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STATISTICAL MODELING OF GaAs MESFETs

J.W. Bandler*, R.M. Biernacki*, S.H. Chen*, J. Song*, S. Ye* and Q.J. Zhang**

Abstract This paper contrasts the statistical extraction of GaAs MESFET equivalent circuit model parameters and physical model parameters from wafer measurements. We observe that the Materka and Kacprzak model based on equivalent circuit parameters provides a better match for individual devices, but the Ladbrooke model based on physical parameters provides a better estimate of device statistics.

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I. INTRODUCTION

Statistical modeling is a prerequisite for yield-driven and cost-driven circuit optimization [1-3]. Purviance *et al.* [3] investigated the use of FET equivalent circuit model parameter statistics in circuit design. However, the ability of equivalent circuit models to reflect the actual device statistics is questionable [4]. Purviance *et al.* proposed [4] to rely on individual models obtained from measured data without extracting sample statistics. This approach, however, is limited by the actual measurement sample size.

In this paper, we present a study on statistical modeling of GaAs MESFETs at the equivalent circuit model parameter level and the physical model parameter level. We contrast the Materka and Kacprzak model [5], which is based on equivalent circuit parameters, with the Ladbrooke model [6] which is defined in terms of physical parameters. The model parameters are extracted from GaAs MESFET wafer measurements provided by Plessey Research Caswell [7]. The measurements consist of DC bias data and multi-bias S parameters from a sample of GaAs MESFET devices. The parameter extraction and statistical postprocessing are automated by the statistical modeling features of HarPE™ [8].

Our results show that modeling at the equivalent circuit parameter level is more flexible and therefore may provide a better match for individual devices. But the Materka and Kacprzak model with equivalent circuit parameter statistics failed to reproduce the sample statistics of the measured data. In contrast, the Ladbrooke model at the physical parameter level can provide a better estimate of the statistical spread of the measurements.

II. THE GAAS MESFET MODELS

The Materka and Kacprzak model [5] is an equivalent circuit model. The equivalent circuit is defined directly by the model parameters. The model parameters to be extracted include the nonlinear intrinsic FET parameters

$$\{I_{DSS}, V_{p0}, \gamma, E, K_E, \tau, S_S, R_{10}, K_R, C_{10}, C_{1S}, K_1, C_{F0}, K_F\}$$

and the linear extrinsic parameters

$$\{L_G, R_G, R_D, L_D, R_S, L_S, G_{DS}, C_{DS}\}.$$

Some of the model parameters are not involved because they are related to the large-signal nonlinear characteristics of the model and have little influence on the responses of interest here.

The Ladbrooke model [6] also uses an equivalent circuit, as shown in Fig. 1. But the model is defined in terms of the device physical parameters, such as the MESFET gate length and doping profile. The equivalent circuit is derived from the physical parameters and the bias conditions. In Fig. 1, G_m , τ , r_0 , C_{GS} , C_{DG} , R_i , R_D , R_S , and L_G are functions of bias conditions and the physical parameters. The model parameters to be extracted are

$$\{L_{g0}, W, N, v_{sat}, V_{B0}, a_0, r_{10}, r_{20}, r_{30}, R_G, L_D, L_S, G_{DS}, C_{DS}\}$$

where L_{g0} is the gate length, W the channel thickness, N the doping density, v_{sat} the saturated value of electron drift velocity, V_{B0} the zero-bias barrier potential, a_0 is a proportionality coefficient, r_{10} , r_{20} and r_{30} are coefficients of r_0 , and the others are linear extrinsic parameters.

III. STATISTICAL MODELING

From the sample of GaAs MESFET measurements provided by Plessey Research Caswell [7], we use 69 individual devices (data sets) from two wafers. Each device represents a four finger $0.5\mu\text{m}$ GaAs MESFET with equal finger width of $75\mu\text{m}$. Each data set contains small-signal S parameters measured under three different bias conditions and at frequencies from 1GHz to 21GHz with a 0.4GHz step. DC drain bias current is also included in the measurements.

We use HarPE [8] to extract the statistical device models. The measurements used for parameter extraction include DC bias currents at three bias points and the S parameters for those bias points at frequencies from 1GHz to 21GHz with a 2GHz step.

