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LARGE SCALE OPTIMIZATION OF ANALOG CIRCUITS WITH MICROWAVE APPLICATIONS

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LARGE SCALE OPTIMIZATION OF ANALOG CIRCUITS WITH MICROWAVE APPLICATIONS

LARGE SCALE OPTIMIZATION OF ANALOG CIRCUITS WITH MICROWAVE APPLICATIONS

By

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ABSTRACT

This thesis addresses itself to computer oriented techniques for large scale optimization of analog circuits. New techniques for simulation and sensitivity analysis are described and are used to improve the performance of circuit optimization. A powerful automatic decomposition technique is developed directly enabling a normal optimizer to solve large circuit problems. Our theory is applied to the design of microwave circuits.

The status of large scale circuit optimization and the state-of-the-art of microwave CAD are reviewed. The necessity of circuit oriented optimization techniques is demonstrated by formulating design, modelling, diagnosis and tuning into optimization problems.

A comprehensive treatment of large change sensitivity computation for linearized circuits using generalized Householder formulas is presented. A technique for circuit response updating via a minimum order reduced system is developed. By avoiding re-analysis of the complete circuit, our method is responsible for efficient simulation of large circuits when a subset of the circuit parameters is frequently perturbed.

An elegant theory for simulation and exact sensitivity analysis of branched cascaded networks is described. Our approach explicitly takes the circuit structure into consideration and does not deteriorate as the overall network becomes large. The practicality of the theory is illustrated by efficient optimization of microwave multiplexers consisting of multi-cavity filters distributed along a waveguide manifold. Examples of optimizing 12- and 16-channel multiplexers are provided.

A novel and general automatic decomposition technique for large scale optimization of microwave circuits is presented. The partitioning approach proposed by Kondoh for FET modelling problems is verified. The application of our technique is demonstrated by the large scale optimization of a 16-channel multiplexer involving 399 nonlinear functions and 240 variables.

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INTRODUCTION

Circuit oriented optimization techniques have been instrumental in advancing the state-of-the-art in computer-aided circuit design. During the past two decades of active research and development by both numerical analysts and engineering professionals, optimization techniques have gained popularity and appreciation by circuit designers. The power of these techniques is further enhanced due to the astonishing progress made in the computer industry. With their high speed and vast capacity in data processing, computers are now being used to optimize circuits with accuracies and sophistication that were only dreams of the previously unaided designers.

Interestingly enough, the ambition of electrical engineers grows as fast as their computing capability. A serious advance they have made is the increased size and complexity of today's analog systems. On the other hand, requirements on the accuracy and practicality of design and modelling methods become more stringent, which in turn necessitates the use of sophisticated techniques such as multi-circuit modelling and yield maximization. Large scale problems become a critical consequence due to the increase in both the size of circuits and the complexity of design methods. The solution of large circuit problems challenges researchers even equipped with up-to-date computers. Considerable efforts by software engineers are inevitable before a complicated system can be designed.

The immediate difficulty with large scale problems is due to the limitation of computer hardware. Prohibitive CPU times and storage requirement often make ordinary CAD software balk at large problems. A frequent frustration with large scale optimization is the increased likelihood of stopping at an undesired local minimum. Other difficulties, especially in prototype and postproduction tuning, are due to human inability to cope with problems involving large numbers of independent variables to be adjusted simultaneously to meet a specified response pattern over a wide frequency range (Bandler and Zhang 1987c).

Fundamental to all circuit optimization procedures is an efficient circuit simulator. A vital mechanism for a powerful gradient based optimizer is a circuit sensitivity analyzer. The preliminary step towards large scale optimization is the development of elegant simulation and sensitivity analysis techniques. Lastly and most importantly, the mathematical optimizer itself must be made capable of handling the large numbers of variables and functions.

Consider the effort of solving N linear equations with N unkowns as an example. As N increases, the storage requirement increases quadratically and the computational effort increases cubically. The trouble often associated with much existing software is that some simple but redundant operations, almost trivial for ordinary problems, become unbearable for large scale problems. Three such situations are worth serious consideration.

One situation occurs when a program, designed to be powerful for a complete circuit simulation, is used very repetitively. Examples can be found in sensitivity approximation, design, tuning and yield optimization where only a few elements or only a subnetwork are frequently adjusted. The straightforward approach is to simply repeat the entire circuit simulation each time even when a majority of circuit subnetworks remain unperturbed. For large circuits, this becomes extremely inefficient.

The second situation is when a general circuit analysis method is applied to a specially structured network. Obviously, a certain amount of operations and storage will become redundant. For example, large microwave circuits belonging to the category of branched cascaded structures should not be treated using the general nodal analysis method and the adjoint network approach, unless special mathematical tools, e.g., the sparse matrix technique is used.

The third situation can be observed directly in an optimization procedure. A general optimizer takes all given functions and variables into consideration. For large scale optimization problems, this is not always necessary. For example, there often exist weak interconnections between certain variables and functions that could be decoupled during initial optimization stages.

The realization of the above facts has prompted investigations into a number of approaches to improve large scale CAD techniques. We can exploit possible properties of the particular circuit, e.g., physical or topological properties. We can use advanced mathematical tools. We can rearrange CAD software into suitable formats for special computers, e.g., the vector processors.

From the simulation and analysis points of view, large scale circuit problems have been fairly treated in the literature. But from the optimization point of view, only sparse and loosely related materials are available.

This thesis attempts to offer a formal treatment to the problem of large scale optimization of analog circuits. We propose new approaches to improve simulation, sensitivity evaluation and optimization, which are all essential for executing a circuit optimization. The approaches include a comprehensive treatment of large change sensitivity computation for repeated circuit analysis, an elegant method for simulation and sensitivity analysis of branched cascaded networks and an automatic decomposition technique for circuit optimzation. The thesis is based on frequency domain equivalent circuit models. The automatic decomposition technique directly enables a mathematical optimizer to handle large circuit problems. The technique is used together with our branched cascaded analysis method to produce the optimal solution of a 16-channel microwave multiplexer involving 240 variables and 399 nonlinear functions, representing the state-of-the-art in circuit optimization.

The next two chapters serve as general review and introduction to some important aspects of contemporary circuit oriented optimization techniques, namely, optimization techniques for design, modelling, diagnosis and tuning. In Chapter 2, we review the state-of-the-art in large scale circuit optimization and in microwave CAD. The design centering problem is formulated as a minimax optimization problem.

Chapter 3 offers a review of optimization techniques for modelling, diagnosis and tuning (MDT) of electrical circuits. A general formulation of circuit diagnosis as an optimization problem is introduced. It is followed by a detailed investigation into three specific formulation cases. Optimization methods for modelling and tuning are presented and compared with those for diagnosis.

Chapter 4 presents an efficient approach to large change sensitivity analysis in linear systems. The approach is based upon a set of generalized Householder formulas. Efficient schemes for computing large change sensitivities of a linear system with different numbers of inputs and outputs are developed. The concept of response updating via solving a minimum order reduced system is introduced. A systematic approach to formulating a minimum order reduced system for linear circuits is devised.

In Chapter 5, we describe a novel approach to the simulation and sensitivity analysis of branched cascaded networks. Formulas are derived for such responses as input or output reflection coefficient, common port and branch output port return loss, insertion loss, gain slope and group delay. Exact sensitivities w.r.t. all variables of interest, including frequency, are evaluated. An explicit algorithm is provided describing the details of the computational aspects of our theory. Our approach is used in the optimal design of microwave multiplexers consisting of multi-cavity filters distributed along a waveguide manifold.

In Chapter 6, we describe a powerful and general decomposition technique for optimization of large microwave systems. Using sensitivity information, variables and functions are systematically grouped following the construction of a decompostion dictionary. The overall problem is automatically separated into a sequence of sub-optimizations. The partitioning approach proposed by Kondoh for FET modelling problems is verified. The technique is successfully tested on large scale optimization of microwave multiplexers involving 16 channels, 399 nonlinear functions and 240 variables.

We conclude in Chapter 7 with some suggestions for further research.

The author contributed substantially to the following original developments presented in this thesis:

(1) The use of generalized Householder formulas for large change sensitivity analysis, and a comprehensive treatment to the efficient computation of response changes of linear systems with different numbers of inputs and outputs.

(2) A systematic scheme for direct formulation of a minimum order reduced system for linear circuit response updating.

(3) A simple algebraic treatment to branched cascaded networks and a set of formulas for calculating various responses and their sensitivities for such networks.

(4) An algorithm for systematic simulation and exact sensitivity analysis of branched cascaded networks.

(5) A theory and an algorithm for automatic decomposition in large scale microwave optimization problems.

(6) A theoretical description of multiplexer decomposition properties and their use in optimal design of practical multiplexers.

2

LARGE SCALE CIRCUIT OPTIMIZATION – REVIEW AND BASIC CONCEPTS

2.1 INTRODUCTION

The use of optimization techniques in circuit design has been advocated for over 20 years. These techniques are now widely appreciated as essential CAD tools in the design of analog circuits. In the case of large scale circuit design, however, direct use of optimization is only sparsely reported in the literature.

In the same period, the use of computers in microwave circuit design also received serious attention. In 1969, the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES Special Issue on Computer-Oriented Microwave Practices summarized the early developments in this area. Since then, extensive research has been performed resulting in various successful CAD techniques for analysis, modelling and design. Sophisticated microwave CAD software is being marketed and used.

In this chapter, we review the state-of-the-art in large scale circuit optimization and in microwave CAD. The basic mathematical formulation of circuit design as an optimization problem is introduced.

2.2 REVIEW OF LARGE SCALE CIRCUIT OPTIMIZATION

Temes and Calahan (1967) are among the earliest to formally advocate the use of iterative optimization in circuit design. Waren, Lasdon and Suchman (1967) illustrated that optimization methods can be applied to a wide range of engineering problems. The introduction of adjoint network methods for circuit sensitivity computation by Director and Rohrer (1969) facilitated the use of powerful gradient based optimization methods in circuit design. Bandler (1969, 1973) systematically treated the formulation of error functions, the least pth objective, nonlinear constraints, optimization methods and circuit sensitivity analysis. Further activities in this area are summarized by Director (1971), Charalambous (1974), Bandler and Rizk (1979), Brayton, Hachtel and Sangiovanni-Vincentelli (1981). The recent review paper of Bandler and Chen (1987) addressed a variety of circuit optimization techniques for realistic design and modelling problems. Dennis Jr. (1984) gave electrical engineers a user's guide to nonlinear optimization algorithms.

The evolution of very large scale integrated circuits (VLSI) prompted the research in solving large scale problems. The immediate work is to simulate and to analyze such circuits. One major effort in this area has been the development of network partitioning and various tearing methods (e.g., Sangiovanni-Vincentelli, Chen and Chua 1977). Another effort is to use vector computers (e.g., Calahan and Ames 1979; Yamamoto and Takahashi 1985) and parallel processors (e.g., Huang and Wing 1979). Sparse matrix techniques have been often involved (e.g., Huang and Wing 1979). More details of large scale simulation are available in Newton (1981), Hachtel and Sangiovanni-Vincentelli (1981) and Pederson (1984).

Other works done for large scale circuits are the computation of poles and zeros (Wehrhahn R. 1979) and yield estimation (Downs, Cook and Rogers 1984). Large scale networks is also considered from the graph theory point of view, e.g., Boesch (1976).

Sparse matrix techniques and decomposition techniques are two powerful mathematical tools for solving large scale problems. Both techniques take advantage of the fact that as a problem becomes large, more weak interactions between subproblems are likely to occur. The sparse matrix techniques are used to avoid unnecessary storage and computation of zero components of a matrix Work in this area can be found in books of Reid (1971) and Duff (1981). The decomposition techniques are generally used to solve decoupled subproblems separately and then to solve the overall problem using the knowlege of subproblem solutions. An excellent survey of decomposition for large scale problems is provided by Himmelblau (1973).

Large scale mathematical programming became an important topic for operations researchers in the 1960's. A major inspiration in this field has been the discovery of the decomposition principle by Dantzig and Wolfe (1960). The work of Geoffrion (1970) and Lasdon (1970) summarized the major pioneering activities in this field. More recent reviews are available in Haimes (1982) and Luna (1984). Many people have used decomposition approaches, e.g., Bunch and Kaufman (1981), Shapiro and White (1982), Borison, Morris and Oren (1984) and Mandakovic and Souder (1985). Others used sparse matrix techniques (Murtagh and Saunders 1978; Coleman 1984), matrix splitting (O'Leary 1981), recursive quadratic programming (Biggs and Laughton 1977) and dual optimization methods (Templeman 1979).

Although the simulation and analysis of large scale analog circuits have been fairly treated in the literature, the direct optimization of such circuits however, is a much open subject. Bandler, Chen, Daijavad, Kellermann, Renault and Zhang (1986) successfully optimized a 16-channel microwave multiplexer involving as many as 240 nonlinear design variables. The first formal attempt to large scale microwave optimization is made by Bandler and Zhang (1987a) who developed an automatic decomposition technique for device modelling and large circuit design.

2.3 REVIEW OF MICROWAVE CAD

The early stages of computer-oriented microwave practices can be represented by the comments of Getsinger (1969). As he pointed out, in the late 1960's, within microwaves, the electromagnetic field analysts were the group most fully converted to the computer. In microwave circuits, old design methods were adapted to the computer and new design approaches were devised. Computer programs were used to do such things as analyze arbitrary microwave circuits, design filters, transistor amplifiers and other components.

Bandler (1974) edited the second Special Issue of Computer-Oriented Microwave Practices of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES. A wide range of opinions held by contributors to the field as well as users were revealed from the panel discussion on the status of computer-oriented microwave practices (Cermak, Getsinger, Leake, Vander Vorst and Varon 1974). The gap between numerical techniques and real engineering problems was brought into serious consideration. Rigorously derived methods for design rather than only for analysis were increasingly used (Bandler 1974).

Without attempting to trace all historical details of microwave CAD, here we devote more attention to the current state-of-the-art in this area.

A wide range of microwave circuits can be covered as candidates for the tool of CAD. As stated by Hoffman (1984), essentially it does not matter very much if the circuit is passive or active, linear or nonlinear, whether it is a single component or a subassembly. Also, the frequency of operation of the circuit is in general not a significant parameter. A treatment of the subject area is also compiled into a textbook by Gupta, Gary and Chadha (1981).

The contemporary and recognized industry-standard in microwave CAD can be represented by the commercially available software SUPER-COMPACT (1986) and TOUCHSTONE (1985). SUPER-COMPACT provides the necessary tool for choosing a design topology and active elements, synthesizing amplifier matching circuits, and can optimize complete circuits having as many as four external ports. Circuit elements include lumped, microstrip and stripline, filter, thin film, coupler and active devices. SUPER-COMPACT can run under mainframe operating systems such as VAX/VMS, HP-UX(UNIX) and IBM/CMS. It can also be run simultaneously by more than one user, i.e., as a time sharing software. On the other hand, TOUCHSTONE (1985) is the most advanced software for RF/microwave CAD, running on personal computers such as IBM PC-XT, AT and IBM-compatibles. With a complete element and measurement catalogue, TOUCHSTONE can be treated as a laboratory instrument to set up a microwave circuit and to perform simulation, optimization and tuning. Other commercially available microwave CAD software also exist, e.g., CIAO (1985) for circuit analysis and optimization and CADEC+ (1987) for computer aided design of electronic circuits, both running on desk top computers. MIDAS (1987) is a microwave/RF CAD program incorporating a Network Descriptive Language, which allows the use of algebraic expressions to define any values for network analysis. Allen and Medley Jr. (1980) developed a set of network analysis programs for microwave circuit design using programmable calculators.

Today's research of microwave CAD continues to be active in a broad subject area. Topics of major interest include modelling of active and passive microwave components (e.g., Salmer 1987; Bandler, Chen and Daijavad 1986b), characterization and modelling of transmission structures and discontinuities (e.g., Pramanick and Bhartia 1986; Koster and Jansen 1986), linear and nonlinear analysis of devices and circuits (e.g., Rizzoli and Lipparini 1985; Gilmore 1986; Curtice 1987), large scale numerical simulation and design of devices and circuits (e.g., Rizzoli, Ferlito and Neri 1986; Bandler and Zhang 1987a) and optimization techniques applicable to microwave CAD (e.g., Bandler, Kellermann and Madsen 1985; Bandler and Chen 1987). In addition to the popular frequency domain, fixed topology and equivalent circuit model description of microwave circuits, methods using time domain (Sobhy and Hosny 1981), changable topology (Dowson 1985), and physical device models (Snowden 1986) are also developed. Besides the scattering parameter approach used in software such as SUPER-COMPACT (1986) and TOUCHSTONE (1985), the wave analysis approach has been adopted in CAD programs for noise analysis of interconnected multiport networks (Kanaglekar, McIntosh and Bryant 1987). In Europe, extensive research in microwave CAD is currently underway as evidenced by the survey paper of Gardiol (1986) and by the IEE PROCEEDINGS-H Special Issue on Computer-Aided Design of Microwave Circuits edited by Pengelly (1986).

In dealing with problems of large numerical size, Rizzoli, Ferlito and Neri (1986) exploited the hardware capability of vector processors (supercomputers) such as the Cray X-MP. They presented a possible approach for vectorization of microwave CAD programs. As opposed to the conventional way of processing a circuit via a sequence of single frequency analysis, they perform a single multifrequency analysis of the circuit. Common computational operations of the circuit at different frequencies are exploited. According to their report, speed up factors of the order of 50 were obtained. However, the memory requirements of this method are increased significantly.

Bandler and Zhang (1987a) treated large scale microwave CAD problems by exploiting advanced mathematical tools. Their approach used an automatic decomposition scheme. Large scale optimization problems were solved using ordinary mainframe computers with memory limitations within reasonable computer time. Their method is effective on microwave circuits having decomposition properties.

Optimization methods are now considered important tools in the microwave CAD community. The survey papers by Bandler (1969), Charalambous (1974) and Bandler and Chen (1987) summarized mathematical programming methods for solving microwave circuit design problems. Optimization methods were used in integrated design centering, tolerancing and tuning (Bandler, Liu and Tromp 1976b), in device modelling (Bandler, Chen and Daijavad 1986b) and postproduction tuning (Bandler and Salama 1985b) of microwave devices. As a general CAD tool, optimization techniques have also been used for diagnosis (e.g., Bandler and Zhang 1987b) and yield maximization (e.g., Hocevar, Lightner and Trick 1984) of electrical circuits. Mathematical programming techniques involved ranging from the random optimization method (TOUCHSTONE 1985) which does not use any derivative information, to various gradient methods using either approximated gradient (Bandler, Chen, Daijavad and Madsen 1986), or exact first-order derivatives (Bandler, Kellermann and Madsen 1985), or exact second-order derivatives (Iobost and Zaki 1982). Particularly, the minimax (Hald and Madsen 1981) and the ℓ_1 (Hald and Madsen 1985) optimization algrithms developed by Hald and Madsen of the Technical University of Denmark have been very practical for modelling and design of microwave circuits (Kellermann 1986; Daijavad 1986).

2.4 FORMULATION OF CIRCUIT DESIGN AS AN OPTIMIZATION PROBLEM

Basically, a design problem is to find a set of designable parameter values which let the circuit respose or performance optimally meet some given specifications (Bandler and Rizk 1979). In this section, the circuit design problem is formulated.

2.4.1 The Circuit Model

In computer-aided design, a circuit is usually described by a mathematical model. Let

$$\boldsymbol{\phi} \triangleq [\phi_1 \ \phi_2 \cdots \phi_n]^T \tag{2.1}$$

represent the design parameters. The circuit responses F_k, $k = 1, 2, \ldots, n_F$, are functions of parameters $\boldsymbol{\phi}$ and of other independent variables $\boldsymbol{\psi}$, i.e.,

$$F_k \triangleq F_k(\boldsymbol{\phi}, \boldsymbol{\psi}). \tag{2.2}$$

Fig. 2.1 depicts a general circuit with multi-inputs and multi-outputs. The response functions F_k are evaluated or measured at output ports and can represent, e.g., voltage, current, insertion loss, return loss, group delay and S parameters. Two responses, e.g., F_1 and F_2 are distinguished either by two different output ports or by two different types of responses at the same port or by a mixture of both. The circuit topology is usually fixed. The design parameters $\boldsymbol{\phi}$ can be accessed either directly (physical parameters), e.g., length of a waveguide, or indirectly (model parameters), e.g., coupling parameters of a cavity filter. The independent variables $\boldsymbol{\psi}$ represent, e.g., frequency, time, tempreture, etc. The functions $F_k(\boldsymbol{\phi}, \boldsymbol{\psi})$ are assumed continuous in the ranges of $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ of interest. Performance specifications are usually functions of $\boldsymbol{\psi}$ only (Bandler and Rizk 1979).

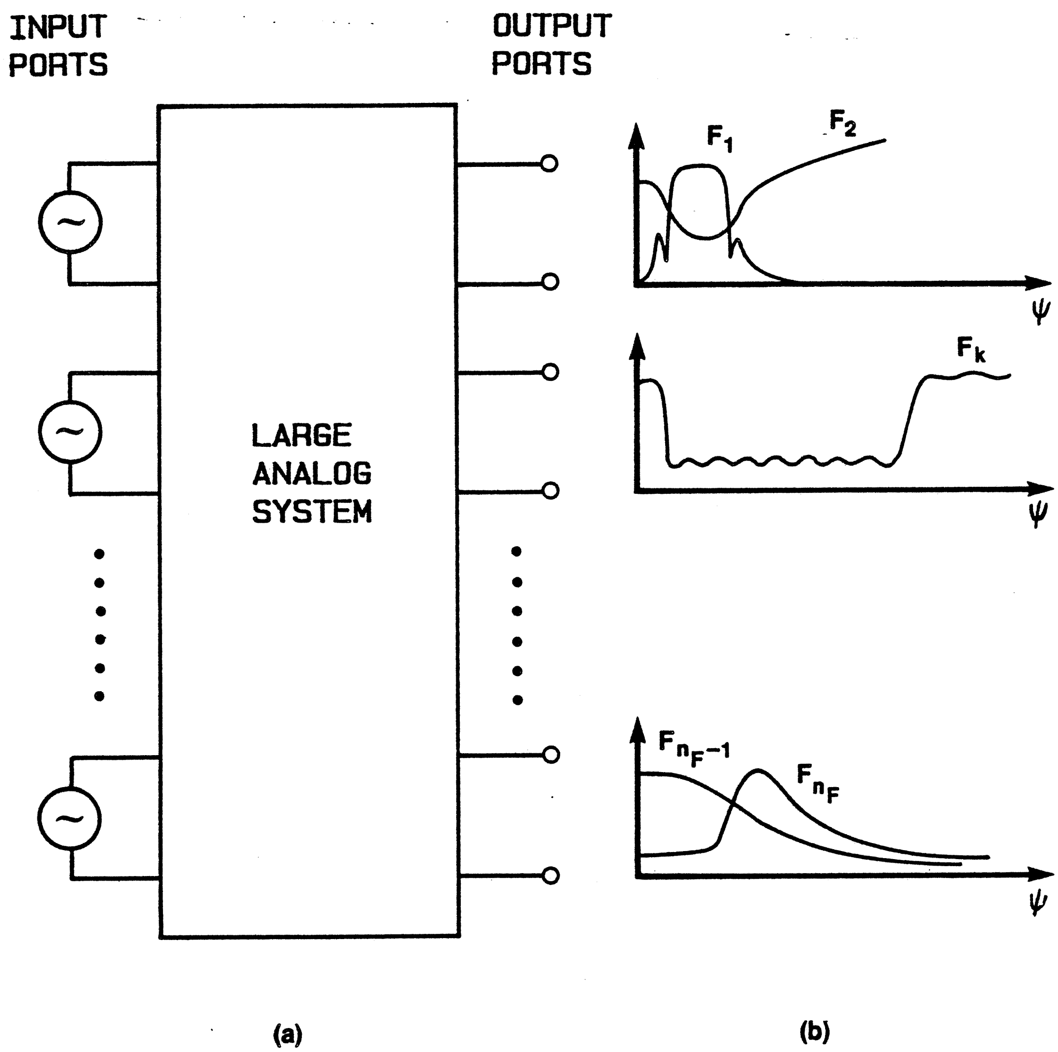


Fig. 2.1 A general representation of multi-input and multi-output analog system. F_k, $k = 1, 2, ..., n_F$ are responses being measured, monitored or used as outputs subject to design specifications. Different types of responses (e.g., voltage, current, return loss, insertion loss, S parameters) may exist at the same output port. (a) system representation. (b) responses corresponding to each output port.

2.4.2 Design Specifications and Error Functions

In this thesis, we will treat circuit problems mainly in the frequency domain. In such a case, the independent variables ψ are commonly substituted by a frequency parameter ω. Let $S_{Uk}(\omega)$ and $S_{Lk}(\omega)$ represent the upper and the lower specifications for the response $F_k(\phi, \omega)$, respectively, $k = 1, 2, ..., n_F$. Let ω_ℓ, $\ell = 1, 2, ..., n_\omega$ be a set of frequency points sampled in the frequency range of interest. In an optimal design problem, the objective function usually involves a set of nonlinear error functions $f_j(\phi)$, $j=1,2, ..., m$. Typically, the error functions represent the weighted differences between circuit responses and given specifications in the form

$$w_{Uk}(\omega_\ell)(F_k(\phi, \omega_\ell) - S_{Uk}(\omega_\ell)) , \tag{2.3a}$$

$$- w_{Lk}(\omega_\ell)(F_k(\phi, \omega_\ell) - S_{Lk}(\omega_\ell)) , \tag{2.3b}$$

$$k \in \{1, 2, ..., n_F\} , \tag{2.3c}$$

$$\ell \in \{1, 2, ..., n_\omega\}, \tag{2.3d}$$

where w_{Uk} and w_{Lk} are (non-negative) weighting factors for upper and lower specifications, respectively.

Let J be an index set defined as

$$J \triangleq \{1, 2, ..., m\} . \tag{2.4}$$

Let

$$M_f(\phi) \triangleq \max_{j \in J} f_j(\phi) . \tag{2.5}$$

Then the sign of $M_f(\phi)$ indicates whether the specifications are satisfied or violated. As described by Bandler and Rizk (1979), if

$$M_f(\phi) \begin{cases} > 0 & \text{the specifications are violated,} \\ = 0 & \text{the specifications are just met,} \\ < 0 & \text{the specifications are satisfied.} \end{cases}$$

An optimization approach to the circuit design problem is to find $\boldsymbol{\phi}$ such that $M_f(\boldsymbol{\phi})$ is minimized. This corresponds to the effort to meet (when specifications are violated) or to exceed (when specifications are satisfied) design specifications as much as possible. Mathematically, this is a minimax problem where the maximum of all error functions is minimized. A practical solver to the minimax problem is the 2-stage algorithm developed by Hald and Madsen (1981).

2.4.3 The Coding Scheme Between Indices of Error Functions, Responses and Frequency Points

There exists a coding scheme representing the one-to-one correspondence between the index of f_j and the indices of the pair (F_k, ω_ℓ) for the error functions of both (2.3a) and (2.3b). We define weighting factor matrices $\mathbf{W}_U$ (for upper specification) and $\mathbf{W}_L$ (for lower specification). Both matrices are n_F by n_ω. The (k, ℓ)th component of $\mathbf{W}_U$ and $\mathbf{W}_L$ are the weighting factors $w_{Uk}(\omega_\ell)$ and $w_{Lk}(\omega_\ell)$, respectively. $w_{Uk}(\omega_\ell)$ or $w_{Lk}(\omega_\ell)$ is zero if no upper or lower specification is imposed on $F_k(\boldsymbol{\phi}, \omega_\ell)$. The coding scheme relating the index of f_j to the indices of nonzeros in $\mathbf{W}_U$ and $\mathbf{W}_L$ are constructed by systematically scanning through $\mathbf{W}_U$ and then $\mathbf{W}_L$, respectively (Bandler and Zhang 1987c).

2.5 CONCLUDING REMARKS

In this chapter, we provided a review of large scale circuit optimization and of microwave CAD. Fairly treated in the literature are the simulation and sensitivity analysis of large scale circuits. The optimization of such circuits remains a much open subject.

Also in this chapter, circuit design has been formulated as a minimax optimization problem. Such a formulation will be used for circuit design problems throughout the thesis.

3

OPTIMIZATION TECHNIQUES FOR MODELLING, DIAGNOSIS AND TUNING

3.1 INTRODUCTION

This chapter deals with the application of optimization techniques for modelling, diagnosis and tuning (MDT) of electrical circuits. A conventional interpretation of such techniques for modelling and diagnosis is the determination of appropriate network parameters leading to the best match between circuit responses and measured data. When the measurements are insufficient to evaluate all network elements, the most likely faults may be located. Otherwise, if the measurements are sufficient, parameter identification is initiated, resulting in a circuit model whose performance best fits the measurement data in the presence of uncertainties and noise. Closely related is the tuning problem which has been approached mostly from the optimization point of view. Existing software for mathematical programming can be readily exploited in this case.

This chapter is based on the work of Bandler and Zhang (1987b). The presentation is tutorial, but designed to be helpful for a state-of-the-art understanding. We first review circuit oriented optimization methods with emphasis on aspects important to MDT. A general formulation of circuit diagnosis as an optimization problem is introduced. It is followed by a detailed investigation into three specific formulation cases. Optimization methods for modelling and tuning are presented and compared with those for diagnosis. Illustrative examples are provided.

3.2 CIRCUIT ORIENTED OPTIMIZATION TECHNIQUES

Optimization methods have played an important role in computer-aided design of circuits and systems (see, e.g., Bandler and Rizk 1979; Brayton, Hachtel and Sangiovanni-Vincentelli 1981; Bandler and Chen 1987). Typical circuit design objectives are to satisfy or to exceed design specifications as much as possible. The MDT problems, however, are usually oriented either towards (response) data fitting or towards "parameter fitting" or a combination of both. The "parameter fitting" can be interpreted as forcing parameters to approach a desired pattern. Such a pattern is constructed to best represent:

1) an estimation of the parameters, e.g., results from a deliberate perturbation to the circuit (for more measurement information), a projected target parameter point for tuning;
2) an assumption of the circuit philosophy, e.g., type of faults, whether catastrophic or soft;
3) a criterion for optimality, e.g., the objective for minimum parameter adjustment in tuning.

3.2.1 Introduction to Mathematical Programming

An optimization problem can be stated as

$$\underset{\boldsymbol{\phi}}{\text{minimize}}\ U(\boldsymbol{\phi}) \tag{3.1a}$$

subject to constraints

$$\mathbf{g}(\boldsymbol{\phi}) \geq \mathbf{0} \tag{3.1b}$$

and

$$\mathbf{h}(\boldsymbol{\phi}) = \mathbf{0}\,, \tag{3.1c}$$

where $\boldsymbol{\phi} \triangleq [\phi_1\ \phi_2\ \dots\ \phi_n]^T$, $\mathbf{g} \triangleq [g_1\ g_2\ \dots\ g_{n_g}]^T$, and $\mathbf{h} \triangleq [h_1\ h_2\ \dots\ h_{n_h}]^T$.

When U, **g** and **h** are all linear functions of $\boldsymbol{\phi}$, (3.1) is a linear programming problem (LP), readily solvable by the simplex method (see Luenberger 1984), a classical approach being currently challenged by Karmarkar's algorithm (Karmarkar 1984).

To handle the nonlinear programming problem (NLP), i.e., the nonlinear case of (3.1), a variety of methods have been developed. The unconstrained NLP can be solved by conjugate direction methods and quasi-Newton methods. The constrained NLP can be handled using, e.g., penalty and barrier methods, and augmented Lagrangian methods.

A systematic treatment to (3.1) can be found in many text books, e g., Luenberger (1984). A comprehensive examination of optimization from the circuit design point of view is provided in e.g., Temes and Calahan (1967); Bandler (1973); Charalambous (1974); Director (1971); Brayton, Hachtel and Sangiovanni-Vincentelli (1981); Bandler and Chen (1987). In this section, we highlight those aspects of optimization which are relevant to MDT.

3.2.2 Least pth Optimization

A frequently encountered objective $U(\boldsymbol{\phi})$ is the pth norm of $\mathbf{f}(\boldsymbol{\phi}) \triangleq [f_1(\boldsymbol{\phi})\ f_2(\boldsymbol{\phi})\ \ldots\ f_m(\boldsymbol{\phi})]^T$, i.e.,

$$U(\boldsymbol{\phi}) = \left(\sum_{i=1}^{m} |f_i(\boldsymbol{\phi})|^p \right)^{1/p}, \quad p \geq 1. \tag{3.2}$$

The larger the value of p, the more emphasis is being put on $\max\{|f_1|, |f_2|, \ldots, |f_m|\}$. At the solution, large (small) p typically produces many $|f_i|$'s which are equal to $\max\{|f_1|, |f_2|, \ldots, |f_m|\}$ (equal to zero).

The p=1 case of (3.2) corresponds to the ℓ_1 norm optimization, solvable by the two-stage algorithm of Hald and Madsen (1985). The algorithm combines a first-order method that approximates the solution by successive linear programming, with a quasi-Newton method that uses approximate second-order information to solve the system of nonlinear equations arising from the necessary first-order conditions at a solution.

The p=2 case of (3.2) (least-squares or ℓ_2 approximation) is a problem of wide publicity. Both first-order and second-order methods have been derived for general nonlinear ℓ_2 problems (see Marquardt 1963 and Dennis Jr. 1977). For certain linear ℓ_2 problems, a closed form solution is obtainable by invoking generalized matrix inversion (Rao and Mitra 1971; Nashed 1976).

The objective function defined in (3.2) is used to penalize the modulus of f_i. To penalize the value of f_i, we use the generalized least pth function (Bandler and Rizk 1979)

$$U(\phi) = \begin{cases} M_f\left(\sum_{i \in K} (f_i(\phi)/M_f)^q \right)^{1/q} & \text{if } M_f \neq 0 \\ 0 & \text{if } M_f = 0 \end{cases} , \tag{3.3}$$

where

$$M_f \triangleq \max_{i \in J} f_i(\phi) \qquad J \triangleq \{1, 2, \ldots, m\} \tag{3.4}$$

and

$$\begin{aligned} &\text{if } M_f > 0\text{, then, } K = \{i \mid f_i \geq 0, i \in J\} && \text{and } q = p \\ &\text{if } M_f < 0\text{, then, } K = J && \text{and } q = -p\,. \end{aligned} \tag{3.5}$$

In the case of $M_f > 0$ ($M_f < 0$), the larger the value of p, the more nearly would we expect the maximum (minumum) $|f_i|$ to be emphasized. Therefore, the minimization of (3.3) corresponds to the effort to meet (when $M_f > 0$) or to exceed (when $M_f < 0$) a design specification as much as possible.

As $p \rightarrow \infty$, the generalized least pth optimization approaches the minimax optimization, the latter being effectively solved by the combined LP and quasi-Newton method of Hald and Madsen (1981). The algorithm is a two-stage one similar to the ℓ_1 optimization algorithm of Hald and Madsen (1985). Initially, Stage 1 is used and at each point, **f** is approximated by linear functions using first-order information. In Stage 2, the quasi-Newton iteration is used to solve a set of nonlinear equations that necessarily hold at a local minimum. Usually, Stage 1 is used to obtain fast convergence to the neighbourhood of the solution. Stage 2 is used to obtain super-linear final convergence, but several switches between the two stages may take place.

The two-stage algorithms for ℓ_1 and minimax optimizations are computationally practical and have been implemented by Bandler, Kellermann and Madsen (1985, 1987).

3.2.3 Quadratic Programming

In a quadratic programming problem (QP), the objective function is defined as

$$U(\boldsymbol{\phi}) \triangleq \Lambda + \mathbf{s}^T \boldsymbol{\phi} + \frac{1}{2} \boldsymbol{\phi}^T \mathbf{H} \boldsymbol{\phi}, \tag{3.6}$$

where Λ is a scalar, **s** is a n-vector, and **H** is a $n \times n$ matrix.

The QP problems arise both in their own right and as subproblems within general nonlinear optimization methods. Typically, a QP problem is to minimize the function of (6) subject to linear equality and/or inequality constraints. Such a problem can be solved, e.g., using the iterative methods described by Gill and Murray (1977) and Gill, Murray, Saunders and Wright (1984). The linear inequality constraints are treated using the active-set methods in which a prediction of the set of constraints that are active at the solution is maintained. This prediction is called the working set and is updated by adding or deleting constraints as the iterations proceed. By treating the working set as equality constraints, the constrained QP problem is transformed into an unconstrained one. The problem is relatively easy to solve if the original **H** is positive definite (Gill and Murray 1977).

For unconstrained QP problems, with **H** as positive definite, the minimum can be uniquely located in a finite number of steps, using, e.g., Newton's method and the conjugate gradient method.

3.2.4 MINMAX and MINBOX Approaches in Linearization

Linearization is often used in solving nonlinear programming problems. Hachtel, Scott and Zug (1980) described the MINMAX and MINBOX approaches where the range of the validity of a linear approximation is specified in the variable domain and the function domain, respectively. Used in nonlinear minimax optimization, the MINMAX approach resembles the conventional way of locating the minimax point of linearized functions subject to a prescribed "box constraint" on $\boldsymbol{\phi}$. The MINBOX approach, on the other hand, either produces a smallest step $\Delta\boldsymbol{\phi}$ which achieves user-specified levels of improvement in **f**, or states that the levels are infeasible.

3.2.5 Gradient and Nongradient Approaches

The employment of exact gradient information $\partial U/\partial\boldsymbol{\phi}$ significantly improves the effectiveness of an optimization algorithm. The well-known adjoint network method developed by Director and Rohrer (1969a, 1969b) remains a powerful tool for sensitivity calculation. An equivalent, but pure algebraic approach has also been studied (Branin Jr. 1973; Bandler and Zhang 1986). For special types of networks, e.g., branched cascaded networks, more effective methods can be derived (Bandler, Daijavad and Zhang 1986).

Not infrequently, the gradient is difficult or even impossible to obtain. Approximate gradient methods have been developed, in addition to the direct search methods which do not depend explicitly on evaluation or estimation of gradients. The theoretical background is the Broyden formula (Broyden 1965), which utilizes function values to improve the gradient estimation as the optimization proceeds. This feature has been implemented in nonlinear ℓ_1 and minimax optimization packages (Bandler, Chen, Daijavad and Madsen 1986).

3.3 GENERAL FORMULATION OF DIAGNOSIS AS OPTIMIZATION PROBLEMS

3.3.1 Introduction

The analog diagnosis techniques are described here using a single frequency measurement. Such a description offers both conceptual and notational simplicity. Particular mathematical manipulations required for multi-frequency cases are illustrated whenever necessary.

Suppose from the circuit under test (CUT), we obtain a set of measurements represented by a n_F-vector $\mathbf{F}^M$. The corresponding responses as functions of circuit parameters $\boldsymbol{\phi} \triangleq [\phi_1\ \phi_2\ \ldots\ \phi_n]^T$ are given by $\mathbf{F} \triangleq \mathbf{F}(\boldsymbol{\phi}, \omega)$. For single frequency cases, $\mathbf{F} \triangleq \mathbf{F}(\boldsymbol{\phi})$ is used for notational convenience. A nominal design of the circuit is characterized by $\boldsymbol{\phi}^0$ and $\mathbf{F}^0$.

When the measurements are insufficient to identify all parameters, e.g., when $n_F < n$, the equation

$$\mathbf{F}^M = \mathbf{F}(\boldsymbol{\phi}^0 + \Delta\boldsymbol{\phi}) \ . \tag{3.7}$$

is an underdetermined one. An optimization technique can be used to find the most likely $\Delta\boldsymbol{\phi}$, among an infinite number of solutions to (3.7). Such a problem can be stated as

$$\underset{\Delta\boldsymbol{\phi}}{\text{minimize}}\ U(\Delta\boldsymbol{\phi}) \tag{3.8a}$$

$$\text{s.t.}\ \ \mathbf{h}(\mathbf{F}^M, \Delta\boldsymbol{\phi}) \triangleq \mathbf{F}(\boldsymbol{\phi}^0 + \Delta\boldsymbol{\phi}) - \mathbf{F}^M = \mathbf{0}\ , \tag{3.8b}$$

where U is an increasing function of $|\Delta\phi_i|$, $i = 1, 2, \ldots, n$.

A convenient approach to solving (3.8) is to use penalty methods. For example, a least pth formulation is

$$\underset{\Delta\boldsymbol{\phi}}{\text{minimize}}\ \left(\sum_{i=1}^{n} w_i |\Delta\phi_i|^p + \sum_{i=1}^{n_F} \beta_i | F_i(\boldsymbol{\phi}^0 + \Delta\boldsymbol{\phi}) - F_i^M |^p \right)^{1/p} , \tag{3.9}$$

where w_i, $i=1, 2, \ldots, n$ and β_i, $i=1, 2, \ldots, n_F$ are appropriate weighting factors (Bandler, Kellermann and Madsen 1987).

3.3.2 Constraint Equation

Suppose the N-node circuit is characterized by its nodal equation

$$\mathbf{Y}\,\mathbf{V} = \mathbf{I} \tag{3.10}$$

where **Y**, **V** and **I** are the nodal admittance matrix, voltage vector and current excitation vector, respectively. We assume, for convenience, that the measurable responses of the CUT, namely **F**, can be represented by linear combinations of nodal voltages using a $N \times n_F$ matrix **C** such that

$$\mathbf{F} = \mathbf{C}^T \mathbf{V}\ . \tag{3.11}$$

Thus,

$$\mathbf{F} = \mathbf{F}(\boldsymbol{\phi}) = \mathbf{C}^T[\mathbf{Y}(\boldsymbol{\phi})]^{-1}\mathbf{I}. \tag{3.12}$$

To simplify the nonlinear optimization of (3.8) and (3.9), researchers have employed two effective formulations transforming the constraint equation into linear forms by introducing intermediate parameters. These formulations are the current/-voltage source substitution model and the component connection model. The former model will be used throughout this chapter. A comprehensive treatment to the latter can be found in Ransom and Saeks (1973), and in DeCarlo and Saeks (1981).

3.3.3 The Current/Voltage Source Substitution Model

The current/voltage source substitution model was used by Bandler, Biernacki and Salama (1981), Bandler, Biernacki, Salama and Starzyk (1982), and Bandler and Salama (1985a) for fault diagnosis. In such a model, changes in element values are equivalently characterized by current or voltage sources. Fig. 3.1 shows equivalent representations for some typical elements in linear circuits. Without loss of generality, we assume that the changes are represented by current sources only. Let $\Delta\mathbf{I}^b$ be a n-vector containing such sources corresponding to the n variable elements, and $\mathbf{Q}$ be a $N \times n$ incidence matrix relating the n branches containing variables to the N nodes of the circuit. By invoking the superposition theorem, we may write

$$\mathbf{Y}(\boldsymbol{\phi}^0)\,\Delta\mathbf{V} = -\mathbf{Q}\Delta\mathbf{I}^b. \tag{3.13}$$

where $\Delta\mathbf{V}$ is the deviation of actual nodal voltages from their nominal values. Also,

$$\Delta\mathbf{F} \triangleq \mathbf{F} - \mathbf{F}^0 = \mathbf{C}^T\Delta\mathbf{V} = -\mathbf{C}^T[\mathbf{Y}(\boldsymbol{\phi}^0)]^{-1}\mathbf{Q}\,\Delta\mathbf{I}^b. \tag{3.14}$$

Denote

$$\mathbf{A}' \triangleq -\mathbf{C}^T[\mathbf{Y}(\boldsymbol{\phi}^0)]^{-1}\mathbf{Q}. \tag{3.15}$$

Then we have the constraint equation in linear form as

$$\mathbf{A}'\,\Delta\mathbf{I}^b = \mathbf{F}^M - \mathbf{F}^0, \tag{3.16}$$

or, in real form, as

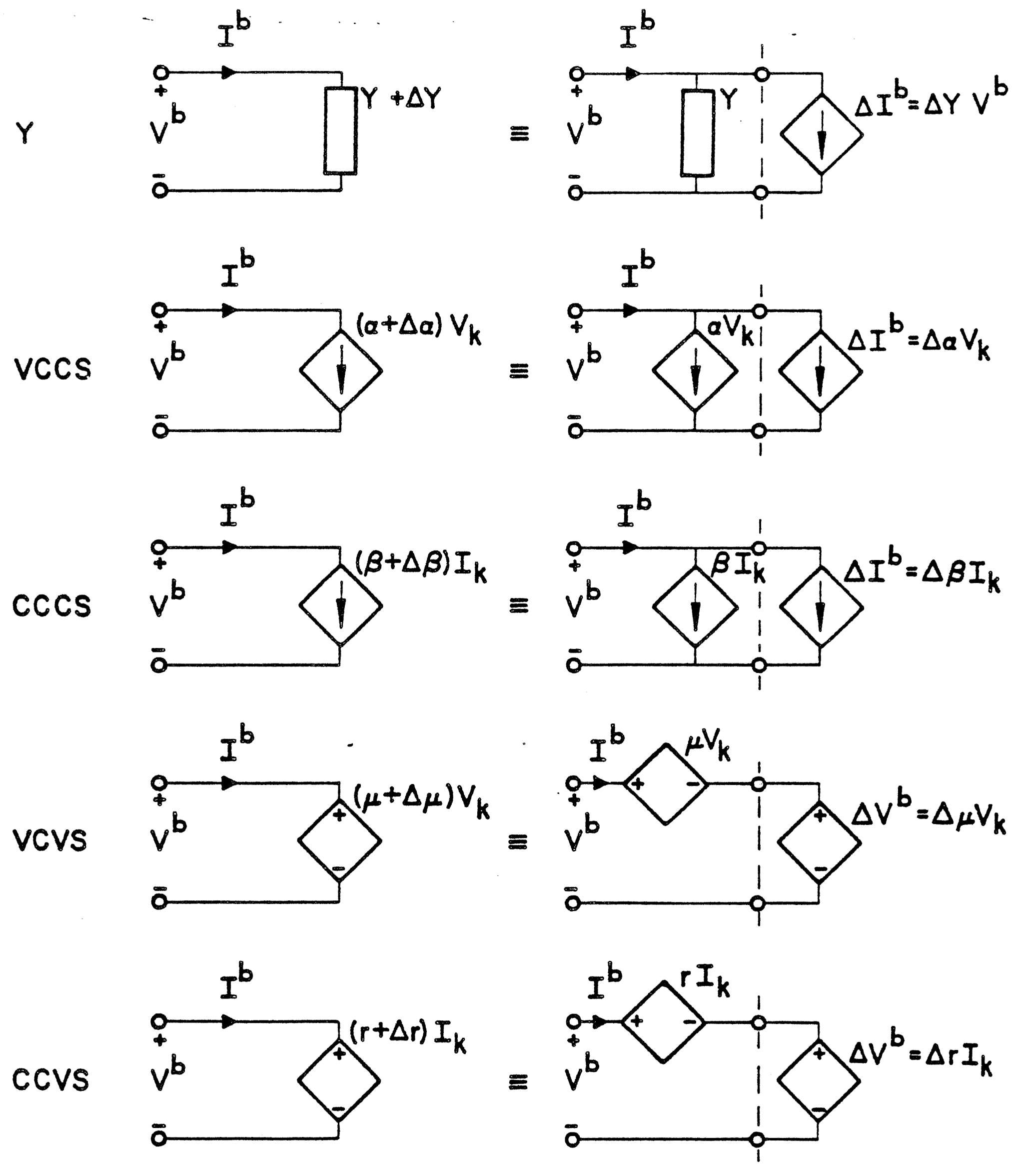


Fig. 3.1 An equivalent representation of changes in element values.

