A UNIFIED FRAMEWORK FOR HARMONIC BALANCE SIMULATION AND SENSITIVITY ANALYSIS

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Abstract

In this paper, a novel theory for exact sensitivity analysis of nonlinear circuits based on harmonic balance simulation is derived. A framework unifying many existing concepts of frequency domain simulation and sensitivity analysis of linear/nonlinear circuits is established. The proposed sensitivity analysis is verified by a MESFET mixer example exhibiting 98% saving of CPU time over the prevailing perturbation method.

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SUMMARY

Introduction

In this paper, we present a unified approach to the simulation and sensitivity analysis of linear/nonlinear circuits in the frequency domain. The linear part of the circuit can be large and can be hierarchically decomposed, highly suited to modern microwave CAD. Analysis of the nonlinear part is performed in the time domain and the large signal steady-state periodic analysis of the overall circuit is carried out by means of the harmonic balance (HB) method.

The HB method has become an important tool for the analysis of nonlinear circuits. The work of Rizzoli et al. [1], Curtice and Ettenberg [2], Curtice [3], Gilmore and Rosenbaum [4], Gilmore [5], Camacho-Penalosa and Aitchison [6] stimulated work on HB in the microwave CAD community. The excellent paper of Kundert and Sangiovanni-Vincentelli [7] provided systematic insight into the HB method. Many others, e.g., [8-14], have also contributed substantially to the state-of-the-art of the HB technique. The first step towards design optimization was made by Rizzoli et al. [1] who used the perturbation method to approximate the gradients.

In our paper, we extend to nonlinear circuits the powerful adjoint network concept, a standard sensitivity analysis approach in linear circuits. The concept involves solving a set of linear equations whose coefficient matrix is available in many existing HB programs. The solution of a single adjoint system is sufficient for the computation of sensitivities w.r.t. all parameters in both the linear and nonlinear subnetworks, as well as in bias, driving sources and terminations. No parameter perturbation or iterative simulations are required.

The sensitivities we propose are exact in terms of the harmonic balance method itself. Our exact adjoint sensitivity analysis can be used with various existing HB simulation techniques, e.g., the basic HB [7], the modified HB [5] and the APFT HB [14]. Computational effort includes solving the adjoint linear equations and calculating the Fourier transforms of all time-domain derivatives at the nonlinear element level. Significant CPU time savings are achieved over the perturbation method.

A MESFET mixer example is used to verify our theory.

Notation and Definition

We follow the notation of [7]. Let $\mathbf{v}(t)$ or $\mathbf{i}(t)$ ($\mathbf{V}(k)$ or $\mathbf{I}(k)$) be real (complex) vectors containing all nodal voltages or currents at time t (harmonic k). A bar denotes the split real and imaginary parts of a complex vector. The hat distinguishes quantities of the adjoint system. In particular, $\overline{\mathbf{V}}$ or $\overline{\mathbf{I}}$ are real vectors containing the real and the imaginary parts of $\mathbf{V}(k)$ or $\mathbf{I}(k)$ for all harmonics k, $k = 0, 1, ..., H-1$. A detailed definition of the notation is available in Table I.

Hierarchical Simulation of the Linear Network

Consider the arbitrary circuit hierarchy of Fig. 1. A typical subnetwork containing internal and external nodes is shown in Fig. 2. A general representation of a terminated circuit is depicted in Fig. 3. An unpartitioned or nonhierarchical approach is a special case of Fig. 1 when only one level exists. In any case, however, we consider an unterminated N-port circuit at the highest level of hierarchy because of the importance of the reference plane in microwave circuits. On the other hand the N-port description is needed for the harmonic balance equations.

By unifying various existing approaches, we have derived a comprehensive set of formulas, systematically computing voltage responses at any nodes (internal or external) for any subnetwork at any level. For example, to compute all (both internal and external) nodal voltages $\mathbf{V}_t(k)$ of a subnetwork using the result of a higher level simulation, i.e., using the external voltages

V(k), we solve

$$A(k) \begin{bmatrix} V_t(k) \\ I(k) \end{bmatrix} = \begin{bmatrix} 0 \\ V(k) \end{bmatrix}, \qquad (1)$$

where the matrix A(k) contains the nodal admittance matrix of the subnetwork and some 0's and 1's as defined in Table I.

