DESIGN CENTERING USING TOLCAD,

PART I: REPORT

OSA-86-TC-8-R

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INDEX

ABSTRACT

This report describes an introduction of the fixed tolerance problem (FTP) option into an existing software package (COMPSHT) for computerized optimization of microwave passive and active circuits. The new software package (TOLCAD) implements five formulations of the FTP, all using an expedient least squares objective function within the restrictions of COMPSHT. For each of the FTP formulations quadratic approximation of the overall least squares objective function is available. The FTP definition is effected via the circuit data file, where the toleranced parameters are specified, and via run-time interaction with the user.

Part I of this report presents the FTP formulations, introduces the concept of quadratic approximation and contrasts the gradient requirements of TOLCAD to those of COMPSHT. An outline of new data file features is given. TOLCAD restrictions and limitations are also described.

A 6-element LC impedance matching circuit is used as a test circuit throughout the course of this report. Sample runs and post-analysis of results are made to illustrate the validity of the FTP design approach. Various other runs are made to illustrate the flexibility permitted in the data file specification of toleranced and/or variable parameters.

Another section is used to present results from a test program used to examine the premise that a Monte Carlo design may be performed by using a few randomly selected sample frequency points in the optimization process. The last section in this part of the report is used for discussion and suggestions.

Part II of this report assembles all individual graphs and associated data of Monte Carlo and vertex analyses mentioned in Part I.

Part III of this report lists all program source codes which were changed or added in the process of creating the current version of TOLCAD from COMPSHT.

INTRODUCTION

The original purpose of the project described in this report was to introduce the fixed tolerance problem (FTP) into an existing software package (COMPSHT) for computerized optimization of microwave passive and active circuits. The package under consideration, namely TOLCAD, performs the following main functions: (1) circuit analysis, (2) sensitivity analysis, (3) optimization (conventional nominal design), (4) sweep option, (5) analyses for various source and load elements, (6) variable analysis, (7) Monte Carlo analysis, (13) analysis and optimization and (44) random grid search. As a result of this work a new option has been added, namely (8), the fixed tolerance problem.

It is impractical to manufacture circuits using precise parameter values obtained from the process of nominal design. Circuit parameter values are subject to random fluctuations from known nominal values. Therefore, the performance of a manufactured circuit may differ from the one predicted by nominal design and quite often a performance specification violation may result.

If a procedure known as design centering is performed it may lead to yield improvement in the presence of tolerances by moving the original nominal design to a point where it is "better centered" w.r.t. constraints resulting from design specifications.

It is assumed that the fixed tolerance problem (FTP) will usually be solved after an acceptable nominal design has been achieved by traditional CAD. Therefore, all the information contained in the output (results) data file should be accessible to the package when solving the FTP.

The TOLCAD package solves the nominal design problem via a least squares objective function of a fixed format. It was determined that additional software created to solve the FTP should be minimally intrusive to the existing software (COMPSHT) and should utilize the existing subroutines, variables and arrays already available. Therefore, formulations for the FTP in TOLCAD have been developed to make use of a sum of the sum of squares type of objective function.

Apart from the TOLCAD program itself, a smaller test program was developed to emulate TOLCAD for specific problems. It is easier to modify this smaller program when testing new FTP formulations. In this report the test program was used to examine the premise that a very good Monte Carlo design (yield optimization) may be performed using a few randomly selected sample frequency points from the range of interest. Results from the modified program can, therefore, be used to decide on the desirability of further development of TOLCAD itself.

FIXED TOLERANCE PROBLEM: AN OPTION OF TOLCAD

The complete set of functions that TOLCAD can perform is

AN(1) - analysis

SEN(2) - sensitivity analysis

OPT(3) - optimization (conventional nominal design)

SW(4) - sweep

MAP(5) - analysis for various source and load elements

VAR(6) - variable analysis

MC(7) - Monte Carlo analysis

TOL(8) - fixed tolerance problem (NEW FEATURE)

ANOPT(13) - analysis and optimization

RND(44) - random grid search.

We consider the problem of designing a circuit from components whose values are known only to certain tolerances. Let x^0 denote the nominal value of the parameter x and t the relative tolerance, so that the actual value of the parameter x lies in a known tolerance region which, for one variable, is the interval $[x^0-tx^0, x^0+tx^0]$. Analogously, we may consider the case where x^0 denotes the nominal value of the parameter x and ε the absolute tolerance, so that the actual value of the parameter x lies in a known tolerance region $[x^0-\varepsilon, x^0+\varepsilon]$. The extreme points of the interval are called vertices. In general, for an n-parameter design, the tolerance region has 2^n vertices. A solution to the FTP should give a nominal point for which the yield, defined as

$$Y = \frac{\text{number of outcomes which meet specifications}}{\text{total number of outcomes}},$$

is greater than the yield calculated at the solution to the nominal design problem.

FIXED TOLERANCE PROBLEM FORMULATIONS

If the user selects TOLCAD option TOL(8), the fixed tolerance problem will be considered and the following options are available:

(1) fixed tolerance problem with all vertices of the tolerance region (Subroutine FTPAQ);

(2) fixed tolerance problem with specified number of worst vertices to be considered (Subroutine FTPBQ);

(3) fixed tolerance problem with a threshold value on the objective function characterizing a given vertex (Subroutine FTPCQ);

(4) fixed tolerance problem with weighted vertices of the tolerance region plus weighted nominal point (Subroutine FTPDQ);

(5) Monte Carlo design which is a fixed tolerance problem where a specified number of uniformly distributed random outcomes within the tolerance region are considered (Subroutine FTPEQ).

<u>Subroutine FTPAQ</u>

The subroutine FTPAQ is designed to solve the FTP using all vertices of the tolerance region.

The problem is formulated using a least squares objective function defined as

$$EF_{tol} = \sum_{v=1}^{NVERT} EF_v \ , \tag{1}$$

where

EF_{tol} is the overall objective function

NVERT is the number of vertices of the tolerance region

EF_v is the function corresponding to vertex v.

The summation is performed outside Subroutine ANALYZ, which is called NVERT times from the subroutine FTPAQ to evaluate EF_{tol}. With the print option "IR" or "OR" the error function generated by the subroutine ANALYZ is of the form (see [1] p.9 and p.23)

$$EF = \frac{1}{NF} \sum_{FI}^{FF} W_1 \mid S_{ii} \mid^2 + W_2 \mid VSWR_i \mid^2 +$$

$$+ W_3 \mid RLG_A - RLG_D \mid^2 + W_5 \mid PAD \mid^2,$$

where

- NF is the number of sample frequency points
- FI is the first sample frequency
- FF is the last sample frequency
- i is 1 for the "IR" and 2 for the "OR" option
- RLG_A is the actual dB reflection gain
- RLG_D is the desired dB reflection gain
- PAD is the phase angle difference of S_{ii} at FI and FF.

Subroutine FTPBQ

The subroutine FTPBQ is designed to solve the FTP using an apriori specified number of worst vertices selected from all vertices at the point corresponding to the optimal nominal design. The selection is based on the value of EF_V characterizing a given vertex (or circuit). At the starting point Subroutine ANALYZ is called from the subroutine FTPBQ for all vertices of the tolerance region and the error function values corresponding to all vertices are stored in the array FERR(I), I=1,...,NVERT. Then all components of the vector FERR(I) are ordered in descending order (according to their value) and vector NRF(I), I=1,...,NVERT, contains the original indices (vertex numbers) of the ordered functions. A limited number (or all) of the worst vertices (NRANK is the number decided by the user) is used in the subsequent iterations of optimization.

The objective function for the problem is defined as

$$EF_{tol} = \sum_{I=1}^{NRANK} FERR(I). \tag{2}$$

The summation is performed for the top NRANK functions characterizing the worst vertices (worst circuits) of the tolerance region. The initial selection of vertices remains constant throughout the optimization.

Subroutine FTPCQ

The subroutine FTPCQ is designed to solve the FTP using a threshold value of the function EF_v characterizing a given vertex. Only functions with values greater than the threshold contribute to the overall objective function. The user decides on the value of the threshold by specifying the variable TRES. The objective function for the problem is defined as

$$EF_{tol} = \sum_{v=1}^{NVERT} (EF_v - TRES), \tag{3}$$

where

$$EF_v - TRES = \begin{cases} 0 \text{ if } EF_v \leqslant TRES \\ EF_v - TRES \text{ if } EF_v \geqslant TRES. \end{cases} \tag{4}$$

Subroutine FTPDQ

The subroutine FTPDQ is designed to solve the FTP using all vertices of the tolerance region and the nominal point. The functions characterizing the vertices and the nominal point are weighted appropriately to emphasize the relative contribution of the vertices and the nominal point to the overall objective function of the FTP.

The objective function for the FTP is defined as

$$EF_{tol} = \sum_{v=1}^{NVERT+1} W_v{*}EF_v, \tag{5}$$

where v=NVERT+1 denotes the nominal point. It is felt that the nominal point should be weighted more heavily than the vertices since the solution to the FTP should not drift far away from the solution to the solution to the nominal design problem. In the existing TOLCAD version, the following weighting factors are used: for v = 1 to NVERT $W_v = 1$ and $W_{v=NVERT+1}=NVERT$.

Subroutine FTPEQ

In order to perform a Monte Carlo design it is desired to consider parameter outcomes having values distributed within the assumed fixed tolerance range rather than at the strictly extreme values given by vertices. Letting x denote the actual outcome of a toleranced parameter we may write

$x = x_0 + \mu t x_0$ for relative tolerances

or

$x = x_0 + \mu\varepsilon$ for absolute tolerances

where

- x_0 denotes the nominal parameter value
- t denotes the relative tolerance
- ε denotes the absolute tolerance
- μ denotes the actual outcome which may have any value on the interval [-1.0, +1.0].

A user specified number of random outcomes NRAND is used to determine the number of uniformly distributed points randomly selected from the set of possible outcomes of μ taken from the interval [-1.0, +1.0] for each parameter x. Hence, a Monte Carlo design is performed by considering these random outcomes throughout the optimization.

The objective function for the problem is defined as

$$EF_{tol} = \sum_{r=1}^{NRAND} EF_r , \qquad (6)$$

where

NRAND	is the user specified number of random outcomes
EF_{tol}	is the overall objective function
EF_r	is the function corresponding to random outcome r

The summation is performed outside the subroutine ANALYZ, which is called NRAND times from the subroutine FTPEQ to evaluate EF_{tol}.

<u>Reference</u>

[1] COMPACT, Version 5.1, User's Manual.

QUADRATIC APPROXIMATION

Worst case and Monte Carlo designs for fixed tolerance problems are inherently computationally intensive. Unlike nominal designs, which only consider one point in the parameter space, these problems consider vertices or random outcomes in the parameter space and circuit simulations must be performed for all of these points. In TOLCAD, this problem is compounded by the fact that gradients are obtained by performing numerical perturbations on the variables.

An attempt has been made to expedite the FTP by using a quadratic polynomial to approximate the error function computed by Subroutine ANALYZ for a region of interest in the parameter space. Gradients computed by Subroutine ANALYZ are approximated by the gradients of the quadratic polynomial. The quadratic polynomial coefficients are established by TOLCAD at the design starting point by calling Subroutine ANALYZ a number of times and solving a set of simultaneous linear equations. Since it is the error function rather than the gradients computed by Subroutine ANALYZ which are being modelled, the gradient computations by Subroutine ANALYZ have been disabled during this approximation process in order to further expedite this process.

Once the quadratic model has been established, TOLCAD proceeds with the FTP design as described in the FIXED TOLERANCE PROBLEM FORMULATIONS section of this report except that instead of calling Subroutine ANALYZ, a quadratic polynomial evaluation routine is called.

Quadratic approximation is available as a run-time option in TOLCAD. Once the user selects TOLCAD option TOL(8), the program will issue the following prompt:

```
DO YOU WANT TO USE QUADRATIC APPROXIMATION FOR THE
FIXED TOLERANCE PROBLEM  (1/0 FOR YES/NO) ?
```

If the user answers affirmatively, TOLCAD will proceed to establish the quadratic model and when completed, the polynomial coefficients will be displayed on the screen. A message will also be issued to warn the user that since this model is not automatically updated through the course of an FTP design, a limited number of iterations are advised. The user should stop the optimization after these few iterations and then save the data to be able to re-start the design process, thereby updating the quadratic model, if desired.

GRADIENT CALCULATIONS IN TOLCAD

Gradients w.r.t. optimization parameters are calculated in Subroutine ANALYZ by perturbations using a relative perturbation factor σ. The subroutine SEARCH uses transformed optimization variables, so the gradients supplied to SEARCH should also be w.r.t. transformed variables. This section explains what ANALYZ computes in terms of gradients and how Subroutine SEARCH is supplied with the required gradients for the cases of nominal design and of fixed tolerance problems with either relative or absolute tolerances specified. The discussion is presented for one variable for simplicity. The extension to n variables is obvious.

Let

x denote the toleranced, untransformed circuit parameter variable

x^0 denote the nominal, untransformed circuit parameter variable

y denote the toleranced, transformed circuit parameter variable, where the transformation is: $y = \ln x$

y^0 denote the nominal, transformed circuit parameter variable, where the transformation is: $y^0 = \ln x^0$

Of course, the reverse transformation gives

$$x = e^y \quad \text{or} \quad x^0 = e^{y^0}.$$

Gradients for Nominal Design

Given a nominal untransformed circuit parameter varible x^0, Subroutine ANALYZ is known to return a numerically computed gradient GG which is defined as

$$GG = \frac{f(x^0+\sigma x^0) - f(x^0)}{\sigma}.$$

Since the numerically computed gradient g_x0 of function $f(x^0)$ w.r.t. x^0 is

$$g_x0 = \frac{df(x^0)}{dx^0} = \frac{f(x^0+\sigma x^0) - f(x^0)}{\sigma x^0}$$

$$= \frac{GG}{x^0}$$

and since Subroutine SEARCH requires the numerically computed gradient g_y0 of function $f(y^0)$ w.r.t. to the nominal transformed circuit parameter variable y^0, we may write

$$g_y0 = \frac{df(y^0)}{dy^0} = \frac{df(x^0)}{dx^0} \cdot \frac{dx^0}{dy^0} = g_x0 \cdot \frac{dx^0}{dy^0}$$

$$= \frac{GG}{x^0} \cdot x^0$$

$$= GG.$$

Hence, for nominal parameter values, the gradient returned from Subroutine ANALYZ may be transferred directly to Subroutine SEARCH.