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}\ , \tag{3.17}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathrm{Re}(\mathbf{A}') & -\mathrm{Im}(\mathbf{A}') \\ \mathrm{Im}(\mathbf{A}') & \mathrm{Re}(\mathbf{A}') \end{bmatrix}, \tag{3.18}$$

$$\mathbf{b} = [\mathrm{Re}(\mathbf{F}^M - \mathbf{F}^0)^T \qquad \mathrm{Im}(\mathbf{F}^M - \mathbf{F}^0)^T]^T \tag{3.19}$$

and

$$\mathbf{x} = [\mathrm{Re}(\Delta\mathbf{I}^b)^T \qquad \mathrm{Im}(\Delta\mathbf{I}^b)^T]^T . \tag{3.20}$$

To compute $\Delta\boldsymbol{\phi}$ from $\mathbf{x}$, we simulate the network with all components held at nominal values and with additional current excitations $\Delta I_i{}^b = x_i + jx_{i+n}$, $i = 1, 2, \ldots, n$ connected across corresponding components. After measuring or calculating branch voltages $V_i{}^b$, $i = 1, 2, \ldots, n$, the component change is evaluated as

$$\Delta\phi_i = \frac{x_i + j\,x_{i+n}}{V_i^b}\,(j\omega)^{-a},\ \ i = 1, 2, \ldots, n \tag{3.21}$$

where $a \equiv a_i$, whose value can be 0, 1 or -1 depending upon whether the ith component is resistive, capactive or inductive.

For multi-frequency diagnosis, we use $\Delta\boldsymbol{\phi}$ as optimization variables directly. $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{x}$ are redefined accordingly. For example,

$$\mathbf{A}_i = -\mathbf{C}^T[\mathbf{Y}(\boldsymbol{\phi}^0, \omega_i)]^{-1}\mathbf{Q}\,\mathrm{diag}\left\{(j\,\omega_i)^{a_1} V_1^b(\omega_i),\ (j\,\omega_i)^{a_2} V_2^b(\omega_i),\ \ldots,\ (j\,\omega_i)^{a_n} V_n^b(\omega_i)\right\}$$

$$i = 1, 2 \tag{3.22}$$

$$\mathbf{A}'' = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}, \quad \mathbf{b}'' = \begin{bmatrix} \mathbf{F}^M(\omega_1) - \mathbf{F}^0(\omega_1) \\ \mathbf{F}^M(\omega_2) - \mathbf{F}^0(\omega_2) \end{bmatrix} \tag{3.23}$$

$$\mathbf{A} = \begin{bmatrix} \mathrm{Re}(\mathbf{A}'') \\ \mathrm{Im}(\mathbf{A}'') \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathrm{Re}(\mathbf{b}'') \\ \mathrm{Im}(\mathbf{b}'') \end{bmatrix} \tag{3.24}$$

and

$$\mathbf{x} = \Delta\boldsymbol{\phi}\ , \tag{3.25}$$

where we have assumed that two frequency points are taken. The branch voltages $V_k^b(\omega_i)$ $k = 1, 2, \ldots, n$ are initially assumed. An iterative procedure updates $V_k^b(\omega_i)$ and at the same time computes the changes in $\mathbf{\Phi}$.

If the nodal equation of (3.10) is replaced by a hybrid equation, a more general form of (3.17) can be similarly deduced where both current and voltage sources exist for an equivalent representation of $\Delta\mathbf{\Phi}$.

3.3.4 The Component Connection Model

The component connection model was used by Ransom and Saeks (1973, 1975). We assume that the system topology is described by a matrix relation

$$\begin{bmatrix} \mathbf{u}' \\ \mathbf{F} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{21} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{u} \end{bmatrix} . \tag{3.26}$$

Here, $\mathbf{u}'$ and $\mathbf{v}$ are the component input and output variables, respectively, related by

$$\mathbf{v} = \mathbf{Z}\,\mathbf{u}' , \tag{3.27}$$

where $\mathbf{Z}$ is the component parameter matrix. The $\mathbf{u}$ and $\mathbf{F}$ in (3.26) are the system input and output variables related using the system matrix $\mathbf{\Gamma}$ as

$$\mathbf{F} = \mathbf{\Gamma}\,\mathbf{u} . \tag{3.28}$$

By introducing intermediate variables $\mathbf{R}$, we have linear relation

$$\mathbf{\Gamma} = \mathbf{L}_{21}\,\mathbf{R}\,\mathbf{L}_{12} , \tag{3.29}$$

where $\mathbf{R}$ is related to $\mathbf{Z}$, using

$$\mathbf{R} = (\mathbf{1} - \mathbf{Z}\,\mathbf{L}_{11})^{-1}\,\mathbf{Z} . \tag{3.30}$$

It has been shown (see Ransom and Saeks 1973) that for small changes in $\mathbf{Z}$,

$$\Delta\mathbf{Z} \approx \Delta\mathbf{R} . \tag{3.31}$$

As such, **R** can be used instead of **Z** for optimization. Final results of **Z** can be computed using either the exact (i.e., deduced form (3.30)) or the approximate (i.e., deduced from (3.31)) relation between **R** and **Z**.

3.3.5 General Formulation

The intermediate variables **x** defined in (3.20) exhibit a similar pattern to the parameters $\Delta\boldsymbol{\phi}$ since an equivalent source current ΔI^b increases as the corresponding $\Delta\phi$ increases. Also, $\Delta I^b = 0$ if and only if $\Delta\phi = 0$. Now, we can solve the optimization problem with **x** as variables and use the solution to find $\Delta\boldsymbol{\phi}$. A simple yet reasonable objective function is the least pth function of **x**. A general formulation of diagnosis as an optimization problem is

$$\underset{\mathbf{x}}{\text{minimize}} \; U(\mathbf{x}) \triangleq \left(\sum_{i=1}^{2n} w_i |x_i|^p \right)^{1/p} \tag{3.32a}$$

$$\text{s.t.} \; \mathbf{A}\mathbf{x} - \mathbf{b} = \mathbf{0} \, , \tag{3.32b}$$

where w_i, $i = 1, 2, \ldots, 2n$ are weighting factors and the constraint (3.32b) is derived from (3.17-3.20). For the multi-frequency case, (3.22)-(3.25) can be used to define **A**, **b**, and **x** for the constraint equation (3.32b). In this case, the objective function U is the weighted least pth function of x_i, $i = 1, 2, \ldots, n$. After solving (3.32), $\Delta\boldsymbol{\phi}$ can be found using (3.21) or (3.25).

3.4 DIAGNOSIS USING THE LEAST-SQUARES METHOD

The diagnosis technique using least squares optimization was suggested by Ransom and Saeks (1973). It is based on the assumption that the catastrophic faults have been eliminated and the circuit failure is due to components drifting out of tolerance (as from age, temperature changes, etc.).

The optimization problem can be stated as

$$\underset{\mathbf{x}}{\text{minimize}}\ U(\mathbf{x}) \triangleq \mathbf{x}^T \mathbf{W}\, \mathbf{x} \tag{3.33a}$$

$$\text{s.t. } \mathbf{A}\,\mathbf{x} - \mathbf{b} = \mathbf{0}\,, \tag{3.33b}$$

where the constraint equation (3.33b) is defined consistently with (3.32b). **W** is a diagonal matrix containing weighting factors w_i, $i = 1, 2, \ldots, 2n$. An appropriate choice of the weightings can be such that the U of (33a) approximates

$$\sum_{i=1}^{n} \Delta\phi_i^2$$

under the assumption that $\Delta\phi_i$, $i = 1, 2, \ldots, n$ are quite small. For example (see Bandler and Salama 1985a), for $1 \leq i \leq n$,

$$\begin{aligned} w_i &= \frac{1}{2}\,(\mathrm{Re}[(j\omega)^{\alpha_i} V_i^b])^{-2}\,, \\ w_{i+n} &= \frac{1}{2}\,(\mathrm{Im}[(j\omega)^{\alpha_i} V_i^b])^{-2}\,. \end{aligned} \tag{3.34}$$

The solution of the ℓ_2 problem is directly obtained using generalized matrix inversion (see Rao and Mitra 1971), e.g.,

$$\mathbf{x} = \mathbf{W}^{-1}\mathbf{A}^T(\mathbf{A}\ \mathbf{W}^{-1}\mathbf{A}^T)^{-1}\mathbf{b}\ . \tag{3.35}$$

Such a technique using a component connection model has been presented in Ransom and Saeks (1973). The variables **x** consist of elements of the matrix $\Delta\mathbf{R}$. The optimization problem is to minimize the ℓ_2 norm of $\Delta\mathbf{R}$ subject to

$$\Delta\boldsymbol{\Gamma} = \mathbf{L}_{21}\,\Delta\mathbf{R}\,\mathbf{L}_{12}\ , \tag{3.36}$$

where $\Delta\boldsymbol{\Gamma}$ is the difference between the measured values and the nominal values of $\boldsymbol{\Gamma}$. The solution is the generalized inverse of the matrix in (3.36). The component connection model is effective here since $\Delta\mathbf{R} \approx \Delta\mathbf{Z}$ under the assumption that no parameters have significant deviation from nominal.

3.5 DIAGNOSIS USING THE QUADRATIC PROGRAMMING METHOD

The quadratic programming technique for diagnosis was suggested by Merrill (1973). He considered such a class of situations where a system becomes inoperative due to the failure of one or a few components. He pointed out that because the individual system components are generally highly reliable and well maintained, a diagnosis that implicates many components as having failed is probably not correct. Therefore, contrary to the ℓ_2 optimization technique, the main assumption here is that the difference between the actual and the nominal values for a few elements, which correspond to the faulty elements, is much greater than that for the remaining elements that are nonfaulty.

The optimization problem can be described as

$$\underset{\mathbf{x}}{\text{minimize}} \; U(\mathbf{x}) \triangleq \sum_{i=1}^{2n} w_i \sqrt{|x_i| + \delta} \tag{3.37a}$$

$$\text{s.t.} \; \mathbf{A}\mathbf{x} - \mathbf{b} = \mathbf{0} \,, \tag{3.37b}$$

where the constraint equation (3.37b) is defined consistently with (3.32b). The δ under the radical prevents the derivative of the objective function from becoming unbounded.

To solve (3.37) efficiently, Merrill put the constraint (3.37b) into the objective function in a quadratic form as a penalty term, applied uniform weightings, $w_i = 1, i = 1, 2, \ldots, 2n$ and transformed the problem into

$$\underset{\mathbf{y}}{\text{minimize}} \; U(\mathbf{y}) \triangleq \sum_{i=1}^{4n} \sqrt{y_i + \delta} + \frac{1}{2}\beta(\bar{\mathbf{A}}\mathbf{y} - \mathbf{b})^T(\bar{\mathbf{A}}\mathbf{y} - \mathbf{b}) \tag{3.38a}$$

$$\text{s.t.} \; \mathbf{y} \geq \mathbf{0} \,, \tag{3.38b}$$

where $\mathbf{A} \triangleq [\mathbf{A} \quad -\mathbf{A}]$ and $\mathbf{y}$ is a 4n-vector related to $\mathbf{x}$ via

$$y_i = x_i \text{ and } y_{2n+i} = 0 \qquad \text{if } x_i \geq 0$$

$$y_i = 0 \text{ and } y_{2n+i} = -x_i \quad \text{if } x_i < 0 \tag{3.39}$$

$$i = 1, 2, \ldots, 2n.$$

Also, **x** can be calculated from y using

$$x_i = y_i - y_{2n+i}, \qquad i = 1, 2, \ldots, 2n. \tag{3.40}$$

Furthermore, the square root portion of U(y) is linearized at $y = y^j$, resulting in

$$U_j(y) = \Lambda + s^T y + \frac{1}{2} y^T H y, \tag{3.41}$$

where

$$s = \frac{1}{2} [(y_1^j + \delta)^{-1/2} \quad (y_2^j + \delta)^{-1/2} \quad \ldots \quad (y_{4n}^j + \delta)^{-1/2}]^T - \beta \, \bar{A}^T b \tag{3.42}$$

and

$$H = \beta \begin{bmatrix} A^T A & -A^T A \\ -A^T A & A^T A \end{bmatrix}. \tag{3.43}$$

The scalar Λ is also a function of β, δ, y^j and **b**, but as its value is irrelevant to the minimization of $U_j(y)$, it will never actually have to be calculated.

As Merrill indicated, the use of variables y, instead of **x**, can eliminate the difficulty of derivative discontinuity of U at $x_i = 0$. The quasi-linearization of U from (3.38a) to U_j of (3.41) leads to the natural application of powerful quadratic programming methods (see Gill and Murray 1977). The optimization problem of (3.38) is solved iteratively by the following steps.

Step 1 $j = 0, y^j = 0$.

Step 2 Compute s as a function of y^j using (3.42).

Step 3 Minimize $U_j(y)$ of (41), subject to $y \geq 0$ using the quadratic programming method. The solution is defined as y^{j+1}.

<u>Step 4</u> If $U(y^{j+1}) \approx U(y^j)$, then calculate $\mathbf{x}$ using (3.40) and stop; otherwise, $j \leftarrow j+1$ and go to Step 2.

3.6 DIAGNOSIS USING THE LINEAR PROGRAMMING METHOD

Bandler, Biernacki and Salama (1981), and Bandler, Biernacki, Salama and Starzyk (1982) proposed the diagnosis technique using the ℓ_1 norm optimization. The main assumption is similar to that for the quadratic programming approach. However, instead of solving a sequence of quadratic optimization problems, a linear programming problem is formulated, taking advantage of the nature of the ℓ_1 norm as well as the linearity of the constraint equation. A solution to such a problem tends to satisfy the constraint with minimum number of parameters different from zero. This is consistent with the assumption that a few elements are actually faulty.

The optimization problem can be expressed as

$$\underset{\mathbf{x}}{\text{minimize}}\ U(\mathbf{x}) \triangleq \sum_{i=1}^{2n} w_i |x_i| \tag{3.44a}$$

$$\text{s.t. } \mathbf{A}\mathbf{x} - \mathbf{b} = \mathbf{0}\,, \tag{3.44b}$$

where the constraint equation (3.44b) is defined consistently with (3.32b).

Such a problem can be solved directly using ℓ_1 optimization algorithms, e.g., Hald and Madsen (1985). It can also be handled by using a regular linear programming solver in a similar manner to that of Barrodale and Roberts (1978). Let $\mathbf{y}$ be defined by (3.39). The problem of (3.44) is transformed into a standard LP problem as

$$\underset{\mathbf{y}}{\text{minimize}} \;\; U(\mathbf{y}) \triangleq [w_1 \;\; w_2 \;\; \cdots \;\; w_{2n} \qquad w_1 \;\; w_2 \;\; \cdots \;\; w_{2n}]\,\mathbf{y} \tag{3.45a}$$

$$\text{s.t. } [\mathbf{A} \quad -\mathbf{A}]\,\mathbf{y} = \mathbf{b} \tag{3.45b}$$

$$\mathbf{y} \geq \mathbf{0}\,. \tag{3.45c}$$

At the solution of (3.45), x can be calculated from (3.40).

3.7 MODELLING USING OPTIMIZATION METHODS

In a modelling problem, it is required to find parameter values of an equivalent device model to best fit measurement data. As Hachtel, Scott and Zug (1980) have described, the problem is of a type that is frequently encountered by product assurance engineers. These engineers are faced with the fact that the circuits which come off the product line differ from the circuits designed with circuit simulation programs. Consequently, they need device models which agree with on-chip measurements in order to estimate the statistics of the on-chip circuit performance.

3.7.1 Basic Formulation

Let $\mathbf{f} = \mathbf{f}(\boldsymbol{\phi})$ be a m-vector containing the weighted difference between calculated response $\mathbf{F}(\boldsymbol{\phi}, \omega)$ and measured data $\mathbf{F}^{M}(\omega)$ in the form of

$$w_i(\omega_j)\,(F_i(\boldsymbol{\phi},\omega_j) - F_i^{M}(\omega_j))\,, \qquad i \in \{1, 2, \ldots, n_F\}, \;\; j \in \{1, 2, \ldots, n_\omega\}\,. \tag{3.46}$$

Due to measurement errors and nonideal effects, $\mathbf{f} = \mathbf{0}$ may not be possible. Therefore, the modelling problem can be stated as

$$\underset{\boldsymbol{\phi}}{\text{minimize}} \;\; U(\boldsymbol{\phi}) \tag{3.47a}$$

$$\text{s.t. } \boldsymbol{\phi}_L \leq \boldsymbol{\phi} \leq \boldsymbol{\phi}_U\,, \tag{3.47b}$$

where U is an increasing function of $|f_i(\phi)|$, $i = 1, 2, \ldots, m$. ϕ_L and ϕ_U are lower and upper bounds, respectively for ϕ.

A reasonable objective function $U(\phi)$ can be the least pth function of $f(\phi)$ in the form of (3.2).

With a small value of p, the objective function tends to accommodate measurements which may contain accidental large errors. Large values of p produce satisfactory results when all measurement errors and nonideal effects are small. Successfully implemented algorithms have used $p = 1$ (e.g., Bandler, Kellermann and Madsen 1987), $p = 2$ (e.g. Kondoh 1986), and $p = \infty$ (e.g., Hachtel, Scott and Zug 1980).

3.7.2 Limitations of the Basic Formulation

The basic formulation of modelling problems is a traditional approach which is almost entirely directed at achieving the best possible match between measured and calculated responses. This approach has serious shortcomings in two frequently encountered cases. The first case is when the equivalent circuit parameters are not unique w.r.t. the responses selected and the second is when nonideal effects are not modelled adequately, the latter causing an imperfect match, even if small measurement errors are allowed for. In both cases, a family of solutions for circuit model parameters may exist which produce a reasonable and similar match between measured and simulated responses (Bandler, Chen and Daijavad, 1986b). Such problems become more difficult to handle with a large number of variables where a direct optimization is hopeless unless started with accurate estimates of most circuit element values from independent measurements or calculations (Tsironis and Meierer 1982; Curtice and Camisa 1984).

Efforts in alleviating those difficulties have been made in several directions. Straightforward approaches include seeking additional independent measurements and/or predetermining some variables. Since both actions reduce the freedom of variables, they can be effectively applied if a further exploitation of physical properties of a given device is permitted. However, when faced with a prescribed set of possible measurements and variables, we can proceed to general approaches such as decomposition and multi-circuit measuring.

3.7.3 Reduction of Model Parameters and Decomposition Approaches

Reduction of model parameters may be possible by full investigation of physical properties of the device to be modelled. Such an approach was demonstrated by Curtice and Camisa (1984) in a FET modelling problem. Using dc and zero bias measurements, they reduced the number of variables from 16 to 8. The final results of the modelling was reported to be accurate and unique.

In laboratory experiments, a repeated trial and error procedure may be necessary. Reduction of model variables can be achieved by exploiting the lab experience with sample devices. Insensitive variables should be removed at initial stages of an optimization process. Variables tending to reach the upper or lower bounds during the optimization can be fixed in an appropriate manner (Hachtel, Scott and Zug 1980).

Tsironis and Meierer (1982) and Kondoh (1986) have suggested to decompose the overall optimization problem of (3.47) into a sequence of suboptimizations. They illustrated successful FET modelling by properly defining and ordering subsets of parameters and responses. Insensitively related parameters and responses are separated into different subproblems. A series of suboptimizations can provide a good starting point for the overall optimization. It also improves model accuracy and reduces the possibility of stopping at an undesired local minimum.

An automatic decomposition approach for device modelling and large circuit design was developed by Bandler and Zhang (1987a). Using this approach, suboptimizations for FET modelling problems have been formulated automatically using computerized sensitivity analysis of the device. The results were consistent with those of Kondoh (1986).

3.7.4 Multi-Circuit Approach

This approach was proposed by Bandler, Chen and Daijavad (1986b). The ℓ_1-norm objective function was used. Suppose that after taking measurements on a device at a number of frequency points, we make an easy-to-achieve physical adjustment, such that one or a few components of $\boldsymbol{\phi}$ are changed in a dominant fashion and the rest remain constant or change slightly. Consider the following optimization problem

$$\underset{\boldsymbol{\phi}^1,\,\boldsymbol{\phi}^2}{\text{minimize}} \quad \sum_{k=1}^{2} \sum_{i=1}^{m^k} |f_i^k| + \sum_{j=1}^{n} \beta_j |\phi_j^1 - \phi_j^2| \tag{3.48}$$

with superscript k identifying the original network model (k = 1) or the model after physical adjustment (k = 2). β_j represents an appropriate weighting factor and m^k is an index whose value depends on k, i.e., a different number of frequencies may be used for the original and the perturbed model. $\boldsymbol{\phi}^1$ and $\boldsymbol{\phi}^2$ are vectors containing circuit parameters of the original and perturbed networks, respectively.

By adding the second segment to the objective function, we take advantage of the knowledge that only one or a few components of $\boldsymbol{\phi}$ should change dominantly by perturbing a physical component of the device. Therefore, we penalize the objective function for any change in $\boldsymbol{\phi}$. However, by cleverly selecting the ℓ_1 norm, we still allow for one or a few large changes in $\boldsymbol{\phi}$.

The confidence in the validity of the equivalent circuit parameters increases if 1) an optimization using the objective function of (3.48) results in a reasonable match between calculated and measured responses for both circuits 1 and 2 (original and perturbed) and 2) the examination of the solution reveals changes from $\boldsymbol{\phi}^1$ to $\boldsymbol{\phi}^2$ which are consistent with the physical adjustment, i.e., only the expected components have changed significantly. We can build upon our confidence even more by expanding the technique to more adjustments, i.e., formulating the optimization problem as

$$\underset{\boldsymbol{\phi}'}{\text{minimize}} \quad \sum_{k=1}^{n_c} \sum_{i=1}^{m^k} |f_i^k| + \sum_{k=2}^{n_c} \sum_{j=1}^{n} \beta_j^k |\phi_j^1 - \phi_j^k| , \tag{3.49}$$

where n_c circuits and their corresponding sets of responses, measurements and parameters are considered and the first circuit is the reference model before any physical adjustment. $\boldsymbol{\phi}'$ contains all $\boldsymbol{\phi}^k$, $k = 1, 2, \ldots, n_c$.

3.8 TUNING USING OPTIMIZATION METHODS

Postproduction tuning is often essential in the manufacturing of electrical circuits. Tolerances on the circuit components, parasitic effects and uncertainties in the circuit model cause deviations in the manufactured circuit performance, and violation of the design specifications may result. Therefore, postproduction tuning is included in the final stages of the production process to readjust the network performance in an effort to meet the specifications.

Computer-aided designers have approached the tuning problem in two ways, each emphasizing one distinct facet. Before production, at the time of designing a circuit, one can consider tuning as an integral part of the design process (Bandler, Liu and Tromp 1976; Polak and Sangiovanni-Vincentelli 1979), the objective being to relax the tolerances on the circuit components and compensate for the uncertainties in the model parameters. The integral design problem is formulated and solved using optimization such that the essential demand of production cost reduction is optimally met. The solution of the design problem provides the manufacturer with the allowed design tolerances and the tunable parameters.

In the final production stages, the manufactured circuit is usually tested to check whether or not it meets design specifications. Tuning is often needed. Here, it is required to implement necessary changes in the tunable parameters to adjust the manufactured circuit to satisfy the design requirements (Bandler and Salama 1981).

3.8.1 Preproduction Tuning

Suppose $\boldsymbol{\varepsilon} \triangleq [\varepsilon_1 \ \varepsilon_2 \ \ldots \ \varepsilon_n]^T$ and $\mathbf{t} \triangleq [t_1 \ t_2 \ \ldots \ t_n]^T$ are vectors containing tolerances and maximum tuning amounts, respectively, for the parameter $\boldsymbol{\phi} \triangleq [\phi_1 \ \phi_2 \ \ldots \ \phi_n]^T$. A nonlinear programming problem integrating design centering, tolerancing and tuning can be stated as:

$$\underset{\boldsymbol{\phi}^0, \boldsymbol{\varepsilon}, \mathbf{t}}{\text{minimize}} \ U(\boldsymbol{\phi}^0, \boldsymbol{\varepsilon}, \mathbf{t}) \tag{3.50a}$$

$$\text{s.t.} \quad \boldsymbol{\phi} = \boldsymbol{\phi}^0 + \mathbf{E}\boldsymbol{\mu} + \mathbf{T}\boldsymbol{\rho} \in R_c$$

$$\text{for all } \boldsymbol{\mu}, \ \boldsymbol{\mu} \in R_\mu \quad \text{and} \tag{3.50b}$$

$$\text{some } \boldsymbol{\rho}, \ \boldsymbol{\rho} \in R_\rho,$$

where $\mathbf{E}$ and $\mathbf{T}$ are $n \times n$ diagonal matrices containing ε_i, $i = 1, 2, ..., n$ and t_i, $i = 1, 2, ..., n$, respectively, and

$$\boldsymbol{\mu} \triangleq [\mu_1 \ \ \mu_2 \ \cdots \ \ \mu_n]^T \tag{3.51}$$

$$\boldsymbol{\rho} \triangleq [\rho_1 \ \ \rho_2 \ \cdots \ \ \rho_n]^T . \tag{3.52}$$

Also, R_c is a constraint region in which all responses satisfy their specifications. R_μ is a region in which $|\mu_i| \le 1$, $i = 1, 2, ..., n$. R_ρ is defined as the region $\{\boldsymbol{\rho} | -1 \le \rho_i \le 1, i = 1, 2, ..., n\}$ for two way tuning and, $\{\boldsymbol{\rho} | \ 0 \le \rho_i \le 1, \ i = 1, 2, ..., n\}$ or $\{\boldsymbol{\rho} | -1 \le \rho_i \le 0, i = 1, 2, ..., n\}$ for one way tuning. The objective function can be an increasing function of $|t_i/\phi_i^0|$ and a decreasing function of $|\varepsilon_i/\phi_i^0|$, respectively. A detailed treatment of the preproduction tuning was presented in Bandler, Liu and Tromp (1976) and in Polak and Sangiovanni-Vincentelli (1979).

3.8.2 Postproduction Tuning: Problem Formulation

Prior to postproduction tuning, the manufactured circuit is characterized by the actual parameter values given by

$$\boldsymbol{\phi}^a = \boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}^a . \tag{3.53}$$

Suppose, for convenience, that the preproduction stage resulted in $t_i > 0$ for $i = 1, 2, \ldots, n_t$ and $t_i = 0$ for $i = n_t + 1, \ldots, n$. Therefore, the tunable parameters are ϕ_i, $i = 1, 2, \ldots, n_t$. A set of circuit performance functions given by

$$\mathbf{F}(\boldsymbol{\phi}, \omega) = \mathbf{F}(\boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}^a + \mathbf{T}\,\boldsymbol{\rho}, \omega) \tag{3.54}$$

are usually monitored during the tuning process. The desired values for $\mathbf{F}$, denoted as $\mathbf{F}^d$ can be either an optimal response or a design specification. Define $\mathbf{f} = \mathbf{f}(\boldsymbol{\phi})$ as a m-vector whose elements are in the form of

$$w_{Ui}(\omega_j)(F_i(\boldsymbol{\phi}, \omega_j) - S_{Ui}(\omega_j))$$

$$\tag{3.55}$$

$$-\,w_{Li}(\omega_j)(F_i(\boldsymbol{\phi}, \omega_j) - S_{Li}(\omega_j))$$

where $i \in \{1, 2, ..., n_F\}$, $j \in \{1, 2, ..., n_\omega\}$ and $\boldsymbol{\phi} \equiv \boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}^a + \mathbf{T}\,\boldsymbol{\rho}$. S_{Ui} and S_{Li} are upper and lower specifications, respectively. w_{Ui} and w_{Li} are weighting factors and are nonnegative. If it is required to match $F_i(\boldsymbol{\phi}, \omega)$ with its desired value $F_i^d(\omega)$, one can either use (3.55) by setting

$$S_{Ui}(\omega) = S_{Li}(\omega) = F_i^d(\omega) \tag{3.56}$$

or define elements of **f** as

$$w_i(\omega_j)\,|\,F_i(\boldsymbol{\phi}, \omega_j) - F_i^d(\omega_j)\,|\,. \tag{3.57}$$

The postproduction tuning can be formulated as the optimization problem

$$\underset{\boldsymbol{\rho}'}{\text{minimize}}\;\; U(\boldsymbol{\rho}) \tag{3.58a}$$

$$\text{s.t.}\;\; |\rho_j| \le 1\,,\quad j = 1, 2, ..., n_t\,, \tag{3.58b}$$

where $\boldsymbol{\rho}'$ is a n_t– vector containing the first n_t elements in $\boldsymbol{\rho}$. The objective function can be a least pth or a generalized least pth function of $\mathbf{f}(\boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}^a + \mathbf{T}\,\boldsymbol{\rho})$, i.e., in the forms of (3.2) and (3.3), respectively.

3.8.3 Postproduction Tuning: Functional Approach

Functional tuning is a traditional approach. The tunable parameters are sequentially adjusted until the circuit specifications are met. Here, the network elements are generally assumed unknown.

Let **J** be a $m \times n_t$ Jacobian matrix whose (i, j)th element is defined by

$$J_{ij} = \frac{\partial f_i}{\partial \rho_j} = \frac{\partial f_i}{\partial \phi_j}\, t_j\,, \tag{3.59}$$

$$i = 1,2,...,m \quad \text{and} \quad j = 1,2,...,n_t\,.$$

The least squares optimization of (3.58), namely, taking $U = \mathbf{f}^T\,\mathbf{f}$, was proposed by Antreich, Gleissner and Muller (1975) and Adams and Manaktala (1975). The solution is given by

$$\Delta \boldsymbol{\rho}' = -(\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \mathbf{f}(\boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}^a + \mathbf{T}\,\boldsymbol{\rho}). \tag{3.60}$$

The minimax optimization of (3.58), namely, taking $U = \max f_i$, was approximated by Bandler and Salama (1981, 1985b) who solved the following linear programming problem

$$\underset{\Delta\boldsymbol{\rho}', z}{\text{minimize}} \; z \tag{3.61a}$$

$$\text{s.t. } f_i(\boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}^a + \mathbf{T}\,\boldsymbol{\rho}) + \sum_{j=1}^{n_t} J_{ij} \Delta\rho_j \le z \tag{3.61b}$$

$$\rho_{Lj} \le \Delta\rho_j \le \rho_{Uj}, \quad i=1,2,\ldots,m, \;\; j=1,2,\ldots,n_t.$$

$\boldsymbol{\rho}$ is initially set to $\mathbf{0}$. After each solution of (3.60) or (3.61), $\boldsymbol{\rho}$ is updated using $\Delta\boldsymbol{\rho}'$. As proposed by Bandler and Salama, simulated sensitivities and Broyden formula can be used for obtaining and updating $\mathbf{J}$.

3.8.4 Postproduction Tuning: Deterministic Approach

In contrast to the functional tuning approach, deterministic tuning requires that all circuit parameters $\boldsymbol{\phi}$ and possible parasitic parameters $\boldsymbol{\zeta}$ (or its effects) can be either measured or identified. By utilizing this information, the optimization of (3.58) becomes faster.

A sequential tuning algorithm has been introduced by Lopresti (1977). Let f be the m-vector defined in (3.55) or (3.57). Initially, we set $\boldsymbol{\rho} = \mathbf{0}$ and define

$$\mathbf{f}^1 \triangleq \sum_{i=n_t+1}^{n} \phi_i \frac{\partial \mathbf{f}}{\partial \phi_i} \frac{\Delta\phi_i}{\phi_i} + \sum_i \zeta_i \frac{\partial \mathbf{f}}{\partial \zeta_i} \frac{\Delta\zeta_i}{\zeta_i}. \tag{3.62}$$

which represents the deviation of f from $\mathbf{f}(\boldsymbol{\phi}^0)$ due to parasitic effects and tolerances in untunable parameters. In the kth iteration, we have

$$\mathbf{f}^{k+1} = \mathbf{f}^k + [J_{1k}\, J_{2k} \ldots J_{mk}]^T \Delta\rho_k, \quad k = 1, 2, \ldots, n_t. \tag{3.63}$$

By defining U of (3.58) as a quadratic function of $\mathbf{f}^{n_t+1}$ and adding a term penalizing large changes in $\Delta\boldsymbol{\rho}'$, we obtain an optimal control problem, i.e., finding $\Delta\boldsymbol{\rho}'$ such that

$$U = \left(\mathbf{f}^{n_t+1}\right)^T \mathbf{B}\,\mathbf{f}^{n_t+1} + \sum_{j=1}^{n_t} \beta_j (\Delta\rho_j)^2 \tag{3.64}$$

is minimized subject to (3.63). **B** of (3.64) is a positive semidefinite matrix and $\beta_j > 0$, $j = 1, 2, \ldots, n_t$. A closed form solution can be obtained in a form as

$$\Delta\rho_k = \boldsymbol{\gamma}_k^T \mathbf{f}^k, \tag{3.65}$$

where $\boldsymbol{\gamma}_k$ is a m-vector calculated using Riccatti equation (see Lopresti 1977).

Instead of using first-order sensitivity information **J** which becomes invalid when components of $\Delta\boldsymbol{\rho}'$ are not small enough, Alajajian, Trick and El-Masry (1980) have suggested a large change sensitivity method for deterministic tuning. The resulting equation is

$$[\mathbf{J}^L \;\; -\mathbf{f}(\boldsymbol{\phi}^0)] \begin{bmatrix} \Delta\boldsymbol{\rho}' \\ c \end{bmatrix} = -\mathbf{f}(\boldsymbol{\phi}^a), \tag{3.66}$$

where $\mathbf{J}^L$ is the large change sensitivity matrix of **f** w.r.t. $\boldsymbol{\rho}'$ and c is an unknown variable.

3.9 EXAMPLES

In this section, we first present the application of optimization techniques for circuit diagnosis through a simple illustrative example followed by selected problems of practical interest for diagnosis, modelling and tuning.

3.9.1 Diagnosis Using Optimization: An Illustrative Example

Consider the passive resistive network of Fig. 3.2. Nominal values for elements G_i, $i = 1, 2, \ldots, 5$ are equal to 1. Each element has $\pm 5\%$ tolerance. The measurable responses are nodal voltages, i.e., $\mathbf{F} = [V_1 \;\; V_2 \;\; V_3]^T$, causing the **C** of

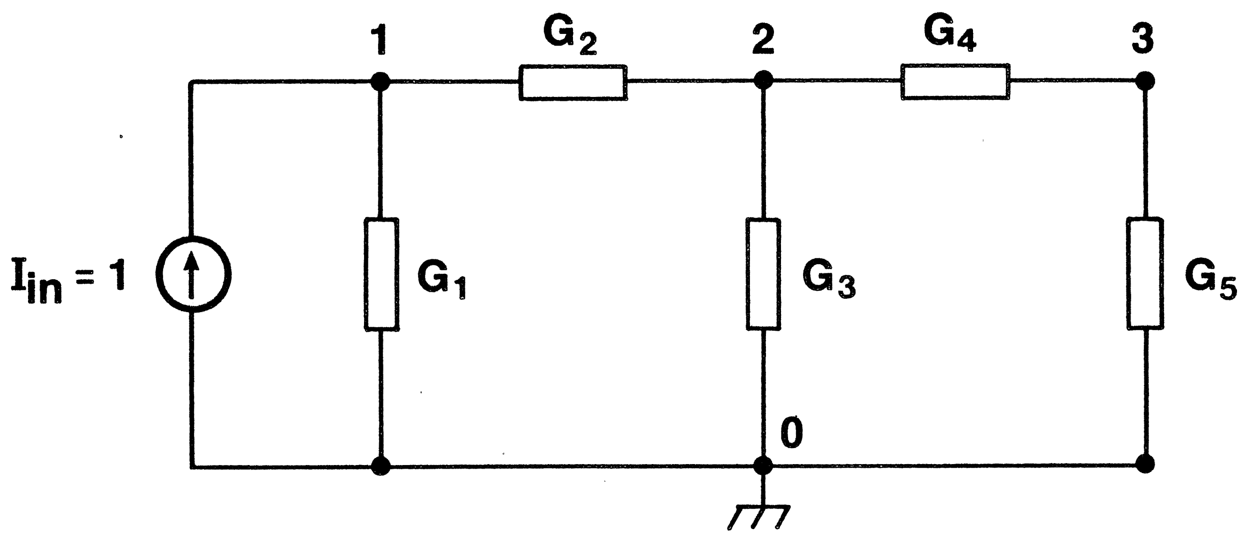


Fig. 3.2 A passive resistive circuit as an example for diagnosis using optimization techniques.

(3.11) to be a 3×3 identity matrix. Also for the example, $N = 3$, $n_F = 3$ and $n = 5$. The incidence matrix is given by

$$\mathbf{Q} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}. \tag{3.67}$$

The variable parameters are defined as $\boldsymbol{\phi} = [G_1 \;\; G_2 \;\; G_3 \;\; G_4 \;\; G_5]^T$. The nodal admittance matrix at nominal point $\boldsymbol{\phi}^0 = [1 \;\; 1 \;\; 1 \;\; 1 \;\; 1]^T$ is

$$\mathbf{Y}(\boldsymbol{\phi}^0) = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}. \tag{3.68}$$

For such a circuit, all quantities are real. Therefore, the constraint equations as well as the related definitions (3.17)–(3.20) becomes

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}\,, \tag{3.69}$$

where

$$\mathbf{A} = -\mathbf{C}^T[\mathbf{Y}(\boldsymbol{\phi}^0)]^{-1}\mathbf{Q}$$

$$= \frac{-1}{8}\begin{bmatrix} 5 & 3 & 2 & 1 & 1 \\ 2 & -2 & 4 & 2 & 2 \\ 1 & -1 & 2 & -3 & 5 \end{bmatrix}, \tag{3.70}$$

$$\mathbf{b} = \mathbf{F}^M - \mathbf{F}^0 = [V_1^M - V_1^0 \quad V_2^M - V_2^0 \quad V_3^M - V_3^0]^T \tag{3.71}$$

and

$$\mathbf{x} = [\Delta I_1^b \quad \Delta I_2^b \quad \Delta I_3^b \quad \Delta I_4^b \quad \Delta I_5^b]^T, \tag{3.72}$$

where $\Delta I_i{}^b$, $i = 1, 2, \ldots, 5$ are the equivalent current sources representing ΔG_i, $i = 1, 2, \ldots, 5$ shown in Fig. 3.3. The nominal responses $\mathbf{F}^0 = [V_1{}^0 \;\; V_2{}^0 \;\; V_3{}^0]^T$ can be calculated as $\mathbf{F}^0 = [5/8 \;\; 2/8 \;\; 1/8]^T$.

Case 1: Here, we assume that no elements have much greater deviation from nominal than others. Table 3.1 shows the results of diagnosis using the ℓ_1, ℓ_2 and the

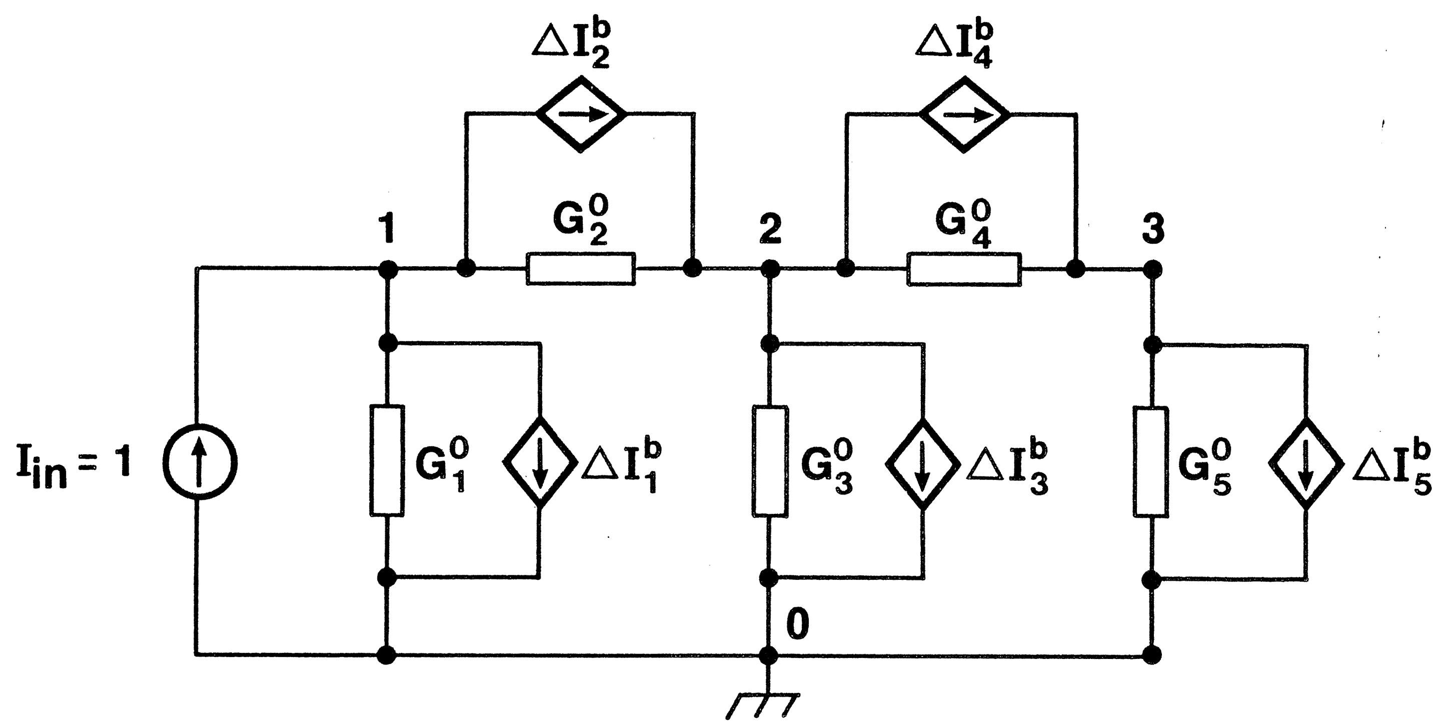


Fig. 3.3 Equivalent current sources representing the effect of changes in G_i, $i = 1, 2, \ldots, 5$ for the circuit of Fig. 3.2. $\Delta I_i^b = \Delta G_i V_i^b$, where V_i^b is the voltage across the ith element.

TABLE 3.1

RESULTS OF DIAGNOSIS USING OPTIMIZATION TECHNIQUES FOR THE CIRCUIT OF FIG. 3.2, CASE 1

CUT	Measurement V^M	Actual $\Delta G_i/G_i^0\%$ $i=1,2,\ldots,5$	Detected $\Delta G_i/G_i^0\%$, $i=1,2,\ldots,5$		
			ℓ_2 Optim.	Quadr. Program.	ℓ_1 Optim.
#1	.5730	4.4	4.36	14.02	13.23
	.2326	18.0	18.07	1.80	3.14
	.1186	9.0	7.61	0.00	0.00
		30.0	33.04	0.00	4.00
		25.0	27.93	−3.85	0.00
#2	.6437	−2.0	−2.38	0.00	0.00
	.2241	−12.0	−11.41	−15.07	−15.07
	.1145	6.0	5.28	7.92	7.92
		20.0	23.73	4.35	4.35
		15.0	18.58	0.00	0.00
#3	.6266	3.0	−0.42	0.00	0.00
	.2412	−8.0	−2.44	−3.12	−3.12
	.1307	−3.4	1.96	0.60	0.60
		10.0	17.69	18.28	18.28
		−7.0	−0.50	0.00	0.00

quadratic programming method. It is demonstrated that the least squares method gives a more reasonable solution, while the other two methods have mistakenly detected, e.g., G_4 as nonfaulty while this element actually changed 30% for CUT #1. However, the ℓ_2 optimization method may also fail to give correct results, see CUT #3 where the G_2 and the G_5 are not detected as out of tolerance.

Case 2: In this case, we assume that only a few elements are faulty having much greater deviation from nominal than the rest element that are within the specified tolerance of $\pm 5\%$. Table 3.2 shows the results of diagnosis using the three optimization techniques presented. It can be seen that both the ℓ_1 and the quadratic techniques give much sharper results than the ℓ_2 technique. In many cases, both ℓ_1 and the quadratic optimization produce the same solution. In some cases, as shown for CUT #2 and CUT #3 in Table 3.2, one method yields a better solution than the other.

For the quadratic programming technique, we have used $\delta = 10^{-6}$ and $\beta = 10^{10}$. The QPSOL Fortran package for quadratic programming (Gill et al 1984) was utilized to perform Step 3 in Section 3.5 with a limit on the number of iterations for each quadratic programming as 3.

3.9.2 Diagnosis of a 28 Node Circuit

Kellermann (1986) experimented with the nonlinear optimization problem of (3.9) with $p = 1$, on a 28-node circuit shown in Fig. 3.4. The nominal values of the elements $G_i = 1.0$ and tolerances $\varepsilon_i = \pm 0.05$, $i = 1, 2, ..., 52$. All outside nodes are assumed to be accessible for measurements. The actual circuit includes four faults where elements G_{41}, G_{44}, G_{45} and G_{48} have -50% deviation from nominal. All other

TABLE 3.2

RESULTS OF DIAGNOSIS USING OPTIMIZATION TECHNIQUES FOR THE CIRCUIT OF FIG. 3.2, CASE 2

CUT	Measurement V^M	Actual $\Delta G_i/G_i^0\%$ i=1,2,...,5	Detected $\Delta G_i/G_i^0\%$, i=1,2,...,5		
			ℓ_2 Optim.	Quadr. Program.	ℓ_1 Optim.
#1	.5000	0.0	16.98	0.00	0.00
	.3333	200.0	149.06	200.00	200.00
	.1667	0.0	−8.49	0.00	0.00
		0.0	−33.96	0.00	0.00
		0.0	−33.96	0.00	0.00
#2	.5933	2.0	1.95	1.77	5.77
	.2207	6.0	6.08	6.36	0.00
	.1755	−3.0	9.68	0.00	0.00
		300.0	238.72	288.35	235.89
		3.0	−12.78	0.00	−13.51
#3	.2688	200.0	63.71	199.62	199.04
	.1304	40.0	304.67	40.73	41.87
	.0660	−3.0	93.19	0.00	0.00
		4.5	378.57	0.00	2.45
		2.0	367.12	−2.39	0.00

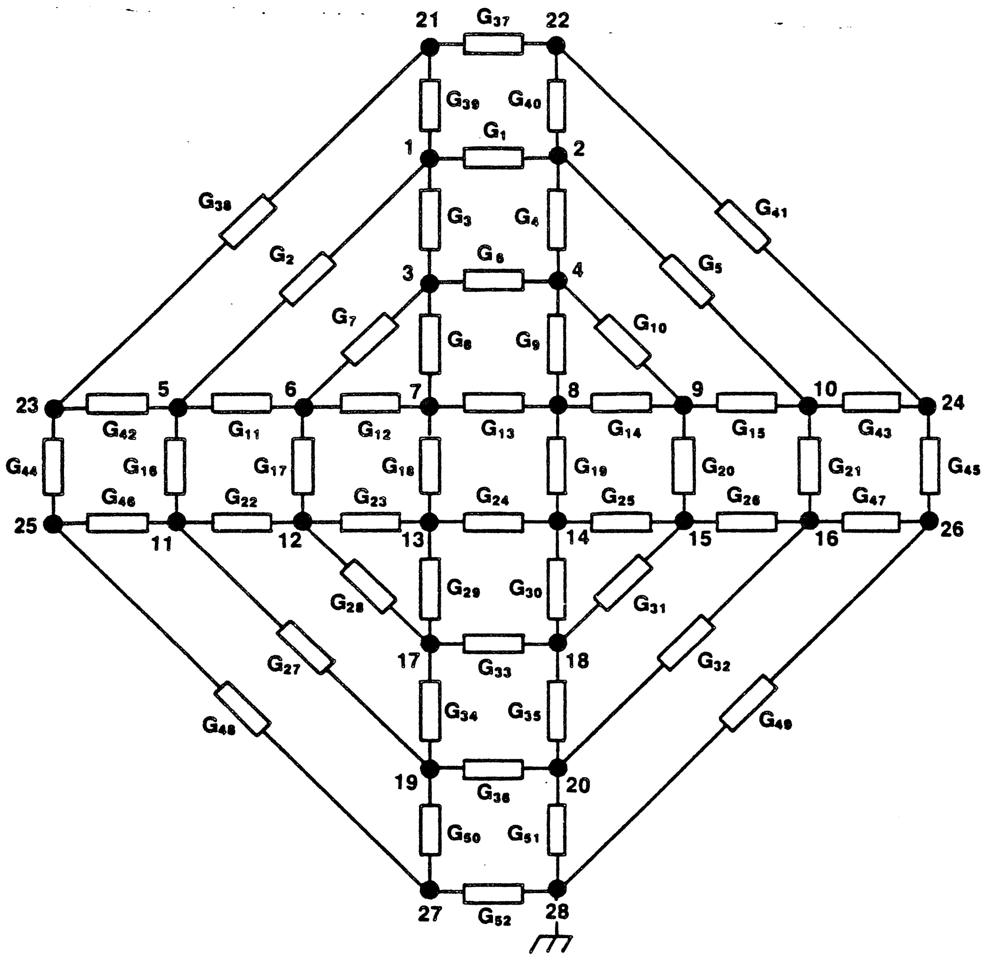


Fig. 3.4 A resistive mesh network (28 nodes).

element values are within their tolerances. The diagnosis was performed successfully with only one excitation. Resulting deviations for G_{41}, G_{44}, G_{45} and G_{48} are -46%, -54%, -45% and -53%, respectively. Deviations for other elements are mostly zero except for a few small nonzero values.

