SIMULATION OPTIMIZATION SYSTEMS
Research Laboratory

COMPUTER AIDED DESIGN OF BRANCHED CASCADED NETWORKS

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Summary

A new and attractive theory is presented for computer orientated simulation, sensitivity analysis and design of branched cascaded circuits. The forward and reverse analysis approach developed by Bandler et al. [1] for cascaded circuit analysis is extended and applied to general branched cascaded circuits. This theory permits an efficient and fast analytical and numerical investigation of responses and sensitivities of all functions of interest w.r.t. any variable parameter, including frequency. Thevenin equivalent circuits at any reference plane and their sensitivities are also expressed analytically and calculated systematically. Thus, responses such as common port return loss, branch output return loss, insertion or transducer loss, gain slope and group delay can be handled exactly and efficiently. The theory is presented together with an arbitrary example of a 4-branched cascaded circuit, followed by a practical 12-channel multiplexer.

To apply forward and reverse analysis, 3-port junctions are reduced to 2-port representations so that the cascaded analysis can be readily carried through these junctions in different desired directions. Consider the 3-port network shown in Fig. 1. To carry the

analysis through the junction along the main cascade, we terminate port 3 and represent the transmission matrix between ports 1 and 2 by **A**. The linear combination between the voltages and currents at ports 2 and 3 can be expressed as

$$\boldsymbol{\alpha}^T \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \boldsymbol{\beta}^T \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix} . \tag{1}$$

The analysis can also be carried through the junctions into any desired branch by terminating port 2 and denoting the transmission matrix between ports 1 and 3 by **D**.

Consider a network consisting of N sections, as shown in Fig. 2. A typical section has a junction, n(k) cascaded elements of branch k and a subsection along the main cascade. All reference planes in the entire network are defined uniformly and numbered consecutively beginning from the main cascade termination, which is designated reference plane 1. The source port is designated reference plane 2N + 2. The termination of the kth branch is called reference plane $\tau(k)$ and the branch main cascade connection is reference plane $\sigma(k)$, k = 1, 2, ..., N, where

$$\begin{aligned} \tau(1) &= 2N + 3, \\ \sigma(k) &= \tau(k) + n(k), \qquad k = 1, 2, \ldots, N, \\ \tau(k) &= \sigma(k-1) + 1, \qquad k = 2, 3, \ldots, N. \end{aligned} \tag{2}$$

Two-port matrix and vector representations **A**, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and **D** are calculated for each branch/junction combination and are denoted as $\mathbf{A}_{2k}$, $\boldsymbol{\alpha}_{2k}$, $\boldsymbol{\beta}_{2k}$ and $\mathbf{D}_{2k}$ for the kth junction. Elements in every branch and subsection in every section are represented by chain matrices $\mathbf{A}_i$, where i is the index of the reference plane at the output of the corresponding element or subsection.

Let

$$I_r = \{1, 2, 3, \ldots, \sigma(N)\} \tag{3}$$

be the index set containing indexes of all reference planes and

$$I = \{i \mid i \in I_r, i \neq 2N+2, i \neq \sigma(k), k = 1, 2, \ldots, N\} \tag{4}$$

be the index set containing subscripts of all **A** matrices which can logically be defined using the subscript of the associated output reference plane.

The forward analysis $(\mathbf{u}^{xi})^T$ (reverse analysis $\mathbf{v}^{ix}$) is the result of a row (column) vector initialized at reference plane x as either $[1 \quad 0]$, $[0 \quad 1]$ ($[1 \quad 0]^T$, $[0 \quad 1]^T$) or a suitable linear combination and successively premultiplying (postmultiplying) each corresponding chain matrix by the resulting row (column) vector until reference plane i is reached.

