MICROWAVE DEVICE MODELLING USING EFFICIENT ℓ_1 OPTIMIZATION: A NOVEL APPROACH

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Abstract

A powerful modelling technique which exploits the unique properties of the ℓ_1 norm is presented. Self-consistent models for passive and active devices are achieved by an approach that automatically checks the validity of model parameters obtained from optimization. Practical use of an efficient ℓ_1 algorithm in complicated problems, for which gradient evaluation may not be feasible, is discussed. Examples in modelling of multi-coupled cavity filters and GaAs FET's are presented.

SUMMARY

The problem of approximating a measured response by a network or system response has been formulated as an optimization problem w.r.t. the equivalent circuit parameters of a proposed model. The traditional approach in modelling is almost entirely directed at achieving the best possible match between measured and calculated responses. This approach has serious shortcomings in two frequently encountered cases. The first case is when the equivalent circuit parameters are not unique w.r.t. the responses selected and the second is when nonideal effects are not modelled adequately, the latter causing an imperfect match, even if small measurement errors are allowed for. In both cases, a family of solutions for circuit model parameters may exist which produce reasonable and similar match between measured and calculated responses.

In this paper, we present a new formulation for modelling that automatically checks the validity of the circuit parameters, with a simultaneous attempt in matching measured and calculated responses. If successful, the method provides confidence in the validity of the model parameters, otherwise it proves their incorrectness. The use of the ℓ_1 norm, with its unique properties, is an integral part of the approach. We discuss the use of an efficient ℓ_1 algorithm [1-3] both in problems for which response gradients can be evaluated, and in complicated problems for which gradient evaluation is not feasible. The use of a gradient-based ℓ_1 algorithm and utilizing a variation of Broyden's formula to update gradients internally [3], makes it possible to employ a state-of-the-art optimization algorithm with any simulation package capable simply of providing responses. Therefore, widely used microwave design programs, e.g., SUPER-COMPACT [4] and TOUCHSTONE [5] which do not calculate exact gradients, could employ such an algorithm with an appropriate interface. As a result, it is conceivable that the modelling technique described could find its way to practicing microwave engineers in a near future.

Two examples of practical interest, namely, modelling of a narrowband multi-coupled cavity filter and a wideband GaAs FET follow the theoretical description of both the

traditional and the new approaches. In both examples, a large number of variables are considered.

The Traditional Approach

The traditional approximation problem is stated as follows

$$\underset{\mathbf{x}}{\text{minimize}} \, \| \mathbf{F} \| \tag{1}$$

where a typical component of vector $\mathbf{F}$, namely F_i evaluated at the frequency point ω_i, is given by

$$F_i \triangleq w_i \, (f_i^c(\mathbf{x}) - f_i^m), \qquad i=1,2,...,k. \tag{2}$$

f_i^m is a measured response at ω_i and f_i^c is the response of an appropriate network which depends nonlinearly on a vector of model parameters $\mathbf{x} \triangleq [x_1 \;\; x_2 \;\; ... \;\; x_n]^T$ and w_i denotes a nonnegative weighting factor.

It is usually assumed that the expected values of the components of $\mathbf{F}$ are zero. However, this cannot be realized in practice due to the presence of measurement errors in observing f_i^m and more importantly, as a result of the imperfections and nonideal effects which may not have been accounted for in the topology of the equivalent circuit. Since the components of $\mathbf{F}$ cannot go to zero, different norms of $\mathbf{F}$ may give different results for $\mathbf{x}$. The use of the ℓ_1 norm, besides the advantages which will be described in the new modelling technique, has the advantage over other norms that some isolated large errors in measurement data, as reflected in large values of components of $\mathbf{F}$, will be automatically ignored.