Simulation of Nonlinear Circuits [7]

The simulation of a nonlinear circuit is to find a $\overline{V}$ such that

$$\overline{F}(\overline{V}) \triangleq \overline{I}_{NL}(\overline{V}) + \overline{I}_{L}(\overline{V}) = 0, \qquad (2)$$

where the vectors $\overline{I}_L$ and $\overline{I}_{NL}$ are defined as the currents into the linear and nonlinear parts at the nodes of their connection. The Newton update for solving (2) is

$$\overline{V}_{new} = \overline{V}_{old} - \overline{J}^{-1}\overline{F}(\overline{V}_{old}), \qquad (3)$$

where $\overline{J}$ is the Jacobian matrix.

Adjoint System for Linear Networks

At the highest level of the hierarchy, the adjoint system is excited at the output port. At other levels, the circuit equation is

$$A^T(k) \begin{bmatrix} \hat{V}_t(k) \\ -\hat{I}(k) \end{bmatrix} = \begin{bmatrix} 0 \\ -\hat{V}(k) \end{bmatrix}. \qquad (4)$$

Notice that the LU factors of A(k) are already available from (1). This equation is used iteratively for the 2nd, 3rd, ... levels of the hierarchy until all desired adjoint voltages are found.

Adjoint System for Nonlinear Networks

Suppose $\overline{V}_{out}$ is the real or imaginary part of output voltage V_{out} and can be selected from the voltage vector $\overline{V}$ by a vector $\overline{e}$ as

$$\overline{V}_{out} = \overline{e}^T\overline{V}. \qquad (5)$$

The adjoint system is the linear equation

$$\overline{J}^T\hat{\overline{V}} = \overline{e}, \qquad (6)$$

where $\overline{J}$ is the Jacobian at the solution of (2). Notice that the LU factors of

$\overline{\mathbf{J}}$ is available from the last iteration of (3). Therefore, to obtain $\hat{\mathbf{V}}$ from (6), we need only the forward and backward substitutions. The adjoint voltages at other nodes (internal nodes of the linear part) can be obtained using (4).

The adjoint voltages can be computed even if the output port is suppressed from the harmonic equation (2). In this case we first compute the adjoint voltages at the external nodes of the linear subnetwork. This can be done by disconnecting the nonlinear part and then solving the linear part for individual harmonics separately. The resulting vector, denoted by $\hat{\mathbf{V}}_L$, is then transformed to the actual adjoint excitations of the overall circuit (including both linear and nonlinear parts) to be incorporated in (6) instead of $\overline{\mathbf{e}}$. The final equation takes the form

$$\overline{\mathbf{J}}^T\hat{\mathbf{V}} = \overline{\mathbf{Y}}^T\hat{\mathbf{V}}_L \, . \tag{7}$$

<u>Sensitivity Expressions</u>

Suppose a variable x belongs to branch b. We have derived the following formula for computing the exact sensitivity of V_{out} w.r.t. x,

$$\frac{\partial \overline{V}_{out}}{\partial x} = \begin{cases} -\sum_k \mathrm{Real}\,[\hat{V}_b(k)V_b^*(k)G_b^*(k)] & \text{if } x \in \text{linear subnetwork} \quad (8a) \\ -\sum_k \mathrm{Real}\,[\hat{V}_b(k)G_b^*(k)] & \text{if } x \in \text{nonlinear VCCS or nonlinear resistor or real part of a complex driving source} \quad (8b) \\ -\sum_k \mathrm{Imag}\,[\hat{V}_b(k)G_b^*(k)] & \text{if } x \in \text{nonlinear capacitor or imaginary part of a complex driving source,} \quad (8c) \end{cases}$$

where * denotes the complex conjugate. Complex quantities $V_b(k)$ and $\hat{V}_b(k)$ are the voltages of branch b at harmonic k and are obtained from vectors $\overline{\mathbf{V}}$ and $\hat{\mathbf{V}}$, respectively. $G_b(k)$ denotes the sensitivity expression of the element containing variable x. For example, if x is the conductance of a linear resistor, $G_b(k) = 1$. If x belongs to a nonlinear resistor represented by $i = i(v(t), x)$, $G_b(k)$ is the <u>kth Fourier coefficient</u> of $\partial i/\partial x$. A list of various cases of $G_b(k)$ is given in Table II.

Our sensitivity formula (8) has no restrictions on the selection of harmonic frequencies or the time samples. In a multi-tone case, the index k in (8) corresponds to all the harmonics used in the harmonic equation (2). When the multidimensional Fourier transform is used, we simply place a multi-dimensional summation in (8).