<u>Gradients for Fixed Tolerance Problems</u>

Given a toleranced untransformed circuit parameter variable x, Subroutine ANALYZ is known to return a numerically computed gradient GG, which is defined as

$$GG = \frac{f(x+\sigma x) - f(x)}{\sigma}.$$

Since the numerically computed gradient g_x of function $f(x)$ w.r.t. x is

$$g_x = \frac{df(x)}{dx} = \frac{f(x+\sigma x) - f(x)}{\sigma x}$$

$$= \frac{GG}{x}$$

and since Subroutine SEARCH requires the numerically computed gradient $g_{y}0$ of function $f(y^0)$ w.r.t. to the nominal transformed circuit parameter variable y^0, we may write

$$g_{y}0 = \frac{df(y^0)}{dy^0} = \frac{df(x)}{dx} \cdot \frac{dx}{dx^0} \cdot \frac{dx^0}{dy^0}$$

$$= g_x \cdot \frac{dx}{dx^0} \cdot x^0$$

$$= \frac{GG}{x} \cdot \frac{dx}{dx^0} \cdot x^0.$$

For relative tolerances we have

$$x = x^0 + \mu t x^0$$

and

$$\frac{dx}{dx^0} = (1+\mu t).$$

Therefore,

$$g_{y}0 = \frac{GG}{x} \cdot \frac{dx}{dx^0} \cdot x^0$$

$$= \frac{GG}{x} \cdot (1+\mu t) \cdot x^0 = \frac{GG}{x} \cdot x$$

$$= GG.$$

Hence, for parameters with relative tolerances, the gradient returned from Subroutine ANALYZ may be transferred directly to Subroutine SEARCH.

For absolute tolerances we have

$$x = x^0 + \mu\varepsilon$$

and

$$\frac{dx}{dx^0} = 1.$$

Therefore,

$$g_y{}^0 = \frac{GG}{x} \cdot \frac{dx}{dx^0} \cdot x^0$$

$$= \frac{GG}{x} \cdot 1 \cdot x^0$$

$$= GG \cdot \frac{x^0}{x}.$$

Hence, for parameters with absolute tolerances, the gradient returned from Subroutine ANALYZ must be multiplied by the factor x^0/x before being transferred to Subroutine SEARCH.

THE FTP DATA FILE

The data file is used to specify which circuit parameters are toleranced and the value of the tolerances. Lower and upper tolerances should be specified as bracketed pairs following a model parameter value in the data file. There must be a third item in the parentheses and it must be either "%T" or "T" to denote relative or absolute tolerances, respectively.

For example,

CAP AA SE 10.0 (10,20,%T)

specifies a relative lower tolerance of 10% and a relative upper tolerance of 20%. This capacitor, therefore, has a lower value of 9.0 or an upper value of 12.0 under worst case conditions. Consider another example, where

IND AA SE 10.0 (4,3,T)

specifies an absolute lower tolerance of 4.0 and an absolute upper tolerance of 3.0. This inductor, therefore, has a lower value of 6.0 or an upper value of 13.0 under worst case conditions.

Since TOLCAD considers independently the concept of a design parameter and the concept of a toleranced parameter, the following flexibility is permitted.

RES AA SE 10.0 (5,5,%T)	to denote a toleranced fixed parameter
RES AA SE -10.0 (5,5,%T)	to denote a toleranced variable
RES AA SE -10.0	to denote an untoleranced variable
RES AA SE 10.0	to denote an untoleranced fixed parameter

A few other points should be made with respect to the TOLCAD interaction with the data file.

Even though the data file is used to specify tolerances, if the user chooses a TOLCAD option other that TOL(8), which is the fixed tolerance problem, the tolerance information will simply be ignored.

TOLCAD has an interactive feature which permits the user to save a design solution in a data file. Unfortunately, for the time being, the tolerance information is not written to a data file. The implications to the user is that the data file will have to be manually edited to reinstate the tolerance information.

Another restriction on a toleranced parameter is that it may currently neither be a candidate for constrained optimization nor for Monte Carlo analysis, two options otherwise available in TOLCAD.

The last restriction is that a maximum of 6 toleranced parameters are allowed. If more than 6 are specified in the data file, the program will issue an appropriate message. This limitation is due to the existing array dimensions in TOLCAD.

RESTRICTIONS AND LIMITATIONS OF TOLCAD

The purpose of the project described in this report was to investigate the feasibility of using a least squares type of objective function for the fixed tolerance problem. Therefore, the software developed and integrated with the existing package for computerized optimization of microwave circuits is experimental in nature. This results in certain restrictions and limitations of which the user should be aware in order to avoid problems when using TOLCAD. The following restrictions apply when this version of TOLCAD is run in the tolerance mode of operation (option 8).

Limitation due to Array Dimensions

15 is the maximum number of variable parameters allowed

6 is the maximum number of toleranced parameters allowed

64 is the maximum number of random outcomes allowed to be specified for fixed tolerance problems using Monte Carlo design.

If any of these limits are exceeded, TOLCAD will issue appropriate messages.

Limitations of Quadratic Approximation

There is currently a bug in the quadratic approximation routine when approximation is attempted in one dimension only (i.e., if only one parameter is toleranced). A message will be issued if this is the case and the program will stop.

Limitations on Toleranced Parameters

A toleranced parameter may not be a candidate for constrained optimization or for Monte Carlo analysis, two options which are otherwise available in TOLCAD.

Limitations with Respect to the Data File

TOLCAD has an interactive feature which permits the user to save a design solution in a data file. At this time being, the tolerance information is not written to a data file.

TEST CIRCUIT

A 6-element LC matching circuit (Fig. 1) to match a 1 ohm load to a 3 ohm generator over the frequency range 0.0796 - 0.1876 MHz has been used as a test problem.

The data file shown in Fig. 2 was used as an input data file for the tolerance problem defined for this LC matching circuit considered. All 6 parameters are variables with 5% relative tolerances. Twenty-one sample frequency points are specified. In Fig. 3 we show a typical run of TOLCAD using option TOL(8) and FTPA. In Fig. 4 we show a TOLCAD run using the same options TOL(8) and FTPA, but also using the quadratic approximation option.

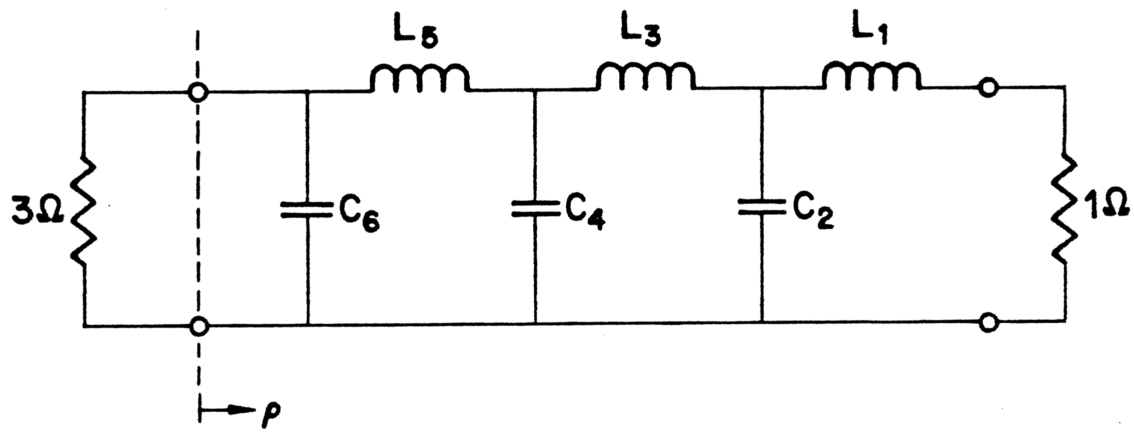


Fig. 1 Six element LC matching circuit used as a test problem for the FTP.

```
* P6F21 DATA FILE:
*
CAP AA SE -.32285E+06 (5,5,%T)
IND DD SE -2896.4 (5,5,%T)
CAP BB SE -.7609E+06 (5,5,%T)
IND EE SE -2281.2 (5,5,%T)
CAP CC SE -.96591E+06 (5,5,%T)
IND FF SE -967.59 (5,5,%T)
CON AA T2 1 0
CON BB T2 2 0
CON CC T2 3 0
CON DD T2 1 2
CON EE T2 2 3
CON FF T2 3 4
DEF AA T2 1 4
PRI AA IR 3 1
END
0.0795774 0.1876436 0.00540331
END
0.000001
1 0 0 0
END
```

Fig. 2 Data file for the FTP. Starting values correspond to the conventional optimal nominal design with zero tolerances.

```
TEST CIRCUIT-4

  'FILE NAME' OR 'QUIT'
? P6F21

  AN(1),SEN(2),OPT(3),SW(4),MAP(5),VAR(6),MC(7),TOL(8),ANOPT(13),RND(44)
? 8

  DO YOU WANT TO USE QUADRATIC APPROXIMATION FOR THE
  FIXED TOLERANCE PROBLEM (1/0 FOR YES/NO) ?
? 0

  SELECT A VERSION OF THE FTP :
      1 - FTP WITH ALL VERTICES OF THE TOLERANCE REGION
      2 - FTP WITH SPECIFIED NUMBER OF WORST VERTICES
      3 - FTP WITH A THRESHOLD VALUE ON THE OBJECTIVE FUNCTION
      4 - FTP WITH WEIGHTED VERTICES PLUS WEIGHTED NOMINAL
      5 - FTP WITH MONTE-CARLO DESIGN
? 1

  OPTIMIZATION BEGINS WITH FOLLOWING VARIABLES AND GRADIENTS

       VARIABLES                     GRADIENTS
    ( 1):  .32285E+06             ( 1):  .50970E-01
    ( 2):  2896.4                 ( 2):  .50692
    ( 3):  .76090E+06             ( 3):  .81277
    ( 4):  2281.2                 ( 4):  .82049
    ( 5):  .96591E+06             ( 5):  .52214
    ( 6):  967.59                 ( 6):  .45962E-01
   ERR. F.=       .6854711
          ____****____
  HOW MANY ITERATIONS BEFORE NEXT STOP ('0' RESULTS IN FINAL ANALYSIS.)
  WANT INTERMEDIATE PRINTS (YES=1,NO=0). TYPE TWO NUMBERS; I,J
? 30,1

    ( 1):  .32227E+06             ( 1):  .81235
    ( 2):  2845.2                 ( 2):  1.4415
    ( 3):  .73943E+06             ( 3): -1.6397
    ( 4):  2216.2                 ( 4): -1.6862
    ( 5):  .94831E+06             ( 5):  1.4016
    ( 6):  966.03                 ( 6):  .82774
   ERR. F.=       .6791969
          ____****____
    ( 1):  .31935E+06             ( 1):  .68504
    ( 2):  2758.8                 ( 2):  .27091
    ( 3):  .72927E+06             ( 3): -.49467
    ( 4):  2186.1                 ( 4): -.53509
    ( 5):  .91937E+06             ( 5):  .23469
    ( 6):  957.30                 ( 6):  .69999
   ERR. F.=       .6437066
          ____****____
```

Fig. 3 Printout of a typical run using option 8 of TOLCAD.

```
( 1):  .30120E+06           ( 1):  .51309E-01
( 2):  2705.3               ( 2):  .26898
( 3):  .71056E+06           ( 3): -.45003
( 4):  2136.3               ( 4): -.15909
( 5):  .90358E+06           ( 5):  .46875
( 6):  901.98               ( 6): -.10043
ERR. F.=       .6138338
      ____****____
( 1):  .30083E+06           ( 1): -.29929E-01
( 2):  2706.9               ( 2):  .36102
( 3):  .71128E+06           ( 3): -.30181
( 4):  2133.1               ( 4): -.29767
( 5):  .90259E+06           ( 5):  .37145
( 6):  902.05               ( 6): -.32893E-01
ERR. F.=       .6135072
      ____****____
( 1):  .30118E+06           ( 1):  .32241E-02
( 2):  2687.2               ( 2):  .24784E-01
( 3):  .70786E+06           ( 3):  .28392E-01
( 4):  2128.0               ( 4):  .88148E-01
( 5):  .89212E+06           ( 5): -.66915E-01
( 6):  902.43               ( 6): -.36364E-01
ERR. F.=       .6111938
      ____****____
( 1):  .30099E+06           ( 1): -.14275E-01
( 2):  2680.5               ( 2): -.41071E-01
( 3):  .70836E+06           ( 3):  .74062E-01
( 4):  2125.5               ( 4):  .78681E-01
( 5):  .89339E+06           ( 5): -.42038E-01
( 6):  903.09               ( 6): -.16481E-01
ERR. F.=       .6110591
      ____****____
( 1):  .30084E+06           ( 1): -.13567E-01
( 2):  2678.8               ( 2): -.20231E-01
( 3):  .70782E+06           ( 3):  .46770E-01
( 4):  2123.5               ( 4):  .48047E-01
( 5):  .89297E+06           ( 5): -.18214E-01
( 6):  902.52               ( 6): -.14187E-01
ERR. F.=       .6110020
      ____****____
( 1):  .30047E+06           ( 1): -.56213E-02
( 2):  2672.7               ( 2):  .56359E-02
( 3):  .70598E+06           ( 3):  .12101E-02
( 4):  2118.1               ( 4): -.52169E-03
( 5):  .89099E+06           ( 5):  .75557E-02
( 6):  901.65               ( 6): -.39223E-02
ERR. F.=       .6109298
      ____****____
```

Fig. 3 (continued) Printout of a typical run using option 8 of TOLCAD.

TEST CIRCUIT-6

```
( 1):  .30052E+06          ( 1): -.66713E-03
( 2):  2670.9              ( 2):  .20723E-02
( 3):  .70563E+06          ( 3): -.14866E-02
( 4):  2116.5              ( 4): -.17602E-02
( 5):  .89034E+06          ( 5):  .32707E-02
( 6):  901.29              ( 6): -.12437E-02
ERR. F.=      .6109245
    ----****----
( 1):  .30052E+06          ( 1): -.66713E-03
( 2):  2670.9              ( 2):  .20723E-02
( 3):  .70563E+06          ( 3): -.14866E-02
( 4):  2116.5              ( 4): -.17602E-02
( 5):  .89034E+06          ( 5):  .32707E-02
( 6):  901.29              ( 6): -.12437E-02
ERR. F.=      .6109245
    ----****----
GRADIENT TERMINATION WITH ABOVE VALUES. FINAL ANALYSIS FOLLOWS
```

.080	.157	88.7	1.37:1	-16.09	2.88	.92
.085	.120	81.6	1.27:1	-18.44	3.02	.73
.090	.084	74.5	1.18:1	-21.48	3.10	.51
.096	.052	67.3	1.11:1	-25.76	3.11	.30
.101	.022	60.1	1.05:1	-33.12	3.06	.12
.107	.003	-127.8	1.01:1	-49.46	2.99	-.02
.112	.024	-134.6	1.05:1	-32.28	2.90	-.10
.117	.040	-141.9	1.08:1	-27.87	2.81	-.14
.123	.051	-149.2	1.11:1	-25.77	2.74	-.14
.128	.057	-156.7	1.12:1	-24.82	2.70	-.12
.134	.058	-164.2	1.12:1	-24.68	2.68	-.09
.139	.054	-171.8	1.12:1	-25.28	2.69	-.04
.144	.046	-179.6	1.10:1	-26.69	2.73	-.00
.150	.034	172.5	1.07:1	-29.27	2.80	.03
.155	.020	164.4	1.04:1	-34.07	2.89	.03
.161	.004	156.5	1.01:1	-48.76	2.98	.01
.166	.013	-32.6	1.03:1	-38.04	3.06	-.04
.171	.027	-41.2	1.06:1	-31.40	3.12	-.11
.177	.037	-50.1	1.08:1	-28.57	3.14	-.18
.182	.041	-59.4	1.09:1	-27.71	3.12	-.22
.188	.035	-69.0	1.07:1	-29.00	3.07	-.20

```
 SAVE OPTIMIZED RESULTS (Y/N)
? N

 'FILE NAME' OR 'QUIT'
? QUIT
```