New Approach Using Multiple Sets of Measurements

The model parameters $\mathbf{x}$ are generally controlled by some physical parameters $\boldsymbol{\phi} \triangleq [\phi_1 \;\; \phi_2 \;\; ... \;\; \phi_\ell]^T$. For instance, in active device modelling intrinsic network parameters are controlled by bias voltages or currents. Although the actual functional relationship between $\boldsymbol{\phi}$ and $\mathbf{x}$ may not be known, the correspondence betwen $\mathbf{x}$ and $\boldsymbol{\phi}$ is usually known, i.e., we know which element or elements of $\mathbf{x}$ are affected by an adjustment on an element of $\boldsymbol{\phi}$. We exploit this knowledge to propose the following formulation.

Suppose that after taking measurements on a microwave device at a number of frequency points, we make an easy-to-achieve adjustment on an element of $\boldsymbol{\phi}$ such that one or a few components of $\mathbf{x}$ are changed in a dominant fashion and the rest remain constant or change slightly. Consider the following optimization problem

$$\underset{\mathbf{x}^{1,2}}{\text{minimize}} \quad \sum_{t=1}^{2} \sum_{i=1}^{k_t} \left| F_i^t \right| + \sum_{j=1}^{n} \beta_j \left| x_j^1 - x_j^2 \right| \tag{3}$$

where

$$F_i^t \triangleq w_i^t [(f_i^c(\mathbf{x}^t) - f_i^m)]^t, \tag{4}$$

with superscript t identifying the original network model ($t=1$) or the model after adjustment on $\boldsymbol{\phi}$ ($t=2$). β_j represents an appropriate weighting factor and k_t is an index whose value depends on t, i.e., a different number of frequencies may be used for the original and the perturbed model. $\mathbf{x}^{1,2}$ is a vector which contains circuit parameters of both the original and perturbed networks, i.e.,

$$\mathbf{x}^{1,2} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \end{bmatrix}. \tag{5}$$

The above formulation has the following properties:

1) Considering only the first segment of the objective function, the formulation is equivalent to performing two optimizations, i.e., matching the calculated repsonse of the original circuit model with its corresponding measurements and repeating the procedure for the perturbed circuit.

2) By adding the second segment to the objective function, we take advantage of the knowledge that only one or a few components of $\mathbf{x}$ should change dominantly by perturbing a component of $\boldsymbol{\phi}$. Therefore, we penalize the objective function for any

change in $\mathbf{x}$. However, by cleverly selecting the ℓ_1 norm, we still allow for one or a few large changes in $\mathbf{x}$. This is completely consistent with the previous assumptions.

The confidence in the validity of the equivalent circuit parameters increases if a) an optimization using the objective function of (3) results in a reasonable match between calculated and measured responses for both circuits 1 and 2 (original and perturbed) and, b) the examination of the solution vector $\mathbf{x}^{1,2}$ reveals changes from $\mathbf{x}^1$ to $\mathbf{x}^2$ which are consistent with the adjustment on $\boldsymbol{\phi}$, i.e., only the expected components have changed significantly. We can build upon our confidence even more by generalizing the technique to more adjustments on $\boldsymbol{\phi}$, i.e., formulating the optimization problem as

$$\underset{\mathbf{x}^{1,\ldots,n_c}}{\text{minimize}} \sum_{t=1}^{n_c} \sum_{i=1}^{k_t} \left| F_i^t \right| + \sum_{t=2}^{n_c} \sum_{j=1}^{n} \beta_j^t \left| x_j^1 - x_j^t \right| , \tag{6}$$

where n_c circuits and their corresponding sets of responses, measurements and parameters are considered and the first circuit is the reference model before any adjustment on $\boldsymbol{\phi}$.

Two common mistakes, which may not be detected easily by the traditional modelling technique, are discovered by observing inconsistencies in changes of $\mathbf{x}$ with the actual change in $\boldsymbol{\phi}$. They are:

1) Neglecting nonideal effects which may not be evident by comparing selected responses at the particular frequencies used, i.e., a reasonable match is observed, although the parameters are incorrect.

2) Selecting an alternative set of parameters capable of producing the original circuit responses, which are nonetheless invalid if different responses or different frequency ranges are used.