3.9.3 GaAs FET Modelling: Multi-Circuit Approach

This example is due to Bandler, Chen and Daijavad (1986b). They used the equivalent circuit at normal operating bias (including the carrier), as illustrated in Fig. 3.5, and created artificial measurements using TOUCHSTONE (1985). Two sets of *S*-parameter (scattering) measurements were created; one set using the parameters reported by Curtice and Camisa (1984) (operating bias V_{ds} = 8.0 V, V_{gs} = -2.0 V and I_{ds} = 128.0 mA) and the other by changing the values of C_1, C_2, L_g and L_d to simulate the effect of taking different reference planes for the carriers. Both sets of data are shown in Fig. 3.6, where the *S*-parameters of the two circuits are plotted on a Smith Chart. Although the maximum number of possible variables, namely 32 (16 for each circuit), were allowed for in the optimization, the intrinsic parameters were found to be the same between the two circuits, and as expected, C_1, C_2, L_g and L_d changed from circuit 1 to 2. Table 3.3 summarizes the parameter values obtained. The problem involved 128 nonlinear functions (real and imaginary parts of 4 *S*-parameters, at 8 frequencies, for two circuits), 16 linear functions and 32 variables.

3.9.4 A Highpass Filter Example for Postproduction Tuning

The highpass notch filter circuit shown in Fig. 3.7 was used by Bandler and Salama (1981) to demonstrate postproduction tuning algorithms. The circuit example

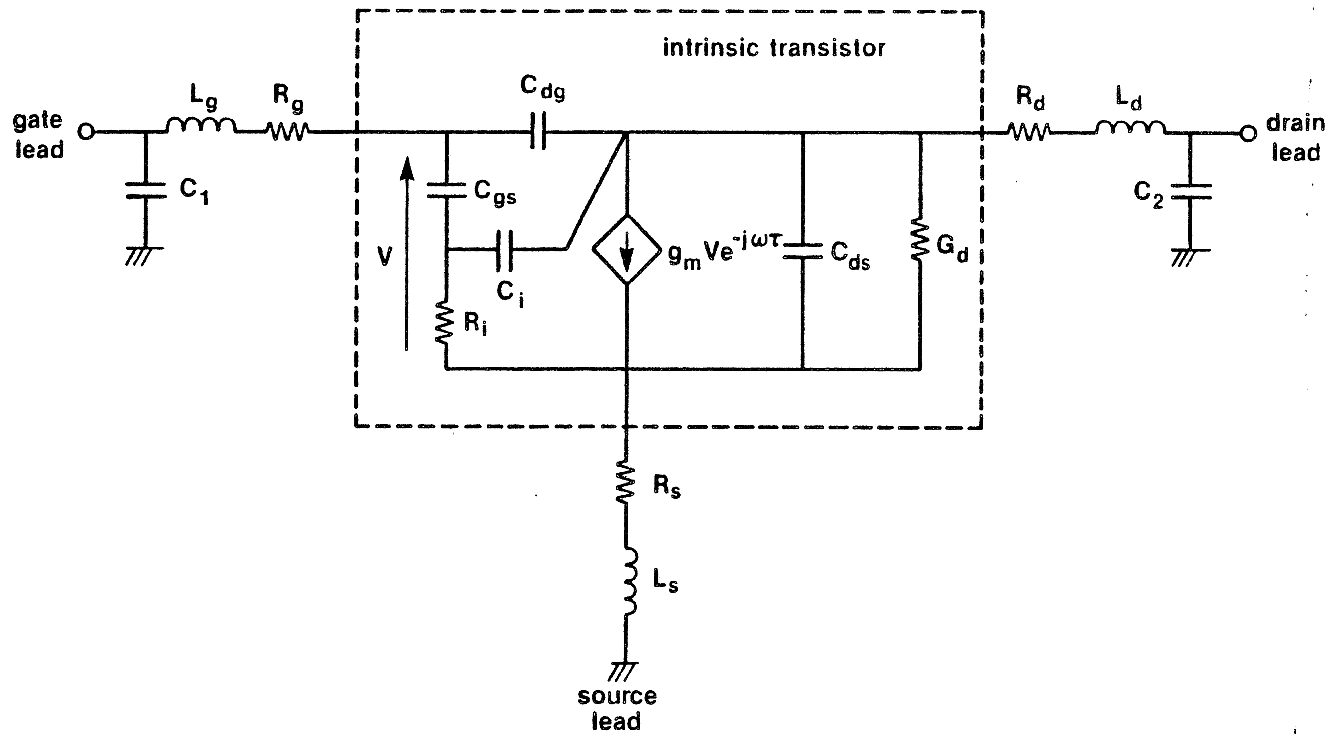


Fig. 3.5 Equivalent circuit of carrier-mounted FET (Device model B1824-20C).

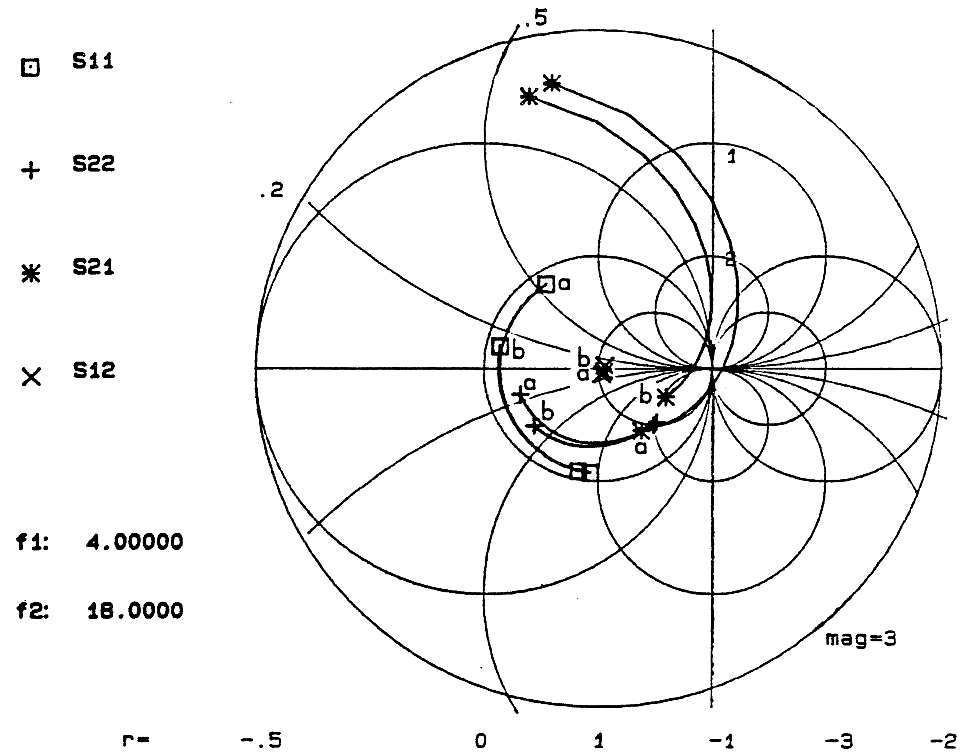


Fig. 3.6 Smith Chart display of scattering parameters S_{11}, S_{22}, S_{12} and S_{21}, for the carrier-mounted FET, before and after adjustments on parameters. Points a and b mark the high frequency end of original and perturbed network responses, respectively.

TABLE 3.3

RESULTS FOR THE GaAs FET EXAMPLE

Parameter		Original Circuit	Perturbed Circuit
C_1	(pF)	0.0440	0.0200*
C_2	(pF)	0.0389	0.0200*
C_{dg}	(pF)	0.0416	0.0416
C_{gs}	(pF)	0.6869	0.6869
C_{ds}	(pF)	0.1900	0.1900
C_i	(pF)	0.0100	0.0100
R_g	(Ω)	0.5490	0.5490
R_d	(Ω)	1.3670	1.3670
R_s	(Ω)	1.0480	1.0480
R_i	(Ω)	1.0842	1.0842
G_d^{-1}	(kΩ)	0.3761	0.3763
L_g	(nH)	0.3158	0.1500*
L_d	(nH)	0.2515	0.1499*
L_s	(nH)	0.0105	0.0105
g_m	(S)	0.0423	0.0423
τ	(ps)	7.4035	7.4035

* significant change in parameter value

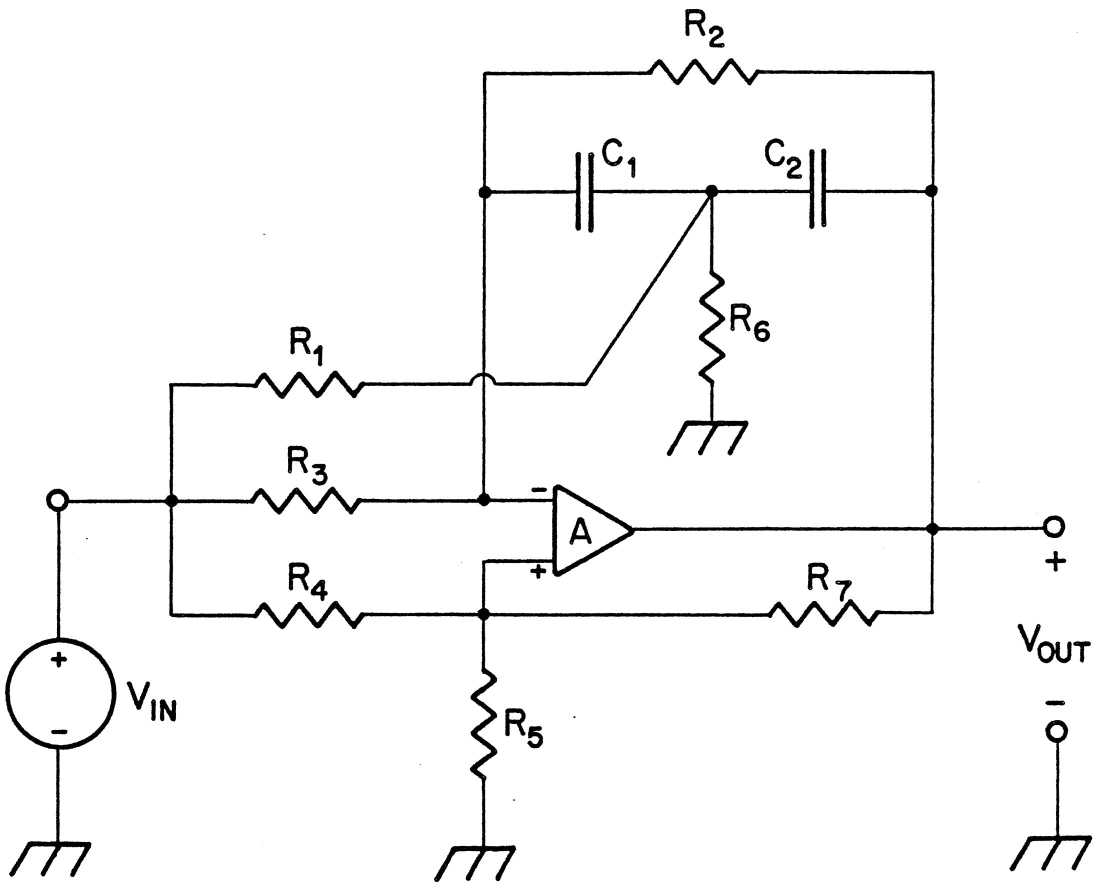


Fig. 3.7 The highpass notch filter circuit.

was originally employed by Alajajian (1979). R_3, R_5, R_6 and R_7 are tunable parameters. The nominal and actual element values are given in Table 3.4.

To use the functional tuning approach of (3.61), Bandler and Salama defined f_i as the absolute value of V_{out} from its nominal, i.e., using (3.57) with $F(\phi, \omega) = V_{out}(\phi, \omega)$ and $F^d(\omega) = V_{out}(\phi^0, \omega)$. Twenty frequencies on the interval 410–505 Hz were used. The limits in (3.61b) are $\rho_{Uj} = -\rho_{Lj} = 0.02$. After 11 iterations, the tuned responses very closely approached the nominal responses, as shown in Fig. 3.8(a). After tuning, the values for tunable parameters $[R_3\ \ R_5\ \ R_6\ \ R_7] = [201.952\ \ 2.115\ \ 13.061\ \ 0.973]$.

The deterministic approach of (3.62)–(3.65) was performed with $\mathbf{F} = [F_1\ \ F_2\ \ ...\ \ F_5]^T$, where the F_i are coefficients in the transfer function of the filter $\Gamma = (s^2+F_1s+F_2)^{-1}\ (F_3s^2+F_4s+F_5)$. $\mathbf{B}$ of (3.64) was taken as diag. $\{4,\ 0.04,\ 4,\ 10^{12},\ 0.0625\}$ and $\beta_j = 0.001$. The responses associated with the tuning is shown in Fig. 3.8(b). After tuning, the values for tunable parameters $[R_3\ \ R_5\ \ R_6\ \ R_7]$ are $[184.487\ \ 2.241\ \ 13.747\ \ 0.9993]$.

3.10 DISCUSSIONS

Close links and similarities exist between optimization techniques for modelling, diagnosis and tuning. In this section, relevant common aspects are discussed.

3.10.1 Use of Sensitivity Information

<u>Sensitivity Matrix</u>

Suppose $f(\phi)$ is defined by (3.46) for modelling and diagnosis and by (3.55) or (3.57) for design and tuning. Let ϕ^0 be the design nominal. Define the $n \times m$

TABLE 3.4

ELEMENT VALUES FOR THE HIGHPASS FILTER OF FIG. 3.7

Element	Nominal Value	Actual Value	Percentage Deviation
R_1 (kΩ)	13.260	13.260	0.0
R_2 (kΩ)	93.0	93.0	0.0
R_3 (kΩ)	214.0	192.6	−10.0
R_4 (kΩ)	2.0	2.0	0.0
R_5 (kΩ)	2.0	1.8	−10.0
R_6 (kΩ)	12.467	11.221	−10.0
R_7 (kΩ)	10.00	9.00	−10.0
C_1 (μF)	0.01	0.00973	−2.07
C_2 (μF)	0.01	0.00965	−3.35
A	10000.0	10000.0	0.0

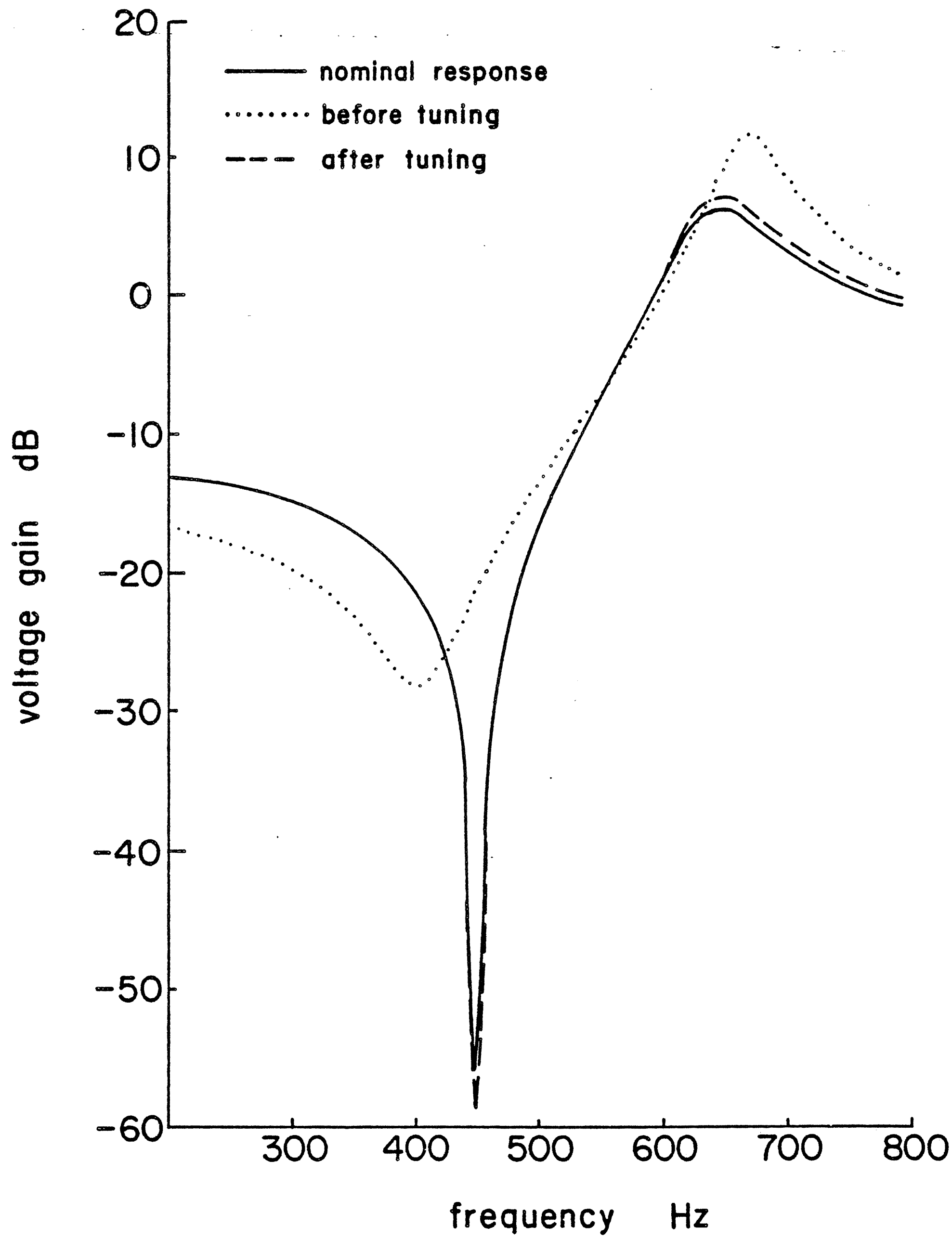


Fig. 3.8(a) The responses for the tuning of the highpass notch filter using functional tuning.

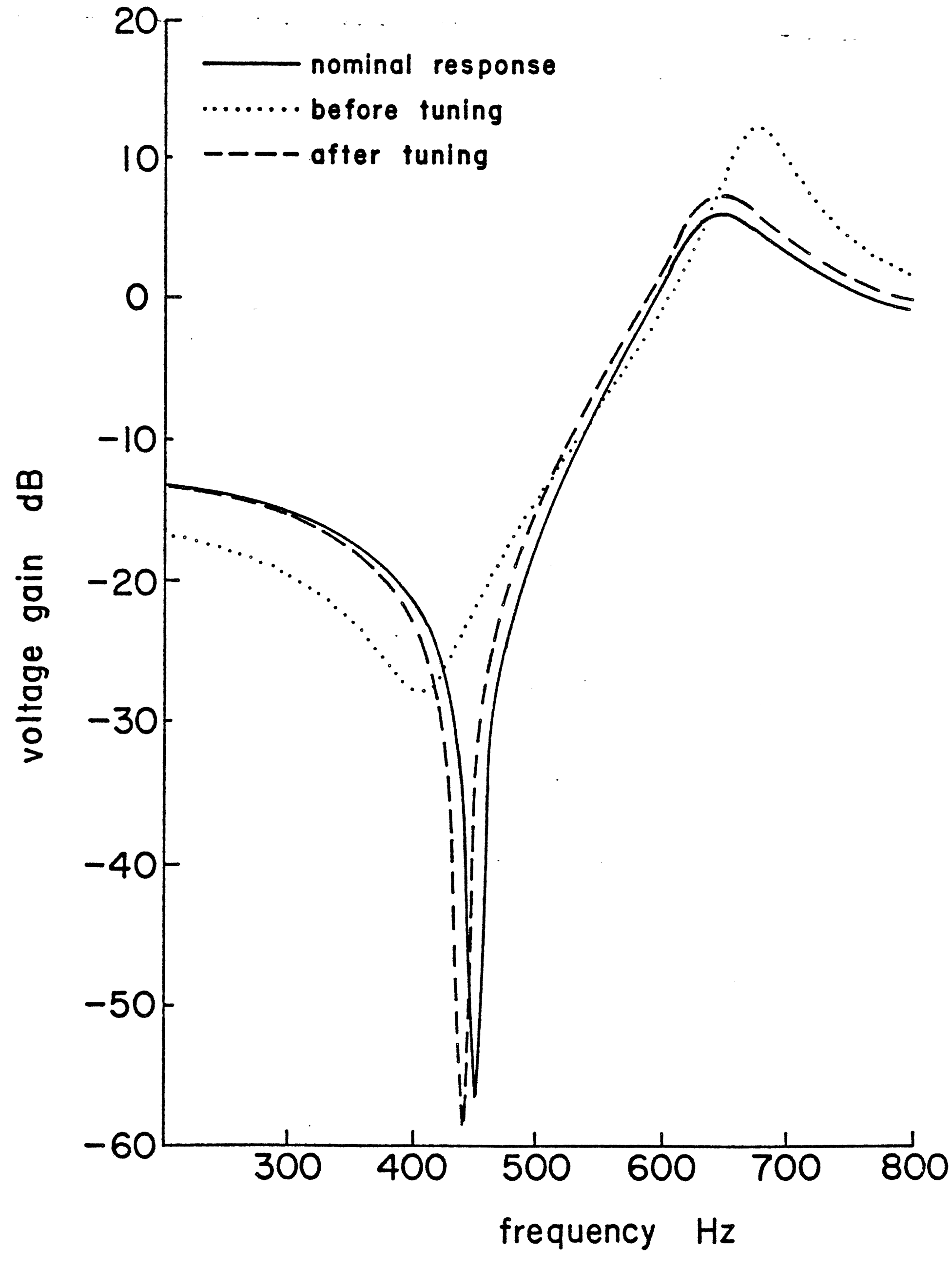


Fig. 3.8(b) The responses for the tuning of the highpass notch filter using deterministic tuning.

sensitivity matrix as

$$S(\phi) \triangleq \text{diag}\{\phi^0\} \frac{\partial f^T(\phi)}{\partial \phi} \text{diag}\{f(\phi^0)\}^{-1}. \tag{3.73}$$

ϕ^* is said to be a regular point (see Saeks, Sangiovanni-Vincentelli and Visvanathan 1981) of $S(\phi)$ if there exists an open neighbourhood of ϕ^* in which $S(\phi)$ has constant rank. Parameter identification (or modelling) is usually performed with the assumption that the actual parameter ϕ^a is at a regular point and $\text{Rank}[S(\phi^a)] = n$. Otherwise, if $\text{Rank}[S(\phi^a)] < n$, i.e., the measurement is not sufficient, we should either use the diagnosis technique introduced in Sections 3.3 to 3.6, or seek possible additional measurements by creating any or a combination of 1) more accessible nodes for excitation and/or measurement, 2) more frequency points, 3) other types of responses (e.g., voltage and current), 4) additional circuits obtained by perturbing a few parameters in the CUT. Research has been performed on the selection of excitation and measurement ports and frequencies (Bandler and Salama 1985a) as well as the multi-type response and multi-circuit concepts, e.g. (Bandler, Chen and Daijavad 1986b).

In tuning problems, it is desired that the submatrix containing the first n_t rows of S (assuming that only the first n_t elements in $[\phi_1 \;\; \phi_2 \;\; \ldots \;\; \phi_n]^T$ are tunable) has a rank which should be as high as the rank of S. Such rank comparison implicates the degree of difficulty to achieve the desired response by tuning ϕ_i, $i = 1, 2, \ldots, n_t$ only.

By checking the S matrix, possible decomposition can be carried out, sequentially optimizing subsets of responses vs. variables which are sensitively related (Bandler, Chen, Daijavad, Kellermann, Renault and Zhang 1986).

Large Change Sensitivity

The embedding of large change sensitivity calculations in an optimization procedure, where only a small subset of circuit parameters are updated each iteration, can greatly increase the efficiency. The application of Householder's formula in fault diagnosis was reported by Temes (1977), Johnson Jr. (1979) and Chen and Saeks (1979). Such application can reduce re-evaluation of $F(\phi)$ from the order of n to r, r being a rank measure of the subcircuit to be updated. r is less than or equal to the number of parameters in the subcircuit (Haley and Current 1985; Bandler and Zhang 1986).

3.10.2 Convergence and Possible Difficulties Using Optimization Techniques

For problems using the ℓ_1 and minimax optimization method of Hald and Madsen (1981, 1985), superlinear or quadratic convergence are guaranteed. The convergence for Merrill's quadratic approach was reported to be about 2 or 3 iterations. For a decomposed problem, sequential optimization may diverge if the subproblems are not well defined or not reasonably ordered. Therefore, it may be desirable to have the system less decomposed as the solution is being approached. Usually, an optimization converges only to a local minimum unless the objective and the constraints satisfy certain conditions. Global optimization methods are being studied (Groch, Vidigal and Director 1985).

Poor or unacceptable results in computer-aided circuit optimization are felt to be most likely due to bad preparation of the problem, a lack of understanding of the hazards that can be encountered and the wrong choice of algorithm (Bandler 1973). Compared with other techniques for modelling, diagnosis and tuning (if applicable), optimization techniques often require more computer time and storage. The choice of starting point is often a demanding task for satisfactory solution and fast convergence.

3.11 CONCLUSION

We have presented basic principles of optimization techniques for modelling, diagnosis and tuning. Emphasis is centered on the problem formulation and related properties, rather than mathematical sophistication of optimization procedures and detailed circuit aspects of MDT. Further research can be directed toward effective modelling techniques to improve the validity of identified parameters. The use and organization of decomposition needs further investigation. The desired outcome is an automatic procedure capable of identifying circuit parameters and making decisions of physical adjustments upon monitored responses and identified parameters.

4

LARGE CHANGE SENSITIVITY ANALYSIS OF LINEAR SYSTEMS

4.1 INTRODUCTION

In computer aided circuit design, it is often required to calculate network responses after a certain set of parameters are changed. This problem, referred to as large change sensitivity problem, becomes especially important when the network is large and/or when a large number of repeated circuit analysis is needed. Without solving the entire network equations for every set of parameter changes, one can update the network responses more efficiently by using large change sensitivity analysis methods. This approach has been studied by many people. Fidler (1976) and Singhal, Vlach and Bryant (1973), considered single and multiple parameter changes, respectively, and developed methods to calculate the response function as a multi-linear form in variable parameters Another method is to formulate a reduced system, whose solutions are then used to update the responses. This method has been treated from different angles, e.g., the current source substitution approach of Leung and Spence (1975), the adjoint network approach of Temes and Cho (1978), the Householder formula approach (Leung and Spence 1975; Hajj 1981), the scattering matrix approach of Haley (1980) and the matrix partitioning approach of Vlach and Singhal (1983). Hajj (1981) derived and summarized a set of algorithms where finite, infinite and zero parameter changes are all permitted and sparsity is exploited. Rauscher and Epprecht (1974) (also, see Gupta, Gary and Chadha 1981) used the concept of large change sensitivities to update wave variables in analyzing perturbed microwave networks. A recent overview of this area is given by Haley and Current

(1985) who presented general approaches encompassing most of the previous methods. A survey of updating matrix inverse formulas was given by Henderson and Searle (1981).

As already noticed, large change analysis algorithms will lose efficiency when too many parameters are changed. This is mainly because the algorithms involve the solution of a reduced system of order n, the number of variables. However, cases exist where this system is larger than needed. Also, in a Monte-Carlo analysis or in an optimization procedure, it is possible that some variables change slightly while others change substantially. In this case, the small parameter changes may cause ill-conditioning in a non-iterative method (e.g., Leung and Spence 1975) and the large parameter changes may affect the convergence rate in an iterative method (Hajj 1981).

In this chapter, we present a set of generalized Householder formulas which is capable of handling complicated cases encountered in practice. The problem of determining a minimum reduced system is investigated. Different aspects of the basic set of formulas are discussed in terms of duality property and operational count. Applications to general linear systems are considered for original and adjoint responses with single and multiple input and output situations. Also, as a special case, a series of first-order sensitivity expressions are obtained without reference to Tellegen's theorem. Numerical examples are given for a general system of linear equations and for two electrical circuits.

4.2 A SET OF GENERALIZED HOUSEHOLDER FORMULAS

4.2.1 Generalized Householder Formulas

Let the linear system be characterized by a $N \times N$ matrix $\mathbf{A}$. Suppose the parameters $\boldsymbol{\phi}$ of the system are changed by $\Delta\boldsymbol{\phi}$. The system matrix $\mathbf{A}$ will then be affected by $\Delta\mathbf{A}$. We can express

$$\Delta\mathbf{A} = \mathbf{V}\,\mathbf{D}\,\mathbf{W}^T, \tag{4.1}$$

where $\mathbf{V}$, $\mathbf{D}$ and $\mathbf{W}$ are $N \times r_1$, $r_1 \times r_2$ and $N \times r_2$ matrices, respectively. For a network example, $\mathbf{D}$ can be a $n \times n$ diagonal matrix containing variables and $\mathbf{V}$ and $\mathbf{W}$ are $N \times n$ matrices containing $+1$ and -1 (Vlach and Singhal 1983).

The effect of $\Delta\boldsymbol{\phi}$ in the response matrix $\mathbf{A}^{-1}$ is defined as

$$\Delta(\mathbf{A}^{-1}) \triangleq (\mathbf{A} + \Delta\mathbf{A})^{-1} - \mathbf{A}^{-1}. \tag{4.2}$$

For the calculation of $\Delta(\mathbf{A}^{-1})$, commonly suggested is the Householder formula (Householder 1957), which can be represented by

$$\Delta(\mathbf{A}^{-1}) = -\mathbf{A}^{-1}\,\mathbf{V}(\mathbf{D}^{-1} + \mathbf{W}^T\,\mathbf{A}^{-1}\,\mathbf{V})^{-1}\,\mathbf{W}^T\,\mathbf{A}^{-1}. \tag{4.3}$$

Notice that in order to obtain $\Delta(\mathbf{A}^{-1})$, one needs to deal with a separate linear system characterized by matrix $(\mathbf{D}^{-1} + \mathbf{W}^T\,\mathbf{A}^{-1}\,\mathbf{V})$. This is usually called a reduced system and its size is usually smaller than the original system.

In (4.3), $\mathbf{D}$ is required to be a square and non-singular matrix. Even if this can be satisfied, ill-conditioning may still happen when $\mathbf{D}$ is inverted. In fact, cases exist where $\mathbf{D}$ is simply not invertible and additional measures such as the partitioning procedures developed by Hajj (1981), Vlach and Singhal (1983) must be applied. Another formula by Householder (1953) is

$$\Delta(\mathbf{A}^{-1}) = -\mathbf{A}^{-1}\,\mathbf{V}\mathbf{D}(\mathbf{D} + \mathbf{D}\mathbf{W}^T\,\mathbf{A}^{-1}\,\mathbf{V}\mathbf{D})^{-1}\,\mathbf{D}\mathbf{W}^T\,\mathbf{A}^{-1}. \tag{4.4}$$

This formula avoids actually performing the inversion of $\mathbf{D}$. But it still has the same limitation as that of (4.3).

According to the formulation of **D**, we refer to (4.3) as Square with Inversion Formula (SIF) and (4.4) as Square without Inversion Formula (SF).

To alleviate the limitations, different variations of the Householder matrix inversion formulas have been derived (Henderson and Searle 1981). Bandler and Zhang (1986) considered two important variations and applied them to linear network sensitivity analysis. The two variations are

$$\Delta(\mathbf{A}^{-1}) = -\mathbf{A}^{-1}\,\mathbf{V}\mathbf{D}(\mathbf{1} + \mathbf{W}^T\,\mathbf{A}^{-1}\,\mathbf{V}\mathbf{D})^{-1}\,\mathbf{W}^T\,\mathbf{A}^{-1} \tag{4.5}$$

and

$$\Delta(\mathbf{A}^{-1}) = -\mathbf{A}^{-1}\,\mathbf{V}(\mathbf{1} + \mathbf{D}\mathbf{W}^T\,\mathbf{A}^{-1}\,\mathbf{V})^{-1}\,\mathbf{D}\mathbf{W}^T\,\mathbf{A}^{-1} \tag{4.6}$$

These two formulas permit **D** to be singular or even rectangular. Thus, more freedom can be exploited using different formulations of **D** and ill-conditioning can be avoided.

The reduced systems in (4.5) and (4.6) are the order of r_2 and r_1, respectively, where r_1 is the number of rows of **D** and r_2 is the number of columns of **D**. Therefore, (4.5) may be preferred if $r_1 > r_2$, otherwise (4.6) should be used. It is reasonable to refer to (4.5) as Vertical Rectangular Formula (VRF) and (4.6) as Horizontal Rectangular Formula (HRF), respectively, reflecting the form of **D**. Other variations of the Householder formula also exist (Henderson and Searle 1981), but the reduced systems are as large as the original system.

The case of a rectangular **D** may occur, e.g., when we construct a minimum order reduced system involving variables that are active element parameters, and when large change algorithms are applied to algebraic linear systems other than electrical networks (Bandler and Zhang 1986). In those cases, the rectangular **D** may be used in VRF and HRF without modification leaving **V** and **W** free of values $\Delta\boldsymbol{\phi}$. Hence, **V** and **W** need to be preprocessed only once.

It should be noted that mathematically, the Square Formulas are special cases of the Rectangular ones. Computationally, the latter have good stability.

The various forms of Householder formulas presented above were also studied by Tylavsky and Sohie (1986). They attempted a generalized representation and connected the Householder formulas with other methods of solving linear equations, e.g., the lower-diagonal-upper (LDU) decomposition method.

4.2.2 Properties of Generalized Householder Formulas

Duality Property

The HRF and the VRF can be considered as dual to each other. If we apply the following interchanges

$$\mathbf{A} \leftrightarrow \mathbf{A}^T, \tag{4.7}$$

$$\mathbf{D} \leftrightarrow \mathbf{D}^T \tag{4.8}$$

and

$$\mathbf{V} \leftrightarrow \mathbf{W}, \tag{4.9}$$

then the two formulas, i.e. (4.5) and (4.6), are completely interchanged.

This duality property can be employed to save our analytical effort by half. Unless otherwise stated, we will focus on the Vertical Formula in the ensuing sections. Results for the Horizontal ones can be similarly obtained.

The Minimum Order of the Reduced System

Using the scattering theory approach, Haley and Current (1985) have found that the order of the reduced system can be as low as rank (ΔA). Using our approach of only simple matrix manipulations one can also verify that

$$\min_{(\mathbf{V},\mathbf{D},\mathbf{W})} r_1 = \min_{(\mathbf{V},\mathbf{D},\mathbf{W})} r_2 = \mathrm{rank}(\Delta\mathbf{A})\,. \tag{4.10}$$

TABLE 4.1

OPERATIONAL COUNT FOR THE GENERALIZED HOUSEHOLDER FORMULAS

Cases	Square with Inversion Formula (SIF)	Vertical Rectangular Formula (VRF)	Horizontal Rectangular Formula (HRF)	Square without Inversion Formula (SF)
<u>Case 1</u>				
$r_1 \neq r_2$				
preparatory calculation	-	C_P	C_P	-
calculation for each set of parameter changes	-	C_2	C_1	-
<u>Case 2</u>				
$r_1 = r_2 = r$				
preparatory calculation	C_P	C_P	C_P	C_P
calculation for each set of parameter changes	$2C_A + C_B$	$3C_A + C_B$	$3C_A + C_B$	$5C_A + C_B$

$C_P = N^2(r_1 + r_2) + Nr_1r_2$

$C_1 = r_1(2r_1r_2 + r_1{}^2 + r_2N + N^2)$, $C_2 = r_2(2r_1r_2 + r_2{}^2 + r_1N + N^2)$

$C_A = r^3$, $C_B = rN(r + N)$

are represented by **P** and **p** for the original system (coefficient matrix **A**) and by **Q** and **q** for the adjoint system (coefficient matrix $\mathbf{A}^T$). To distinguish these solutions for different R.H.S., we use the characters, similar to the R.H.S., as subscript. For example, $\mathbf{P}_V$ is the solution of

$$\mathbf{A}\mathbf{P}_V = \mathbf{V} \tag{4.11}$$

and $\mathbf{q}_b$ is the solution of

$$\mathbf{A}^T \mathbf{q}_b = \mathbf{b} \,. \tag{4.12}$$

4.3.1 Different Cases for Computing Response Changes

Case 1: Response Matrix $\mathbf{A}^{-1}$

$$\Delta(\mathbf{A}^{-1}) = -\mathbf{P}_V \mathbf{D}\mathbf{S}\mathbf{Q}_W{}^T \,, \tag{4.13}$$

where **S** is the inverse of $(1 + \mathbf{W}^T \mathbf{P}_V \mathbf{D})$.

Case 2: System Responses for a Single Excitation Vector **c**

Suppose the response vector corresponding to excitation **c** is $\mathbf{x} = [x_1 \; x_2 \; ... \; x_N]^T$, i.e.,

$$\mathbf{A}\mathbf{x} = \mathbf{c} \,. \tag{4.14}$$

We have

$$\begin{aligned} \Delta\mathbf{x} &= \Delta(\mathbf{A}^{-1}\mathbf{c}) \\ &= -\mathbf{P}_V \mathbf{D}\mathbf{s} \,, \end{aligned} \tag{4.15}$$

where **s** is the solution of

$$(1 + \mathbf{W}^T \mathbf{P}_V \mathbf{D})\mathbf{s} = \mathbf{W}^T \mathbf{x} \,. \tag{4.16}$$

Case 3: Adjoint Responses for a Single Excitation Vector **b**

Suppose the adjoint response vector corresponding to excitation **b** is $\mathbf{y} = [y_1\ y_2 \ldots y_N]^T$, i.e.,

$$\mathbf{A}^T \mathbf{y} = \mathbf{b}\,. \tag{4.17}$$

We have

$$\begin{aligned} \Delta \mathbf{y}^T &= \Delta(\mathbf{b}^T \mathbf{A}^{-1}) \\ &= -\mathbf{s}'^T \mathbf{Q}_W^T\,, \end{aligned} \tag{4.18}$$

where $\mathbf{s}'$ is the solution of

$$(1 + \mathbf{Q}_W{}^T \mathbf{V}\mathbf{D})^T \mathbf{s}' = \mathbf{D}^T \mathbf{V}^T \mathbf{q}_b\,. \tag{4.19}$$

Case 4: Response of Single-Input and Single-Output (SISO) System

If we use vector **b** to select the desired output from response vector **x**, then

$$\begin{aligned} \Delta(\mathbf{b}^T \mathbf{x}) &= \Delta(\mathbf{b}^T \mathbf{A}^{-1} \mathbf{c}) \\ &= -\mathbf{b}_1^T \mathbf{D}\mathbf{s}\,, \end{aligned} \tag{4.20}$$

where $\mathbf{s}$ is defined in (4.16) and $\mathbf{b}_1$ equals $\mathbf{P}_V{}^T\mathbf{b}$ and is obtained in the preparatory step.

Case 5: Responses of Multi-Input and Multi-Output (MIMO) System

Suppose **C** is a $N \times n'$ matrix whose columns represent different excitation vectors and **B** is an $N \times m'$ matrix whose columns select the desired output measurements. Then the n′–input m′–output case can be expressed, formally, by $\mathbf{B}^T \mathbf{A}^{-1} \mathbf{C}$. Thus

$$\Delta(\mathbf{B}^T \mathbf{A}^{-1} \mathbf{C}) = -\mathbf{B}^T \mathbf{A}^{-1} \mathbf{V}\mathbf{D}(1 + \mathbf{W}^T \mathbf{A}^{-1} \mathbf{V}\mathbf{D})^{-1} \mathbf{W}^T \mathbf{A}^{-1} \mathbf{C}\,. \tag{4.21}$$

We notice that the term $\mathbf{B}^T\mathbf{A}^{-1}\mathbf{V}$ can be computed either as $\mathbf{B}^T \mathbf{P}_V$ or $\mathbf{Q}_B{}^T\mathbf{V}$ with a difference of operational count as $N^2 (r_1 - m')$. Therefore, comparing r_1 and m', we can calculate $(\mathbf{B}^T \mathbf{A}^{-1} \mathbf{V})$ as

$$B^T A^{-1} V = \begin{cases} B^T P_V, & \text{if } r_1 \le m' \quad (4.22a) \\ Q_B^T V, & \text{if } r_1 > m'. \quad (4.22b) \end{cases}$$

Similarly,

$$W^T A^{-1} C = \begin{cases} W^T P_C, & \text{if } r_2 > n' \quad (4.23a) \\ Q_W^T C, & \text{if } r_2 \le n'. \quad (4.23b) \end{cases}$$

Also, at least one of (4.22a) and (4.23b) should be used in order to yield either P_V or Q_W which is required in calculating

$$(1 + W^T A^{-1} VD) = (1 + Q_W{}^T VD)$$
$$= (1 + W^T P_V D) . \qquad (4.24)$$

Hence, according to the values of r_1, r_2, m' and n', we can choose appropriate formulations. For example, when $m' < n'$ and $m' < r_2$, we use

$$\Delta(B^T A^{-1} C) = - S^T Q_W{}^T C , \qquad (4.25)$$

where S is the solution to

$$(1 + Q_W{}^T VD)^T S = (Q_B{}^T VD)^T . \qquad (4.26)$$

This approach requires $m' + r_2$ FBS in the adjoint system for Q_B and Q_W as preparatory calculations, one LU factorization and m' FBS in the reduced system of (4.26).

Expressions for Different Cases of Large Change Evaluation

In Table 4.2, we summarize the various cases of the above discussion. Different situations of the MIMO case are distinguished so that the number of FBS in the $N \times N$ system equals the minimum of $m' + r_2$, $n' + r_1$ and $r_1 + r_2$ and the number of FBS in the reduced system equals the minimum of r_1, r_2, m' and n', as shown in

TABLE 4.2

FORMULAS FOR THE COMPUTATION OF LARGE CHANGES WHEN $\mathbf{A}^{-1}$ IS INVOLVED AND WHEN $r_1 \geq r_2$

Identification		Formula	Definition of S or s
$\Delta(\mathbf{A}^{-1})$		$-\mathbf{P_V}\,\mathbf{D}\,\mathbf{S}\,\mathbf{Q_W}^T$	$\mathbf{H_1}\,\mathbf{S} = \mathbf{1}$ or $\mathbf{H_2}\,\mathbf{S} = \mathbf{1}$
$\Delta(\mathbf{b}^T\mathbf{A}^{-1})$		$-\mathbf{s}^T\mathbf{Q_W}^T$	$\mathbf{H_1}^T\mathbf{s} = \mathbf{D}^T(\mathbf{V}^T\mathbf{q_b})$
$\Delta(\mathbf{A}^{-1}\mathbf{c})$		$-\mathbf{P_V}\,\mathbf{D}\,\mathbf{s}$	$\mathbf{H_2}\,\mathbf{s} = \mathbf{W}^T\mathbf{p_c}$
$\Delta(\mathbf{b}^T\mathbf{A}^{-1}\mathbf{c})$		$-(\mathbf{b}^T\mathbf{P_V})\,\mathbf{D}\,\mathbf{s}$	$\mathbf{H_2}\,\mathbf{s} = \mathbf{W}^T\mathbf{p_c}$
† $\Delta(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{C})$	(1)	$-\mathbf{S}^T(\mathbf{Q_W}^T\mathbf{C})$	$\mathbf{H_1}^T\mathbf{S} = \mathbf{D}^T(\mathbf{V}^T\mathbf{Q_B})$
	(2)	$-(\mathbf{B}^T\mathbf{P_V})\mathbf{D}\,\mathbf{S}$	$\mathbf{H_2}\,\mathbf{S} = \mathbf{W}^T\mathbf{P_C}$
	(3)	$-(\mathbf{Q_B}^T\mathbf{V})\mathbf{D}\,\mathbf{S}$	$\mathbf{H_1}\,\mathbf{S} = \mathbf{Q_W}^T\mathbf{C}$
	(4)	$-(\mathbf{B}^T\mathbf{P_V})\mathbf{D}\,\mathbf{S}(\mathbf{Q_W}^T\mathbf{C})$	$\mathbf{H_1}\,\mathbf{S} = \mathbf{1}$ or $\mathbf{H_2}\,\mathbf{S} = \mathbf{1}$
	(5)	$-(\mathbf{Q_B}^T\mathbf{V})\mathbf{D}\,\mathbf{S}(\mathbf{Q_W}^T\mathbf{C})$	$\mathbf{H_1}\,\mathbf{S} = \mathbf{1}$

where $\mathbf{H_1} = (\mathbf{1} + \mathbf{Q_W}^T\mathbf{V}\,\mathbf{D})$, $\mathbf{H_2} = (\mathbf{1} + \mathbf{W}^T\mathbf{P_V}\,\mathbf{D})$

† Table 4.3 can be used as a guide to select among (1) to (5) by the minimum FBS criterion.

Table 4.3. This minimum FBS criterion can be used as a guide to select appropriate expressions for the calculation of $\Delta(\mathbf{B}^T \mathbf{A}^{-1} \mathbf{C})$.

When the number of FBS exceeds the order of the system, a matrix inversion may be directly performed.

4.3.2 Discussions

Computational Cost Consideration

In Section 4.2.2, the operational count has been discussed for a general linear system of equations. However, when an electric circuit is concerned, the cost is much less. We consider the SISO network as an example. Suppose the reduced system is of order r. In the preparatory step, we calculate $\mathbf{P}_V$ whose operational count is rN^2 and $\mathbf{P}^T{}_V\, \mathbf{b}$, $\mathbf{W}^T\, \mathbf{P}_V$ and $\mathbf{W}^T\, \mathbf{x}$ which are simply element selections and additions. Then, for each set of parameter changes, we formulate and solve the reduced system by at worst $4r^3/3 - r/3 + r^2$ operations. The operational count for updating the output is r for the SIF and $r + r^2$ for the HRF and the VRF.

Special Case: First-Order Sensitivity

As a special case of large change sensitivity analysis, small change sensitivity computations can be deduced from our large change formulas without reference to Tellegen's theorem. Table 4.4 gives examples of such first-order sensitivities w.r.t. components of a matrix. These results are obtained by putting $\Delta\phi$ into the denominator of large change formulas and then letting the parameter change $\Delta\phi$ approach zero. The formulas in Table 4.4 are consistent with the existing ones derived using other approaches, e.g., Bandler (1973).