Comparison with the Perturbation Method

To approximate the sensitivities using the traditional perturbation method, one needs a circuit simulation for each variable. The best possible situation for this method is that all simulations finish in one iteration. For our exact adjoint sensitivity analysis, the major computation, i.e., solving the adjoint equations, is done only once for all variables. A detailed comparison reveals that the worst case for our approach takes less computation than the best situation of the perturbation method. In our experiment, we used only 1.6% of the CPU time required by the perturbation method to obtain all sensitivities.

Gradient Vector for Optimization

The novel formula (8) can be used as a key to formulate the gradient vectors for design optimization and yield maximization of nonlinear circuits. Table III lists the gradients of a FET mixer conversion gain w.r.t. various variables, expressed as simple functions of $\partial V_{out}/\partial x$.

A MESFET Mixer Example

The MESFET mixer example reported in [6] was used to verify our theory. Figs. 4 and 5 show the large-signal MESFET model and the DC characteristics of the device. The frequencies are f_{LO} = 11 GHz, f_{RF} = 12 GHz and f_{IF} = 1 GHz. The DC bias voltages are V_{GS} = -0.9 V and V_{DS} = 3.0 V. With LO power P_{LO} = 7 dBm and RF power P_{RF} = -15 dBm, the conversion gain was 6.4 dB. 26 variables were considered including all parameters in the linear as well as the nonlinear parts, DC bias, LO power, RF power, IF, LO and RF terminations. Exact sensitivities of the conversion gain w.r.t. all the variables are computed using our novel theory. The results were in excellent agreement with those from the perturbation method, as shown in Table IV. The circuit was solved in 22 seconds on a VAX 8600. The CPU time for sensitivity analysis using our method and the perturbation method are 3.7 seconds and 240 seconds, respectively.

The dangling node between the nonlinear elements C_{gs} and R_i, a case which could cause trouble in HB programs, is directly accommodated in our approach.

We have plotted selected sensitivities vs. LO power in Fig. 6. For example, as LO power is increased, conversion gain becomes less sensitive to changes in gate bias V_{GS}.

Conclusion

Our formula (8a) encompasses the adjoint network approach in the frequency domain [15], a standard for exact sensitivity analysis of linear circuits, as a special case. Since the simulation of nonlinear circuits is expensive, gradient approximations for nonlinear circuits using repeated simulation is very costly. Consequently, the adjoint sensitivity analysis becomes far more significant for nonlinear circuits than for linear ones. Our theory will greatly facilitate the design optimization and yield maximization of nonlinear circuits.

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TABLE I

NOTATION AND DEFINITION

Notation	Definition
N_t	total number of nodes (internal and external) of a linear subnetwork.
N	number of circuit nodes (or ports) used in harmonic analysis. Also, it is the number of external nodes for a typical subnetwork of Fig. 2.
H	number of harmonics, including DC.
k	harmonic index. $k = 0$ for DC, $k = 1$ for the fundamental harmonic, $k = 2, 3, \ldots, H-1$ for other harmonics.
$\mathbf{V}_t(k)$, $\mathbf{I}_t(k)$	complex N_t-vectors indicating kth harmonic voltages or currents at all nodes (both internal and external) of a linear subnetwork.
$\mathbf{V}(k)$, $\mathbf{I}(k)$	complex N-vectors indicating kth harmonic voltages or currents at all external nodes of any linear subnetwork (at the highest level of hierarchy the nodes at which the harmonic balance equations are formulated).
$\overline{\mathbf{V}}$, $\overline{\mathbf{I}}$	real 2HN-vectors containing real and imaginary parts of $\mathbf{V}(k)$ or $\mathbf{I}(k)$ at all harmonics k, $k = 0, 1, \ldots, H-1$.
$\mathbf{Y}_t(k)$	N_t by N_t matrix representing the unreduced nodal admittance matrix of a linear subnetwork at harmonic k.
$\mathbf{Y}(k)$	N by N matrix representing the reduced nodal admittance matrix of a linear subnetwork at harmonic k.
$\overline{\mathbf{Y}}$	2HN by 2HN real matrix obtained by splitting the real and imaginary parts of $\mathbf{Y}(k)$ for all harmonics k, $k = 0, 1, \ldots, H-1$.
$\mathbf{A}(k)$	$\begin{bmatrix} \mathbf{Y}_t(k) & -\mathbf{U} \\ \mathbf{U}^T & \mathbf{0} \end{bmatrix}$ where $\mathbf{U}$ is $\begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}$ and $\mathbf{1}$ is an N by N identity matrix.

