EFFICIENT APPROACHES TO THE SIMULATION OF NARROW-BAND MULTI-CAVITY FILTERS

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Abstract

We present three exact approaches utilizing (a) the unterminated circuit model, (b) the canonical representation, and (c) the loaded filter network, as well as two approximate approaches, namely, the approximate determinant method and the Neumann series method, to the simulation and sensitivity analysis of narrow-bandpass multi-cavity microwave filters. The principal aim is to highlight the efficiency of these approaches in computation and flexibility in exploiting various network structures. Detailed formulas of filter responses, including group delay and gain slope, and their sensitivities w.r.t. design variables are presented and tabulated in detail.

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I. INTRODUCTION

The frequent application of multi-coupled cavity narrow-bandpass filters in microwave communication systems and their advantages have been extensively discussed in the literature. See, for example, Atia and Williams [1-2], Chen et al. [3] and Cameron [4].

As in many other situations where the complexity and requirements of the problem are ever-increasing, the commonly accepted approach is to employ modern computer-aided design techniques. For example, when asymmetric, nonminimum-phase characteristics are of interest, the traditional approach to an analytical solution may be inappropriate. Furthermore, the CAD techniques can be utilized to predict the effects of nonideal factors such as modeling and manufacturing imperfections which are not to be ignored in microwave devices. Needless to say, a systematic and efficient simulation and sensitivity evaluation method is essential here.

In this report, three approaches to the exact simulation and sensitivity analysis of narrow-bandpass multi-coupled cavity microwave filters are presented. The formulation and description of each method are self-contained, yet they are related in a unified manner. Two approximate methods are also derived, aimed at minimizing the computational effort, wherever possible. The structure of the filter as well as the terminations considered could be arbitrary. Various types of responses, including group delay and gain slope, and optimization variables, including all possible couplings, cavity resonant frequencies, and cavity dissipations, are taken into account. The solutions of the network involve only real operations. The algorithms presented can be easily implemented on a digital computer.

II. NOTATION

Unless otherwise specified the notation used in this report is the following: Boldface lower-cases **b**, **p**, etc., denote column vectors. Boldface capitals, **M**, **Z**, etc., denote matrices, except **I** and **V** which are reserved to denote current and voltage vectors, respectively. A letter with a double subscript denotes an element of the corresponding matrix, e.g., $M_{\ell k}$

denotes the element at the ℓth row, kth column of matrix **M**. A letter with a single subscript denotes a component of the corresponding vector, e.g., p_n denotes the nth component of vector **p**. **1** denotes an identity matrix of appropriate dimension. $\mathbf{e}_k$ denotes a vector whose components are zero except that the kth component is 1. $\bar{1}$ denotes a rotation matrix of zero entries except the anti-diagonal elements which are 1.

III. THE STRUCTURE AND MODEL OF THE FILTER

A narrow-band lumped model of an unterminated multi-cavity filter has been given by Atia and Williams [1] as

$$j\,\mathbf{ZI} = \mathbf{V} \ , \tag{1}$$

where

$$\mathbf{Z} \triangleq (s\,\mathbf{1} + \mathbf{M}) \ , \tag{2}$$

1 denotes an nxn identity matrix and **M** the nxn coupling matrix whose (i, j) element represents the normalized coupling between the ith and jth cavities, as illustrated in Fig. 1, and the diagonal entries M_{ii} represent the deviations from the synchronous tuning. The normalized frequency variable s in (2) is given by

$$s \triangleq \frac{\omega_0}{\Delta\omega}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) , \tag{3}$$

where ω_0 is the synchronously tuned cavity resonant frequency, or the center frequency as it is sometimes called, and $\Delta\omega$ is the bandwidth parameter. The physical configuration of the filter could be either symmetric or asymmetric. Figure 2 shows the shorthand notation for a sixth-order example.

The matrix **M** is always symmetrical w.r.t. its diagonal, i.e.,

$$M_{\ell k} = M_{k\ell} \ , \tag{4}$$

or, in matrix notation,

$$\mathbf{M}^T = \mathbf{M} \ . \tag{5}$$

In the case of a symmetric realization as shown in Fig. 2(a) **M** could be also symmetric w.r.t. its anti-diagonal if

$$M_{\ell k} = M_{\sigma\tau} \ , \tag{6}$$

where

$$\sigma \triangleq n+1-\ell \ \text{and} \ \tau \triangleq n+1-k \ , \tag{7}$$

or, using the notation defined in Section II, such a symmetry can be implied by

$$\bar{\mathbf{1}}\mathbf{M}\bar{\mathbf{1}} = \mathbf{M} \ . \tag{8}$$

From (2), another identity immediately follows, namely,

$$\bar{\mathbf{1}}\mathbf{Z}\bar{\mathbf{1}} = \mathbf{Z} \ . \tag{9}$$

In this report, a filter coupled in such a pattern is said to be dual-symmetric. We will see later that significant computational advantage can be exploited in the analysis of the dual-symmetric structure.

As has been indicated by Cameron [4], the synchronously tuned configuration, although it may be physically asymmetric, always realizes electrically symmetric characteristics. The feasibility of configurations that could realize asymmetric characteristics has been demonstrated in [4], where couplings other than those shown in Fig. 2 are introduced.

Imbedded into a microwave system the filter is terminated through the input and output couplings, by a source and a load as shown in Fig. 3. The first two approaches, developed hereafter, first reduce the unterminated filter to a two-port (Fig. 4) based on which the overall network is analyzed. The third approach solves the filter terminated by the load for its input-impedance Z_{in} as shown in Fig. 5.

IV. APPROACH 1: THE UNTERMINATED EQUIVALENT CIRCUIT MODEL

Two-Port Model

Recall the system defined in (1) as

$$j\,\mathbf{ZI} = \mathbf{V} \ .$$

The system can be reduced to a two-port model whose parameters and sensitivity expressions can be obtained by solving the real systems

$$\mathbf{Zp} = \mathbf{e}_1 \; , \tag{10}$$

$$\mathbf{Zq} = \mathbf{e}_n \; , \tag{11}$$

$$\mathbf{Z\bar{p}} = \mathbf{p} \tag{12}$$

and

$$\mathbf{Z\bar{q}} = \mathbf{q} \; . \tag{13}$$

It is clear that the solutions of (10) - (13) require only one real LU factorization of the matrix $\mathbf{Z}$ and four forward-backward-substitutions (FBS) with appropriate right-hand side vectors. For the dual-symmetrical systems described in (8), the computational efforts could be further reduced by solving only (10) and (12) and then taking

$$\mathbf{q} = \bar{\mathbf{1}}\mathbf{p} \tag{14-a}$$

and

$$\bar{\mathbf{q}} = \bar{\mathbf{1}}\bar{\mathbf{p}} \; , \tag{14-b}$$

i.e.,

$$q_i = p_{n+1-i} \; , i = 1, 2, ..., n \tag{15-a}$$

and

$$\bar{q}_i = \bar{p}_{n+1-i} \; , i = 1, 2, ..., n \, . \tag{15-b}$$

Equations (14) - (15) are verified by considering the fact that, as given in (9),

$$\bar{\mathbf{1}}\mathbf{Z}\bar{\mathbf{1}} = \mathbf{Z}$$

and

$$\mathbf{e}_n = \bar{\mathbf{1}}\mathbf{e}_1 \; ,$$

hence

$$\mathbf{Z}\bar{\mathbf{1}}\mathbf{p} = \bar{\mathbf{1}}\bar{\mathbf{1}}\mathbf{Z}\bar{\mathbf{1}}\mathbf{p} = \bar{\mathbf{1}}\mathbf{Z}\mathbf{p} = \bar{\mathbf{1}}\mathbf{e}_1 = \mathbf{e}_n \; . \tag{16}$$

Comparing (16) with (11), we get (14-a). The verification of (14-b) is similar.

Having the solutions of (10) and (11), we can model the network (1) as a two-port, as also shown in Fig. 4(a), in the form

$$\begin{bmatrix} I_1' \\ I_n' \end{bmatrix} = -j \begin{bmatrix} y_{11}' & y_{12}' \\ y_{21}' & y_{22}' \end{bmatrix} \begin{bmatrix} V_1' \\ V_n' \end{bmatrix} , \tag{17}$$

where the short-circuit admittance parameters are given by

$$y_{11}' = p_1 , \tag{18}$$

$$y_{12}' = y_{21}' = p_n = q_1 \tag{19}$$

and

$$y_{22}' = q_n . \tag{20}$$

Furthermore, the two-port model including the input and output couplings, as shown in Fig. 4(b), is readily obtained as

$$\begin{bmatrix} I_1 \\ I_n \end{bmatrix} = -j\mathbf{y} \begin{bmatrix} V_1 \\ V_n \end{bmatrix} = -j \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_n \end{bmatrix} , \tag{21}$$

where the elements of $\mathbf{y}$ are given by

$$y_{11} = n_1^2 y_{11}' , \tag{22}$$

$$y_{12} = y_{21} = n_1 n_2 y_{12}' \tag{23}$$

and

$$y_{22} = n_2^2 y_{22}' . \tag{24}$$

It is evident that the sensitivity expressions of $\mathbf{y}$ can be obtained via the corresponding sensitivities of p_1, p_n and q_n.

The general formulas for calculating the first-order derivative of p_1, p_n and q_n w.r.t. parameter ϕ in $\mathbf{Z}$ are given by

$$\frac{\partial p_1}{\partial \phi} = - \mathbf{e}_1^T \mathbf{Z}^{-1} \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{Z}^{-1} \mathbf{e}_1 = - \mathbf{p}^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{p} , \tag{25}$$

$$\frac{\partial p_n}{\partial \phi} = - \mathbf{e}_n^T \mathbf{Z}^{-1} \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{Z}^{-1} \mathbf{e}_1 = - \mathbf{q}^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{p} \tag{26}$$

and

$$\frac{\partial q_n}{\partial \phi} = - \mathbf{e}_n^T \mathbf{Z}^{-1} \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{Z}^{-1} \mathbf{e}_n = - \mathbf{q}^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{q} . \tag{27}$$

If the parameter ϕ enters $\mathbf{Z}$ linearly, (25) - (27) become

$$\frac{\partial p_1}{\partial \phi} = - c \sum_{\ell, k \in I_\phi} p_\ell p_k , \tag{28}$$

$$\frac{\partial p_n}{\partial \phi} = - c \sum_{\ell, k \in I_\phi} q_\ell p_k \tag{29}$$

and

$$\frac{\partial q_n}{\partial \phi} = - c \sum_{\ell, k \in I_\phi} q_\ell q_k , \tag{30}$$

where index set I_ϕ and coefficient c are defined by

$$I_\phi \triangleq \left\{ \ell, k \,\middle|\, \frac{\partial Z_{\ell k}}{\partial \phi} = c \right\} . \tag{31}$$

Besides the first-order sensitivities, a particular set of second-order sensitivities, namely,

$$\frac{\partial}{\partial \phi}\left(\frac{\partial p_1}{\partial \omega}\right) , \quad \frac{\partial}{\partial \phi}\left(\frac{\partial p_n}{\partial \omega}\right) \text{ and } \frac{\partial}{\partial \phi}\left(\frac{\partial q_n}{\partial \omega}\right) ,$$

might be of interest in the computation of the sensitivities of the group delay and the gain slope.