Fig. 3 (continued) Printout of a typical run using option 8 of TOLCAD.

```
  'FILE NAME' OR 'QUIT'
? P6F21

  AN(1),SEN(2),OPT(3),SW(4),MAP(5),VAR(6),MC(7),TOL(8),ANOPT(13),RND(44)
? 8

  DO YOU WANT TO USE QUADRATIC APPROXIMATION FOR THE
  FIXED TOLERANCE PROBLEM (1/0 FOR YES/NO) ?
? 1

     STEPS AROUND CENTER OF INTERPOLATION
        .16142500E+05
        .14482000E+03
        .38045000E+05
        .11406000E+03
        .48295500E+05
        .48379500E+02

     ER( 1)
     ------

  .16671634E-11    .72634365E-07    .16242800E-11    .18071026E-06
  .65262694E-12    .18531450E-06   -.55981852E-10   -.24263616E-11
 -.50545677E-10    .57703188E-12    .67555851E-10   -.58549878E-10
 -.16298458E-06   -.71222876E-10    .66889656E-07    .38489399E-09
 -.14848657E-11   -.60061678E-10   -.82094506E-10   -.27012343E-06
 -.56961398E-10    .42427075E-06    .17716135E-04   -.90473943E-06
 -.28829236E-03    .13164669E-06    .14262053E-03    .44859237E+00

  SELECT A VERSION OF THE FTP :
      1 - FTP WITH ALL VERTICES OF THE TOLERANCE REGION
      2 - FTP WITH SPECIFIED NUMBER OF WORST VERTICES
      3 - FTP WITH A THRESHOLD VALUE ON THE OBJECTIVE FUNCTION
      4 - FTP WITH WEIGHTED VERTICES PLUS WEIGHTED NOMINAL
      5 - FTP WITH MONTE-CARLO DESIGN
? 1

  THE OPTION YOU HAVE SELECTED REQUIRES SOME
  COMMENTS. SINCE THE QUADRATIC MODEL DEVELOPED EARLIER IN
  THIS RUN IS VALID ONLY FOR A FEW ITERATIONS IT IS RECOMMENDED
  THAT YOU RUN OPTIMIZATION ONLY FOR 5 ITERATIONS. IN ORDER
  TO CONTINUE OPTIMIZATION USING QUADRATIC APPROXIMATION QUIT
  AFTER 5 ITERATIONS, SAVE RESULTS AND RESTART THE PROCESS
```

Fig. 4 Printout of a typical quadratic approximation run using option 8 of TOLCAD.

```
  OPTIMIZATION BEGINS WITH FOLLOWING VARIABLES AND GRADIENTS

       VARIABLES                  GRADIENTS
    ( 1):  .32285E+06          ( 1):  .51809E-01
    ( 2):  2896.4              ( 2):  .19028
    ( 3):  .76090E+06          ( 3):  .29185
    ( 4):  2281.2              ( 4):  .30007
    ( 5):  .96591E+06          ( 5):  .20633
    ( 6):  967.59              ( 6):  .46910E-01
   ERR. F.=      .6896011
          ____****____
  HOW MANY ITERATIONS BEFORE NEXT STOP ('0' RESULTS IN FINAL ANALYSIS.)
  WANT INTERMEDIATE PRINTS (YES=1,NO=0). TYPE TWO NUMBERS; I,J
? 5,1

    ( 1):  .32250E+06          ( 1):  .23468
    ( 2):  2885.0              ( 2):  .40380
    ( 3):  .75632E+06          ( 3): -.32309
    ( 4):  2267.1              ( 4): -.34173
    ( 5):  .96180E+06          ( 5):  .40477
    ( 6):  966.65              ( 6):  .24401
   ERR. F.=      .6870689
          ____****____
    ( 1):  .32039E+06          ( 1):  .23383
    ( 2):  2839.7              ( 2): -.10712
    ( 3):  .75085E+06          ( 3):  .39251E-01
    ( 4):  2250.6              ( 4):  .95535E-02
    ( 5):  .94598E+06          ( 5): -.13806
    ( 6):  960.37              ( 6):  .25278
   ERR. F.=      .6816481
          ____****____
    ( 1):  .31325E+06          ( 1):  .67088E-01
    ( 2):  2819.7              ( 2): -.15270
    ( 3):  .74330E+06          ( 3): -.14727
    ( 4):  2232.5              ( 4):  .10849
    ( 5):  .94072E+06          ( 5):  .12860
    ( 6):  937.77              ( 6): -.27928E-01
   ERR. F.=      .6766563
          ____****____
    ( 1):  .31273E+06          ( 1):  .32634E-02
    ( 2):  2821.7              ( 2): -.78224E-02
    ( 3):  .74360E+06          ( 3): -.14217E-01
    ( 4):  2229.6              ( 4): -.19588E-01
    ( 5):  .93954E+06          ( 5):  .16195E-03
    ( 6):  936.79              ( 6):  .72154E-02
   ERR. F.=      .6763795
          ____****____
```

Fig. 4 (continued) Printout of a typical quadratic approximation run using option 8 of TOLCAD.

```
( 1):  .31276E+06          ( 1):  .68257E-02
( 2):  2822.2              ( 2): -.30358E-02
( 3):  .74352E+06          ( 3): -.17150E-01
( 4):  2229.9              ( 4): -.15327E-01
( 5):  .93943E+06          ( 5): -.42018E-02
( 6):  936.73              ( 6):  .34419E-02
ERR. F.=      .6763781
      ____****____
HOW MANY ITERATIONS BEFORE NEXT STOP ('0' RESULTS IN FINAL ANALYSIS.)
WANT INTERMEDIATE PRINTS (YES=1,NO=0). TYPE TWO NUMBERS; I,J
? 0,0

SEARCH INTERRUPTED, FINAL ANALYSIS FOLLOWS
```

.080	.134	83.2	1.31:1	-17.44	2.99	.81
.085	.097	75.6	1.21:1	-20.26	3.09	.59
.090	.062	67.9	1.13:1	-24.10	3.12	.36
.096	.031	60.0	1.06:1	-30.10	3.09	.17
.101	.004	47.6	1.01:1	-47.24	3.02	.02
.107	.018	-133.3	1.04:1	-34.99	2.93	-.08
.112	.035	-141.6	1.07:1	-29.19	2.84	-.12
.117	.046	-149.6	1.10:1	-26.71	2.77	-.13
.123	.052	-157.5	1.11:1	-25.65	2.72	-.11
.128	.053	-165.5	1.11:1	-25.54	2.71	-.07
.134	.048	-173.6	1.10:1	-26.29	2.72	-.03
.139	.040	178.1	1.08:1	-28.01	2.77	.01
.144	.027	169.7	1.06:1	-31.22	2.84	.03
.150	.013	160.8	1.03:1	-37.82	2.93	.02
.155	.003	-24.7	1.01:1	-51.79	3.01	-.01
.161	.017	-36.0	1.03:1	-35.45	3.08	-.06
.166	.028	-45.3	1.06:1	-31.14	3.12	-.12
.171	.032	-54.6	1.07:1	-29.80	3.11	-.16
.177	.028	-63.7	1.06:1	-31.20	3.07	-.15
.182	.010	-67.5	1.02:1	-40.39	3.02	-.05
.188	.026	88.0	1.05:1	-31.59	3.00	.16

```
SAVE OPTIMIZED RESULTS (Y/N)
? N

'FILE NAME' OR 'QUIT'
? QUIT
```

Fig. 4 (continued) Printout of a typical quadratic approximation run using option 8 of TOLCAD.

RESULTS PRODUCED BY TOLCAD AND POST ANALYSIS

It is important to examine the validity of the options included at this time in TOLCAD for the fixed tolerance problem. We do this by investigating the computed yield versus reflection coefficient specification achievable for each candidate design solution. The yield is calculated after a suitable Monte Carlo analysis. This Monte Carlo analysis and the graphical output are obtained by software separate from TOLCAD.

For the purpose of this comparison a relative tolerance of 5% of the nominal value was assumed for each element of the circuit under consideration. A Monte Carlo analysis is performed for each design using 1000 random outcomes with circuit parameters having actual values within the assumed fixed tolerance range. The random number generator used for this purpose generates uniformly distributed points. A yield is obtained by computing the percentage of the 1000 circuits that satisfy a particular reflection coefficient specification.

For Example, in Fig. 1a, representing the zero tolerance nominal solution obtainable by COMPSHT or TOLCAD using option OPT(3), we see that 78.1% of the possible outcomes are capable of satisfying a specification of 0.1525 on their reflection coefficient. A "normalized distribution" as shown in Fig. 1b is obtained by taking the derivative of the yield curve.

A similar analysis can be performed on circuits representing the vertices of the tolerance region, where instead of Monte Carlo generated circuits we use circuits whose parameter values correspond to vertices of the tolerance region. The results of the so-called "vertex analysis" are shown in Fig. 2a and Fig. 2b for the zero tolerance nominal design. This type of yield and distribution analysis is used as a basis for the comparison of different options of the fixed tolerance problem.

Figs. 3 and 4 show the results of Monte Carlo and vertex analysis, respectively, for the solution to the FTP obtained using all vertices of the tolerance region (option (1), Subroutine FTPAQ), and 21 sample frequency points. By comparing the distribution of "Monte Carlo outcomes" for the zero tolerance nominal design where the distribution spans the range of reflection coefficient values from 0.1010 to 0.2369, to that of the FTP solution using all vertices where the distribution spans the range from 0.1210 to 0.1924, we conclude that the least squares objective function approach to the fixed tolerance problem is valid.

Figs. 5 and 6 compare the results of Monte Carlo and vertex analyses, respectively, for the solution to the FTPA design obtained using 21, 7 or 4 sample frequency points.

Figs. 7 and 8 show the results of Monte Carlo and vertex analyses, respectively, for the solution to the FTP obtained with weighting factors of 1.0 for each of the 64 vertices and a weight of 64.0 for the nominal point (option (4), Subroutine FTPDQ). The three curves on each plot compare the designs obtained using 21, 7 or 4 sample frequency points.

Figs. 9 and 10 show the results of Monte Carlo and vertex analyses, respectively, for the solution to the FTP while performing Monte Carlo design using 50 random outcomes (option(5), Subroutine FTPEQ). The three curves on each plot compare the designs obtained using 21, 7 or 4 sample frequency points.

Two examples which use TOLCAD option TOL(8) with quadratic approximation are also reported. The first example performed FTP design while considering all vertices (Subroutine FTPAQ). The procedure of starting the problem, performing 5 iterations and saving the data was repeated three times. The result is displayed as one of the curves in Figs. 11 and 12. Analogously, a second example which uses quadratic approximation was performed while considering 50 random outcomes (Subroutine FTPEQ). The result is displayed as one of the curves in Figs. 13 and 14.

All plots are accompanied by data files showing the specific numerical values used to produce the plots. These data files are in Part II of this report.

It should be very clearly understood that the Monte Carlo and vertex analyses presented in this section are not options of TOLCAD. They were performed using specialized software independent of TOLCAD. It is possible, however, that future versions of TOLCAD could contain such analyses as additional options to the existing ones.

CPU Execution Times

The execution times (in seconds) on the CYBER 170/730 computer for different options of the fixed tolerance problem compared with the zero tolerance nominal design problem are as in Tables I and II.

TABLE I

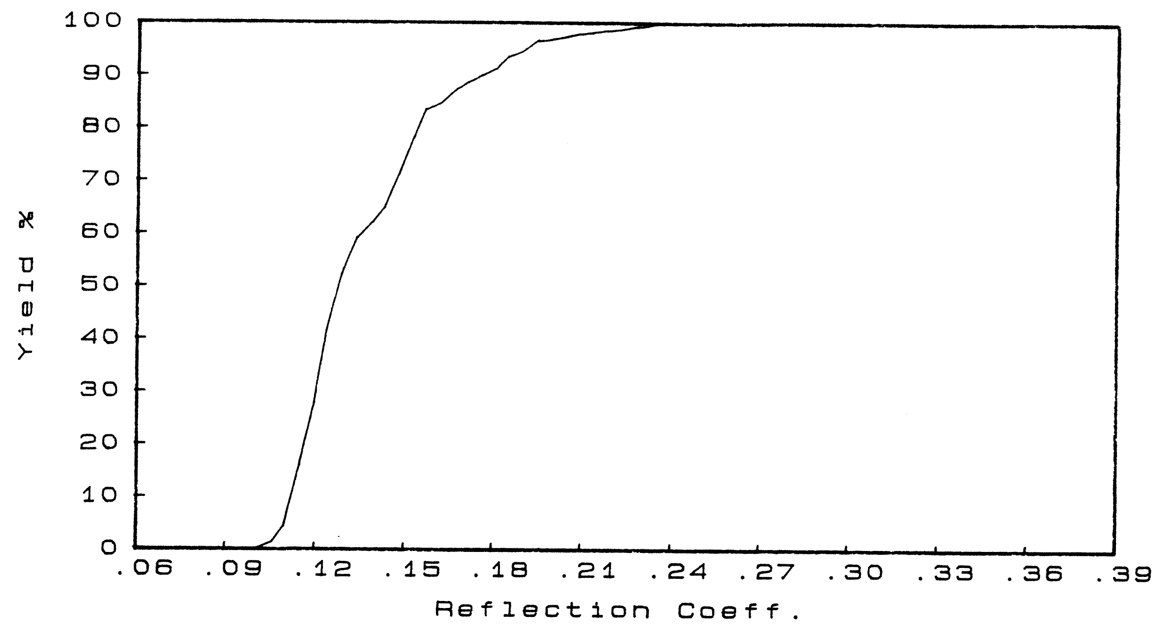
EXACT DESIGN WITH ALL 6PARAMETERS VARIABLE AND TOLERANCED (5%)