We first extract model parameters for each individual device by matching simultaneously the DC and small-signal S parameter responses to the corresponding measurements [9]. The resulting sample of 69 models is postprocessed to obtain the parameter statistics, including the mean value, standard deviation and discrete distribution function (DDF) for each parameter, as

well as the correlations among the parameters. The postprocessing is automated by the statistical modeling feature of HarPE.

The parameter statistics (mean values and standard deviations) of the Ladbroke model and those of the Materka and Kacprzak model are listed in Table I. Fig. 2 illustrates the histograms of the FET gate length L_{g0} (a parameter of the Ladbroke model) and I_{DSS} (a parameter of the Materka and Kacprzak model).

The postprocessed statistical model can be used for nominal and Monte Carlo simulation. In a Monte Carlo simulation, statistical outcomes are generated from the parameter statistics. In a nominal simulation, the parameters assume their mean values. Fig. 3 shows the match between the S parameters computed from a nominal simulation (i.e., the parameters assume their mean values) and the mean values of the measured S parameters at the bias point $V_{GB}=0V$ and $V_{DB}=5V$. Excellent fit by the Materka and Kacprzak model and a good agreement by the Ladbroke model can be observed. This indicates that modeling at the equivalent circuit parameter level is more flexible and therefore can provide a better match for individual devices.

IV. MODEL VERIFICATION

For statistical modeling to be useful in yield analysis and optimization, we must be able to predict the statistical behaviour of the actual devices through Monte Carlo simulation, i.e., the model responses and the actual device responses must be statistically consistent.

To this end we compare the statistical characteristics of the S parameters of the extracted MESFET models with the measurements. The comparison is made at the bias point $V_{GB}=0V$ and $V_{DB}=5V$ and at the frequency 11GHz. For Monte Carlo simulation, we generate 400 outcomes from the mean values, standard deviations, correlations and DDFs of the model parameters [10].

The mean values and standard deviations of the measured S parameters and the simulated S parameters from the Ladbroke model and from the Materka and Kacprzak model are listed in Table II. We can see from Table II that the standard deviation match given by the

Ladbrooke model is good, while large mismatches by the Materka and Kacprzak model exist. On the other hand, the mean value match for the Materka and Kacprzak model appears to be much better than that for the Ladbrooke model. However, the mean value discrepancies for the Ladbrooke model are consistent with the S parameter match shown in Fig. 3. In other words, the error in the mean value estimate is largely due to the deficiency of the model in matching the measurements of individual devices. Such deficiency can be viewed as a *deterministic* factor resulting in a deterministic shift in the estimated mean value. If adjusted for such a shift, the discrepancies in the mean values estimated by the Ladbrooke model would be reduced.

We also plot histograms of one S parameter as shown in Fig. 4, to further explore the validity of the models as suggested in [4]. It is very interesting to see that the Ladbrooke model closely reproduces the distribution pattern (spread) of the S parameters.

V. CONCLUSIONS

We have presented a case study, based on 69 devices, of statistical GaAs MESFET device modeling. We have shown that the Ladbrooke model based on physical parameters can preserve the statistical characteristics of the actual device. We could, therefore, use it in statistical circuit designs. We have also shown that the Materka equivalent circuit model can accurately fit the data from which the model parameters are extracted, because it has fewer constraints than the physical model.

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TABLE I
 STATISTICAL PARAMETERS FOR THE LADBROOKE
 AND THE MATERKA AND KACPRZAK MODELS