The result of the analysis between reference planes i and j is defined as

$$\mathbf{Q}_{ij} \triangleq [\mathbf{p}_{ij} \quad \mathbf{q}_{ij}] \triangleq \begin{bmatrix} A_{ij} & B_{ij} \\ C_{ij} & D_{ij} \end{bmatrix}, \tag{5}$$

where

$$\mathbf{p}_{ij} \triangleq \begin{bmatrix} A_{ij} \\ C_{ij} \end{bmatrix}, \mathbf{q}_{ij} \triangleq \begin{bmatrix} B_{ij} \\ D_{ij} \end{bmatrix} \tag{6}$$

and where A_{ij}, B_{ij}, C_{ij} and D_{ij} are the equivalent chain matrix elements between reference planes i and j and are expressed in the form $\mathbf{u}^T \mathbf{A} \mathbf{v}$ to facilitate sensitivity, first-order change, and large change analysis [1]. For example, we have

$$\frac{\partial Q_{ij}}{\partial \phi} = \sum_{\ell \in I_\phi} \frac{\partial Q_{ij}^{\ell}}{\partial \phi}, \tag{7}$$

where I_ϕ is an index set whose elements identify the chain matrices between reference planes i and j containing the variable parameter ϕ and Q represents A, B, C or D.

Having performed the appropriate forward and reverse analysis, the branch output responses and their sensitivities are readily calculated. For example, for a short-circuit main cascade termination (at reference plane 1), we can calculate the output voltage of the kth branch and its sensitivities as

$$V = \frac{\boldsymbol{\alpha}^T \mathbf{q}_{2k,1} V_S}{\boldsymbol{\beta}^T \mathbf{p}_{\sigma\tau} B_{2N+2,1}} \tag{8}$$

and

$$V' = \frac{(\boldsymbol{\alpha}^T \mathbf{q}_{2k,1} V_S)' - (\boldsymbol{\beta}^T \mathbf{p}_{\sigma\tau} B_{2N+2,1})' V}{\boldsymbol{\beta}^T \mathbf{p}_{\sigma\tau} B_{2N+2,1}}. \tag{9}$$

Similar response and sensitivity formulas are also available for different excitations and terminations [2]. For different ϕ, appearing in different parts of the network, the sensitivity formulas can be simplified. For example, if ϕ is in a section to the left of the branch for which the output voltage sensitivity is calculated, (9) is reduced to

$$V' = \frac{-B'_{2N+2,1} V}{B_{2N+2,1}} . \qquad (10)$$

The branch output voltages can be utilized to compute insertion loss between output and common ports. Sensitivities of insertion loss for each branch output w.r.t. all variables are also computed using the sensitivities of the corresponding branch output voltage. In particular, the sensitivities w.r.t. frequency can result in the exact calculation of group delay and gain slope for each branch [3-4].

The group delay from the source port to the kth branch output port, namely T_G^k is calculated as

$$T_G = -\mathrm{Im}\left\{ \frac{(\boldsymbol{\alpha}^T \mathbf{q}'_{2k,1} + \mathbf{q}^T_{2k,1} \boldsymbol{\alpha}')}{\boldsymbol{\alpha}^T \mathbf{q}_{2k,1}} - \frac{(\boldsymbol{\beta}^T \mathbf{p}'_{\sigma\tau} + \mathbf{p}^T_{\sigma\tau} \boldsymbol{\beta}')}{\boldsymbol{\beta}^T \mathbf{p}_{\sigma\tau}} - \frac{B'_{2N+2,1}}{B_{2N+2,1}} \right\}, \qquad (11)$$

where $T_G \equiv T_G^k$, $\boldsymbol{\beta} \equiv \boldsymbol{\beta}_{2k}$, $\boldsymbol{\alpha} \equiv \boldsymbol{\alpha}_{2k}$, $\sigma \equiv \sigma(k)$, $\tau \equiv \tau(k)$ and $\partial/\partial\omega$ is denoted as $'$.

The gain slope for the kth branch output port is

$$S_G^k = \frac{\partial}{\partial\omega} L_I^k$$

$$= \frac{-20}{\ell n\, 10} \mathrm{Re}\left\{ \frac{(\boldsymbol{\alpha}^T \mathbf{q}'_{2k,1} + \mathbf{q}^T_{2k,1} \boldsymbol{\alpha}')}{\boldsymbol{\alpha}^T \mathbf{q}_{2k,1}} - \frac{(\boldsymbol{\beta}^T \mathbf{p}'_{\sigma\tau} + \mathbf{p}^T_{\sigma\tau} \boldsymbol{\beta}')}{\boldsymbol{\beta}^T \mathbf{p}_{\sigma\tau}} - \frac{B'_{2N+2,1}}{B_{2N+2,1}} \right\} \qquad (12)$$