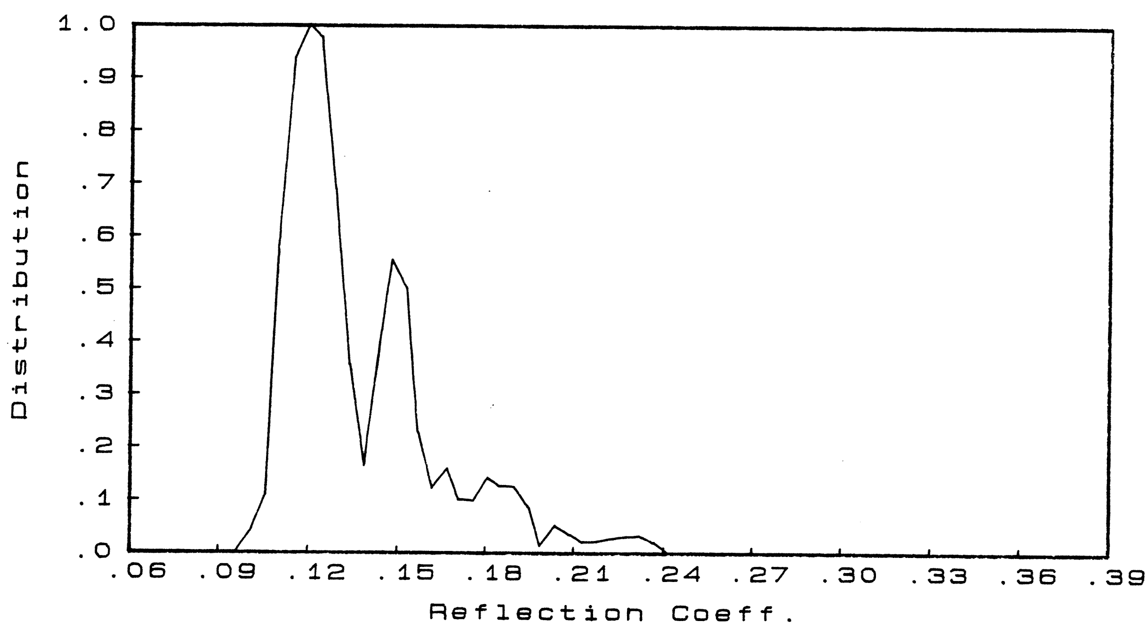
design option	number of sample frequency points for design		
	21	7	4
Zero tolerance nominal design	41	-	-
FTPAQ (N=64 vertices)	1367	491	246
FTPDQ (N=64 vertices + weighted nominal point)	1463	432	266
FTPEQ (R=50 random outcomes)	793	287	192

TABLE II

QUADRATICAPPROXIMATION DESIGN WITH ALL 6 PARAMETES VARIABLE AND TOLERANCED (5%) AND WITH 21 SAMPLE FREQUENCY POINTS

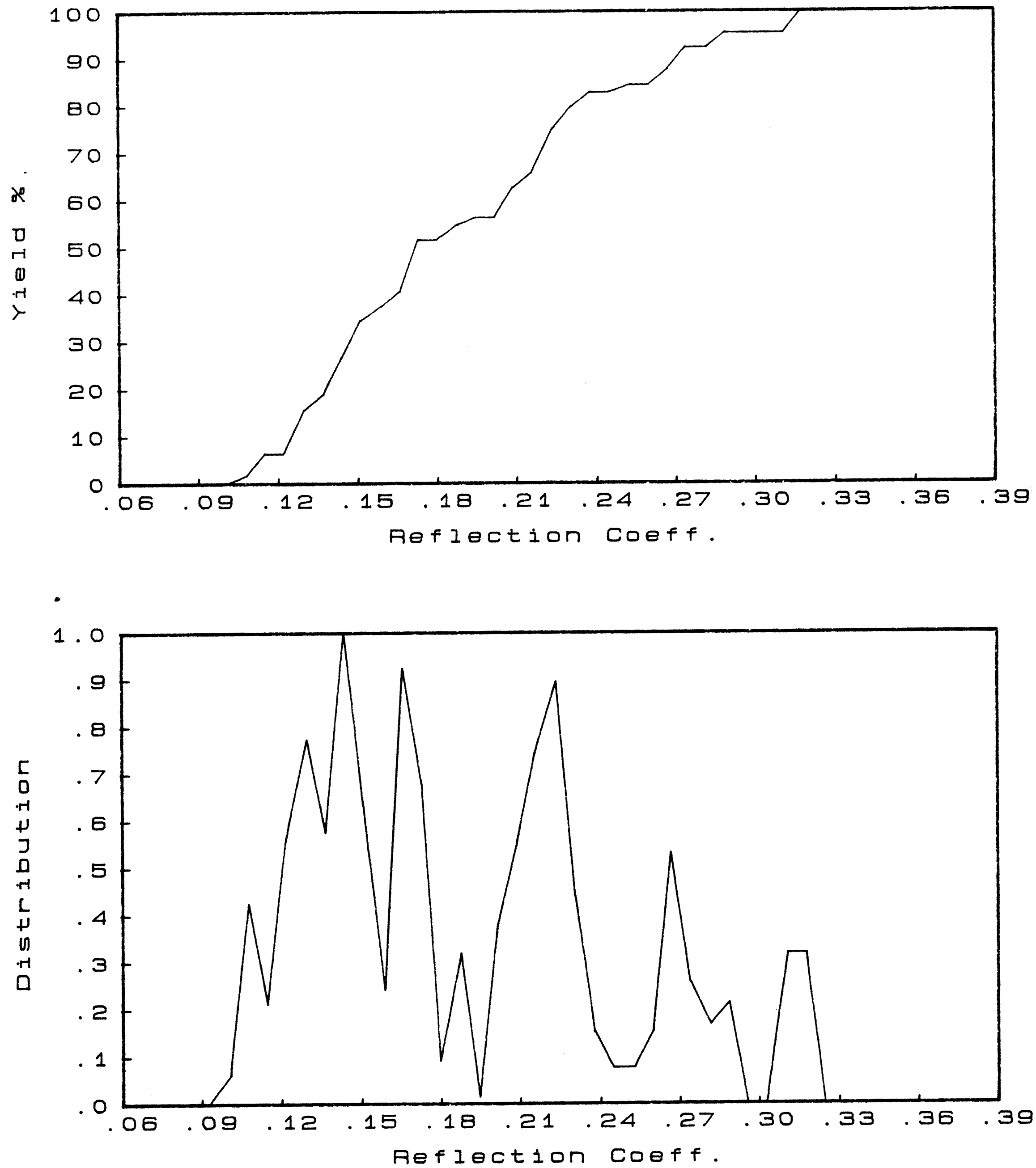
design option	Step 1	Step 2	Step 3
	(5 iter)	(5 iter)	(5 iter)
FTPAQ (N=64)	8	8	8
FTPEQ (R=50)	7	7	7





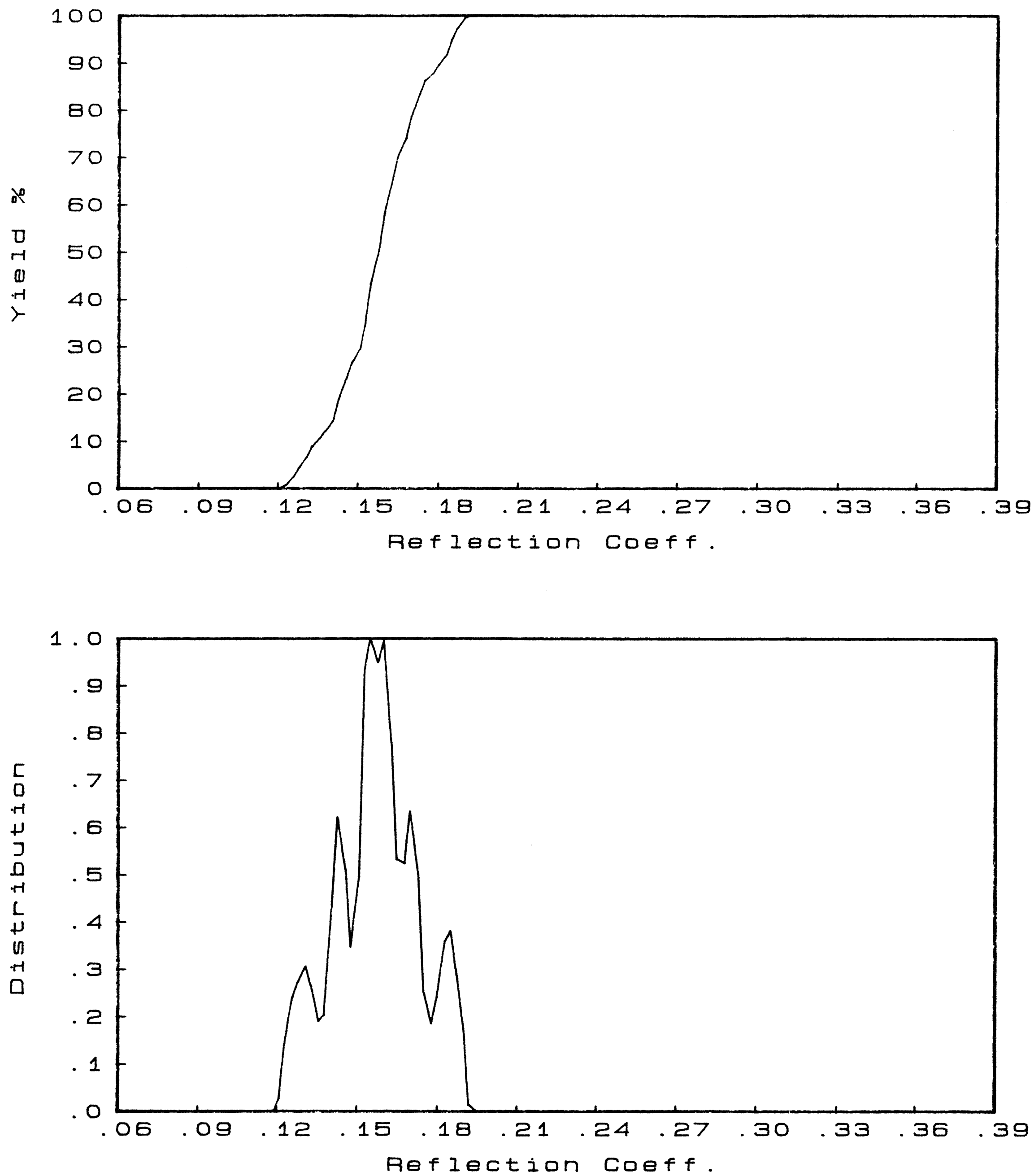
FILE: MCNOM
RANGES OF REFLECTION COEFFICIENTS : .1010 .2369

Fig. 1 Monte Carlo analysis for the conventional nominal design obtained using a least squares objective function with zero tolerances.



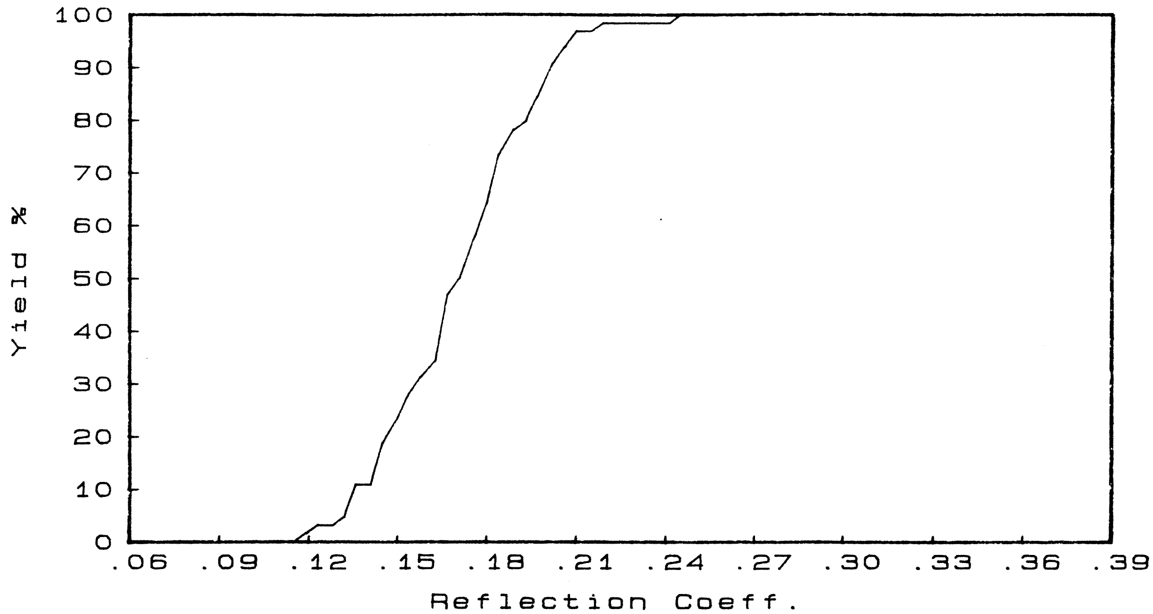
FILE: VANOM
RANGES OF REFLECTION COEFFICIENTS : .1078 .3178

Fig. 2 Vertex analysis for the conventional nominal design obtained using a least squares objective function with zero tolerances.



FILE: MCOAF21
RANGES OF REFLECTION COEFFICIENTS : .1210 .1924

Fig. 3 Monte Carlo analysis of the solution to the FTP obtained using all vertices of the tolerance region and 21 sample frequency points.



FILE: VAOAF21
RANGES OF REFLECTION COEFFICIENTS : .1192 .2450

Fig. 4 Vertex analysis of the solution to the FTP obtained using all vertices of the tolerance region and 21 sample frequency points.

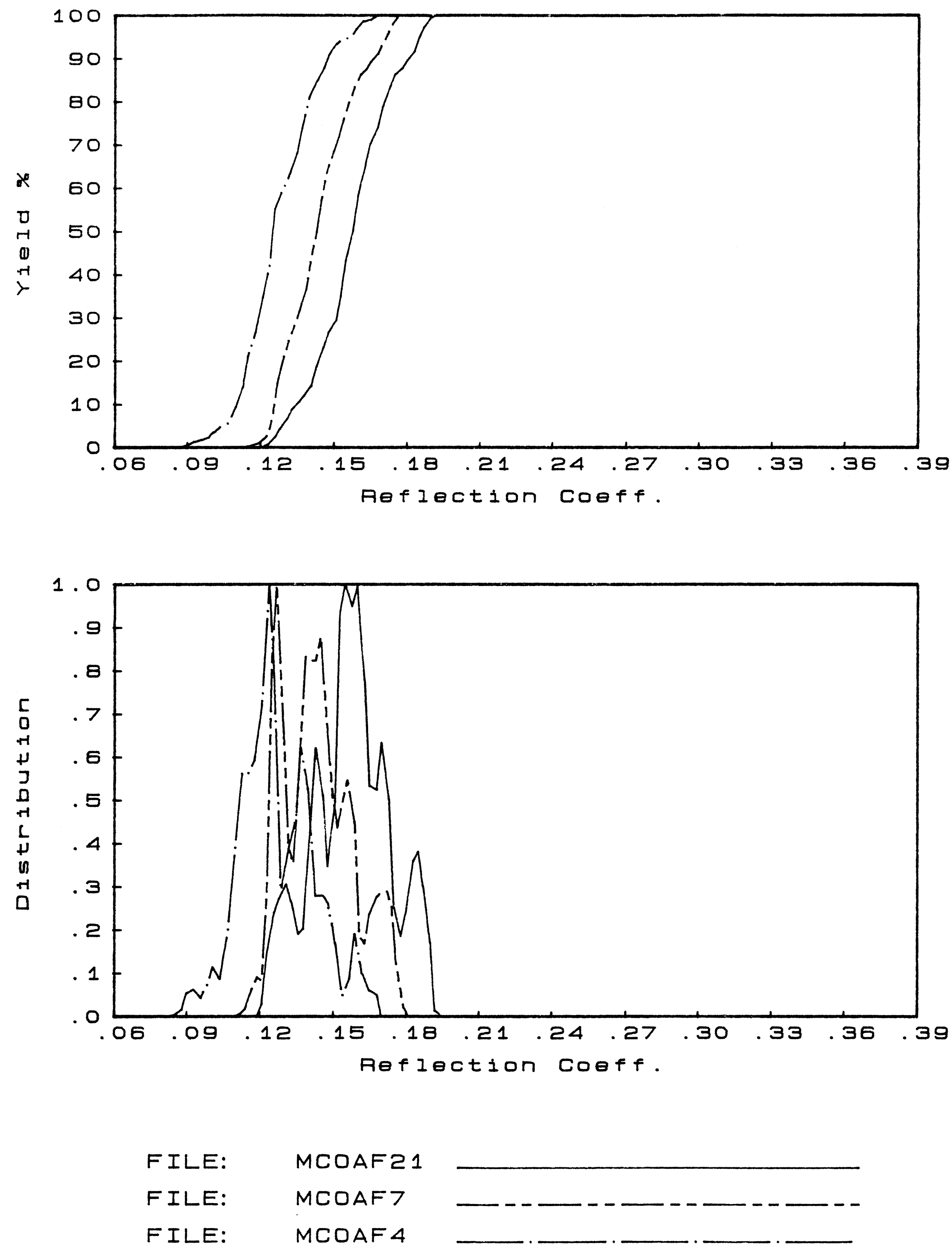


Fig. 5 Monte Carlo analyses of the solution to the FTP obtained using all vertices of the tolerance region. Curves MCOAF21, MCOAF7 and MCOAF4 correspond to designs using 21, 7 and 4 sample frequency points, respectively.