Applying formulas (28) - (30) for $\phi = \omega$, we find that

$$\frac{\partial p_1}{\partial \omega} = - \frac{\partial s}{\partial \omega} \mathbf{p}^T \mathbf{p} , \tag{32}$$

$$\frac{\partial p_n}{\partial \omega} = - \frac{\partial s}{\partial \omega} \mathbf{q}^T \mathbf{p} \tag{33}$$

and

$$\frac{\partial q_n}{\partial \omega} = - \frac{\partial s}{\partial \omega} \mathbf{q}^T \mathbf{q} . \tag{34}$$

Differentiating (32), we have

$$\frac{\partial}{\partial \phi}\left(\frac{\partial p_1}{\partial \omega}\right) = - 2 \frac{\partial s}{\partial \omega} \mathbf{p}^T \frac{\partial \mathbf{p}}{\partial \phi} = - 2 \frac{\partial s}{\partial \omega} \mathbf{p}^T \left(- \mathbf{Z}^{-1} \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{Z}^{-1} \mathbf{e}_1 \right)$$

$$= 2 \frac{\partial s}{\partial \omega} \mathbf{p}^T \mathbf{Z}^{-1} \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{Z}^{-1} \mathbf{e}_1 = 2 \frac{\partial s}{\partial \omega} \bar{\mathbf{p}}^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{p} \ , \tag{35}$$

where $\bar{\mathbf{p}}$ is defined in (12). Equation (35) can be expanded, similarly to (28), as

$$\frac{\partial}{\partial \phi}\left(\frac{\partial p_1}{\partial \omega} \right) = 2 \frac{\partial s}{\partial \omega} c \sum_{\ell, k \in I_\phi} \bar{p}_\ell p_k \ , \tag{35-a}$$

where I_ϕ and c are as defined previously in (31).

Following the same procedure, we can derive that

$$\frac{\partial}{\partial \phi}\left(\frac{\partial p_n}{\partial \omega} \right) = \frac{\partial s}{\partial \omega} \left[\bar{\mathbf{q}}^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{p} + \mathbf{q}^T \frac{\partial \mathbf{Z}}{\partial \phi} \bar{\mathbf{p}} \right] \ , \tag{36}$$

$$\frac{\partial}{\partial \phi}\left(\frac{\partial p_n}{\partial \omega} \right) = \frac{\partial s}{\partial \omega} c \sum_{\ell, k \in I_\phi} (\bar{q}_\ell p_k + q_\ell \bar{p}_k) \tag{36-a}$$

and

$$\frac{\partial}{\partial \phi}\left(\frac{\partial q_n}{\partial \omega} \right) = 2 \frac{\partial s}{\partial \omega} \bar{\mathbf{q}}^T \frac{\partial \mathbf{Z}}{\partial \phi} \mathbf{q} \ , \tag{37}$$

$$\frac{\partial}{\partial \phi}\left(\frac{\partial q_n}{\partial \omega} \right) = 2 \frac{\partial s}{\partial \omega} c \sum_{\ell, k \in I_\phi} \bar{q}_\ell q_k . \tag{37-a}$$

Detailed sensitivity expressions of the matrix **y**, utilizing the above results, are given in Tables 1-3, w.r.t. specific design variables.

The Terminated Filter

Having solved the unterminated network, we continue with the analysis of the filter terminated with arbitrary source and load, as shown in Fig. 4, where the voltage source is normalized.

The input and output currents can be solved from the two-port equations

$$\begin{bmatrix} I_1 \\ I_n \end{bmatrix} = -j\mathbf{y} \begin{bmatrix} V_1 \\ V_n \end{bmatrix} \ , \tag{38}$$

subject to the terminating conditions

$$V_1 = 1 - Z_S I_1 \tag{39}$$

and

$$V_n = -Z_L I_n \,, \tag{40}$$

or in a more compact form,

$$\begin{bmatrix} V_1 \\ V_n \end{bmatrix} = -\hat{\mathbf{Z}}\,\mathbf{I}_p + \mathbf{e}_1 \,, \tag{41}$$

where

$$\hat{\mathbf{Z}} \triangleq \begin{bmatrix} Z_S & 0 \\ 0 & Z_L \end{bmatrix} \,, \tag{42}$$

and

$$\mathbf{I}_p \triangleq \begin{bmatrix} I_1 \\ I_n \end{bmatrix} \,. \tag{43}$$

Substituting (41) into the two-port equations, we get

$$\mathbf{I}_p = \mathrm{j}\,\mathbf{y}\,\hat{\mathbf{Z}}\,\mathbf{I}_p - \mathrm{j}\,\mathbf{y}\,\mathbf{e}_1 \,. \tag{44}$$

Defining

$$\hat{\mathbf{Y}} \triangleq \mathbf{1} - \mathrm{j}\,\mathbf{y}\,\hat{\mathbf{Z}} \,, \tag{45}$$

we can write the solution of (44) as

$$\mathbf{I}_p = -\mathrm{j}\,\hat{\mathbf{Y}}^{-1}\,\mathbf{y}\,\mathbf{e}_1 \,, \tag{46}$$

where

$$\hat{\mathbf{Y}}^{-1} = \frac{1}{D_y} \begin{bmatrix} 1 - \mathrm{j}Z_L y_{22} & \mathrm{j}Z_L y_{12} \\ \mathrm{j}Z_S y_{12} & 1 - \mathrm{j}Z_S y_{11} \end{bmatrix} \,, \tag{47}$$

and

$$D_y \triangleq 1 + Z_L Z_S \left(y_{12}^2 - y_{11} y_{22} \right) - \mathrm{j}\left(Z_S y_{11} + Z_L y_{22} \right) \,. \tag{48}$$

Equation (46) can be expanded explicitly as

$$I_1 = \frac{Z_L (y_{12}^2 - y_{11} y_{22}) - \mathrm{j}\,y_{11}}{D_y} \,, \tag{49}$$

$$I_n = \frac{-\,\mathrm{j}\,y_{12}}{D_y} \,. \tag{50}$$

Sometimes, as for the evaluation of the output reflection coefficient, it is also of interest to solve the network excited at the output port instead of the input port. The solution, denoted by $\hat{\mathbf{I}}_p$, can be obtained by simply replacing $\mathbf{e}_1$ in (46) by $\mathbf{e}_n$. Thus,

$$\hat{\mathbf{I}}_p = \begin{bmatrix} \hat{I}_1 \\ \hat{I}_n \end{bmatrix} = -j\,\hat{\mathbf{Y}}^{-1}\,\mathbf{y}\,\mathbf{e}_n\,, \tag{51}$$

where, to avoid any confusion, we define

$$\mathbf{e}_n \triangleq \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{52}$$

The derivation of the sensitivity expressions of $\mathbf{I}_p$ and $\hat{\mathbf{I}}_p$, utilizing Tables 1-3, from equations (46) and (51) is straightforward. For convenient reference, the results are tabulated in Table 4.

The Reflection Coefficient and the Return Loss

The input reflection coefficient of the network is defined as

$$\rho_{in} \triangleq \frac{Z_{in} - Z_S^*}{Z_{in} + Z_S}\,. \tag{53}$$

It can be expressed in terms of I_1, which is given in (46) and (49), by

$$\rho_{in} = 1 - \frac{2\mathrm{Re}(Z_S)}{Z_{in} + Z_S} = 1 - 2\mathrm{Re}(Z_S)\,I_1\,. \tag{54}$$

Usually only the modulus of ρ_{in} is used, that is

$$|\rho_{in}| = (\rho_{in}^*\,\rho_{in})^{1/2}\,, \tag{55}$$

whose sensitivities are given by

$$\frac{\partial|\rho_{in}|}{\partial\phi} = \frac{1}{|\rho_{in}|}\,\mathrm{Re}\left[\rho_{in}^*\,\frac{\partial\rho_{in}}{\partial\phi}\right] = \frac{-2\mathrm{Re}(Z_S)}{|\rho_{in}|}\,\mathrm{Re}\left[\rho_{in}^*\,\frac{\partial I_1}{\partial\phi}\right]. \tag{56}$$

Usually $\mathrm{Re}(Z_S)$ is frequency independent, i.e., $\partial\mathrm{Re}(Z_S)/\partial\omega = 0$ even if $\partial\mathrm{Im}(Z_S)/\partial\omega \neq 0$. This may not be true when Z_S represents an adjacent network, in which case, (56) needs to be augmented by

$$\frac{\partial|\rho_{in}|}{\partial\omega} = \frac{-2}{|\rho_{in}|}\,\mathrm{Re}\left[\rho_{in}^*\left(\frac{\partial I_1}{\partial\omega}\,\mathrm{Re}(Z_S) + I_1\,\frac{\partial\mathrm{Re}(Z_S)}{\partial\omega}\right)\right]. \tag{57}$$

For a full description of the filter characteristics, we can define the output reflection coefficient as

$$\rho_{out} \triangleq \frac{Z_{out} - Z_L^*}{Z_{out} + Z_L} = 1 - 2\mathrm{Re}(Z_L)\hat{I}_n , \tag{58}$$

where $\hat{I}_n$ is given in equation (51). Similar to (55) - (57), we have

$$|\rho_{out}| = (\rho_{out}^* \rho_{out})^{1/2} , \tag{59}$$

$$\frac{\partial|\rho_{out}|}{\partial\phi} = \frac{-2\mathrm{Re}(Z_L)}{|\rho_{out}|} \mathrm{Re}\left[\rho_{out}^* \frac{\partial\hat{I}_n}{\partial\phi} \right] , \tag{60}$$

and

$$\frac{\partial|\rho_{out}|}{\partial\omega} = \frac{-2}{|\rho_{out}|} \mathrm{Re}\left[\rho_{out}^* \left(\frac{\partial\hat{I}_n}{\partial\omega} \mathrm{Re}(Z_L) + \hat{I}_n \frac{\partial\mathrm{Re}(Z_L)}{\partial\omega} \right) \right] . \tag{61}$$

The input return loss and the output return loss, also commonly used, are defined as

$$\Gamma_{in,out} \triangleq -20 \log_{10}|\rho_{in,out}| \text{ dB} , \tag{62}$$

where $\Gamma_{in,out}$ and $\rho_{in,out}$ mean Γ_{in} or Γ_{out} and ρ_{in} or ρ_{out}, for brevity. Their sensitivities are

$$\frac{\partial\Gamma_{in,out}}{\partial\phi} = -\frac{20}{\ell n10} \frac{1}{|\rho_{in,out}|} \frac{\partial|\rho_{in,out}|}{\partial\phi} = -\frac{20}{\ell n10} \mathrm{Re}\left[\frac{1}{\rho_{in,out}} \frac{\partial\rho_{in,out}}{\partial\phi} \right] . \tag{63}$$

The Transducer Loss and the Insertion Loss

The transducer loss, which is also referred to as the discrimination loss, is defined as the logarithmic ratio of the real power transmitted to the load under matched condition, i.e., the maximum transmission, to that of the actual network under consideration. The former is given by

$$P_{Lmax} = \frac{1}{4\,\mathrm{Re}(Z_S)} , \tag{64}$$

when

$$Z_L = Z_S^* \text{ and } E_S = 1 , \tag{65}$$

and the latter by

$$P_L = |I_n|^2 \mathrm{Re}(Z_L) . \tag{66}$$

Therefore, the transducer loss is

$$\Lambda \triangleq 10 \log_{10} \frac{P_{Lmax}}{P_L} = 10 \log_{10} \left[\frac{1}{4|I_n|^2 \operatorname{Re}(Z_S) \operatorname{Re}(Z_L)} \right] \text{ dB} . \tag{67}$$

Its sensitivities are evaluated by

$$\frac{\partial \Lambda}{\partial \phi} = \frac{-20}{\ell n 10} \operatorname{Re} \left[\frac{1}{I_n} \frac{\partial I_n}{\partial \phi} \right] . \tag{68}$$

If $\operatorname{Re}(Z_S)$ and/or $\operatorname{Re}(Z_L)$ are frequency dependent, (68) should be modified for $\partial\Lambda/\partial\omega$ as

$$\frac{\partial \Lambda}{\partial \omega} = \frac{-20}{\ell n 10} \operatorname{Re} \left[\frac{1}{I_n} \frac{\partial I_n}{\partial \omega} \right] - \frac{10}{\ell n 10} \left[\frac{1}{\operatorname{Re}(Z_S)} \frac{\partial \operatorname{Re}(Z_S)}{\partial \omega} + \frac{1}{\operatorname{Re}(Z_L)} \frac{\partial \operatorname{Re}(Z_L)}{\partial \omega} \right] . \tag{69}$$

For a lossless network, the transducer loss is related to the input reflection coefficient by

$$\Lambda = -10 \log_{10} (1 - |\rho_{in}|^2) . \tag{70}$$

Another filter response, namely the insertion loss, is defined as the logarithmic ratio of the real power absorbed by the load when the load is directly connected to the source, which is given by

$$P_L' = \frac{\operatorname{Re}(Z_L)}{|Z_S + Z_L|^2} , \; (E_S = 1) \tag{71}$$

to that when the filter is inserted in between, as given by (66).