The common port and branch output port return loss responses are evaluated using the Thevenin equivalent approach originated by Bandler et al. [1]. Denoting the Thevenin equivalent voltages and impedances at reference planes i and j by V_S^i, Z_S^i, V_S^j and Z_S^j, we have

$$V_S^j = \frac{V_S^i}{A_{ij} + Z_S^i C_{ij}} \qquad (13)$$

and

$$Z_S^j = \frac{B_{ij} + Z_S^i D_{ij}}{A_{ij} + Z_S^i C_{ij}} . \tag{14}$$

The sensitivities are obtained as

$$(V_S^j)' = \frac{(V_S^i)' - [A_{ij}' + Z_S^i C_{ij}' + (Z_S^i)' C_{ij}] V_S^j}{A_{ij} + Z_S^i C_{ij}} \tag{15}$$

and

$$(Z_S^j)' = \frac{[1 \quad Z_S^i] \mathbf{Q}_{ij}' \begin{bmatrix} -Z_S^j \\ 1 \end{bmatrix} + (Z_S^i)' (D_{ij} - Z_S^j C_{ij})}{A_{ij} + Z_S^i C_{ij}} . \tag{16}$$

If the reflection coefficient at the branch output port is to be calculated (evaluation of branch output return loss), then (14) and (16) are specialized to

$$Z_S^{\tau+1} = \frac{B_{2N+2,\tau+1}}{A_{2N+2,\tau+1}} \tag{17}$$

and

$$(Z_S^{\tau+1})' = \frac{B_{2N+2,\tau+1}' - A_{2N+2,\tau+1}' Z_S^{\tau+1}}{A_{2N+2,\tau+1}} . \tag{18}$$

Norton equivalent admittances and current sources are calculated similar to the Thevenin equivalents. The Norton equivalent admittance at the common port, given by

$$Y_L^{2N+2} = \frac{D_{2N+2,1}}{B_{2N+2,1}} , \tag{19}$$

is used in computation of common port reflection coefficient and return loss [2].

The theory discussed above has been implemented into a computer program for simulation and sensitivity analysis of branched cascaded networks with an arbitrary number of sections and arbitrary number of branch elements. Exact sensitivity analysis can be performed w.r.t. any variable, including frequency.

Consider an arbitrary 4-branch cascaded circuit depicted in Fig. 3. All element values are normalized. The normalized frequency is 1 Hz. Table I shows results of the simulation. Tables II to VII summarize the sensitivities of various responses w.r.t. variables ϕ_1, ϕ_2, ..., ϕ_8. Sensitivities of responses w.r.t. frequency are given in Table VIII. Table IX gives numerical values for the gain slope and group delay of all branches. The units for all quantities are SI units except as noted.

A practical application of the theory which we have presented, is the optimal design of contiguous-band multiplexers. We have used our simulation and sensitivity formulas in conjunction with the powerful gradient-based minimax optimization procedure of Hald and Madsen [5] to optimize a 12-channel, 12 GHz multiplexer without dummy channels [6]. The structure under consideration consists of synchronously and asynchronously tuned multi-coupled cavity filters distributed along a waveguide manifold. Waveguide spacings, input and output transformer ratios, cavity resonant frequencies as well as intercavity couplings are used as optimization variables. A lower specification of 20 dB on return loss has been imposed. The problem involves 60 nonlinear design variables. The filters are assumed lossy and dispersive; waveguide junctions are assumed nonideal. The results of optimization are shown in Fig. 4.

References

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TABLE I

NUMERICAL VALUES OF THE RESPONSES FOR THE 4-BRANCH CASCADED NETWORK OF FIG. 3

Type of Response	Branch 1†	Branch 2	Branch 3	Branch 4
output voltage	0.03624 -j0.07487	-0.07595 -j0.06875	0.05983 -j0.04039	-15.00361 +j1.16405
Thevenin equivalent voltage*	0.03008 -j0.07785	0.03529 -j0.30176	0.03193 -j0.08172	-15.65346 -j2.31876
Thevenin equivalent impedance*	0.00003 -j0.08225	0.72129 +j2.41490	0.00004 -j0.69080	0.02515 +j0.23408
insertion loss (dB)	55.57892	53.76940	56.81050	10.42942
branch port return loss (dB)	0.00055	1.72670	0.00052	0.41430

common port return loss = 0.41243 dB

† Branch 1 is the furthest from the common port.