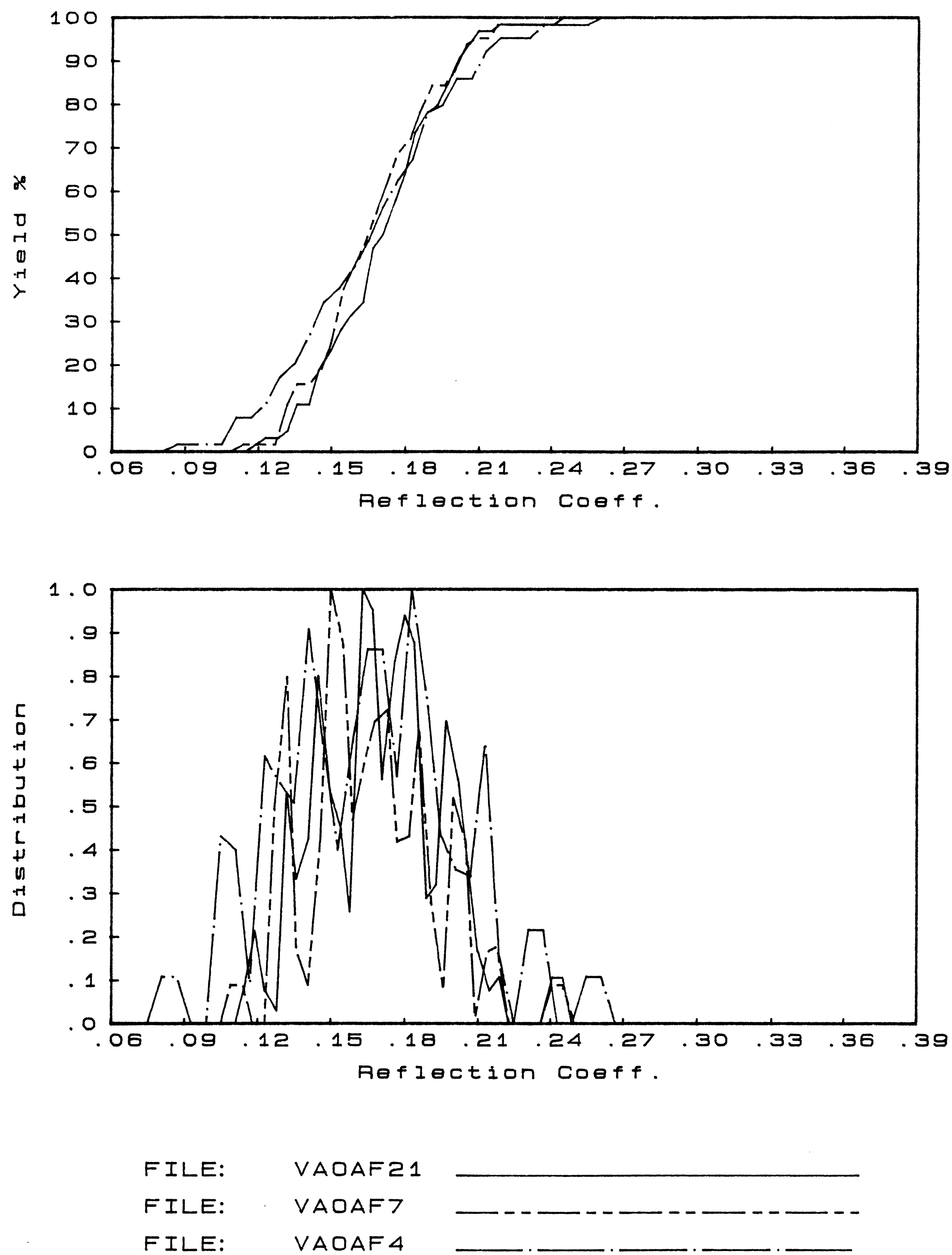


Fig. 6 Vertex analyses of the solution to the FTP obtained using all vertices of the tolerance region. Curves VAOAF21, VAOAF7 and VAOAF4 correspond to designs using 21, 7 and 4 sample frequency points, respectively.

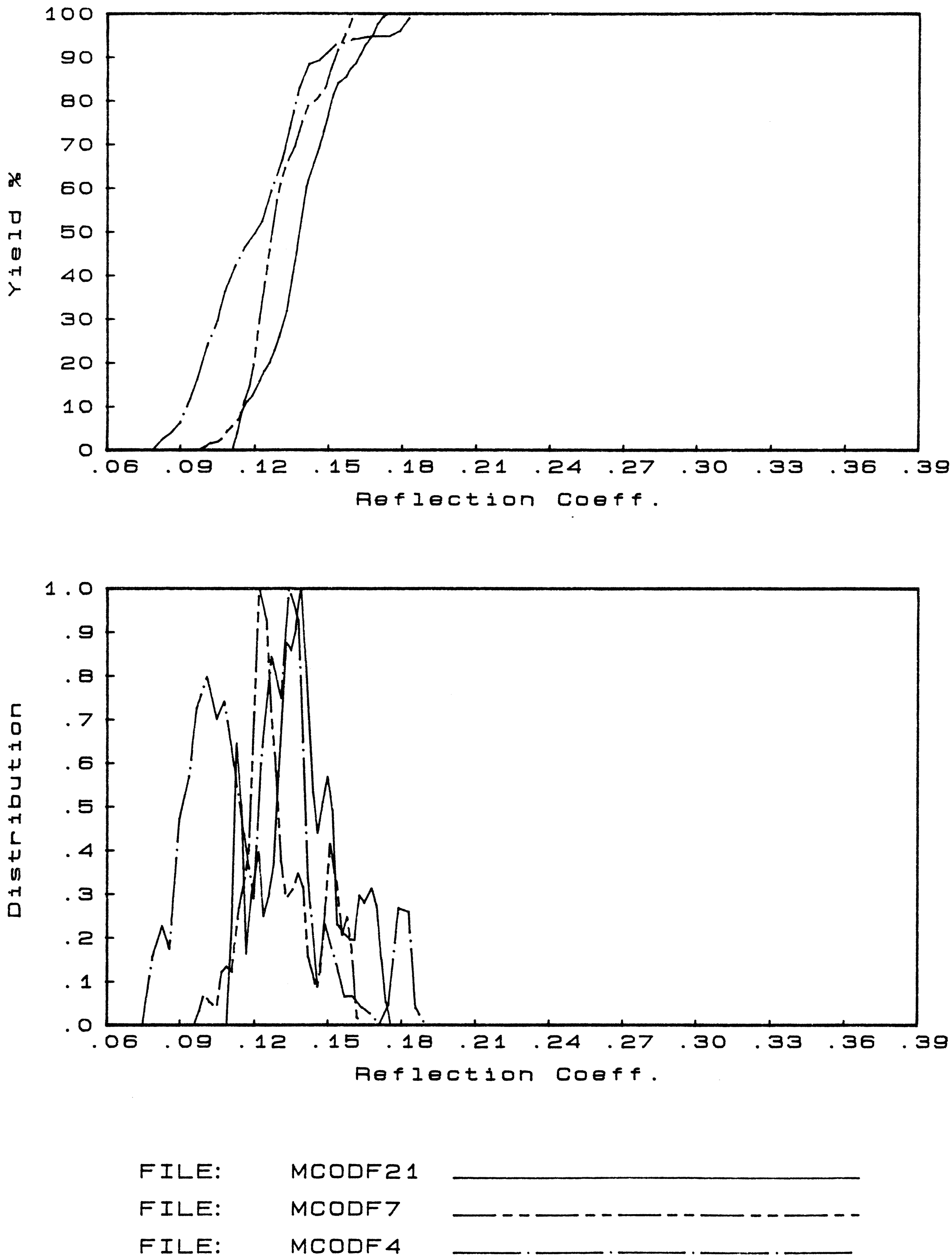


Fig. 7 Monte Carlo analyses of the solution to the FTP obtained using weighting factors of 1.0 for each of the 64 vertices and a weight of 64.0 for the nominal point. Curves MCODF21, MCODF7 and MCODF4 correspond to designs using 21, 7 and 4 sample frequency points, respectively.

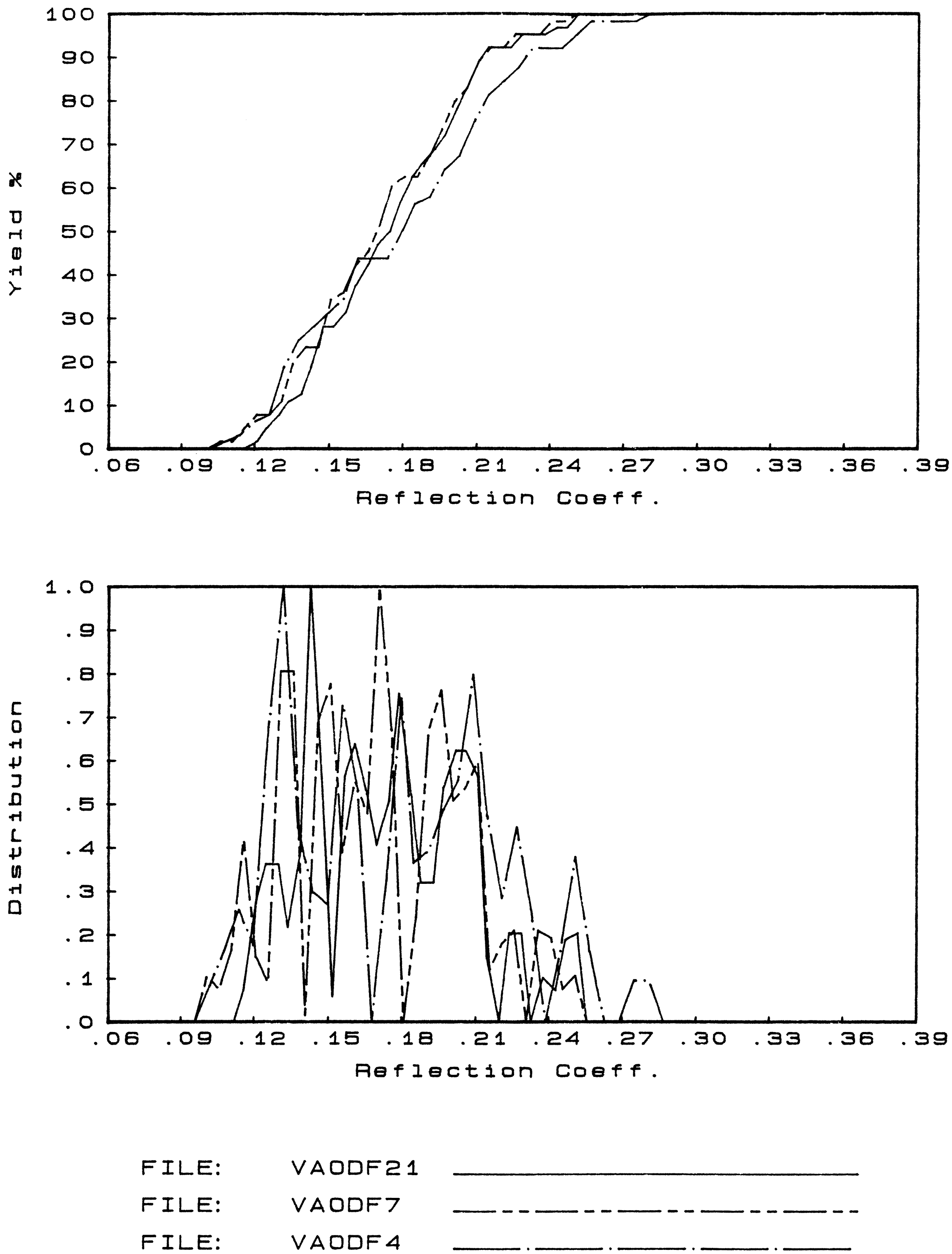
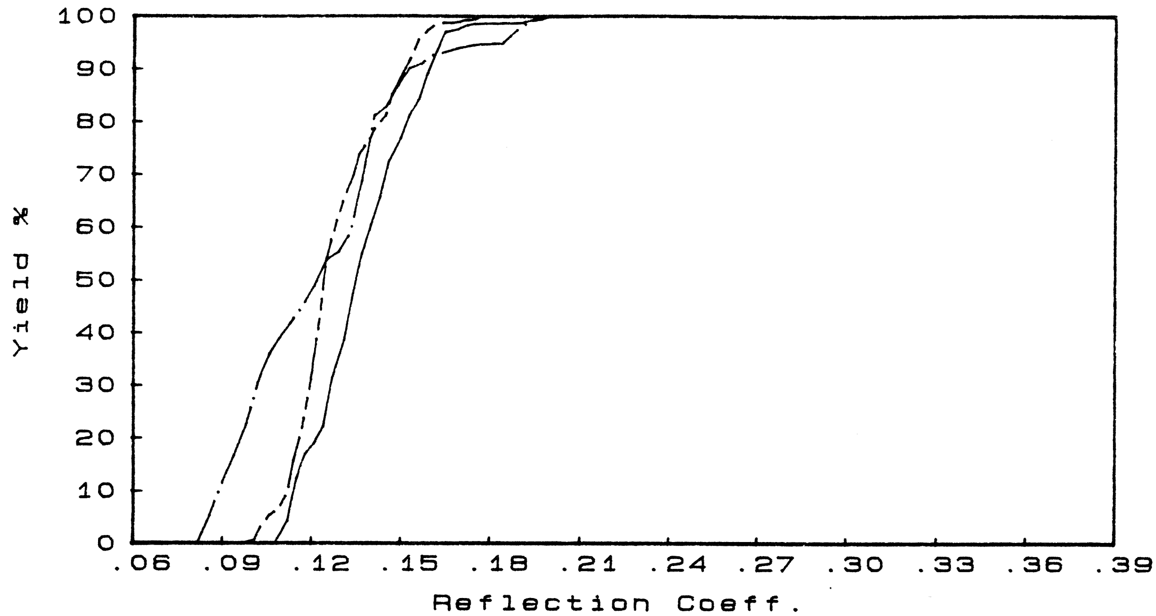


Fig. 8 Vertex analyses of the solution to the FTP obtained using weighting factors of 1.0 for each of the 64 vertices and a weight of 64.0 for the nominal point. Curves VAODF21, VAODF7 and VAODF4 correspond to designs using 21, 7 and 4 sample frequency points, respectively.