TABLE 4.3

MAJOR COMPUTATIONAL EFFORT FOR CALCULATING $\Delta(B^T A^{-1} C)$ BY FORMULAS IN TABLE 4.2 WHERE $r_1 \geq r_2$

Category	Corresponding Case in Table 4.2	The $N \times N$ System Represented By **A**	The $r_2 \times r_2$ System Represented By H_1 or H_2
No. of LU Factorizations	(1)–(5)	1	1
No. of FBS	(1)	$m' + r_2$	m'
	(2)	$n' + r_1$	n'
	(3)	$m' + r_2$	n'
	(4)	$r_1 + r_2$	r_2
	(5)	$m' + r_2$	r_2

TABLE 4.4

EXPRESSIONS APPROPRIATE FOR COMPUTATIONS FOR SENSITIVITIES W.R.T. COMPONENTS OF MATRIX A WHEN A^{-1} IS INVOLVED

Identification	Sensitivity Expression	
	(a) General	(b) when $\mathbf{A} = \mathbf{A}^T$ and $i \neq j$
$\dfrac{\partial \mathbf{A}^{-1}}{\partial A_{ij}}$	$-\mathbf{p}_{ui}\,\mathbf{q}_{uj}^T$	$-(\mathbf{p}_{ui}\,\mathbf{p}_{uj}^T + \mathbf{p}_{uj}\,\mathbf{p}_{ui}^T)$
$\dfrac{\partial(\mathbf{b}^T\mathbf{A}^{-1}\mathbf{c})}{\partial \mathbf{A}}$	$-\mathbf{q}_b\,\mathbf{p}_c^T$	$-(\mathbf{p}_b\,\mathbf{p}_c^T + \mathbf{p}_c\,\mathbf{p}_b^T)$
$\dfrac{\partial(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{C})}{\partial A_{ij}}$	$-\mathbf{B}^T\,\mathbf{p}_{ui}\,\mathbf{q}_{uj}^T\,\mathbf{C}$	$-\mathbf{B}^T(\mathbf{p}_{ui}\,\mathbf{p}_{uj}^T + \mathbf{p}_{uj}\,\mathbf{p}_{ui}^T)\mathbf{C}$
$\dfrac{\partial[\mathbf{B}^T\mathbf{A}^{-1}\mathbf{C}]_{\ell k}}{\partial \mathbf{A}}$ †	$-\mathbf{q}_b\,\mathbf{p}_c^T$	$-(\mathbf{p}_b\,\mathbf{p}_c^T + \mathbf{p}_c\,\mathbf{p}_b^T)$ ††

$\mathbf{u}_i$ ($\mathbf{u}_j$) is a unit N-vector containing 1 at the ith (jth) row and zeros everywhere else.

† where $[*]_{\ell k}$ is the (ℓ,k) th element of matrix *.

†† where **b** is the ℓth column of **B** and **c** is the kth column of **C**. Both **b** and **c** are used as the R.H.S. of the system involving **A** for original solutions $\mathbf{p}_b$, $\mathbf{p}_c$ and adjoint solution $\mathbf{q}_b$.

4.4 LARGE CHANGE SENSITIVITY ANALYSIS OF LINEARIZED CIRCUITS

In this section, we illustrate how to formulate the large change sensitivity problem of an electrical circuit into the algebraic representations presented in Sections 4.2 and 4.3. Here, we introduce the conventional formulation. A new formulation is presented in the next section.

Suppose a linear circuit is represented by

$$\mathbf{A}\,\mathbf{x} = \mathbf{b}\,, \tag{4.27}$$

where **A** is a N×N matrix characterizing the network, **b** is a N-vector representing the excitation and **x** is a N-vector containing system responses. A simple form of equation (4.27) is the nodal equations of the linear circuit.

When system parameters $\phi_1, \phi_2, \ldots, \phi_n$ are changed, causing the change of **A** by $\Delta\mathbf{A}$, response changes can be calculated by large change formulas. The commonly used method is to express $\Delta\mathbf{A}$ as a triple product as (Hajj 1981)

$$\Delta\mathbf{A} = \mathbf{V}\,\mathbf{D}\,\mathbf{W}^T \tag{4.28}$$

or using parameter matrix decomposition of $\Delta\mathbf{A}$ as (Haley 1980; Haley and Current 1985)

$$\Delta\mathbf{A} = \sum_{i=1}^{r} \mathbf{v}_i\,\Delta\phi_i\,\mathbf{w}_i^T\,,\quad r \leq n\,. \tag{4.29}$$

The response changes $\Delta\mathbf{x}$ are then calculated using the Householder formula or its various equivalents. These calculations involve the solution of a reduced system whose size is determined from the formulation of **V**, **D** and **W**. We focus on this formulation. Subsequent calculations leading to $\Delta\mathbf{x}$ can be performed according to the presentation in Section 4.3.

Using the well-established methods (e.g., Hajj 1981; Vlach and Singhal 1983; Haley and Current 1985), one can generate a $n \times n$ reduced system for an arbitrary linear network by choosing

$$\mathbf{D} = \mathrm{diag.}\{\Delta\phi_1, \Delta\phi_2, \ldots, \Delta\phi_n\}, \tag{4.30}$$

$$\mathbf{V} = [\mathbf{v}_1 \ \ \mathbf{v}_2 \cdots \mathbf{v}_n] \tag{4.31}$$

and

$$\mathbf{W} = [\mathbf{w}_1 \ \ \mathbf{w}_2 \cdots \mathbf{w}_n] \ , \tag{4.32}$$

where $\mathbf{v}_i$ and $\mathbf{w}_i$ $i=1, \ldots, n$ are N-vectors containing ± 1 and 0. ϕ_i, $i=1, \ldots, n$ represent the value of variable i, being of the type that enter the tableau or modified nodal equations in the form $\mathbf{v}_i \, \phi_i \, \mathbf{w}_i^T$ (Hajj 1981)

It can be seen that this formulation gives each variable an equal treatment and no consideration regarding topological relations of these variables is taken into account. In a case where the number of perturbed variables is not very small compared to the order of the original system, the efficiency of large change algorithms is greatly degenerated. Such a case occurred in Example 8.1.1 of Vlach and Singhal (1983) where a 3×3 system had to be solved in order to update the response of a 2×2 system, merely because 3 variables exist.

A further reduction of the reduced system is made possible by the discovery that the order of such a system can be as low as the rank of the original system deviation matrix (Haley and Current 1985). This manifests itself as a minimum system (Bandler and Zhang 1986). Such a minimum system can be achieved by a thorough exploitation of the topological relations among variables.

4.5 A NEW FORMULATION OF V, D AND W FOR VARIABLES OF RCL TYPES

A new formulation of **V**, **D** and **W** was developed to achieve the minimum order reduced system for large change sensitivity computation. It was briefly introduced by Bandler and Zhang (1986). A systematic description is given here.

Let the network topology be represented by graph G and the edge set of G be represented by E, respectively. Let E' be a subset of E such that an edge in E corresponding to a variable is classified in E'. The induced subgraph of G on edge set E' is denoted as G'. Separate G' into blocks G_1', G_2' ..., G_b', $b \geq 1$, such that $G' = G_1' \cup G_2' \cup \ldots G_b'$ and $G_i' \cap G_j'$ is either null or empty containing only a cut-vertex of G' for all $i,j = 1,2,\ldots,b$ and $i \neq j$. Relevant terminologies used here are defined in Appendix A.

For RCL type variables in a linear network, **V**, **D** and **W** can be formulated using nodal relations instead of the conventional branch relations so as to achieve a minimum order reduced system. **V**, **D** and **W** are $N \times r$, $r \times r$ and $N \times r$ matrices, respectively and are decomposed such that

$$\mathbf{V} = [\mathbf{V}_1 \ \mathbf{V}_2 \ldots \mathbf{V}_b] , \tag{4.33}$$

$$\mathbf{W} = [\mathbf{W}_1 \ \mathbf{W}_2 \ldots \mathbf{W}_b] \tag{4.34}$$

and

$$\mathbf{D} = \text{diag}\{\mathbf{D}_1 , \mathbf{D}_2 , \ldots , \mathbf{D}_b\} , \tag{4.35}$$

where $\mathbf{D}_i$ is the nodal admittance matrix of G_i' using the $\Delta\phi$ as parameter and $\mathbf{V}_i$ and $\mathbf{W}_i$ are incidence matrices of G_i' indicating vertex locations of G_i' as seen from G. Suppose G_i' has m vertices, $m \geq 2$. $\mathbf{D}_i$ is $(m-1)\times(m-1)$ since one vertex can be considered as "ground" and is taken as a reference vertex. $\mathbf{V}_i$ and $\mathbf{W}_i$ are both $N\times(m-1)$ where each column vector corresponds to a non-reference vertex of G_i'. If this non-reference vertex and the reference vertex of G_i' appear in G as the kth and ℓth vertices, respectively, the corresponding columns of $\mathbf{V}_i$ and $\mathbf{W}_i$ are equal to $\mathbf{u}_k - \mathbf{u}_\ell$ or $\mathbf{u}_k$, if the ℓth vertex corresponds to the ground, $\mathbf{u}_k$ being a unit N-vector with 1 in its kth position and zeros everywhere else. For mathematical simplicity, a vertex in G_i' is taken as a reference vertex if it corresponds to the ground of the overall circuit.

4.6 EXAMPLES

4.6.1 A System of Linear Equations With Rectangular D

Consider a 10×10 system of linear equations with coefficient matrix as $\mathbf{A}$. Suppose the intersection elements of rows 2, 5, 9 and columns 3 and 6 are constantly changed. We formulate $\mathbf{V}$, $\mathbf{D}$ and $\mathbf{W}$ such that

$$\mathbf{V} = [\, \mathbf{u}_2 \ \mathbf{u}_5 \ \mathbf{u}_9 \,], \tag{4.36}$$

$$\mathbf{W} = [\, \mathbf{u}_3 \ \mathbf{u}_6] \tag{4.37}$$

and

$$\mathbf{D} = \begin{bmatrix} \Delta A_{23} & \Delta A_{26} \\ \Delta A_{53} & \Delta A_{56} \\ \Delta A_{93} & \Delta A_{96} \end{bmatrix}, \tag{4.38}$$

where $\mathbf{u}_i$, $i=2,3,5,6,9$, is a unit 10-vector containing 1 in the ith row and zeros everywhere else. In this way, no additional effort is involved when applying the VRF and HRF. If we use the Square Formulas, elementary transformations must be employed in order to obtain a square matrix $\mathbf{D}$.

Numerical solutions as well as intermediate results are shown in Fig. 4.2.

MATRIX [A]										VECTOR [B]
1.0	5.0	5.0	1.0	5.0	2.0	1.0	1.0	7.0	2.0	35.0
2.0	3.0	3.0	7.0	0.0	4.0	3.0	6.0	8.0	3.0	32.0
3.0	0.0	2.0	4.0	2.0	6.0	4.0	4.0	9.0	7.0	16.0
6.0	1.0	2.0	5.0	2.0	3.0	3.0	7.0	3.0	5.0	51.0
8.0	1.0	2.0	2.0	4.0	4.0	6.0	8.0	4.0	8.0	42.0
4.0	1.0	6.0	7.0	3.0	5.0	7.0	3.0	5.0	3.0	19.0
7.0	0.0	6.0	5.0	9.0	4.0	8.0	9.0	2.0	9.0	34.0
2.0	0.0	4.0	2.0	2.0	5.0	3.0	5.0	4.0	3.0	71.0
3.0	2.0	0.0	1.0	5.0	3.0	4.0	2.0	3.0	1.0	36.0
4.0	2.0	4.0	4.0	6.0	2.0	9.0	6.0	1.0	7.0	61.0

SOLUTION BEFORE ANY CHANGE :

VECTOR [X]
-8.89217
39.80097
-3.00067
2.31014
-5.40544
48.42778
-12.11626
-3.61726
-32.93004
16.99799

Fig. 4.2(a) The original linear system and its solutions. **A** is a 10×10 matrix containing parameters of the system. **b** is the excitation vector. **x** is the solution vector.

MATRIX [V]

0.0	0.0	0.0
1.0	0.0	0.0
0.0	0.0	0.0
0.0	0.0	0.0
0.0	1.0	0.0
0.0	0.0	0.0
0.0	0.0	0.0
0.0	0.0	0.0
0.0	0.0	1.0
0.0	0.0	0.0

MATRIX [W]

0.0	0.0
0.0	0.0
1.0	0.0
0.0	0.0
0.0	0.0
0.0	1.0
0.0	0.0
0.0	0.0
0.0	0.0
0.0	0.0

MATRIX [PV]

-.03684	.19072	-.00936
-.30799	-.26526	.04838
.00454	.09406	-.15865
-.04645	-.23600	.02949
.02608	-.09579	.12002
-.48948	-.43199	.13012
.18919	.22984	.01487
.27060	.13754	.00846
.36321	.32717	-.02238
-.27658	-.20670	-.09789

VECTOR [RHS]

-3.00067
48.42778

Fig. 4.2(b) Matrices **V**, **W**, $\mathbf{P_V}$ and vector **RHS**, where $\mathbf{P_V}$ is the solution of $\mathbf{A}\ \mathbf{P_V} = \mathbf{V}$ and $\mathbf{RHS} = \mathbf{W}^T\ \mathbf{x}$.

MATRIX [D]

2.00000	3.00000
4.00000	5.00000
2.00000	3.00000

MATRIX [H]

1.06802	.00797
-2.44666	-2.23801

VECTOR [S]

-2.66983

-18.72004

SOLUTION AFTER THE FIRST LARGE CHANGE :

VECTOR [X]

8.15496

-3.82546

-2.66983

-23.34277

-6.40995

-18.72004

24.40133

27.88727

22.14824

-27.58607

Fig. 4.2(c) Results corresponding to the first change of variable parameters represented by **D**. **H** represents $(1 + \mathbf{W}^T \mathbf{A}^{-1} \mathbf{V}\mathbf{D})$ and **s** is the solution of the reduced system $\mathbf{H}\,\mathbf{s} = \mathbf{W}^T \mathbf{x}$.

```
VARIABLES CHANGE AGAIN CAUSING A CHANGE OF [D].
[V] AND [W] REMAIN UNCHANGED.

                 MATRIX [D]
             6.00000      7.00000
             5.00000      4.00000
             3.00000      4.00000

                 MATRIX [H]
             1.02160      -.22657
            -4.70646     -3.63383

                 VECTOR [S]
                  -4.57788
                  -7.39775

       SOLUTION AFTER THE SECOND LARGE CHANGE :
                 VECTOR [X]
                  -2.20815
                   3.56798
                  -4.57788
                 -12.47901
                  -3.16642
                  -7.39775
                  15.58395
                  25.41279
                  12.05534
                 -20.00992
```

Fig. 4.2(d) Results corresponding to the second change of variable parameters. **H** and **s** are similarly defined to those in Fig. 4.2(c).

4.6.2 An Electrical Network with Its Minimum Order Reduced System Achieved

The 10-node circuit of Fig. 4.1 is solved using the generalized Householder formulas with simultaneous changes of 7 variable components. Topological relations showing the network graph G, the induced subgraph of G on edge set E' and the blocks are given in Fig. 4.3. G' is divided into G_1' and G_2'. The minimum order of the reduced system is 4, which is achieved by formulating **V**, **D** and **W** as

$$\mathbf{V} = [\mathbf{V}_1 \quad \mathbf{V}_2]$$

$$= [\mathbf{u}_3 - \mathbf{u}_9 \quad \mathbf{u}_4 - \mathbf{u}_9 \quad \mathbf{u}_8 - \mathbf{u}_9 \quad \mathbf{u}_4 - \mathbf{u}_5]$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (4.39)$$

$$\mathbf{W} = \mathbf{V} \qquad (4.40)$$

and

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & \\ & \mathbf{D}_2 \end{bmatrix}$$

$$= \begin{bmatrix} \Delta\phi_1+\Delta\phi_2+\Delta\phi_4 & -\Delta\phi_4 & -\Delta\phi_2 & 0 \\ -\Delta\phi_4 & \Delta\phi_4+\Delta\phi_5+\Delta\phi_6 & -\Delta\phi_5 & 0 \\ -\Delta\phi_2 & -\Delta\phi_5 & \Delta\phi_2+\Delta\phi_3+\Delta\phi_5 & 0 \\ 0 & 0 & 0 & \Delta\phi_7 \end{bmatrix} . \qquad (4.41)$$

Notice that nodes 9 and 5 have been taken as references for G_1' and G_2',respectively.

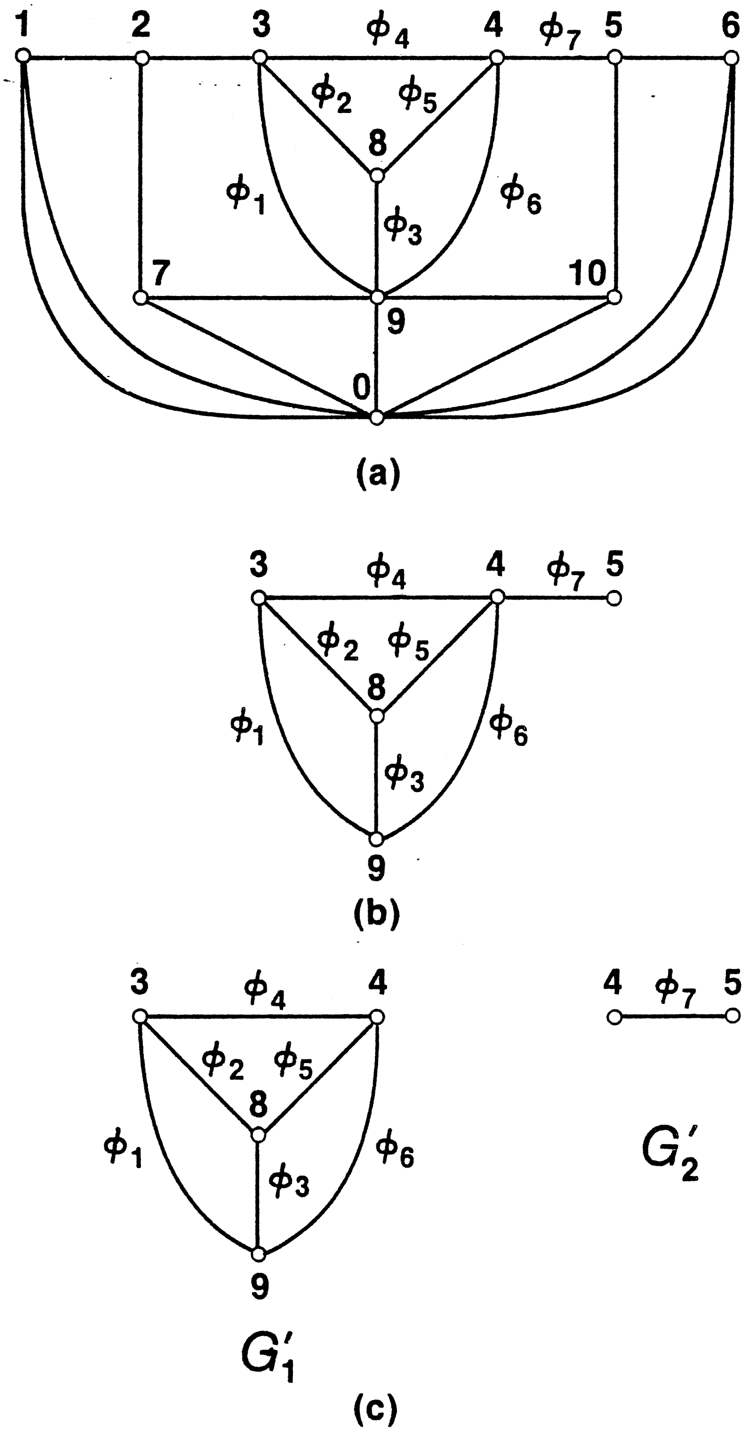


Fig. 4.3 Topological relations for the circuit of Fig. 4.1. (a) Graph G, (b) Edge induced subgraph G' and, (c) Blocks $G_1{}'$ and $G_2{}'$.

The changes of variables range from 0.00001 to 90. Zero changes are also included as shown in Table 4.5. These simultaneous small, large and zero changes are handled directly by the VRF. For the two extreme cases of $\Delta\phi$, the SIF can handle $\Delta\phi \to \infty$ while the VRF and HRF accommodate $\Delta\phi \to 0$. In a Monte-Carlo analysis, network optimization, identification and tuning, various unpredictable patterns of $\Delta\phi \to 0$ in multiparameter changes may be possible while $\Delta\phi \to \infty$ is often limited by, e.g., tolerances and tuning ranges or by step size constraints. For 100 sets of variable changes of ϕ_1 to ϕ_7, the operational count for our method using SIF, VRF, the conventional method and the direct method are in the order of 11230, 14030, 24930 and 43430, respectively.

4.6.3 The Case of Example 8.1.1 of Vlach and Singhal (1983)

Consider the circuit of Fig. 4.4 where G_1, G_2 and G_3 are all variables. Evidently, the "reduced" system is of order 2 using the nodal based approach which gives

$$\mathbf{D} = \Delta\mathbf{A} = \begin{bmatrix} \Delta G_1 + \Delta G_2 & -\Delta G_2 \\ -\Delta G_2 & \Delta G_2 + \Delta G_3 \end{bmatrix} \tag{4.42}$$

and

$$\mathbf{V} = \mathbf{W} = \mathbf{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4.43}$$

Compared with the branch based method which yields a 3×3 system, the operational count is reduced from 23 to 16 for each set of values of ΔG_i, $i = 1,2,3$.

Although for this circuit, one would rather solve the original network equations than use large change formulas, such a variable structure can exist in a large system as a subnetwork where an efficient large change algorithm is extremely important.

TABLE 4.5

PARAMETER CHANGES FOR THE CIRCUIT OF FIG. 4.1

Variable	The First Change (1/Ω)	The Second Change (1/Ω)	The Third Change (1/Ω)
$\Delta\phi_1$	84.0	0.00001	0.2
$\Delta\phi_2$	0.5	0.001	0
$\Delta\phi_3$	0.00001	0.12	3.0
$\Delta\phi_4$	0.02	45.	0
$\Delta\phi_5$	40.	0.00003	0.02
$\Delta\phi_6$	50.	90.	15
$\Delta\phi_7$	0.00002	–2.	0.1

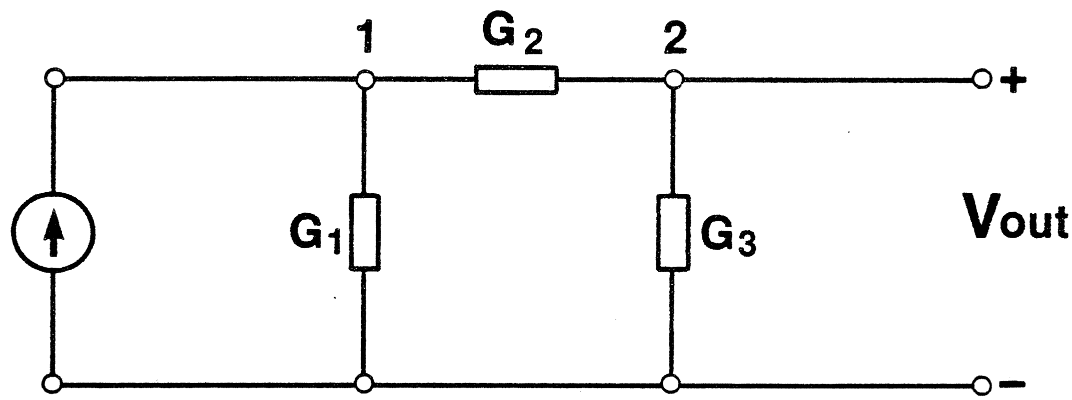


Fig. 4.4 The simple circuit from Vlach and Singhal (1983).

4.7 CONCLUSIONS

We have presented a multiparameter large change sensitivity analysis approach for a general system involving solutions of linear equations. Particular attention has been devoted to the formulation and order of the reduced system, which in turn affects the stability and efficiency of the system response evaluation. The mathematical essence of the generalized Householder formulas also provides basic links with other approaches, indicating their theoretical equivalence. However, our extended formulas accommodate more cases of various formulations of the reduced system which the traditional methods cannot handle directly. For a general circuit with arbitrary distribution of variable components, proper formulations of **V**, **D** and **W** can be used to ensure the large change calculation to be performed via a minimum order reduced system. Thus, under certain circumstances, large change algorithms are still feasible even if many system parameters are changed. Our work was recently referred to by Haley and Pham (1987) as one of the distinct, useful contributions to the analysis of modified systems. It is envisaged that a general formulation of **V**, **D** and **W**, together with the set of Householder formulas, can be embedded into the different iterative and non-iterative methods of Hajj (1981) to yield various powerful design procedures.

5

EXACT SIMULATION AND SENSITIVITY ANALYSIS OF MULTIPLEXING NETWORKS

5.1 INTRODUCTION

Many circuits can be categorized or reduced to the class of branched cascaded networks. The simulation and sensitivity evaluation of such networks can be directly performed using general software which solves nodal equations and adjoint networks. However, when the circuit becomes large, the general methods often deteriorate rapidly. On the other hand, the cascaded structure (without branches) has been treated using 2-port transmission matrices (e.g. Bandler, Rizk and Abdel-Malek 1978; Iobost and Zaki 1982). Such treatment has been very efficient especially for large cascaded circuits.

Bandler, Daijavad and Zhang (1985, 1986) developed a novel and elegant approach to the simulation and sensitivity analysis of branched cascaded circuits. They explicitly took the circuit structure into consideration. The forward and reverse analysis method of Bandler, Rizk and Abdel-Malek (1978) was extended to general branched cascaded netowrks. Our theory permits an efficient and fast analytical and numerical investigation of responses and sensitivities of all functions of interest w.r.t. any variable parameter, including frequency. Thevenin equivalent circuits at any reference plane and their sensitivities are also expressed analytically and calculated systematically. Thus, responses such as common port return loss, branch output return loss, insertion or transducer loss, gain slope and group delay can be handled exactly and efficiently. All analyses are performed in the original circuit and no adjoint networks are needed in sensitivity computation. More importantly, the method does not deteriorate for large circuits since possible redundant storage and computational requirements are eliminated by explicit exploitation of the circuit structure.

In this approach, each basic component of the structure is either a 2-port model or a 3-port junction and can contain variables or be constant. The fundamental requirement for the approach is that the transmission matrix description of all basic components and their derivatives, if they contain variables, are provided. This information is utilized in a systematic and efficient scheme which leads to the evaluation of various responses of the network at all ports of interest.

Microwave multiplexers consisting of multi-coupled cavity filters are structurally branched cascaded. The design of contiguous band multiplexers was a problem of significant theoretical interest for several years (Atia 1974; Chen, Assal and Mahle 1976), however, the manufacturing of such structures with more than 5 channels did not appear to be feasible. Recently, the subject has turned into an important development area in microwave engineering practice due to reports by leading manufacturers of successful production of 12 channel contiguous band multiplexers for satellite applications (Tong et al. 1982; Chen 1983; Egri, Williams and Atia 1983; Holme 1984). The employment of optimization techniques to determine the best multiplexer parameters has been an indispensable part of the design procedures reported. The use of a powerful gradient-based minimax optimization technique has reduced the CPU time required in the design procedure significantly (Bandler, Kellermann and Madsen 1985; Bandler, Daijavad and Zhang 1986).

The implementation of a gradient based optimization technique in multiplexer design requires, as a vital step, a robust and efficient algorithm for simulation and sensitivity analysis. This is achieved by applying our branched cascaded analysis technique.

This chapter is organized in the following way. We first describe the basic cascaded analysis approach as applied to a general branched cascaded circuit. Formulas for Thevenin equivalents, reflection coefficients and branch output voltages as well as their first- and second-order sensitivities w.r.t. design variables and frequency at any reference plane are developed. A 4-branch cascaded circuit is presented to illustrate our theory. We consider multiplexers consisting of multi-cavity filters distributed along a waveguide manifold. Transmission matrices and sensitivity expressions for typical components in a multiplexer, which are required by our approach, are tabulated. The optimization of a 12-channel 12 GHz multiplexer is described.

5.2 BRANCHED CASCADED NETWORKS

The category of a general class of networks, namely, branched cascaded structures, can be depicted as in Fig. 5.1. For such structures, we develop a novel procedure to calculate the reflection coefficients at the common port and branch output ports as well as branch output voltages. Simultaneously, first- and second-order derivatives are evaluated. The approach is based on the computation of Thevenin source and impedance equivalents and their first- and second-order sensitivities w.r.t. design parameters and frequency at the ports of interest.

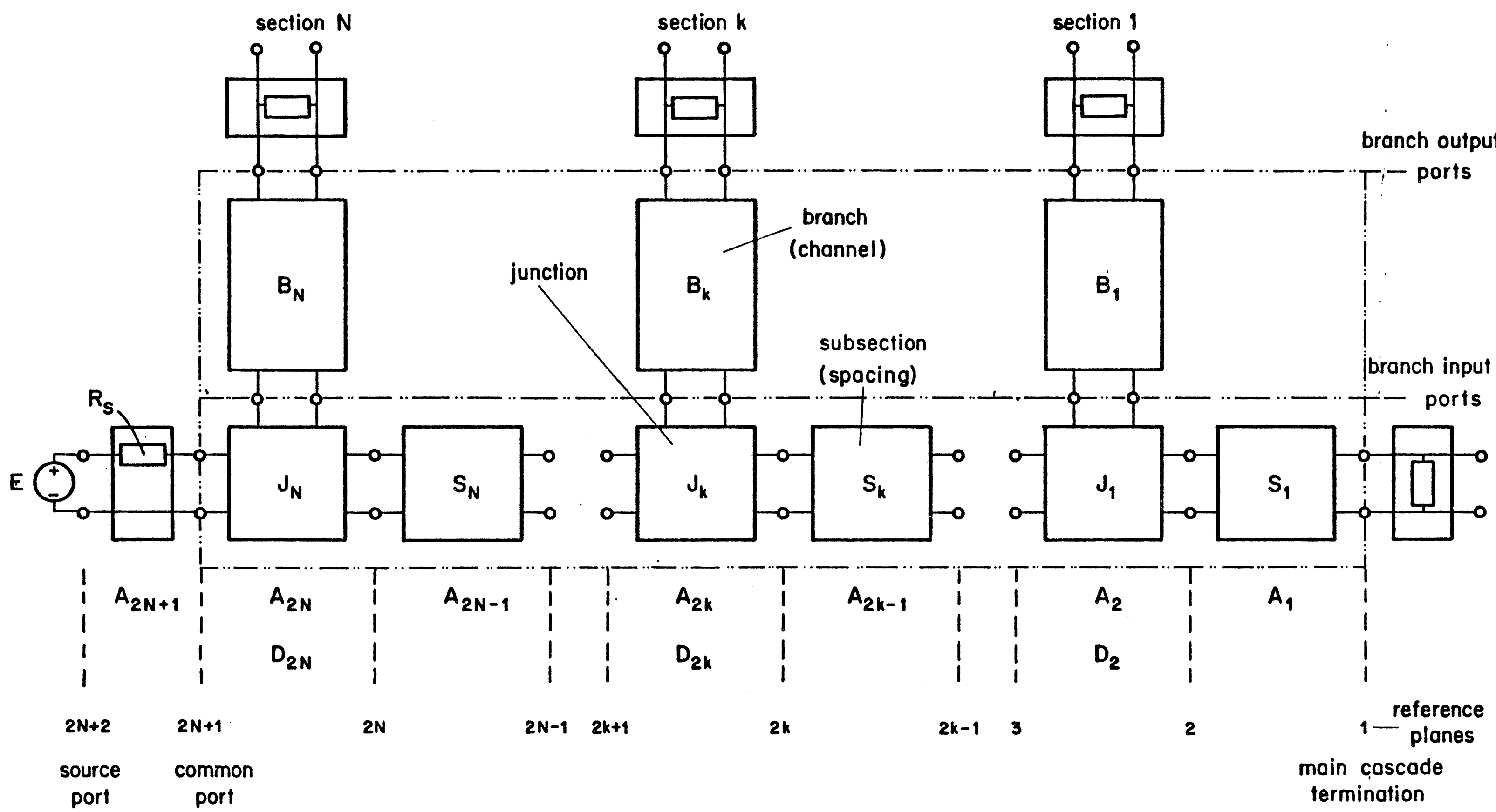


Fig. 5.1 The branched cascaded network under consideration. The junctions are arbitrarily defined 3-port junctions. Branches or channels are represented in reduced cascade forms. Adjacent junctions are separated by a subsection. Principal concepts of reference planes, transmission matrices and typical ports are illustrated.

5.2.1 Preliminary Description of the Network

Models of Basic Components

Although the basic components of a branched cascaded circuit are 2-port elements or 3-port junctions, internally they can be complicated subnetworks characterized by admittance, impedance or hybrid matrices. An example of such a subnetwork is the multi-coupled cavity filter described by an impedance matrix and containing many design variables. As a prerequisite step towards using our theory, the transmission matrix for each 2-port element should be deduced either by a reduction procedure or by direct measurements. Also, if variables exist in a subnetwork, the derivative of the corresponding transmission matrix should be provided. For the 3-port junctions, however, a 3-port description in the form of an arbitrary hybrid matrix, is sufficient.

Reference Planes

Consider the branched cascaded network of Fig. 5.1, which consists of N sections. A typical section, e.g., the kth one, has a junction, n(k) cascaded elements of branch k and a subsection along the main cascade, as shown in Fig. 5.2. All reference planes in the entire network are defined uniformly and numbered consecutively beginning from the main cascade termination, which is designated reference plane 1. The source port is at reference plane $2N+2$. The termination of the kth branch is called reference plane $\tau(k)$ and the branch main cascade connection (branch input port) is reference plane $\sigma(k)$, $k=1,2,\ldots,N$, where

$$\begin{aligned} &\tau(1) = 2N+3 \\ &\sigma(k) = \tau(k) + n(k), \quad k = 1, 2, \ldots, N \\ &\tau(k) = \sigma(k-1) + 1, \quad k = 2, 3, \ldots, N\,. \end{aligned} \tag{5.1}$$

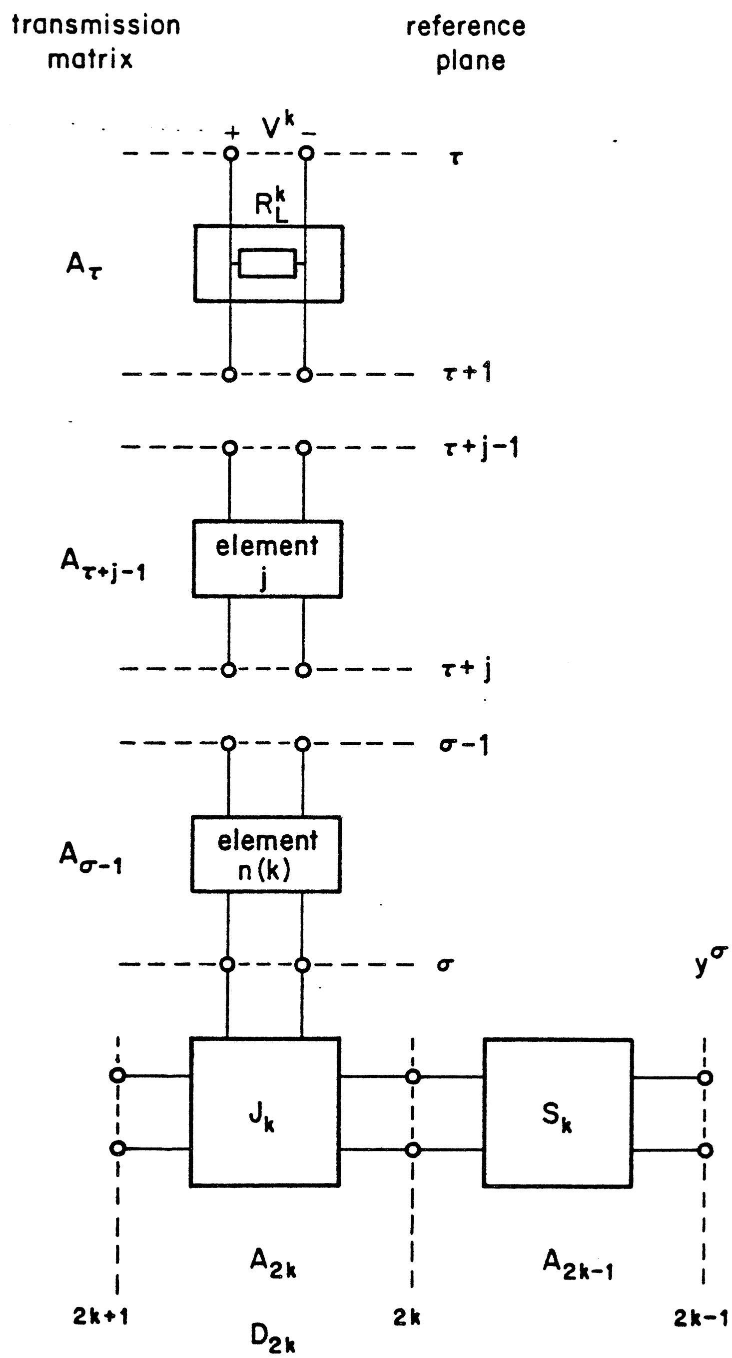


Fig. 5.2 Detail of the kth section of a branched cascaded circuit showing reference planes along the branch where $\tau = \tau(k)$ and $\sigma = \sigma(k)$.

5.2.2 Reduction of Junctions to 2-port Representations

Bandler, Rizk and Abdel-Malek (1978) introduced the concept of forward and reverse analysis for cascaded networks. To simplify the structure under consideration to a cascade of 2-ports for which the forward and reverse analysis is applicable, the 3-port junctions are reduced to 2-port representations.

Consider the 3-port junction shown in Fig. 5.3. To carry the analysis through the junction along the main cascade, we terminate port 3, e.g., by calculating the equivalent admittance seen at this port given by $Y_3 = (-I_3)/V_3$ and represent the transmission matrix between ports 1 and 2 by **A**. The analysis can also be carried through the junction into the branch by terminating port 2, e.g., calculating $Y_2 = (-I_2)/V_2$ and denoting the transmission matrix between ports 1 and 3 by **D**.

As an example, suppose the 3-port junction is characterized by a hybrid matrix **H** such that

$$[V_1 \quad I_1 \quad I_3]^T = \mathbf{H}\,[V_2 \quad I_2 \quad V_3]^T, \tag{5.2}$$

where $\mathbf{H} = [h_{ij}]_{3x3}$. Then $\mathbf{A} = [a_{ij}]_{2x2}$ can be found from

$$a_{ij} = (-1)^{j-1}[h_{ij} - h_{i3}h_{3j}/(Y_3 + h_{33})]. \tag{5.3}$$

For various forms of hybrid matrices **H**, the 2-port representation **A** or **D** is evaluated in a similar manner using elements of **H** and the equivalent termination at port 3 or 2.

5.2.3 Cascaded Analysis

Having reduced the junctions to 2-port representations, the network structure between any two reference planes is transformed to a simple cascade of two-ports. Assuming that the transmission matrices for all 2-ports are given, we define the equivalent transmission matrix between reference planes i and j by

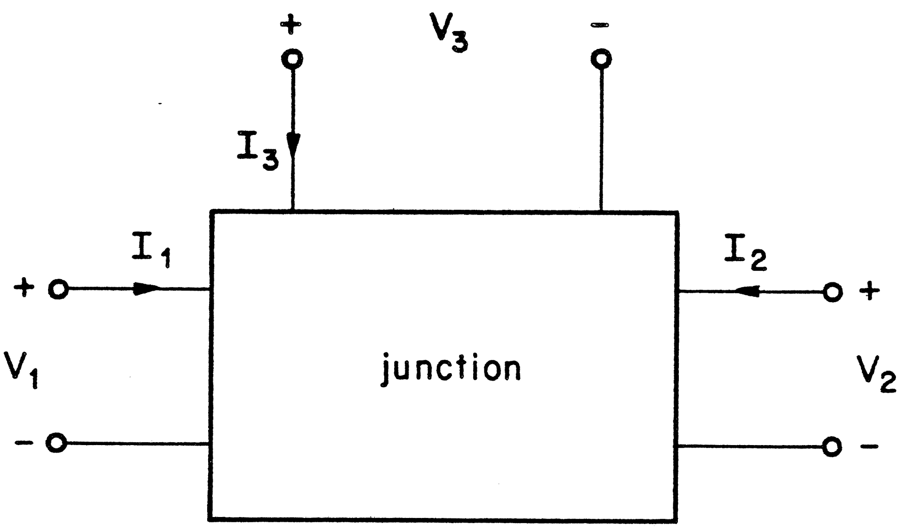


Fig. 5.3 A 3-port junction in which ports 1 and 2 are considered along a main cascade and port 3 represents a channel or branch of the main cascade.

$$\mathbf{Q}_{ij} \triangleq [\mathbf{p}_{ij} \quad \mathbf{q}_{ij}] \triangleq \begin{bmatrix} A_{ij} & B_{ij} \\ C_{ij} & D_{ij} \end{bmatrix}, \tag{5.4}$$

where

$$\mathbf{p}_{ij} \triangleq \begin{bmatrix} A_{ij} \\ C_{ij} \end{bmatrix}, \quad \mathbf{q}_{ij} \triangleq \begin{bmatrix} B_{ij} \\ D_{ij} \end{bmatrix}. \tag{5.5}$$

In a forward (reverse) analysis, $\mathbf{Q}_{ij}$ is computed by initializing row vectors $\mathbf{u}_1^T$ and $\mathbf{u}_2^T$ (column vectors $\mathbf{u}_1$ and $\mathbf{u}_2$) at reference plane i(j) and successively premultiplying (postmultiplying) each transmission matrix by the resulting row (column) vector until reference plane j(i) is reached. $\mathbf{u}_1$ and $\mathbf{u}_2$ are unit vectors given by $[1 \quad 0]^T$ and $[0 \quad 1]^T$, respectively.

Let ϕ be a generic notation that can be used to represent any design variable in the network. Sensitivities of $\mathbf{Q}_{ij}$ w.r.t. any variable ϕ located between reference planes i and j are evaluated as

$$\frac{\partial \mathbf{Q}_{ij}}{\partial \phi} = \sum_{\ell \in I_\phi} \frac{\partial}{\partial \phi} (\mathbf{Q}_{ij}^{\ell}), \tag{5.6}$$

where I_ϕ is an index set whose elements identify the transmission matrices containing ϕ and $\partial \mathbf{Q}_{ij}^{\ell}/\partial \phi$ is the result of a forward or reverse analysis between reference planes i and j with the ℓth matrix replaced by its derivative w.r.t. ϕ. Second-order sensitivities can be derived in a similar manner as

$$\frac{\partial^2 \mathbf{Q}_{ij}}{\partial \phi \, \partial \omega} = \sum_{\ell \in I_\phi} \sum_{k \in I_\omega} \frac{\partial^2 \mathbf{Q}_{ij}^{\ell k}}{\partial \phi \, \partial \omega}, \tag{5.7}$$

where I_ϕ and I_ω are index sets, not necessarily disjoint, identifying those matrices which are functions of ϕ and ω. Also, we define $\partial^2 \mathbf{Q}_{ij}^{\ell k}/(\partial \phi \, \partial \omega)$ as the second-order sensitivity of $\mathbf{Q}_{ij}$ as if ϕ and ω exist only in the ℓth and kth matrices, respectively.

5.2.4 Thevenin Equivalent Circuits and Basic Responses

To calculate the input reflection coefficient at the common port, the output reflection coefficients at the branch output ports, as well as the branch output voltages and their sensitivities in a unified manner, we employ Thevenin equivalents at the ports of interest evaluated by the method of forward and reverse analysis (Bandler, Rizk and Abdel-Malek 1978). Denoting the Thevenin equivalent voltages and impedances at reference planes i and j by V_S^i, Z_S^i, V_S^j and Z_S^j, we have

$$V_S^j = \frac{V_S^i}{A_{ij} + Z_S^i C_{ij}} \tag{5.8}$$

and

$$Z_S^j = \frac{B_{ij} + Z_S^i D_{ij}}{A_{ij} + Z_S^i C_{ij}} , \tag{5.9}$$

where reference plane i is located towards the source w.r.t. j, as shown in Fig. 5.4. The sensitivities are obtained as

$$(V_S^j)_\phi = \frac{(V_S^i)_\phi - [(A_{ij})_\phi + Z_S^i (C_{ij})_\phi + (Z_S^i)_\phi C_{ij}]\, V_S^j}{A_{ij} + Z_S^i C_{ij}} \tag{5.10}$$

and

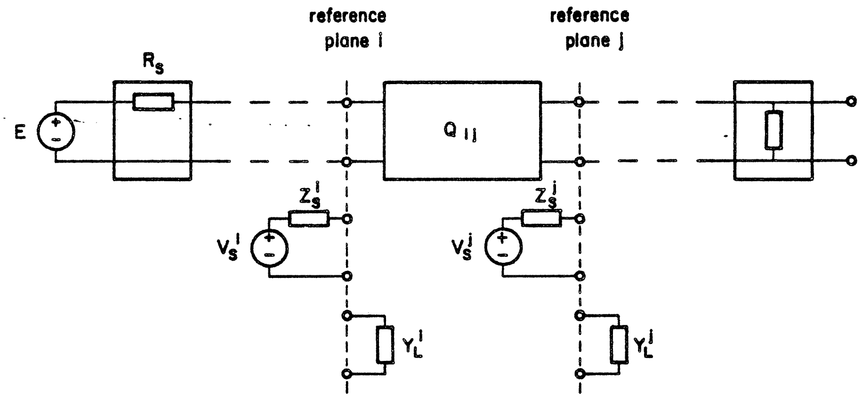
$$(Z_S^j)_\phi = \frac{[1 \quad Z_S^i](Q_{ij})_\phi \begin{bmatrix} -Z_S^j \\ 1 \end{bmatrix} + (Z_S^i)_\phi (D_{ij} - Z_S^j C_{ij})}{A_{ij} + Z_S^i C_{ij}} , \tag{5.11}$$

where subscript ϕ denotes $\partial/\partial\phi$.

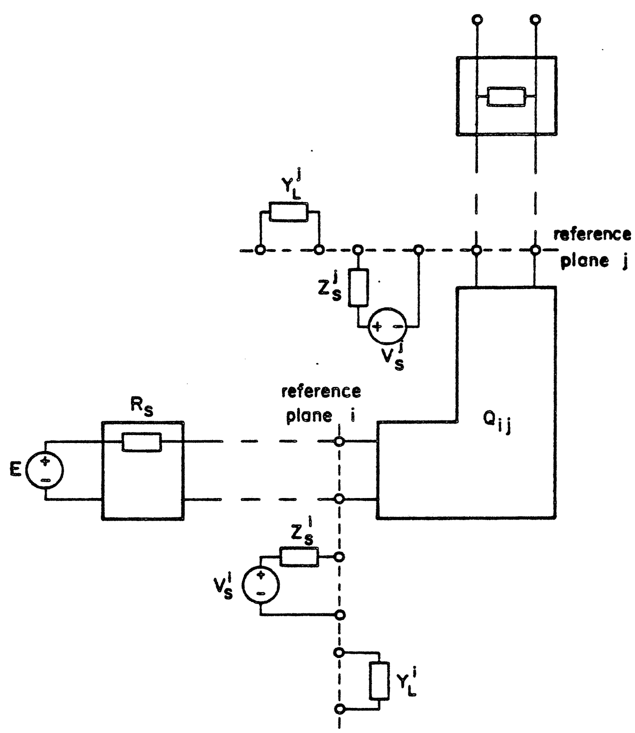
If the reflection coefficient at the kth branch output port and its sensitivities are to be calculated, then (5.9) and (5.11) are specialized to

$$Z_S^{\tau+1} = \frac{B}{A} \tag{5.12}$$

and



(a)



(b)

Fig. 5.4 Thevenin and Norton equivalents at reference planes i and j, where reference plane i is towards the source w.r.t. reference plane j. (a) reference plane j is in the main cascade. (b) reference plane j is in a branch.

$$(Z_S^{\tau+1})_\phi = \frac{(B)_\phi - (A)_\phi Z_S^{\tau+1}}{A}, \tag{5.13}$$

where $A \equiv A_{2N+2,\tau+1}$, $B \equiv B_{2N+2,\tau+1}$ and $\tau \equiv \tau(k)$. This is simply due to the fact that there is no impedance to the left of reference plane 2N+2, i.e., $Z_S^{2N+2} = 0$. The corresponding output reflection coefficient is defined as

$$\rho^k \triangleq \frac{Z_S^{\tau+1} - R_L^k}{Z_S^{\tau+1} + R_L^k}, \tag{5.14}$$

where R_L^k is the load resistance at the kth channel output. Clearly, (5.13) is utilized in the evaluation of $(\rho^k)_\phi$ as

$$(\rho^k)_\phi = \frac{(Z_S^{\tau+1})_\phi (1-\rho^k)}{Z_S^{\tau+1} + R_L^k}. \tag{5.15}$$

Branch output voltage is also computed by utilizing the Thevenin equivalent voltage source and impedance at the branch output port. At the kth channel we have

$$V^k = \frac{R_L^k}{A(R_L^k + Z_S^{\tau+1})}, \tag{5.16}$$

assuming a normalized excitation at the source port. This can be easily explained by noticing that V^k is evaluated using a voltage divider once $V_S^{\tau+1}$ is known. Using (5.8) and taking into account that $V_S^{2N+2} = 1$ and $Z_S^{2N+2} = 0$, we have $V_S^{\tau+1} = 1/A$. Also

$$(V^k)_\phi = -R_L^k V^k \left[\frac{(A)_\phi}{A} + \frac{(Z_S^{\tau+1})_\phi}{R_L^k + Z_S^{\tau+1}} \right]. \tag{5.17}$$

The second-order sensitivity of V^k w.r.t. ϕ and ω, i.e., $\partial^2 V^k/(\partial\phi\partial\omega)$ is obtained via evaluation of $\partial^2 Z_S^{\tau+1}/(\partial\phi\,\partial\omega)$. Substituting ω for ϕ in (5.13) and differentiating w.r.t. ϕ, gives

$$(Z_S^{\tau+1})_{\phi\omega} = \frac{(B)_{\phi\omega} - Z_S^{\tau+1}(A)_{\phi\omega} - (A)_\omega (Z_S^{\tau+1})_\phi - (Z_S^{\tau+1})_\omega (A)_\phi}{A}, \tag{5.18}$$

where double subscript $\phi\omega$ denotes $\partial^2/(\partial\phi\,\partial\omega)$.