TABLE II

SENSITIVITY EXPRESSIONS AT THE ELEMENT LEVEL

Type of Element*	Expression for $G_b(k)$	Applicable Equation
linear conductor G	1	(8a)
linear resistor R	$-1/R^2$	(8a)
linear capacitor C	$j\omega_k$	(8a)
linear inductor L	$-1/(j\omega_k L^2)$	(8a)
nonlinear VCCS or nonlinear resistor described by $i = i(\mathbf{v}(t),x)$	[kth Fourier coefficient of $\partial i/\partial x$]	(8b)
nonlinear capacitor described by $q = q(\mathbf{v}(t),x)$	ω_k [kth Fourier coefficient of $\partial q/\partial x$]	(8c)
current driving source	1	(8b) or (8c)+
voltage driving source	1/(source impedance)	(8b) or (8c)+

* the element is located in branch b and contains the variable x.

\+ (8b) is used if x is the real part of the driving source.
(8c) is used if x is the imaginary part of the driving source.

ω_k is the kth harmonic frequency used in the harmonic equation (2)

TABLE III

GRADIENTS OF MIXER CONVERSION GAIN

Variable x	Gradient Expression
RF power	$c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}\} - 1$
$R_g(f_{RF})$	$c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}\} + c/(2R_g(f_{RF}))$
$R_d(f_{IF})$	$c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}$
	$- 1/(R_d(f_{IF}) + jX_d(f_{IF}))\} + c/(2R_d(f_{IF}))$
$X_d(f_{IF})$	$c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}$
	$- j/(R_d(f_{IF}) + jX_d(f_{IF}))\}$
any parameter other than above	$c\ \mathrm{Real}\{(\partial V_{out}/\partial x)/V_{out}\}$

$c = 20/\ell n10$

R and X represent the real and the imaginary parts of the impedance terminations, respectively. Subscripts g and d represent the gate and the drain terminations, respectively.

complex quantity $\partial V_{out}/\partial x$ is obtained by solving (5) - (8) twice, once for the real part and the other for the imaginary part. The LU factors of $\bar{J}$ and the Fourier transforms of element sensitivities are common between the two operations.

TABLE IV

NUMERICAL VERIFICATION OF SENSITIVITIES OF THE MIXER

Location of Variables	Variable	Exact Sensitivity	Numerical Sensitivity	Difference (%)
linear	C_{ds}	2.23080	2.23042	0.02
subnetwork	C_{gd}	-29.44595	-29.44659	0.00
	C_{de}	0.00000	0.00000	0.03
	R_g	3.17234	3.17214	0.01
	R_d	6.42682	6.42751	0.01
	R_s	11.50766	11.50805	0.00
	R_{de}	-0.02396	-0.02412	0.66
	L_g	-0.50245	-0.50346	0.20
	L_d	-0.20664	-0.20679	0.07
	L_s	1.15334	1.15333	0.00
nonlinear	C_{gs0}	-6.17770	-6.17786	0.00
subnetwork*	τ	0.49428	0.49414	0.03
	V_ϕ	-20.85730	-20.85758	0.00
	V_{p0}	-26.48210	-26.48041	0.01
	V_{dss}	0.01064	0.01028	3.33
	I_{dsp}	9.93696	9.93680	0.00
bias and	V_{GS}	-31.62080	-31.62423	0.01
driving	V_{DS}	-2.17821	-2.17823	0.00
sources	P_{LO}	2.76412	2.76412	0.00
	P_{RF}	-0.05401	-0.05392	0.16

TABLE IV (continued)

NUMERICAL VERIFICATION OF SENSITIVITIES OF THE MIXER

Location of Variables	Variable	Exact Sensitivity	Numerical Sensitivity	Difference (%)
termina-tions	$R_g(f_{LO})$	0.06671	0.06657	0.22
	$X_g(f_{LO})$	0.37855	0.37854	0.00
	$R_g(f_{RF})$	0.78812	0.78798	0.02
	$X_g(f_{RF})$	0.45120	0.45119	0.00
	$R_d(f_{IF})$	0.71451	0.71436	0.02
	$X_d(f_{IF})$	0.10886	0.10871	0.14

* Nonlinear elements are characterized by

$$C_{gs}(v_1) = C_{gs0} \; / \; \sqrt{1-v_1/V_\phi},$$

$$R_i(v_1)C_{gs}(v_1) = \tau$$

and the function for $i_m(v_1, v_2)$ is shown in Fig. 5, whose mathematical expression is consistent with [6]. V_ϕ, V_{p0}, V_{dss} and I_{dsp} are parameters in the function $i_m(v_1, v_2)$.