Denoting the insertion loss by Δ, we have

$$\Delta = 10 \log_{10} \frac{P_L'}{P_L} = 10 \log_{10} \left[\frac{1}{|(Z_S + Z_L) I_n|^2} \right] \text{ dB} . \tag{72}$$

Comparing (72) with (67), we observe that

$$\Delta = \Lambda - \Lambda_0 , \tag{73}$$

where

$$\Lambda_0 \triangleq 10 \log_{10} \left[\frac{|Z_S + Z_L|^2}{4 \operatorname{Re}(Z_S) \operatorname{Re}(Z_L)} \right] \geq 0 \tag{74}$$

represents the extra loss due to the mismatched terminations.

From (73) it is clear that

$$\frac{\partial \Delta}{\partial \phi} = \frac{\partial \Lambda}{\partial \phi} = \frac{-20}{\ell n 10} \operatorname{Re} \left[\frac{1}{I_n} \frac{\partial I_n}{\partial \phi} \right] , \tag{75}$$

except for $\partial\Delta/\partial\omega$, which is

$$\frac{\partial \Delta}{\partial \omega} = - \frac{20}{\ell n 10} \mathrm{Re}\left[\frac{1}{I_n}\frac{\partial I_n}{\partial \omega} + \frac{1}{Z_S + Z_L}\left(\frac{\partial Z_S}{\partial \omega} + \frac{\partial Z_L}{\partial \omega} \right) \right] . \tag{76}$$

The Gain Slope

Sometimes we are interested not only in the gain response at given frequencies but also in its variation with the frequency. A measure of this variation is the gain slope, defined by

$$S_G \triangleq \frac{\partial \Delta}{\partial \omega} \ \mathrm{dB/Hz} . \tag{77}$$

Actually it can be computed using equation (76) as

$$S_G = - \frac{20}{\ell n 10} \mathrm{Re}\left[\frac{1}{I_n}\frac{\partial I_n}{\partial \omega} \right] - \frac{20}{\ell n 10} \mathrm{Re}\left[\frac{1}{Z_S + Z_L}\left(\frac{\partial Z_S}{\partial \omega} + \frac{\partial Z_L}{\partial \omega} \right) \right] = S_{G1} + S_{G2} , \tag{78}$$

where $S_{G2} = 0$ if the terminations are frequency independent. Following (78) we have

$$\frac{\partial S_{G1}}{\partial \phi} = \frac{-20}{\ell n 10} \mathrm{Re}\left[\frac{1}{I_n}\frac{\partial^2 I_n}{\partial \phi \partial \omega} - \frac{1}{I_n^2}\frac{\partial I_n}{\partial \phi}\frac{\partial I_n}{\partial \omega} \right] , \tag{79}$$

where the second-order derivative term can be found in Table 4. Furthermore, we have

$$\frac{\partial S_{G2}}{\partial \omega} = \frac{-20}{\ell n 10} \mathrm{Re}\left[\frac{1}{Z_S + Z_L}\left(\frac{\partial^2 Z_S}{\partial \omega^2} + \frac{\partial^2 Z_L}{\partial \omega^2} \right) - \frac{1}{(Z_S + Z_L)^2}\left(\frac{\partial Z_S}{\partial \omega} + \frac{\partial Z_L}{\partial \omega} \right)^2 \right] . \tag{80}$$

Obviously $\partial S_{G2}/\partial \phi = 0$ for $\phi \neq \omega$.

The Group Delay

Unlike all the responses discussed before, the group delay is related to the network phase characteristics. It is defined as [6]

$$T_G \triangleq \frac{\partial}{\partial \omega}\left[\angle E_S - \angle V_n \right] = - \frac{\partial}{\partial \omega} \angle V_n , \tag{81}$$

or, equivalently, by

$$T_G = - \mathrm{Im}\left[\frac{1}{V_n}\frac{\partial V_n}{\partial \omega} \right] . \tag{82}$$

Notice that $V_n = -I_n Z_L$, therefore

$$T_G = -\,\mathrm{Im}\left[\frac{1}{I_n Z_L}\,\frac{\partial(I_n Z_L)}{\partial\omega}\right] = -\,\mathrm{Im}\left[\frac{1}{I_n}\frac{\partial I_n}{\partial\omega}\right] - \mathrm{Im}\left[\frac{1}{Z_L}\,\frac{\partial Z_L}{\partial\omega}\right] = T_{G1} + T_{G2}\,, \tag{83}$$

where $T_{G2}=0$ if Z_L is independent of ω.

Note the similarity between (78) and (83), which suggests that the evaluations of the gain slope and the group delay be unified as

$$C_S\,S_{G1} + j\,T_{G1} = -\,\frac{1}{I_n}\frac{\partial I_n}{\partial\omega}\,, \tag{84}$$

where the coefficient

$$C_S \triangleq \frac{\ell n 10}{20}\,. \tag{85}$$

Their sensitivities are related by

$$C_S\,\frac{\partial S_{G1}}{\partial\phi} + j\,\frac{\partial T_{G1}}{\partial\phi} = -\,\frac{1}{I_n}\,\frac{\partial^2 I_n}{\partial\phi\partial\omega} + \frac{1}{I_n^2}\frac{\partial I_n}{\partial\phi}\,\frac{\partial I_n}{\partial\omega}\,. \tag{86}$$

In (83), we have

$$\frac{\partial T_{G2}}{\partial\omega} = -\,\mathrm{Im}\left[\frac{1}{Z_L}\,\frac{\partial^2 Z_L}{\partial\omega^2} - \frac{1}{Z_L^2}\left(\frac{\partial Z_L}{\partial\omega}\right)^2\right] \tag{87}$$

and

$$\frac{\partial T_{G2}}{\partial\phi} = 0 \quad \text{for } \phi \neq \omega\,. \tag{88}$$

V. APPROACH 2: ANALYSIS IN CANONICAL FORM

In the previous sections we constructed the two-port model of the filter by solving the linear network directly. Alternatively, a second approach accomplishes the analysis by decomposing the matrix **Z** as

$$\mathbf{Z} = \mathbf{T}(s\,\mathbf{1} + \mathbf{D})\,\mathbf{T}^T\,, \tag{89}$$

where the diagonal matrix **D** contains the eigenvalues of **M**, and the orthogonal matrix **T** contains the corresponding eigenvectors of **M**, i.e., the ith column vector of **T**, namely $\mathbf{t}_i$, is an eigenvector of **M** corresponding to d_i, which is an eigenvalue of **M** as well as the ith diagonal element of **D**.

Then, the two-port matrix $\mathbf{y}'$, corresponding to formulas (17)-(20), can be constructed as follows. We have, as given in [1] and [3]

$$\mathbf{y}' = \sum_{i=1}^{n} \left(\frac{1}{(s+d_i)} \begin{bmatrix} t_{1i}^2 & t_{1i}t_{ni} \\ t_{1i}t_{ni} & t_{ni}^2 \end{bmatrix} \right), \tag{90}$$

from which

$$\frac{\partial \mathbf{y}'}{\partial \omega} = -\frac{\partial s}{\partial \omega} \sum_{i=1}^{n} \left(\frac{1}{(s+d_i)^2} \begin{bmatrix} t_{1i}^2 & t_{1i}t_{ni} \\ t_{1i}t_{ni} & t_{ni}^2 \end{bmatrix} \right). \tag{91}$$

The solutions of equations (10) and (11) can be obtained by

$$p_\ell = \sum_{i=1}^{n} \left(\frac{t_{1i}\, t_{\ell i}}{(s+d_i)} \right) \tag{92}$$

and

$$q_k = \sum_{i=1}^{n} \left(\frac{t_{ni}\, t_{ki}}{(s+d_i)} \right), \tag{93}$$

respectively, which can be used to compute the results in Table 1.

We observe that in formulas (90) and (91), $\mathbf{y}'$ and its sensitivities w.r.t. frequency are expressed explicitly in ω, so this approach possesses advantages in some simulation cases. On the other hand, it should be noted that a numerical decomposition of $\mathbf{M}$ (e.g., the QR algorithm) usually requires much more computational effort than the LU factorization of the matrix.

VI. APPROACH 3: SOLUTION OF THE LOADED FILTER

The Basic Analyses

The third approach, as mentioned in Section I, simulates the network by solving the loaded filter, i.e.,

$$\mathbf{Z}'\mathbf{I} = \mathbf{V}, \tag{94}$$

where

$$\mathbf{Z}' = j\mathbf{Z} + \text{diag}\,\{0, 0, \ldots, 0, n_2^2 Z_L\}, \tag{95}$$

for its input impedance, instead of obtaining the two-port representation (cf. Fig. 5). It is shown in Appendix 1 that a solution of the complex equations (94) requires only real LU factorization of **Z**.

We define the original and adjoint analyses, after the pattern of the equations (10)-(13), as

$$\mathbf{Z}'\mathbf{p} = \mathbf{e}_1 , \tag{96}$$

$$\mathbf{Z}'\mathbf{q} = \mathbf{e}_n , \tag{97}$$

$$\mathbf{Z}'\bar{\mathbf{p}} = \mathbf{p} , \tag{98}$$

and

$$\mathbf{Z}'\bar{\mathbf{q}} = \mathbf{q} . \tag{99}$$

For brevity we use the notation $\mathbf{p}$, $\mathbf{q}$, $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ to denote the solutions of (96)-(99) throughout this section. They should not be confused with those defined in Section IV.

The definitions and derivation of the corresponding sensitivity expressions are so similar to those presented in (25)-(37) that they are not repeated here. The results are simply tabulated in Tables 5 and 6.

It should be noted that, due to the presence of Z_L, $\mathbf{Z}'$ can not be dual-symmetrical, i.e.,

$$\bar{\mathbf{1}}\,\mathbf{Z}'\bar{\mathbf{1}} \neq \mathbf{Z}' \tag{100}$$

even if the cavities are coupled in the dual-symmetrical pattern, therefore the relationship defined by equations (14)-(15) does not apply here.

The solutions of (96)-(99) can be used directly to formulate various responses of interest.

<u>The Input Reflection Coefficient and the Return Loss</u>

Referring to Fig. 5, we know that

$$Z_{in}' = \frac{1}{p_1} , \tag{101}$$

and the input-impedance of the loaded filter is

$$Z_{in} = \frac{Z'_{in}}{n_1^2} = \frac{1}{n_1^2 p_1} . \tag{102}$$

The input reflection coefficient, as has been defined in (53), is now

$$\rho_{in} = \frac{Z_{in} - Z_S^*}{Z_{in} + Z_S} = \frac{1 - n_1^2 p_1 Z_S^*}{1 + n_1^2 p_1 Z_S} . \tag{103}$$

Its sensitivities are

$$\frac{\partial \rho_{in}}{\partial \phi} = -2\alpha_v^2 \, \mathrm{Re}(Z_S) \frac{\partial p_1}{\partial \phi} , \; \phi \neq n_1, \omega , \tag{104}$$

$$\frac{\partial \rho_{in}}{\partial \omega} = -2\alpha_v^2 \left[\mathrm{Re}(Z_S) \frac{\partial p_1}{\partial \omega} + \frac{\partial \mathrm{Re}(Z_S)}{\partial \omega} p_1 - j\, n_1^2 p_1^2 \, \mathrm{Im}\left(Z_S^* \frac{\partial Z_S}{\partial \omega} \right) \right] , \tag{105}$$

and

$$\frac{\partial \rho_{in}}{\partial n_1} = -4 \frac{\alpha_v^2}{n_1} p_1 \, \mathrm{Re}(Z_S) , \tag{106}$$

where α_v is the voltage divider ratio (cf. Fig. 5) given by

$$\alpha_v = \frac{V'_1}{E_S} = n_1 \frac{Z_{in}}{Z_{in} + Z_S} = \frac{n_1}{1 + n_1^2 p_1 Z_S} . \tag{107}$$

Equations (103)-(107) can be used to compute $|\rho_{in}|$, the return loss Γ_{in} and their sensitivities as have been shown in (55), (56), (62) and (63).