Now, replacing ϕ by ω in (5.17) and differentiating w.r.t. ϕ, we have

$$(V^k)_{\phi\omega} = \frac{(V^k)_\phi (V^k)_\omega}{V^k} - R_L^k\, V^k \left[\frac{A\,(A)_{\phi\omega} - A_\phi A_\omega}{A^2} + \right.$$

$$\left. \frac{(Z_S^{\tau+1})_{\phi\omega}(R_L^k + Z_S^{\tau+1}) - (Z_S^{\tau+1})_\omega (Z_S^{\tau+1})_\phi}{(R_L^k + Z_S^{\tau+1})^2} \right] \tag{5.19}$$

Norton equivalent admittances and current sources are calculated similarly to the Thevenin equivalents. Denoting the Norton equivalent currents and admittances at reference planes i and j by $I_L{}^i$, $Y_L{}^i$, $I_L{}^j$ and $Y_L{}^j$, we have

$$Y_L^i = \frac{C_{ij} + Y_L^j D_{ij}}{A_{ij} + Y_L^j B_{ij}} \tag{5.20}$$

and

$$I_L^i = I_L^j = 0\,. \tag{5.21}$$

Also,

$$(Y_L^i)_\phi = \frac{[-Y_L^i \quad 1](Q_{ij})_\phi \begin{bmatrix} 1 \\ Y_L^j \end{bmatrix} + (Y_L^j)_\phi\,(D_{ij} - Y_L^i\, B_{ij})}{A_{ij} + Y_L^j\, B_{ij}}\,. \tag{5.22}$$

As special cases of (5.20), the equivalent admittances Y_3 and Y_2 required in the reduction of junctions to 2-port representations are calculated as

$$Y_3^k = Y_L^{\sigma(k)} = \frac{C_{\sigma(k),\,\tau(k)}}{A_{\sigma(k),\,\tau(k)}}\,, \qquad k = 1, 2, \ldots, N \tag{5.23}$$

and, for a short-circuit main cascade termination,

$$Y_2^k = Y_L^{2k} = \frac{D_{2k,1}}{B_{2k,1}}\,, \qquad k = 1, 2, \ldots, N\,. \tag{5.24}$$

The common port reflection coefficient is also computed using the Norton equivalent (at the source reference plane) as

$$\rho^0 = 1 - \frac{2\,R_S\,D_{2N+2,1}}{B_{2N+2,1}} \quad . \tag{5.25}$$

Its sensitivity is given by

$$(\rho^0)_\phi = 2\,R_S\,\frac{(B)_\phi D - (D)_\phi B}{B^2} \tag{5.26}$$

where $B \equiv B_{2N+2,1}$ and $D \equiv D_{2N+2,1}$.

5.2.5 Responses of Interest

Suppose the frequency responses of interest include return loss, insertion loss, transducer loss, gain slope and group delay for each individual branch and the return loss at the common port. Table 5.1 provides equations for calculating the responses and their sensitivities w.r.t. design parameter ϕ. It is clear that the evaluation of reflection coefficients at the common port and branch output ports (ρ^0 and ρ^k), branch output voltages (V^k) and the first- and second-order sensitivities

$$\left(\frac{\partial \rho^0}{\partial \phi}, \frac{\partial \rho^k}{\partial \phi}, \frac{\partial V^k}{\partial \phi}, \frac{\partial V^k}{\partial \omega} \text{ and } \frac{\partial^2 V^k}{\partial \phi\, \partial \omega} \right),$$

as described in Section5.2.4, is sufficient to compute all responses and sensitivities tabulated.

5.3 ALGORITHM FOR CALCULATION OF THEVENIN EQUIVALENTS AND THEIR SENSITIVITIES

The following algorithm can be used to obtain Thevenin equivalents at output ports and their sensitivities w.r.t. any variable. The algorithm assumes that the transmission matrices for all 2-port elements and the hybrid matrices for all junctions, as well as their sensitivities are given. The reverse analysis along the main cascade is initialized by $\mathbf{u}_2$ for a short circuit termination or, $\mathbf{u}_1$ for an open circuit

TABLE 5.1

VARIOUS FREQUENCY RESPONSES AND THEIR SENSITIVITIES EXPRESSED IN TERMS OF BASIC VOLTAGE RESPONSE AND REFLECTION COEFFICIENT

Response Type	Response Formula	Expression for Sensitivity w.r.t. ϕ
return loss (common port or channel output port)	$-20\log_{10}\lvert\rho\rvert$	$c\mathrm{Re}\left[\dfrac{\rho_\phi}{\rho}\right]$
transducer loss †	$-10\log_{10}\left(\dfrac{4\lvert V^k\rvert^2 R_S}{R_L^k}\right)$	$c\mathrm{Re}\left[\dfrac{(V^k)_\phi}{V^k}\right]$
insertion loss †	$-20\log_{10}\left[\dfrac{\lvert V^k\rvert(R_S+R_L^k)}{R_L^k}\right]$	$c\mathrm{Re}\left[\dfrac{(V^k)_\phi}{V^k}\right]$
gain slope †	$c\mathrm{Re}\left[\dfrac{(V^k)_\omega}{V^k}\right]$	$c\mathrm{Re}\left[\dfrac{(V^k)_{\phi\omega}}{V^k}-\dfrac{(V^k)_\phi(V^k)_\omega}{(V^k)^2}\right]$
group delay †	$-\mathrm{Im}\left[\dfrac{(V^k)_\omega}{V^k}\right]$	$-\mathrm{Im}\left[\dfrac{(V^k)_{\phi\omega}}{V^k}-\dfrac{(V^k)_\phi(V^k)_\omega}{(V^k)^2}\right]$

$$c = -\frac{20}{\ell n\, 10}$$

† between common port and channel k output port

termination. Correspondingly, the resulting analysis is represented by **q** vectors (as in the algorithm) or **p** vectors.

Step 1 — For $k = 1, 2, \ldots, N$, set σ and τ to $\sigma(k)$ and $\tau(k)$, respectively, and execute Steps 1.1 to 1.7.

Step 1.1 — Calculate $\mathbf{Q}_{i,\tau+1}$ by reverse analysis for $i = \tau+1, \tau+2, \ldots, \sigma$. Calculate $\mathbf{Q}_{\sigma j}$ by forward analysis for $j = \sigma, \sigma-1, \ldots, \tau+1$.

Comment — Cascaded analysis is performed on the kth branch. The reverse (forward) analysis starts from the branch output (input) port and is carried to the branch input (output) port.

Step 1.2 — $\mathbf{p}_{\sigma\tau} \leftarrow \mathbf{Q}_{\sigma,\tau+1} \mathbf{A}_\tau \mathbf{e}_1$.

Calculate $Y_3{}^k$ using (5.23).

Comment — The equivalent admittance of the kth branch, looking from the branch input port, is computed. This admittance is utilized in the 2-port representation of the kth junction.

Step 1.3 — Calculate $\partial \mathbf{p}_{\sigma\tau}/\partial\phi$ using (5.6) and $\partial Y_3{}^k/\partial\phi$ from (5.22) for all the variable ϕ's in the kth branch.

Comment — Sensitivities of the branch equivalent admittance w.r.t. all variables in the branch are calculated. In evaluating $\partial Y_3{}^k/\partial\phi$, we use a special case for (5.22) which corresponds to $Y_3{}^k$ given in (5.23).

Step 1.4 — Calculate $\mathbf{A}_{2k}$ using (5.3). Calculate $\partial \mathbf{A}_{2k}/\partial\phi$ for all the variable ϕ's in the kth junction and the kth branch.

Comment — The 2-port representation of the kth junction, when terminating its port 3, is computed. The sensitivities of the resulting transmission matrix are readily obtained.

Step 1.5 — Calculate $\mathbf{q}_{2k,1}$ by reverse analysis and $Y_2{}^k$ from (5.24).

<u>Comment</u> The equivalent admittance at port 2 of the kth junction, looking towards the main cascade termination, is calculated after a reverse analysis from reference plane 1 to reference plane 2k.

<u>Step 1.6</u> Calculate $\partial \mathbf{q}_{2k,1}/\partial \phi$ using (5.6) and $\partial Y_2{}^k/\partial \phi$ using (5.22) for all variables ϕ in section k′, k′ < k and in the kth spacing.

<u>Comment</u> The sensitivities of the equivalent admittance $Y_2{}^k$, w.r.t. all variables geometrically located to the right of junction k, are computed. In evaluating $\partial Y_2{}^k/\partial \phi$, we use a special case for (5.22) which corresponds to $Y_2{}^k$, given in (5.24).

<u>Step 1.7</u> Calculate $\mathbf{D}_{2k}$ using the method described in Section 5.2.2. Calculate $\partial \mathbf{D}_{2k}/\partial \phi$ for all the variable ϕ's in the kth junction and spacing, as well as in all k′ sections, k′ < k.

<u>Comment</u> The 2-port representation of junction k when terminating its port 2, is computed. The sensitivities of the resulting transmission matrix w.r.t. all variables, included in or located to the right of the junction, are computed.

<u>Step 2</u> Calculate $\mathbf{q}_{2N+2,1}$ by extending the reverse analysis already performed up to reference plane 2N in Step 1.5, to reference plane 2N + 2. Note that $\mathbf{A}_{2N}$ has been evaluated in Step 1.4.

Calculate $\partial \mathbf{q}_{2N+2,1}/\partial \phi$ using $\partial \mathbf{q}_{2N,1}/\partial \phi$ (ϕ belongs to the set of all variables to the right of section N and the Nth spacing), which has been evaluated in Step 1.6 and $\partial \mathbf{A}_{2N}/\partial \phi$ (ϕ belongs to the set of all variables in the Nth branch and Nth junction), which has been evaluated in Step 1.4.

<u>Comment</u> The reverse analysis from the main cascade termination is carried back to the source port. The corresponding sensitivities are also calculated. These results are used to calculate the common port reflection coefficient and its sensitivities w.r.t. all variables in the entire network.

<u>Step 3</u> For $k = N, N-1, \ldots, 1$, set σ and τ to $\sigma(k)$ and $\tau(k)$, respectively and execute Steps 3.1 to 3.3.

<u>Step 3.1</u> Calculate $\mathbf{Q}_{2N+2,2k+1}$ by forward analysis.

<u>Comment</u> The forward analysis is carried along the main cascade from the source port to the input port of junction k.

<u>Step 3.2</u> $\mathbf{Q}_{2N+2,\tau+1} \leftarrow \mathbf{Q}_{2N+2,2k+1}\, \mathbf{D}_{2k}\, \mathbf{Q}_{\sigma,\tau+1}$.

Calculate $\partial \mathbf{Q}_{2N+2,\tau+1}/\partial\phi$ using (5.6) for all the variable ϕ's in the entire multiplexer.

<u>Comment</u> A cascaded analysis from the source port is carried through the kth junction into the kth branch. The sensitivities w.r.t. all variables are computed.

<u>Step 3.3</u> Calculate $V_S^{\tau+1}$ and $Z_S^{\tau+1}$ using (5.8) and (5.9). Also, calculate $\partial V_S^{\tau+1}/\partial\phi$ and $\partial Z_S^{\tau+1}/\partial\phi$ using (5.10) and (5.11) for all variables ϕ in the entire network.

<u>Comment</u> Thevenin equivalents and their sensitivities are computed for the kth branch output port.

The theory and the algorithm have been implemented into a computer program for simulation and sensitivity analysis of branched cascaded networks. The number of branches and the numbers of branch elements are user defined. Exact sensitivity analysis can be performed w.r.t. any variable, including frequency. A brief description of the computer program is available in Appendix B.

5.4 A 4-BRANCH CASCADED NETWORK EXAMPLE

This example was presented by Bandler, Daijavad and Zhang (1985a) to illustrate the basic concepts for simulation and sensitivity analysis of branched cascaded networks. The circuit diagram is shown in Fig. 5.5. The circuit has 4 branches, i.e., $N = 4$. The numbers of cascaded elements for the 4 branches are $n(1) = 3$, $n(2) = 4$, $n(3) = 3$ and $n(4) = 2$. According to Eq. (5.1), the reference planes for branch terminations are $\tau(1) = 11$, $\tau(2) = 15$, $\tau(3) = 20$ and $\tau(4) = 24$. The reference planes for branch-main cascade connections are $\sigma(1) = 14$, $\sigma(2) = 19$, $\sigma(3) = 23$ and $\sigma(1) = 26$.

The computer program described in Appendix B was used to calculate all responses of interest and their sensitivities. Table 5.2 lists values of computed responses (e.g., output voltage, Thevenin equivalent voltage and impedance, insertion and return loss) for each branch. The common port (i.e. at reference plane 9) return loss is also evaluated. Tables 5.3–5.8 provide sensitivities for each response in Table 5.2. These sensitivities are evaluated w.r.t. circuit variables ϕ_i, $i = 1, 2, \ldots, 8$. Table 5.9 shows sensitivities of the circuit responses w.r.t. frequency ω. Gain slope and group delay responses are listed in Table 5.10.

5.5 COMPUTER-AIDED DESIGN OF MICROWAVE MULTIPLEXERS

5.5.1 Analysis of Specific Multiplexer Structures

While the approach developed in Section 5.2 is general, as a special case, the design of multiplexers consisting of coupled cavity filters distributed along a

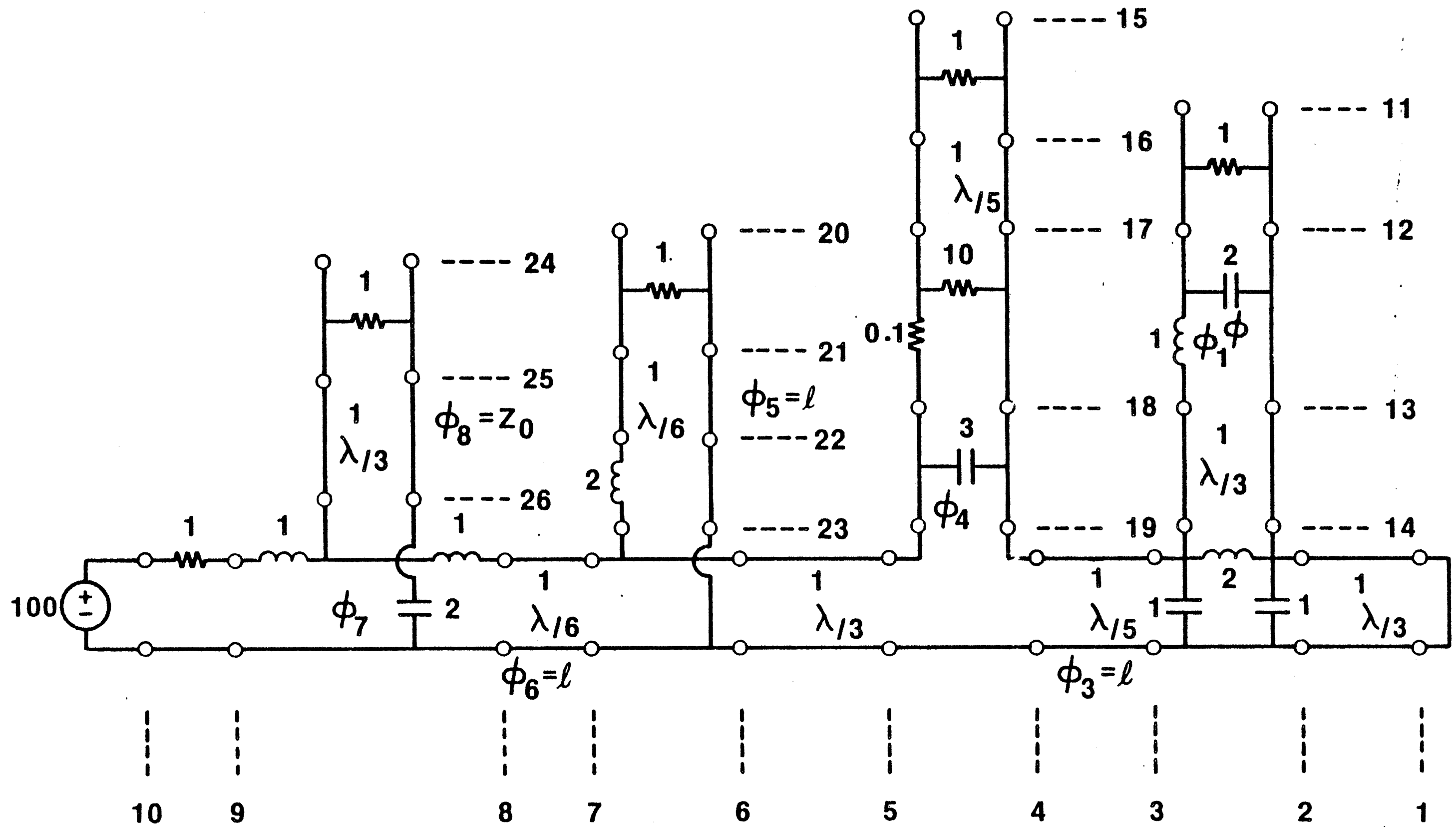


Fig. 5.5 Illustration of an arbitrary 4-branch cascaded circuit with short-circuit termination of the main cascade. Lossy elements as well as transmission lines are included.

TABLE 5.2

NUMERICAL VALUES OF THE RESPONSES FOR THE 4-BRANCH CASCADED NETWORK OF FIG. 5.5

Type of Response	Branch 1†	Branch 2	Branch 3	Branch 4
output voltage	0.03624 -j0.07487	-0.07595 -j0.06875	0.05983 -j0.04039	-15.00361 +j1.16405
Thevenin equivalent voltage*	0.03008 -j0.07785	0.03529 -j0.30176	0.03193 -j0.08172	-15.65346 -j2.31876
Thevenin equivalent impedance*	0.00003 -j0.08225	0.72129 +j2.41490	0.00004 -j0.69080	0.02515 +j0.23408
insertion loss (dB)	55.57892	53.76940	56.81050	10.42942
branch port return loss (dB)	0.00055	1.72670	0.00052	0.41430
common port return loss = 0.41243 dB				

† Branch 1 is the furthest from the common port.

* Thevenin equivalents for each branch are evaluated at the reference plane just before the load corresponding to that branch.

TABLE 5.3

SENSITIVITIES OF BRANCH OUTPUT VOLTAGES W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 5.5

Variable	Branch 1	Branch 2	Branch 3	Branch 4
Φ_1	−0.09888 +j0.19690	0.01602 +j0.01904	−0.12152 +j0.07920	−0.06148 +j0.43382
Φ_2	−0.02178 +j0.03689	0.00008 +j0.00013	−0.00083 +j0.00037	−0.00081 +j0.00263
Φ_3 (per Gm)	0.41840 −j1.02730	0.42340 +j0.49683	−3.17461 +j2.09775	−1.54074 +j11.39034
Φ_4	−0.00015 +j0.00018	0.02421 +j0.02442	−0.00152 +j0.00078	−0.00123 +j0.00500
Φ_5 (per Gm)	0.00000 +j0.00000	0.00000 +j0.00000	−0.84583 −j1.25308	0.00000 +j0.00000
Φ_6 (per Gm)	0.42131 −j1.01718	−1.05647 −j0.85004	0.75952 −j0.57964	−1.32161 +j10.30781
Φ_7	0.00216 +j0.00231	0.00347 −j0.00175	0.00061 +j0.00267	0.16241 +j0.04932
Φ_8	0.03997 −j0.13157	−0.14168 −j0.09279	0.08734 −j0.08130	−12.42431 +j0.17372

TABLE 5.4

SENSITIVITIES OF THEVENIN EQUIVALENT VOLTAGE SOURCES W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 5.5

Variable	Branch 1	Branch 2	Branch 3	Branch 4
Φ_1	−0.08217 +j0.20531	−0.01837 +j0.07139	−0.06664 +j0.16342	−0.20599 −j0.03020
Φ_2	−0.01556 +j0.04023	−0.00019 +j0.00043	−0.00058 +j0.00095	−0.00125 −j0.00039
Φ_3 (per Gm)	0.33572 −j1.06095	−0.47001 +j1.87577	−1.72106 +j4.29805	−5.40867 −j0.75803
Φ_4	−0.00014 +j0.00020	−0.01427 +j0.08775	−0.00097 +j0.00183	−0.00237 −j0.00060
Φ_5 (per Gm)	0.00000 +j0.00000	0.00000 +j0.00000	−0.46211 +j1.18227	0.00000 +j0.00000
Φ_6 (per Gm)	0.33964 −j1.05097	0.24151 −j4.04905	0.36033 −j1.10264	−4.89480 −j0.65144
Φ_7	0.00235 +j0.00213	0.01021 +j0.00537	0.00246 +j0.00225	0.15861 +j0.03550
Φ_8	0.02916 −j0.13486	−0.01986 −j0.50186	0.03119 −j0.14165	−13.35199 −j1.80362

TABLE 5.5

SENSITIVITIES OF THEVENIN EQUIVALENT IMPEDANCES W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 5.5

Variable	Branch 1	Branch 2	Branch 3	Branch 4
Φ_1	−0.00017 +j0.00707	0.00019 +j0.00077	−0.00016 +j0.00450	0.00038 +j0.03072
Φ_2	−0.00003 +j0.04250	0.00000 +j0.00000	−0.00001 +j0.00003	−0.00003 +j0.00019
Φ_3 (per Gm)	0.00085 +j0.02364	0.00501 +j0.02006	−0.00334 +j0.11797	0.01498 +j0.80592
Φ_4	0.00000 +j0.00000	0.06160 +j0.11222	−0.00001 +j0.00005	−0.00003 +j0.00036
Φ_5 (per Gm)	0 00000 +j0.00000	0.00000 +j0.00000	−0.00129 +j30.93848	0.00000 +j0.00000
Φ_6 (per Gm)	0.00066 +j0.02605	0.17452 +j0.29796	0.00051 +j0.02879	0.01860 +j0.72854
Φ_7	0.00000 +j0.00000	0.00000 +j0.00001	0.00000 +j0.00000	−0.00052 +j0.00350
Φ_8	0.00005 +j0.00000	0.00058 −j0.00028	0.00005 +j0.00000	0.04283 −j0.05844

TABLE 5.6

SENSITIVITIES OF INSERTION LOSS W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 5.5

Variable	Branch 1	Branch 2	Branch 3	Branch 4
ϕ_1	23.00568	2.09034	17.45152	–0.05475
ϕ_2	4.45823	0.01270	0.10816	–0.00059
ϕ_3 (per Gm)	–115.59096	54.88169	457.83905	–1.39517
ϕ_4	0.02412	2.91144	0.20388	–0.00093
ϕ_5 (per Gm)	0.00000	0.00000	0.00000	0.00000
ϕ_6 (per Gm)	–114.77168	–114.77168	–114.77168	–1.22074
ϕ_7	0.11859	0.11859	0.11859	0.09126
ϕ_8	–14.18466	–14.18466	–14.18466	–7.15740

TABLE 5.7

SENSITIVITIES OF BRANCH PORT RETURN LOSS W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 5.5

Variable	Branch 1	Branch 2	Branch 3	Branch 4
ϕ_1	−0.00292	−0.00050	−0.00183	0.00055
ϕ_2	−0.00057	0.00000	−0.00006	−0.00050
ϕ_3 (per Gm)	0.01465	−0.01278	−0.03917	0.09851
ϕ_4	0.00000	−0.00071	−0.00008	−0.00059
ϕ_5 (per Gm)	0.00000	0.00000	0.00000	0.00000
ϕ_6 (per Gm)	0.01133	0.02123	0.00598	0.17232
ϕ_7	−0.00001	−0.00002	−0.00001	−0.00913
ϕ_8	0.00084	0.00155	0.00063	0.71641

TABLE 5.8

SENSITIVITIES OF COMMON PORT RETURN LOSS W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 5.5

Variable	Sensitivity
ϕ_1	0.00533
ϕ_2	0.00004
ϕ_3 (per Gm)	0.13797
ϕ_4	0.00008
ϕ_5 (per Gm)	0.00000
ϕ_6 (per Gm)	0.12286
ϕ_7	–0.00909
ϕ_8	0.71310

TABLE 5.9

SENSITIVITIES OF VARIOUS RESPONSES W.R.T. ANGULAR FREQUENCY ω FOR THE CIRCUIT OF FIG. 5.5

Type of Response	Branch 1	Branch 2	Branch 3	Branch 4
output voltage	–0.17778 +j0.33120	0.03944 +j0.08791	–0.44100 +j0.26906	2.39068 +j7.36235
Thevenin equivalent voltage	–0.14889 +j0.34666	–0.10880 +j0.09567	–0.24384 +j0.59055	0.15888 +j0.51010
Thevenin equivalent impedance	–0.00028 +j0.02219	0.73081 +j1.32535	–0.00051 +j0.28101	–0.00138 +j0.50624
branch port return loss	–0.00484	–0.00153	–0.00590	–0.11597
sensitivity of common port return loss = –0.10460				

TABLE 5.10

GAIN SLOPE AND GROUP DELAY FOR THE CIRCUIT OF FIG. 5.5

Type of Response	Branch 1	Branch 2	Branch 3	Branch 4
gain slope (dB/Hz)	246.411	47.006	390.162	6.579
group delay (s)	0.18892	0.37785	0.32862	0.50006

waveguide manifold is considered here in more detail. Contiguous or non-contiguous band multiplexers are treated in a similar manner. Fig. 5.6, which is a special case for the structure in Fig. 5.1, illustrates a typical circuit equivalent for a multiplexer. A branch consists of a coupled-cavity filter, together with input-output transformers, and an impedance inverter. A subsection is the waveguide section separating two adjacent filters and the junction is the equivalent circuit model for the physical junction between channel filters and the manifold. The main cascade is short-circuited and the responses of interest are common port return loss, channel output return loss, insertion or transducer loss, gain slope and group delay between common port and channel output ports.

To apply the general method of Section 5.2, the subnetworks, namely, channel filters, waveguide spacings and junctions should be represented by 2-port transmission matrices. Recently, a comprehensive set of formulas for reduction of multi-cavity filters to two-port equivalents which also provides sensitivities w.r.t. variables of interest in the filter structure, has been presented by Bandler, Chen and Daijavad (1986a). The formulas evaluate short-circuit admittance parameters and their sensitivities w.r.t. all couplings as well as frequency for the unterminated filter model. Evaluation of transmission matrices from short-circuit admittance matrices is straight-forward.

In Table 5.11, the transmission matrices for the individual components of the multiplexer structure shown in Fig. 5.6, have been listed. The series 3-port junctions are reduced to 2-port equivalents using the method described in Section 5.2. Table 5.12 lists the sensitivities of transmission matrices in Table 5.11 w.r.t. relevant parameters and frequency.

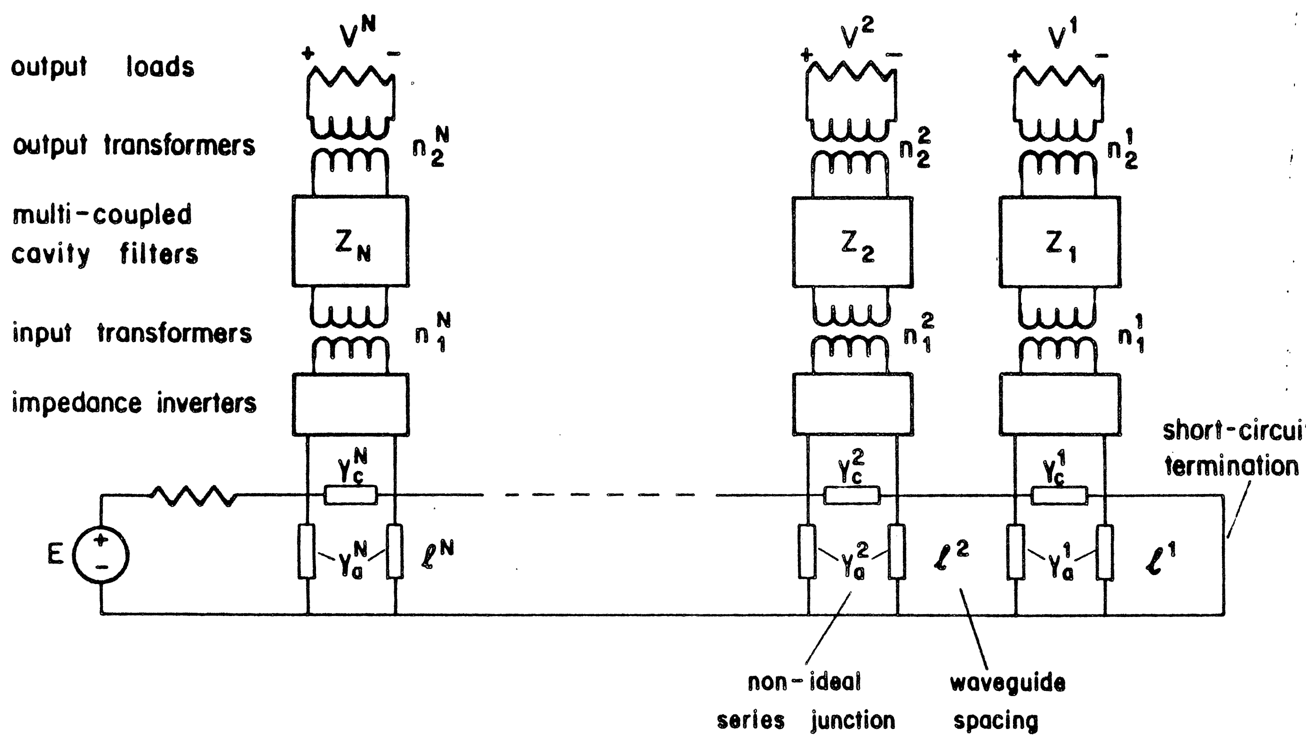


Fig. 5.6 Equivalent circuit of a contiguous band multiplexer. Each channel has a multi-coupled cavity filter with input and output transformers as well as an impedance inverter. The main cascade is a waveguide manifold with a short circuit termination. Branches are connected to the main cascade through nonideal series junctions.

TABLE 5.11

EXAMPLES OF TRANSMISSION MATRICES FOR SUBNETWORKS IN THE MULTIPLEXER OF FIG. 5.6

Subnetwork	Transmission Matrix: Expression	Transmission Matrix: Notation
output transformer n_2:1	$\begin{bmatrix} n_2 & 0 \\ 0 & \frac{1}{n_2} \end{bmatrix}$	A
multi-coupled cavity filter†	$\frac{1}{q_1}\begin{bmatrix} -q_n & -1 \\ q_1^2 - p_1 q_n & -p_1 \end{bmatrix}$	A
input transformer 1:n_1	$\begin{bmatrix} \frac{1}{n_1} & 0 \\ 0 & n_1 \end{bmatrix}$	A
series junction terminated at port 3 by Y_3, $(Y = Y_c + Y_3)$	$\frac{1}{Y}\begin{bmatrix} Y + Y_a & 1 \\ 2Y_aY + Y_a^2 & Y + Y_a \end{bmatrix}$	A
series junction terminated at port 2 by Y_2, $(Y = Y_a + Y_2)$	$\frac{1}{Y}\begin{bmatrix} Y + Y_c & 1 \\ Y(Y_a + Y_c) + Y_aY_c & Y + Y_a \end{bmatrix}$	D

TABLE 5.11 (continued)

Subnetwork	Transmission Matrix	
	Expression	Notation
waveguide spacing††	$\begin{bmatrix} \cos\theta & j Z_0 \sin\theta \\ \frac{j \sin\theta}{Z_0} & \cos\theta \end{bmatrix}$	**A**

† $p_i(q_i)$ is the ith element of vector $\mathbf{p}(\mathbf{q})$ which is the solution of $\mathbf{Zp} = \mathbf{u}_1$ ($\mathbf{Zq} = \mathbf{u}_n$), where $\mathbf{Z} = j(s\mathbf{1} + \mathbf{M}) + r\mathbf{1}$ and $s = (\omega_0/\Delta\omega)(\omega/\omega_0 - \omega_0/\omega)$ for a nth order filter with coupling matrix **M** centered at ω_0 and having a bandwidth parameter $\Delta\omega$ and a uniform cavity dissipation parameter r. **1** is a n×n identity matrix.

†† a waveguide section has a characteristic impedance Z_0 and $\theta = \beta\ell$, $\beta = 2\pi/\lambda_g$, where ℓ is the section length and λ_g is the guide wavelength.

TABLE 5.12

FIRST-ORDER SENSITIVITIES OF THE TRANSMISSION MATRICES IN TABLE 5.11

Subnetwork	Identification	Sensitivity of the Transmission Matrix
output transformer	$\dfrac{\partial \mathbf{A}}{\partial n_2}$	$\begin{bmatrix} 1 & 0 \\ 0 & -\dfrac{1}{n_2^2} \end{bmatrix}$
multi-coupled cavity filter	$\dfrac{\partial \mathbf{A}}{\partial M_{ab}}$	$\dfrac{j c^{\dagger}}{2 q_1} (p_a q_b + q_a p_b) \mathbf{A} + \dfrac{j c}{q_1} \begin{bmatrix} q_a q_b & 0 \\ p_1 q_a q_b + q_n p_a p_b - q_1 (p_a q_b + p_b q_a) & p_a p_b \end{bmatrix}$
	$\dfrac{\partial \mathbf{A}}{\partial \omega}$	$\dfrac{s_\omega}{q_1} \left(\mathbf{p}^T \mathbf{q} \mathbf{A} + \begin{bmatrix} \mathbf{q}^T \mathbf{q} & 0 \\ p_1 \mathbf{q}^T \mathbf{q} + q_n \mathbf{p}^T \mathbf{p} - 2 q_1 \mathbf{p}^T \mathbf{q} & \mathbf{p}^T \mathbf{p} \end{bmatrix} \right)$
input transformer	$\dfrac{\partial \mathbf{A}}{\partial n_1}$	$\begin{bmatrix} -\dfrac{1}{n_1^2} & 0 \\ 0 & 1 \end{bmatrix}$
series junction terminated at port 3 ††	$\dfrac{\partial \mathbf{A}}{\partial \phi}, \ \phi \in Y_3$	$(Y_3)_\phi \mathbf{K}_1$
	$\dfrac{\partial \mathbf{A}}{\partial \phi}, \ \phi \in J$	$(Y_c)_\phi \mathbf{K}_1 + (Y_a)_\phi \mathbf{K}_2$
	$\dfrac{\partial \mathbf{A}}{\partial \omega}$	$(Y_3 + Y_c)_\omega \mathbf{K}_1 + (Y_a)_\omega \mathbf{K}_2$

TABLE 5.12 (continued)

Subnetwork	Identification	Sensitivity of the Transmission Matrix
series junction terminated at port 2 †††	$\frac{\partial \mathbf{D}}{\partial \phi}$, $\phi \in Y_2$	$(Y_2)_\phi \mathbf{L}_1$
	$\frac{\partial \mathbf{D}}{\partial \phi}$, $\phi \in J$	$(Y_a)_\phi (\mathbf{L}_1 + \mathbf{L}_2) + (Y_c)_\phi \mathbf{L}_3$
	$\frac{\partial \mathbf{D}}{\partial \omega}$	$(Y_2 + Y_a)_\omega \mathbf{L}_1 + (Y_a)_\omega \mathbf{L}_2 + (Y_c)_\omega \mathbf{L}_3$
waveguide spacing	$\frac{\partial \mathbf{A}}{\partial \ell}$	$\beta \begin{bmatrix} -\sin\theta & j Z_0 \cos\theta \\ \frac{j \cos\theta}{Z_0} & -\sin\theta \end{bmatrix}$
	$\frac{\partial \mathbf{A}}{\partial \omega}$	$\ell(\beta)_\omega \begin{bmatrix} -\sin\theta & j Z_0 \cos\theta \\ \frac{j \cos\theta}{Z_0} & -\sin\theta \end{bmatrix}$

† $c = \begin{cases} 2 & \text{if } a \neq b \\ 1 & \text{if } a = b \end{cases}$

†† $\mathbf{K}_1 = -\frac{1}{Y^2} \begin{bmatrix} Y_a & 1 \\ Y_a^2 & Y_a \end{bmatrix}, \quad \mathbf{K}_2 = \frac{1}{Y} \begin{bmatrix} 1 & 0 \\ 2(Y_a + Y) & 1 \end{bmatrix}$

††† $\mathbf{L}_1 = -\frac{1}{Y^2} \begin{bmatrix} Y_c & 1 \\ Y_a Y_c & Y_a \end{bmatrix}, \quad \mathbf{L}_2 = \frac{1}{Y} \begin{bmatrix} 0 & 0 \\ Y_c + Y & 1 \end{bmatrix}, \quad \mathbf{L}_3 = \frac{1}{Y} \begin{bmatrix} 1 & 0 \\ Y_a + Y & 0 \end{bmatrix}$

5.5.2 Optimization of a 12-Channel 12 GHz Multiplexer

A wide range of possible multiplexer optimization problems can be formulated and solved by appropriately defining specifications on various frequency responses of interest. The sensitivities are used in conjunction with the gradient-based minimax algorithm of Hald and Madsen (1981) to ensure the fastest possible solutions.

As an example, we have used our simulation and sensitivity formulas to optimize a 12-channel, 12 GHz multiplexer without dummy channels. Waveguide spacings, input and output transformer ratios, cavity resonant frequencies as well as intercavity couplings are used as optimization variables. The optimization was executed by Kellermann (1986) and Daijavad (1986) and described in Bandler, Chen, Daijavad and Kellermann (1984) and in Bandler, Daijavad and Zhang (1986).

The problem is described as follows. There are twelve 6th-order multi-cavity filters mounted on the waveguide manifold. An optimization on a singly terminated filter was performed to obtain the starting values for the non-zero couplings M_{12}, M_{23}, M_{34}, M_{36}, M_{45}, M_{56} and the same values were assumed for all filters. The model for the nonideal junctions, i.e., the equivalent admittances Y_a and Y_c of Fig. 5.6, which have also been assumed in the transmission matrix description of junctions as appearing in Table 5.11, are consistent with the models suggested by Chen, Assal and Mahle (1976). Fig. 5.7 shows the common port return loss and channel insertion loss responses at the starting point for the optimization of the whole structure.

The specific optimization problem considered in this example was to satisfy a lower specification of 20 dB on the common port return loss over the entire frequency band of interest for the multiplexer. From Table 5.1 it is clear that the

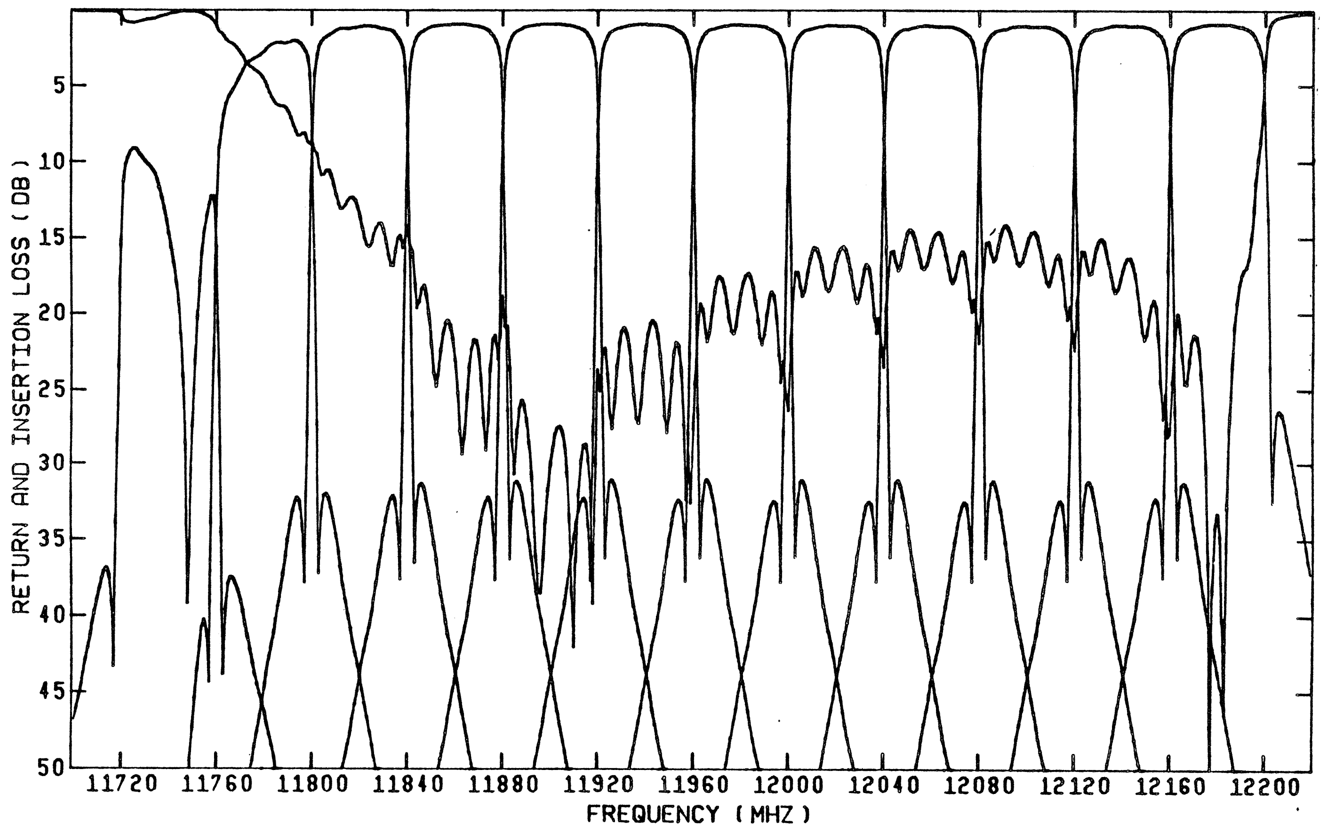


Fig. 5.7 Common port return loss and channel output port insertion loss responses of the 12-channel multiplexer before optimization.

evaluation of common port return loss and its sensitivities w.r.t. the generic optimization variable ϕ is straight-forward once the common port reflection coefficient ρ^o and its sensitivities $(\rho^o)_\phi$ are known. We will describe the particular variables considered in this example later. Recalling equations (5.25) and (5.26) and the definition of $\mathbf{q}_{ij}$ in (5.5), ρ^o and $(\rho^o)_\phi$ are evaluated from $\mathbf{q}_{2N+2,1}$ and its sensitivities. Finally, by referring to the algorithm and specifically Step 2 in this case, $\mathbf{q}_{2N+2,1}$, $\partial \mathbf{q}_{2N+2,1}/\partial\phi$ are calculated.

The optimization involved 60 variables, namely, 12 section lengths, 14 variables for each of channels 1 and 12 (all 6 possible intercavity couplings, 6 cavity resonant frequencies, input and output transformer ratios) and 4 variables for each of channels 2, 8, 9, 10, and 11 (input and output transformer ratios, resonant frequency of the first cavity and coupling M_{12}). The total CPU time on the Cyber 170/815 system was about ten minutes. The results of the final optimization are shown in Fig. 5.8. Equi-ripple return loss response satisfying the requirements over the entire communication band has been achieved.

5.6 CONCLUDING REMARKS

We have presented a new approach to simulation and sensitivity analysis of branched cascaded networks. By utilizing our formulas of Thevenin equivalents and their sensitivities w.r.t. network parameters as well as frequency, various frequency responses and their sensitivities at arbitrarily chosen reference planes are evaluated. The method presented has been utilized in the optimal design of a state of the art 12 channel contiguous band multiplexer. Attractive and fast computer results obtained using a gradient-based optimization technique justify our treatment of sensitivity evaluation as an integral part of the analysis. All the sensitivity formulas presented

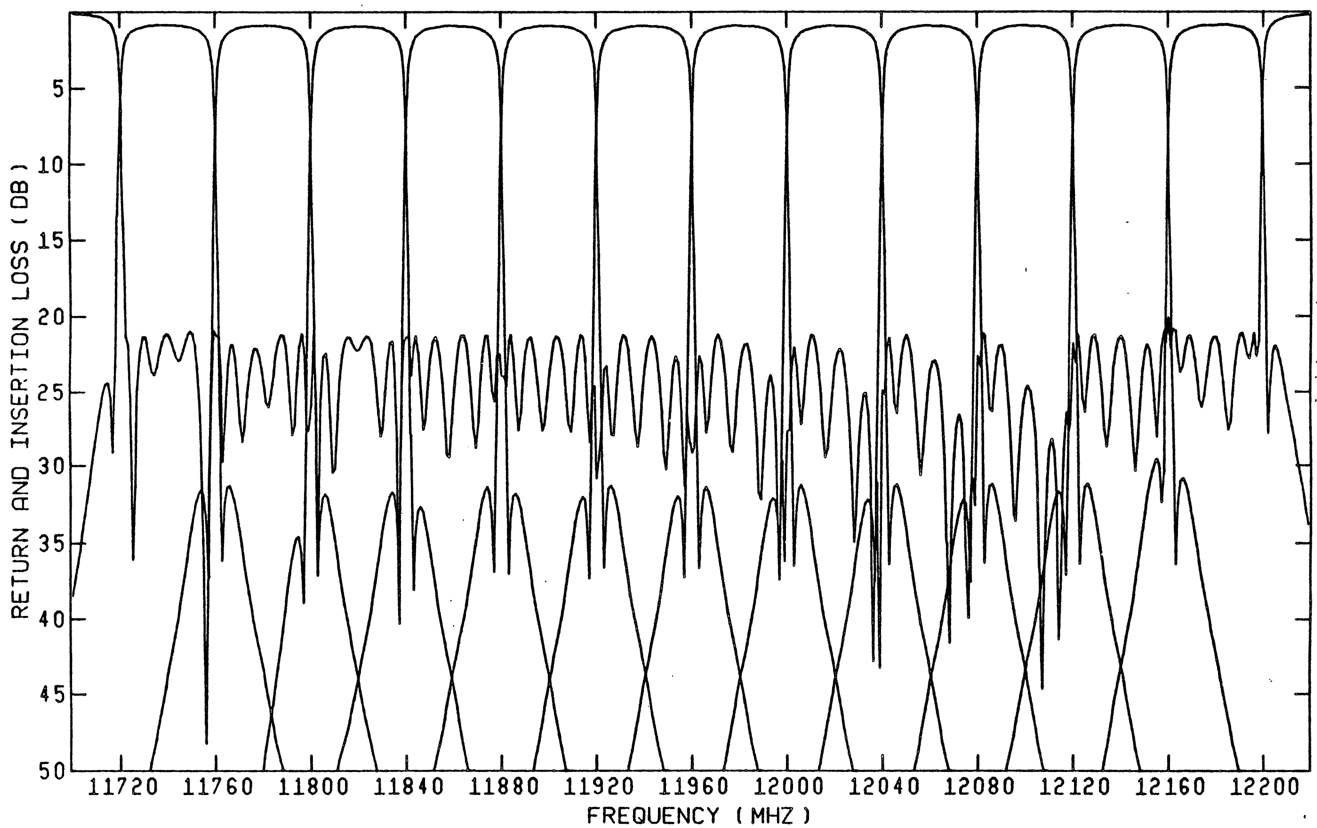


Fig. 5.8 Common port return loss and channel output port insertion loss responses of the 12-channel multiplexer with optimized spacings, input-output transformer ratios, cavity resonances and coupling parameters.

in this chapter can be verified independently. Actual implementation of our approach, however, requires only an understanding of the definitions of the responses, formulas for which are available in Table 5.1. For more theoretically oriented researchers or engineers, our method of dealing with the sensitivities (Section 5.2) is straightforward and should be applicable to almost any complex linear circuit structure in the frequency domain.