* Thevenin equivalents for each branch are evaluated at the reference plane just before the load corresponding to that branch.

TABLE II

SENSITIVITIES OF BRANCH OUTPUT VOLTAGES W.R.T.

VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 3

Variable	Branch 1	Branch 2	Branch 3	Branch 4
ϕ_1	−0.09888 +j0.19690	0.01602 +j0.01904	−0.12152 +j0.07920	−0.06148 +j0.43382
ϕ_2	−0.02178 +j0.03689	0.00008 +j0.00013	−0.00083 +j0.00037	−0.00081 +j0.00263
ϕ_3 (per Gm)	0.41840 −j1.02730	0.42340 +j0.49683	−3.17461 +j2.09775	−1.54074 +j11.39034
ϕ_4	−0.00015 +j0.00018	0.02421 +j0.02442	−0.00152 +j0.00078	−0.00123 +j0.00500
ϕ_5 (per Gm)	−0.00000 −j0.00000	−0.00000 +j0.00000	−0.84583 −j1.25308	−0.00000 −j0.00000
ϕ_6 (per Gm)	0.42131 −j1.01718	−1.05647 −j0.85004	0.75952 −j0.57964	−1.32161 +j10.30781
ϕ_7	0.00216 +j0.00231	0.00347 −j0.00175	0.00061 +j0.00267	0.16241 +j0.04932
ϕ_8	0.03997 −j0.13157	−0.14168 −j0.09279	0.08734 −j0.08130	−12.42431 +j0.17372

TABLE III

SENSITIVITIES OF THEVENIN EQUIVALENT VOLTAGE SOURCES W.R.T.

VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 3

Variable	Branch 1	Branch 2	Branch 3	Branch 4
ϕ_1	−0.08217 +j0.20531	−0.01837 +j0.07139	−0.06664 +j0.16342	−0.20599 −j0.03020
ϕ_2	−0.01556 +j0.04023	−0.00019 +j0.00043	−0.00058 +j0.00095	−0.00125 −j0.00039
ϕ_3 (per Gm)	0.33572 −j1.06095	−0.47001 +j1.87577	−1.72106 +j4.29805	−5.40867 −j0.75803
ϕ_4	−0.00014 +j0.00020	−0.01427 +j0.08775	−0.00097 +j0.00183	−0.00237 −j0.00060
ϕ_5 (per Gm)	−0.00000 −j0.00000	−0.00000 −j0.00000	−0.46211 +j1.18227	0.00000 −j0.00000
ϕ_6 (per Gm)	0.33964 −j1.05097	0.24151 −j4.04905	0.36033 −j1.10264	−4.89480 −j0.65144
ϕ_7	0.00235 +j0.00213	0.01021 +j0.00537	0.00246 +j0.00225	0.15861 +j0.03550
ϕ_8	0.02916 −j0.13486	−0.01986 −j0.50186	0.03119 −j0.14165	−13.35199 −j1.80362

TABLE IV

SENSITIVITIES OF THEVENIN EQUIVALENT IMPEDANCES W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 3

Variable	Branch 1	Branch 2	Branch 3	Branch 4
ϕ_1	-0.00017 +j0.00707	0.00019 +j0.00077	-0.00016 +j0.00450	0.00038 +j0.03072
ϕ_2	-0.00003 +j0.04250	0.00000 0.00000	-0.00001 +j0.00003	-0.00003 +j0.00019
ϕ_3 (per Gm)	0.00085 +j0.02364	0.00501 +j0.02006	-0.00334 +j0.11797	0.01498 +j0.80592
ϕ_4	-0.00000 +j0.00000	0.06160 +j0.11222	-0.00001 +j0.00005	-0.00003 +j0.00036
ϕ_5 (per Gm)	-0.00000 -j0.00000	-0.00000 +j0.00000	-0.00129 +j30.93848	-0.00000 -j0.00000
ϕ_6 (per Gm)	0.00066 +j0.02605	0.17452 +j0.29796	0.00051 +j0.02879	0.01860 +j0.72854
ϕ_7	-0.00000 +j0.00000	-0.00000 +j0.00001	-0.00000 +j0.00000	-0.00052 +j0.00350
ϕ_8	0.00005 +j0.00000	0.00058 -j0.00028	0.00005 +j0.00000	0.04283 -j0.05844

TABLE V

SENSITIVITIES OF INSERTION LOSS W.R.T.

VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 3

Variable	Branch 1	Branch 2	Branch 3	Branch 4
ϕ_1	23.00568	2.09034	17.45152	–0.05475
ϕ_2	4.45823	0.01270	0.10816	–0.00059
ϕ_3 (per Gm)	–115.59096	54.88169	457.83905	–1.39517
ϕ_4	0.02412	2.91144	0.20388	–0.00093
ϕ_5 (per Gm)	–0.00000	–0.00000	0.00000	–0.00000
ϕ_6 (per Gm)	–114.77168	–114.77168	–114.77168	–1.22074
ϕ_7	0.11859	0.11859	0.11859	0.09126
ϕ_8	–14.18466	–14.18466	–14.18466	–7.15740

TABLE VI

SENSITIVITIES OF BRANCH PORT RETURN LOSS W.R.T.

VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 3

Variable	Branch 1	Branch 2	Branch 3	Branch 4
ϕ_1	–0.00292	–0.00050	–0.00183	0.00055
ϕ_2	–0.00057	–0.00000	–0.00006	–0.00050
ϕ_3 (per Gm)	0.01465	–0.01278	–0.03917	0.09851
ϕ_4	–0.00000	–0.00071	–0.00008	–0.00059
ϕ_5 (per Gm)	–0.00000	–0.00000	0.00000	–0.00000
ϕ_6 (per Gm)	0.01133	0.02123	0.00598	0.17232
ϕ_7	–0.00001	–0.00002	–0.00001	–0.00913
ϕ_8	0.00084	0.00155	0.00063	0.71641

TABLE VII

SENSITIVITIES OF COMMON PORT RETURN LOSS W.R.T.

VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 3

Variable	Sensitivity
Φ_1	0.00533
Φ_2	0.00004
Φ_3 (per Gm)	0.13797
Φ_4	0.00008
Φ_5 (per Gm)	0.00000
Φ_6 (per Gm)	0.12286
Φ_7	−0.00909
Φ_8	0.71310

TABLE VIII

SENSITIVITIES OF VARIOUS RESPONSES W.R.T.

ANGULAR FREQUENCY ω FOR THE CIRCUIT OF FIG. 3

Type of Response	Branch 1	Branch 2	Branch 3	Branch 4
output voltage	−0.17778 +j0.33120	0.03944 +j0.08791	−0.44100 +j0.26906	2.39068 +j7.36235
Thevenin equivalent voltage	−0.14889 +j0.34666	−0.10880 +j0.09567	−0.24384 +j0.59055	0.15888 +j0.51010
Thevenin equivalent impedance	−0.00028 +j0.02219	0.73081 +j1.32535	−0.00051 +j0.28101	−0.00138 +j0.50624
branch port return loss	−0.00484	−0.00153	−0.00590	−0.11597

sensitivity of common port return loss = −0.10460

TABLE IX

GAIN SLOPE AND GROUP DELAY FOR

THE CIRCUIT OF FIG. 3

Type of response	Branch 1	Branch 2	Branch 3	Branch 4
gain slope (dB/Hz)	246.411	47.006	390.162	6.579
group delay (s)	0.18892	0.37785	0.32862	0.50006

