SOLCH - A FORTRAN PACKAGE FOR SOLUTIONS OF PERTURBED LINEAR EQUATIONS

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Abstract

SOLCH is a package of subroutines for evaluating solutions of perturbed linear equations. Appropriate large change formulas are used to ensure efficient computation. The user should supply the coefficient matrix, the right-hand-side and the coefficient deviation matrix in a product form. The method is described in detail by our previous report. The package and documentation have been developed for the CDC 170/815 system with the NOS 2.2-602/587 operating system and the Fortran Extended (FTN) version 4.8 compiler.

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I. INTRODUCTION

The process of solving an n x n system of linear equations under perturbation can be essentially reduced to solving a smaller system of dimension r x r, where r is the rank of the deviation matrix of the original system. The method is described in detail in our previous report [1]. SOLCH is a package using this method to calculate solutions of linear equations affected by various large changes in system parameters. Appropriate large change formulas are used which require the user to supply the large changes of the system coefficient matrix in the form of $\mathbf{VDW}^T$, where **V**, **D** and **W** are matrices of dimensions n x r_1, r_1 x r_2 and n x r_2, respectively.

There are two different entries to the package. The user can call from either Entry 1 or Entry 2.

Calling from Entry 1, the user will go through an interactive procedure with the computer for easy and direct access to large change computations. The calling procedure is very simple. Entry 1 is recommended when the package is used directly for the solution of perturbed linear equations as final output results and when the numerical perturbations can be practically entered from the computer terminal.

Calling from Entry 2, the user will use the basic subroutines for large change calculations. The package can be used at this entry as a subsection of a more comprehensive program which requires the solution of linear equations only as intermediate steps. Also, at this entry, the perturbations of the linear system can be supplied from, e.g., variable updating calculations, not necessarily from a computer terminal.

The whole package is written in Fortran IV for the CDC 170/815 system. At McMaster University is is available in the form of a library of binary relocatable subroutines which is linked with the user's program by an appropriate call to the package. The name of the library is LIBSLCH. The library is available as a group indirect file under the charge RJWBAND. The general sequence of NOS commands to use the package can be as follows:

/GET (LIBSLCH/GR) - fetch the library,

/LIBRARY (LIBSLCH) - indicate the library to the loader.

The user's program should be composed (at least) of the main segment which prepares parameters and calls the subroutines of the package corresponding to either Entry 1 or Entry 2.

This document includes the user's manual of the SOLCH package. In Section II, we give a brief theoretical review of the problem formulation and the use of large change formulas. Section III describes the structure of the package. Sections IV and V provide argument lists for package Entries 1 and 2, respectively. Examples are given in Section VI to provide numerical results and to show how the package can be used.

A Fortran listing of the package is found in [2].

II. THEORETICAL REVIEW

In our investigation of sensitivity computations in linear systems, we have concluded that the calculation of large change sensitivity of an n x n linear system can be essentially reduced to the computation of a smaller system depending upon the rank of the n x n deviation matrix [1].

Formulation of the Problem

Consider

$$\mathbf{A}\mathbf{x} = \mathbf{b}\,, \qquad (1)$$

where $\mathbf{A}$ is an n x n matrix containing variables, $\mathbf{b}$ is a constant n-vector and $\mathbf{x}$ is an n-vector containing solutions of the linear system. Suppose the values of variable parameters are changed causing changes of matrix $\mathbf{A}$ such that

$$\mathbf{A}_{\text{new}} = \mathbf{A} + \Delta\mathbf{A}\,. \qquad (2)$$

Consequently, these large changes will affect the solution which can be represented by

$$\mathbf{x}_{\text{new}} = \mathbf{x} + \Delta\mathbf{x}\,. \qquad (3)$$

Also, the inverse of $\mathbf{A}$ will be affected such that

$$(\mathbf{A} + \Delta\mathbf{A})^{-1} = \mathbf{A}^{-1} + \Delta(\mathbf{A}^{-1}), \tag{4}$$

where $\Delta(\mathbf{A}^{-1})$ is defined as [1]

$$\Delta(\mathbf{A}^{-1}) \triangleq (\mathbf{A} + \Delta\mathbf{A})^{-1} - \mathbf{A}^{-1}. \tag{5}$$

If we use direct methods, $\mathbf{x}_{new}$ can be obtained by solving the new n x n system

$$\mathbf{A}_{new}\,\mathbf{x}_{new} = \mathbf{b}. \tag{6}$$

However, when changes in the system only affect a submatrix of **A**, a more elegant approach for $\mathbf{x}_{new}$ is to use large change formulas which can essentially reduce the computation from solving an n x n system to solving an r x r system where r is the rank of **A**,

$$r \triangleq \text{Rank}\,(\Delta\mathbf{A}). \tag{7}$$

In our previous report [1] we introduced a series of formulas for large change calculations, among which formulas (47) and (48) are used in package SOLCH.