The Transducer Loss and the Insertion Loss

From (96) we know that p_n is the output current when $V'_1 = 1$, therefore the output current when the filter is connected to the actual source is

$$I_n = \alpha_v \, p_n , \tag{108}$$

where α_v is given by (107). Furthermore, since the network model considered is lossless, the fact that the input real power is wholly absorbed by the load leads to the identity

$$\mathrm{Re}(p_1) = |p_n|^2 \, \mathrm{Re}(Z_L) . \tag{109}$$

The transducer loss, defined by (67), is then given by

$$\Lambda = 10\log_{10}\left[\frac{1}{4|\alpha_v p_n|^2 \operatorname{Re}(Z_S)\operatorname{Re}(Z_L)}\right]$$

$$= 10\log_{10}\left[\frac{1}{4|\alpha_v|^2 \operatorname{Re}(Z_S)\operatorname{Re}(p_1)}\right]. \tag{110}$$

From (110), we have

$$\frac{\partial\Lambda}{\partial\phi} = \frac{20}{\ell n 10}\left[\operatorname{Re}\left(\alpha_v n_1 Z_S \frac{\partial p_1}{\partial\phi}\right) - \frac{1}{2\operatorname{Re}(p_1)}\operatorname{Re}\left(\frac{\partial p_1}{\partial\phi}\right)\right], \quad \phi \neq \omega, n_1, \tag{111}$$

$$\frac{\partial\Lambda}{\partial\omega} = \frac{20}{\ell n 10}\left\{\operatorname{Re}\left[\alpha_v n_1\left(Z_S\frac{\partial p_1}{\partial\omega} + p_1\frac{\partial Z_S}{\partial\omega}\right)\right] - \frac{1}{2\operatorname{Re}(p_1)}\operatorname{Re}\left(\frac{\partial p_1}{\partial\omega}\right) - \frac{1}{2\operatorname{Re}(Z_S)}\frac{\partial\operatorname{Re}(Z_S)}{\partial\omega}\right\} \tag{112}$$

and

$$\frac{\partial\Lambda}{\partial n_1} = \frac{20}{n_1\,\ell n 10}\left[1 - 2\operatorname{Re}\left(\frac{\alpha_v}{n_1}\right)\right]. \tag{113}$$

The definition of the insertion loss Δ and the formulas for the evaluation of Δ and $\partial\Delta/\partial\phi$ from Λ and $\partial\Lambda/\partial\phi$, as given in (72)-(76) also apply here.

The Gain Slope

Using equations (108), (109) and the definition (77), we have

$$S_G = \frac{\partial\Delta}{\partial\omega} = \frac{20}{\ell n 10}\left\{\operatorname{Re}\left[\alpha_v n_1\left(Z_S\frac{\partial p_1}{\partial\omega} + p_1\frac{\partial Z_S}{\partial\omega}\right) - \frac{1}{Z_S + Z_L}\left(\frac{\partial Z_L}{\partial\omega} + \frac{\partial Z_S}{\partial\omega}\right)\right]\right.$$

$$\left. + \frac{1}{2}\left[\frac{1}{r_L}\frac{\partial r_L}{\partial\omega} - \frac{1}{g_1}\frac{\partial g_1}{\partial\omega}\right]\right\}, \tag{114}$$

where

$$r_L \triangleq \operatorname{Re}(Z_L) \text{ and } g_1 \triangleq \operatorname{Re}(p_1). \tag{115}$$

Differentiating (114), we get

$$\frac{\partial S_G}{\partial\phi} = \frac{20}{\ell n 10}\left\{\operatorname{Re}\left[\alpha_v\left(n_1 Z_S\frac{\partial^2 p_1}{\partial\phi\partial\omega} + \alpha_v\left(\frac{\partial p_1}{\partial\phi}\frac{\partial Z_S}{\partial\omega} - n_1^2 Z_S^2\frac{\partial p_1}{\partial\phi}\frac{\partial p_1}{\partial\omega}\right)\right)\right]\right.$$

$$+\frac{1}{2g_1}\left[\frac{1}{g_1}\frac{\partial g_1}{\partial\phi}\frac{\partial g_1}{\partial\omega}-\frac{\partial^2 g_1}{\partial\phi\partial\omega}\right]\Bigg\}\ , \tag{116}$$

$$\frac{\partial S_G}{\partial n_1}=\frac{40}{n_1\ell n10}\,\mathrm{Re}\left[\alpha_v^2\left(Z_S\frac{\partial p_1}{\partial\omega}+p_1\frac{\partial Z_S}{\partial\omega}\right)\right]\ . \tag{117}$$

The following extra terms should be added to (116) for $\partial S_G/\partial\omega$ if Z_S and Z_L are frequency dependent,

$$\frac{20}{\ell n10}\,\mathrm{Re}\Bigg\{\alpha_v\left[n_1p_1\frac{\partial^2 Z_S}{\partial\omega^2}+\alpha_v\frac{\partial Z_S}{\partial\omega}\left(\frac{\partial p_1}{\partial\omega}-n_1^2p_1^2\frac{\partial Z_S}{\partial\omega}\right)\right]$$

$$-\frac{1}{Z_S+Z_L}\left[\frac{\partial^2 Z_L}{\partial\omega^2}+\frac{\partial^2 Z_S}{\partial\omega^2}-\frac{1}{Z_S+Z_L}\left(\frac{\partial Z_L}{\partial\omega}+\frac{\partial Z_S}{\partial\omega}\right)^2\right]+\frac{1}{2r_L}\left[\frac{\partial^2 r_L}{\partial\omega^2}-\frac{1}{r_L}\left(\frac{\partial r_L}{\partial\omega}\right)^2\right]\Bigg\}\ . \tag{118}$$

<u>The Group Delay</u>

In equation (83), we have shown that the group delay is given by

$$T_G=-\,\mathrm{Im}\left(\frac{1}{I_n}\frac{\partial I_n}{\partial\omega}\right)-\mathrm{Im}\left(\frac{1}{Z_L}\frac{\partial Z_L}{\partial\omega}\right).$$

Substituting (108) into the last equation, we have

$$T_G=-\,\mathrm{Im}\left[\frac{1}{p_n}\frac{\partial p_n}{\partial\omega}-\alpha_v n_1\left(Z_S\frac{\partial p_1}{\partial\omega}+p_1\frac{\partial Z_S}{\partial\omega}\right)+\frac{1}{Z_L}\frac{\partial Z_L}{\partial\omega}\right]\ . \tag{119}$$

Consequently,

$$\frac{\partial T_G}{\partial\phi}=-\,\mathrm{Im}\Bigg\{\frac{1}{p_n}\left(\frac{\partial^2 p_n}{\partial\phi\partial\omega}-\frac{1}{p_n}\frac{\partial p_n}{\partial\phi}\frac{\partial p_n}{\partial\omega}\right)$$

$$-\alpha_v\left[n_1Z_S\frac{\partial^2 p_1}{\partial\phi\partial\omega}+\alpha_v\left(\frac{\partial p_1}{\partial\phi}\frac{\partial Z_S}{\partial\omega}-n_1^2Z_S^2\frac{\partial p_1}{\partial\phi}\frac{\partial p_1}{\partial\omega}\right)\right]\Bigg\} \tag{120}$$

and

$$\frac{\partial T_G}{\partial n_1}=-\,\mathrm{Im}\left[\frac{2\alpha_v^2}{n_1}\left(Z_S\frac{\partial p_1}{\partial\omega}+p_1\frac{\partial Z_S}{\partial\omega}\right)\right]\ . \tag{121}$$

Similarly to (118), the supplementary terms for $\partial T_G/\partial\omega$ in (120) are

$$\mathrm{Im}\left\{ \alpha_v \left[n_1 p_1 \frac{\partial^2 Z_S}{\partial\omega^2} + \alpha_v \frac{\partial Z_S}{\partial\omega} \left(\frac{\partial p_1}{\partial\omega} - n_1^2 p_1^2 \frac{\partial Z_S}{\partial\omega} \right) \right] - \frac{1}{Z_L} \left[\frac{\partial^2 Z_L}{\partial\omega^2} - \frac{1}{Z_L} \left(\frac{\partial Z_L}{\partial\omega} \right)^2 \right] \right\} . \tag{122}$$

Notice that $\alpha_v n_1 (Z_S(\partial p_1/\partial\omega) + p_1(\partial Z_S/\partial\omega))$ and its derivatives are common factors in the corresponding expressions of both S_G and T_G. Also, in both cases, the second-order sensitivities of p_1 and p_n appear only in (116) and (120), respectively.

VII. A BRIEF COMPARISON OF THE EXACT METHODS

Three different approaches to the exact simulation of the filter network have been described in detail.

For the first approach, the basic solutions and the resultant two-port parameters are independent of the terminations. Therefore, this approach has greater flexibility in accommodating various external network structures, e.g., multiplexers.

The second approach analyzes the system in its canonical form, in which the traditional design parameters such as the eigenvalues, eigenvectors, loss poles and zeros are explicitly available. Although having obvious significance in the traditional analysis and synthesis, it requires much more computational effort and does not treat the couplings as variables directly.

The main advantage of the third approach lies in the simplicity and explicitness of formulating the responses from the basic solution. As has been shown, only one solution is needed to evaluate the gain responses and the corresponding sensitivities, compared with the two solutions needed in the first approach. But we have also seen that when the load is frequency dependent some formulas appear bulky. For this reason, this method is more suitable for a resistive load.

Actually, the solutions of the unterminated filter and those of the loaded filter are related as follows. Consider the systems defined in (1) and (94), as

$$j\mathbf{Z}\mathbf{I} = \mathbf{V} , \tag{123}$$

and

$$\mathbf{Z}'\hat{\mathbf{I}} = (j\mathbf{Z} + \text{ diag}\{0, 0, \ldots, 0, n_2^2 Z_L\})\hat{\mathbf{I}} = \mathbf{V}. \tag{124}$$

Equation (124) can be thought of as resulting from perturbing (123) by

$$(j\,\mathbf{Z} + \Delta\mathbf{Z})\mathbf{I} = \mathbf{V}$$

with

$$\Delta\mathbf{Z} \triangleq n_2^2 Z_L \,\mathbf{e}_n \mathbf{e}_n^T = \Delta z\ \mathbf{e}_n \mathbf{e}_n^T. \tag{125}$$

From matrix theory, we know that the perturbed solution is given by

$$\hat{\mathbf{I}} = (j\mathbf{Z})^{-1}\mathbf{V} - \frac{(j\mathbf{Z})^{-1}\mathbf{e}_n \mathbf{e}_n^T (j\mathbf{Z})^{-1}\mathbf{V}\,\Delta z}{1 + \Delta z\,\mathbf{e}_n^T (j\mathbf{Z})^{-1}\mathbf{e}_n}. \tag{126}$$

Letting $\mathbf{V}$ in (126) be $\mathbf{e}_1$, $\mathbf{e}_n$, $\mathbf{p}$ and $\mathbf{q}$, we can relate the solutions of the loaded filter to those, as defined in (10)-(13), of the unterminated filter by

$$\hat{\mathbf{I}} = -\,j\mathbf{I} + \frac{n_2^2 Z_L I_n}{1 - j n_2^2 Z_L q_n}\,\mathbf{q}\ , \tag{127}$$

where

$$\mathbf{I} = \mathbf{p}, \mathbf{q}, \bar{\mathbf{p}} \text{ and } \bar{\mathbf{q}}$$

represent the solutions of the unterminated filter, as given in (10)-(13), and the corresponding results $\hat{\mathbf{I}}$ are the solutions of the loaded filter, as defined in (96)-(99).