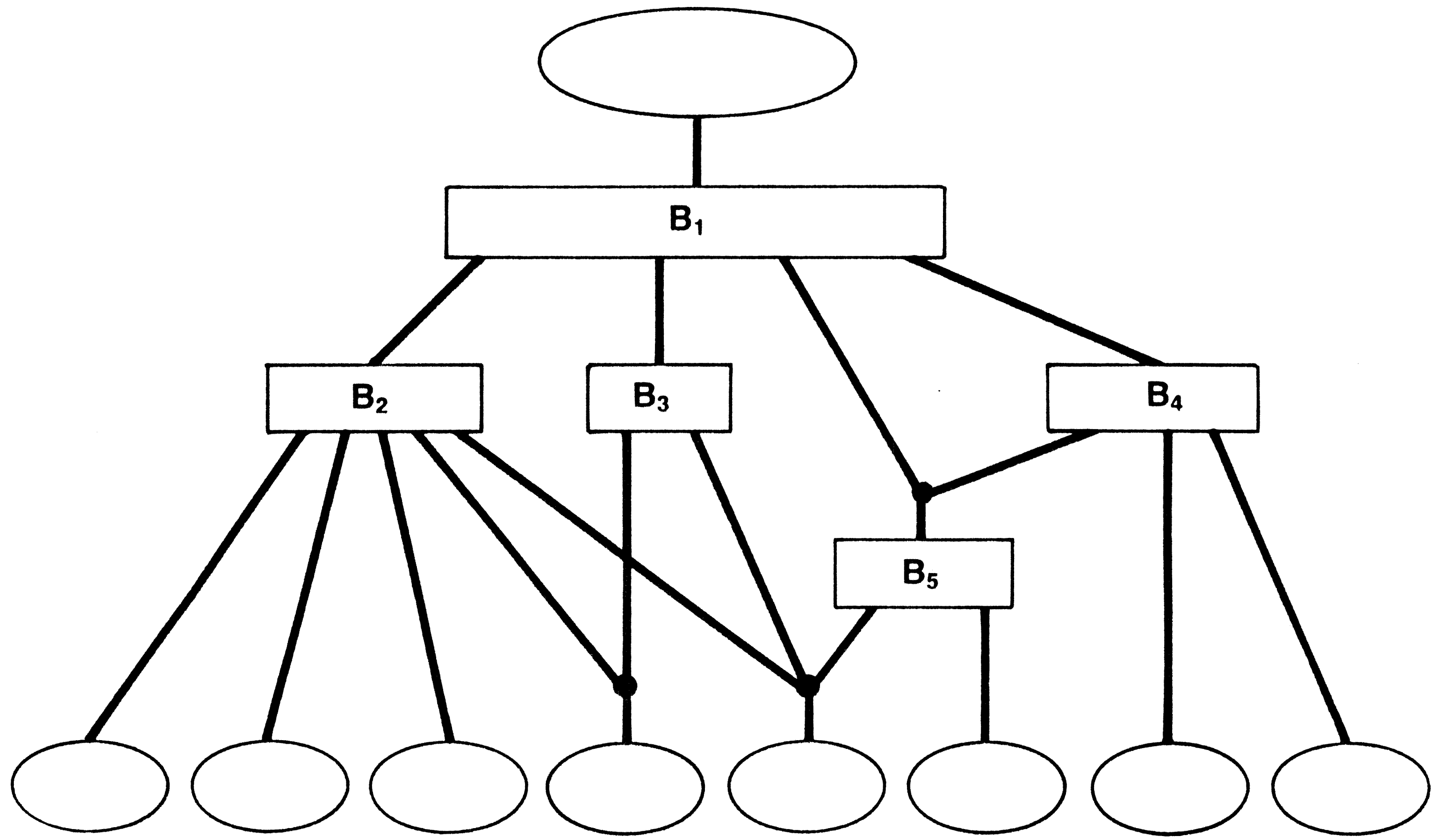
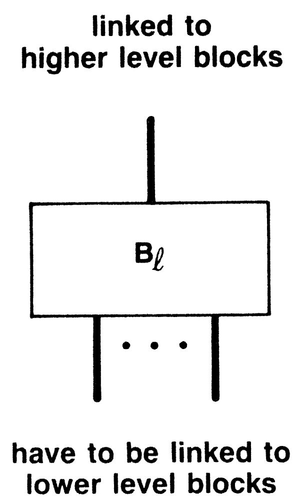
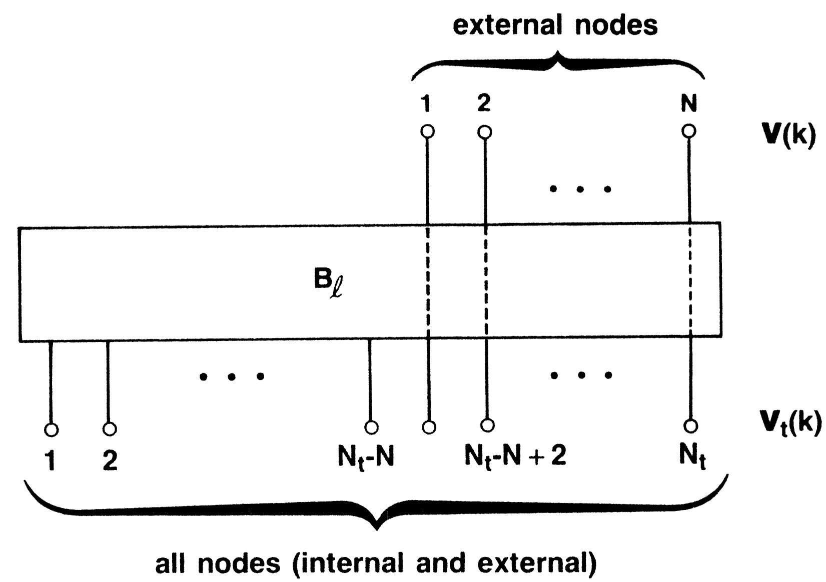


