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FUNCTIONAL APPROACH TO MICROWAVE POSTPRODUCTION TUNING

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Abstract

This paper deals with the postproduction tuning problem in microwave circuits using the functional approach. The main aspects of the problem are addressed. In particular, we consider the choice of the critical samples of the response, the choice of the most effective tunable parameters, and the description of two functional tuning algorithms. Least-one and minimax optimizations are utilized in the formulation of the considered problems. Minimax optimization is used to identify the tuning frequencies, and least-one optimization is employed to minimize the number of tunable parameters. Worst-case analysis is utilized to reduce the size of the problem. The proposed techniques are applied, using recent, well documented and highly efficient optimization packages, in tuning a microwave amplifier.

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I. INTRODUCTION

Postproduction tuning is often essential in the manufacturing of electrical circuits. Tolerances on the circuit components, parasitic effects and uncertainties in the circuit model cause deviations in the manufactured circuit performance and violation of the design specifications may result. Therefore, postproduction tuning is included in the final stages of the production process to readjust the network performance in an effort to meet the specifications.

Tuning has formally been considered as an integral part of the design process [1], the objective being to relax the tolerances and compensate for the uncertainties in the model parameters. We give here a unified and integrated approach to the postproduction tuning problem. Minimax optimization is used in the nominal design stage to provide us with the critical active functions. As such, the tuning frequencies are identified. The least-one optimization is used to minimize the number of tunable parameters needed to tune all possible outcomes of a manufactured circuit. Worst-case analysis is employed to reduce the size of the problem. Finally, two functional tuning algorithms are presented. Both algorithms are based on measuring the response of the circuit at a number of critical frequencies and formulating the postproduction tuning as an optimization problem, which is solved on-line for the required changes in the tunable parameter values.

A microwave amplifier example is included to illustrate the application of the approach to real examples.

II. FUNDAMENTAL CONCEPTS AND DEFINITIONS

The actual values of the p circuit parameters can be expressed as

$$\boldsymbol{\phi} \triangleq \boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}\,, \tag{1a}$$

where

$$\boldsymbol{\phi}^0 \triangleq [\,\phi_1^0\;\phi_2^0 \cdots \phi_p^0\,]^T, \tag{1b}$$

$$\mathbf{E} \triangleq \mathrm{diag}\,\{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p\}, \tag{1c}$$

$$-1 \leq \mu_i \leq 1, i \in I_\phi, \tag{1d}$$

and

$$I_\phi \triangleq \{1, 2, \ldots, p\}. \tag{1e}$$

$\boldsymbol{\phi}^0$ is the nominal vector and ε_i is the tolerance associated with the ith element ϕ_i [1,2].

The designer has no control over $\boldsymbol{\mu}$ and this leads to the concept of tolerance region R_ε defined by

$$R_\varepsilon \triangleq \{\boldsymbol{\phi} \mid \phi_i = \phi_i^0 + \varepsilon_i\, \mu_i,\, -1 \leq \mu_i \leq 1, i \in I_\phi\}, \tag{2}$$

which is a convex regular polytype of p dimensions with sides of length $2\varepsilon_i$, $i \in I_\phi$.

The extreme points of R_ε are defined by setting $\mu_i = \pm 1$. Thus, the set of vertices may be defined by

$$R_v \triangleq \{\boldsymbol{\phi} \mid \phi_i = \phi_i^0 + \varepsilon_i\, \mu_i,\, \mu_i \in \{-1, 1\},\ i \in I_\phi\}. \tag{3}$$

The number of points in R_v is 2^p. Let each of these points be indexed by $i \in I_v$, where

$$I_v \triangleq \{1, 2, \ldots, 2^p\}, \tag{4}$$

and the vertex number is given by the formula

$$r \triangleq 1 + \sum_{i=1}^{p} \left[\frac{\mu_i^r + 1}{2} \right] 2^{i-1}, \tag{5a}$$

$$\mu_i^r \in \{-1, 1\},\ i \in I_\phi\ . \tag{5b}$$

and the rth vertex is referred to by $\boldsymbol{\phi}^r$.