VIII. APPROXIMATE METHODS

As has been shown for the exact methods an nxn linear system has to be solved at each frequency. Here, two approximate methods are introduced with the aim of further reducing the computational effort, which becomes especially significant when the filter is considered as a part of a large network, e.g., a multiplexer.

The Diagonally Dominant Z Matrix

The normalized frequency variable

$$s \triangleq \frac{\omega_0}{\Delta\omega}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{(\omega + \omega_0)(\omega - \omega_0)}{\omega\,\Delta\omega}$$

is a unimodal function in ω. Its modulus $|s|$ satisfies the inequality

$$|s| > \left| \frac{\omega + \omega_0}{\omega} \right| \approx 2 \qquad \text{for} \, |\omega - \omega_0| > \Delta\omega \, .$$

In other words, in the frequency regions that are $\Delta\omega$ away from the center frequency the above inequality holds. Consequently, the matrix $\mathbf{Z} = s\mathbf{1} + \mathbf{M}$ becomes diagonally dominant, since its off-diagonal elements, namely the couplings, are usually smaller than 1.

Method 1: Approximate Determinants

By definition we know that the solutions

$$\mathbf{p} = \mathbf{Z}^{-1} \, \mathbf{e}_1$$

and

$$\mathbf{q} = \mathbf{Z}^{-1} \, \mathbf{e}_n$$

are actually the 1st and the last column vectors of $\mathbf{Z}^{-1}$. From matrix theory, we know that the elements of $\mathbf{Z}^{-1}$ can be calculated with the determinants of the corresponding algebraic complementary matrices, known as the cofactors, and the determinant of $\mathbf{Z}$.

In order not to divert attention, we put the mathematical proof of our theory in Appendix 2, where we show that the determinant of the diagonally dominant matrix $\mathbf{Z}$ can be approximated, by neglecting the lower-order terms of s^k, as

$$\det(\mathbf{Z}) \approx s^n - [\Sigma \, (M_{ij})^2] \, s^{n-2} \, , \tag{128}$$

where the summation is taken over all the nonzero elements in the upper-triangular part of $\mathbf{M}$.

The cofactors of the elements of the first column of $\mathbf{Z}$, denoted by $\det(\mathbf{Z}_{\ell 1})$, can also be approximated by

$$\det(\mathbf{Z}_{11}) \approx s^{n-1} - \left[\sum_{i \neq 1} (M_{ij})^2 \right] s^{n-3} \tag{129}$$

and

$$\det(\mathbf{Z}_{\ell 1}) \approx (-1)^{\ell - 2} \, M_{\ell 1} \, s^{n-2} \, , \; \ell \neq 1 \, . \tag{130}$$

Similarly, we have

$$\det(\mathbf{Z}_{nn}) \approx s^{n-1} - \left[\sum_{j \neq n} (M_{ij})^2 \right] s^{n-3} \tag{131}$$

and

$$\det(\mathbf{Z}_{kn}) \approx (-1)^{k-2} M_{kn} s^{n-2} , \quad k \neq n . \tag{132}$$

Eventually, we have

$$p_1 = Z_{11}^{-1} = \frac{\det(\mathbf{Z}_{11})}{\det(\mathbf{Z})} \approx \frac{s^2 - \left[\sum_{i \neq 1} (M_{ij})^2 \right]}{s^3 - [\Sigma (M_{ij})^2] s} , \tag{133}$$

$$p_\ell = Z_{\ell 1}^{-1} = \frac{(-1)^{\ell+1} \det(\mathbf{Z}_{\ell 1})}{\det(\mathbf{Z})} \approx \frac{-M_{\ell 1}}{s^2 - [\Sigma (M_{ij})^2]} , \ \ell \neq 1 , \tag{134}$$

$$q_k = Z_{kn}^{-1} = \frac{(-1)^{k+1} \det(\mathbf{Z}_{kn})}{\det(\mathbf{Z})} \approx \frac{-M_{kn}}{s^2 - [\Sigma (M_{ij})^2]} , \ k \neq n \tag{135}$$

and

$$q_n = Z_{nn}^{-1} = \frac{\det(\mathbf{Z}_{nn})}{\det(\mathbf{Z})} \approx \frac{s^2 - \left[\sum_{j \neq n} (M_{ij})^2 \right]}{s^3 - [\Sigma (M_{ij})^2] s} . \tag{136}$$

The results of (133)-(136) give the approximate solutions of (10) and (11).

Method 2: Neumann Series

The Neumann series [7], arising from matrix perturbation theory, is referred to as the following infinite series expansion,

$$(\mathbf{1} + \mathbf{A})^{-1} = \mathbf{1} - \mathbf{A} + \mathbf{A}^2 - \mathbf{A}^3 + \ldots \tag{137}$$

subject to the condition

$$1 > \max_{1 \leq i \leq n} |\lambda_{ai}| , \tag{138}$$

where the λ_{ai} are the n eigenvalues of the nxn matrix $\mathbf{A}$. It is actually the generalization of the one-dimensional Taylor series expansion of

$$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots , \ 1 > |x| . \tag{139}$$

By definition we have

$$\mathbf{Z} = s\mathbf{1} + \mathbf{M} = s(\mathbf{1} + \frac{1}{s}\mathbf{M}) , \tag{140}$$

hence

$$\mathbf{Z}^{-1} = \frac{1}{s}(\mathbf{1} + \frac{1}{s}\mathbf{M})^{-1} . \tag{141}$$

The right-hand side of (141) can be expanded by the Neumann series as

$$\mathbf{Z}^{-1} = \frac{1}{s}(\mathbf{1} + \frac{1}{s}\mathbf{M})^{-1} = \frac{1}{s}[\mathbf{1} - \frac{1}{s}\mathbf{M} + \frac{1}{s^2}\mathbf{M}^2 - \frac{1}{s^3}\mathbf{M}^3 + ...] , \tag{142}$$

if the condition, corresponding to (138),

$$1 > \max_{1 \le i \le n} |\lambda_i/s| , \tag{143}$$

or

$$|s| > \max_{1 \le i \le n} |\lambda_i| , \tag{144}$$

is satisfied, where λ_i denotes the ith eigenvalue of **M**.

Obviously, matrix **Z** is diagonally dominant if (144) is satisified, in which case we can approximate $\mathbf{Z}^{-1}$ by taking a finite Neumann series. The first and the last columns of the resultant matrix can be used directly as the approximate solutions **p** and **q**, furthermore, the other two solutions, namely $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$, can also be approximated by

$$\bar{\mathbf{p}} = \mathbf{Z}^{-1}\mathbf{p} , \tag{145}$$

and

$$\bar{\mathbf{q}} = \mathbf{Z}^{-1}\mathbf{q} . \tag{146}$$

This actually implies that the second-order sensitivities can also be approximated, which is not possible for the first approximate method. Generally, the Neumann series expansion approach achieves better accuracy and has more flexibility at the cost of more computational effort. This is also verified by the numerical example tried.

IX. CONCLUSION

Various approaches, both exact and approximate, to the simulation and sensitivity evaluation of arbitrarily terminated narrow-band multi-cavity microwave filters have been described and compared. Detailed formulas and, for convenient reference, tables have been presented. Computational efficiency and flexibility have been stressed.

By performing appropriate analyses on the filter network, various responses of interest, including the group delay and the gain slope, and their sensitivities w.r.t. design variables and other network parameters, including the nonideal cavity losses, can be obtained. Consequently, the application of efficient, modern computer-aided design techniques, such as design optimization, automatic modeling, tolerancing and optimal tuning, to multi-cavity microwave filters becomes possible and very practical.

Some of these applications have been actually implemented, yielding excellent results. Further treatment of this subject is to be presented in subsequent reports.

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APPENDIX 1

The Augmented LU Factorization Method

Recall the system defined in (94) as

$$\mathbf{Z'I} = \mathbf{V} \tag{147}$$

where

$$\mathbf{Z}' \triangleq j\mathbf{Z} + \text{ diag } \{0, 0, \ldots, 0, n_2^2 Z_L\} \ .$$

Suppose the real matrix $\mathbf{Z}$ is factorized by, say, the Crout algorithm, into

$$\mathbf{Z} = \mathbf{L}\,\mathbf{U} \ , \tag{148}$$

where $\mathbf{L}$ and $\mathbf{U}$ are real lower- and upper-triangular matrices, respectively. It can be easily verified that

$$\mathbf{Z}' = j\,\mathbf{L}\,\mathbf{U}' \ , \tag{149}$$

where the augmented matrix

$$\mathbf{U}' \triangleq \mathbf{U} + \text{ diag}\left\{ 0, 0, \ldots, 0, -j\,\frac{n_2^2 Z_L}{L_{nn}} \right\} \tag{150}$$

is also an upper-triangular matrix. That is, complex matrix $\mathbf{Z}'$ can be factorized using only a real algorithm. Now, system (147) can be solved by one real forward-substitution

$$\mathbf{L}\,\mathbf{I}' = \mathbf{V} \ , \tag{151}$$

and one backward-substitution involving mostly real operations, as

$$\mathbf{U}'\,\mathbf{I} = -j\,\mathbf{I}' \ . \tag{152}$$

APPENDIX 2

The Determinant of a Matrix [7]

The determinant of a matrix $\mathbf{A} \triangleq [A_{ij}]^{nxn}$ can be expressed by

$$\det(A) = \sum_{\sigma \in \delta_n} \left[\text{sign}(\sigma) \cdot A_{\sigma(1),1} \cdot A_{\sigma(2),2} \cdot \ldots \cdot A_{\sigma(n),n} \right] , \tag{153}$$

where δ_n is the set of permutations on the index set $\{1, 2, ..., n\}$. A permutation σ can be expressed as the product of some elementary transpositions τ_i:

$$\sigma = \tau_1 \, \tau_2 \ldots \tau_m \, . \tag{154}$$

τ_i is a transposition if for distinct integers k and $\ell \in \{1, 2, ..., n\}$, it holds that

$$\tau_i(k) = \ell , \tag{155}$$

$$\tau_i(\ell) = k \tag{156}$$

and

$$\tau_i(j) = j \; , \quad \text{for all } j \neq \ell, j \neq k \text{ and } j \in \{1,2,\ldots,n\} \, . \tag{157}$$

In (153) we have

$$\text{sign}(\sigma) = (-1)^m \, , \tag{158}$$

where m, as in (154), is the number of the elementary transpositions.

The Determinant of a Diagonally Dominant Matrix

Consider the diagonally dominant matrix

$$\mathbf{Z} = s\,\mathbf{1} + \mathbf{M} \, .$$

If we denote the off-diagonal elements by ε, then, corresponding to (153), det($\mathbf{Z}$) is composed of terms of the form

$$\pm \, s^r \, \varepsilon^{(n-r)} \, . \tag{159}$$

We arrange the terms according to the order of s, apparently, the highest-order term is s^n. Also, we know by definition that a transposition involves two integers, should there be a transposition at least two diagonal elements have to be ruled out, therefore no terms such as $s^{n-1}\varepsilon$ could result. The next highest-order terms of det($\mathbf{Z}$) are those of the form

$$-s^{n-2} Z_{ij} Z_{ji} \ , \ i \neq j \tag{160}$$

corresponding to one transposition between index i and j. Actually, in (160) we have

$$Z_{ij} = M_{ij} = M_{ji} = Z_{ji} \ ,$$

hence (160) can be written as

$$-s^{n-2} (M_{ij})^2 . \tag{161}$$

Neglecting the remaining lower-order terms, we come to the final result:

$$\det(\mathbf{Z}) \approx s^n - \left[\sum (M_{ij})^2 \right] s^{n-2} , \tag{162}$$

where the summation is taken over all the off-diagonal elements in the upper-triangular (or, the lower-triangular) part of $\mathbf{M}$.