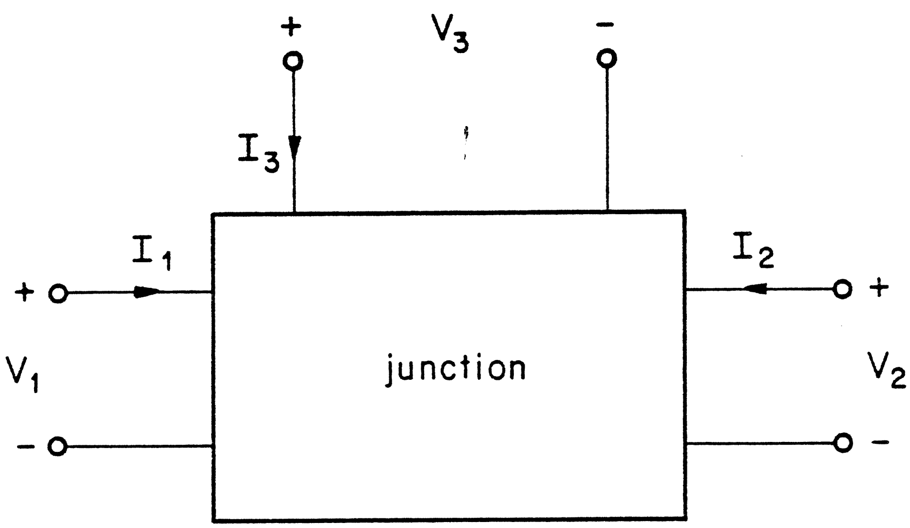

Fig. 1 A 3-port network in which ports 1 and 2 are considered along a main cascade and port 3 represents a branch of the main cascade.

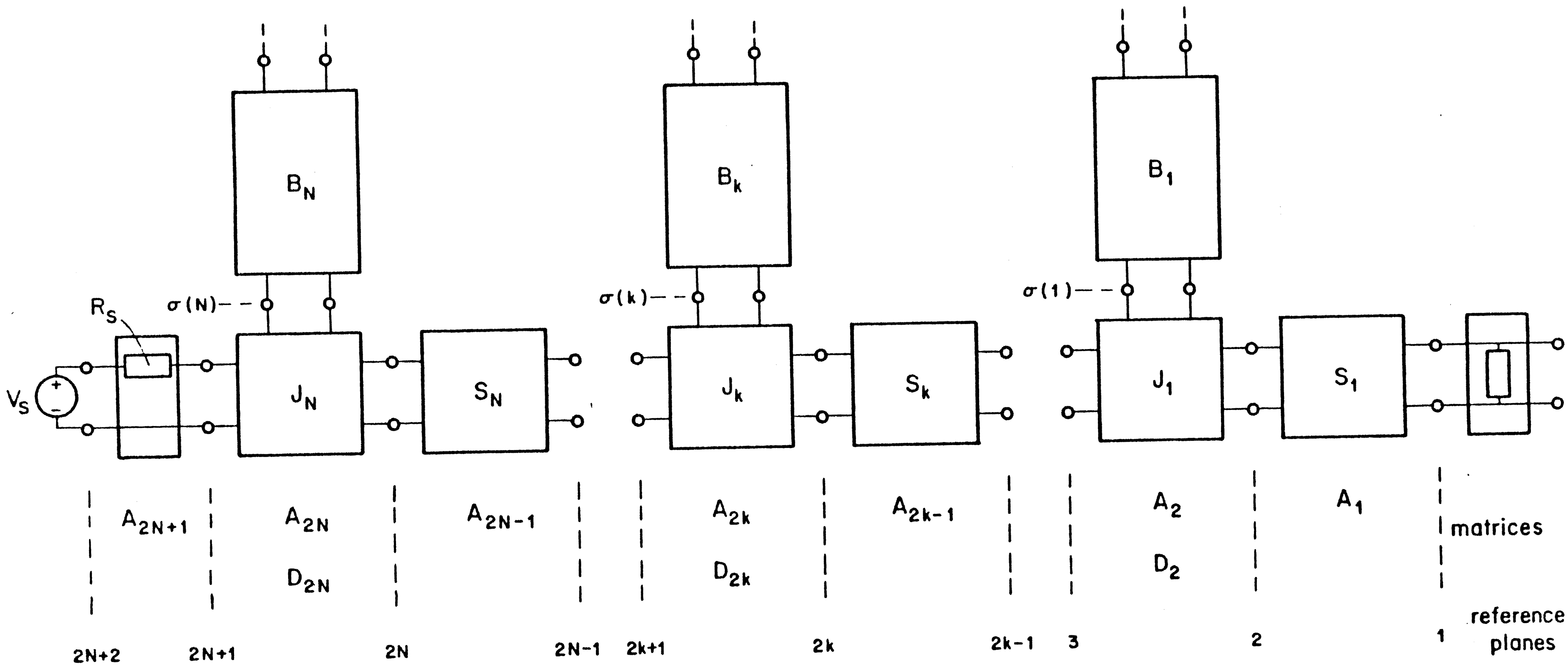

Fig. 2 Illustration of principal concepts involved in branched cascaded network simulation showing reference planes and transmission matrices.

