AGGRESSIVE SPACE MAPPING WITH MULTIPLE POINT PARAMETER EXTRACTION: A STEP BY STEP ILLUSTRATION USING TWO ROSENBROCK FUNCTIONS

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Abstract The Aggressive Space Mapping (ASM) algorithm employing multiple point parameter extraction is illustrated in this report, making use of two analytical Rosenbrock functions as fine and coarse models. The aggressive Space Mapping concept is reviewed. The improvement in the uniqueness of the parameter extraction due to multiple point matching is graphically illustrated. The ASM algorithm is shown in a step by step fashion. The corresponding space mapped solution is obtained and verified with the direct fine solution.

I. INTRODUCTION

Space Mapping (SM) is a novel concept for circuit design and optimization that combines the computational efficiency of coarse models with the accuracy of fine models. The coarse models are typically empirical circuit-equivalent engineering models, which are computationally very efficient but often have a limited validity range for its parameters, beyond which the simulation results become very coarse. On the other hand, the fine models may be provided by an electromagnetic simulator, or even by direct lab measurements; they are very accurate but demand considerable resources.

The SM technique establishes a mathematical link between the coarse and the fine models, and

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directs the bulk of CPU intensive optimization to the coarse model, while preserving the accuracy and confidence offered by the fine model. The SM technique was originally developed by Bandler *et al.* [1].

The Aggressive Space Mapping (ASM) algorithm using multiple point parameter extraction is illustrated in this report making use of two analytical Rosenbrock functions. The original Rosenbrock function is taken as the coarse model, and a perturbed Rosenbrock function as the fine model. The aggressive Space Mapping concept is reviewed. The improvement in the uniqueness of parameter extraction due to multiple point matching is graphically illustrated. The ASM algorithm is shown in a step by step fashion. The corresponding space mapped solution is obtained and verified with the direct fine solution.

II. AGGRESSIVE SPACE MAPPING ALGORITHM

Let the vectors $\boldsymbol{x}_c$ and $\boldsymbol{x}_f$ represent the design parameters of the coarse and fine models, respectively, and $\boldsymbol{R}_c(\boldsymbol{x}_c)$ and $\boldsymbol{R}_f(\boldsymbol{x}_f)$ the corresponding model responses. $\boldsymbol{R}_c$ is much faster to calculate but less accurate than $\boldsymbol{R}_f$.

SM optimization consists in the generation of an appropriate mapping, $\boldsymbol{P}$, from the fine model parameter space $\boldsymbol{x}_f$ to the coarse model parameter space $\boldsymbol{x}_c$

$$\boldsymbol{x}_c = \boldsymbol{P}(\boldsymbol{x}_f) \tag{1}$$

such that

$$\boldsymbol{R}_c(\boldsymbol{P}(\boldsymbol{x}_f)) \approx \boldsymbol{R}_f(\boldsymbol{x}_f) \tag{2}$$

The mapping is established iteratively. In the original work [1], the initial mapping is established by performing upfront fine model analysis at a number of base points. The aggressive SM strategy minimizes the upfront effort by targeting every fine simulation at optimizing the design and progressively refining the mapping [2].

The aggressive SM concept can be illustrated by the use of the diagram in Fig. 1. We initially perform conventional optimization using the coarse model to obtain the optimal coarse solution $\boldsymbol{x}_c^*$. Assuming that $\boldsymbol{x}_f$ and $\boldsymbol{x}_c$ have the same dimension, we choose the coarse optimal solution as a starting point for the fine model

$$\boldsymbol{x}_f = \boldsymbol{x}_c^* \tag{3}$$

and calculate the corresponding fine response $\boldsymbol{R}_f(\boldsymbol{x}_f)$. Then we perform parameter extraction to find the optimal value of the coarse model parameters $\boldsymbol{x}_c$, such that the coarse response sufficiently match the fine response calculated,

$$\boldsymbol{R}_c(\boldsymbol{x}_c) \approx \boldsymbol{R}_f(\boldsymbol{x}_f) \tag{4}$$

If the extracted parameters $\boldsymbol{x}_c$ are approximately the same as the optimal coarse solution $\boldsymbol{x}_c^*$, then we have the space-mapped solution $\bar{\boldsymbol{x}}_f$ and the optimization ends, otherwise, a new value of $\boldsymbol{x}_f$ is generated using Broyden's formula, the mapping is updated, and the optimization continues by calculating the corresponding new fine response (see Fig. 1). To find the space-mapped solution $\bar{\boldsymbol{x}}_f$ means to find the fine model parameters $\boldsymbol{x}_f$ whose response $\boldsymbol{R}_f(\boldsymbol{x}_f)$ matches the optimal coarse response $\boldsymbol{R}_c(\boldsymbol{x}_c^*)$.

