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# A ROBUST PHYSICS-ORIENTED STATISTICAL GaAs MESFET MODEL

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## Abstract

In this paper we present a robust physics-oriented statistical GaAs MESFET model. Our model integrates the DC Khatibzadeh and Trew model for DC simulation with the Ladbrooke formulas for small-signal analysis (KTL). Accuracy of the statistical KTL model is verified by Monte Carlo simulations using device measurements. Statistical extraction and postprocessing of device physical parameters are carried out by HarPE.

## Introduction

In IC manufacturing fabricated device parameter values deviate randomly from their nominal (or designed) values. These random variations result in complicated distributions and correlations of device responses, and affect production yield. Statistical modeling is needed to characterize the device statistics to provide accurate response prediction for statistical analysis and yield optimization.

Statistical models can be based on equivalent circuit models (ECMs), abstract models, data bases and physics-based models (PBMs). The advantages of PBMs for statistical modeling and yield optimization have been discussed by a number of researchers, e.g., [1-7]. The Ladbrooke model [8], in particular, has been used by Bandler *et al.* [1-3] and by Bastida *et al.* [6] for GaAs MESFET statistical modeling and yield optimization. Very promising results have been reported.

In this paper we present a novel robust GaAs MESFET statistical PBM for small-signal applications which we call the KTL (Khatibzadeh-Trew-Ladbrooke) statistical model. It combines the advantages of the Khatibzadeh and Trew model [9] and the small-signal Ladbrooke model [8] while overcoming their respective shortcomings. The KTL model has been implemented in the CAD systems HarPE [10] and OSA90/hope [11].

We also discuss the concept of data alignment to adjust the measured data to meet the requirement of consistent measurement conditions for statistical modeling. Deterministic models are extracted for individual manufactured outcomes from the corresponding measured data. These models are used to generate "pseudo measurements" at some other bias conditions. We apply the Materka and Kacprzak model [12] to this end.

Using HarPE [10] we illustrate the benefit of the KTL model by statistical characterization of a GaAs MESFET from wafer measurements [13]. Model accuracy is demonstrated by good agreement between Monte Carlo simulations using the KTL model and the statistical data.

## The KTL Model for GaAs MESFETs

The Ladbrooke model [8] is a small-signal model which includes element values derived from device physical/geometrical parameters and intrinsic voltages at the DC operating point. Its attractive statistical properties have already been noticed [1,2,6]. However, the intrinsic voltages at the DC operating point must be determined separately.

On the other hand the Khatibzadeh and Trew model [9] is a large-signal (or global) model which is capable of providing accurate DC solutions. However, for small-signal applications, in particular statistical modeling, it is not as accurate as the Ladbrooke model [2].

For complete and accurate DC/small-signal device simulation we create the KTL model by combining the Ladbrooke model with the Khatibzadeh and Trew model. The latter is employed to solve for the DC operating point needed in establishing the former. Both models share the same physical

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parameters, therefore the resulting combined, or integrated, model is consistently defined.

The KTL small-signal equivalent circuit follows the Ladbrooke model and is shown in Fig. 1. The model includes the intrinsic FET parameters, L, Z, a,  $N_d$ ,  $V_{b0}$ ,  $v_{sat}$ ,  $\mu_0$ ,  $\varepsilon$ ,  $L_{G0}$ ,  $a_0$ ,  $r_{01}$ ,  $r_{02}$ ,  $r_{03}$ , and the linear extrinsic elements,  $L_g$ ,  $R_g$ ,  $L_d$ ,  $R_d$ ,  $L_s$ ,  $R_s$ ,  $G_{ds}$ ,  $C_{ds}$ ,  $C_{ge}$ ,  $C_{dc}$ , where L is the gate length, Z the gate width, a the channel thickness,  $N_d$  the doping density,  $V_{b0}$  the zero-bias barrier potential,  $v_{sat}$  the saturation value of electron drift velocity,  $E_c$  the critical electric field,  $\mu_0$  the low-field mobility of GaAs,  $\varepsilon$  the dielectric constant,  $L_{G0}$  the inductance from gate bond wires and pads,  $a_0$  the proportionality coefficient, and  $r_{01}$ ,  $r_{02}$  and  $r_{03}$  the fitting coefficients [1].