6

AN AUTOMATIC DECOMPOSITION APPROACH TO OPTIMIZATION OF LARGE MICROWAVE SYSTEMS

6.1 INTRODUCTION

A serious challenge to researchers in microwave CAD areas is due to the size of practical microwave systems. Existing CAD techniques, mature enough to handle systems of ordinary size, generally balk at large circuits. The reasons for their failure include prohibitive computer storage and CPU times required. A frequent frustration with large scale optimization is the increased likelihood of stopping at an undesired local optimum Other difficulties, especially in prototype and production tuning, are due to human inability to cope with problems involving large numbers of independent variables to be adjusted simultaneously to meet a specified response pattern over a wide frequency range.

Recently, FET modelling (Kondoh 1986) and manifold multiplexer design (Bandler, Chen, Daijavad, Kellermann, Renault and Zhang 1986) problems were solved using appropriate decomposition schemes. The optimization problems were cleverly treated by systematically or repeatedly selecting and adjusting various small sets of parameters and responses until the system becomes acceptably operational. The success of these efforts motivated us to pursue the generalization and automation of decomposition approaches for microwave optimization problems.

The concept of decomposition has been a traditional mathematically based vehicle for approaching large scale problems. Himmelblau (1973) has an excellent collection of surveys from the areas of mathematics, engineering, economics and management sciences. In this chapter, we will be interested in such aspects of decomposition that are beneficial to circuit optimization problems.

Decomposition methods used in mathematical programming theory usually assume certain structures for the objective function and constraints. Theoretical investigations have been performed for linear programming, nonlinear programming and minimax optimizations (e.g., Geoffrion 1970; Lasdon 1970; Himmelblau 1973; Luna 1984).

In circuits and systems, diakoptic analysis, generalized hybrid analysis (e.g. Chua and Chen 1976) and network tearing methods (e.g., Wu 1976; Tong and Chen 1986; Asai, Urano and Tanaka 1986) have been developed. Important to those methods are circuit relations, especially topological relations. In addition to being used for circuit analysis, the decomposition techniques have been used in design (Himmelblau 1973) and fault diagnosis (Salama, Starzyk and Bandler 1984).

Decomposition has also been an active subject in electrical power systems since such problems easily result in thousands of variables and equations. Examples can be found in optimal power flow (Talukdar, Giras and Kalyan 1983; Contaxis, Delkis and Korres 1986), state estimation (Lo and Mahmoud 1986) and real and reactive power optimization problems (Billinton and Shachdeva 1973). The decomposition patterns involved are obtained using both physical and analytical investigations of the systems.

Microwave engineers have their own special difficulties. Thorough laboratory experimentation has to be performed before using certain function structures assumed in mathematical programming theory. They do not take advantage of topological analysis often exploited in the areas of circuits and systems since microwave device models are oriented more to physical than topological analysis. Unlike power systems, most microwave responses are much more complicated and highly nonlinear. It is often difficult for microwave engineers to analytically indicate possible decomposition patterns.

The state-of-the-art in large-scale optimization of microwave circuits is still device dependent and based on heuristic judgement Very recently, Bandler and Zhang (1987a) made a first attempt to develop a general and abstract theory describing a decomposition approach to microwave circuit optimization not requiring particular physical or topological knowledge of the system.

In this chapter, we present the novel technique of Bandler and Zhang (1987a, 1987c) for the optimization of large microwave systems. Using sensitivity information obtained from a suitable Monte-Carlo analysis, we extract possible decomposition properties which could otherwise be deduced only through a physical and topological investigation. The overall problem is automatically separated into a sequence of subproblems, each being characterized by the optimization of a subset of circuit functions w.r.t. variables which are sensitive to the selected responses. Our suggested technique has been successfully tested on microwave multiplexers involving up to 16 channels and 240 variables.

In Section 6.2, we describe the basic concepts of decomposition for circuit optimization problems. Using these concepts, the partitioning approach for FET modelling problems suggested by Kondoh (1986) is verified and the decomposition property of multiplexers is explained, as presented in Section 6.3. Section 6.4 illustrates the automatic determination of suboptimization problems. An automated decomposition algorithm for large scale microwave optimization is presented in Section 6.5. In Section 6.6, the method is applied to the optimization of microwave multiplexers. Interesting results demonstrating the whole procedure of automated decomposition for a 5-channel multiplexer are depicted in illustrative graphs. The results of optimizing a 16-channel multiplexer using our approach are provided.

6.2 THE DECOMPOSITION APPROACH FOR CIRCUIT OPTIMIZATION PROBLEMS

6.2.1 Circuit Optimization Problems

Let

$$\boldsymbol{\phi} = [\phi_1 \ \phi_2 \ \dots \ \phi_n]^T \tag{6.1}$$

represent the system parameters. The circuit responses, denoted as $F_k(\boldsymbol{\phi}, \omega)$, $k = 1, 2, \dots, n_F$, are functions of variables $\boldsymbol{\phi}$ and frequency ω. In an optimization problem for circuit design, the objective function usually involves a set of nonlinear error functions $f_j(\boldsymbol{\phi})$, $j = 1, 2, \dots, m$. Typically, the error functions represent the weighted differences between circuit responses and given specifications in the form defined in (2.3).

Suppose sets I and J are defined as

$$I \triangleq \{1, 2, \dots, n\}, \tag{6.2}$$

$$J \triangleq \{1, 2, \dots, m\}. \tag{6.3}$$

The overall optimization problem, e.g., a minimax optimization, is

$$\underset{\phi_i,\ i \in I}{\text{minimize}} \quad \max_{j \in J} \ f_j(\boldsymbol{\phi}). \tag{6.4}$$

In a decomposition approach, one attempts to reach the overall solution by solving a sequence of subproblems. A typical subproblem is characterized by

$$\underset{\phi_i,\ i \in I^s}{\text{minimize}} \quad \max_{j \in J^s} \ f_j(\boldsymbol{\phi}), \tag{6.5}$$

where I^s and J^s are subsets of I and J, respectively.

The basic idea for decomposition is to decouple a variable ϕ_i from a function f_j if the interaction between them is weak. A subproblem contains only the sensitively related variables and functions. A proper arrangement of the sequence of different subproblems to be solved is often important to ensure convergence and efficiency.

6.2.2 Grouping of Variables and Functions Using Sensitivity Information

The essential task for the automatic decomposition technique is the automatic decision on I^s and J^s, and the automatic sequential arrangement of various subproblems. This is accomplished through an appropriate decomposition dictionary to be introduced in the ensuing text.

Sensitivity Analysis

We perform sensitivity analysis at a set of randomly chosen points $\boldsymbol{\phi}^{\ell}$, $\ell = 1, 2, \ldots$. A measure of the interaction between ϕ_i and f_j is defined as

$$S_{ij} \triangleq \sum_{\ell} \left(\frac{\partial f_j(\boldsymbol{\phi}^{\ell})}{\partial \phi_i} \frac{\phi_i^0}{f_j^0} \right)^2 , \tag{6.6}$$

where ϕ_i^0 and f_j^0 are used for scaling. All the S_{ij}, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$, constitute a nxm sensitivity matrix $\mathbf{S}$. It is reasonable to conclude that ϕ_i and f_j can be decoupled if S_{ij} is very small.

Grouping of Variables and Functions

The examination of various interaction patterns between ϕ_i, $i \in I$, and f_j, $j \in J$, results in the breakdown of all variables $\boldsymbol{\phi}$ into p groups identified by index sets I_1, I_2, ..., I_p, and all functions f into q groups identified by sets $J_1, J_2, \ldots, J_q$. We have

$$I = I_1 \cup I_2 \cup \ldots \cup I_p \tag{6.7}$$

and

$$J = J_1 \cup J_2 \cup \ldots \cup J_q \ . \tag{6.8}$$

The partitioning of $\boldsymbol{\phi}$ or $\mathbf{f}$ can be achieved either manually or automatically. The manual procedure corresponds to the manual determination of variable groups and function groups using a priori knowledge. Such knowledge is typically obtained through extensive laboratory experiment and an excellent understanding of the particular device. The automatic procedure corresponds to the computerized partitioning of $\boldsymbol{\phi}$ or $\mathbf{f}$ based upon the sensitivity matrix $\mathbf{S}$. The partitioning of $\boldsymbol{\phi}$ and $\mathbf{f}$ can be performed 1) both manually, 2) manually for $\boldsymbol{\phi}$ and automatically for $\mathbf{f}$, 3) automatically for $\boldsymbol{\phi}$ and manually for $\mathbf{f}$, 4) both automatically.

As an example for manual partitioning of $\mathbf{f}$, we consider a N-channel multiplexer. The common port return loss and channel insertion loss responses associated with the same channel can be grouped together since their behavior is similarly affected by variables $\boldsymbol{\phi}$. Therefore, we have N groups of functions, i.e., $q = N$. J_ℓ contains indices of error functions related to channel ℓ, $\ell = 1, 2, \ldots, N$.

<u>A Procedure for Automatic Partitioning of Variables $\boldsymbol{\phi}$</u>

Suppose the function groups have been determined, i.e., J has been decomposed into J_ℓ, $\ell = 1, 2, \ldots, q$. We define a $n \times q$ matrix $\mathbf{C}$ whose (i, ℓ)th component is

$$C_{i\ell} = \sum_{j \in J_\ell} (w_{ij} S_{ij}) , \tag{6.9}$$

where w_{ij} is a weighting factor. A very small value of an entry in the $\mathbf{C}$ matrix, say, $C_{i\ell}$, implies that the ith variable and the ℓth function group are weakly interconnected.

Let C_{ave} represent the average value of all components in the C matrix. For a given factor λ, $\lambda \geq 0$, the matrix is made sparse such that $C_{i\ell}$ is set to zero if it is less than λC_{ave}. By making C sparse, insensitive variables are eliminated and weak interactions between variables and function groups are decoupled.

Two variables ϕ_i and ϕ_j belong to the same group if they interact only with the same groups of functions, i.e., if the ith and the jth rows of C have the same zero/nonzero pattern. A thorough computerized checking of the C matrix results in the automatic determination of index sets I_k, $k = 1, 2, ..., p$.

An Illustrative Example of Matrix C

Consider the fictitious relations between variables and function groups shown in Fig. 6.1(a). The functions f have been arranged into 5 groups The C matrix (already made sparse) is

$$\begin{bmatrix} 22. & 100. & 32. & 0. & 0. \\ 0. & 100. & 0. & 0. & 0. \\ 0. & 100. & 0. & 0. & 0. \\ 0. & 0. & 83. & 100. & 0. \\ 0. & 0. & 0. & 0. & 100. \\ 0. & 0. & 100. & 86. & 0. \\ 0. & 0. & 100. & 0. & 0. \\ 0. & 78. & 100. & 55. & 0. \\ 100. & 0. & 0. & 0. & 0. \end{bmatrix} . \qquad (6.10)$$

As seen from Fig. 6.1(a), ϕ_2 and ϕ_3 both affect only the 2nd function group. In the C matrix, rows 2 and 3 both have only one nonzero located at the 2nd column. Therefore, variables ϕ_2 and ϕ_3 are grouped together. Similarly, variables ϕ_4 and ϕ_6

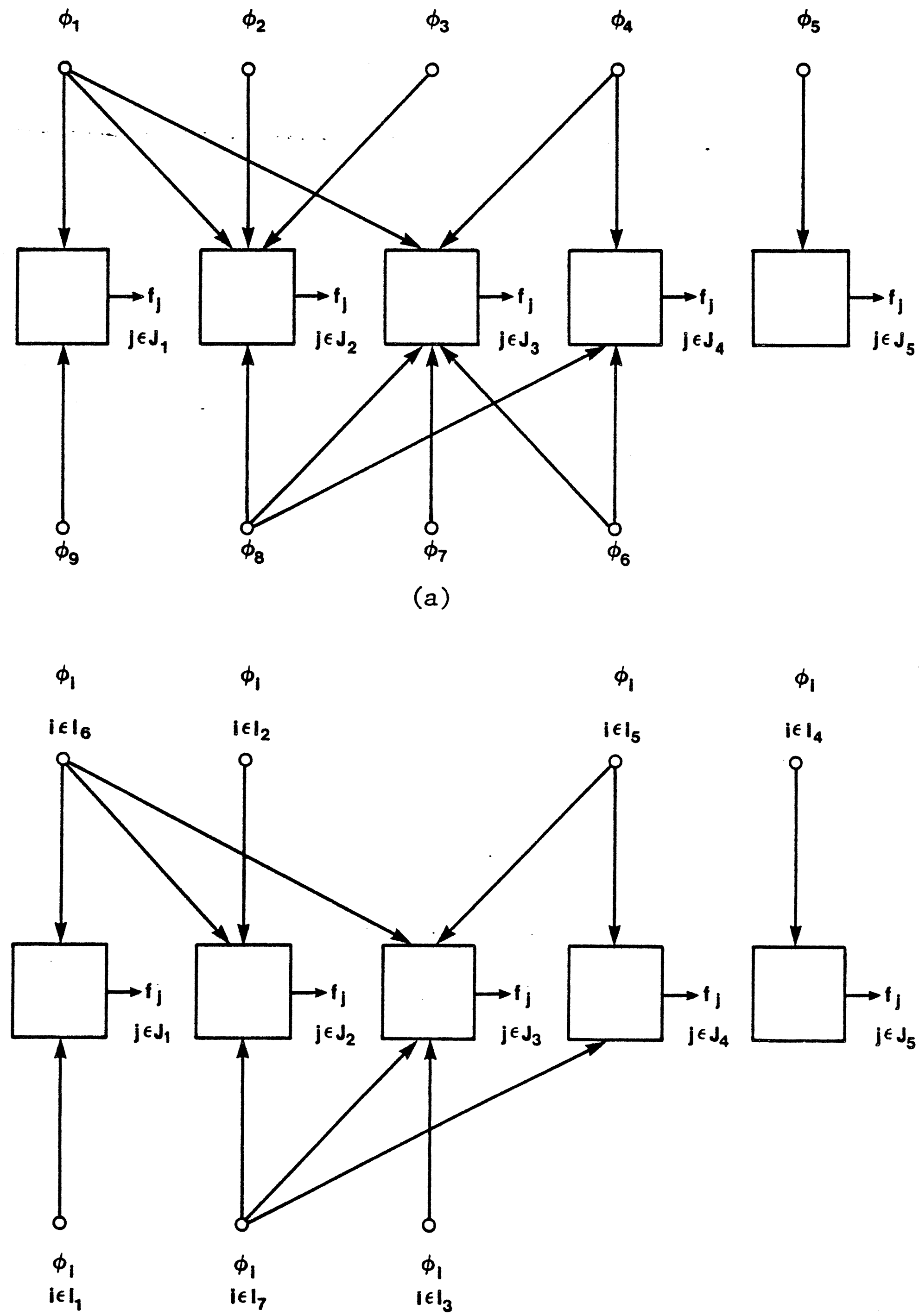


Fig. 6.1 A fictitious example showing only the strong interconnections between variables and function groups. (a) system configuration corresponding to matrix **C**. (b) system configuration corresponding to matrix **D**.

belong to the same group. The resulting index sets for variable groups are $I_1 = \{9\}$, $I_2 = \{2, 3\}$, $I_3 = \{7\}$, $I_4 = \{5\}$, $I_5 = \{4, 6\}$, $I_6 = \{1\}$ and $I_7 = \{8\}$. The index sets have been ordered such that the kth variable group correlates with no more function groups than the (k+1)th variable group does, k = 1, 2, ..., 6. Such an arrangement is made to keep subsequent description simple.

6.2.3 Decomposition Dictionary

To manipulate directly with groups of variables and groups of functions, we construct a *pxq* dictionary decomposition matrix **D**. Define the (k, ℓ)th component of **D** as

$$\begin{aligned} D_{k\ell} &= \sum_{i \in I_k} \sum_{j \in J_\ell} (w_{ij} S_{ij}) \\ &= \sum_{i \in I_k} C_{i\ell} \, . \end{aligned} \tag{6.11}$$

If $D_{k\ell}$ is zero, variables in the kth group are decoupled from functions in the ℓth group. Otherwise if $D_{k\ell} \neq 0$, we say that ϕ_i, $i \in I_k$, and f_j, $j \in J_\ell$, are correlated. The decomposition dictionary gives a clear picture of the correlation patterns between groups of variables and functions, facilitating the automatic determination of suboptimization problems. The ideal dictionary is a diagonal matrix where a subproblem simply corresponds to a diagonal element. In this case, only one variable group and one function group is involved in a subproblem. If a diagonal dictionary can be obtained without artificially making **C** sparse (i.e., using sparse factor $\lambda = 0$), then the system is completely decomposable. For a completely decomposable system, different subproblems can be calculated in parallel. Details of decomposability can be found in Courtois (1977).

Consider the previous example with the resulting **C** matrix defined in (6.10). According to the index sets I_k, $k = 1, 2, ..., 7$, the decomposition dictionary **D** can be obtained from **C** by adding rows 2 and 3, and adding rows 4 and 6, respectively. The relations between groups of variables and functions are shown in Fig. 6.1(b). The resulting dictionary is

$$\begin{bmatrix} 100. & 0. & 0. & 0. & 0. \\ 0. & 200. & 0. & 0. & 0. \\ 0. & 0. & 100. & 0. & 0. \\ 0. & 0. & 0. & 0. & 100. \\ 0. & 0. & 180. & 180. & 0. \\ 20. & 100. & 30. & 0. & 0. \\ 0. & 70. & 100. & 50. & 0. \end{bmatrix} , \qquad (6.12)$$

where each entry has been rounded to multiples of 10.

6.3 PRACTICAL EXAMPLES OF DECOMPOSITION DICTIONARY

6.3.1 Decomposition Dictionary for FET Device Models

Through extensive experiment on practical FET devices, Kondoh (1986) summarized 8 suboptimization problems which can be repeatedly solved to yield a FET model with improved accuracy. The equivalent circuit is shown in Fig. 6.2. Using the theory described in the previous section, Bandler and Zhang (1987a) presented a decomposition dictionary for such devices. The 13 variable parameters were automatically partitioned into 8 groups. Their result was in complete agreement with the 8 subproblems of Kondoh (1986).

Here we describe the experiment reported by Bandler and Zhang (1987a). A set of true parameter values listed in Kondoh (1986) is used as a reference point $\boldsymbol{\phi}^0$,

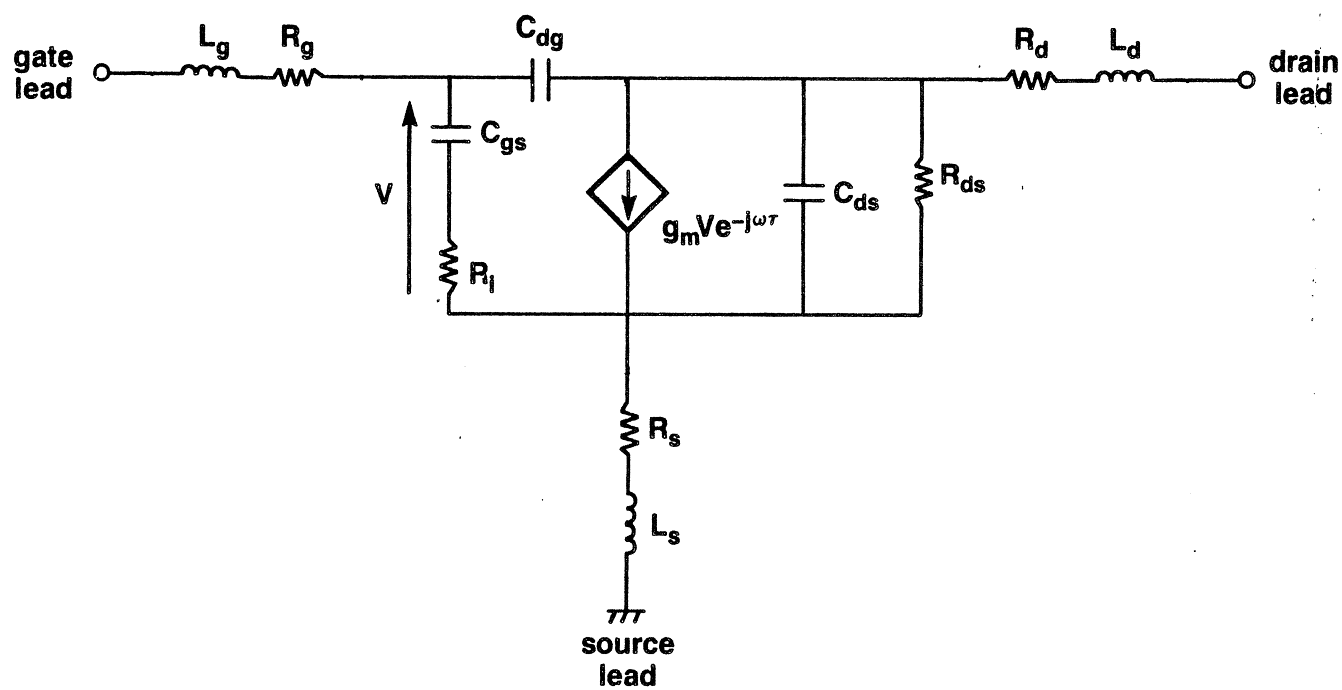


Fig. 6.2 A FET equivalent circuit.

as shown in Table 6.1. The entire frequency range of measurement is 1.5 GHz to 26.5 GHz. We perform sensitivity analysis at 10 randomly chosen parameter points in the 10% neighbourhood of ϕ^0. The function f_j used in (6.6) is defined as the weighted difference between the calculated and the measured values of the modulus or the phase of a particular S parameter. Functions associated with the same S parameter are grouped together. Table 6.2 shows the **C** matrix of (6.9) before being made sparse, indicating strong as well as weak interconnection patterns between each individual parameter and different groups of functions. Table 6.3 provides an example of the decomposition dictionary calculated and normalized from the **C** matrix of Table 6.2. Table 6.3 yields 8 subproblems which agree with and further verify the decomposition scheme proposed by Kondoh (1986). When the dictionary is made sparse, certain entries, whose values are only slightly less than the dominant ones, are also set to zero. Therefore, as mentioned by Kondoh, repeated cycling and careful ordering of the 8 suboptimizations are necessary. The feasibility of computerized automatic decomposition is demonstrated by this example.

6.3.2 Decomposition Dictionary of a 16-Channel Multiplexer

Multiplexers consisting of multicavity filters distributed along a waveguide manifold was introduced in Chapter 5. It has been observed that parameters associated with a particular channel of the multiplexer structure have a strong effect on responses corresponding to that channel and a weak effect on responses related to other channels. The theoretical description of this phenomenon was presented in Bandler, Chen, Daijavad, Kellermann, Renault and Zhang (1986). A prototype decomposition dictionary was constructed from manual partitioning of variables and

TABLE 6.1

PARAMETER VALUES FOR ϕ^0 FOR THE FET CIRCUIT MODEL

i	Parameter ϕ_i	Unit	Value for ϕ_i
1	g_m	mS	50.0
2	τ	ps	3.0
3	C_{gs}	pF	0.25
4	C_{ds}	pF	0.08
5	C_{dg}	pF	0.025
6	R_g	Ohm	4.0
7	R_s	Ohm	4.0
8	R_d	Ohm	3.0
9	R_{ds}	Ohm	250.
10	R_i	Ohm	0.2
11	L_g	pH	60.0
12	L_d	pH	25.0
13	L_s	pH	15.0

TABLE 6.2

THE C MATRIX FOR THE FET MODEL

(a) FUNCTION GROUPS INVOLVING THE ENTIRE FREQUENCY BAND (1.5 GHZ TO 26.5 GHZ)

Variables	Function Groups			
	S_{11} Entire Freq. Band	S_{21} Entire Freq. Band	S_{12} Entire Freq. Band	S_{22} Entire Freq. Band
g_m	18.55	100.00	87.55	68.33
C_{gs}	100.00	89.74	67.98	62.25
C_{ds}	4.88	67.74	45.73	100.00
C_{dg}	4.24	48.88	100.00	81.27
R_s	35.53	37.14	100.00	5.88
R_{ds}	17.44	97.68	70.51	100.00

Each row of the table has been scaled.

TABLE 6.2 (continued)

THE C MATRIX FOR THE FET MODEL

(b) FUNCTION GROUPS INVOLVING ONLY THE UPPER HALF FREQUENCY BAND (14.0 GHZ TO 26.5 GHZ)

Variables	Function Groups			
	S_{11} Upper Freq. Band	S_{21} Upper Freq. Band	S_{12} Upper Freq. Band	S_{22} Upper Freq. Band
τ	31.91	100.00	36.61	59.31
R_g	100.00	50.67	24.87	29.89
R_d	34.65	74.31	85.85	100.00
R_i	100.00	65.63	88.43	39.53
L_g	100.00	87.85	57.16	37.44
L_d	9.99	97.88	61.78	100.00
L_s	62.94	31.31	100.00	21.99

Each row of the table has been scaled.

TABLE 6.3

NORMALIZED DECOMPOSITION DICTIONARY D FOR THE FET MODEL

(a) CORRESPONDING TO THE SENSITIVITY ANALYSIS OF TABLE 6.2(a)

Variable Groups	Function Groups			
	S_{11} Entire Freq. Band	S_{21} Entire Freq. Band	S_{12} Entire Freq. Band	S_{22} Entire Freq. Band
R_{ds}, C_{ds}	0.00	0.00	0.00	1.00
C_{gs}	1.00	0.00	0.00	0.00
C_{dg}, R_s	0.00	0.00	1.00	0.00
g_m	0.00	1.00	0.00	0.00

(b) CORRESPONDING TO THE SENSITIVITY ANALYSIS OF TABLE 6.2(b)

Variable Groups	Function Groups			
	S_{11} Upper Freq. Band	S_{21} Upper Freq. Band	S_{12} Upper Freq. Band	S_{22} Upper Freq. Band
R_d, L_d	0.00	0.00	0.00	1.00
R_g, R_i, L_g	1.00	0.00	0.00	0.00
L_s	0.00	0.00	1.00	0.00
τ	0.00	1.00	0.00	0.00

functions. The following description is based upon their experiment and the sensitivity matrices they used.

Consider a 16-channel, 12 GHz contiguous band multiplexer whose equivalent circuit follows Fig. 5.6. All channel filters are 6th order. We perform sensitivity analysis at 30 points selected randomly within 40% region of the optimal point of the multiplexer. Sixteen function groups are composed, each corresponding to a particular channel of the multiplexer. The kth function group consists of common port return loss functions calculated at 7 frequency points, 3 of which are in the passband of channel k, and the remaining points are in the stopband of channel k.

In the first experiment, we have 16 groups of variables. The kth variable group includes all coupling parameters as well as input and output transformer ratios of the kth channel filter. The decomposition dictionary is shown in Table 6.4. As performed by Bandler et al. (1986), this dictionary matrix was normalized such that each element of the matrix is divided by the average value of the corresponding row before normalization. The matrix is then made sparse by rounding off all entries to integers. In the second experiment, 16 variable groups were used, each group containing only one variable. The variable in the kth group is the distance of the kth channel filter from the short circuit main cascade termination. The corresponding dictionary is shown in Table 6.5. The dictionary is normalized and made sparse similarly to that for Table 6.4.

These dictionaries provided a theoretical background for the large scale minimax optimization of the 16-channel multiplexer reported for the first time by Bandler, Chen, Daijavad, Kellermann, Renault and Zhang (1986). Their approach was a manual manipulation of the decomposition properties discovered. The band form of Table 6.4 and the near band form of Table 6.5 correspond to the phenomenon

TABLE 6.4

DECOMPOSITION DICTIONARY FOR A 16-CHANNEL MULTIPLEXER, WHERE VARIABLES ARE COUPLING PARAMETERS AND TRANSFORMER RATIOS

channels (responses)

variables in channel	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	7	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	4	8	4	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	6	8	3	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	3	8	5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	5	7	3	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	5	7	4	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	5	6	5	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	3	7	6	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	7	8	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	4	7	5	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	5	6	5	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	5	5	5	0	0
14	0	0	0	0	0	0	0	0	0	0	0	1	5	7	3	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	5	7	4
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	10

TABLE 6.5

DECOMPOSITION DICTIONARY FOR A 16-CHANNEL MULTIPLEXER, WHERE VARIABLES ARE THE DISTANCES FROM THE SHORT CIRCUIT

variables in channel	channels (responses)															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	8	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	7	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	2	3	2	0	0	0	0	0	1	1	2	2	2	0	0
4	0	0	4	6	2	0	0	0	0	1	1	1	1	1	0	0
5	0	0	0	4	6	3	0	0	0	1	1	0	0	0	0	0
6	0	0	0	0	5	7	2	0	0	1	1	0	0	0	0	0
7	0	0	0	0	0	4	6	2	0	2	1	0	0	0	0	0
8	0	0	0	0	0	1	3	5	3	2	2	0	0	0	0	0
9	0	0	0	0	0	0	0	3	4	4	1	0	0	1	0	0
10	0	0	0	0	0	0	0	0	4	7	3	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	3	5	3	3	0	0
12	0	0	0	0	0	0	0	0	0	0	3	6	3	3	0	0
13	0	0	0	0	0	0	0	0	0	0	0	5	5	5	0	0
14	0	0	0	0	0	0	0	0	0	0	0	2	5	5	2	1
15	0	0	0	0	0	0	0	0	0	0	0	0	1	5	6	4
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	7

that variables in a channel mainly affect responses at the same channel and adjacent channels. The nonzeros in the far off-diagonal regions in Table 6.5 indicate the effect between non-adjacent channels due to adjustments on the distance of a channel filter from the main cascade.

6.4 AUTOMATIC DETERMINATION OF SUBOPTIMIZATION PROBLEMS

6.4.1 Theoretical Description

The Reference Function Group

Usually, the decomposition dictionary is not diagonal. A suboptimization often involves several function groups and several variable groups. Among the function groups involved, there is a key group which we call the reference group Such a group typically contains the worst error function. The reference function group is used to initiate a subproblem as described in the subsequent text.

Candidate Groups of Variables

Suppose the index set J_ℓ indicates the reference function group. The candidate groups of variables to be used for the suboptimization are those which affect $f_j, j \in J_\ell$.

In the decomposition dictionary, the ℓth column associates with the reference function group. Rows having a nonzero in the ℓth column are candidate rows, each corresponding to a candidate variable group. Take Fig. 6.1(b) as an example. Suppose that the function group associated with index set J_2 is the reference group, i.e., $\ell = 2$. The candidate groups of variables are I_2, I_6 and I_7 since they correlate with the reference function group. Correspondingly, in the **D** matrix of (6.12), rows 2, 6 and 7 are candidate rows since they all have a nonzero in the 2nd column.

Determination of a Suboptimization Problem

An automatic procedure for the determination of I^s and J^s for the sub optimzation of (6.5) has been developed. Suppose J_ℓ indicates the reference function group. For a selected candidate variable group, e.g., the one corresponding to set I_k, the index set J^s indicates the union of all function groups which correlate with variable group k. I^s identifies variables in the kth group, as well as all other variables which correlate with functions only within f_j, $j \in J^s$. Also, I^s excludes variables not correlating with any active functions in $f_j, j \in J^s$. A function f is said to be active if

$$\begin{aligned} f &> 0.8M_f \quad \text{when} \quad M_f > 0 \\ f &> 1.25M_f \quad \text{when} \quad M_f < 0\,, \end{aligned} \tag{6.13}$$

where

$$M_f \triangleq \max_{j \in J^s} f_j\,. \tag{6.14}$$

Priority of Candidate Groups of Variables

It can be seen that a pair of (I^s, J^s) associate with a pair of (I_k, J_ℓ). For a selected reference function group, each candidate variable group leads to a sub-problem. The sequence of subproblems used to penalize f_j, $j \in J_\ell$, are determined by the priority of all resulting candidates.

Since each candidate determines the function set J^s for a suboptimization, the priority of the candidate is based upon the pattern of error functions it will affect, i.e. patterns of f_j, $j \in J^s$. Firstly, the fewer the number of function groups in J^s, the higher the priority. Secondly, the worse the overall error functions in J^s, the higher the priority. The overall error functions in J^s are ranked by the generalized least pth function (GLP) (Bandler and Rizk 1979) as

$$GLP = \begin{cases} M_f \left(\sum_{j \in K} (f_j(\phi)/M_f)^q \right)^{1/q} & \text{if } M_f \neq 0 \\ 0 & \text{if } M_f = 0\,, \end{cases} \tag{6.15}$$

where M_f was defined in (6.14) and

$$\begin{array}{lll} \text{if } M_f > 0, & \text{then } K = \{j \mid f_j \geq 0\,, j \in J^s\} & \text{and } q = p \\ \text{if } M_f < 0, & \text{then } K = J^s & \text{and } q = -p\,. \end{array} \tag{6.16}$$

Typically, we choose $p = 2$.

The priority of candidate variable groups can be similarly determined in the decomposition dictionary. The fewer the number of nonzeros that exist in a candidate row, the higher the priority. For two candidate rows containing an equal number of nonzeros, a higher priority is given to the candidate having a larger value in its generalized least pth function.

6.4.2 An Example for Deciding on a Subproblem and Candidate Priority

For the example of Fig. 6.1, suppose that the maximum error functions within each of the 5 function groups are [3.8 4. 1. -1. 2.]. Suppose that we choose the worst group, i.e., group 2, as the reference function group. According to our previous discussions, the candidate variable groups are I_2, I_6 and I_7. I_2 has the highest priority since it affects fewer (i.e. only one) function groups than I_6 or I_7 does (I_6 and I_7 both affect three function groups). To rank the priority between candidates I_6 and I_7, we compare the overall error functions they will affect. The functions affected by variables in I_6 (or I_7) are $f_j, j \in J^s = J_1 \cup J_2 \cup J_3$ (or $J^s = J_2 \cup J_3 \cup J_4$). I_6 has a higher priority than I_7 since the overall error functions in $J_1 \cup J_2 \cup J_3$ are worse than that in $J_2 \cup J_3 \cup J_4$.

Correspondingly, in the decomposition dictionary of (6.12), rows 2, 6 and 7 are candidates. Row 2 has the highest priority since it contains fewer nonzeros than others. Row 6 has the second highest priority since its GLP value is obviously larger than the GLP value for row 7.

To formulate a suboptimization problem, i.e., to decide I^s and J^s, we choose a pair of (I_k, I_ℓ), e.g., candidate variable group I_6 and reference function group J_2. The index set $J^s = J_1 \cup J_2 \cup J_3$. The variable index set I^s includes I_6 (indicating the candidate variable group), as well as I_1, I_2 and I_3 (indicating all other variables affecting functions only within J^s). Further, I_3 can be excluded from I^s since variables in I_3 do not affect active functions in J^s. Therefore, we have $I^s = I_6 \cup I_1 \cup I_2$.

6.5 AN AUTOMATIC DECOMPOSITION ALGORITHM FOR CIRCUIT OPTIMIZATION

An automatic decomposition algorithm for optimization of microwave systems has been developed and implemented. The algorithm can decide when to update the sensitivity matrix and the decomposition dictionary. The formulation and the sequence of suboptimization problems are dynamically determined. The degree of decomposition is reduced as the system converges to its overall solution. As a special case, if all variables interact with all functions, our approach solves only one subproblem, this being identical to the original overall optimization.

<u>Step 1</u> Initialize sparse factor λ. Calculate the sensitivity matrix **S** and the decomposition dictionary **D**. Calculate **f**.

<u>Comment</u> The initial sensitivity matrix can be obtained from a suitable Monte-Carlo sensitivity analysis performed off-line. All error functions are calculated in this step.

Step 2 Define ℓ such that

$$f_{worst} = \max_{j \in J_\ell} f_j = \max_{j \in J} f_j .$$

Comment The ℓth function group contains the worst response. Such a function group will be frequently chosen as the reference group to be penalized.

Step 3 For the given ℓ, determine the sequence of candidate rows in **D**. Rank the candidates in decreasing priority. Set $k = 0$.

Comment The ℓth function group is the reference group to be penalized. All variable groups correlating with the ℓth function group are considered as candidates.

Step 4 If $k = 0$ then set k to the row index of the first candidate, otherwise set k to the row index of the next candidate. If such a candidate does not exist then go to Step 8.

Comment The candidate groups of variables are sequentially selected. Each entry into this step results in a selection of a candidate with a lower priority than the current one.

Step 5 Define I^s and J^s using the current k, ℓ. If I^s and J^s are identical with their previous values then go to Step 4. Solve the suboptimization problem

$$\underset{\phi_i,\ i \in I^s}{\text{minimize}} \ \max_{j \in J^s} \ f_j(\boldsymbol{\phi}) .$$

Terminate the optimization if

$$\max_{j \notin J^s} \ f_j > \lambda' \ f_{worst} .$$

Comment A subproblem is formulated and solved in this step. By checking the functions not covered in the present suboptimization, any significant deterioration in the overall objective function is prevented. The factor λ' can be, e.g., 1.2.

Step 6 If $I^s = I$ and $J^s = J$ then stop.

Comment The program terminates following the completion of an overall optimization which is considered as the last subproblem.

Step 7 Calculate f. Calculate

$$f_{worst} = \max_{j \in J} f_j .$$

Go to Step 5.

Comment An overall simulation is performed. By going to Step 5, the current reference function group can be continuously penalized in the next subproblem even if this group does not include the worst error functions.

Step 8 If

$$\max_{j \in J^s} f_j < \max_{j \in J} f_j$$

then go to Step 2. If $\lambda \approx 0$ then stop otherwise, update **S**, reduce λ, update dictionary **D** and go to Step 3.

Comment: When the selection of a candidate fails, a new sequence of candidates will be defined by going to Steps 2 or 3. By reducing the sparse factor λ, the degree of decomposition is reduced as the overall solution is being approached. The reference function group will be readjusted if the existing one does not contain the maximum error function. For completely decomposable problems, the terminating conditions in Step 6 will not be satisfied and the program will exit from Step 8.

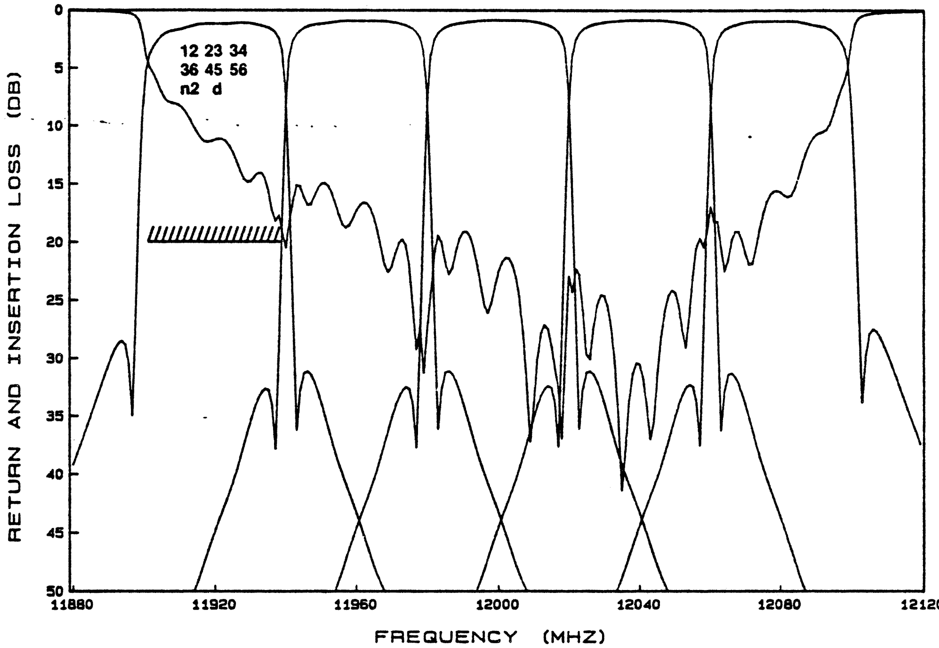
6.6 LARGE SCALE OPTIMIZATION OF MULTIPLEXERS

The automatic decomposition technique was tested on the optimization of microwave multiplexers used in satellite communications. Specifications were imposed on the common port return loss and individual channel insertion loss functions. Each suboptimization was solved using a recent minimax algorithm of Bandler, Kellermann and Madsen (1985). Until our recent paper on multiplexers (Bandler, Chen, Daijavad, Kellermann, Renault and Zhang 1986; Bandler and Zhang 1987a), the reported design and manufacturing of these devices were limited to 12 channels (e.g., Egri, Williams and Atia 1983; Tong and Smith 1984; Holme 1984; Chen 1985; Bandler, Daijavad and Zhang 1986).

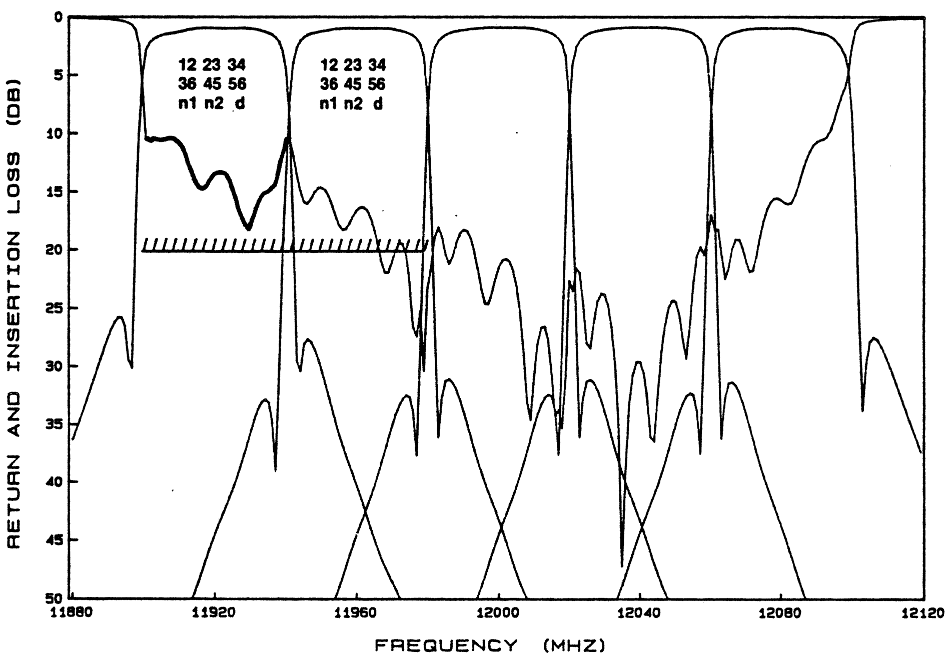
A contiguous band 5-channel multiplexer was specifically optimized to illustrate the novel process of automatic decomposition, as shown in Fig. 6.3. Functions associated with the same channel are grouped together. Variables for each channel include 12 coupling parameters, input and output transformer ratios (n_1 and n_2) and the distance measure from the channel filter to the short circuit main cascade termination. The overall problem involved 75 variables and 124 nonlinear functions. As the parameters approached their solution, weak interactions between variables and functions were also considered. The final subproblem was the overall optimization.

We also tested our approach on a 16-channel multiplexer involving 240 variables and 399 nonlinear functions. The responses at the starting point is shown in Fig. 6.4. Only 10 suboptimizations were performed before reaching the response of Fig. 6.5. Then a full optimization is activated resulting in all responses satisfying their specifications as shown in Fig. 6.6. A comparison between the optimal design with and without decomposition is provided in Table 6.6. When used to obtain a good

Fig. 6.3 Return and insertion loss responses of the 5-channel multiplexer for each suboptimization. The 20 dB specification line indicates which channel(s) is to be optimized in the next subproblem. The variables to be selected are indicated in the graph, e.g., 35 representing coupling M_{35}, d representing the distance of the corresponding channel filter from the short circuit main cascade termination. The previously optimized channels are highlighted by thick response curves. (a) responses at the starting point. (b)–(k) responses for each suboptimization. (ℓ) responses at the final solution.