Following [2], the ASM algorithm can be implemented as follows

Step 0. Initialize $\boldsymbol{x}_f^{(1)} = \boldsymbol{x}_c^*$, $\boldsymbol{B}^{(1)} = \boldsymbol{1}$, $j = 1$.

Step 1. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$.

Step 2. Extract $\boldsymbol{x}_c^{(1)}$ such that $\boldsymbol{R}_c(\boldsymbol{x}_c^{(1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$.

Step 3. Evaluate $\boldsymbol{f}^{(1)} = \boldsymbol{x}_c^{(1)} - \boldsymbol{x}_c^*$. Stop if $\left\| \boldsymbol{f}^{(1)} \right\| \le h$.

Step 4. Solve $\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)} = - \boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$.

Step 5. Set $\boldsymbol{x}_f^{(j+1)} = \boldsymbol{x}_f^{(j)} + \boldsymbol{h}^{(j)}$.

Step 6. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$.

Step 7. Extract $\boldsymbol{x}_c^{(j+1)}$ such that $\boldsymbol{R}_c(\boldsymbol{x}_c^{(j+1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$.

Step 8. Evaluate $\boldsymbol{f}^{(j+1)} = \boldsymbol{x}_c^{(j+1)} - \boldsymbol{x}_c^*$. Stop if $\left\|\boldsymbol{f}^{(j+1)}\right\| \leq h$.

Step 9. Update $\boldsymbol{B}^{(j+1)} = \boldsymbol{B}^{(j)} + \dfrac{\boldsymbol{f}^{(j+1)}\boldsymbol{h}^{(j)^T}}{\boldsymbol{h}^{(j)^T}\boldsymbol{h}^{(j)}}$.

Step 10. Set $j = j+1$; go to *Step* 4.

It is clear that the matrix $\boldsymbol{B}^{(j)}$ obtained from Broyden's formula represents the most up-to-date approximation to the mapping between the two spaces around the optimal coarse solution $\boldsymbol{x}_c^*$. That is

$$\boldsymbol{B}^{(j)}(\boldsymbol{x}_f^{(j+1)} - \boldsymbol{x}_f^{(j)}) = -(\boldsymbol{x}_c^{(j)} - \boldsymbol{x}_c^*) \tag{5}$$

Thus, when the space-mapped solution is reached, $\boldsymbol{x}_f^{(j+1)} = \bar{\boldsymbol{x}}_f$, the mapping can be obtained from (5) using the final value of $\boldsymbol{B}$, as follows

$$\boldsymbol{x}_c = \boldsymbol{P}(\boldsymbol{x}_f) = \boldsymbol{B}(\boldsymbol{x}_f - \bar{\boldsymbol{x}}_f) + \boldsymbol{x}_c^* \tag{6}$$

Using (6) the fine model could be replaced by the coarse model for fast response evaluations.

III. COARSE AND FINE MODELS

Consider a Rosenbrock function as a coarse model, $R_c(\boldsymbol{x})$, given by

$$R_c(\boldsymbol{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \tag{7}$$

where $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ are the design parameters. The-3-D plot and the contours of the coarse model $R_c(\boldsymbol{x})$ are illustrated in Fig. 2, which were obtained using *MatLab*™ [3]. The well-known coarse optimal solution is at point $\boldsymbol{x}_c^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the coarse optimal value is $R_c^* = R_c(\boldsymbol{x}_c^*) = 0$.

A perturbed Rosenbrock function will be considered as a fine model, $R_f(\boldsymbol{x})$, given by

$$R_f\ (\boldsymbol{x})\ =\ \ 100(u_2 - u_1^2)^2 + (1-u_1)^2 \tag{8}$$

where $\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{x} + \begin{bmatrix} -0.2 \\ +0.2 \end{bmatrix}$. The 3-D plot and the corresponding contours of the fine model are illustrated in Fig. 3.