The bias-dependent small-signal parameters, namely,  $g_m$ ,  $C_{gs}$ ,  $C_{gd}$ ,  $R_i$ ,  $L_g$ ,  $r_0$  and  $\tau$ , as shown in Fig. 1, are derived using the modified Ladbrooke formulas once the DC operating point is solved for. For instance,

$$g_{m} = \varepsilon v_{sat} Z/d,$$
  

$$\tau = [0.5X - 2dL/(L+2X)]/v_{sat},$$
  

$$R_{i} = L/[Z\mu_{0}qN_{d}(a-d)],$$
 (1)  

$$C_{gd} = 2\varepsilon Z/(1+2X/L),$$
  

$$r_{0} = r_{01}V_{D'S'}(r_{02} - V_{G'S'}) + r_{03}$$

where  $V_{D'S'}$  and  $V_{G'S'}$  are DC intrinsic voltages from D' to S' and from G' to S', respectively, as shown in Fig. 1.



Fig. 1 Small-signal equivalent circuit, where  $I_d = g_m V_g e^{-j\omega\tau}$ .

The equivalent depletion depth d and the spacecharge layer extension X are defined by

$$d = [2\varepsilon(-V_{G'S'}+V_{b0})/(qN_d)]^{0.5},$$

$$X = a_0 \{2\varepsilon/[qN_d(-V_{G'S'}+V_{b0})]\}^{0.5}(V_{D'G'}+V_{b0}).$$
(2)

#### Measurement Data Alignment

Statistical modeling requires data for different, but supposedly identical, devices to be taken under identical measurement conditions. However, the measurement conditions (e.g., the bias conditions) of different devices may not be identical. We preprocess the data to achieve bias condition alignment.

Measurements on 0.5  $\mu$ m GaAs MESFETs were chosen from the Plessey data [13]. We selected 34 individual devices from Wafer B and 35 individual devices from Wafer D for statistical modeling. Each data set contains small-signal S parameters measured at frequencies from 1 GHz to 21 GHz with 0.4 GHz step under three bias conditions with a fixed drain bias  $V_{DS} = 5$  V. DC drain bias currents are also included in the measurements.

Due to the variations of the measurement conditions the gate bias voltages vary slightly from device to device except for  $V_{GS} = 0$  V. For Wafer B  $V_{GS}$  for the other two bias conditions varies between -0.79 V and -0.59 V (mean value -0.6818 V and standard deviation 6.73 %) and between -1.38 V and -1.09 V (mean values -1.232 V and standard deviation 6.07 %), respectively. For Wafer D  $V_{GS}$  for the other two bias conditions varies between -0.89 V and -0.74 V (mean values -0.8039 V and standard deviation 5.83 %) and between -1.64 V and -1.25 V (mean values -1.347 V and standard deviation 5.93 %), respectively. Therefore, we need to align the different data sets to provide consistent bias points for statistical modeling. It is also desirable to interpolate measured data at other bias points. The Materka and Kacprzak model is a suitable interpolator for this purpose, because of its excellent single device fitting accuracy for these devices.

For each individual device we fitted the Materka and Kacprzak model to its corresponding data set. The resulting models were used to interpolate data for each device at two bias points (gate bias voltages -0.5 V and -0.7 V, drain bias voltage 5 V). In this way we generated 34 data sets for Wafer B and 35 data sets for Wafer D, including DC responses and S parameters from 1 GHz to 21 GHz with 2 GHz step under the two bias conditions.

# Statistical Modeling and Verification

Our statistical modeling technique consists of two stages: multi-device parameter extraction and postprocessing. The two stages, leading to a concise model described by the means, standard deviations, discrete distribution correlation matrix and functions (DDFs), were carried out by HarPE [10]. After alignment of the measurement data, the KTL model parameters were extracted for each device by fitting the model responses to the corresponding Sparameter data and drain bias currents at gate bias -0.5 V and -0.7 V and drain bias 5 V. The 34 (deterministic) models of Wafer B and the 35 models of Wafer D were then postprocessed to obtain the parameter statistics, respectively. The resulting mean values and the standard deviations for Wafer B and Wafer D are listed in Table I and Table II, respectively. Histograms of channel thickness for Wafer B and Wafer D are shown in Fig. 2 and Fig. 3, respectively.