Fig. 1 An arbitrary circuit hierarchy. Each thick line represents a group of nodes. Each rectangular box represents a connection block for a subcircuit. Each bottom circular box represents a circuit element and the top circular box represents the sources and loads.



(a)



(b)

Fig. 2 A typical subcircuit connection block: (a) as seen from Fig. 1, (b) detailed representation of all the nodes of the subnetwork. Nodes at the top (bottom) of the rectangular box are the external (external and internal) nodes of the subnetwork.

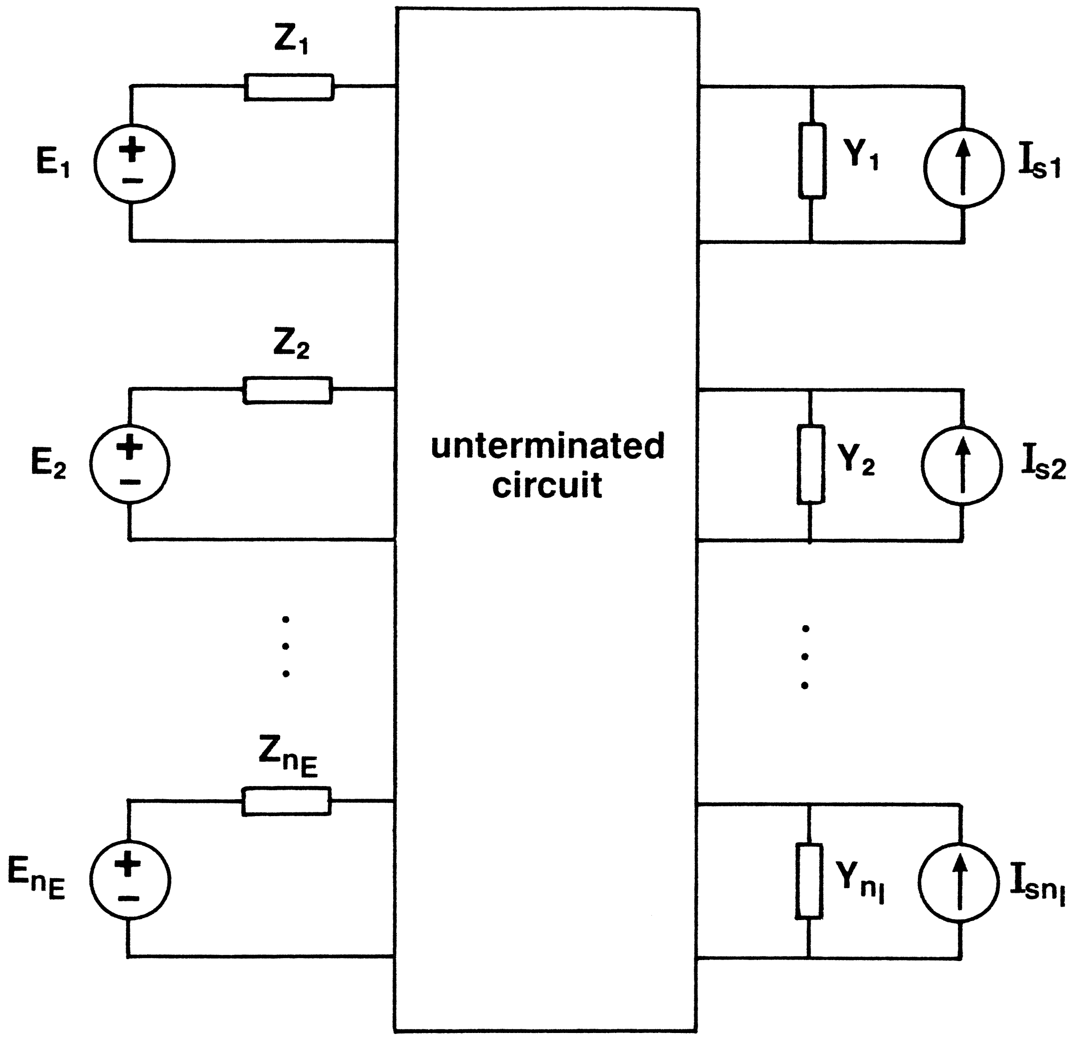


Fig. 3 A representation of a terminated subnetwork. Both current and voltage sources can be accommodated.

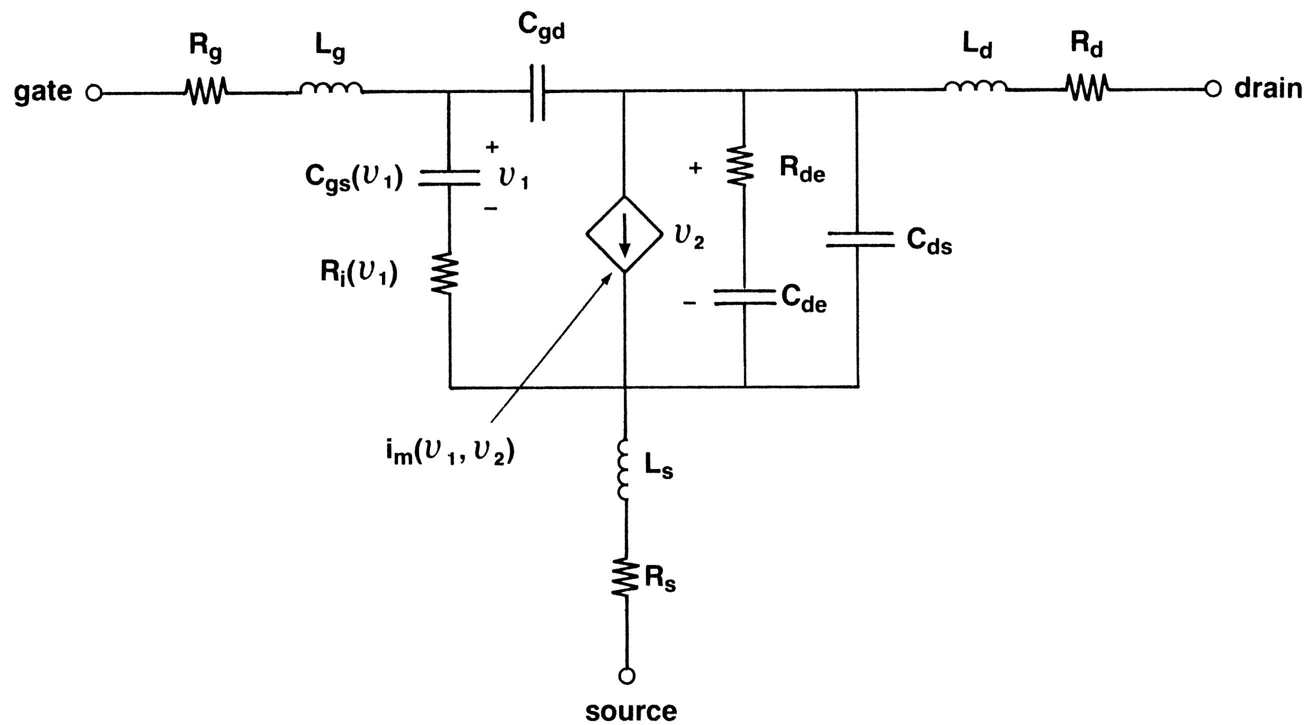


Fig. 4 A large signal MESFET model. All parameter values are consistent with [6].