The Approximate Cofactors

Deleting the jth row and the ith column of $\mathbf{Z}$, we define the remainder, denoted by $\mathbf{Z}_{ij}$, as the algebraic complementary matrix of the element Z_{ij}. The determinant $\det(\mathbf{Z}_{ij})$ is known as the cofactor of Z_{ij}.

Two of these complementary matrices, namely $\mathbf{Z}_{11}$ and $\mathbf{Z}_{nn}$, are two $(n-1)x(n-1)$ principal submatrices of $\mathbf{Z}$ and, therefore, are also diagonally dominant.

Applying formula (162), we have

$$\det(\mathbf{Z}_{11}) \approx s^{n-1} - \left[\sum_{i \neq 1} (M_{ij})^2 \right] s^{n-3} , \tag{163}$$

and

$$\det(\mathbf{Z}_{nn}) \approx s^{n-1} - \left[\sum_{j \neq n} (M_{ij})^2 \right] s^{n-3} . \tag{164}$$

The other cofactors of interest here are $\det(\mathbf{Z}_{\ell 1})$ and $\det(\mathbf{Z}_{kn})$. First consider $\det(\mathbf{Z}_{\ell 1})$, for $\ell \neq 1$. Unlike the principal submatrices discussed above, two diagonal elements, indexed $(1,1)$ and (ℓ,ℓ), are deleted, hence the highest-order term is of the form $\pm \varepsilon s^{n-2}$. This term can be determined by comparing (162)

$$\det(\mathbf{Z}) \approx s^n - \left[\sum (M_{ij})^2 \right] s^{n-2}$$

with the Cramer expansion of det(**Z**) by the first column, i.e.,

$$\det(\mathbf{Z}) = \sum_{\ell=1}^{n} (-1)^{\ell-1} Z_{\ell 1} \det(\mathbf{Z}_{\ell 1}) = \sum_{\ell=1}^{n} (-1)^{\ell-1} M_{\ell 1} \det(\mathbf{Z}_{\ell 1}) \tag{165}$$

as

$$\det(\mathbf{Z}_{\ell 1}) \approx (-1)^{\ell-2} M_{\ell 1} s^{n-2}, \ \ell \neq 1 \ . \tag{166}$$

Similarly, we have

$$\det(\mathbf{Z}_{kn}) \approx (-1)^{k-2} M_{kn} s^{n-2}, \ k \neq n \ . \tag{167}$$

TABLE 1

FIRST-ORDER SENSITIVITY EXPRESSIONS FOR

(a) SYMMETRICAL AND (b) DUAL-SYMMETRICAL COUPLING MATRICES

Basic solutions (a) $\mathbf{Z}\,\mathbf{p} = \mathbf{e}_1$, $\mathbf{Z}\,\mathbf{q} = \mathbf{e}_n$, (b) $\mathbf{Z}\,\mathbf{p} = \mathbf{e}_1$, $\mathbf{q} = \bar{\mathbf{I}}\,\mathbf{p}$

Variable	Derivative	First-order Sensitivity Expressions: Symmetrical case	First-order Sensitivity Expressions: Dual-Symmetrical Case
$\phi = M_{\ell k}$	$\dfrac{\partial \mathbf{y}}{\partial M_{\ell k}}$ †	$C_1\begin{bmatrix} 2n_1^2 p_\ell p_k & n_1 n_2 (p_\ell q_k + p_k q_\ell) \\ n_1 n_2 (p_\ell q_k + p_k q_\ell) & 2n_2^2 q_\ell q_k \end{bmatrix}$	$2C_1\begin{bmatrix} n_1^2 (p_\ell p_k + p_\sigma p_\tau) & n_1 n_2 (p_\ell p_\tau + p_k p_\sigma) \\ n_1 n_2 (p_\ell p_\tau + p_k p_\sigma) & n_2^2 (p_\ell p_k + p_\sigma p_\tau) \end{bmatrix}$
$\phi = \omega, \Delta\omega, \omega_0$	$\dfrac{\partial \mathbf{y}}{\partial \phi}$	$-\dfrac{\partial s}{\partial \phi}\begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2\, \mathbf{q}^T \mathbf{p} \\ n_1 n_2\, \mathbf{q}^T \mathbf{p} & n_2^2\, \mathbf{q}^T \mathbf{q} \end{bmatrix}$	$-\dfrac{\partial s}{\partial \phi}\begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2\, \mathbf{p}^T \bar{\mathbf{I}}\, \mathbf{p} \\ n_1 n_2\, \mathbf{p}^T \bar{\mathbf{I}}\, \mathbf{p} & n_2^2\, \mathbf{p}^T \mathbf{p} \end{bmatrix}$
$\phi = r_i$ ††	$\dfrac{\partial \mathbf{y}}{\partial r_i}$	$j\begin{bmatrix} n_1^2 p_i^2 & n_1 n_2 p_i q_i \\ n_1 n_2 p_i q_i & n_2^2 q_i^2 \end{bmatrix}$	$j\begin{bmatrix} n_1^2 (p_i^2 + p_{n+1-i}^2) & 2n_1 n_2 p_i p_{n+1-i} \\ 2n_1 n_2 p_i p_{n+1-i} & n_2^2 (p_i^2 + p_{n+1-i}^2) \end{bmatrix}$
$\phi = n_1$	$\dfrac{\partial \mathbf{y}}{\partial n_1}$	$\begin{bmatrix} 2n_1 p_1 & n_2 p_n \\ n_2 p_n & 0 \end{bmatrix}$	$\begin{bmatrix} 2n_1 p_1 & n_2 p_n \\ n_2 p_n & 0 \end{bmatrix}$
$\phi = n_2$	$\dfrac{\partial \mathbf{y}}{\partial n_2}$	$\begin{bmatrix} 0 & n_1 p_n \\ n_1 p_n & 2n_2 q_n \end{bmatrix}$	$\begin{bmatrix} 0 & n_1 p_n \\ n_1 p_n & 2n_2 p_1 \end{bmatrix}$

† where $C_1 \triangleq \begin{cases} -\dfrac{1}{2} & \text{if } \ell = k \text{ or } \sigma = k \\ -1 & \text{otherwise} \end{cases}$

and $\sigma \triangleq n+1-\ell$, $\tau \triangleq n+1-k$

†† r_i is the lumped resistive parameter of the ith cavity. Taking the dissipation into account the unterminated filter is described by

$$(\text{diag}\,\{r_1, r_2, \ldots, r_n\} + j\mathbf{Z})\mathbf{I} = \mathbf{V} \;,$$

which is reduced to equation (1) at nominal (zero dissipation)

TABLE 2

SOME SECOND-ORDER SENSITIVITY EXPRESSIONS FOR A SYMMETRICAL COUPLING MATRIX

Basic solutions $\mathbf{Z}\,\mathbf{p} = \mathbf{e}_1$, $\mathbf{Z}\,\mathbf{q} = \mathbf{e}_n$, $\mathbf{Z}\,\bar{\mathbf{p}} = \mathbf{p}$, $\mathbf{Z}\,\bar{\mathbf{q}} = \mathbf{q}$

Variable	Derivative	Second-Order Sensitivity Expression
$\phi = M_{\ell k}$	$\dfrac{\partial^2 \mathbf{y}}{\partial M_{\ell k}\partial\omega}$	$C_2 \begin{bmatrix} 2n_1^2(p_\ell \bar{p}_k + p_k \bar{p}_\ell) & n_1 n_2(p_\ell \bar{q}_k + p_k \bar{q}_\ell + \bar{p}_\ell q_k + \bar{p}_k q_\ell) \\ n_1 n_2(p_\ell \bar{q}_k + p_k \bar{q}_\ell + \bar{p}_\ell q_k + \bar{p}_k q_\ell) & 2n_2^2(q_\ell \bar{q}_k + q_k \bar{q}_\ell) \end{bmatrix}$ $C_2 \triangleq \begin{cases} \dfrac{1}{2}\dfrac{\partial s}{\partial\omega} & \text{if } \ell = k \\ \dfrac{\partial s}{\partial\omega} & \text{otherwise} \end{cases}$
$\phi = \omega, \Delta\omega, \omega_0$	$\dfrac{\partial^2 \mathbf{y}}{\partial\phi\partial\omega}$	$2\left(\dfrac{\partial s}{\partial\phi}\right)\left(\dfrac{\partial s}{\partial\omega}\right)\begin{bmatrix} n_1^2 \mathbf{p}^T\bar{\mathbf{p}} & n_1 n_2 \mathbf{p}^T\bar{\mathbf{q}} \\ n_1 n_2 \mathbf{p}^T\bar{\mathbf{q}} & n_2^2 \mathbf{q}^T\bar{\mathbf{q}} \end{bmatrix}$ $-\dfrac{\partial^2 s}{\partial\phi\partial\omega}\begin{bmatrix} n_1^2 \bar{p}_1 & n_1 n_2 \bar{p}_n \\ n_1 n_2 \bar{p}_n & n_2^2 \bar{q}_n \end{bmatrix}$
$\phi = r_i$	$\dfrac{\partial^2 \mathbf{y}}{\partial r_1 \partial\omega}$	$-j\dfrac{\partial s}{\partial\omega}\begin{bmatrix} 2n_1^2 p_i \bar{p}_i & n_1 n_2(p_i \bar{q}_i + \bar{p}_i q_i) \\ n_1 n_2(p_i \bar{q}_i + \bar{p}_i q_i) & 2n_2^2 q_i \bar{q}_i \end{bmatrix}$
$\phi = n_1$	$\dfrac{\partial^2 \mathbf{y}}{\partial n_1 \partial\omega}$	$-\dfrac{\partial s}{\partial\omega}\begin{bmatrix} 2n_1 \bar{p}_1 & n_2 \bar{p}_n \\ n_2 \bar{p}_n & 0 \end{bmatrix}$
$\phi = n_2$	$\dfrac{\partial^2 \mathbf{y}}{\partial n_2 \partial\omega}$	$-\dfrac{\partial s}{\partial\omega}\begin{bmatrix} 0 & n_1 \bar{p}_n \\ n_1 \bar{p}_n & 2n_2 \bar{q}_n \end{bmatrix}$

TABLE 3

SOME SECOND-ORDER SENSITIVITY EXPRESSIONS FOR

A DUAL-SYMMETRICAL COUPLING MATRIX

Basic solutions $\mathbf{Z}\,\mathbf{p} = \mathbf{e}_1$, $\mathbf{Z}\,\bar{\mathbf{p}} = \mathbf{p}$