Practical Application of the ℓ_1 Algorithm

The ℓ_1 optimization problem is formulated in (6). The success of the new technique described relies upon the use of an efficient and robust ℓ_1 algorithm. Recently, a superlinearly convergent algorithm for nonlinear ℓ_1 optimization has been described [1]. The algorithm, based on the original work of Hald and Madsen [2], is a combination of a first-order method that approximates the solution by successive linear progamming and a quasi-Newton method using approximate second-order information to solve the system of nonlinear equations resulting from the first-order necessary conditions for an optimum.

The most efficient use of the ℓ_1 algorithm requires the user to supply function and gradient values of the individual functions in (6), i.e., network responses as well as their gradients are needed. Starting with the impedance or nodal admittance description of an arbitrary network model, we have derived analytical formulas for evaluation of first-order sensitivities of S-parameters at ports of interest w.r.t. any circuit parameter appearing in the impedance or admittance matrix. Without using the concept of the adjoint network, the computational effort involves solving four systems of linear equations: $\mathbf{A}\,\mathbf{x} = \mathbf{e}_1$, $\mathbf{A}^T \hat{\mathbf{x}} = \mathbf{e}_1$, $\mathbf{A}\,\mathbf{y} = \mathbf{e}_p$ and $\mathbf{A}^T \hat{\mathbf{y}} = \mathbf{e}_p$, where $\mathbf{A}_{pxp}$ is the impedance or admittance matrix, $\mathbf{e}_1 = [1\;\; 0\;\; \ldots\; 0]^T$ and $\mathbf{e}_p = [0\;\; \ldots\; 0\;\; 1]^T$, for the case of two-port S-parameters. Notice that one LU factorization is sufficient. The approach can be extended to multi-ports at the expense of increasing computational effort.

In many practical problems, e.g., in the presence of nonlinear devices or complicated field problems, the evaluation of gradients is not feasible. In such cases, it is possible to estimate the gradients using the numerical difference method. However, this is computationally slow and consequently expensive. To take advantage of a fast gradient-based algorithm, without supplying gradients or using the numerical difference method, the original ℓ_1 algorithm has been modified [3]. Different and flexible versions of the modified algorithm exist. A typical version estimates the gradients using the numerical difference method only once and updates the gradients with minimum extra effort by applying a variation of Broyden's formula as the optimization proceeds. All approximations are performed internally, therefore, the optimization could be linked to any analysis program which provides only the responses.

Examples

A. Modelling of Multi-Coupled Cavity Filters

A 6th order multi-coupled cavity filter centered at 11785.5 MHz with a 56.2 MHz bandwidth is considered. Measurements on input and output return loss, insertion loss and group delay of an optimally tuned filter and the same filter after a deliberate adjustment on the screw which dominantly controls coupling M_{12}, were provided by ComDev Ltd., Cambridge, Canada [6]. Although the pass-band return loss changes significantly, we anticipate that such a physical adjustment affects only model parameters M_{12}, M_{11} and M_{22} (the last two correspond to cavity resonant frequencies) in a dominant fashion, possibly with slight changes in other parameters. Using the new technique described in this paper, we simultaneously processed measurements on pass-band return loss (input reflection coefficient with a weighting of 1), and stop-band insertion loss (with a weighting of 0.05) of both filters, i.e., the original and perturbed models. The ℓ_1 algorithm with exact gradients was used. The evaluation of sensitivities is discussed in detail by Bandler et al. [7]. The model parameters identified for two filters are summarized in Table I. Figs. 1 and 2 illustrate the measured and modelled responses of the original filter. Fig. 3 shows the measured and modelled input return loss for the filter after adjustment. An examination of results in Table I and Figs. 1-3 shows that not only an excellent match between measured and modelled responses has been achieved, but also the changes in parameters are completely consistent with the actual physical adjustment. Therefore, by means of only one optimization, we have built confidence in the validity of equivalent circuit parameters. The problem involved 84 nonlinear functions (42×2 responses for original and perturbed filters) and 12 linear functions (change in parameters of two circuit equivalents) and 24 variables. The solution was achieved in 72 seconds of CPU time on the VAX 11/780 system.