Ladbroke Model			Materka and Kacprzak Model		
Parameter	Mean	Deviation	Parameter	Mean	Deviation
$L_{g0}(\mu\text{m})$	0.561	3.44%	$I_{DSS}(\text{mA})$	47.42	10.6%
$W(\mu\text{m})$	0.106	4.87%	$V_{p0}(\text{V})$	-1.4928	11.9%
$N(\text{m}^{-3})$	3.082	4.13%	γ	-0.1053	7.32%
$v_{sat}(\text{ms}^{-1})$	7.671	4.49%	E	1.6594	2.39%
$V_{B0}(\text{V})$	0.665	6.83%	$K_E(1/\text{V})$	4.819E-3	16.5%
$r_{01}(1/\text{A}^2)$	1.100E-2	1.99%	$\tau(\text{pS})$	2.1901	2.89%
$r_{02}(\text{V})$	589.6	9.03%	$S_S(1/\Omega)$	1.539E-3	11.7%
$r_{03}(\Omega)$	14.07	1.93%	$L_G(\text{nH})$	3.424E-2	17.8%
$R_G(\Omega)$	3.417	10.8%	$R_G(\Omega)$	9.956E-3	22.0%
$L_D(\text{nH})$	5.942E-2	22.2%	$R_D(\Omega)$	2.331	42.8%
$L_S(\text{nH})$	2.153E-2	11.1%	$L_D(\text{nH})$	5.141E-2	32.4%
$G_{DS}(1/\Omega)$	2.069E-3	6.14%	$R_S(\Omega)$	0.7051	60.0%
$C_{DS}(\text{pF})$	5.417E-2	3.21%	$L_S(\text{nH})$	1.455E-2	24.3%
			$G_{DS}(1/\Omega)$	1.853E-3	6.10%
			$C_{DS}(\text{pF})$	5.811E-2	4.24%
			$R_{10}(\Omega)$	7.694	9.11%
			$K_R(1/\text{V})$	0.3332	20.8%
			$C_{10}(\text{pF})$	0.3672	4.70%
			$C_{1S}(\text{pF})$	1.551E-3	75.6%
			$K_1(1/\text{V})$	1.225	10.8%
			$C_{F0}(\text{pF})$	1.631E-2	4.75%
			$K_F(1/\text{V})$	-0.1178	3.44%

TABLE II
 MEAN VALUES AND STANDARD DEVIATIONS OF
 MEASURED AND SIMULATED S PARAMETERS

	Measured S Parameters		Simulated S Parameters			
			Ladbroke Model		Materka and Kacprzak Model	
	Mean	Deviation	Mean	Deviation	Mean	Deviation
MS ₁₁	0.773	0.988%	0.7867	1.25%	0.7707	1.97%
PS ₁₁	-114.3	1.36%	-118.5	0.982%	-114.8	1.71%
MS ₂₁	1.919	0.802%	1.672	1.25%	1.929	14.7%
PS ₂₁	93.35	0.856%	94.61	0.766%	93.41	0.902%
MS ₁₂	0.0765	3.77%	0.07554	3.64%	0.07546	5.4%
PS ₁₂	34.00	2.51%	32.18	2.46%	33.69	2.57%
MS ₂₂	0.5957	1.48%	0.5834	1.63%	0.5935	4.27%
PS ₂₂	-38.69	2.10%	-36.87	1.52%	-37.82	3.48%