Variable	Derivative	Second-Order Sensitivity Expression
$\phi = M_{\ell k}$	$\dfrac{\partial^2 y_{ii}}{\partial M_{\ell k}\partial\omega}$ $i = 1,2$	$2C_2 n_i^2(p_\ell \bar{p}_k + p_k \bar{p}_\ell + p_\sigma \bar{p}_\tau + p_\tau \bar{p}_\sigma)$
	$\dfrac{\partial^2 y_{12}}{\partial M_{\ell k}\partial\omega} = \dfrac{\partial^2 y_{21}}{\partial M_{\ell k}\partial\omega}$	$2C_2 n_1 n_2(p_\ell \bar{p}_\tau + p_k \bar{p}_\sigma + p_\sigma \bar{p}_k + p_\tau \bar{p}_\ell)$
		$C_2 \triangleq \begin{cases} \dfrac{1}{2}\dfrac{\partial s}{\partial\omega} & \text{if } \ell = k \text{ or } \sigma = k \\ \dfrac{\partial s}{\partial\omega} & \text{otherwise} \end{cases}$
		$\sigma \triangleq n+1-\ell$, $\tau \triangleq n+1-k$
$\phi = \omega, \omega_0, \Delta\omega$	$\dfrac{\partial^2 y_{ii}}{\partial\phi\partial\omega}$ $i = 1, 2$	$n_i^2\left[2\left(\dfrac{\partial s}{\partial\phi}\right)\left(\dfrac{\partial s}{\partial\omega}\right)\mathbf{p}^T\bar{\mathbf{p}} - \dfrac{\partial^2 s}{\partial\phi\partial\omega}\bar{p}_1\right]$
	$\dfrac{\partial^2 y_{12}}{\partial\phi\partial\omega} = \dfrac{\partial^2 y_{21}}{\partial\phi\partial\omega}$	$n_1 n_2\left[2\left(\dfrac{\partial s}{\partial\phi}\right)\left(\dfrac{\partial s}{\partial\omega}\right)\mathbf{p}^T\bar{\mathbf{I}}\,\bar{\mathbf{p}} - \dfrac{\partial^2 s}{\partial\phi\partial\omega}\bar{p}_n\right]$
$\phi = r_k$	$\dfrac{\partial^2 y_{ii}}{\partial r_k\partial\omega}$ $i = 1, 2$	$-2j\,\dfrac{\partial s}{\partial\omega}\,n_i^2(p_k \bar{p}_k + p_\tau \bar{p}_\tau)$
	$\dfrac{\partial^2 y_{12}}{\partial r_k\partial\omega} = \dfrac{\partial^2 y_{21}}{\partial r_k\partial\omega}$	$-2j\,\dfrac{\partial s}{\partial\omega}\,n_1 n_2(p_k \bar{p}_\tau + p_\tau \bar{p}_k)$
		$\tau \triangleq n+1-k$

TABLE 3 (continued)

Variable	Derivative	Second-Order Sensitivity Expression
	$\frac{\partial^2 y_{11}}{\partial n_1 \partial \omega}$	$-2 \frac{\partial s}{\partial \omega} n_1 \bar{p}_1$
	$\frac{\partial^2 y_{12}}{\partial n_1 \partial \omega} = \frac{\partial^2 y_{21}}{\partial n_1 \partial \omega}$	$- \frac{\partial s}{\partial \omega} n_2 \bar{p}_n$
$\phi = n_1, n_2$	$\frac{\partial^2 y_{22}}{\partial n_1 \partial \omega} = \frac{\partial^2 y_{11}}{\partial n_2 \partial \omega}$	0
	$\frac{\partial^2 y_{12}}{\partial n_2 \partial \omega} = \frac{\partial^2 y_{21}}{\partial n_2 \partial \omega}$	$- \frac{\partial s}{\partial \omega} n_1 \bar{p}_n$
	$\frac{\partial^2 y_{22}}{\partial n_2 \partial \omega}$	$-2 \frac{\partial s}{\partial \omega} n_2 \bar{p}_1$

TABLE 4

SENSITIVITY EXPRESSIONS OF $\mathbf{I}_p$ AND $\hat{\mathbf{I}}_p$

$$\mathbf{I}_p = \begin{bmatrix} I_1 \\ I_n \end{bmatrix} = -j\hat{\mathbf{Y}}^{-1}\mathbf{y}\,\mathbf{e}_1\,,\ \hat{\mathbf{I}}_p = \begin{bmatrix} \hat{I}_1 \\ \hat{I}_n \end{bmatrix} = -j\hat{\mathbf{Y}}^{-1}\mathbf{y}\,\mathbf{e}_n\,,\ \hat{\mathbf{Y}} \triangleq \mathbf{1} - j\mathbf{y}\hat{\mathbf{Z}}\,,\ \hat{\mathbf{Z}} \triangleq \begin{bmatrix} Z_S & 0 \\ 0 & Z_L \end{bmatrix}$$

$$\phi = M_{\ell k},\ r_i,\ n_1,\ n_2,\ \Delta\omega,\ \omega_0$$

$$\frac{\partial \mathbf{I}_p}{\partial \phi} = j\hat{\mathbf{Y}}^{-1}\frac{\partial \mathbf{y}}{\partial \phi}[\hat{\mathbf{Z}}\mathbf{I}_p - \mathbf{e}_1]$$

$$\frac{\partial \mathbf{I}_p}{\partial \omega} = j\hat{\mathbf{Y}}^{-1}\left[\left(\frac{\partial \mathbf{y}}{\partial \omega}\hat{\mathbf{Z}} + \mathbf{y}\frac{\partial \hat{\mathbf{Z}}}{\partial \omega}\right)\mathbf{I}_p - \frac{\partial \mathbf{y}}{\partial \omega}\mathbf{e}_1\right]$$

$$\frac{\partial^2 \mathbf{I}_p}{\partial \phi \partial \omega} = j\hat{\mathbf{Y}}^{-1}\left[\left(\frac{\partial^2 \mathbf{y}}{\partial \phi \partial \omega}\hat{\mathbf{Z}} + \frac{\partial \mathbf{y}}{\partial \phi}\frac{\partial \hat{\mathbf{Z}}}{\partial \omega}\right)\mathbf{I}_p + \frac{\partial \mathbf{y}}{\partial \phi}\hat{\mathbf{Z}}\frac{\partial \mathbf{I}_p}{\partial \omega} + \left(\frac{\partial \mathbf{y}}{\partial \omega}\hat{\mathbf{Z}} + \mathbf{y}\frac{\partial \hat{\mathbf{Z}}}{\partial \omega}\right)\frac{\partial \mathbf{I}_p}{\partial \phi} - \frac{\partial^2 \mathbf{y}}{\partial \phi \partial \omega}\mathbf{e}_1\right]$$

$$\frac{\partial^2 \mathbf{I}_p}{\partial \omega^2} = j\hat{\mathbf{Y}}^{-1}\left[\left(\frac{\partial^2 \mathbf{y}}{\partial \omega^2}\hat{\mathbf{Z}} + 2\frac{\partial \mathbf{y}}{\partial \omega}\frac{\partial \hat{\mathbf{Z}}}{\partial \omega} + \mathbf{y}\frac{\partial^2 \hat{\mathbf{Z}}}{\partial \omega^2}\right)\mathbf{I}_p + 2\left(\frac{\partial \mathbf{y}}{\partial \omega}\hat{\mathbf{Z}} + \mathbf{y}\frac{\partial \hat{\mathbf{Z}}}{\partial \omega}\right)\frac{\partial \mathbf{I}_p}{\partial \omega} - \frac{\partial^2 \mathbf{y}}{\partial \omega^2}\mathbf{e}_1\right]$$

The sensitivity expressions of $\hat{\mathbf{I}}_p$ are the same as those of $\mathbf{I}_p$ except that $\mathbf{e}_1$ is replaced by $\mathbf{e}_n$ as appropriate.

TABLE 5

FIRST-ORDER SENSITIVITY EXPRESSIONS FOR

A LOADED FILTER

System $\mathbf{Z}'\,\mathbf{I} = \mathbf{V}$, where $\mathbf{Z}' = j\,(s\mathbf{1} + \mathbf{M}) + \mathrm{diag}\,\{0,0,\ldots,0,n_2^2\,Z_L\}$

Basic solutions $\mathbf{Z}'\mathbf{p} = \mathbf{e}_1,\ \mathbf{Z}'\mathbf{q} = \mathbf{e}_n$

Variable	$\dfrac{\partial p_1}{\partial \phi}$	$\dfrac{\partial p_n}{\partial \phi}$
$M_{\ell k}$	$C_1\, p_\ell p_k$ $C_1 \triangleq \begin{cases} -j & \text{if } \ell = k \\ -2j & \text{otherwise} \end{cases}$	$\dfrac{1}{2}\, C_1\,(p_\ell q_k + p_k q_\ell)$
ω	$-\left(j\,\dfrac{\partial s}{\partial \omega}\,\mathbf{p}^T\mathbf{p} + n_2^2\,p_n^2\,\dfrac{\partial Z_L}{\partial \omega}\right)$	$-\left(j\,\dfrac{\partial s}{\partial \omega}\,\mathbf{p}^T\mathbf{q} + n_2^2\,p_n q_n\,\dfrac{\partial Z_L}{\partial \omega}\right)$
$\Delta\omega,\ \omega_0$	$-\,j\,\dfrac{\partial s}{\partial \phi}\,\mathbf{p}^T\mathbf{p}$	$-\,j\,\dfrac{\partial s}{\partial \phi}\,\mathbf{p}^T\mathbf{q}$
n_2	$-\,2\,n_2 Z_L\, p_n^2$	$-\,2\,n_2 Z_L\, p_n q_n$
r_i (cavity losses)	$-\,p_i^2$	$-\,p_i q_i$

TABLE 6

SECOND-ORDER SENSITIVITY EXPRESSIONS FOR

THE LOADED FILTER OF TABLE 5

Basic solutions $\mathbf{Z'p} = \mathbf{e}_1$, $\mathbf{Z'q} = \mathbf{e}_n$, $\mathbf{Z'\bar{p}} = \mathbf{p}$, $\mathbf{Z'\bar{q}} = \mathbf{q}$

Derivative	Second-Order Sensitivity Expression
$\frac{\partial^2 p_1}{\partial M_{\ell k} \partial \omega}$	$C_2(p_\ell \bar{p}_k + p_k \bar{p}_\ell) + C_3 p_n (p_\ell q_k + p_k q_\ell) \frac{\partial Z_L}{\partial \omega}$
$\frac{\partial^2 p_n}{\partial M_{\ell k} \partial \omega}$	$\frac{1}{2} C_2(p_\ell \bar{q}_k + p_k \bar{q}_\ell + \bar{p}_\ell q_k + \bar{p}_k q_\ell) + \frac{1}{2} C_3 [q_n(p_\ell q_k + p_k q_\ell) + 2\, p_n q_k q_\ell] \frac{\partial Z_L}{\partial \omega}$
	where $C_2 \triangleq \begin{cases} -\frac{\partial s}{\partial \omega} & \text{if } \ell = k \\ -2\frac{\partial s}{\partial \omega} & \text{otherwise} \end{cases}$ $\quad C_3 \triangleq \begin{cases} n_2^2 j & \text{if } \ell = k \\ 2\, n_2^2 j & \text{otherwise} \end{cases}$
$\frac{\partial^2 p_1}{\partial \omega^2}$	$- j\frac{\partial^2 s}{\partial \omega^2} \bar{p}_1 - 2\left(\frac{\partial s}{\partial \omega}\right)^2 \mathbf{p}^{\mathbf{T}}\bar{\mathbf{p}}$
	$+ n_2^2 p_n \left[2\, j\left(\frac{\partial s}{\partial \omega}\right)\left(\frac{\partial Z_L}{\partial \omega}\right) \bar{p}_n + 2n_2^2 \left(\frac{\partial Z_L}{\partial \omega}\right)^2 p_n q_n - \frac{\partial^2 Z_L}{\partial \omega^2}\, p_n \right]$
$\frac{\partial^2 p_n}{\partial \omega^2}$	$- j\frac{\partial^2 s}{\partial \omega^2} \bar{p}_n - 2\left(\frac{\partial s}{\partial \omega}\right)^2 \mathbf{p}^{\mathbf{T}}\bar{\mathbf{q}}$
	$+ n_2^2\left[j\left(\frac{\partial s}{\partial \omega}\right)\left(\frac{\partial Z_L}{\partial \omega}\right)\left(q_n \bar{p}_n + p_n \bar{q}_n\right) + 2n_2^2 \left(\frac{\partial Z_L}{\partial \omega}\right)^2 p_n q_n^2 - \frac{\partial^2 Z_L}{\partial \omega^2}\, p_n q_n \right]$

TABLE 6 (continued)