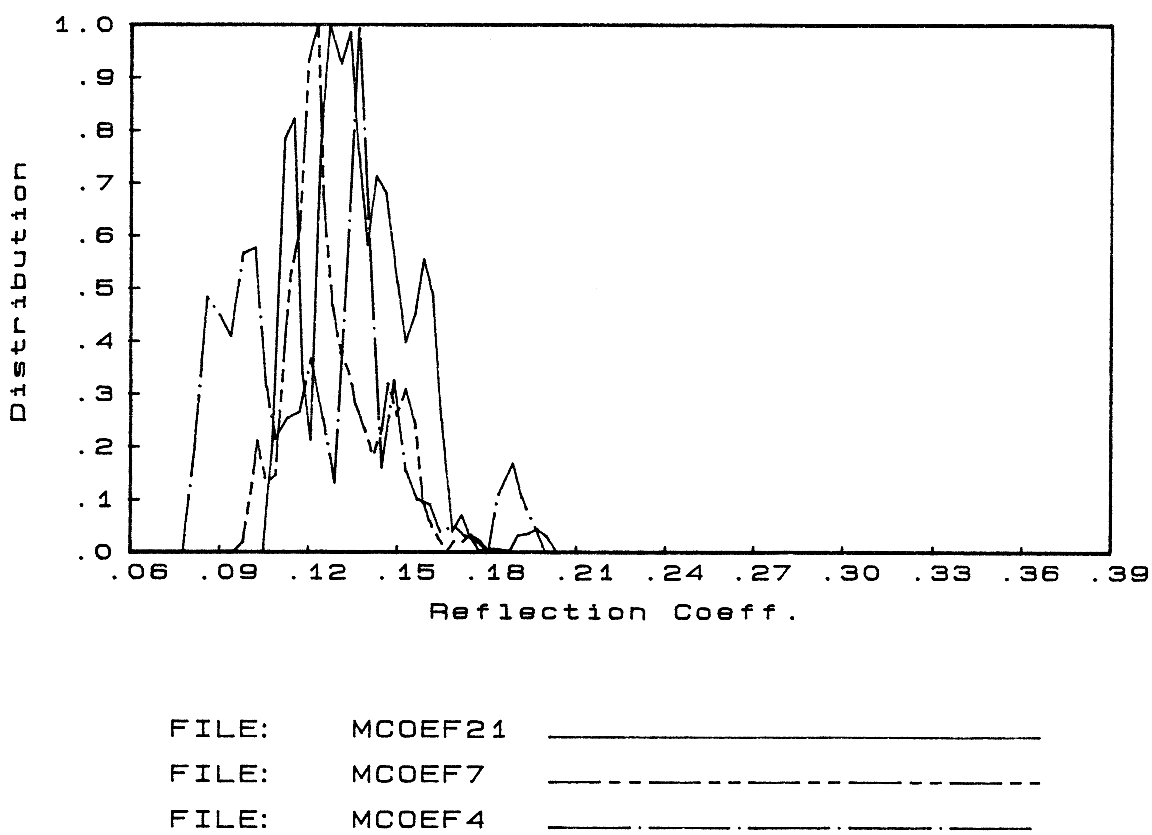


Fig. 9 Monte Carlo analyses of the solution to the FTP obtained using Monte Carlo design with 50 random outcomes. Curves MCOEF21, MCOEF7 and MCOEF4 correspond to designs using 21, 7 and 4 sample frequency points, respectively.

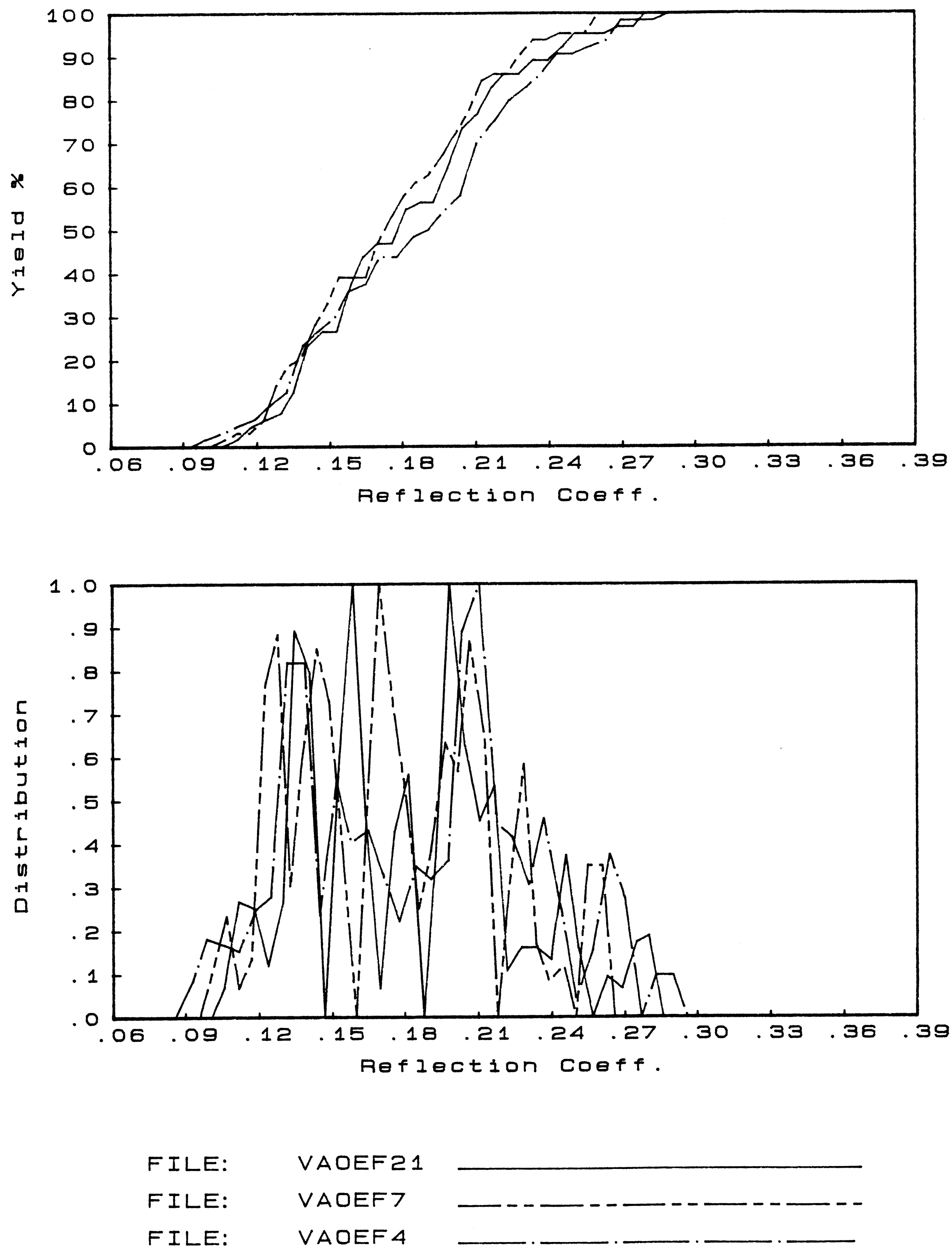


Fig. 10 Vertex analyses of the solution to the FTP obtained using Monte Carlo design with 50 random outcomes. Curves VAOEF21, VAOEF7 and VAOEF4 correspond to designs using 21, 7 and 4 sample frequency points, respectively.

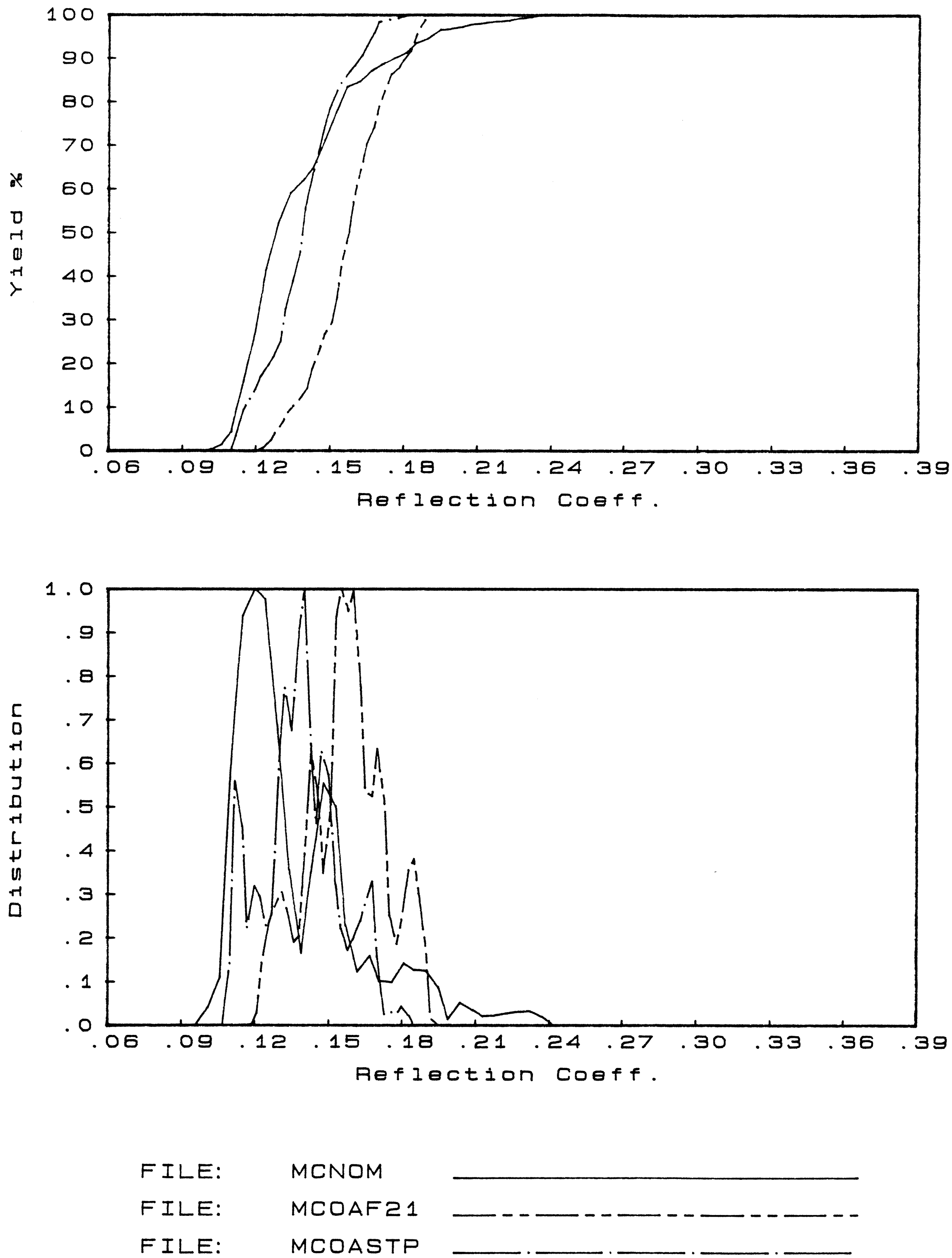


Fig. 11 Monte Carlo analyses comparing the results obtained after nominal design (curve MCNOM); after exact FTP design (curve MCOAF21); and after three stages of 5 iterations with problem re-starts when performing FTP design with the quadratic approximation option (curve MCOASTP). For the latter two designs, all vertices of the tolerance region and 21 sample frequency points were considered.

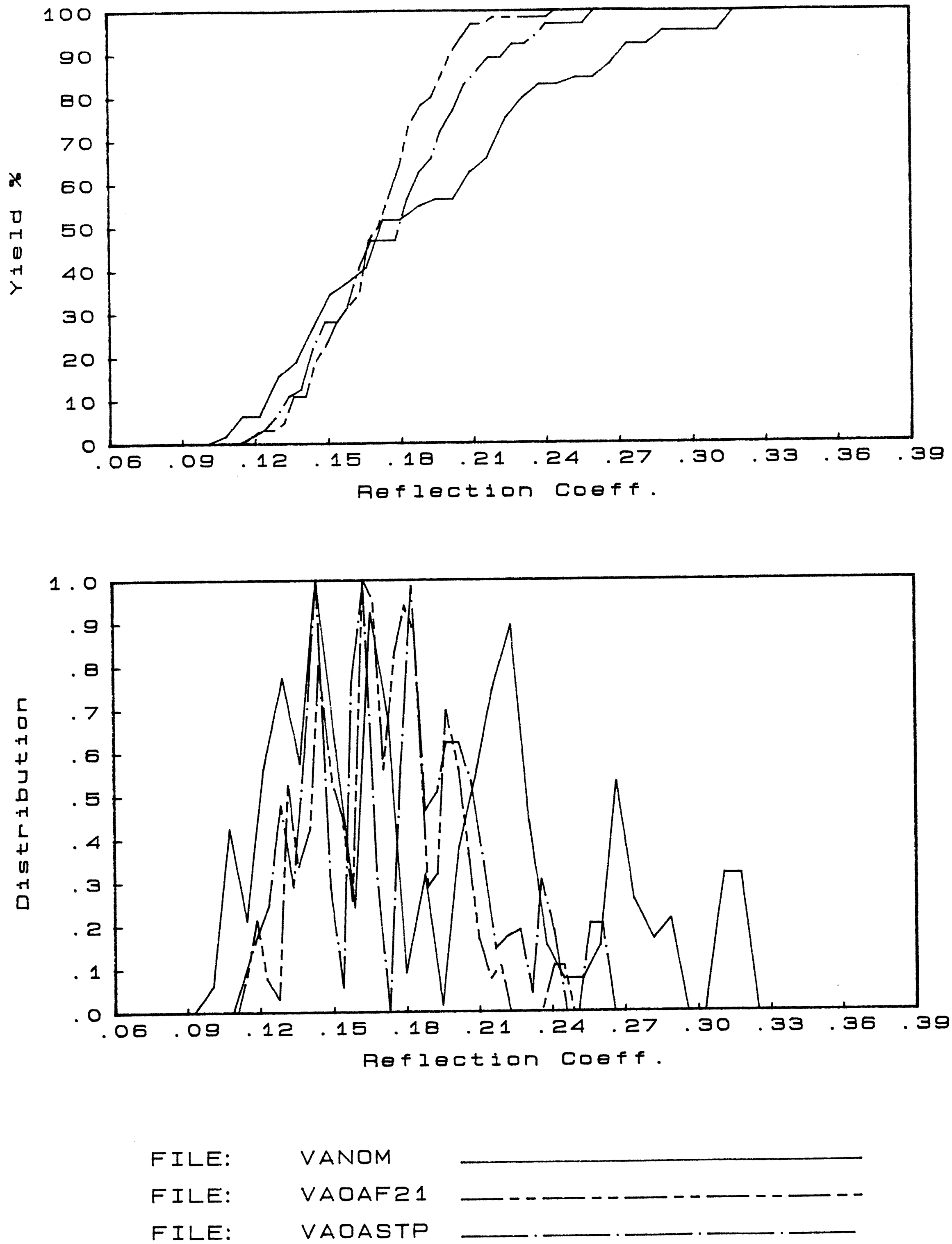


Fig. 12 Vertex analyses comparing the results obtained after nominal design (curve VANOM); after exact FTP design (curve VAOAF21); and after three stages of 5 iterations with problem re-starts when performing a FTP design with the quadratic approximation option (curve VAOASTP). For the latter two designs, all vertices of the tolerance region and 21 sample frequency points were considered.

