on bias of these parameters was assumed to be arbitrary.

The other four parameters, namely $\{R_s,\ R_i,\ C_{dg},\ C_{ds}\}$, are considered bias-dependent. Since we are not imposing constraints, they are treated as separate variables for each kth bias point:

$$\{R_s^k,\ R_i^k,\ C_{dg}^k,\ C_{ds}^k\},\ \ k = 1,2,3. \tag{6}$$

In total, we have 26 optimization variables as given by (3), (5) and (6). Denoting the variable vector by **x**, we define an ℓ_1 optimization problem for the example at hand as

$$\underset{x}{\text{minimize}} \sum_{t=1}^{3} \sum_{i=1}^{17} \sum_{j=1}^{2} \sum_{k=1}^{2} \{|\mathrm{Re}[f_{jk}^t(\omega_i)]| + |\mathrm{Im}[f_{jk}^t(\omega_i)]|\}, \tag{7}$$

where

$$f_{jk}^t(\omega_i) = F_{jk}^t(x^t,\omega_i) - S_{jk}^t(\omega_i), \tag{8}$$

F_{jk}^t and S_{jk}^t are the calculated and measured scattering parameters, respectively, with superscript identifying three different bias points. Having 17 frequency points (2GHZ, 3GHZ, ..., 18GHZ) with real and imaginary parts of the complex S-parameters being treated separately, we have a total of 408 error functions.

In order to supply efficiently the gradients for the ℓ_1 algorithm [6], adjoint analyses were performed to obtain sensitivities of the ac circuit model. To obtain G_{ds}, C_{gs} and g_m from (4), we solved the nonlinear dc circuit equations by the Newton-Raphson method. The required derivatives were provided through dc adjoint analyses.

The solution consists of the common variables given by

$$\{R_g, R_d, L_s, \tau\} = \{2.6008\Omega, 3.1335\Omega, 3.9\text{pH}, 4.2792\text{ps}\}$$

and the bias-dependent variables $\{G_{ds}^k, C_{gs}^k, g_m^k, R_s^k, R_i^k, C_{dg}^k, C_{ds}^k\}$ given by

$$\{5.1\text{mS}, 0.72\text{pF}, 69.5\text{mS}, 0.977\Omega, 0.0001\Omega, 0.0306\text{pF}, 0.2202\text{pF}\},\ k = 1,$$

$$\{5.8\text{mS}, 0.4332\text{pF}, 53.4\text{mS}, 0.6232\Omega, 1.4536\Omega, 0.0488\text{pF}, 0.2185\text{pF}\},\ k = 2,$$

$$\{6.9\text{mS}, 0.3505\text{pF}, 39.3\text{mS}, 0.533\Omega, 0.0492\Omega, 0.0612\text{pF}, 0.2133\text{pF}\},\ k = 3.$$

We also obtained the dc coefficients $\{I_s, \alpha_s, I_{sr}, \alpha_{sr}, I_{dss}, \alpha, V_{po}, \gamma, C_{go}, V_{bi}\}$ as {3pA, 0.6/kV, 1nA, 1.5, 0.2483A, 5.1838, -5.4035V, -0.3832, 0.8353pF, 0.6706V}.

To demonstrate the robustness and uniqueness of the algorithm, we randomly perturb the solution by 20 to 100 percent and restart the modeling optimization. All the parameters, except for R_i, have converged to virtually the same solution. Actually, our sensitivity dictionary indicates that R_i hardly affects any S-parameters, therefore its value would not be unique.

Sequential Model Building

We have started the modeling process using the models in [2] and [7], which did not include L_g and L_d. At the solution, the match between calculated and measured S-parameters is satisfactory except for S_{11}. To further improve the match of S_{11} without destroying the match of the other S-parameters, we can consider a more complex model topology. We expanded the dictionary to include the sensitivities of L_g and L_d. It indicates that L_g is among the dominant variables affecting S_{11}. Consequently, we update the equivalent circuit topology to include L_g, as shown in Fig. 1 and optimize L_g. The solution improved the match between calculated and measured S_{11} without significantly affecting the other S-parameters.

In another experiment, we have further updated the model by including L_d as an additional variable. The optimized L_d is identically zero even when its initial value was nonzero. This verifies that L_d has little influence on S_{11} and including L_g in the model is already adequate.

The ability to isolate a few large errors, such as those observed on S_{11}, and to suppress redundant variables, such as L_d being driven to zero, is a unique property of the ℓ_1 optimization. Without ℓ_1 being an integral part of the algorithm, we might not have observed these results.