IV. MULTIPLE POINT PARAMETER EXTRACTION

According to the step 2 of the ASM algorithm previously described, we want to extract the corresponding parameters for the coarse model, $\boldsymbol{x}$, such that $R_c(\boldsymbol{x}) \approx R_f\ (\boldsymbol{x}_c^*)$. Instead of minimizing

$$\left\| R_c(\boldsymbol{x})\ - R_f\ (\boldsymbol{x}_c^*) \right\|_p \tag{9}$$

we will perform a multiple point parameter extraction to obtain a sharper objective function and improve the uniqueness of the parameter extraction, at the expense of more fine model evaluations. The multiple point parameter extraction approach was introduced by Bandler *et al.* [4]. The new objective function considering five matching points is

$$\left\| R_c(\boldsymbol{x} + \boldsymbol{B}^{(j)}?\ \boldsymbol{x}_i)\ - R_f\ (\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_i) \right\|_p \tag{10}$$

where $\Delta\boldsymbol{x}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\Delta\boldsymbol{x}_2 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, $\Delta\boldsymbol{x}_3 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$, $\Delta\boldsymbol{x}_4 = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$ and $\Delta\boldsymbol{x}_5 = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$ were chosen arbitrary.

The points $\boldsymbol{p}_i = \boldsymbol{x}_c^* + ?\ \boldsymbol{x}_i$ in the fine parameter space are being indicated in Fig. 3b. We are considering that, at the j^{th} iteration of the ASM algorithm, a perturbation in the fine parameter space $\Delta\boldsymbol{x}_i$ corresponds to a perturbation $\boldsymbol{B}^{(j)}?\ \boldsymbol{x}_i$ in the coarse parameter space, as proposed in [5].

Since the matrix $\boldsymbol{B}^{(j)} = \boldsymbol{1}$ for the first parameter extraction optimization, the corresponding five error functions E_i are given by

$$E_i = R_c(\boldsymbol{x} + ?\ \boldsymbol{x}_i)\ - R_f\ (\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_i) \tag{11}$$

If we consider only l_1 and l_2 norms, the corresponding objective functions are

$$ABS_n = |E_1| + |E_2| + \ldots + |E_n| \tag{12}$$

and

$$SQR_n = (E_1)^2 + (E_2)^2 + \ldots + (E_n)^2 \tag{13}$$

The l_1 objective functions for the single point parameter extraction problem (ABS_1) and for the multiple point parameter extraction problem taking four additional points (ABS_5) are illustrated in Fig. 4, from where it can clearly be noticed the improvement in the uniqueness of the parameter extraction solution. A similar result is illustrated in Fig. 5 for the l_2 objective functions SQR_1 and SQR_5. It can be visualized from the plots of ABS_5 and SQR_5 that the solution for the parameter extraction problem is $\boldsymbol{x} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix}$. It can also be verified that

$$\begin{aligned}
&R_c(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_1) = R_f\ (\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_1) = 31.4\,, \\
&R_c(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_2) = R_f\ (\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_2) = 112.4\,, \\
&R_c(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_3) = R_f\ (\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_3) = 24.1\,, \\
&R_c(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_4) = R_f\ (\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_4) = 98.1\,, \text{ and} \\
&R_c(\begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} + ?\ \boldsymbol{x}_5) = R_f\ (\boldsymbol{x}_c^* + ?\ \boldsymbol{x}_5) = 37.7\,.
\end{aligned} \tag{14a-e}$$

V. SPACE MAPPED SOLUTION

Now we will apply the ASM algorithm described in Section II to the coarse and fine models defined in Section III.

Step 0. $\boldsymbol{x}_f^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\boldsymbol{B}^{(1)} = \boldsymbol{1}$, $j = 1$

Step 1. From (14a), $R_f(\boldsymbol{x}_f^{(1)}) = 31.4$

Step 2. From Figs. 4 and 5, when $\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix}$, $R_c(\boldsymbol{x}_c^{(1)}) = R_f(\boldsymbol{x}_f^{(1)})$

Step 3. $\boldsymbol{f}^{(1)} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}$

Step 4. Since $\boldsymbol{B}^{(1)} = \boldsymbol{1}$, $\boldsymbol{h}^{(1)} = \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}$

Step 5. Set $\boldsymbol{x}_f^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$

Step 6. $R_f(\boldsymbol{x}_f^{(2)}) = 0$

Step 7. $\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, because we know $R_c(\boldsymbol{x}_c^{(2)}) = 0$

Step 8. Since $\boldsymbol{f}^{(2)} = \boldsymbol{x}_c^{(2)} - \boldsymbol{x}_c^{*} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, the algorithm ends, and $\bar{\boldsymbol{x}}_f = \boldsymbol{x}_f^{(2)} = \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}$.