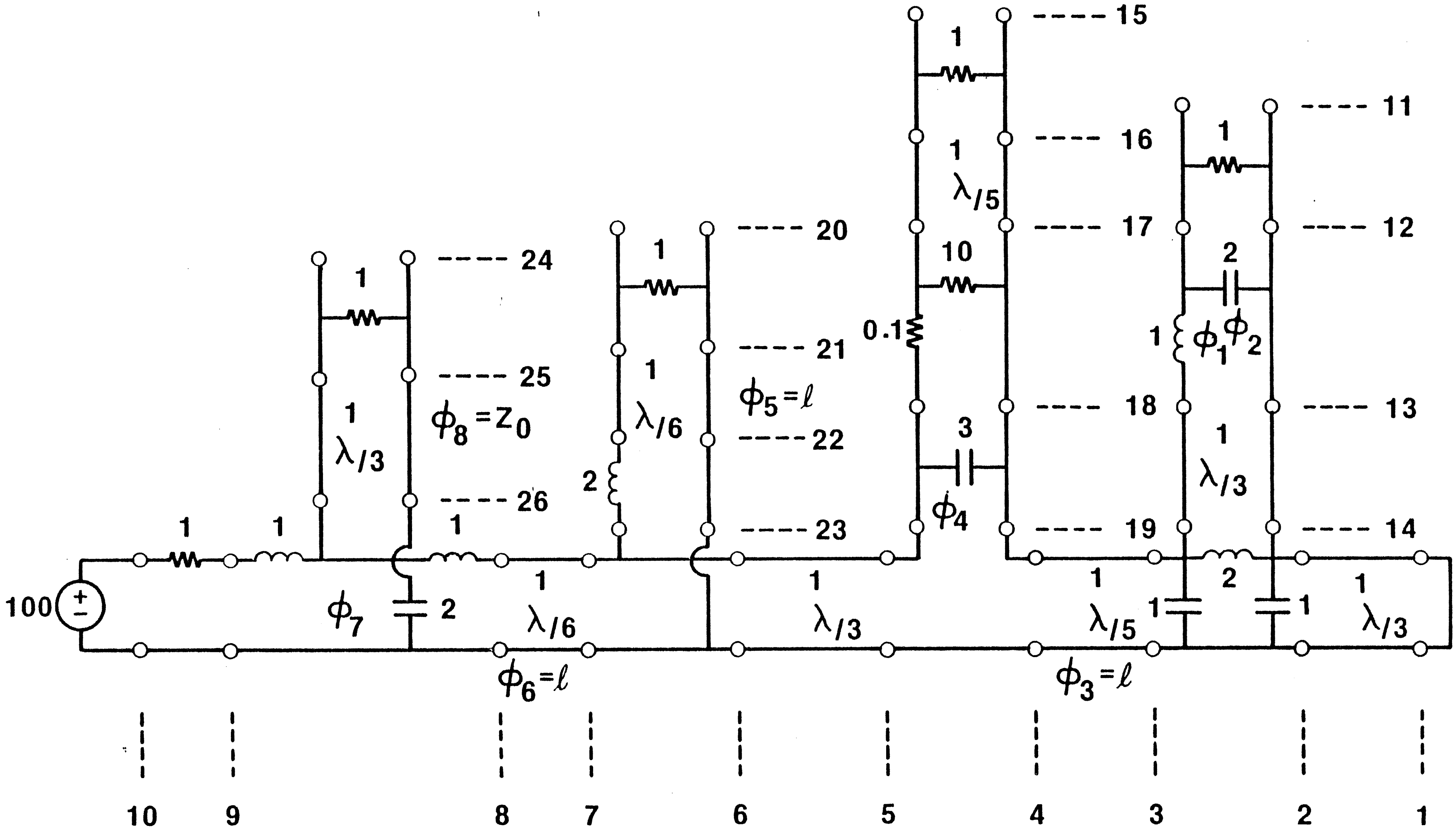

Fig. 3 Illustration of an arbitrary 4-branch cascaded circuit with short-circuit termination of the main cascade. Lossy elements as well as transmission lines are included.