TABLE I

KTL MODEL PARAMETERS FOR WAFER B

Mean

Parameter

TABLE II KTL MODEL PARAMETERS FOR WAFER D

Parameter	Mean	Std. Dev. (%)
L(μm)	0.5055	3.93
<i>a</i> (μm)	0.1337	2.49
$N_d$ (m <sup>-3</sup> )	$2.2885 \times 10^{23}$	2.19
$v_{sal}(m/s)$	$9.8251 \times 10^{4}$	5.22
$L_{G0}(nH)$	0.0375	15.4
$r_{01}(\Omega/V^2)$	0.3463	2.15
$r_{02}(\Omega)$	1979.0	2.15
a	0.9337	5.71
$R_d(\Omega)$	1.0416	1.70
$R_{s}(\Omega)$	3.8814	4.77
$R_{\rho}(\Omega)$	6.5256	0.41
$L_d(nH)$	0.0499	12.7
$L_{\star}(nH)$	0.0359	8.10
$G_{ds}(1/\Omega)$	3.6315×10 <sup>-3</sup>	3.71
$C_{d}(\mathbf{pF})$	0.0517	1.92
$C_{\alpha}(\mathbf{pF})$	0.0733	7.74
$C_{d}(\mathbf{pF})$	0.0106	2.75
$C_{\rm x}(\rm pF)$	3.7355	12.1

Z = 300  $\mu$ m,  $\varepsilon$  = 12.9,  $V_{b0}$  = 0.6 V,  $r_{03}$  = 7.0 V and  $\mu_0$  = 6.0×10<sup>-10</sup> m<sup>2</sup>/Vns are fixed parameters.

The bias-dependent linear extrinsic element  $L_g$  is computed using the Ladbrooke formula [8].



L(μm) 0.5237 2.84

Std. Dev. (%)

a(um)	0 1/38	9 97
$u(\mu m)$	0.1450	2.57
N <sub>d</sub> (m°)	2.1857×10~	1.88
v <sub>sat</sub> (m/s)	$10.6416 \times 10^{4}$	2.85
$\mu_0(m^2/Vns)$	5.8309×10 <sup>-10</sup>	2.26
$L_{G0}(nH)$	0.0355	15.0
$r_{01}(\Omega/V^2)$	0.3525	0.277
$r_{02}(\Omega)$	2014.5	0.276
a <sub>0</sub>	0.9978	1.19
$R_d(\Omega)$	1.0169	1.27
$R_{s}(\Omega)$	3.5209	3.46
$R_{\rho}(\Omega)$	6.5181	0.22
$L_{d}(nH)$	0.0766	9.58
$L_{(nH)}$	0.0382	3.75
$G_{d_{1}}(1/\Omega)$	3.7406×10 <sup>-3</sup>	1.63
$C_{ds}(\mathbf{pF})$	0.0505	1.57
$C_{\mathbf{g}}(\mathbf{pF})$	0.0669	5.84
$C_{de}(\mathbf{pF})$	0.0104	2.16
$C_{\rm x}(\rm pF)$	3.2699	1.69

 $\overline{Z} = 300 \ \mu\text{m}, \ \varepsilon = 12.9, \ V_{b0} = 0.6 \ \text{V} \text{ and } r_{03} = 7.0 \ \text{V} \text{ are fixed (non-statistical) parameters.}$ 

The bias-dependent linear extrinsic element  $L_g$  is computed using the Ladbrooke formula [8].

Fig. 2 Histograms of channel thickness for Wafer B.



Fig. 3 Histograms of channel thickness for Wafer D.