For this particular problem, which involves two optimization variables, the Aggressive Space Mapping algorithm takes only one iteration. It can be verified that the space-mapped solution obtained is equal to the exact fine solution (see Fig. 3b).

The mapping between the coarse and the fine parameter spaces can be obtained using (6) as follows

$$\boldsymbol{x}_c = \boldsymbol{P}(\boldsymbol{x}_f) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\boldsymbol{x}_f - \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix}) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \boldsymbol{x}_f + \begin{bmatrix} -0.2 \\ 0.2 \end{bmatrix}, \tag{15}$$

which corresponds to the original perturbation that relates the coarse and the fine models (8). Therefore, the mapping obtained is valid for the whole range of parameter values.

VI. CONCLUSIONS

The Aggressive Space Mapping (ASM) algorithm with multiple point parameter extraction was illustrated in this report making use of two analytical Rosenbrock functions. The original Rosenbrock function was taken as the coarse model, and a perturbed Rosenbrock function as the fine model. The improvement in the uniqueness of the parameter extraction due to multiple point matching was graphically illustrated. The ASM algorithm was executed in a step by step fashion, and the corresponding space mapped solution was accomplished after only one iteration. The mapping calculated by the ASM algorithm is equal to the original linear mapping between both parameter spaces.

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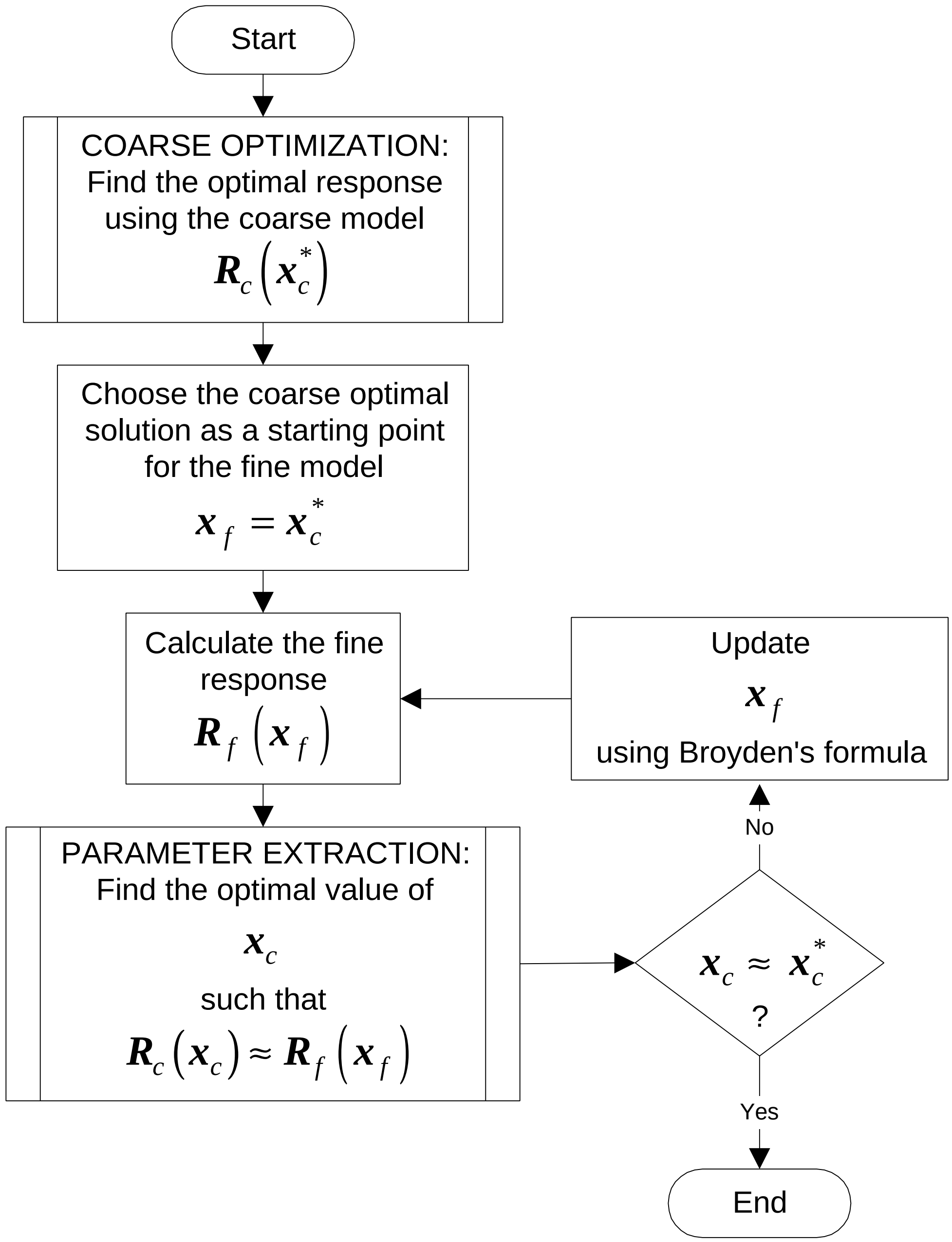
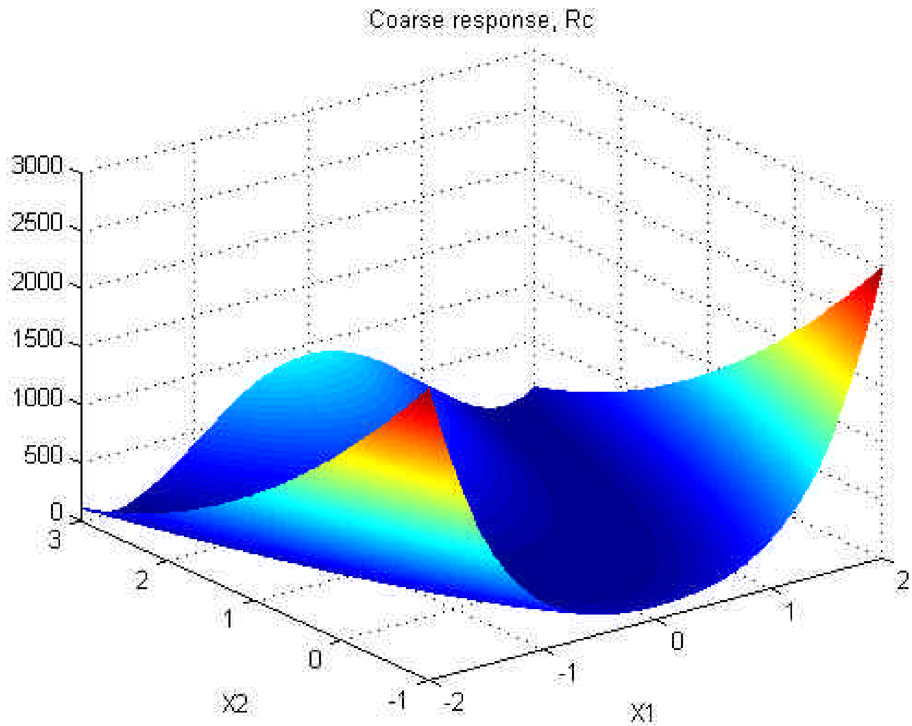
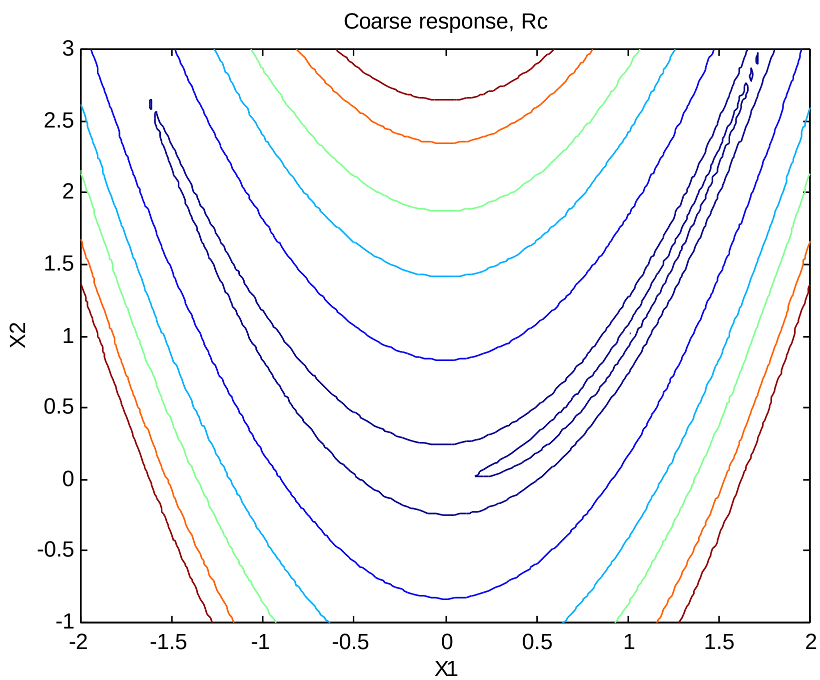


Fig. 1. Space Mapping concept.