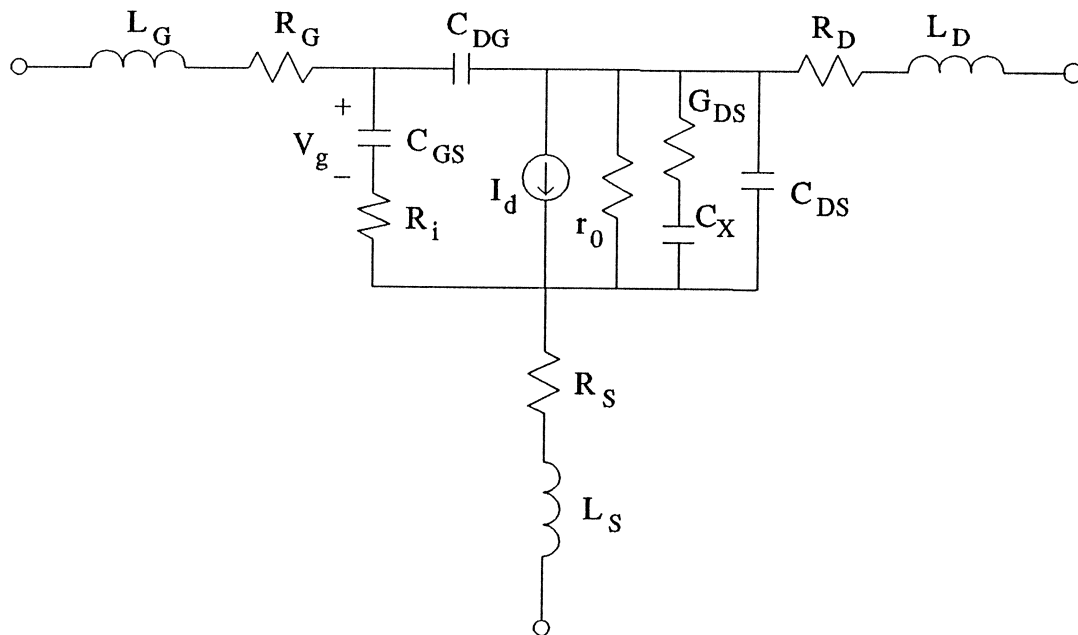
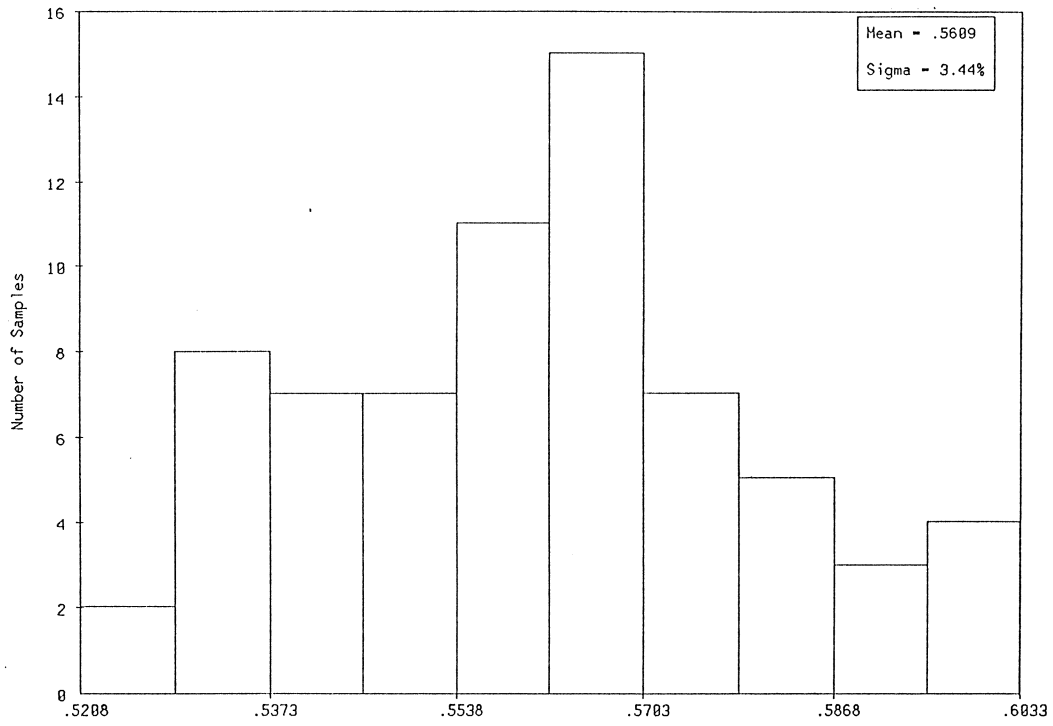
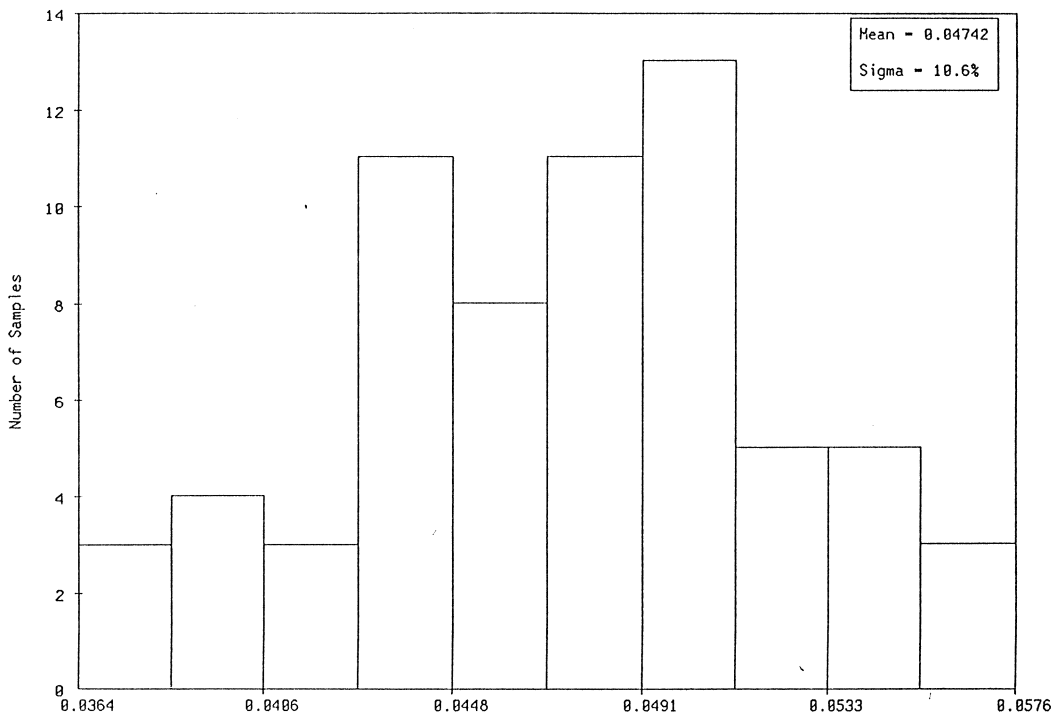