B. GaAs FET Modelling

Using S-parameter data for the device B1824-20C from 4 to 18 GHz, Curtice and Camisa have achieved a very good model for the FET chip [8]. They have used the traditional least squares optimization of responses utilizing SUPER-COMPACT. Their success is due to the fact that they have reduced the number of possible variables from 16 to 8 by using dc and zero-bias measurements. We used their equivalent circuit at normal operating bias (including the carrier), as illustrated in Fig. 4, and created artificial measurements using TOUCHSTONE. Two sets of S-parameter measurements were created; one set using the parameters reported by Curtice and Camisa (operating bias V_{ds} = 8.0 V, $V_{gs} = -2.0$V and $I_{ds} = 128.0$ mA) and the other by changing the values of C_1, C_2, L_g and L_d to simulate the effect of taking different reference planes for the carriers. Both sets of data are shown in Fig. 5, where the S-parameters of the two circuits are plotted on a Smith Chart. Using the technique described in this paper, we processed the measurements on the two circuits simultaneously by minimizing the function defined in (3). The objective of this experiment is to show that even if the equivalent circuit parameters were not known, as is the case using real measurements, the consistency of the results would be proved only if the intrinsic parameters of the FET remain unchanged between the two circuits. This was indeed the case for the experiment performed. Although the maximum number of possible variables, namely 32 (16 for each circuit), were allowed for in the optimization, the intrinsic parameters were found to be the same between the two circuits and, as expected, C_1, C_2, L_g and L_d changed from circuit 1 to 2. Table II summarizes the parameter values obtained. The problem involved 128 nonlinear functions (real and imaginary parts of 4 S-parameters, at 8 frequencies, for two circuits), 16 linear functions and 32 variables. The CPU time on the VAX 11/780 system was 79 seconds.

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TABLE I

RESULTS FOR THE 6TH ORDER FILTER EXAMPLE

Coupling	Original Filter	Perturbed Filter	Change in Parameter
M_{11}	–0.0473	–0.1472	–0.0999*
M_{22}	–0.0204	–0.0696	–0.0492*
M_{33}	–0.0305	–0.0230	0.0075
M_{44}	0.0005	0.0066	0.0061
M_{55}	–0.0026	0.0014	0.0040
M_{66}	0.0177	–0.0047	–0.0224
M_{12}	0.8489	0.7119	–0.1370*
M_{23}	0.6064	0.5969	–0.0095
M_{34}	0.5106	0.5101	–0.0005
M_{45}	0.7709	0.7709	0.0000
M_{56}	0.7898	0.7806	–0.0092
M_{36}	–0.2783	–0.2850	–0.0067

* significant change in parameter value.

TABLE II

RESULTS FOR THE GaAs FET EXAMPLE

Parameter		Original Circuit	Perturbed Circuit
C_1	(pF)	0.0440	0.0200*
C_2	(pF)	0.0389	0.0200*
C_{dg}	(pF)	0.0416	0.0416
C_{gs}	(pF)	0.6869	0.6869
C_{ds}	(pF)	0.1900	0.1900
C_i	(pF)	0.0100	0.0100
R_g	(Ω)	0.5490	0.5490
R_d	(Ω)	1.3670	1.3670
R_s	(Ω)	1.0480	1.0480
R_i	(Ω)	1.0842	1.0842
G_d^{-1}	(kΩ)	0.3761	0.3763
L_g	(nH)	0.3158	0.1500*
L_d	(nH)	0.2515	0.1499*
L_s	(nH)	0.0105	0.0105
g_m	(S)	0.0423	0.0423
τ	(ps)	7.4035	7.4035

* significant change in parameter value