Large Change Formulas (47) and (48)

Formulas (47) and (48) of reference [1] are expressed as

$$\Delta(\mathbf{A}^{-1}) = -\mathbf{A}^{-1}\,\mathbf{V}\,\mathbf{D}\,(\mathbf{1} + \mathbf{W}^{T}\,\mathbf{A}^{-1}\,\mathbf{V}\,\mathbf{D})^{-1}\,\mathbf{W}^{T}\,\mathbf{A}^{-1} \qquad \text{(Formula 47)}$$

and

$$\Delta(\mathbf{A}^{-1}) = -\mathbf{A}^{-1}\,\mathbf{V}\,(\mathbf{1} + \mathbf{D}\,\mathbf{W}^{T}\,\mathbf{A}^{-1}\,\mathbf{V})^{-1}\,\mathbf{D}\,\mathbf{W}^{T}\,\mathbf{A}^{-1}, \qquad \text{(Formula 48)}$$

where **V**, **D** and **W** are n x r_1, r_1 x r_2 and n x r_2 matrices such that

$$\Delta\mathbf{A} = \mathbf{V}\,\mathbf{D}\,\mathbf{W}^{T}. \tag{8}$$

Solving the Perturbed System of Linear Equations by Formula (47)

Using Formula (47), $\mathbf{x}_{new}$ of (3) can be calculated by

$$\mathbf{x}_{new} = \mathbf{x} - \mathbf{P}_{V}\,\mathbf{D}\,\mathbf{s}, \tag{9}$$

where $\mathbf{P}_V$ and **s** are solutions of the n x n system

$$\mathbf{A}\,\mathbf{P}_{V} = \mathbf{V} \tag{10}$$

and the r_2 x r_2 smaller system

$$(\mathbf{1} + \mathbf{W}^{T}\,\mathbf{P}_{V}\,\mathbf{D})\,\mathbf{s} = \mathbf{W}^{T}\,\mathbf{x}, \tag{11}$$

respectively. Notice that only forward and backward substitutions are necessary in solving (10) since **A** has already been LU factorized when solving the original system for **x**.

Solving the Perturbed System of Linear Equations by Formula (48)

Using Formula (48), $\mathbf{x}_{new}$ of (3) can be calculated by

$$\mathbf{x}_{new} = \mathbf{x} - \mathbf{P}_V \mathbf{s}' , \tag{12}$$

where $\mathbf{P}_V$ has been defined by (10) and $\mathbf{s}'$ is the solution of the $r_1 \times r_1$ smaller system

$$(\mathbf{1} + \mathbf{D}\,\mathbf{W}^T \mathbf{P}_V)\,\mathbf{s}' = \mathbf{D}\,\mathbf{W}^T \mathbf{x} . \tag{13}$$

Discussion

According to our investigation of operational count, Formula (47) is preferred when $r_1 \geq r_2$ otherwise Formula (48) is recommended. The SOLCH package has been designed to be able to switch between these two formulas automatically, using the values of r_1 and r_2.

We can always formulate **V**, **D** and **W** such that $r_1 \leq n$ and $r_2 \leq n$. The smaller r_1 and r_2 are, the better. The minimal value of r_1 and r_2 is

$$\min_{(\mathbf{V},\mathbf{D},\mathbf{W})} r_1 = \min_{(\mathbf{V},\mathbf{D},\mathbf{W})} r_2 = r , \tag{14}$$

where r is the rank of $\Delta\mathbf{A}$. Detailed information on the formulation of **V**, **D** and **W** can be found in reference [1].

III. STRUCTURE OF THE PACKAGE

Figures 1 and 2 show possible structures of the SOLCH package corresponding to Entries 1 and 2, respectively.

Let us study Fig. 2 first. We have expressed $\Delta\mathbf{A}$ in the form of $\mathbf{V}\,\mathbf{D}\,\mathbf{W}^T$. In many cases, we can choose **V**, **D** and **W** such that **V** and **W** indicate the positions of variables in **A** and **D** represents values of large changes of variables [1]. Thus, when the values of a certain set of variables are changed many times, we will have many different **D** matrices corresponding to the same set of **V** and **W**. Subroutine SOLCHA is designed to perform all possible calculations with **V** and **W**, but without **D**. Subroutine SOLCHB executes the calculation of $\mathbf{x}_{new}$ using **D** and the results from SOLCHA. Subroutine LUFACT performs LU factorization and/or forward backward substitution (FBS). In the main program, the user must call SOLCHA first and SOLCHB subsequently. As discussed, SOLCHB can be executed many times after only one execution of SOLCHA, corresponding to many **D** matrices for only one pair of **V** and **W**. This logic is shown in Fig. 3.

To provide users easy access to the package for solutions of perturbed linear equations, we have implemented the logic of Fig. 3 into subroutine SOLCH and SOLU0. They create an interactive procedure for the user with the computer. This is done by calling the package from Entry 1 and is shown in Fig. 1. The main program must be supplied by the user. In the main program, the user only needs to call SOLCH. r_1, r_2, **V**, **D** and **W** are entered from the computer terminal through a user-computer dialogue procedure guided by the package.

IV. ARGUMENT LIST FOR ENTRY 1

The subroutine call is

CALL SOLCH (N, LW0, A, B, W0, ICH)

The arguments are as follows.

N — is an INTEGER argument which must be set to n, the number of rows or columns of **A**. Its value must be positive and it is not altered by the package.

LW0 — is an INTEGER argument which must be set to the length of working space W0. Its value must be at least

$$n(3 + n) + 2 r_1(n + r_2) + r_2(1 + n)$$

where r_1 and r_2 are the number of rows and columns of matrix **D**, respectively. LW0 = n(4 + 6n) may be used to provide easy evaluation and sufficient value for LW0.

A is a REAL array of dimensions (N, N). On entry, it must be set to **A**, the original coefficient matrix of the linear equations. On exit, it contains the LU factors of **A**.