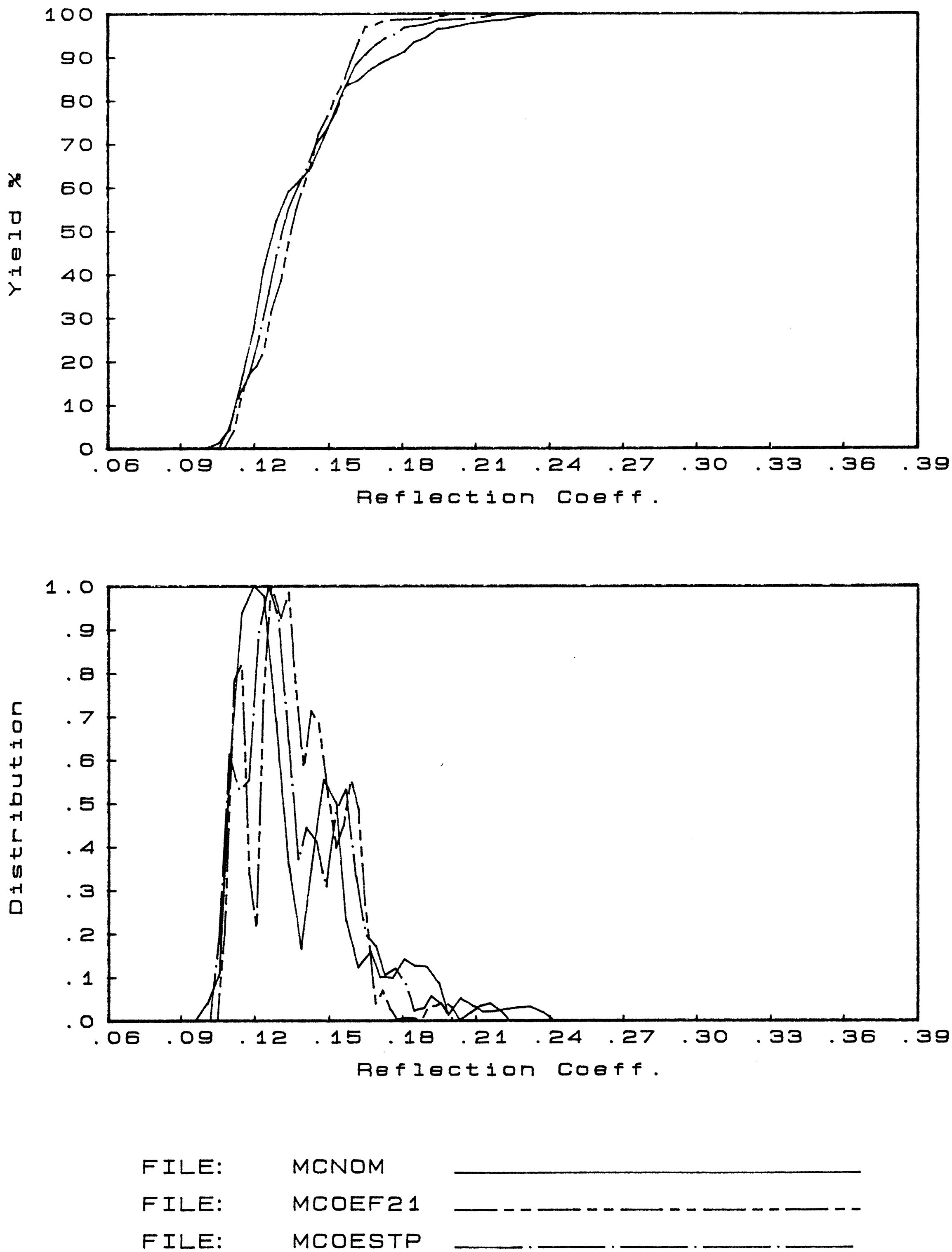


Fig. 13 Monte Carlo analyses comparing the results obtained after nominal design (curve MCNOM); after exact FTP design (curve MCOEF21); and after three stages of 5 iterations with problem re-starts when performing a FTP design with the quadratic approximation option (curve MCOESTP). For the latter two designs, Monte Carlo design with 50 random outcomes and 21 sample frequency points were considered.

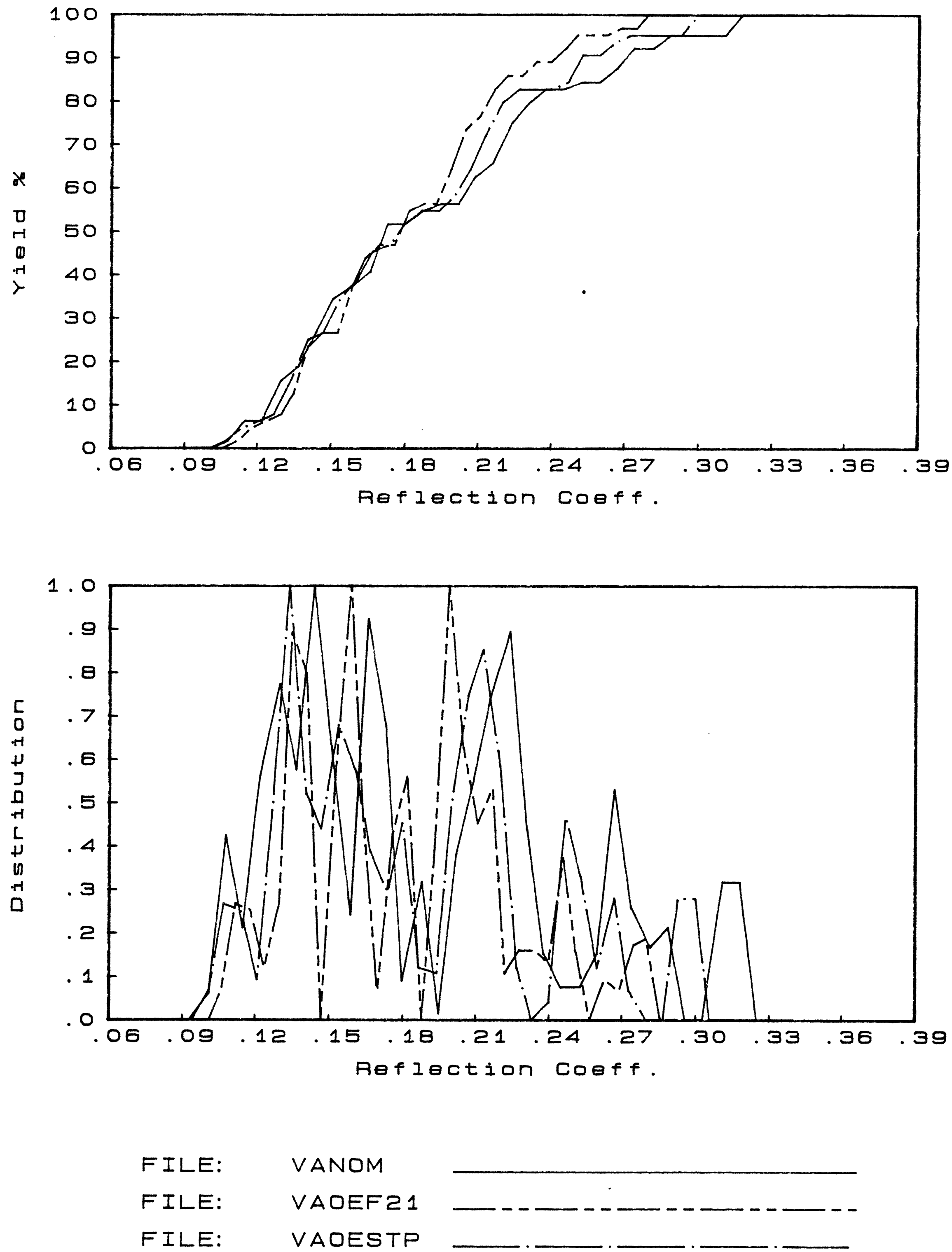


Fig. 14 Vertex analyses comparing the results obtained after nominal design (curve VANOM); after exact FTP design (curve VAOEF21); and after three stages of 5 iterations with problem re-starts when performing a FTP design with the quadratic approximation option (curve VAOESTP). For the latter two designs, Monte Carlo design with 50 random outcomes and 21 sample frequency points were considered.

VARIOUS EXAMPLES OF THE FTP

To illustrate the flexibility of the FTP specifications allowed through the data file, three examples have been included in this section of the report. Each example considers the 6-element LC matching circuit to match a 1 ohm load to a 3 ohm generator over the frequency range 0.0796 - 0.1876 MHz with 7 sample frequency points.

In Example 1, all 6 circuit parameters have 5% relative tolerances but only 3 circuit parameters are variables. In Example 2, 2 circuit parameters are variable and untoleranced and 2 circuit parameters are fixed with 5% relative tolerances. In Example 3, 3 circuit parameters are variable and have absolute tolerances (effectively 5% at the starting point) and 3 parameters are fixed with 5% relative tolerances. Data files and corresponding sample runs for the case of Monte Carlo design follow.

```
* DDFVT1 DATA FILE:
*
CAP AA SE -.32285E+06 (5,5,%T)
IND DD SE -2896.4 (5,5,%T)
CAP BB SE -.7609E+06 (5,5,%T)
IND EE SE 2281.2 (5,5,%T)
CAP CC SE .96591E+06 (5,5,%T)
IND FF SE 967.59 (5,5,%T)
CON AA T2 1 0
CON BB T2 2 0
CON CC T2 3 0
CON DD T2 1 2
CON EE T2 2 3
CON FF T2 3 4
DEF AA T2 1 4
PRI AA IR 3 1
END
0.0795774 0.1876436 0.018011033
END
0.000001
1 0 0 0
END
```

Fig. 1 Data file for Example 1 where all 6 circuit parameters have 5% relative tolerances but only 3 circuit parameters are variables.

```
* DDFVT2 DATA FILE:
*
CAP AA SE -.32285E+06
IND DD SE -2896.4
CAP BB SE .7609E+06
IND EE SE 2281.2
CAP CC SE .96591E+06 (5,5,%T)
IND FF SE 967.59 (5,5,%T)
CON AA T2 1 0
CON BB T2 2 0
CON CC T2 3 0
CON DD T2 1 2
CON EE T2 2 3
CON FF T2 3 4
DEF AA T2 1 4
PRI AA IR 3 1
END
0.0795774 0.1876436 0.018011033
END
0.000001
1 0 0 0
END
```

Fig. 2 Data file for Example 2 where 2 circuit parameters are variable and untoleranced and where 2 circuit parameters are fixed with 5% relative tolerances.

```
* DDFVT3 DATA FILE:
*
CAP AA SE -.32285E+06 (16142.5,16142.5,T)
IND DD SE -2896.4 (144.82,144.82,T)
CAP BB SE -.7609E+06 (38045.,38045.,T)
IND EE SE 2281.2 (5,5,%T)
CAP CC SE .96591E+06 (5,5,%T)
IND FF SE 967.59 (5,5,%T)
CON AA T2 1 0
CON BB T2 2 0
CON CC T2 3 0
CON DD T2 1 2
CON EE T2 2 3
CON FF T2 3 4
DEF AA T2 1 4
PRI AA IR 3 1
END
0.0795774 0.1876436 0.018011033
END
0.000001
1 0 0 0
END
```

Fig. 3 Data file for Example 3 where 3 circuit parameters are variable and have absolute tolerances (effectively 5% at the starting point) and 3 parameters are fixed but with 5% relative tolerances.

```
  'FILE NAME' OR 'QUIT'
? DDFVT1

  AN(1),SEN(2),OPT(3),SW(4),MAP(5),VAR(6),MC(7),TOL(8),ANOPT(13),RND(44)
? 8

  DO YOU WANT TO USE QUADRATIC APPROXIMATION FOR THE
  FIXED TOLERANCE PROBLEM (1/0 FOR YES/NO) ?
? 0

  SELECT A VERSION OF THE FTP :
      1 - FTP WITH ALL VERTICES OF THE TOLERANCE REGION
      2 - FTP WITH SPECIFIED NUMBER OF WORST VERTICES
      3 - FTP WITH A THRESHOLD VALUE ON THE OBJECTIVE FUNCTION
      4 - FTP WITH WEIGHTED VERTICES PLUS WEIGHTED NOMINAL
      5 - FTP WITH MONTE-CARLO DESIGN
? 5

   ENTER THE NUMBER OF RANDOM POINTS DESIRED (64 MAX)
? 50

  OPTIMIZATION BEGINS WITH FOLLOWING VARIABLES AND GRADIENTS

       VARIABLES                    GRADIENTS
    ( 1):  .32285E+06            ( 1): -.30623
    ( 2):  2896.4                ( 2):  .29939
    ( 3):  .76090E+06            ( 3):  .80742
   ERR. F.=       .3257314
          ____****____
  HOW MANY ITERATIONS BEFORE NEXT STOP ('0' RESULTS IN FINAL ANALYSIS.)
  WANT INTERMEDIATE PRINTS (YES=1,NO=0). TYPE TWO NUMBERS; I,J
? 20,1

    ( 1):  .32423E+06            ( 1):  .12068
    ( 2):  2884.3                ( 2): -.23749E-02
    ( 3):  .75239E+06            ( 3): -.50393
   ERR. F.=       .3230705
          ____****____
    ( 1):  .32442E+06            ( 1):  .11153
    ( 2):  2878.6                ( 2): -.13222
    ( 3):  .75294E+06            ( 3): -.44709
   ERR. F.=       .3229210
          ____****____
    ( 1):  .32734E+06            ( 1):  .14821
    ( 2):  2881.8                ( 2): -.68915E-01
    ( 3):  .75596E+06            ( 3): -.31300
   ERR. F.=       .3224498
          ____****____
```