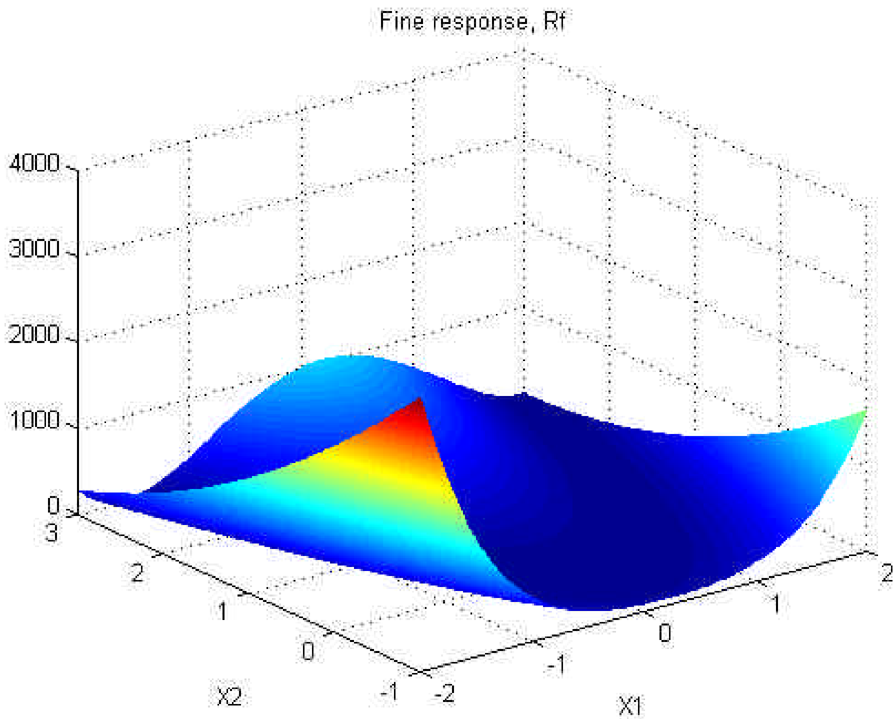


(*a*)



(*b*)

Fig. 2. Original Rosenbrock function or coarse response, R_c: (*a*) surface plot; (*b*) contours for levels 0.0001, 0.7, 7, 70, 200, 350, 550, 700.