B is a REAL array of length N. On entry, it must be set to **b**, the R.H.S. of the linear equations. On exit, it contains **x**, the solution of the original equations.

W0 is a REAL array used as workspace. Its length is given by LW0.

ICH is an INTEGER argument which must be set to the unit number (or channel number) that is to be used for the printed output generated by the package. Usually, it is the unit number of the file OUTPUT.

V. ARGUMENT LIST FOR ENTRY 2

Both subroutines, SOLCHA and SOLCHB should be called. SOLCHB should be called after calling SOLCHA. The subroutine calls are

CALL SOLCHA (N, IR1, IR2, A, X0, V, W, PV, WAV, RHS, W1)

and

CALL SOLCHB (N, IR1, IR2, IR, D, PV, WAV, RHS, X0, X, H, W1, S).

The arguments are as follows.

For Both SOLCHA and SOLCHB

N is an INTEGER argument which must be set to n, the number of rows or columns of **A**. Its values must be positive and it is not altered by the package.

IR1 is an INTEGER argument which must be set to r_1, the number of rows of **D**. Its value must be positive and it is not altered by the package.

IR2 is an INTEGER argument which must be set to r_2, the number of columns of **D**. Its value must be positive and it is not altered by the package.

X0 is a REAL array which must be set to **x**, the solution of the original system, i.e., the solution of the unperturbed linear equations. The length of X0 is N. X0 is not altered by either SOLCHA or SOLCHB.

W1 is a REAL array used as workspace. Its length is N.

For SOLCHA Only

A is a REAL array of dimensions (N, N). It must be set to the LU factors of **A** and is not altered by the package.

V is a REAL array of dimensions (N, IR1). It must be set to **V**. (See the definition of **V** in (8)) and is not altered by the package.

W is a REAL array of dimensions (N, IR2). It must be set to **W** (see the definition of **W** in (8)) and is not altered by the package.

For SOLCHB Only

IR is an INTEGER argument which must be set to IR1 if IR1 $<$ IR2 or to IR2 if IR1 $\geq$ IR2. It is not altered by the package.

D is a REAL array of dimension (IR1, IR2). It must be set to **D** (see the definition of **D** in (8)) and is not altered by the package.

X is a REAL array of length N. On exit, it contains $\mathbf{x}_{new}$, the solution vector of the perturbed linear equations (see (3)).

H is a REAL array used as workspace. Its dimensions are (IR, IR). On exit, it contains the LU factors of the coefficient matrix of (11) if IR1 $\geq$ IR2 or (13), if IR1 $<$ IR2.

S is a REAL array used as workspace. Its length is IR. On exit, it contains the solution of (11) if IR1 $\geq$ IR2 or (13), if IR1 $<$ IR2.

Output Arguments for SOLCHA and Input Arguments for SOLCHB

PV is a REAL array of dimensions (N, IR1). On exit from SOLCHA, it contains $\mathbf{P}_V$, the solutions of linear equation with coefficient matrix as **A** and the R.H.S. as **V** (see (10)). These values of PV must be preserved between calls SOLCHA and SOLCHB. It is not altered by SOLCHB.

WAV is a REAL array of dimensions (IR2, IR1). On exit from SOLCHA, it contains $\mathbf{W}^T \mathbf{P}_V$. These values must be preserved between calls SOLCHA and SOLCHB. It is not altered by SOLCHB.

RHS is a REAL array of length IR2. On exit from SOLCHA, it contains $\mathbf{W}^T\mathbf{x}$. These values must be preserved between calls SOLCHA and SOLCHB. It is not altered by SOLCHB.

VI. EXAMPLES

Example 1 (The SOLCH package is called at Entry 1)

Consider the 4 x 4 system of linear equations,

$$\begin{bmatrix} 1 & 4 & 2 & 4 \\ 2 & 3 & 0 & 8 \\ 3 & 2 & 9 & 1 \\ 4 & 1 & 5 & 9 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 10 \\ 13 \\ 4 \\ 5 \end{bmatrix}. \tag{15}$$