Derivative	Second-Order Sensitivity Expression
$\phi = \omega_0, \Delta\omega$	
$\frac{\partial^2 p_1}{\partial\phi\partial\omega}$	$-j\frac{\partial^2 s}{\partial\phi\partial\omega}\bar{p}_1 - 2\left(\frac{\partial s}{\partial\phi}\right)\left(\frac{\partial s}{\partial\omega}\right)\mathbf{p}^T\bar{\mathbf{p}} + 2jn_2^2\left(\frac{\partial s}{\partial\phi}\right)\left(\frac{\partial Z_L}{\partial\omega}\right)p_n\bar{p}_n$
$\frac{\partial^2 p_n}{\partial\phi\partial\omega}$	$-j\frac{\partial^2 s}{\partial\phi\partial\omega}\bar{p}_n - 2\left(\frac{\partial s}{\partial\phi}\right)\left(\frac{\partial s}{\partial\omega}\right)\mathbf{p}^T\bar{\mathbf{q}} + jn_2^2\left(\frac{\partial s}{\partial\phi}\right)\left(\frac{\partial Z_L}{\partial\omega}\right)\left(p_n\bar{q}_n + q_n\bar{p}_n\right)$
$\frac{\partial^2 p_1}{\partial n_2\partial\omega}$	$j4n_2Z_L\frac{\partial s}{\partial\omega}p_n\bar{p}_n + 2n_2p_n^2[2n_2^2Z_Lq_n - 1]\frac{\partial Z_L}{\partial\omega}$
$\frac{\partial^2 p_n}{\partial n_2\partial\omega}$	$j2n_2Z_L\frac{\partial s}{\partial\omega}(p_n\bar{q}_n + \bar{p}_nq_n) + 2n_2p_nq_n[2n_2^2Z_Lq_n - 1]\frac{\partial Z_L}{\partial\omega}$
$\frac{\partial^2 p_1}{\partial r_i\partial\omega}$	$j2\frac{\partial s}{\partial\omega}p_i\bar{p}_i + 2n_2^2p_np_iq_i\frac{\partial Z_L}{\partial\omega}$
$\frac{\partial^2 p_n}{\partial r_i\partial\omega}$	$j\frac{\partial s}{\partial\omega}(p_i\bar{q}_i + \bar{p}_iq_i) + n_2^2(p_iq_n + p_nq_i)q_i\frac{\partial Z_L}{\partial\omega}$

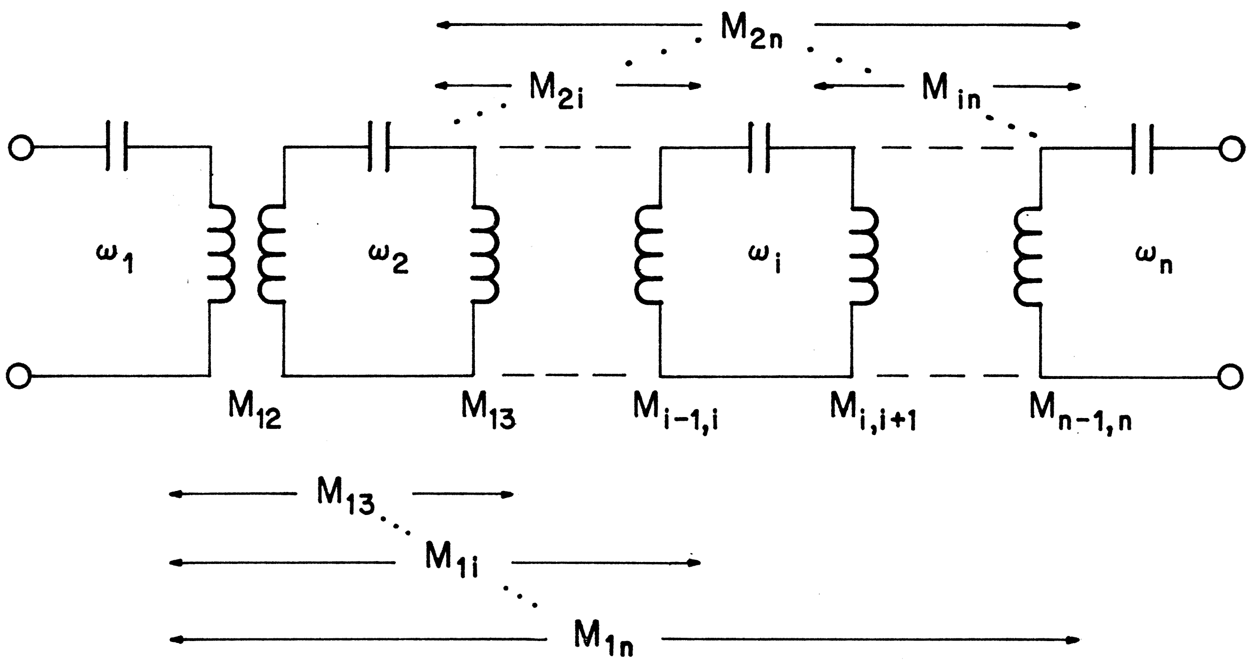
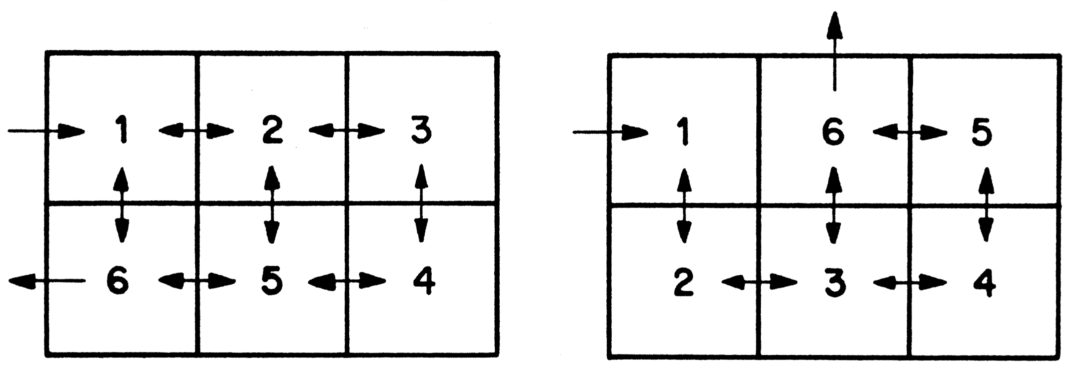


Fig. 1 Unterminated coupled-cavity filter illustrating the coupling coefficients.



$$\begin{bmatrix} & M_{12} & & & & M_{16} \\ M_{21} & & M_{23} & & M_{25} & \\ & M_{32} & & M_{34} & & \\ & & M_{43} & & M_{45} & \\ & M_{52} & & M_{54} & & M_{56} \\ M_{61} & & & & M_{65} & \end{bmatrix}$$

Symmetrical Realization

$$\begin{bmatrix} & M_{12} & & & & \\ M_{21} & & M_{23} & & & \\ & M_{32} & & M_{34} & & M_{36} \\ & & M_{43} & & M_{45} & \\ & & & M_{54} & & M_{56} \\ & & M_{63} & & M_{65} & \end{bmatrix}$$

Asymmetrical Realization

Fig. 2 Notation for symmetric and asymmetric filter realization examples [5].

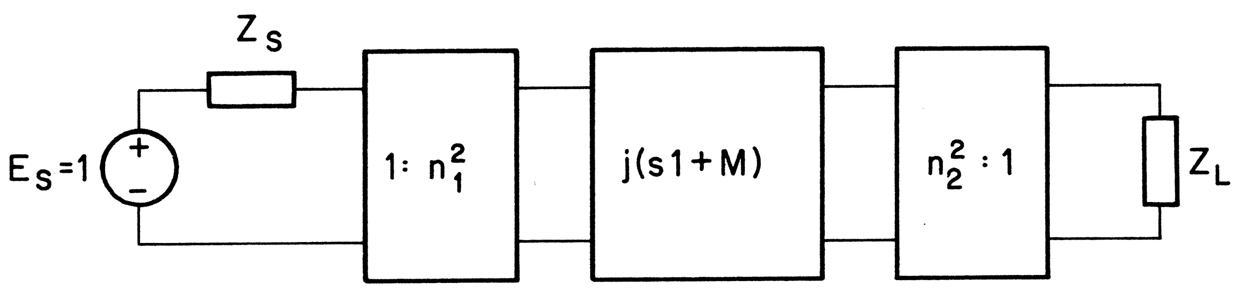
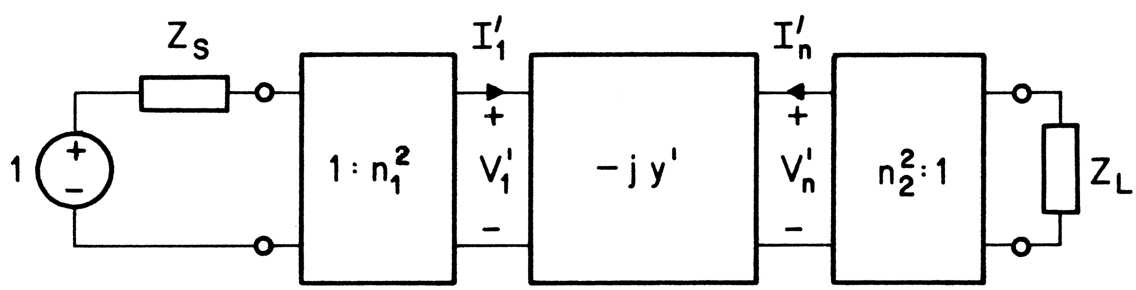
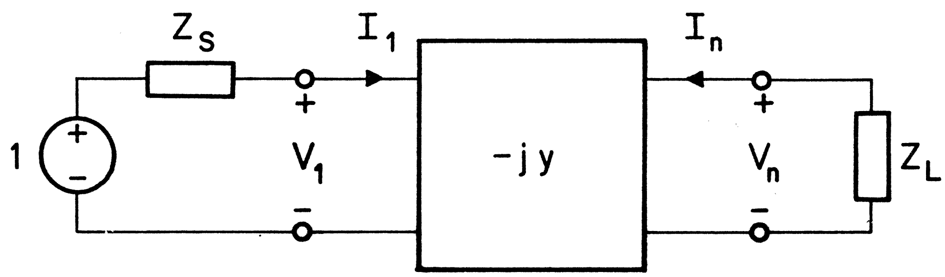


Fig. 3 Block representation of the overall network considered.



(a)



(b)

Fig. 4 Two-port representations of the coupled-cavity filter.

(a) the **y**′ matrix. (b) the **y** matrix.

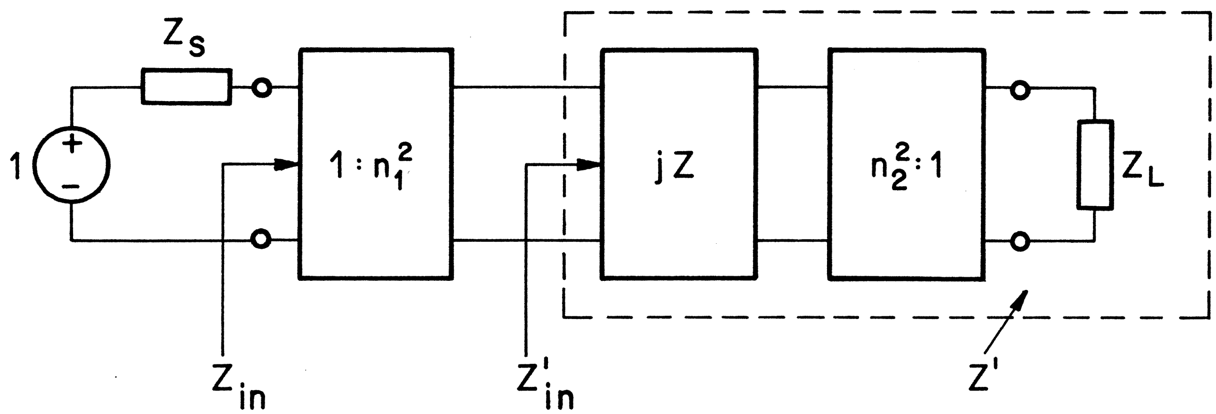


Fig. 5 The input impedance of the loaded filter illustrating **Z**, **Z**′, Z'_{in} and Z_{in}.