For verification, 400 Monte Carlo outcomes were generated using the statistical KTL model. The statistics of the simulated S parameters and DC drain currents for those 400 outcomes were compared with the statistics of the data. The mean values and standard deviations from the data and the KTL model including S parameters at frequency 11 GHz and the DC drain currents at both bias points for Wafer B and Wafer D are listed in Table III and Table IV, respectively. Note that the statistics of the data and the KTL model responses are consistent.

#### Conclusions

We have presented the KTL model: a physicsoriented model for GaAs MESFETs particularly suitable for robust statistical device characterization. Our experiments demonstrate its ability to accurately represent the statistical properties of MESFETs. KTL has been implemented in HarPE and OSA90/hope and is suitable for both nominal design and yield optimization of small-signal circuits. Using KTL, exciting results have been achieved in yield-driven amplifier design [3].

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TABLE III MEAN VALUES AND STANDARD DEVIATIONS OF DATA AND KTL MODEL RESPONSES FOR WAFER B

	Bias 1				Bias 2			
	Data		KTL		Data		KTL	
	Mean	Dev. (%)	Mean	Dev. (%)	Mean	Dev. (%)	Mean	Dev. (%)
S <sub>11</sub>	0.777	0.83	0.778	0.63	0.780	0.81	0.788	0.61
$\sum S_{11}$	-104.7	1.32	-105.8	1.00	-101.3	1.38	-102.7	1.07
$ S_{21} $	1.793	1.17	1.739	1.44	1.703	1.61	1.700	1.47
$\sum_{i=1}^{n}$	96.80	0.61	96.8 <b>2</b>	0.56	97. <b>78</b>	0.60	98.54	0.57
$ S_{12} $	0.090	2.46	0.092	1.28	0.095	2.49	0.096	1.22
$/S_{12}$	35.30	1.35	35.64	1.58	35.95	1.30	34.80	1.59
$\overline{ S_{22} }$	0.571	0.90	0.574	0.7 <b>2</b>	0.572	0.91	0.576	0.71
/S22	-39.58	1.23	-40.03	1.21	-39.91	1.21	-40.53	1.20
$\overline{I}_d(\mathbf{\tilde{A}})$	0.040	8.16	0.0398	7.71	0.033	9.51	0.033	8.76
Bias 1: $V_{GS} = -0.5 \text{ V}, V_{DS} = 5 \text{ V}.$								
$P_{inc} $ $\mathcal{H} = 0.7 V \mathcal{H} = 5 V$								

Bias 2:  $V_{GS} = -0.7$  V,  $V_{DS} = 5$  V.

Frequency is 11 GHz for S parameters.

TABLE IV MEAN VALUES AND STANDARD DEVIATIONS OF DATA AND KTL MODEL RESPONSES FOR WAFER D

	Bias 1				Bias 2			
	Data		KTL		Data		KTL	
	Mean	Dev. (%)	Mean	Dev. (%)	Mean	Dev. (%)	Mean	Dev. (%)
S11	0.784	0.44	0.785	0.59	0.787	0.45	0.794	0.59
$/S_{11}$	-103.9	2.24	-105.8	1.81	-100.4	2.37	-102.7	1.91
$\overline{ S_{21} }$	1.725	2.14	1.648	2.88	1.612	3.0	1.608	2.95
/S21	97.06	0.99	96.96	0.82	97.91	0.94	9 <b>8.69</b>	0.84
S <sub>12</sub>	0.096	3.35	0.095	3.11	0.102	3.45	0.100	3.07
$/S_{12}$	34.51	1.78	33.97	1.99	35.25	1.76	34.19	2.08
$\overline{\left[S_{22}\right]}$	0.583	1.18	0.591	1.01	0.588	1.09	0.593	1.00
/Sm	-40.51	0.97	-40.40	0.92	-40.47	0.84	-40.88	<b>0.92</b>
$\overline{I_d}(\mathbf{A})$	0.031	9.54	0.031	9.73	0.025	11.2	0.025	11.0
Bias 1: $V_{GS} = -0.5 \text{ V}, V_{DS} = 5 \text{ V}.$								
Bias 2: $V_{CS} = -0.7 \text{ V}, V_{DS} = 5 \text{ V}.$								
Frequency is 11 GHz for S parameters.								

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