Various perturbations to the coefficient matrix are expressed by different sets of **V**, **D** and **W**. Together with r_1 and r_2, the number of rows and columns of **D** respectively, **V**, **D** and **W** are entered through an interactive procedure. The following shows the main program and the user-computer dialogue procedure.

```
      PROGRAM EXAMP1(INPUT,OUTPUT,TAPE6=OUTPUT)
C
C     EXAMPLE 1.
C
      DIMENSION A(4,4),B(4),W0(120)
C
      DATA A/1.,2.,3.,4.,
     +       4.,3.,2.,1.,
     +       2.,0.,9.,5.,
     +       4.,8.,1.,9./
      DATA B/10.,13.,4.,5./
      DATA N,LW0,ICH/4,120,6/
C
      WRITE(6,60) ((A(I,J),J=1,N),B(I),I=1,N)
      CALL SOLCH(N,LW0,A,B,W0,ICH)
      STOP
   60 FORMAT(1H1,10X,"MATRIX [A]",13X,"VECTOR [B]"//
     +4(5X,4F5.1,F15.1//)//)
      END
```

```
        MATRIX [A]                    VECTOR [B]

   1.0   4.0   2.0   4.0                 10.0

   2.0   3.0   0.0   8.0                 13.0

   3.0   2.0   9.0   1.0                  4.0

   4.0   1.0   5.0   9.0                  5.0


*******************************************************
SOLVING ([A]+[V][D][W]')[X]=[B] BY LARGE CHANGE FORMULA

   SOLUTIONS BEFORE LARGE CHANGE :

                                  VECTOR [X]

                                   8.94545

                                   3.94545

                                  -3.18182

                                  -2.09091


 ENTER  IR1 AND IR2   ( SUCH THAT [D] IS OF IR1 BY IR2 )
*INPUT*  3,2
 [V], [D] AND [W] ARE MATRICES OF DIMENSIONS N BY IR1,
                  IR1 BY IR2 AND N BY IR2 RESPECTIVELY
                  SUCH THAT  DELTA( [A] )=[V][D][W]'

ENTER [V]             ( COLUMN BY COLUMN )
*INPUT*  1.,0.,0.,0.
*INPUT*  0.,1.,0.,0.
*INPUT*  0.,0.,0.,1.
ENTER [W]             ( COLUMN BY COLUMN )
*INPUT*  0.,1.,0.,0.
*INPUT*  0.,0.,1.,0.
    NUMBER OF ROWS OR COLUMNS OF [A] ( N )  .........  4

    NUMBER OF ROWS OF [D] ( IR1 )           .........  3

    NUMBER OF COLUMNS OF [D] ( IR2 )        .........  2

     MATRIX [V] :

            1.00000   0.00000   0.00000

            0.00000   1.00000   0.00000

            0.00000   0.00000   0.00000

            0.00000   0.00000   1.00000
```

```
MATRIX [W] :

        0.00000   0.00000

        1.00000   0.00000

        0.00000   1.00000

        0.00000   0.00000


[D] IS OF 3 BY 2.
ENTER [D]               ( COLUMN BY COLUMN )
*INPUT*  1.,2.,3.
*INPUT*  4.,5.,6.

     MATRIX [D] :

        1.00000   4.00000

        2.00000   5.00000

        3.00000   6.00000


   SOLUTIONS AFTER LARGE CHANGE :

                                  VECTOR [X]

                                   5.21538

                                   4.41538

                                  -2.15385

                                  -1.09231

 TRY ANOTHER [D] ?   ENTER 1 OR 2  ( Y OR N )
*INPUT*  1
[D] IS OF 3 BY 2.
ENTER [D]               ( COLUMN BY COLUMN )
*INPUT*  3.,4.,5.
*INPUT*  0.,8.,3.

     [V] AND [W] ARE THE SAME AS THOSE WITH THE PREVIOUS [D].

     MATRIX [D] :

        3.00000   0.00000

        4.00000   8.00000

        5.00000   3.00000
```

```
SOLUTIONS AFTER LARGE CHANGE :

                              VECTOR [X]

                              -3.09262

                               1.64494

                               1.12864

                               -.16981

 TRY ANOTHER [D] ?   ENTER 1 OR 2  ( Y OR N )
*INPUT*  2
 TRY ANOTHER SET OF [V], [D] AND [W] ?
  ENTER 1 OR 2  ( Y OR N )
*INPUT*  1
 ENTER  IR1 AND IR2   ( SUCH THAT [D] IS OF IR1 BY IR2 )
*INPUT*  1,2
 [V], [D] AND [W] ARE MATRICES OF DIMENSIONS N BY IR1,
                  IR1 BY IR2 AND N BY IR2 RESPECTIVELY
                  SUCH THAT  DELTA( [A] )=[V][D][W]'

ENTER [V]              ( COLUMN BY COLUMN )
*INPUT*  1.,-2.,4.,7.
ENTER [W]              ( COLUMN BY COLUMN )
*INPUT*  2.,0.,4.,5.
*INPUT*  3.,1.,1.,2.
    NUMBER OF ROWS OR COLUMNS OF [A] ( N )  .........  4

    NUMBER OF ROWS OF [D] ( IR1 )           .........  1

    NUMBER OF COLUMNS OF [D] ( IR2 )        .........  2

     MATRIX [V] :

            1.00000

           -2.00000

            4.00000

            7.00000


     MATRIX [W] :

            2.00000   3.00000

            0.00000   1.00000

            4.00000   1.00000

            5.00000   2.00000


[D] IS OF 1 BY 2.
ENTER [D]              ( COLUMN BY COLUMN )
*INPUT*  1.,3.

     MATRIX [D] :

            1.00000   3.00000
```

```
SOLUTIONS AFTER LARGE CHANGE :

                         VECTOR [X]

                          -3.09165

                           1.00509

                           1.50438

                           1.76832

 TRY ANOTHER [D] ?    ENTER 1 OR 2  ( Y OR N )
*INPUT*   1
[D] IS OF 1 BY 2.
ENTER [D]             ( COLUMN BY COLUMN )
*INPUT*  3.,7.

    [V] AND [W] ARE THE SAME AS THOSE WITH THE PREVIOUS [D].

    MATRIX [D] :

          3.00000   7.00000


SOLUTIONS AFTER LARGE CHANGE :

                         VECTOR [X]

                          -3.46347

                            .91427

                           1.64914

                           1.88752

 TRY ANOTHER [D] ?    ENTER 1 OR 2  ( Y OR N )
*INPUT*   2
 TRY ANOTHER SET OF [V], [D] AND [W] ?
  ENTER 1 OR 2  ( Y OR N )
*INPUT*   2
```

Example 2 (The SOLCH package is called at Entry 2)