Fig. 1 Topology for the Ladbrooke GaAs MESFET small-signal model where $I_d = G_m V_g e^{-j\omega\tau}$.

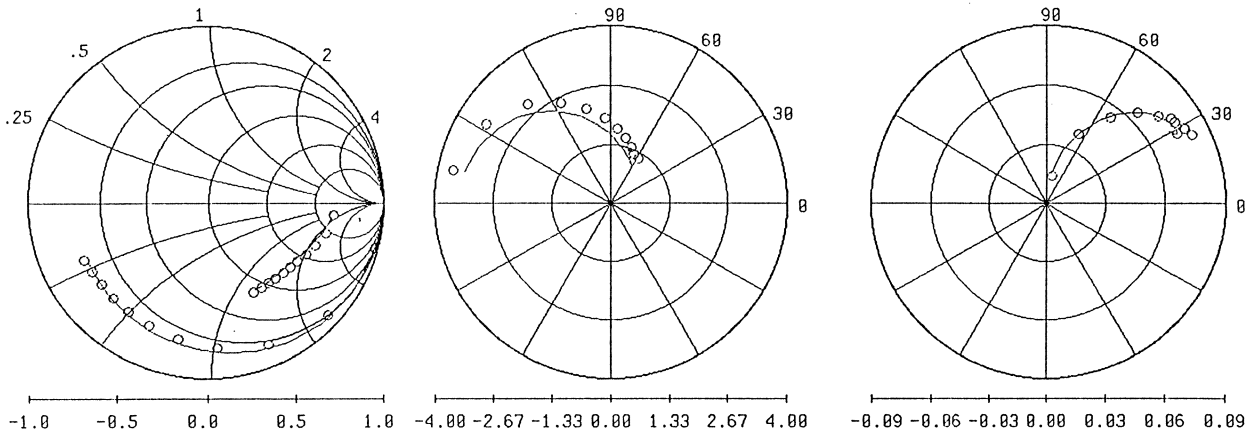


(a)

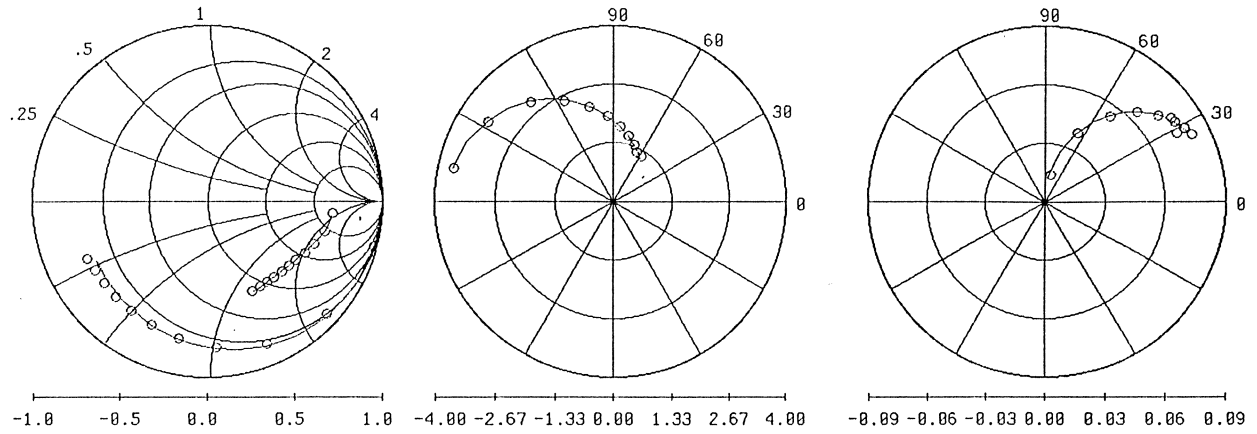


(b)

Fig. 2 Model parameter histograms. (a) Gate length L_{g0} in the Ladbrooke model. (b) I_{DSS} in the Materka model.