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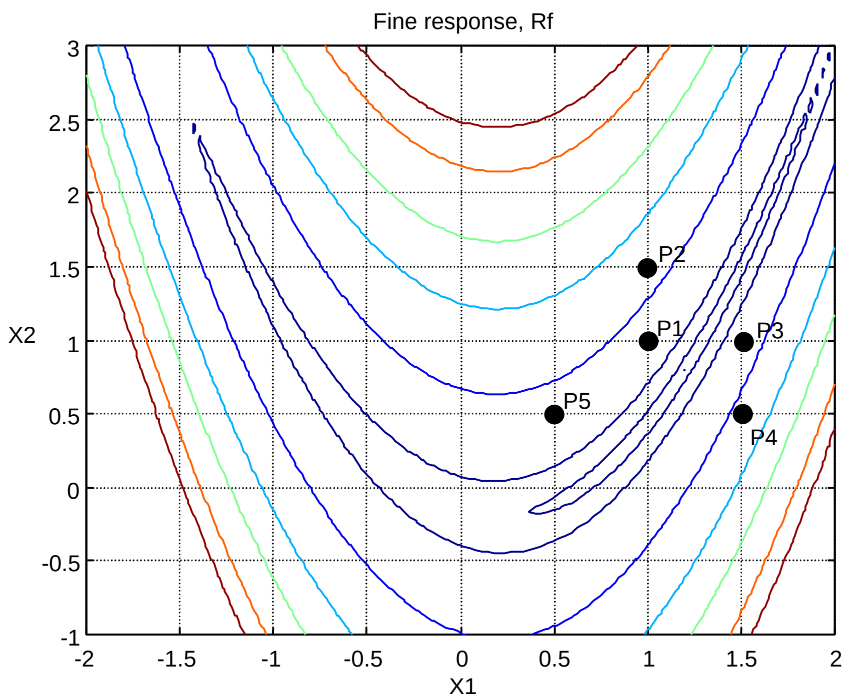
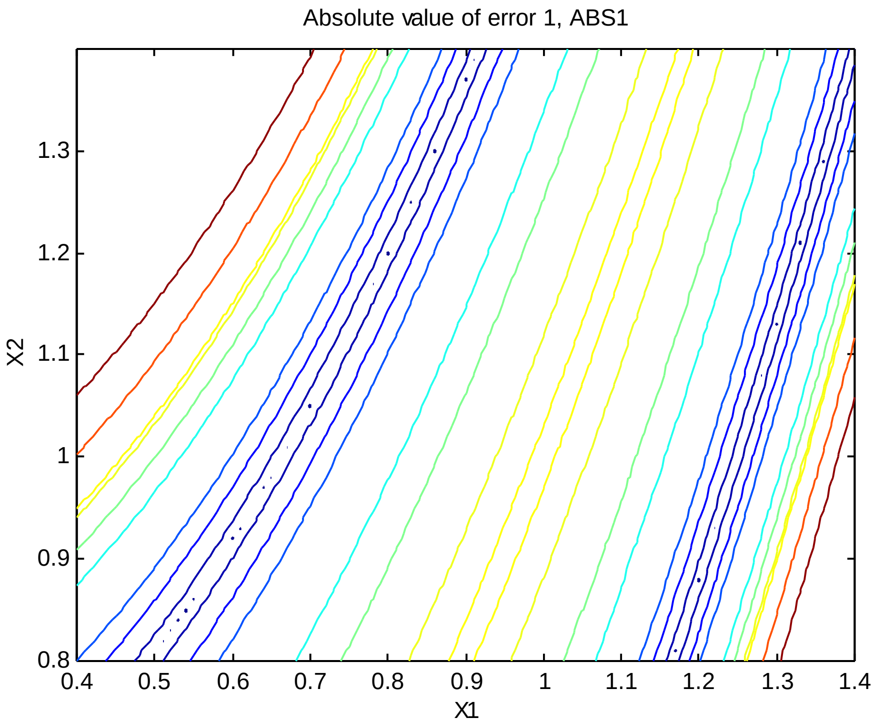


(*b*)

Fig. 3. Perturbed Rosenbrock function or fine response, R_f: (*a*) surface plot; (*b*) contours for levels 0.0001, 0.7, 7, 70, 200, 350, 550, 700.



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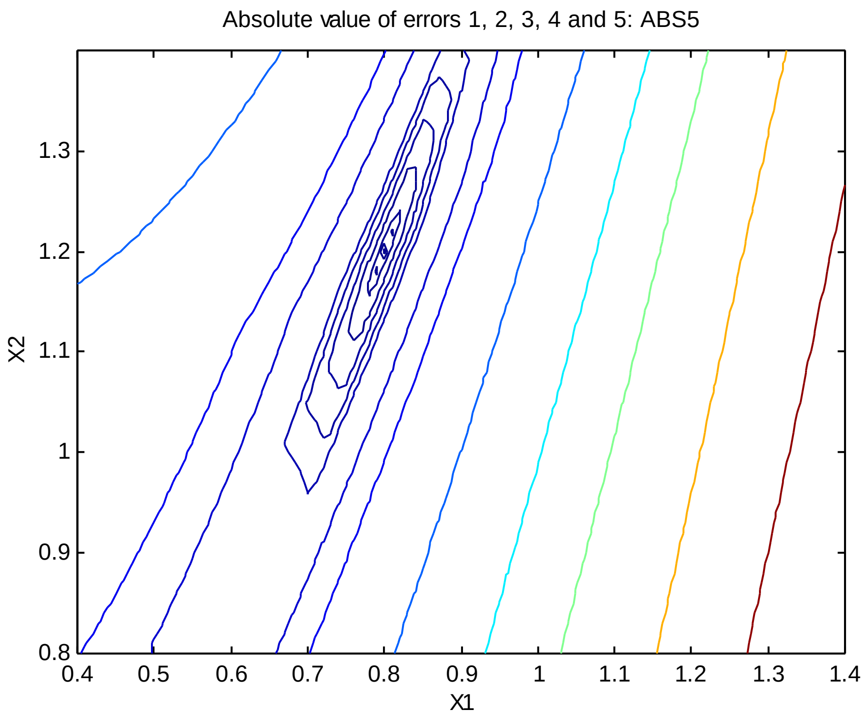
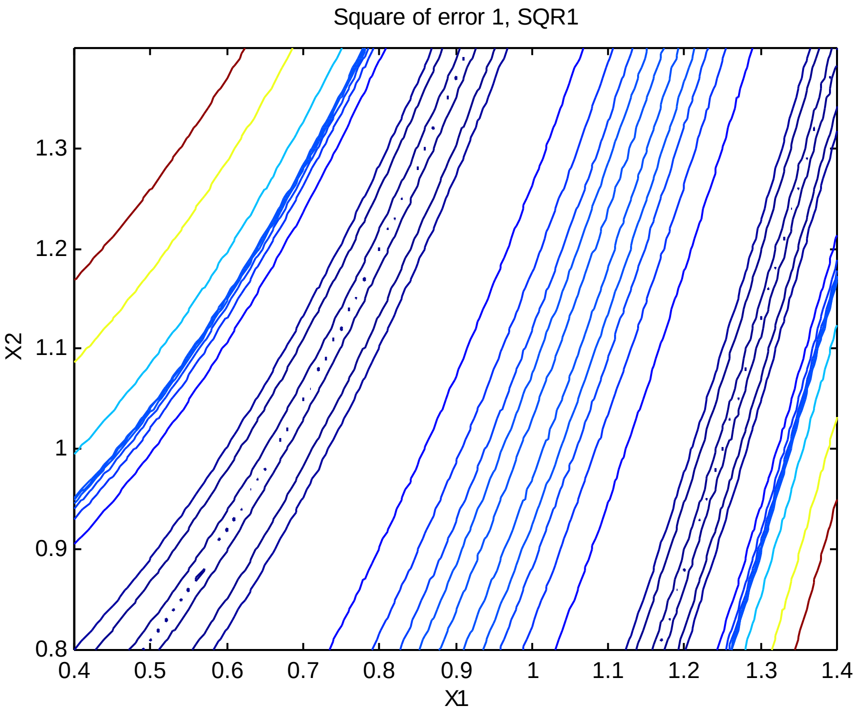