Consider the linear equations of (1) with

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 & 3 & 4 \\ 1 & 6 & 9 & 6 & 0 \\ 5 & 7 & 2 & 5 & 9 \\ 0 & 2 & 1 & 4 & 3 \\ 9 & 1 & 0 & 1 & 6 \end{bmatrix} \qquad (16)$$

and

$$\mathbf{b} = [14 \quad 18 \quad 42 \quad 90 \quad 21]^T .$$

The variable parameters of the system are A_{12}, A_{14}, A_{32}, A_{34}, A_{42} and A_{44}. An easy expression of $\Delta\mathbf{A}$ in the form of $\mathbf{V}\ \mathbf{D}\ \mathbf{W}^T$ can be such that

$$\mathbf{V} = [\mathbf{u}_1\ \mathbf{u}_3\ \mathbf{u}_4], \qquad (17)$$

$$\mathbf{W} = [\mathbf{u}_2\ \mathbf{u}_4] \qquad (18)$$

and

$$\mathbf{D} = \begin{bmatrix} \Delta A_{12} & \Delta A_{14} \\ \Delta A_{32} & \Delta A_{34} \\ \Delta A_{42} & \Delta A_{44} \end{bmatrix}, \qquad (19)$$

where $\mathbf{u}_i$, i = 1, 2, 3, 4, are unit 5-vectors with 1 at its ith row and zeros everywhere else. Suppose the variable parameters are perturbed 4 times according to Table I. Therefore, we have 4 different $\mathbf{D}$ matrices corresponding to the same set of $\mathbf{V}$ and $\mathbf{W}$. SOLCHA is called only once and SOLCHB is called 4 times afterwards. The values of arrays PV, WAV and RHS are preserved after exiting from SOLCHA all through the 4 callings of SOLCHB.

```
      PROGRAM EXAMP2(INPUT,OUTPUT,TAPE6=OUTPUT)
C
C     EXAMPLE 2.
C
      DIMENSION A(5,5),B(5),X(5),V(5,3),D(3,2),W(5,2),DD(3,2,4)
      DIMENSION PV(5,3),WAV(2,3),RHS(2),H(2,2),W1(5),W2(2)
C
      DATA N,IR1,IR2,IR/5,3,2,2/
      DATA A/2.,1.,5.,0.,9.,
     +       4.,6.,7.,2.,1.,
     +       3.,9.,2.,1.,0.,
     +       3.,6.,5.,4.,1.,
     +       4.,0.,9.,3.,6./
      DATA B/14.,18.,42.,90.,21./
      DATA V,W/1.,0.,0.,0.,0.,    0.,0.,1.,0.,0.,
     +         0.,0.,0.,1.,0.,
     +         0.,1.,0.,0.,0.,    0.,0.,0.,1.,0./
      DATA DD/2.,3.,4., 5.,3.,8.,    4.,6.,9., 1.,2.,4.,
     +        3.,4.,7., 1.,2.,9.,    2.,4.,9., 0.,4.,1./
C
      CALL LUFACT(N,A,B,1)
C
      PRINT*,"    SOLUTIONS BEFORE LARGE CHANGE :"
      WRITE(6,50) B
      CALL SOLCHA(N,IR1,IR2,A,B,V,W,PV,WAV,RHS,W1)
C
      DO 20 K=1,4
      DO 10 I=1,IR1
      DO 10 J=1,IR2
   10 D(I,J)=DD(I,J,K)
C
      CALL SOLCHB(N,IR1,IR2,IR,D,PV,WAV,RHS,B,X,H,W1,W2)
      PRINT*,"    SOLUTIONS AFTER LARGE CHANGE (",K,") :"
   20 WRITE(6,50) X
      STOP
C
   50 FORMAT(1H1/31X,"VECTOR [X]"//10(22X,F18.5//)//)
      END
```

```
SOLUTIONS BEFORE LARGE CHANGE :

                         VECTOR [X]

                          -1.69859

                         -17.45070

                          -5.99155

                          29.72113

                           4.00282
```

SOLUTIONS AFTER LARGE CHANGE (1) :

VECTOR [X]

48.73250

59.12208

-43.84201

1.51885

-79.70557

SOLUTIONS AFTER LARGE CHANGE (2) :

VECTOR [X]

8.71290

2.57559

-9.69487

13.51456

-12.25104

SOLUTIONS AFTER LARGE CHANGE (3) :

VECTOR [X]

-2.05340

-6.53883

-.13107

10.07767

5.99029

SOLUTIONS AFTER LARGE CHANGE (4) :

VECTOR [X]

11.71383

13.44651

-8.14236

-.18527

-16.28095

<u>Example 3</u> (The SOLCH package is called at Entry 2)

Consider the linear equations

$$\mathbf{Y}\,\mathbf{X} = \mathbf{I}\,, \tag{20}$$

where

$$\mathbf{Y} = \begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \tag{21}$$

and

$$\mathbf{I} = [1 \quad 0 \quad 0 \quad 0]^T\,. \tag{22}$$

Suppose variable parameters of the system are G_2 and G_6 affecting **Y** such that

$$\Delta\mathbf{Y} = \begin{bmatrix} \Delta G_2 & -\Delta G_2 & 0 & 0 \\ -\Delta G_2 & \Delta G_2 & 0 & 0 \\ 0 & 0 & \Delta G_6 & -\Delta G_6 \\ 0 & 0 & -\Delta G_6 & \Delta G_6 \end{bmatrix}. \tag{23}$$

It should be noticed that if we choose

$$\mathbf{V} = \mathbf{W} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \tag{24}$$

and

$$\mathbf{D} = \begin{bmatrix} \Delta G_2 & 0 \\ 0 & \Delta G_6 \end{bmatrix}, \tag{25}$$

then

$$\Delta\mathbf{Y} = \mathbf{V}\,\mathbf{D}\,\mathbf{W}^T\,. \tag{26}$$

The variables G_2 and G_6 are perturbed 5 times as shown in Table II. Also, SOLCHA is used only once, whereas SOLCHB is called 5 times corresponding to different values of **D**.