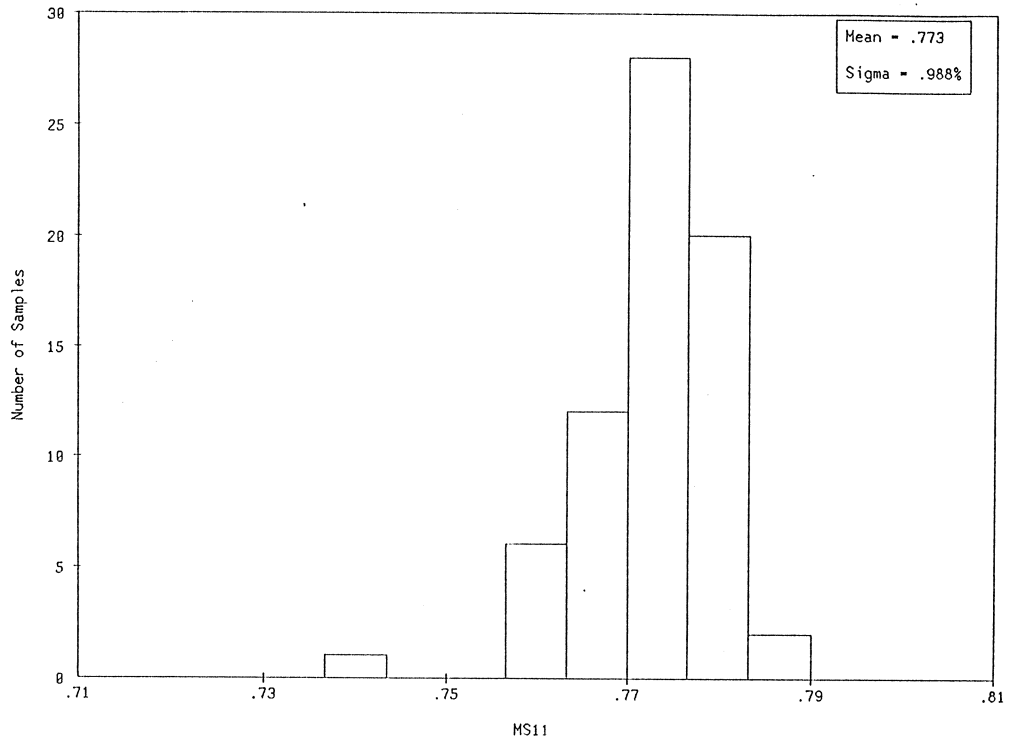


(a)