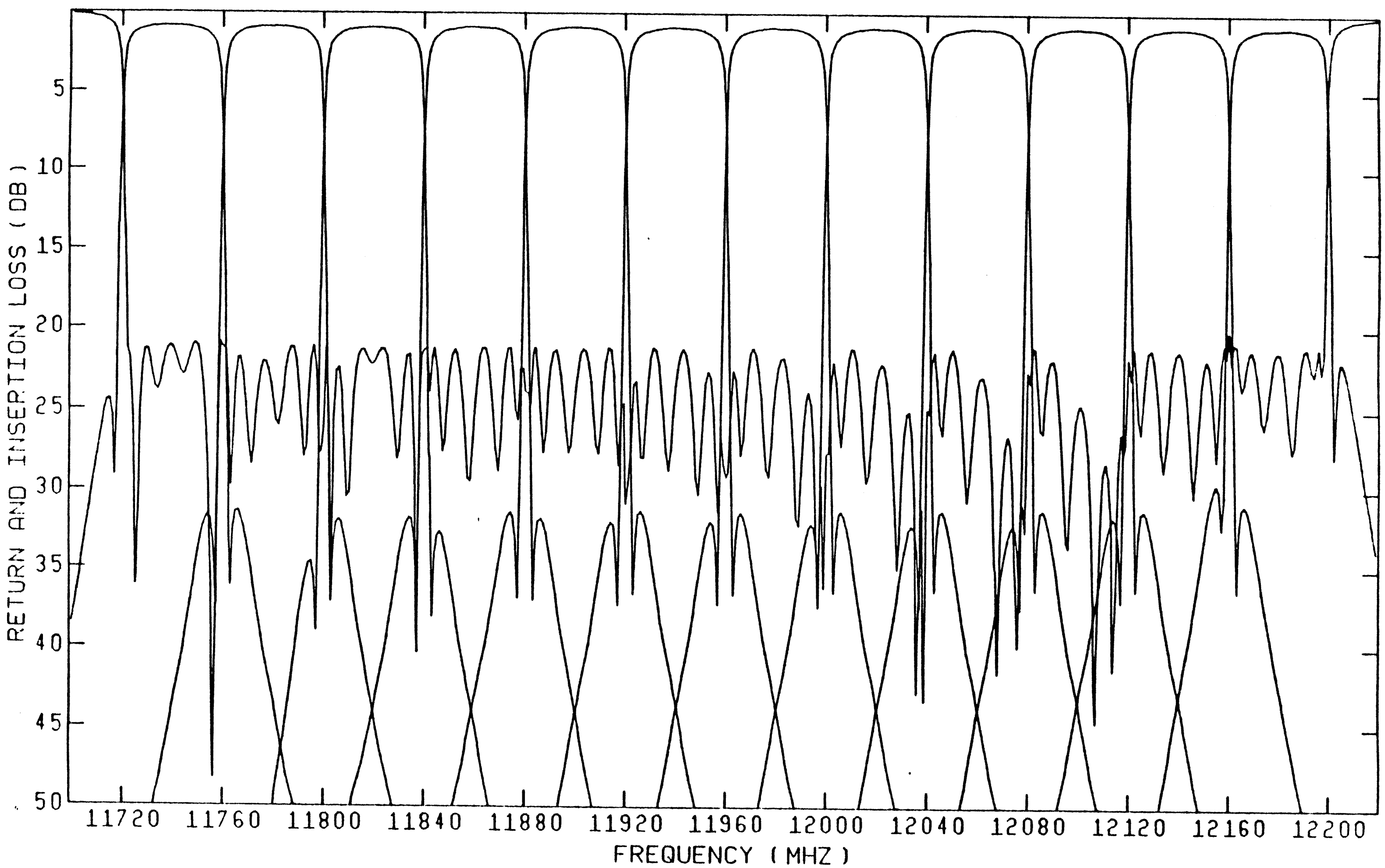

Fig. 4 Responses of a 12-channel multiplexer without dummy channels with optimized spacings, input-output transformer ratios, cavity resonances and coupling parameters.