The reader may notice that Example 3 corresponds to the electrical circuit of Fig. 4. The results show different values of every nodal voltage for different values of conductances G_2 and G_6.

```
      PROGRAM EXAMP3( INPUT, OUTPUT, TAPE6=OUTPUT)
C
C     EXAMPLE 3.
C
      DIMENSION Y(4,4),AI(4),X(4),V(4,2),D(2,2),W(4,2),DG26(2,5)
      DIMENSION PV(4,2),WAV(2,2),RHS(2),H(2,2),W1(4),W2(2)
C
      DATA N,IR1,IR2,IR/4,2,2,2/
      DATA Y/ 3.,-1., 0.,-1.,
     +       -1., 3.,-1., 0.,
     +        0.,-1., 3.,-1.,
     +       -1., 0.,-1., 2./
      DATA AI/1.,0.,0.,0./
      DATA V,W/1.,-1.,0.,0.,    0.,0.,1.,-1.,
     +         1.,-1.,0.,0.,    0.,0.,1.,-1./
      DATA DG26/2.,-1.,    1.5,.8,    -.7,.6,    .4,-.3,    .2,.1/
      DATA D/4*0./
C
      CALL LUFACT(N,Y,AI,1)
C
      PRINT*,"    NODAL VOLTAGES BEFORE LARGE CHANGE :"
      WRITE(6,50) AI
C
      CALL SOLCHA(N,IR1,IR2,Y,AI,V,W,PV,WAV,RHS,W1)
C
      DO 20 K=1,5
      D(1,1)=DG26(1,K)
      D(2,2)=DG26(2,K)
C
      CALL SOLCHB(N,IR1,IR2,IR,D,PV,WAV,RHS,AI,X,H,W1,W2)
      PRINT*,"    NODAL VOLTAGES AFTER LARGE CHANGE (",K,") :"
   20 WRITE(6,50) X
      STOP
C
   50 FORMAT(/20X,F18.5/)
      END
```

NODAL VOLTAGES BEFORE LARGE CHANGE :

.54167

.25000

.20833

.37500

NODAL VOLTAGES AFTER LARGE CHANGE (1) :

.50000

.33333

.16667

.50000

NODAL VOLTAGES AFTER LARGE CHANGE (2) :

.46282

.30804

.22914

.31259

NODAL VOLTAGES AFTER LARGE CHANGE (3) :

.61684

.17219

.21097

.36708

NODAL VOLTAGES AFTER LARGE CHANGE (4) :

.52218

.27474

.20307

.39078

```
NODAL VOLTAGES AFTER LARGE CHANGE (5) :
                          .52398

                          .26305

                          .21298

                          .36107
```

REFERENCES

[1] J.W. Bandler and Q.J. Zhang, "A unified approach to first-order and large change sensitivity computations in linear systems", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-84-20-R, 1984.

[2] J.W. Bandler and Q.J. Zhang, "SOLCH - A Fortran package for solutions of perturbed linear equations", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-84-21-L, 1984.

TABLE I

DIFFERENT PERTURBATIONS TO VARIABLES IN EXAMPLE 2

Large Change	ΔA_{12}	ΔA_{14}	ΔA_{32}	ΔA_{34}	ΔA_{42}	ΔA_{44}
1	2	5	3	3	4	8
2	4	1	6	2	9	4
3	3	1	4	2	7	9
4	2	0	4	4	9	1