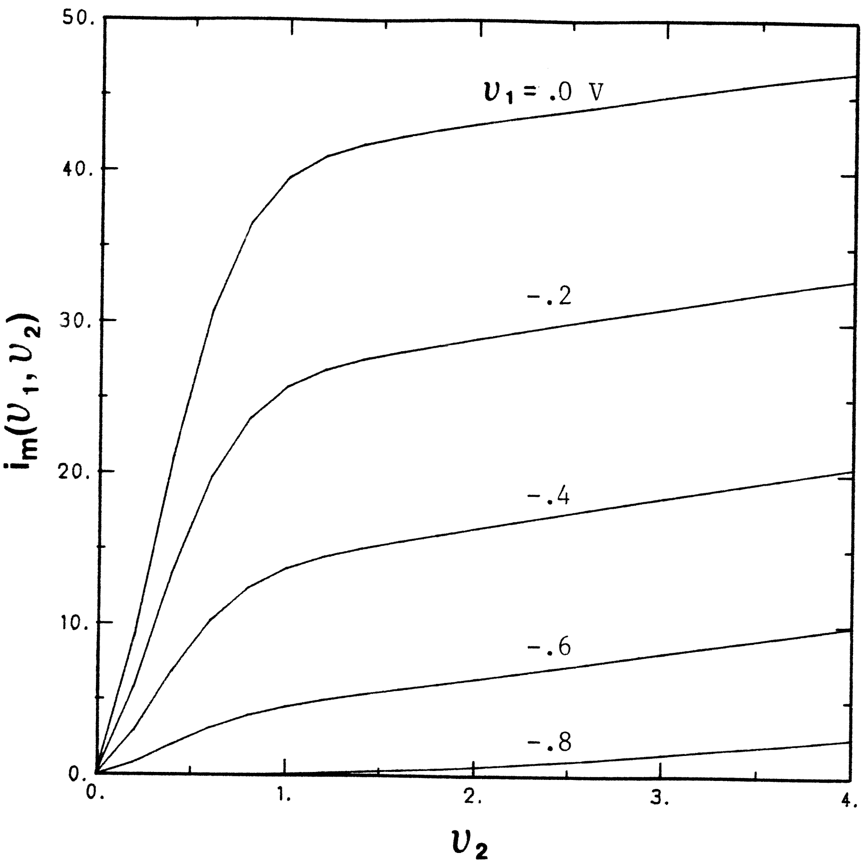


Fig. 5 The DC characteristics of the MESFET model.

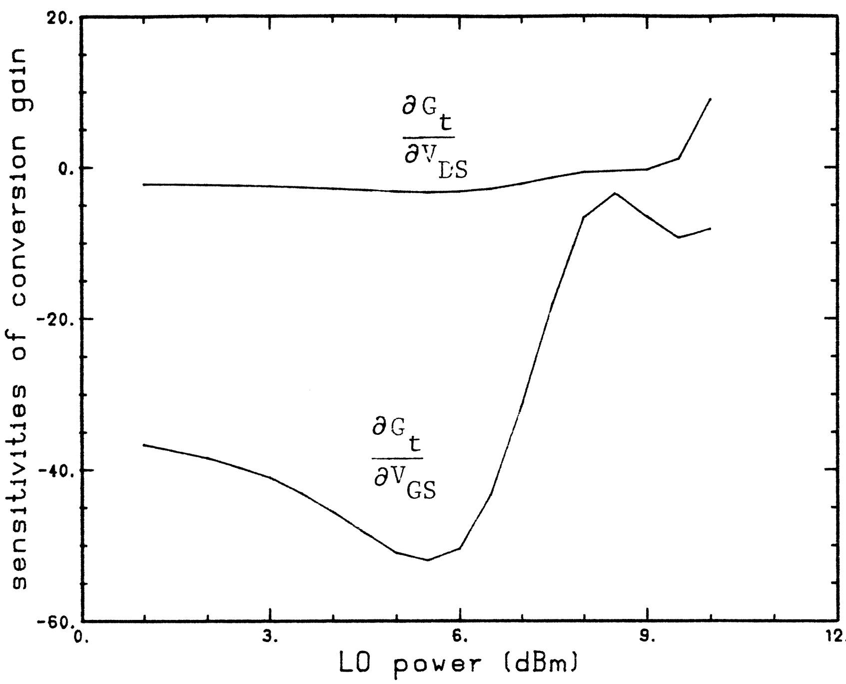


Fig. 6 Sensitivities of conversion gain w.r.t. bias voltages as functions of LO power.