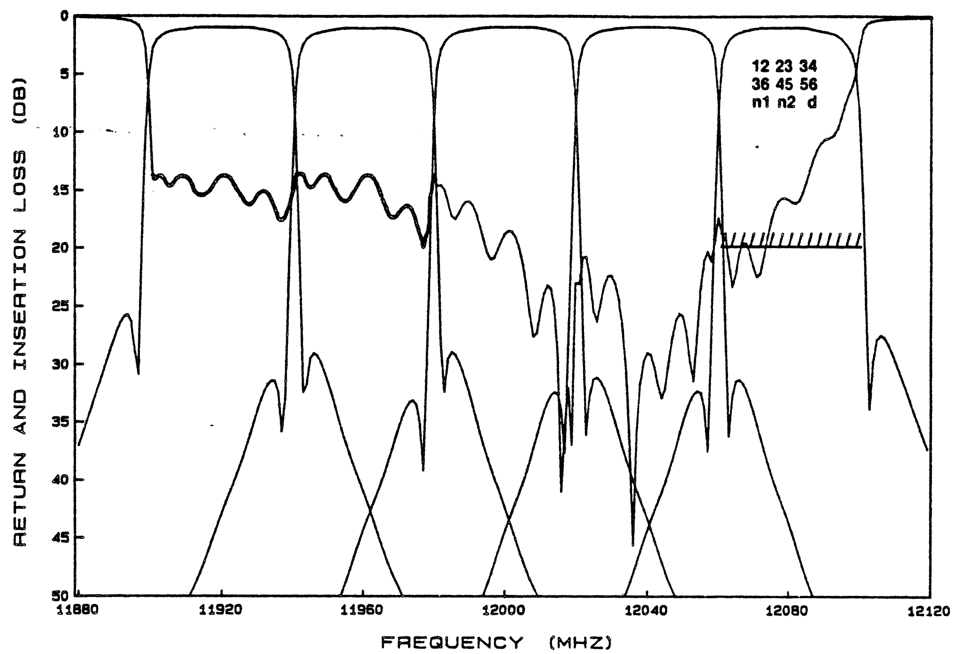


(a)



(b)

Fig. 6.3 (continued)



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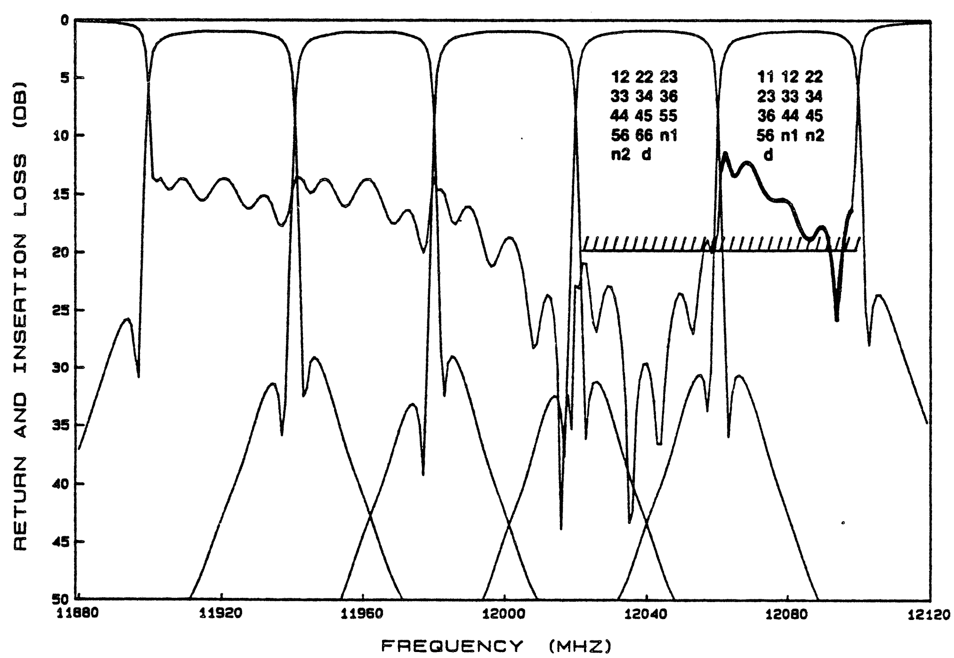
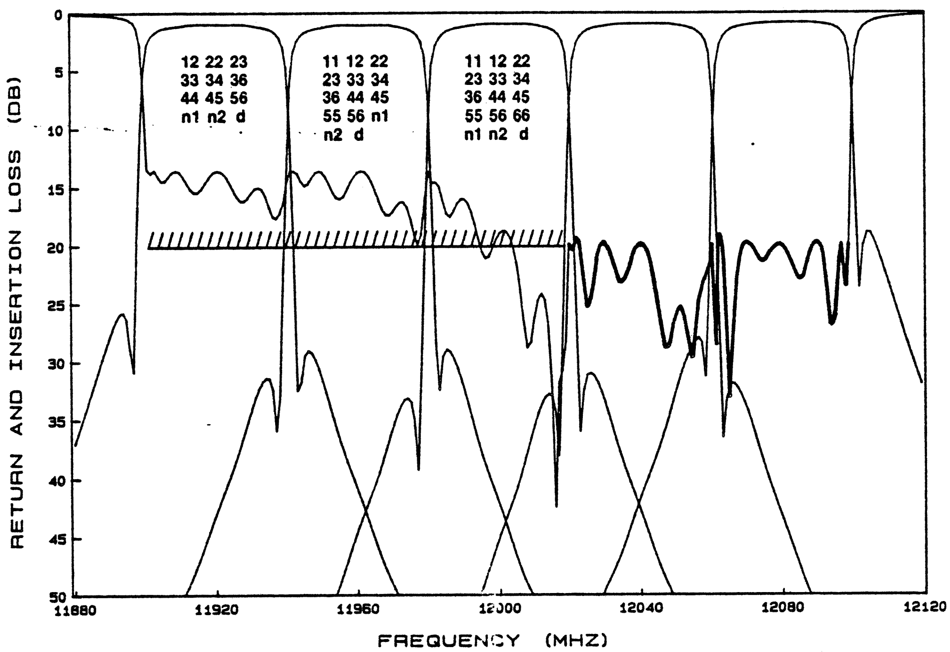
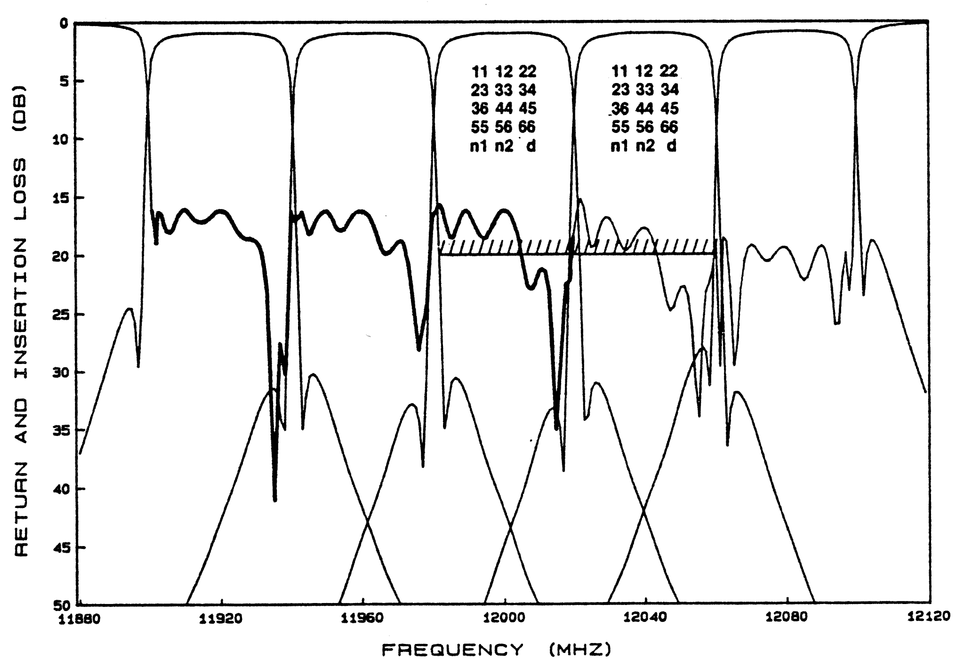


(d)

Fig. 6.3 (continued)

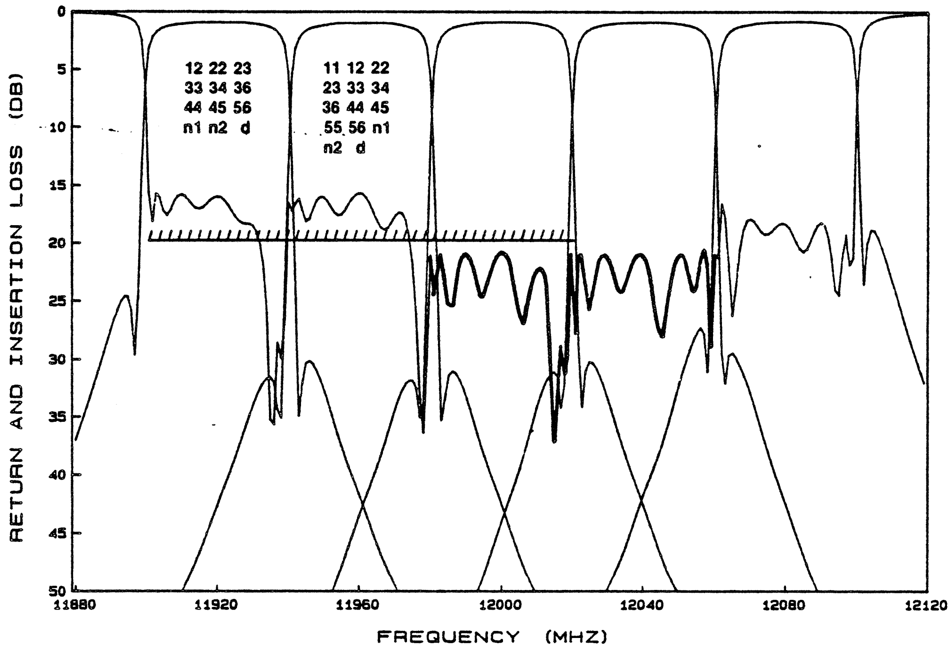


(e)

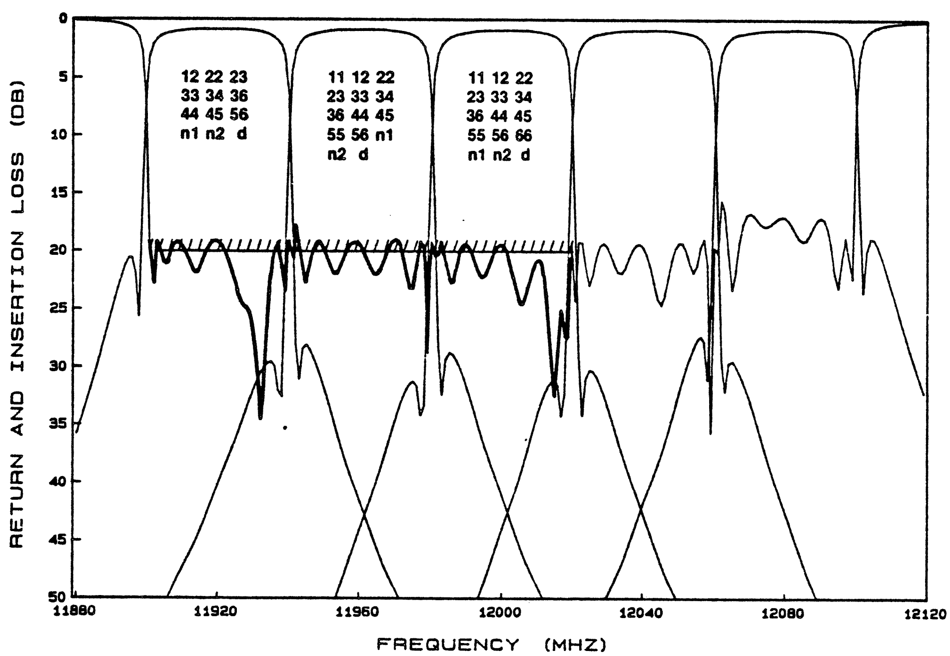


(f)

Fig. 6.3 (continued)

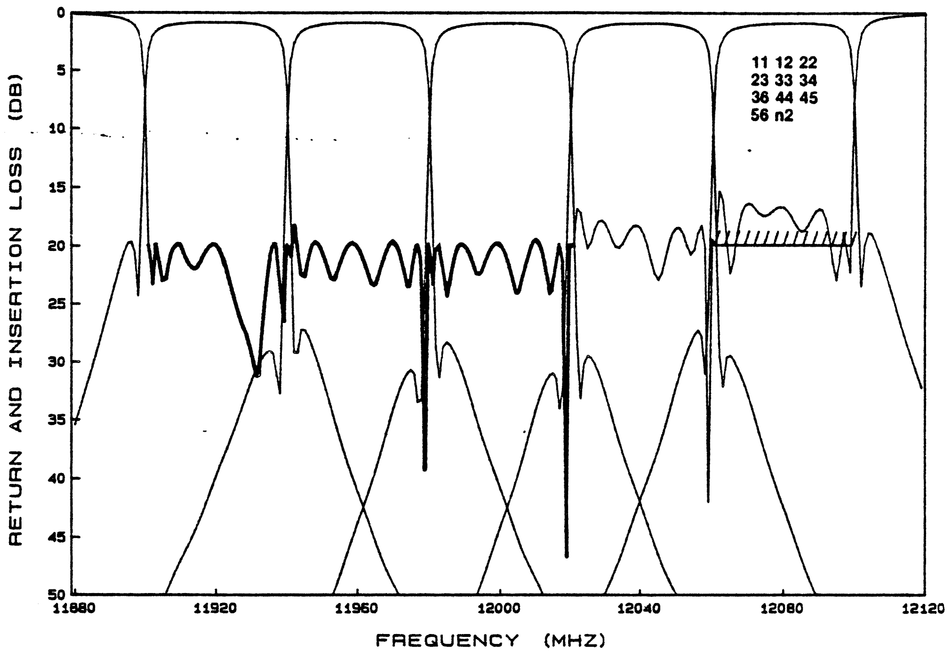


(g)

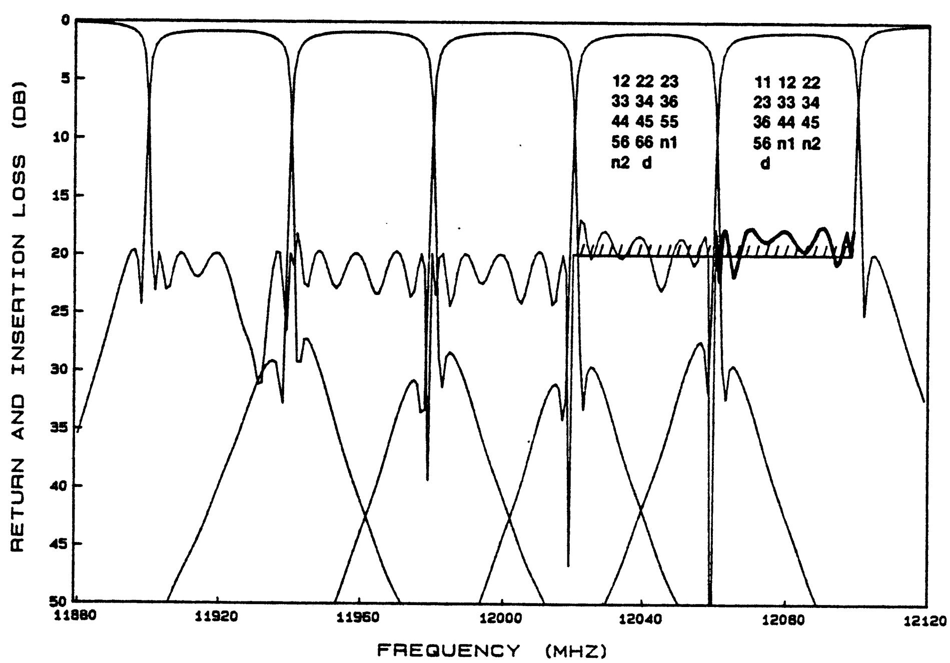


(h)

Fig. 6.3 (continued)

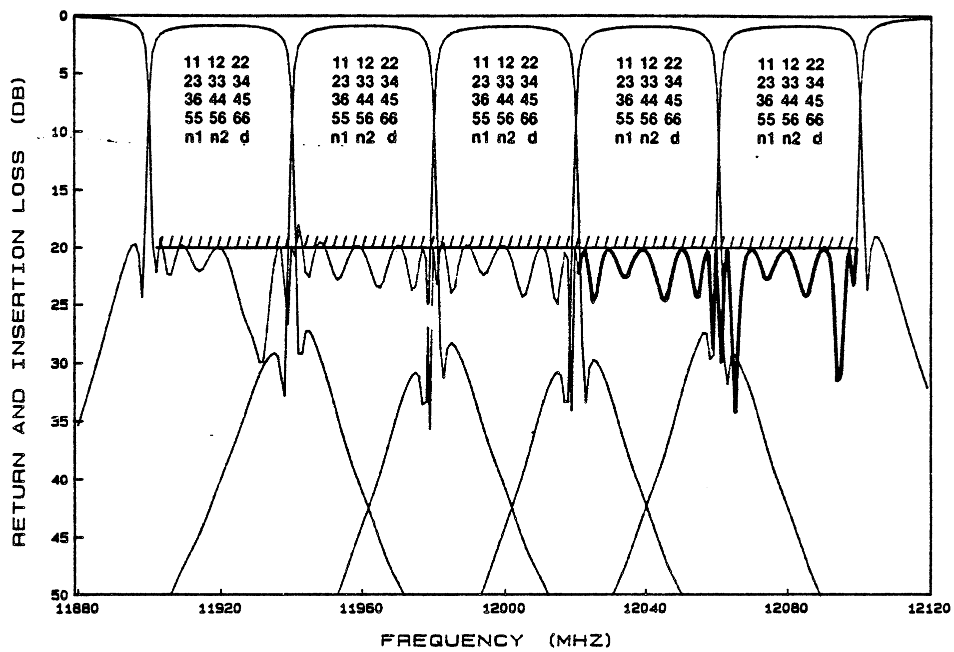


(i)

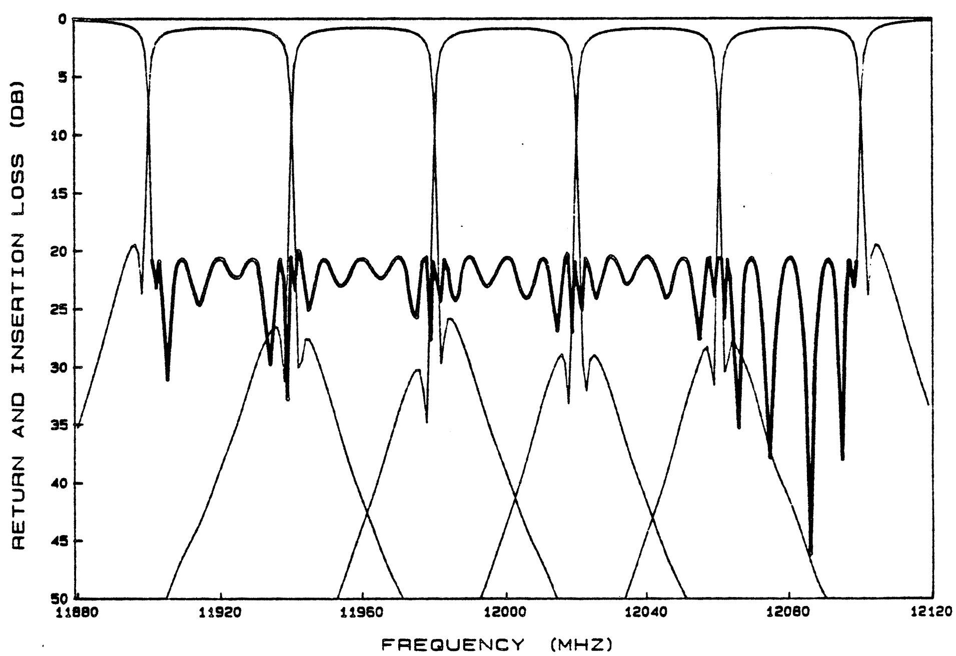


(j)

Fig. 6.3 (continued)



(k)



(ℓ)

Fig. 6.3 (continued)

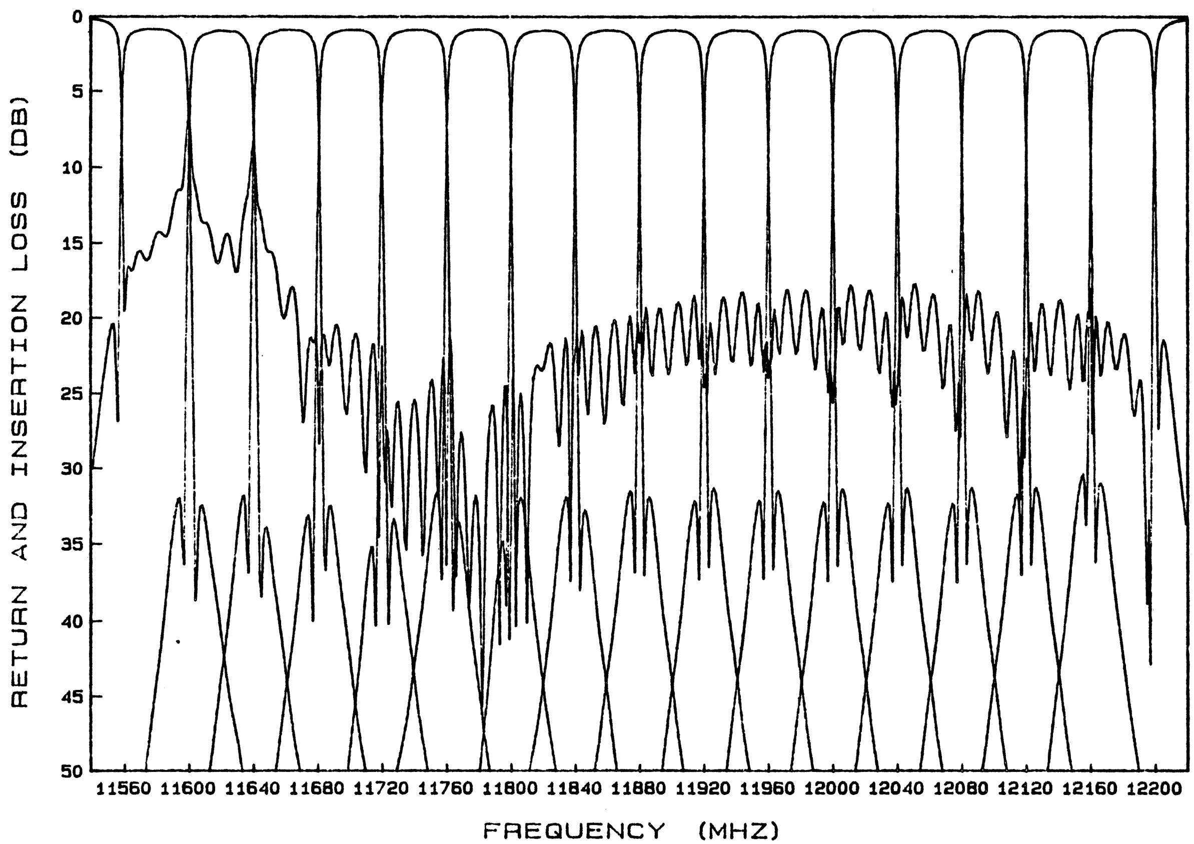


Fig. 6.4 Return and insertion loss responses of the 16-channel multiplexer before optimization.

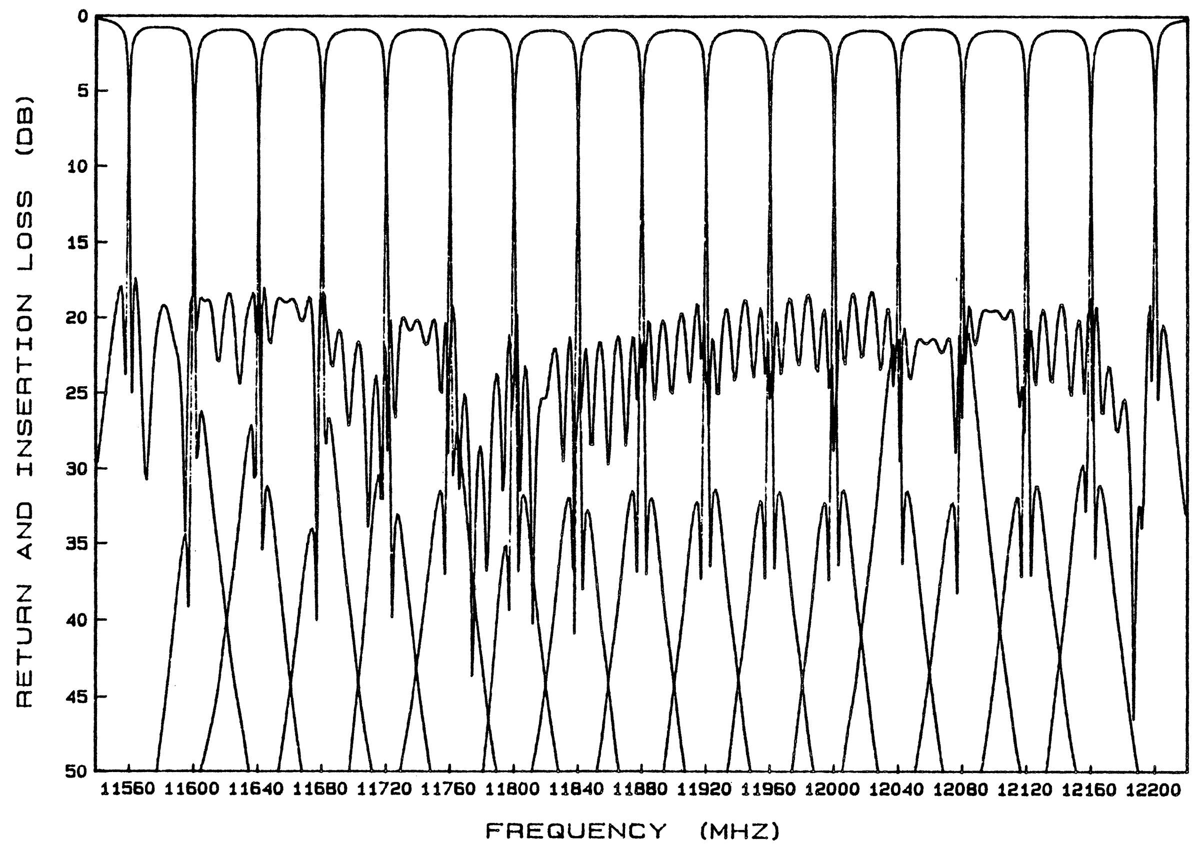


Fig. 6.5 Return and insertion loss responses of the 16-channel multiplexer after 10 suboptimizations. Each of the 10 suboptimizations involved responses associated with only one channel and no more than 15 variables.

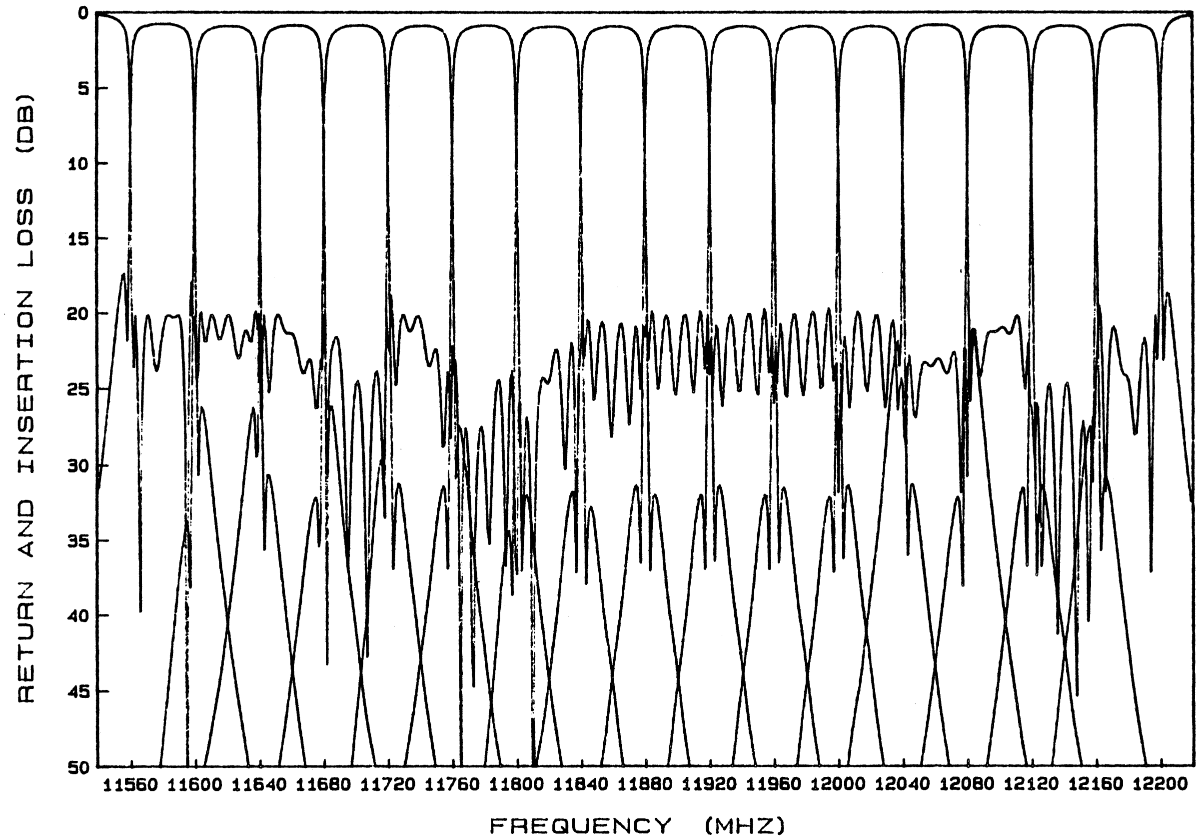


Fig. 6.6 Return and insertion loss responses of the 16-channel multiplexer at the overall solution. All design specifications are satisfied.

TABLE 6.6

COMPARISON OF 16-CHANNEL MULTIPLEXER OPTIMIZATION WITH AND WITHOUT DECOMPOSITION

Purpose of Optimization +	Reduction in Objective Function	Criteria for Comparison	With Decomp.	Without Decomp.
to provide a good starting point for further optimization	from 13.46	CPU time *	99	250
		working space needed†	2,197	483,036
	to 2.4	number of suboptimizations	10	–
to obtain a near optimum solution	from 13.46	CPU time *	651	553
		working space needed†	73,972	483,036
	to 0.32	number of suboptimizations	51	–
to obtain optimum solution	from 13.46	CPU time *	1045	1289
		working space needed†	483,036	483,036
	to −0.09	number of suboptimizations	11	–

\+ different sparse factors λ have been used to control the degree of decomposition for the three different purposes.

* seconds on the FPS-264 mainframe.

† of machine memory units (one unit per real number) required by the minimax optimization package (Bandler, Kellermann and Madsen 1985).

starting point for subsequent optimization, the decomposition approach offers considerable reductions in both CPU time and storage. The feasibility of obtaining a near optimum for large problems using computers with memory limitations is observed from the table. However, when close to the desired solution, the sizes of the subproblems may approach those of the overall problem. In this case, the performance of optimization does not differ significantly with or without decomposition, unless the original problem is almost completely decomposable.

6.7 CONCLUDING REMARKS

We have presented an automated decomposition approach for optimization of large microwave systems. The approach is generally applicable to the optimal design of large analog circuits. Compared with the existing decomposition methods, the novelty of our approach lies in its generality in terms of device independency and its automation. Advantages of the approach are 1) a very significant saving of CPU time and/or computer storage and 2) efficient decomposition by automation. By partitioning the overall problem into smaller ones, the approach promises to provide a basis for computer-assisted tuning. It contributes positively towards future general computer software for large-scale optimization of microwave systems.

7

CONCLUSIONS

This thesis addressed three important phases in large scale optimization of analog circuits, namely, simulation, sensitivity analysis and optimization. Novel approaches have been described for large change sensitivity analysis, branched cascaded network analysis, and automatic decomposition of circuit optimization. The use of microwave circuit examples demonstrated the practicality of our new approaches.

Our approaches offer immediate reductions in computer storage and CPU times required, enabling an engineer to optimize a large circuit with existing computers. The large change sensitivity analysis method is used to perform repeated circuit simulation. When used to solve adjoint systems, the method is also applicable to repeated sensitivity evaluation. The branched cascaded analysis method is a clever alternative to general simulation and sensitivity analysis methods such as the nodal equation approach and the adjoint network approach. The use of our methods in simulation and sensitivity evaluation significantly speeds up a circuit optimization procedure. The automatic decomposition technique directly partitions the large scale problem into small ones manageable by a mathematical programming software.

In addition to their computational efficiency, our novel approaches also provide better insight into the various effects between variables and responses, as discussed in Chapters 4, 5 and 6, respectively.

Logically, the thesis intends to fill the gap between the scale of circuit design problems and the practical limit of available computers. It is interesting to notice that this gap remains widely open even though the power of computers has increased dramatically over the past two decades. It is worthwhile to mention that the methods described in the thesis are directed towards large scale problems. In case of small circuit problems, the efficiency of our methods may be less than desired. The implementation of these methods is generally much more complicated than that of conventional methods.

The application of our methods is possible in many situations. Repeated simulation and gradient evaluation are often essential in optimization problems arising from modelling, design, yield maximization etc. A large number of repeated circuit simulations also exist in constructing a fault dictionary for circuit diagnosis (Bandler and Salama 1985a) and in constructing the database for statistical design using the parametric sampling method (Singhal and Pinel 1981). All these situations provide background for use of the large change sensitivity analysis method. The efficiency of the method increases if the number of perturbed variables is small whereas the overall problem is large.

The branched cascaded analysis method is directly applicable to any circuits structurally branched cascaded, or any circuits reducible to such a structure. We have illustrated how to extend the forward and reverse analysis method of Bandler, Rizk and Abdel-Malek (1978) to branched cascaded structures. The critical step for such an extension is the reduction of 3-port junctions into suitable 2-port representations. Using this idea, it is also natural to extend the method to cascaded networks having multiple levels of branches, formulating a possible direction for further research.

The application of automatic decomposition has been demonstrated through a FET modelling problem and the minimax design of a 16-channel multiplexer. General applications in minimax optimization problems originating from design and tuning can be envisaged. It is also worthwhile to embed the automatic decomposition into ℓ_1 and least squares optimization used in large scale modelling problems. The efficiency of our method increases as the circuit becomes highly decomposable.

The sparse matrix technique is another important tool for solving large scale problems. The essence of our branched cascaded analysis method can be considered as explicitly taking the topological sparsity of the circuit into consideration. The sparse matrix technique can be directly used to analyze branched cascaded networks by solving the nodal equations for the original and the adjoint networks. As a further research effort, it is worthwhile to compare the efficiency of the sparse matrix approach with the branched cascaded analysis approach. On the other hand, the automatic decomposition technique is to systematically exploit the sparse pattern of the Jacobian matrix obtained from differentiating error functions w.r.t. circuit variables. It is profitable to use appropriate ideas from the sparse matrix technique to improve the effect of automatic decomposition, e.g., the sequential arrangement of subproblems.

We have not considered another alternative of treating large scale problems, i.e., exploiting special computers such as vector processors (Calahan and Ames 1979; Yamamoto and Takahashi 1985; Rizzoli, Ferlito and Neri 1986) and parallel processors (Huang and Wing 1979; Jacob, Newton and Pederson 1986). For example, circuit analysis at different frequencies, simulation of different circuits in a multi-circuit approach (Bandler, Chen and Daijavad 1986b), circuit simulation at different parameter points in a statistical design, and different suboptimizations in a highly decomposable problem, are all suitable situations for vector and/or parallel processing. Research in this area is currently active in the circuits and systems community.

The fundamental mechanisms for executing a circuit optimization are a circuit simulator, a sensitivity analyzer and a mathematical optimizer. General efforts towards large scale circuit optimization can be considered as the embedding of common approaches such as decomposition, sparse matrix manipulation and vectorization into the three fundamental mechanisms. The automatic decomposition algorithm described in the thesis has been designed to operate externally to a mathematical optimizer. It is envisaged that future optimization of large analog circuits can be performed by optimizers having internal capability of decomposition, sparse matrix manipulation or vectorization.

A number of other problems are also worth further research and development.

(a) We have considered theoretical and computational aspects of large change sensitivity evaluations with relatively simple algebraic and electric circuit examples. The application in practical circuit design problems should be fully tested. For instance, in a quadratic approximation to a circuit response, one needs to solve a large set of linear equations. The repeated solution of the linear equations is necessary if the quadratic approximation is to be updated with replacements of sampling parameter points. Large change sensitivity formulas in this algebraic case can be used to minimize the effort of solving the updated linear equations.

(b) The automatic decomposition theory has been tested through automatic partitioning of variables and manual partitioning of functions. Further research is needed to examine automatic partitioning of functions and completely automated grouping of both functions and variables.

(c) When we tested the automatic decomposition algorithm on multiplexers, we have assumed that the decomposition dictionary takes a band matrix form. This assumption may not be true for general circuits. The arrangement for different subproblems needs to be further tested with decomposition dictionaries having forms other than the band matrix form.

(d) The multiplexer problem has been used to demonstrate the practical use of the branched cascaded analysis and the automatic decomposition techniques presented in the thesis. During our experiments with such a device, it was discovered that as the number of channels increases, strong interactions exist not only between adjacent channels, but also between certain non-adjacent channels, e.g., those about 7 channels apart. Abnormalities in the response curve, particularly, sharp kinks, are likely to occur for large multiplexers. Such a phenomenon has plagued our experiment with multiplexers having more than 16 channels. How to control the occurrence of such abnormalities is still unknown. Further research is needed.

APPENDIX A

SOME DEFINITIONS IN GRAPH THEORY

Let $G = (V, E)$ denote a graph where V and E are the vertex set and the edge set, respectively. Let V' and E' be subsets of V and E, respectively.

Definition 1: $G' = (V', E')$ is an edge-induced subgraph of G if every vertex in V' is the end vertex of some edge in E'.

Definition 2: A vertex v is a cut vertex of a connected graph G if and only if there exist two vertices u and w distinct from v such that v is on every u-w path.

Definition 3: A block of a separable graph G is a maximal nonseparable subgraph of G.

The books by Swamy and Thulasiraman (1981) and by Chen (1976) can be referred to for the relevant definitions.

APPENDIX B

BRIEF DESCRIPTION OF THE COMPUTER PROGRAM FOR SIMULATION, SENSITIVITY ANALYSIS AND OPTIMIZATION OF BRANCHED CASCADED NETWORKS

A computer program has been developed implementing the branched cascaded analysis method described in Chapter 5. The program can be used to perform simulation and sensitivity analysis for a general branched cascaded circuit. Limited optimization capability is also available. Circuit elements are either 2-port subnetworks or 3-port junctions. A catalogue of some frequently used elements are coded. The option of user-defined elements is also available.

There are three entries to the program. The first entry is used to perform simulation and sensitivity analysis at the element level. It is designed to analyze circuit subnetworks individually or to help checking the correctness of user-defined elements. The second entry is used to perform simulation and/or sensitivity analysis of a general branched cascaded network. The third entry is used to perform design optimization of branched cascaded circuits.

The program is written in Fortran-77. The block diagram of the program is shown in Fig. B.1. Here we briefly describe each of the blocks.

MAIN1 is a main program defined by the user. It is used to execute the program through Entry 1. In Fig. B.2., a list of the main program and an illustrative session of execution is provided. In the execution, the element tested is a simple 2-port containing only a seriesly connected resistor.

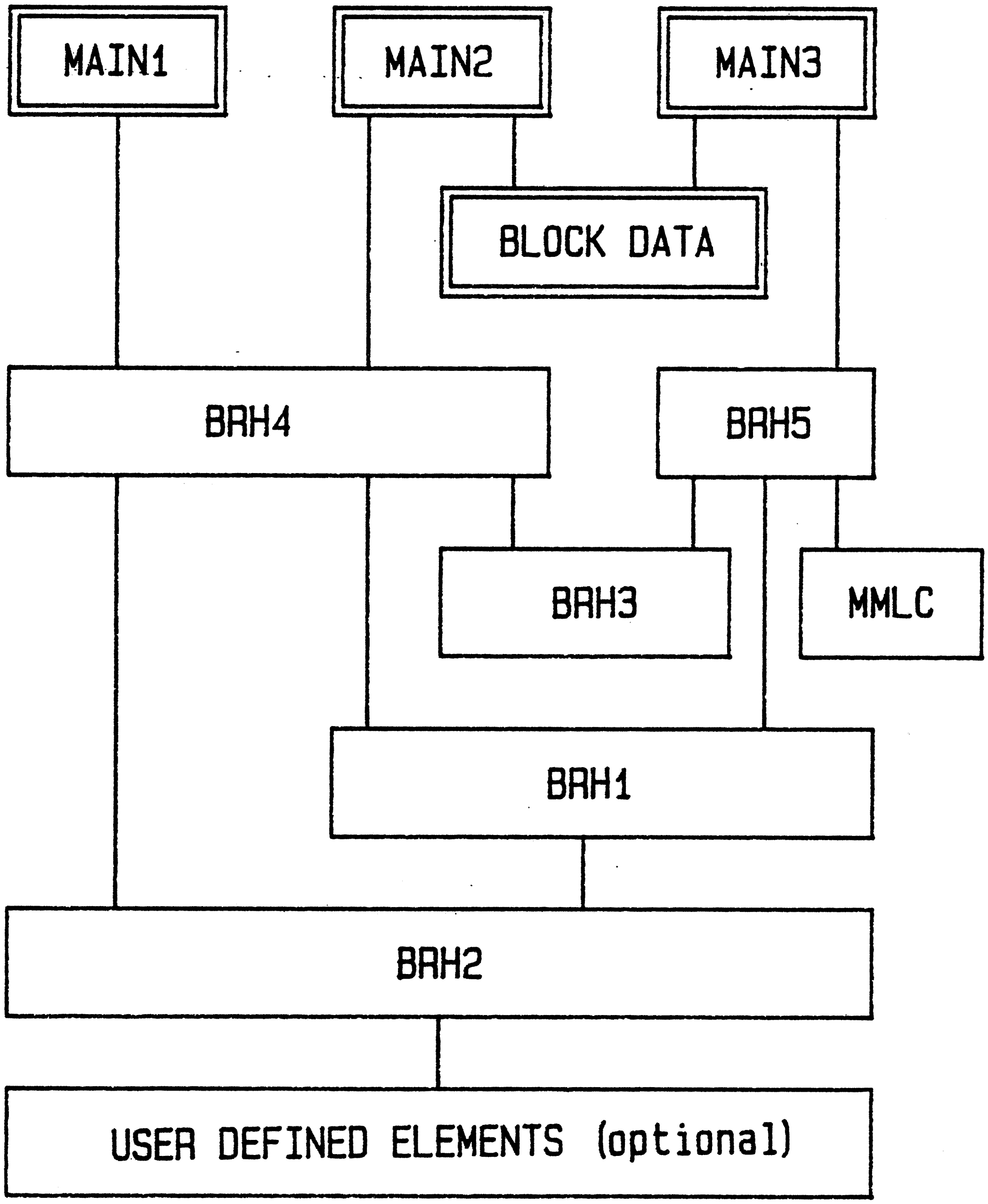


Fig. B.1 Block diagram of the computer program for simulation, sensitivity analysis and optimization of branched cascaded networks.

```
C------------------------------------------------------------------
C
C       SIMULATION AND SENSITIVITY ANALYSIS OF 2- AND 3-PORT
C       ELEMENTS
C
C------------------------------------------------------------------

        PROGRAM ENT1
        CALL TESTCH
        STOP
        END
```

```
ELEMENT TYPE = ?
Input : 1
NO. OF VARIABLES  ( NX ) = ?
Input : 1
PARAMETER VALUE ( 1 - 3 ) = ?
Input : 1, 1, 1
CODINGS FOR PARAMETERS ( 1 - 3 ) = ?
Input : 1, 1, 1
WHICH PARAMETERS ARE VARIABLES (            1 INDICES ) =?
Input : 1
TYPE   2  FOR 2-PORT ELEMENT.
       3  FOR 3-PORT ELEMENT.
Input : 2
FREQUENCY = ?
Input : 2
*********************************************************
 CHAIN MATRIX  ( A,B,C,D )  FOR ELEMENT TYPE            1

       A                         1.00000     0.00000

       C                         0.00000     0.00000

       B                         1.00000     0.00000

       D                         1.00000     0.00000
```

Fig. B.2 Main program and the computer output for the simulation and sensitivity analysis of a 2-port element.

```
-------- PERTURBATION CHECK OF SENSITIVITY --------

VAR   ABCD    ABS(A,,D)  DIFF(%)        SENSITIVITY
--- -------- --------- --------  ------------------------
 0       A    0.10E-05   0.00      0.00000      0.00000

 0       C    0.10E-05   0.00      0.00000      0.00000

 0       B    0.10E-05   0.00      0.00000      0.00000

 0       D    0.10E-05   0.00      0.00000      0.00000

 1       A    0.10E-05   0.00      0.00000      0.00000

 1       C    0.10E-05   0.00      0.00000      0.00000

 1       B    0.10E+01   0.00      1.00000      0.00000

 1       D    0.10E-05   0.00      0.00000      0.00000

 VARIABLE  0  REF.  TO  FREQUENCY.
-------------------------------------------------------
ANOTHER ELEMENT ?   Y/N  .....  1 / 2
Input : 2
FORTRAN STOP
```

Fig. B.2 (continued)

MAIN2 is a main program defined by the user. It is used to execute the program through Entry 2. In Fig. B.3, is a list of the main program for the 4-branch cascaded network of Fig. 5.5. This network has been described in Section 5.4. The output of an interactive session is also included in Fig. B.3.

MAIN3 is a main program defined by the user. It is used to execute the program through Entry 3. Fig. B.4 gives an example of the main program for optimizing a multicavity filter, a device considered as a branched cascaded network with 1 branch. Such a filter has been described in Table 5.11. The filter is 6th order with center frequency as 4 GHz and bandwidth as 40 MHz. The output of the optimization is also included in the figure.

BLOCK DATA is a Fortran data block defined by the user. In this block, the user is required to define the network structure, the elements involved, the source and the loads, the variables, and the optimization specifications. The following is a brief description of the arguments in this block.

(a) Structure of the network: N is the total number of branches. NIR is the total number of reference planes. NK(k) is the number of elements in branch k.