(*b*)

Fig. 4. l_1 objective function for the parameter extraction problem: (*a*) single point parameter extraction; (*b*) multiple point parameter extraction (with 4 additional points).



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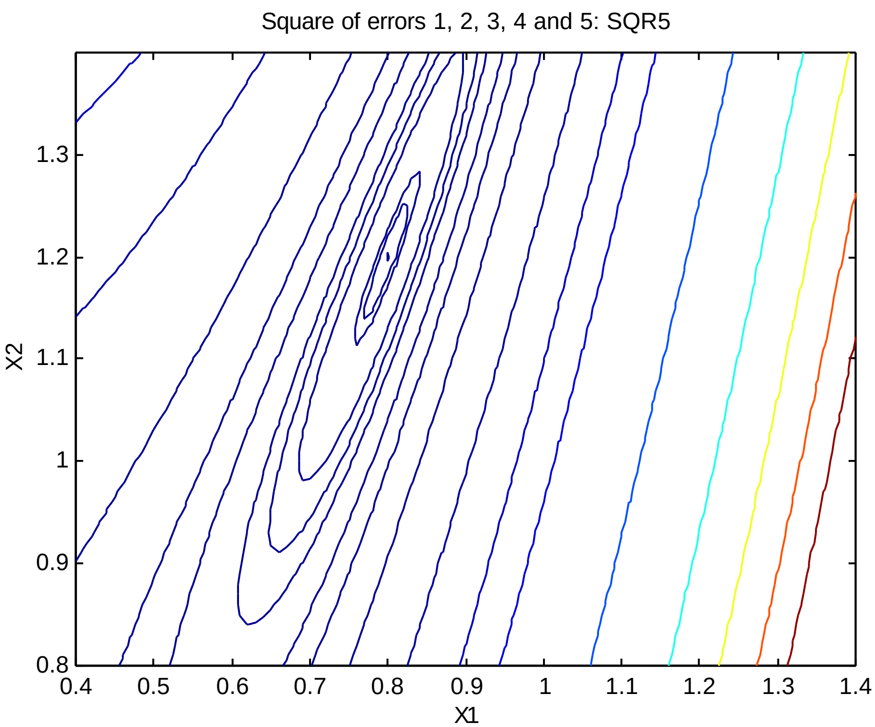

(*b*)

Fig. 4. l_2 objective function for the parameter extraction problem: (*a*) single point parameter extraction; (*b*) multiple point parameter extraction (with 4 additional points).