References

[1] J.W. Bandler, S.H. Chen and S. Daijavad, "Microwave device modelling using efficient ℓ_1 optimization: a novel approach", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, 1986, pp. 1282-1293.

[2] J.W. Bandler and S.H. Chen, "Circuit optimization: the state of the art", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, February 1988.

[3] J.W. Bandler and Q.J. Zhang, "An automatic decomposition approach to optimization of large microwave systems", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, December 1987.

[4] H. Kondoh, "An accurate FET modelling from measured S-parameters", *IEEE Int. Microwave Symp. Digest* (Baltimore, MD), 1986, pp. 377-380.

[5] A. Materka and T. Kacprzak, "Computer calculation of large-signal GaAs FET amplifier characteristics", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, 1985, pp. 129-135.

[6] J.W. Bandler, W. Kellermann and K. Madsen, "An nonlinear ℓ_1 optimization algorithm for design, modelling and diagnosis of networks", *IEEE Trans. Circuits and Systems*, vol. CAS-34, 1987, pp. 174-181.

[7] J.W. Bandler, S.H. Chen, S. Daijavad and K. Madsen, "Efficient optimization with integrated gradient approximations", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, February 1988.

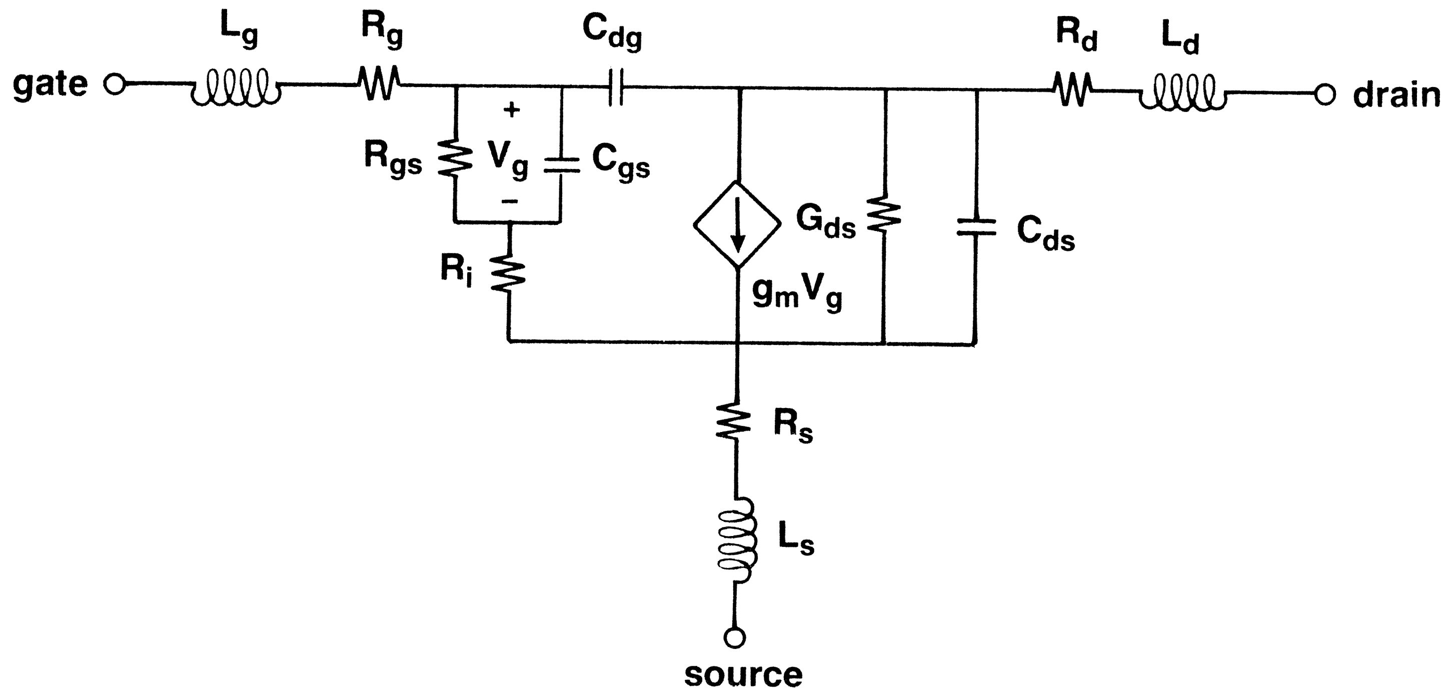

Fig. 1 The FET small-signal equivalent circuit model that includes L_g and L_d.