(b) Circuit elements: ITYP(j) is the index for the type of the jth element. RDAT(i,j) or RDATB(i,j) or RDATC(i,j) contains the value of the ith parameter in the jth element. An element may have up to 3, 17 and 10 parameters for RDAT, RDATB and RDATC, respectively. IDAT(i,j) or IDATB(i,j) or IDATC(i,j) is an index for the ith parameter in the jth element (e.g., whether inductive, resistive or capacitive) and corresponds to RDAT or RDATB or RDATC, respectively.

```
C----------------------------------------------------------------------
C
C       SIMULATION AND SENSITIVITY ANALYSIS OF BRANCHED
C       CASCADED NETWORKS.
C
C----------------------------------------------------------------------

      PROGRAM ENT2
      CALL BRANCH
      STOP
      END

      BLOCK DATA
      IMPLICIT REAL*8 (A-H,O-Z)
C**********************************************************************
C  >>>>>>>    ADJUST :
      PARAMETER (N=4,NIR=26,NX=8,NINT=3)
C**********************************************************************
C     DO NOT ALTER :
      PARAMETER (MN=6,MNIR=27,MNX=36,MNFR=91)
      PARAMETER (NDATB=17,NIRB=6,NDATC=10,NIRC=6,MNFLT=6)
      PARAMETER (MNINT=10)
      CHARACTER*10 INFL,OUTFL,IFILE1,IFILE2,OFILE1,OFILE2,FLNAM
      COMPLEX*16 VS,RS,RL
      COMMON /BLOK1/  ITYP(MNIR),NK(MN)
      COMMON /BLOK2/  RDAT(3,MNIR),IDAT(3,MNIR)
      COMMON /BLOK3/  IXX(2,MNX)
      COMMON /BLOK4/  VS,RS,RL(MN),ISO
      COMMON /BLOK5/  OMG1,OMG2,NOMG,MODOMG
      COMMON /BLOK7/  OMGA(2,MNINT),NPO(MNINT),ISPEC(MNINT),
     +                SPEC(2,MNINT)
      COMMON /BLK1/   VLIGHT
      COMMON /BLK2/   INFL,OUTFL,IFILE1,IFILE2,OFILE1,OFILE2
      COMMON /BLK3/   RDATB(NDATB,NIRB),IDATB(2,NDATB,NIRB),
     +                RDATC(NDATC,NIRC),IDATC(2,NDATC,NIRC)
      COMMON /BLK33/  OMG(MNFR)
      COMMON /BLK36/  SPE(2,MNFR),ISPE(MNFR)
      COMMON /BLK37/  NT,NIRT,NXT,NINTT
      DATA NT,NIRT,NXT,NINTT/N,NIR,NX,NINT/
C**********************************************************************
C  >>>>>>>   SET DATA :

      DATA (NK(I),I=1,N)/3,4,3,2/
      DATA VS,RS,(RL(I),I=1,N),ISO/100.,5*1.,2/
```

Fig. B.3 Main program block data and the computer output for the simulation and sensitivity computation of the 4-branch cascaded network.

```
      DATA (ITYP(I),I=1,NIR)/
     +      105,7,105,2,105,1,105,8,41,0,  42,3,105,0, 42,105,3,2,0,
     +      42,105,1,0,   42,105,0/
      DATA ((IXX(J,I),J=1,2),I=1,NX)/
     +      12,1, 12,2, 3,1, 18,1, 21,1, 7,1, 8,3, 25,2/
      DATA ((IDAT(I,J),I=1,3),J=1,NIR)/
     +      3*0, 2,2,3, 3*0, 3*5, 3*0, 3*1, 3*0, 3,3,2, 3*1, 3*0,
     +      3*1, 3,2,0, 3*0, 3*0,      3*1, 3*0, 1,1,1, 3*2, 3*0,
     +      3*1, 0,0,0, 3*3, 3*0,      3*1, 3*0, 0,0,0/
      DATA ((RDAT(I,J),I=1,3),J=1,NIR)/
     +    .1,1.,0., 1.,1.,2., .06,1.,0., 0.,0.,0., .1,1.,0., 0.,0.,0.,
     +   .05,1.,0., 1.,1.,2.,  1.,1.,1., 0.,0.,0., 1.,1.,1., 1.,2.,2.,
     +    .1,1.,0., 0.,0.,0.,  1.,1.,1.,.06,1.,0., .1,10.,0.,3.,3.,3.,
     +    0.,0.,0., 1.,1.,1., .05,1.,0., 2.,2.,2., 0.,0.,0., 1.,1.,1.,
     +    .1,1.,0., 0.,0.,0./
      DATA OMG1,OMG2,NOMG,MODOMG/6.2831853,6.2831853,1,2/
      DATA VLIGHT/.3/
      DATA INFL/'SYS$INPUT'/
      DATA OUTFL/'SYS$OUTPUT'/
      END
```

```
 OUTPUT FILE NAME  ?
 Input : SYS$OUTPUT

*********************************************************************

 EXACT SIMULATION AND SENSITIVITY ANALYSIS OF MULTIPLEXING NETWORKS


  NUMBER OF BRANCHES     ( N  )   .....    4

  NUMBER OF VARIABLES    ( NX )   .....    8

---------------------------------------------------------------------

 (1) >>   1. SIMULATION,  OR
          2. SIMULATION & SENSITIVITY .
 Input : 2
 (2) >>   SENSITIVITY W.R.T. :
                   1.  [X],  OR
                   2.  [X] & FREQUENCY,  OR
                   3.  FREQUENCY.
 Input : 2
 (4) >> CHANGE FREQUENCY ?  Y/N  ...1/2
            ( PREVIOUSLY, FREQ=   1.000000000000000      )
 Input : 2

---------------------------------------------------------------------
```

Fig. B.3 (continued)

```
************************
*                      *
*  SIMULATION RESULTS  *
*                      *
************************
```

VARIABLES :

1.000 2.000 0.060 3.000 0.050 0.050 2.000 1.000

FREQUENCY :

1.00000

BRANCH VOLTAGES :

0.03624	-0.07595	0.05983	-15.00361
-0.07487	-0.06875	-0.04039	1.16405

THEVENIN VOLTAGES :

0.03008	0.03529	0.03193	-15.65346
-0.07785	-0.30176	-0.08172	-2.31876

THEVENIN IMPEDANCES :

0.00003	0.72129	0.00004	0.02515
-0.08225	2.41490	-0.69080	0.23408

INSERTION LOSS :

55.57892	53.76940	56.81050	10.42942

RETURN LOSS :

0.00055	1.72670	0.00052	0.41430

COMMON PORT RETURN LOSS :

0.41243

Fig. B.3 (continued)

```
(6) >> PRINT  DATA :
              0                 NONE, OR
              1, 2, 3, 4        BRIEF ,,,, DETAIL.
Input : 0
 CONTINUE FOR SENSITIVITY ?     Y/N   ....  1 / 2
Input : 1
```

```
************************************
*                                  *
*  SENSITIVITIES W.R.T. VARIABLES  *
*                                  *
************************************
```

SENSITIVITIES OF BRANCH VOLTAGES :

-0.09888	0.01602	-0.12152	-0.06148
0.19690	0.01904	0.07920	0.43382
-0.02178	0.00008	-0.00083	-0.00081
0.03689	0.00013	0.00037	0.00263
0.41840	0.42340	-3.17461	-1.54074
-1.02730	0.49683	2.09775	11.39034
-0.00015	0.02421	-0.00152	-0.00123
0.00018	0.02442	0.00078	0.00500
0.00000	0.00000	-0.84583	0.00000
0.00000	0.00000	-1.25308	0.00000
0.42131	-1.05647	0.75952	-1.32161
-1.01718	-0.85004	-0.57964	10.30781
0.00216	0.00347	0.00061	0.16241
0.00231	-0.00175	0.00267	0.04932
0.03997	-0.14168	0.08734	-12.42431
-0.13157	-0.09279	-0.08130	0.17372

Fig. B.3 (continued)

SENSITIVITIES OF INSERTION LOSS :

23.00568	2.09034	17.45152	-0.05475
4.45823	0.01270	0.10816	-0.00059
-115.59096	54.88169	457.83905	-1.39517
0.02412	2.91144	0.20388	-0.00093
0.00000	0.00000	0.00000	0.00000
-114.77168	-114.77168	-114.77168	-1.22074
0.11859	0.11859	0.11859	0.09126
-14.18466	-14.18466	-14.18466	-7.15740

SENSITIVITIES OF RETURN LOSS :

-0.00292	-0.00050	-0.00183	0.00055
-0.00057	0.00000	-0.00006	-0.00050
0.01465	-0.01278	-0.03917	0.09851
0.00000	-0.00071	-0.00008	-0.00059
0.00000	0.00000	0.00000	0.00000
0.01133	0.02123	0.00598	0.17232
-0.00001	-0.00002	-0.00001	-0.00913
0.00084	0.00155	0.00063	0.71641

SENSITIVITIES OF COMMON PORT RETURN LOSS :

0.00533
0.00004
0.13797
0.00008

Fig. B.3 (continued)

```
        0.00000

        0.12286

       -0.00909

        0.71310
```

```
(7) >> CHECK SENSITIVITY ?    Y/N  .....   1 / 2
Input : 2
CONTINUE SENSITIVITY W.R.T. OMEG ? Y/N .. 1/2
Input : 1
```

```
***********************************
*                                 *
*  SENSITIVITIES W.R.T. FREQUENCY *
*                                 *
***********************************


SENSITIVITIES OF BRANCH VOLTAGES :

       -0.17778     0.03944    -0.44100     2.39068
        0.33120     0.08791     0.26906     7.36235


SENSITIVITIES OF RETURN LOSS :

       -0.00484    -0.00153    -0.00590    -0.11597


SENSITIVITY OF COMMON PORT RETURN LOSS :

       -0.10460


GAIN SLOPE :

       39.21750     7.48130    62.09628     1.04703


GROUP DELAY :

        0.18892     0.37785     0.32862     0.50006
```

Fig. B.3 (continued)

```
(8) >> CHECK SENSITIVITY ?    Y/N  .....   1 / 2
Input : 2
(9) >>    1. SIMULATION AND SENSITIVITY CONTINUED, OR
          2. EXIT
Input : 2
FORTRAN STOP
```

Fig. B.3 (continued)

```
C-----------------------------------------------------------------
C
C      OPTIMIZATION OF MULTIPLEXING NETWORKS
C
C-----------------------------------------------------------------

      PROGRAM ENT3
      IMPLICIT REAL*8 (A-H,O-Z)
*******************************************************************
*    >>>>>>> ADJUST :
      PARAMETER (NX=6,L=6,NFR=60)
*     NX, L, and NFR are the number of variables, the number of
*     linear constraints and the number of sample frequencies,
*     respectively, for the circuit optimization.
*******************************************************************
C     DO NOT ALTER :
      PARAMETER (IW=2*NFR*NX+5*NX*NX+5*NFR+10*NX+4*L)
      DIMENSION C(L,NX),B(L),W(IW),X(NX)
*******************************************************************
*   >>>>>>>   ADJUST :
*
      DATA C,B/42*0./
      DATA LEQ,IPR,ICH,KEQS/0,10,6,3/
      DATA EPS,DX/1.E-6,1.E-2/
      DATA MAXF/40/
      DATA MODE,MODX/2,0/

*     C,B,LEQ,IPR,ICH,KEQS,EPS,DX and MAXF are defined consistently
*     with the MMLC package.
*     MODE = 1 or 2 for  L1 or minimax optimization.
*     If MODX = 0, the initial values of variables are defined in
*     the block data, otherwise they are defined in a file.

      DO 20 I=1,L
   20 C(I,I)= 1.
      C(6,6)=-1.
C
      CALL MULOP(NX,NFR,L,LEQ,B,C,L,X,DX,EPS,MAXF,KEQS,W,IW,ICH,IPR,
     +           MODE,MODX)
      STOP
      END
```

Fig. B.4 Main program, block data and the computer output for optimization of a 6th order multicavity filter.

```
      BLOCK DATA
      IMPLICIT REAL*8 (A-H,O-Z)
C*************************************************************
C  >>>>>>>    ADJUST :
      PARAMETER (N=1,NIR=8,NX=6,NINT=3,NFR=60)
C*************************************************************
C     DO NOT ALTER :
      PARAMETER (MN=6,MNIR=27,MNX=36,MNFR=91)
      PARAMETER (NDATB=17,NIRB=6,NDATC=10,NIRC=6,MNFLT=6)
      PARAMETER (MNINT=10)
      CHARACTER*10 INFL,OUTFL,IFILE1,IFILE2,OFILE1,OFILE2,FLNAM
      COMPLEX*16 VS,RS,RL
      COMMON /BLOK1/  ITYP(MNIR),NK(MN)
      COMMON /BLOK2/  RDAT(3,MNIR),IDAT(3,MNIR)
      COMMON /BLOK3/  IXX(2,MNX)
      COMMON /BLOK4/  VS,RS,RL(MN),ISO
      COMMON /BLOK5/  OMG1,OMG2,NOMG,MODOMG
      COMMON /BLOK7/  OMGA(2,MNINT),NPO(MNINT),ISPEC(MNINT),
     +                SPEC(2,MNINT)
      COMMON /BLK1/   VLIGHT
      COMMON /BLK2/   INFL,OUTFL,IFILE1,IFILE2,OFILE1,OFILE2
      COMMON /BLK3/   RDATB(NDATB,NIRB),IDATB(2,NDATB,NIRB),
     +                RDATC(NDATC,NIRC),IDATC(2,NDATC,NIRC)
      COMMON /BLK33/  OMG(MNFR)
      COMMON /BLK36/  SPE(2,MNFR),ISPE(MNFR)
      COMMON /BLK37/  NT,NIRT,NXT,NINTT
      DATA NT,NIRT,NXT,NINTT/N,NIR,NX,NINT/
C*************************************************************
C  >>>>>>>   SET DATA :
C
      DATA (NK(I),I=1,N)/3/
      DATA VS,RS,(RL(I),I=1,N),ISO/1.,1.,1.,2/
      DATA (ITYP(I),I=1,NIR)/
     +    5,2,41,0,  42,402,301,0/
      DATA ((IXX(I,J),J=1,NX),I=1,2)/
     +    6*6, 4,6,7,8,9,10/
      DATA ((IDAT(I,J),I=1,3),J=1,NIR)/
     +     15*0, 6,7,0, 6*0/
      DATA ((RDAT(I,J),I=1,3),J=1,NIR)/
     +     3*0, 3*0, 3*1., 3*0, 3*1., 4000.,40.,0., 6*0./
      DATA (RDATB(I,1),I=1,7)/
     +     .979796, .979796, .8101, .4894, .8450, .1197, -.4010/
      DATA (RDATB(I,2),I=1,8)/0.,0.,  0.,-1.,  0.,-1.,  0.,0./
      DATA ((IDATB(J,I,2),J=1,2),I=1,8)/16*0/
      DATA ( IDATB(1,I,1),IDATB(2,I,1),I=1,7)/
     +     4*0, 1,2,  2,3,  3,4,  1,6,  2,5/
      DATA VLIGHT/11802.85/
      DATA OMG1,OMG2,NOMG,MODOMG/3940., 4060., 121, 2/
```

Fig. B.4 (continued)

```
      DATA (OMGA(1,I),OMGA(2,I),I=1,NINT)/
     +      3950.,3970., 3970.,3976., 3980.,4001./
      DATA (NPO(I),I=1,NINT)/21,17,22/
      DATA (ISPEC(I),I=1,NINT)/0,0,0/
      DATA (SPEC(1,I),SPEC(2,I),I=1,NINT)/
     +     -100., .9993,   -100.,.9993,  100.,.1/
      DATA INFL,OUTFL,IFILE1,OFILE1/'SYS$INPUT','SYS$OUTPUT',
     +     'FLXXX','FLXXC'/
      DATA OFILE2/'FLRSPC'/
      END
```

```
*******************************************************************

 COMPUTER AIDED DESIGN OF MULTIPLEXING NETWORKS


***********************
*                     *
* NETWORK DESCRIPTION *
*                     *
***********************

  NUMBER OF SECTIONS (N) . . . . . . . . . . . . . . . . . . .    1

  NUMBER OF BRANCH ELEMENTS IN SECTION  1  . . . . . . . . . .    3


***********************
*                     *
* DESIGN OPTIMIZATION *
*                     *
***********************

METHOD          MINIMAX OPTIMIZATION
======
```

Fig. B.4 (continued)

SPECIFICATION
=============

FREQUENCY					OUTPUT	RESP.	WEIGHT	SPEC.
INTERV.	# OF PT.	LOWER	UPPER		PORT	TYPE		VALUE
1	21	3950.	3970.	REF. COEF	COM. PT	LOWER	100.0	0.999
2	17	3970.	3976.	REF. COEF	COM. PT	LOWER	100.0	0.999
3	22	3980.	4001.	REF. COEF	COM. PT	UPPER	100.0	0.100

OPTIMIZATION CONTROL DATA
=========================

NUMBER OF VARIABLES (NX) 6

NUMBER OF FUNCTIONS (M) 60

TOTAL NUMBER OF LINEAR CONSTRAINTS (L) 6

NUMBER OF EQUALITY CONSTRAINTS (LEQ) 0

STEP LENGTH (DX) 1.000E-02

ACCURACY (EPS) 1.000E-06

MAX NUMBER OF FUNCTION EVALUATIONS (MAXF) 40

NUMBER OF SUCCESSIVE ITERATIONS (KEQS) 3

WORKING SPACE (IW) 1284

PRINTOUT CONTROL (IPR) 10

Fig. B.4 (continued)

STARTING POINT --- VARIABLES AND FUNCTIONS

VARIABLE	IDENTIFICATION		VALUE
	PARAMETER	OF ELEMENT	
1	4	6	9.797960000E-01
2	6	6	8.101000000E-01
3	7	6	4.894000000E-01
4	8	6	8.450000000E-01
5	9	6	1.197000000E-01
6	10	6	-4.010000000E-01

FUNCTION	IDENTIFICATION					VALUE
	FREQUENCY	RESPONSE	OUTPUT PORT	SPEC. TYPE	WEIGHT	
1	3950.00	REF. COEF	COM. PT	LOWER	100.0	-3.8468E-02
2	3951.00	REF. COEF	COM. PT	LOWER	100.0	-3.7153E-02
3	3952.00	REF. COEF	COM. PT	LOWER	100.0	-3.5855E-02
4	3953.00	REF. COEF	COM. PT	LOWER	100.0	-3.4594E-02
5	3954.00	REF. COEF	COM. PT	LOWER	100.0	-3.3399E-02
6	3955.00	REF. COEF	COM. PT	LOWER	100.0	-3.2305E-02
7	3956.00	REF. COEF	COM. PT	LOWER	100.0	-3.1354E-02
8	3957.00	REF. COEF	COM. PT	LOWER	100.0	-3.0598E-02
9	3958.00	REF. COEF	COM. PT	LOWER	100.0	-3.0101E-02
10	3959.00	REF. COEF	COM. PT	LOWER	100.0	-2.9939E-02
11	3960.00	REF. COEF	COM. PT	LOWER	100.0	-3.0198E-02
12	3961.00	REF. COEF	COM. PT	LOWER	100.0	-3.0981E-02
13	3962.00	REF. COEF	COM. PT	LOWER	100.0	-3.2396E-02
14	3963.00	REF. COEF	COM. PT	LOWER	100.0	-3.4558E-02
15	3964.00	REF. COEF	COM. PT	LOWER	100.0	-3.7572E-02
16	3965.00	REF. COEF	COM. PT	LOWER	100.0	-4.1514E-02
17	3966.00	REF. COEF	COM. PT	LOWER	100.0	-4.6391E-02
18	3967.00	REF. COEF	COM. PT	LOWER	100.0	-5.2082E-02
19	3968.00	REF. COEF	COM. PT	LOWER	100.0	-5.8244E-02
20	3969.00	REF. COEF	COM. PT	LOWER	100.0	-6.4179E-02
21	3970.00	REF. COEF	COM. PT	LOWER	100.0	-6.8671E-02
22	3970.00	REF. COEF	COM. PT	LOWER	100.0	-6.8671E-02
23	3970.38	REF. COEF	COM. PT	LOWER	100.0	-6.9625E-02
24	3970.75	REF. COEF	COM. PT	LOWER	100.0	-6.9998E-02
25	3971.13	REF. COEF	COM. PT	LOWER	100.0	-6.9661E-02

Fig. B.4 (continued)

26	3971.50	REF. COEF	COM. PT	LOWER	100.0	-6.8480E-02
27	3971.87	REF. COEF	COM. PT	LOWER	100.0	-6.6333E-02
28	3972.25	REF. COEF	COM. PT	LOWER	100.0	-6.3124E-02
29	3972.63	REF. COEF	COM. PT	LOWER	100.0	-5.8812E-02
30	3973.00	REF. COEF	COM. PT	LOWER	100.0	-5.3455E-02
31	3973.38	REF. COEF	COM. PT	LOWER	100.0	-4.7263E-02
32	3973.75	REF. COEF	COM. PT	LOWER	100.0	-4.0677E-02
33	3974.13	REF. COEF	COM. PT	LOWER	100.0	-3.4463E-02
34	3974.50	REF. COEF	COM. PT	LOWER	100.0	-2.9802E-02
35	3974.88	REF. COEF	COM. PT	LOWER	100.0	-2.8331E-02
36	3975.25	REF. COEF	COM. PT	LOWER	100.0	-3.1982E-02
37	3975.63	REF. COEF	COM. PT	LOWER	100.0	-4.2260E-02
38	3976.00	REF. COEF	COM. PT	LOWER	100.0	-5.8074E-02
39	3980.00	REF. COEF	COM. PT	UPPER	100.0	2.5537E+00
40	3981.00	REF. COEF	COM. PT	UPPER	100.0	-3.2249E+00
41	3982.00	REF. COEF	COM. PT	UPPER	100.0	-2.2767E+00
42	3983.00	REF. COEF	COM. PT	UPPER	100.0	-6.7620E+00
43	3984.00	REF. COEF	COM. PT	UPPER	100.0	-8.0003E+00
44	3985.00	REF. COEF	COM. PT	UPPER	100.0	-3.7464E+00
45	3986.00	REF. COEF	COM. PT	UPPER	100.0	-9.8285E-01
46	3987.00	REF. COEF	COM. PT	UPPER	100.0	3.0138E-01
47	3988.00	REF. COEF	COM. PT	UPPER	100.0	3.0779E-01
48	3989.00	REF. COEF	COM. PT	UPPER	100.0	-7.0967E-01
49	3990.00	REF. COEF	COM. PT	UPPER	100.0	-2.4955E+00
50	3991.00	REF. COEF	COM. PT	UPPER	100.0	-4.8093E+00
51	3992.00	REF. COEF	COM. PT	UPPER	100.0	-7.4311E+00
52	3993.00	REF. COEF	COM. PT	UPPER	100.0	-9.8356E+00
53	3994.00	REF. COEF	COM. PT	UPPER	100.0	-7.1611E+00
54	3995.00	REF. COEF	COM. PT	UPPER	100.0	-4.6872E+00
55	3996.00	REF. COEF	COM. PT	UPPER	100.0	-2.5274E+00
56	3997.00	REF. COEF	COM. PT	UPPER	100.0	-7.6811E-01
57	3998.00	REF. COEF	COM. PT	UPPER	100.0	5.2872E-01
58	3999.00	REF. COEF	COM. PT	UPPER	100.0	1.3220E+00
59	4000.00	REF. COEF	COM. PT	UPPER	100.0	1.5889E+00
60	4001.00	REF. COEF	COM. PT	UPPER	100.0	1.3222E+00

VALUE OF OBJECTIVE FUNCTION . . . . 2.55371E+00

Fig. B.4 (continued)

AT SOLUTION --- VARIABLES AND FUNCTIONS

VARIABLE	IDENTIFICATION PARAMETER	OF ELEMENT	VALUE
1	4	6	9.898047042E-01
2	6	6	8.159142169E-01
3	7	6	5.105933013E-01
4	8	6	8.235514350E-01
5	9	6	9.236507325E-02
6	10	6	-3.557522286E-01

FUNCTION	IDENTIFICATION					VALUE
	FREQUENCY	RESPONSE	OUTPUT PORT	SPEC. TYPE	WEIGHT	
1	3950.00	REF. COEF	COM. PT	LOWER	100.0	-5.3588E-02
2	3951.00	REF. COEF	COM. PT	LOWER	100.0	-5.3085E-02
3	3952.00	REF. COEF	COM. PT	LOWER	100.0	-5.2624E-02
4	3953.00	REF. COEF	COM. PT	LOWER	100.0	-5.2219E-02
5	3954.00	REF. COEF	COM. PT	LOWER	100.0	-5.1888E-02
6	3955.00	REF. COEF	COM. PT	LOWER	100.0	-5.1655E-02
7	3956.00	REF. COEF	COM. PT	LOWER	100.0	-5.1544E-02
8	3957.00	REF. COEF	COM. PT	LOWER	100.0	-5.1586E-02
9	3958.00	REF. COEF	COM. PT	LOWER	100.0	-5.1815E-02
10	3959.00	REF. COEF	COM. PT	LOWER	100.0	-5.2268E-02
11	3960.00	REF. COEF	COM. PT	LOWER	100.0	-5.2985E-02
12	3961.00	REF. COEF	COM. PT	LOWER	100.0	-5.4004E-02
13	3962.00	REF. COEF	COM. PT	LOWER	100.0	-5.5360E-02
14	3963.00	REF. COEF	COM. PT	LOWER	100.0	-5.7074E-02
15	3964.00	REF. COEF	COM. PT	LOWER	100.0	-5.9143E-02
16	3965.00	REF. COEF	COM. PT	LOWER	100.0	-6.1521E-02
17	3966.00	REF. COEF	COM. PT	LOWER	100.0	-6.4090E-02
18	3967.00	REF. COEF	COM. PT	LOWER	100.0	-6.6629E-02
19	3968.00	REF. COEF	COM. PT	LOWER	100.0	-6.8766E-02
20	3969.00	REF. COEF	COM. PT	LOWER	100.0	-6.9940E-02
21	3970.00	REF. COEF	COM. PT	LOWER	100.0	-6.9413E-02
22	3970.00	REF. COEF	COM. PT	LOWER	100.0	-6.9413E-02
23	3970.38	REF. COEF	COM. PT	LOWER	100.0	-6.8614E-02
24	3970.75	REF. COEF	COM. PT	LOWER	100.0	-6.7432E-02
25	3971.13	REF. COEF	COM. PT	LOWER	100.0	-6.5851E-02

Fig. B.4 (continued)

26	3971.50	REF.	COEF	COM.	PT	LOWER	100.0	-6.3880E-02
27	3971.87	REF.	COEF	COM.	PT	LOWER	100.0	-6.1564E-02
28	3972.25	REF.	COEF	COM.	PT	LOWER	100.0	-5.9005E-02
29	3972.63	REF.	COEF	COM.	PT	LOWER	100.0	-5.6384E-02
30	3973.00	REF.	COEF	COM.	PT	LOWER	100.0	-5.3979E-02
31	3973.38	REF.	COEF	COM.	PT	LOWER	100.0	-5.2192E-02
32	3973.75	REF.	COEF	COM.	PT	LOWER	100.0	-5.1544E-02
33	3974.13	REF.	COEF	COM.	PT	LOWER	100.0	-5.2638E-02
34	3974.50	REF.	COEF	COM.	PT	LOWER	100.0	-5.5985E-02
35	3974.88	REF.	COEF	COM.	PT	LOWER	100.0	-6.1575E-02
36	3975.25	REF.	COEF	COM.	PT	LOWER	100.0	-6.7844E-02
37	3975.63	REF.	COEF	COM.	PT	LOWER	100.0	-6.9343E-02
38	3976.00	REF.	COEF	COM.	PT	LOWER	100.0	-5.1544E-02
39	3980.00	REF.	COEF	COM.	PT	UPPER	100.0	-5.1544E-02
40	3981.00	REF.	COEF	COM.	PT	UPPER	100.0	-1.7255E+00
41	3982.00	REF.	COEF	COM.	PT	UPPER	100.0	-5.1544E-02
42	3983.00	REF.	COEF	COM.	PT	UPPER	100.0	-4.0692E+00
43	3984.00	REF.	COEF	COM.	PT	UPPER	100.0	-9.3586E+00
44	3985.00	REF.	COEF	COM.	PT	UPPER	100.0	-5.9621E+00
45	3986.00	REF.	COEF	COM.	PT	UPPER	100.0	-2.5942E+00
46	3987.00	REF.	COEF	COM.	PT	UPPER	100.0	-6.6540E-01
47	3988.00	REF.	COEF	COM.	PT	UPPER	100.0	-5.1544E-02
48	3989.00	REF.	COEF	COM.	PT	UPPER	100.0	-5.3600E-01
49	3990.00	REF.	COEF	COM.	PT	UPPER	100.0	-1.8782E+00
50	3991.00	REF.	COEF	COM.	PT	UPPER	100.0	-3.8407E+00
51	3992.00	REF.	COEF	COM.	PT	UPPER	100.0	-6.1989E+00
52	3993.00	REF.	COEF	COM.	PT	UPPER	100.0	-8.7470E+00
53	3994.00	REF.	COEF	COM.	PT	UPPER	100.0	-8.6979E+00
54	3995.00	REF.	COEF	COM.	PT	UPPER	100.0	-6.2926E+00
55	3996.00	REF.	COEF	COM.	PT	UPPER	100.0	-4.1653E+00
56	3997.00	REF.	COEF	COM.	PT	UPPER	100.0	-2.4159E+00
57	3998.00	REF.	COEF	COM.	PT	UPPER	100.0	-1.1178E+00
58	3999.00	REF.	COEF	COM.	PT	UPPER	100.0	-3.2037E-01
59	4000.00	REF.	COEF	COM.	PT	UPPER	100.0	-5.1544E-02
60	4001.00	REF.	COEF	COM.	PT	UPPER	100.0	-3.2024E-01

VALUE OF OBJECTIVE FUNCTION . . . . -5.15445E-02

OPTIMIZATION CONCLUDING DATA
============================

TYPE OF SOLUTION (IFALL) 0

NUMBER OF FUNCTION EVALUATIONS 16

Fig. B.4 (continued)

```
  NUMBER OF SHIFTS TO STAGE-2 . . . . . . . . . . . . . . . .      2
  STEP LENGTH (DX) . . . . . . . . . . . . . . . . . . . . 4.030E-09
  EXECUTION TIME (IN SECONDS) . . . . . . . . . . . . . . .   54.200

FORTRAN STOP
```

Fig. B.4 (continued)

(c) Source and loads: VS contains the value of the voltage excitation. RS contains the value of source impedance. RL(k) contains the load impedance of the kth branch. ISO is 1 (or 2) if the main cascade termination is open (or short) circuited.

(d) Variables: NX is the number of variables. The jth variable is identified as the IXX(2,j)th parameter in the IXX(1,j)th element.

(e) Optimization: NINT is the number of frequency subintervals. Within each subinterval, a uniform design specification is imposed. NPO(k) is the number of frequency points in subinterval k. OMGA(1,k) and OMGA(2,k) contain the lower and the upper frequencies for subinterval k. SPEC(1,k) and SPEC(2,k) contain the weighting and the specification for subinterval k. ISPEC(k) equals 0, i or $-i$ if the specification SPEC(2,k) is imposed on the common port reflection coefficient, the ith branch reflection coefficient or insertion loss, respectively. SPE(i,j) and ISPE(j) are reserved for ℓ_1 optimization. OMG1, OMG2, NOMG are the lower frequency, the upper frequency and the number of frequency points used for a complete circuit simulation (obtaining all circuit responses) at the optimum solution. MODOMG indicates the mode of such a simulation and is usually set to 2.

(f) Files: INFL and OUTFL are character strings containing the input and output file names, respectively. IFILE1, OFILE1, IFILE2 and OFILE2 are also character strings reserved for file names for complicated use of the program.

(g) Constant: VLIGHT is the velocity of light.

In Fig. B.3, the data block for the 4 branch cascaded network of Fig. 5.5 is provided. In Fig. B.4, the data block for optimization of the multicavity filter example is listed.

BRH1 contains a set of subroutines used to perform various forward and reverse analysis in the overall circuit. This block incorporates the branched cascaded analysis technique described in Chapter 5, formulating the heart of the entire program. It exists in a library form.

BRH2 contains a set of subroutines used to perform simulation and sensitivity calculation at the element level. A catalogue of standard 2-port and 3-port elements are coded here. BRH2 exists in a library form.

BRH3 contains a set of subroutines used to initialize various arrays for simulation and sensitivity analysis of the overall circuit. It exists in a library form.

BRH4 contains a set of subroutines which provide interactive access to the program for simulation and sensitivity analysis both at the element level and at the overall circuit level. It exists in a library form.

BRH5 contains a set of subroutines used to perform circuit optimization, specifically, to formulate the optimization problem, to call the minimax optimizer and to print both the initial data and the optimal solutions. It exists in a library form.

USER DEFINED ELEMENTS is an optional block which contains a set of subroutines written by the user to define his or her own 2-port or 3-port elements. Users are responsible themselves to represent their 3-port elements in two port forms using the method of Section 5.2.2. All elements should be defined in the form of transmission matrices.

MMLC is a minimax optimizer. It was developed by Hald and Madsen based on their 2-stage algorithm for nonlinear minimax optimization (Hald and

Madsen 1981). It is available as a standard software (Bandler and Zuberek 1983).

APPENDIX C

BRIEF DESCRIPTION OF THE PROGRAM FOR MULTIPLEXER OPTIMIZATION USING AUTOMATIC DECOMPOSITION

A computer program has been developed for minimax optimization of microwave multiplexers using automatic decomposition. The theory and the algorithm have been described in Chapter 6. The program is written to be compatible with the MXSOS2 (1984) package developed by Optimization Systems Associates Inc. Users are required only to define parameters for the multiplexer before executing the program. All control parameters for decomposition are prompted interactively.

The program is written in Fortran-77. The block diagram of the program is shown in Fig. C.1. Here we briefly describe each of the blocks.

MAIN is the main program used to open necessary files and to initialize necessary parameters. Users are not required to alter this part.

SETMUX is a subroutine in which users are required to define all necessary parameters and codes for the multiplexer device. This subroutine is completely consistent with the main program of the MXSOS2 package. The MXSOS2 user's manual can be referred to for all detailed definitions of arguments in this subroutine.

PARAIO is a subroutine in which users are required to define initial values or default values of all variables of the multiplexer. This subroutine is completely consistent with the main program of MXSOS2 package. The MXSOS2 user's manual can be referred to for all necessary definitions of arguments in this subroutine.

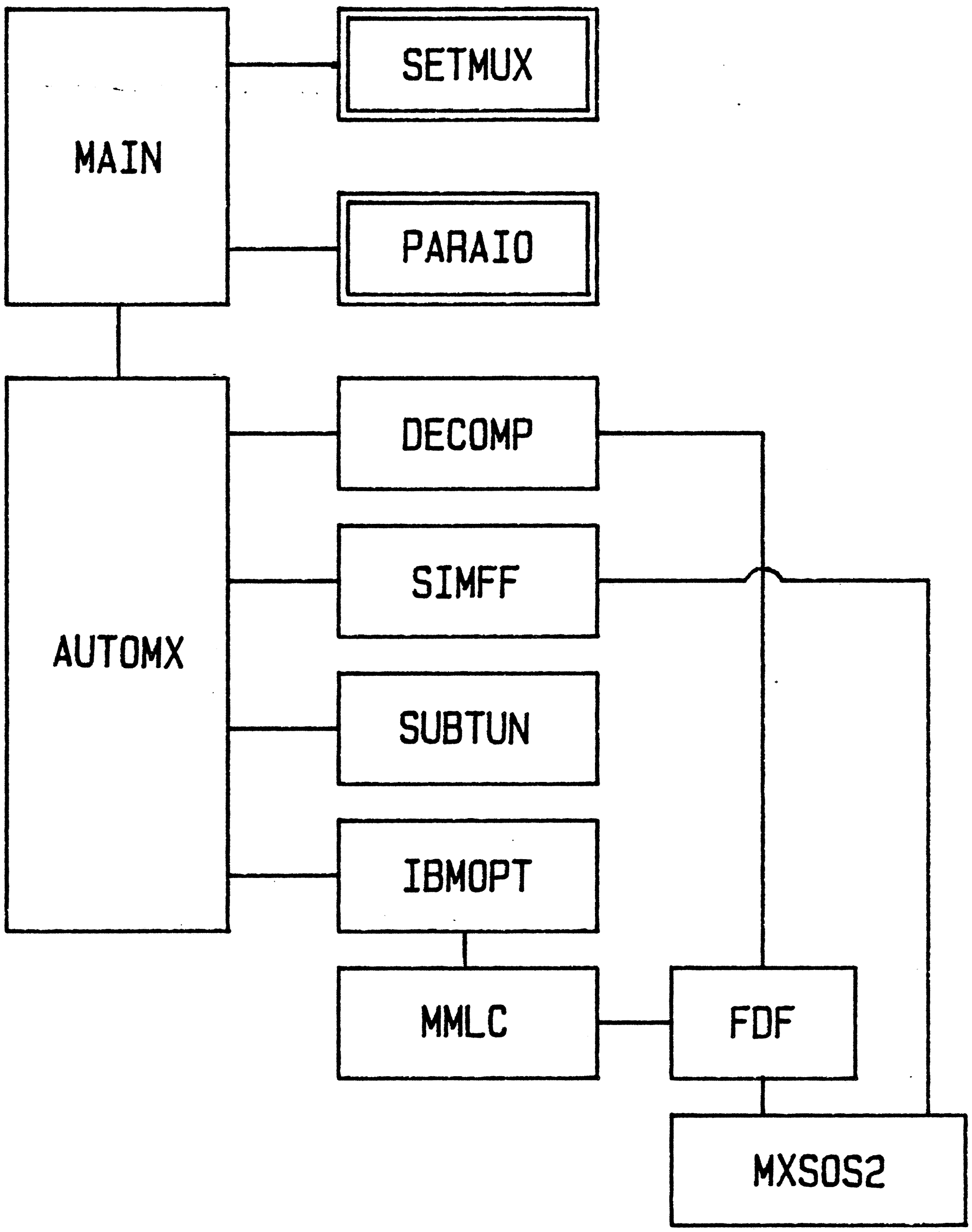


Fig. C.1 Block diagram of the computer program for optimization of multiplexers using automatic decomposition.

AUTOMX is the major subroutine for sequential arrangement of the automatic decomposition strategy.

DECOMP contains a set of subroutines for Monte-Carlo sensitivity analysis and for the construction of a decomposition dictionary based upon a specified sparse factor.

SIMFF contains a set of subroutines for calculating necessary function values. It is used to check the pattern of all error functions after every suboptimization or during a suboptimization.

SUBTUN contains a set of subroutines for choosing the most suitable suboptimization problem to solve. The selection procedure is based upon the pattern of all error functions and the current decomposition dictionary.

IBMOPT is the shortened version of the main program of the MXSOS2 package. It makes appropriate arrangement for calling the minimax optimizer.

MMLC is a minimax optimizer. It was developed by Hald and Madsen based on their 2-stage algorithm for nonlinear minimax optimization (Hald and Madsen 1981). It is available as a standard software (Bandler and Zuberek 1983).

FDF contains a set of subroutines for calculating the selected subset of functions and their sensitivities w.r.t. the selected subset of variables. The distances of channel filters from the main cascade termination are converted to waveguide spacings here.

MXSOS2 is a computer package for simulation, sensitivity analysis and optimization of microwave multiplexers. It was developed by Optimization Systems Associates Inc.

Fig. C.2 gives the computer output of optimizing the 5-channel multiplexer described in Section 6.6. In this output, a SUB-DESIGN means solving a sub-

```
*********************************************************************

           OPTIMAL DESIGN AND TUNING USING DECOMPOSITION

               A MULTIPLEXER EXAMPLE :

               NUMBER OF CHANNELS                   5
               ORDER  OF FILTERS                    6
               NUMBER OF VAR. PER CHANNEL          15
               TOTAL NUMBER OF VARIABLES           75

*********************************************************************

AUTOMATED DESIGN


SELECT :  SEND OPTIMIZATION OUTPUT TO
           1.  A SEPARATE  FILE
           2.  SCREEN OUTPUT
           3.  OUTPUT FILE
           4.  NOT NEEDED
     1
SELECT :    0. PRINT BRIEFLY
            1. PRINT IN DETAIL
            2. SAVE X FOR SUB-DESIGN
     0
DERIVATIVE VERIFICATION REQUIRED ?  ( Y / N )
N
MAXOPT = ?  ( MAX # OF SUB-DESIGNS )
    11
MCHK = ?  ( SUGGEST :  5 <= MCHK <= 15 )
     6
MINIMUM # OF VAR PER SUB-DESIGN =?  ( E.G. 5 )
     5
WANT TO READ: CM,PN1,PN2,WGL FOR EACH CHANNEL ?
N
WORST OBJ COMPARED BY RATIO  ( E.G,  .99, 1., 1.01 )
RATIO = ?
     1.3000000
NEXT WORST OBJ COMPARED BY  ( E.G,  .4 )
RATIO = ?
     0.4000000
```

Fig. C.2 Computer output for the optimization of the 5-channel multiplexer using automatic decomposition.

```
THRESHOLD LAMD FOR LEVEL UPDATE ( E.G.  2.E-1 )
LAMD = ?
     0.2000000
NO. OF POINTS FOR EACH MONTE-CARLO ANALYSIS = ?
     3
INITIAL SPARSE FACTOR = ?  ( E.G. .6, MUST < 1 )
     0.6000000
SELECT :  INITIALIZING DECOMPOSITION DICTIONARY (D) USING
     1.   OLD RESULTS ( IN FILE MONTE )
     2.   NEW MONTE-CARLO ANALYSIS ( TO BE PERFORMED )

     2

---------------------------------------------------------------------
CHANNEL #       PARAMETER VALUES

     1          0.00000   0.59395   0.00000   0.53514   0.00000
                0.42471  -0.39967   0.00000   0.83371   0.00000
                0.76313   0.00000   0.83646   1.04618   0.64385

     2          0.00000   0.59395   0.00000   0.53514   0.00000
                0.42471  -0.39967   0.00000   0.83371   0.00000
                0.76313   0.00000   0.83646   1.04618   1.29143

     3          0.00000   0.59395   0.00000   0.53514   0.00000
                0.42471  -0.39967   0.00000   0.83371   0.00000
                0.76313   0.00000   0.83646   1.04618   1.94280

     4          0.00000   0.59395   0.00000   0.53514   0.00000
                0.42471  -0.39967   0.00000   0.83371   0.00000
                0.76313   0.00000   0.83646   1.04618   2.59800

     5          0.00000   0.59395   0.00000   0.53514   0.00000
                0.42471  -0.39967   0.00000   0.83371   0.00000
                0.76313   0.00000   0.83646   1.04618   3.25709

*********************************************************************

  A SUB-DESIGN IS CHARACTERIZED BY
     (1)  RESPONSES OF CHANNELS  N1 --- N2.
     (2)  NX VAR.'S AS A SUBSET OF ALL VAR.'S.

  RESULTS OF A SUB-DESIGN ARE GIVEN BY
*********************************************************************
              IFALL MAXF  OBJ       OBJ( 1 ---  5 )
*********************************************************************

                0    0  16.025   15.117  3.037  0.898  5.076 16.025
```

Fig. C.2 (continued)

```
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
    -1   0   0
 NO. OF MONTE-CARLO POINTS ?
     3
 SPARSE FACTOR = ?  ( E.G.   .01  OR >1 TO EXIT)
       0.6000000

 GRP    N1   N2   NX        VAR.'S IN GROUP
-----------------------------------------------------------------------
  1      1    1    9          2   4   6   7   9  11  13  14  15
  2      2    2    9         17  19  21  22  24  26  28  29  30
  3      3    3    9         32  34  36  37  39  41  43  44  45
  4      4    4    9         47  49  51  52  54  56  58  59  60
  5      5    5    8         62  64  66  67  69  71  74  75
  6      4    5    1         73

                 0   0  16.025   15.117  3.037  0.898  5.076 16.025

N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     5   5   8
  8 VAR. INDICES = ?
    62  64  66  67  69  71  74  75
                 1  12  15.143   15.143  3.028  1.936  9.605  9.605

SUBDESIGN #    1 IMPROVE 0.88+000
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     4   5  18
 18 VAR. INDICES = ?
    47  49  51  52  54  56  58  59  60  62
    64  66  67  69  71  73  74  75
                 1   8  15.187   15.187  2.595  6.378  6.433  6.433

SUBDESIGN #    2 IMPROVE-0.44-001
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     1   1   9
  9 VAR. INDICES = ?
     2   4   6   7   9  11  13  14  15
                 1  15   7.674    7.674  7.674  6.409  6.437  6.437
```

Fig. C.2 (continued)

```
SUBDESIGN #   3 IMPROVE 0.75+001
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
    -1   0   0
 NO. OF MONTE-CARLO POINTS ?
     3
 SPARSE FACTOR = ?  ( E.G.  .01  OR >1 TO EXIT)
       0.3000000

 GRP    N1   N2   NX       VAR.'S IN GROUP
----------------------------------------------------------------------
  1      1    1   11        1   2   3   4   5   6   7   8   9  11
                           14
  2      2    2   14       17  18  19  20  21  22  23  24  25  26
                           27  28  29  30
  3      3    3   15       31  32  33  34  35  36  37  38  39  40
                           41  42  43  44  45
  4      4    4   14       46  47  48  49  50  51  52  53  54  55
                           56  58  59  60
  5      5    5   11       62  63  64  65  66  67  68  69  71  74
                           75
  6      1    2    2       13  15
  7      2    3    1       16
  8      3    5    1       73

                 0   0   7.674    7.674  7.674  6.409  6.437  6.437

N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     1   2  27
 27 VAR. INDICES = ?
     1   2   3   4   5   6   7   8   9  11
    13  14  15  17  18  19  20  21  22  23
    24  25  26  27  28  29  30
                 1  15   6.489    0.121  0.118  6.354  6.409  6.489

SUBDESIGN #   4 IMPROVE 0.12+001
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     3   5  41
 41 VAR. INDICES = ?
    31  32  33  34  35  36  37  38  39  40
    41  42  43  44  45  46  47  48  49  50
    51  52  53  54  55  56  58  59  60  62
    63  64  65  66  67  68  69  71  73  74
    75
```

Fig. C.2 (continued)

```
                 1   5   4.028    1.167  4.028  3.801  3.801  3.808

SUBDESIGN #   5 IMPROVE 0.25+001
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     2   3  30
 30 VAR. INDICES = ?
    16  17  18  19  20  21  22  23  24  25
    26  27  28  29  30  31  32  33  34  35
    36  37  38  39  40  41  42  43  44  45
                 1   5   4.292    3.519 -0.476 -0.922  4.206  4.292

SUBDESIGN #   6 IMPROVE-0.26+000
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     3   5  26
 26 VAR. INDICES = ?
    46  47  48  49  50  51  52  53  54  55
    56  58  59  60  62  63  64  65  66  67
    68  69  71  73  74  75
                 1  31   3.322    3.322  0.975  0.664  0.668  0.668

SUBDESIGN #   7 IMPROVE 0.97+000
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     3   5  41
 41 VAR. INDICES = ?
    31  32  33  34  35  36  37  38  39  40
    41  42  43  44  45  46  47  48  49  50
    51  52  53  54  55  56  58  59  60  62
    63  64  65  66  67  68  69  71  73  74
    75
                 1   5   3.735    3.735  2.729  0.163  0.156  0.169

SUBDESIGN #   8 IMPROVE-0.41+000
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     1   1  11
 11 VAR. INDICES = ?
     1   2   3   4   5   6   7   8   9  11
    14
                 1   8   2.726    2.248  2.726  0.159  0.152  0.176
```

Fig. C.2 (continued)

```
SUBDESIGN #   9 IMPROVE 0.10+001
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     1   2  27
 27 VAR. INDICES = ?
     1   2   3   4   5   6   7   8   9  11
    13  14  15  17  18  19  20  21  22  23
    24  25  26  27  28  29  30
                 1   9   0.657    0.333  0.323  0.323  0.298  0.657

SUBDESIGN #  10 IMPROVE 0.21+001
N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
    -1   5  11
 NO. OF MONTE-CARLO POINTS ?
     3
 SPARSE FACTOR = ?  ( E.G.  .01  OR >1 TO EXIT)
       1.0000000E-023

 GRP    N1   N2   NX       VAR.'S IN GROUP
----------------------------------------------------------------------
  1      1    4    2       10  25
  2      2    5    1       70
  3      1    5   72        1   2   3   4   5   6   7   8   9  11
                           12  13  14  15  16  17  18  19  20  21
                           22  23  24  26  27  28  29  30  31  32
                           33  34  35  36  37  38  39  40  41  42
                           43  44  45  46  47  48  49  50  51  52
                           53  54  55  56  57  58  59  60  61  62
                           63  64  65  66  67  68  69  71  72  73
                           74  75

                 0   0   0.657    0.333  0.323  0.323  0.298  0.657

N1 = 0 EXIT
N1 < 0 NEW D
N1,N2,NX = ?
     1   5  73
 73 VAR. INDICES = ?
```

Fig. C.2 (continued)

```
 1   2   3   4   5   6   7   8   9  11
12  13  14  15  16  17  18  19  20  21
22  23  24  26  27  28  29  30  31  32
33  34  35  36  37  38  39  40  41  42
43  44  45  46  47  48  49  50  51  52
53  54  55  56  57  58  59  60  61  62
63  64  65  66  67  68  69  70  71  72
73  74  75
             3  64  -0.025   -0.025 -0.693 -0.695 -0.695 -0.695

SUBDESIGN #  11 IMPROVE 0.68+000

NO. OF SUB-DESIGNS PERFORMED :    11

SUGGEST TO PERFORM A SIMULATION AT THE SOLUTION

DETAILS OF OPTIMIZATIONS SAVED IN FILE :  MXDOPT

WRITE: CM,PN1,PN2,WGL FOR EACH CHANNEL ?
Y

WRITING CHANNEL DATA: ENTER LOCAL FILE NAME
FLXX

RESULTS OF MONTE-CARLO ANALYSIS SAVED IN FILE :  MONTE

---------------------------------------------------------------------
CHANNEL #      PARAMETER VALUES

     1        -0.04718   0.70920  -0.10560   0.55167  -0.04397
               0.42314  -0.37637  -0.02669   0.79935   0.00000
               0.62737  -0.00779   1.05925   0.91543   0.73072

     2         0.08258   0.61762  -0.02519   0.53718  -0.02078
               0.40834  -0.43245  -0.02801   0.83029   0.00612
               0.73380   0.00847   0.90316   1.06973   1.33536

     3         0.11558   0.62033  -0.02077   0.53476  -0.01795
               0.41570  -0.41449  -0.02229   0.82319   0.00430
               0.74930   0.00624   0.92125   1.07112   1.95865

     4         0.28426   0.66443  -0.00799   0.54219  -0.00957
               0.39010  -0.48177  -0.01631   0.86340   0.00991
               0.74938   0.00187   0.99681   1.10682   2.56751
```

Fig. C.2 (continued)

5	0.08014	0.64788	0.03984	0.55282	0.01415
	0.40545	-0.45203	0.00330	0.84896	-0.02884
	0.71103	-0.02860	0.79837	1.06257	3.08771

TOTAL TIME SPENT (SECONDS) 179.2657

Fig. C.2 (continued)

optimization problem. During a suboptimization, MCHK is the number of iterations required per occurrence of an overall function check. The maximum number of iterations for one suboptimization is automatically set to MCHK*MCHK. The term OBJ denotes the objective function of the overall optimization or of a suboptimization as the case may be. The first ratio to compare objective functions is used to compare the overall objective function before (old OBJ) and after (new OBJ) each sub-design. The comparison results in the rejection or acceptance of the sub-design depending upon whether the division of the new OBJ by the old OBJ is greater or less than the specified ratio. The second ratio is usually set to 0.4 and is used to check the deterioration of the overall error functions during a suboptimization. The "threshold LAMD for level update" is usually set to 0.2. A large value of LAMD leads to the quick and premature termination of suboptimizations. A Monte-Carlo analysis will be activated if the decomposition dictionary is to be updated. Each occurrence of such an updating causes a reduction in the sparse factor which is used in constructing the decomposition dictionary. The initial sparse factor is specified by the user. The suggested value for this factor is about 0.6. IFALL indicates the type of a sub-optimization solution, being consistent with the IFALL in the MMLC package (Bandler and Zuberek 1983). MAXF gives the number of iterations actually performed in a suboptimization. OBJ(k) indicate the objective function for the subset of functions associated with channel k, k = 1, 2, ..., 5.

In Table C.1, the indices of variables appeared in Fig. C.2 are interpreted into specific variables for the 5-channel multiplexer.

TABLE C.1

INTERPRETATION OF VARIABLE INDICES IN FIG. C.2
FOR THE 5-CHANNEL MULTIPLEXER

Channel Number	Standard Notation for Variables +														
	M_{11}	M_{12}	M_{22}	M_{23}	M_{33}	M_{34}	M_{36}	M_{44}	M_{45}	M_{55}	M_{56}	M_{66}	n_1	n_2	d
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
3	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
4	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
5	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75

\+ where M_{ij} is the cavity resonance or the coupling parameter, $i, j \in \{1, 2, ..., 6\}$. n_1 and n_2 are the input and the output transformer ratios. d is the distance of a channel filter from the short circuited main cascade termination.

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