TABLE II

DIFFERENT PERTURBATIONS TO VARIABLES IN EXAMPLE 3

Large Change	ΔG_2	ΔG_6
1	2.0	–1.0
2	1.5	0.8
3	–0.7	0.6
4	0.4	–0.3
5	0.2	0.1

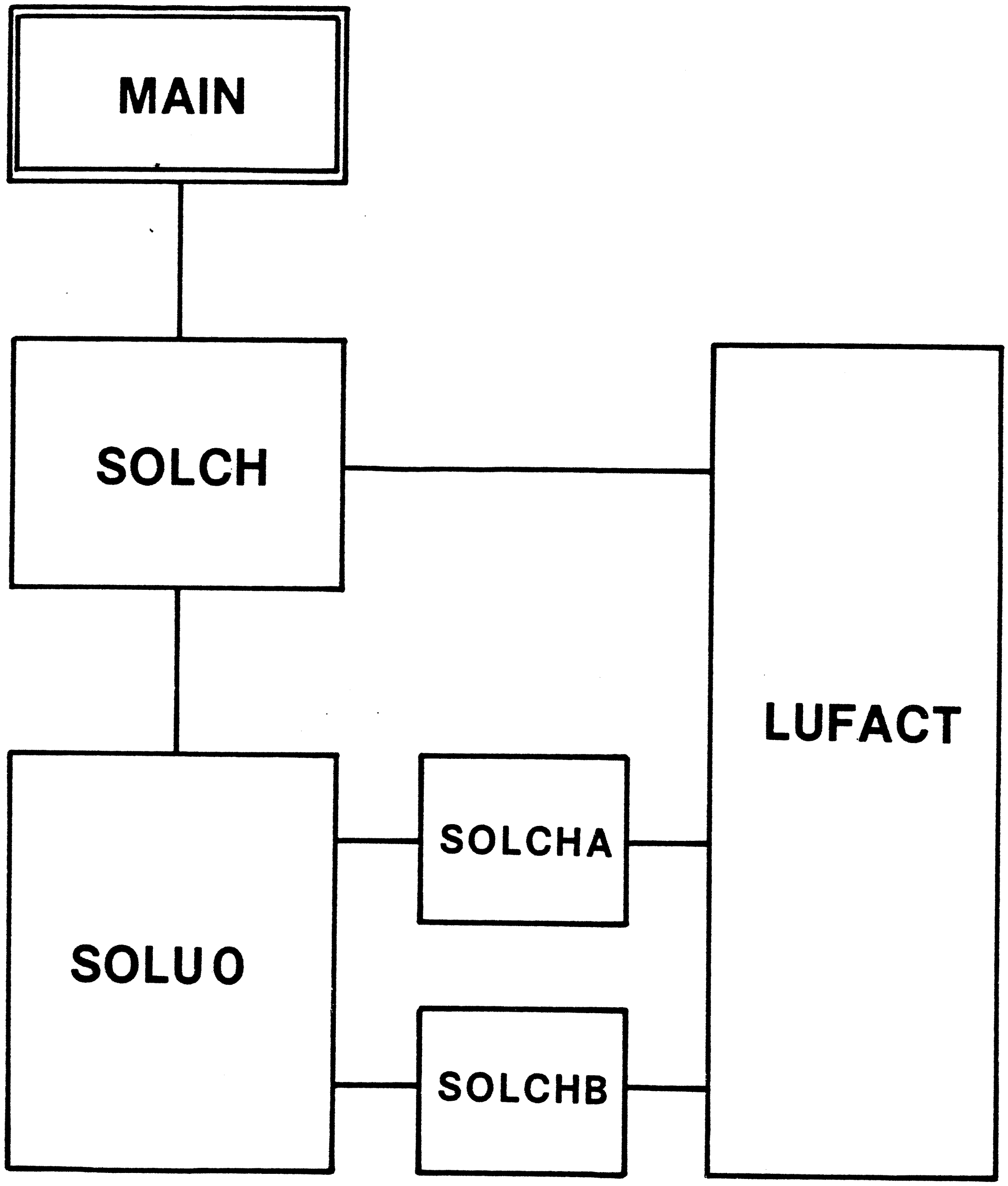

Fig. 1 Structure of the SOLCH package corresponding to Entry 1.

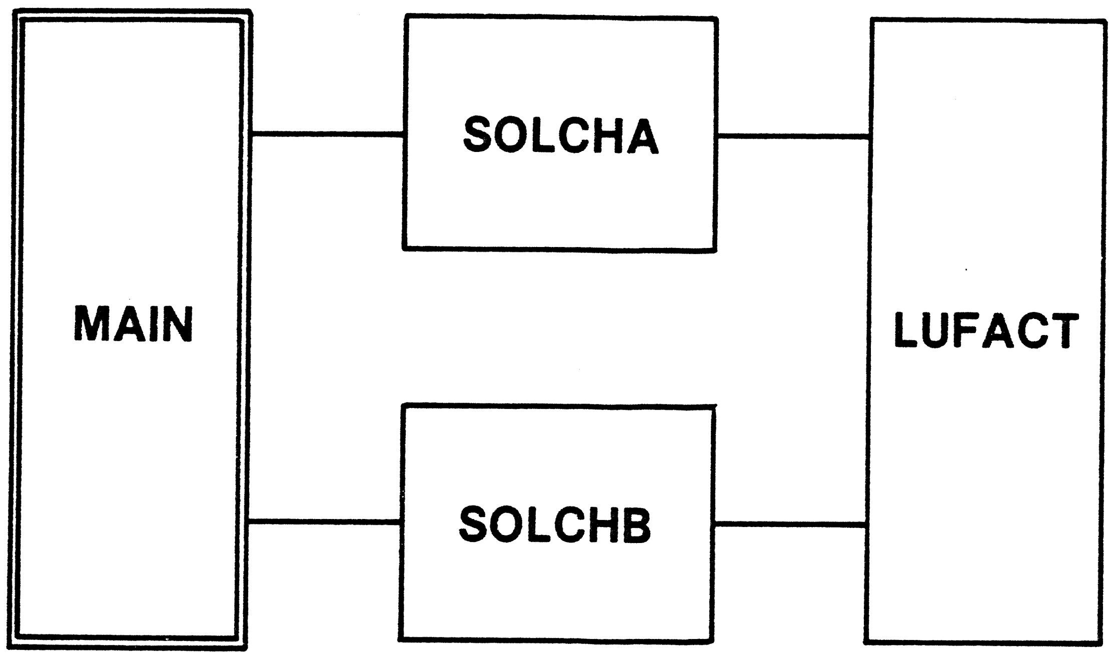

Fig. 2 Structure of the SOLCH package corresponding to Entry 2.