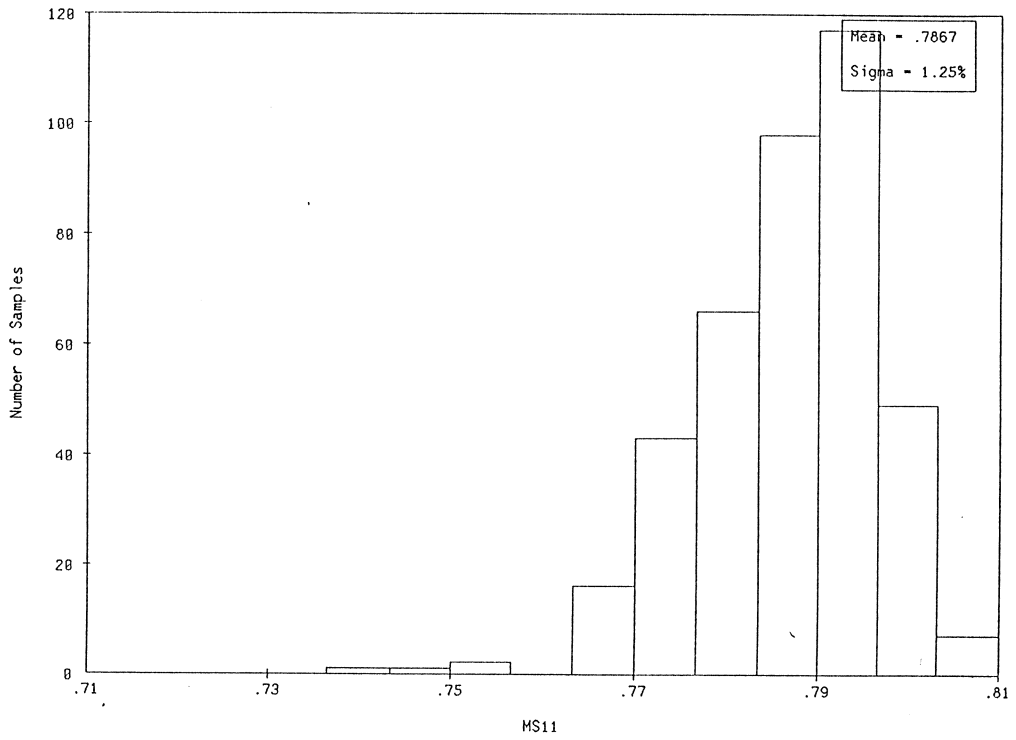


(b)

Fig. 3 S-parameter fit. Circles represent the mean-valued S parameters at bias $V_{GD}=0V$ and $V_{DB}=5V$. Solid lines are model responses simulated from the mean parameter values. (a) Fit by the Ladbroke model. (b) Fit by the Materka model.



(a)



(b)

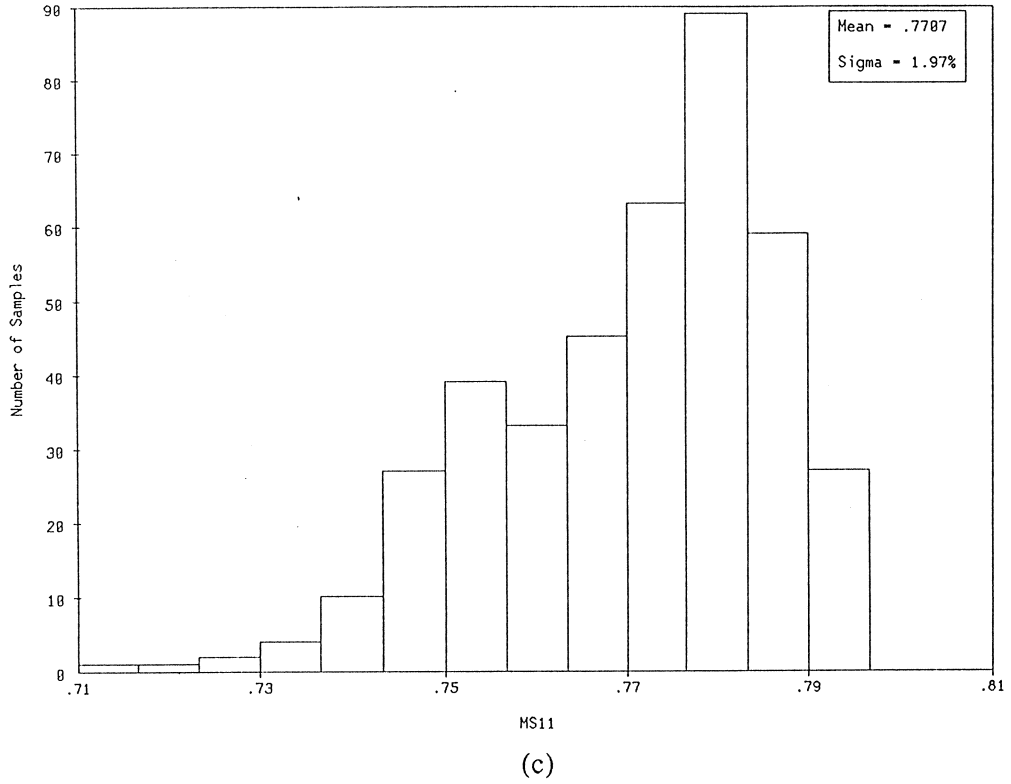


Fig. 4 Measured and simulated S-parameter distributions at bias $V_{GB}=0V$ and $V_{DB}=5V$ and at frequency 11GHz. Histograms from (a) measurements, (b) the Ladbrooke model, and (c) the Materka model.