Fig. 4 Sample run for Example 1 where all 6 circuit parameters have 5% relative tolerances but only 3 circuit parameters are variables.

```
( 1):  .32502E+06          ( 1): -.26052E-01
( 2):  2884.9              ( 2):  .14456E-01
( 3):  .75704E+06          ( 3):  .57109E-01
ERR. F.=      .3218043
      ____****____
( 1):  .32535E+06          ( 1): -.19141E-03
( 2):  2884.4              ( 2): -.14244E-03
( 3):  .75685E+06          ( 3): -.28869E-04
ERR. F.=      .3217826
      ____****____
( 1):  .32535E+06          ( 1): -.19141E-03
( 2):  2884.4              ( 2): -.14244E-03
( 3):  .75685E+06          ( 3): -.28869E-04
ERR. F.=      .3217826
      ____****____
GRADIENT TERMINATION WITH ABOVE VALUES. FINAL ANALYSIS FOLLOWS

   .080    .117     80.0    1.27:1   -18.60    3.04    .71
   .098    .008     35.6    1.02:1   -42.01    3.04    .03
   .116    .048   -146.3    1.10:1   -26.40    2.77   -.15
   .134    .039   -169.4    1.08:1   -28.07    2.78   -.04
   .152    .017    -59.5    1.03:1   -35.54    3.05   -.09
   .170    .052    -71.3    1.11:1   -25.70    3.09   -.30
   .188    .048     92.8    1.10:1   -26.36    2.97    .29
SAVE OPTIMIZED RESULTS (Y/N)
? N

'FILE NAME' OR 'QUIT'
? QUIT
```

Fig. 4 (continued) Sample run for Example 1 where all 6 circuit parameters have 5% relative tolerances but only 3 circuit parameters are variables.

```
  'FILE NAME' OR 'QUIT'
? DDFVT2

  AN(1),SEN(2),OPT(3),SW(4),MAP(5),VAR(6),MC(7),TOL(8),ANOPT(13),RND(44)
? 8

  DO YOU WANT TO USE QUADRATIC APPROXIMATION FOR THE
  FIXED TOLERANCE PROBLEM (1/0 FOR YES/NO) ?
? 0

  SELECT A VERSION OF THE FTP :
      1 - FTP WITH ALL VERTICES OF THE TOLERANCE REGION
      2 - FTP WITH SPECIFIED NUMBER OF WORST VERTICES
      3 - FTP WITH A THRESHOLD VALUE ON THE OBJECTIVE FUNCTION
      4 - FTP WITH WEIGHTED VERTICES PLUS WEIGHTED NOMINAL
      5 - FTP WITH MONTE-CARLO DESIGN
? 5

   ENTER THE NUMBER OF RANDOM POINTS DESIRED (64 MAX)
? 50


  OPTIMIZATION BEGINS WITH FOLLOWING VARIABLES AND GRADIENTS

       VARIABLES                    GRADIENTS
    ( 1):  .32285E+06             ( 1): -.43937
    ( 2):  2896.4                 ( 2):  .49090E-01
   ERR. F.=       .2040557
          ____****____
  HOW MANY ITERATIONS BEFORE NEXT STOP ('0' RESULTS IN FINAL ANALYSIS.)
  WANT INTERMEDIATE PRINTS (YES=1,NO=0). TYPE TWO NUMBERS; I,J
? 20,1

    ( 1):  .33041E+06             ( 1): -.26657E-01
    ( 2):  2888.9                 ( 2): -.16239
   ERR. F.=       .1987610
          ____****____
    ( 1):  .33082E+06             ( 1): -.68996E-02
    ( 2):  2895.9                 ( 2): -.12503E-03
   ERR. F.=       .1985427
          ____****____
    ( 1):  .33095E+06             ( 1):  .23173E-03
    ( 2):  2895.9                 ( 2): -.57756E-04
   ERR. F.=       .1985414
          ____****____
    ( 1):  .33095E+06             ( 1):  .23173E-03
    ( 2):  2895.9                 ( 2): -.57756E-04
   ERR. F.=       .1985414
          ____****____
```

Fig. 5 Sample run for Example 2 where 2 circuit parameters are variable and untoleranced and where 2 circuit parameters are fixed with 5% relative tolerances.

GRADIENT TERMINATION WITH ABOVE VALUES. FINAL ANALYSIS FOLLOWS

.080	.113	79.3	1.25:1	-18.95	3.05	.68
.098	.005	-1.6	1.01:1	-45.87	3.03	-.00
.116	.051	-144.2	1.11:1	-25.81	2.76	-.17
.134	.040	-165.7	1.08:1	-27.88	2.77	-.06
.152	.020	-60.7	1.04:1	-33.84	3.06	-.11
.170	.056	-69.1	1.12:1	-25.05	3.10	-.33
.188	.047	77.1	1.10:1	-26.59	3.05	.28

SAVE OPTIMIZED RESULTS (Y/N)
? N

'FILE NAME' OR 'QUIT'
? QUIT

Fig. 5 (continued) Sample run for Example 2 where 2 circuit parameters are variable and untoleranced and where 2 circuit parameters are fixed with 5% relative tolerances.

```
  'FILE NAME' OR 'QUIT'
? DDFVT3

  AN(1),SEN(2),OPT(3),SW(4),MAP(5),VAR(6),MC(7),TOL(8),ANOPT(13),RND(44)
? 8

  DO YOU WANT TO USE QUADRATIC APPROXIMATION FOR THE
  FIXED TOLERANCE PROBLEM (1/0 FOR YES/NO) ?
? 0

  SELECT A VERSION OF THE FTP :
      1 - FTP WITH ALL VERTICES OF THE TOLERANCE REGION
      2 - FTP WITH SPECIFIED NUMBER OF WORST VERTICES
      3 - FTP WITH A THRESHOLD VALUE ON THE OBJECTIVE FUNCTION
      4 - FTP WITH WEIGHTED VERTICES PLUS WEIGHTED NOMINAL
      5 - FTP WITH MONTE-CARLO DESIGN
? 5

   ENTER THE NUMBER OF RANDOM POINTS DESIRED (64 MAX)
? 50

  OPTIMIZATION BEGINS WITH FOLLOWING VARIABLES AND GRADIENTS

       VARIABLES                      GRADIENTS
    ( 1):  .32285E+06             ( 1): -.31272
    ( 2):  2896.4                 ( 2):  .25440
    ( 3):  .76090E+06             ( 3):  .71546
   ERR. F.=       .3257314
          ____****____
  HOW MANY ITERATIONS BEFORE NEXT STOP ('0' RESULTS IN FINAL ANALYSIS.)
  WANT INTERMEDIATE PRINTS (YES=1,NO=0). TYPE TWO NUMBERS; I,J
? 20,1

    ( 1):  .32435E+06             ( 1):  .98356E-01
    ( 2):  2885.5                 ( 2): -.17964E-01
    ( 3):  .75286E+06             ( 3): -.54645
   ERR. F.=       .3239415
          ____****____
    ( 1):  .32490E+06             ( 1):  .95381E-01
    ( 2):  2878.7                 ( 2): -.17193
    ( 3):  .75375E+06             ( 3): -.47675
   ERR. F.=       .3237207
          ____****____
    ( 1):  .32835E+06             ( 1):  .15958
    ( 2):  2883.3                 ( 2): -.79075E-01
    ( 3):  .75675E+06             ( 3): -.39061
   ERR. F.=       .3231390
          ____****____
```

Fig. 6 Sample run for Example 3 where 3 circuit parameters are variable and have absolute tolerances (effectively 5% at the starting point) and 3 parameters are fixed but with 5% relative tolerances.

```
( 1):  .32668E+06           ( 1): -.85744E-02
( 2):  2886.6               ( 2):  .75687E-02
( 3):  .75860E+06           ( 3):  .23660E-01
ERR. F.=       .3222671
          ____****____
( 1):  .32674E+06           ( 1): -.68745E-04
( 2):  2886.3               ( 2): -.15916E-03
( 3):  .75846E+06           ( 3): -.12708E-03
ERR. F.=       .3222639
          ____****____
( 1):  .32674E+06           ( 1): -.68745E-04
( 2):  2886.3               ( 2): -.15916E-03
( 3):  .75846E+06           ( 3): -.12708E-03
ERR. F.=       .3222639
          ____****____
GRADIENT TERMINATION WITH ABOVE VALUES. FINAL ANALYSIS FOLLOWS
```

.080	.116	80.0	1.26:1	-18.71	3.04	.70
.098	.006	30.8	1.01:1	-43.75	3.03	.02
.116	.049	-146.3	1.10:1	-26.17	2.76	-.15
.134	.040	-169.2	1.08:1	-27.91	2.77	-.04
.152	.017	-60.2	1.03:1	-35.52	3.05	-.09
.170	.052	-70.7	1.11:1	-25.71	3.09	-.30
.188	.049	88.5	1.10:1	-26.22	2.99	.29

```
  SAVE OPTIMIZED RESULTS (Y/N)
? N

  'FILE NAME' OR 'QUIT'
? QUIT
```

Fig. 6 (continued) Sample run for Example 3 where 3 circuit parameters are variable and have absolute tolerances (effectively 5% at the starting point) and 3 parameters are fixed but with 5% relative tolerances.

MONTE CARLO DESIGN WITH RANDOM FREQUENCY POINTS

A specialized test program, not TOLCAD, was used to test the premise that a Monte Carlo design of a fixed tolerance problem could be performed by selecting a number of random frequency points. The LC filter test circuit with all six parameters being variable with 5% relative tolerances was used for the experiment. The experiment consisted in trying various combinations of parameter outcomes and frequency points.

The special test program prompted the user for the number of random parameter outcomes P and for the number of random sample frequency points F. Prior to starting the design, the program selects for each toleranced parameter a different set of P uniformly distributed outcomes within the full set of outcomes $\mu = [-1.0,...,+1.0]$ where $x = x^0 + \mu x^0$. The set of selected outcomes is kept constant throughout the design process. Also before starting the design, the program selects for each parameter outcome a different set of F uniformly distributed sample frequency points within the full set of $\eta = [0.0,...,+1.0]$. A randomly selected frequency, FREQ, is given by FREQ = FRST + ηFDEL where FRST is the first frequency point on the frequency interval of interest and FDEL=FLST-FRST where FLST is the last frequency point on the interval. The set of random frequencies for each random outcome is kept constant throughout the design process.

Results are summarized in Table I.

TABLE I

SUMMARY OF RESULTS FOR MONTE CARLO FTP DESIGN USING VARIOUS COMBINATIONS OFRANDOMLY SELECTED PARAMETER OUTCOMES AND RANDOMLY SELECTED SAMPLE FREQUENCY POINTS

P	F	Monte Carlo analysis	Vertex analysis	Parameters AA [μF]	DD [μH]	BB [μF]	EE [μH]	CC [μF]	FF [μH]
50	21	.1047-.1806	.1224-.2626	.310	2.83	.741	2.23	.944	.941
	10	.1066-.1832	.1183-.2673	.311	2.83	.742	2.24	.945	.951
	5	.1040-.1756	.1207-.2616	.309	2.83	.738	2.24	.944	.952
	2	.1069-.1839	.1166-.2660	.313	2.83	.744	2.24	.946	.951
	1	.1091-.1979	.1106-.2833	.314	2.85	.749	2.25	.952	.963
30	21	.1034-.1969	.1207-.2769	.312	2.85	.746	2.24	.951	.942
	10	.1046-.1990	.1175-.2774	.313	2.85	.747	2.24	.951	.946
	5	.1043-.2002	.1129-.2832	.314	2.87	.747	2.26	.954	.961
	2	.1068-.2040	.1114-.2830	.318	2.86	.751	2.25	.953	.954
	1	.0976-.2701	.1069-.3586	.322	2.93	.768	2.30	.973	.979
10	21	.1061-.2186	.1085-.3029	.317	2.89	.756	2.26	.963	.963
	10	.1029-.2150	.1063-.3013	.319	2.88	.756	2.27	.963	.974
	5	.1031-.2210	.1072-.3110	.318	2.88	.754	2.29	.966	.991
	2	.1766-.6850	.1215-.6987	.340	3.17	.821	2.51	1.05	1.08
	1	.1827-.7105	.1333-.7182	.351	3.19	.832	2.51	1.06	1.10
5	21	.1043-.2395	.1085-.3176	.320	2.90	.758	2.28	.958	.964
	10	.0917-.2796	.1012-.3646	.326	2.93	.772	2.31	.968	.977
	5	.1334-.6095	.0854-.6375	.343	3.13	.819	2.45	1.03	1.03
	2	.1473-.6326	.0941-.6562	.320	3.18	.792	2.55	1.03	1.10
	1	.1325-.6312	.0889-.6529	.327	3.18	.793	2.56	1.02	1.12

Legend:
P = number of uniformly distributed random parameter outcomes
F = number of uniformly distributed random frequency points

Experimental results indicate that valid designs are obtained when a total of approximately 70 overall response evaluations are performed at each iteration. The implication is that it should be possible to effectively reduce the number of sample frequency points when many parameter outcomes are considered.

DISCUSSION AND SUGGESTIONS

The TOLCAD package in its present form does not allow the user to define general lower and upper specifications on a performance function of interest to formulate a general design problem. All it offers is a fixed set of least squares objective functions for certain response functions or S parameters of the circuit. From the work done up to now it seems that several ideas and suggestions, if implemented, would significantly increase the capabilities of the package in terms of the range and type of problems that could be solved.

One suggestion is to create access to individual responses at user defined frequency points and to store that information in a separate array or arrays. This will facilitate improvements to the quadratic approximation. It would also be desirable to create an option for the user to define lower and upper specifications for any performance function of interest at arbitrarily defined frequency points or intervals. This, in conjunction with the access to individual responses, would lead to the possibility of creating error functions at user defined frequencies, which in turn makes possible, for example, the definition of a design problem in terms of alternative norms.

The next logical step would be to incorporate minimax optimization or optimization w.r.t. other norms for the nominal design problem as well as for the fixed tolerance problem. Another very significant benefit would be the opportunity to replace the existing optimizer with more effective-state-of-the-art optimizers.

In order to be able to evaluate solutions to the FTP in terms of yield it would be very useful to have the option of performing a Monte Carlo analysis for a given design in a way that would lead directly to the calculation of yield of circuit outcomes. At present, the "Monte Carlo" analysis in TOLCAD, being the original COMPACT formulation, appears to be carried out at a single frequency at a time. It is not employed in any of the analyses reported here.

Another suggestion for further development is to create an automatic scheme for reducing the number of frequency points used in optimization retaining at the same time the original frequency points defined by the user for analysis purposes. Such a scheme is presented in the section where a test program is used to perform Monte Carlo FTP design with randomly selected sample frequency points. Results are encouraging.

Since it is now possible to disable gradient calculations in Subroutine ANALYZ, the FTPB option which consider "N worst vertices" could be improved by (1) calling ANALYZ for each vertex while disabling gradient calculations, (2) sorting the N worst vertices from the set of all vertices, (3) calling Subroutine ANALYZ N times, with gradient calculations enabled. This would allow FTPB to continually update the set of worst vertices, unlike what it is doing in the present version of TOLCAD. An analogous scheme could be introduced for FTPE, where Monte Carlo design could proceed while considering only "N worst (or even best!) outcome points" which would be a subset of the total R random outcomes.

Further test examples must use larger parameter tolerances than the 5% value used in this report. We must solve a wider variety of test circuits and check if some observations can be reproduced. For instance, with the test circuit of this report worst case designs with a least squares objective function performed with a large number of frequency points tend towards minimax solutions. For the least squares type objective function used in TOLCAD, "worst case" designs performed with fewer and fewer sample frequency points tend to improve the Monte Carlo yield more and more. In other words, design emphasis tends to shift away from the worst cases.

Another desirable feature in future versions of TOLCAD would be the ability to specify functional relationships between design variables.