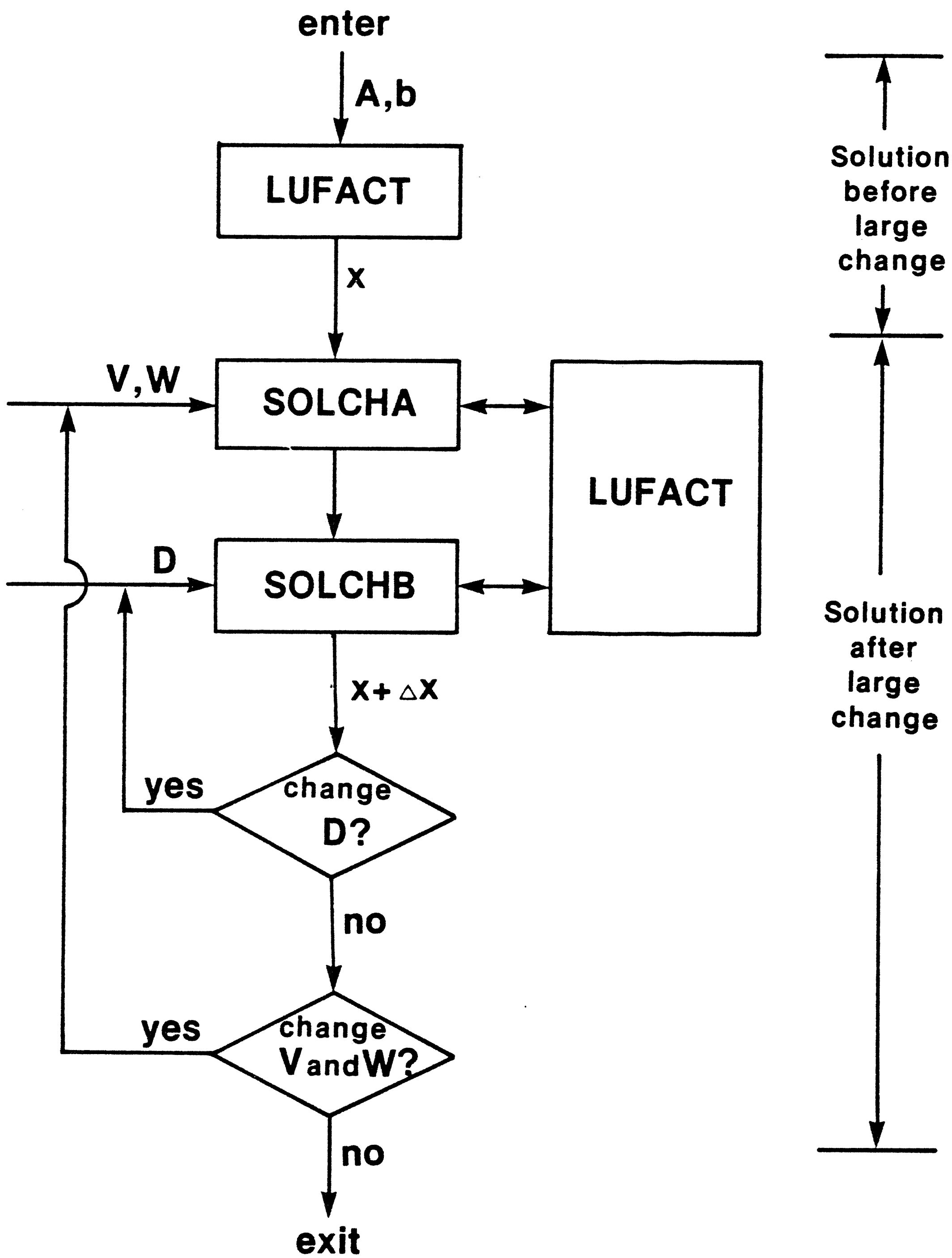

Fig. 3 The logical relation between subroutines SOLCHA, SOLCHB and LUFACT.

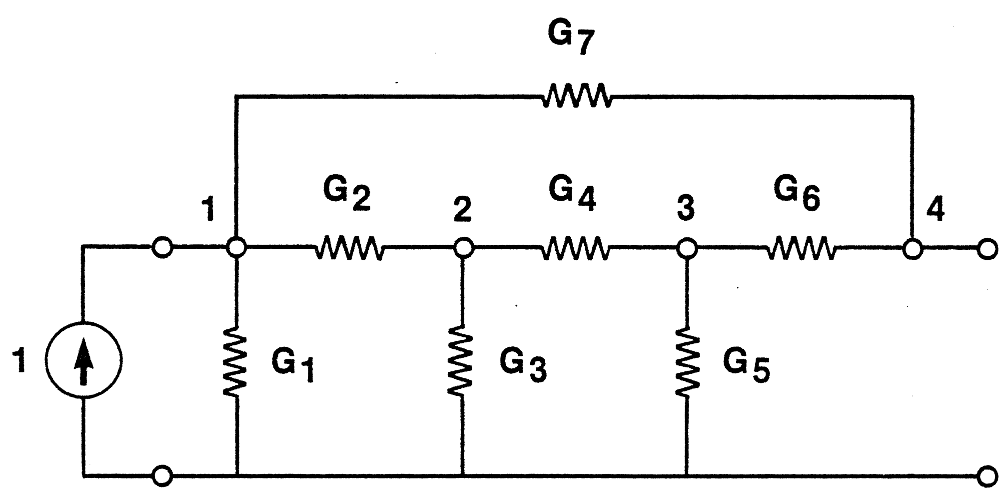

Fig. 4 The electrical circuit configuration for Example 3.