A subset of design parameters is often used for tuning the manufactured circuits to the design specifications. In the <u>tuning parameter selection problem</u>, of the tuning parameter set $I_t \subseteq I_\phi$ it is required to find the subset I_t^* of minimum cardinality, say k, such that all possible outcomes of R_ε region are tunable to satisfy the design specifications.

The circuit parameters (1) with tuning taken into consideration are given by

$$\phi_i \triangleq \left\{ \begin{array}{ll} \phi_i^0 + \mu_i\, \varepsilon_i & , i \notin I_t^* \\ \phi_i^0 + \mu_i\, \varepsilon_i + \rho_i t_i & , i \in I_t^* \end{array} \right\} \quad i \in I_\phi, \tag{6a}$$

where

$$-1 \leq \rho_i \leq 1, \quad i \in I_t^*, \tag{6b}$$

for two-way tuning. Or, more compactly,

$$\boldsymbol{\phi} = \boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu} + \mathbf{T}\,\boldsymbol{\rho}, \tag{6c}$$

where

$$\mathbf{T} \triangleq \mathrm{diag}\{t_1, t_2, \ldots, t_p\} \tag{6d}$$

and t_i is the tuning amount associated with the ith element, $t_i = 0$ if $i \notin I_t^*$.

The objective of the <u>postproduction tuning assignment problem</u> is to find the changes in the tunable parameters such that the manufactured circuit is tuned to satisfy the design specifications.

In a typical tuning assignment problem, the output response of a circuit $F(\boldsymbol{\phi}, \psi)$ is required to meet upper specification $S_u(\psi)$ and lower specification $S_\ell(\psi)$ at a number of discrete frequency points ψ_i. Without loss of generality, we consider the following error functions [3].

$$e_{ui}(\boldsymbol{\phi}) \triangleq e_u(\boldsymbol{\phi}, \psi_i) = w_{ui}(F_i(\boldsymbol{\phi}) - \mathbf{S}_{ui}),\ i \in \mathbf{I}_u, \tag{7a}$$

$$e_{\ell i}(\boldsymbol{\phi}) \triangleq e_\ell(\boldsymbol{\phi}, \psi_i) = w_{\ell i}(F_i(\boldsymbol{\phi}) - S_{\ell i}),\ i \in I_\ell, \tag{7b}$$

where

$$F_i(\boldsymbol{\phi}) \triangleq F(\boldsymbol{\phi}, \psi_i) \tag{7c}$$

and w_{ui}, $w_{\ell i}$ are positive weighting functions. I_u and I_ℓ are index sets, not necessarily disjoint.

Let

$$f_i \triangleq \begin{cases} e_{uj}, & j \in I_u \\ -e_{\ell k}, & k \in I_\ell \end{cases} \quad i \in I_c, \tag{8a}$$

where

$$I_c \triangleq \{1, 2, \ldots, m\}. \tag{8b}$$

The m functions

$$\mathbf{f} = [f_1\ f_2 \ldots f_m]^T \tag{8c}$$

characterize the circuit performance, which is monitored during the tuning process. Accordingly, the feasible region is defined as

$$R_c \triangleq \{\boldsymbol{\phi} \mid f_i(\boldsymbol{\phi}) \leq 0 , \ i \in I_c\}. \tag{8d}$$

Fig. 1 illustrates a tolerance region R_ε inscribed in the constraint region R_c.

III. SELECTION OF TUNING FREQUENCIES

It is required to find a subset $I_c^* \subset I_c$ of critical error functions, which are used in selecting the tunable parameters and could be the only monitored functions during the tuning assignment process.

The effect of including a particular frequency point is to greatly reduce the error f_i at that frequency. Since the response gradients for two closely spaced frequencies will be almost collinear, the frequencies should be reasonably spaced and placed in areas where tight control over the response is desired [4].

The nominal network design problem can be formulated as a minimax optimization problem as follows.

$$\underset{\boldsymbol{\phi}^0}{\text{Minimize}} \quad M_f(\boldsymbol{\phi}^0) \tag{9a}$$

where

$$M_f(\boldsymbol{\phi}^0) \triangleq \max_{i \in I_c} f_i(\boldsymbol{\phi}^0), \tag{9b}$$

which is converted to a regular nonlinear programming problem as follows.

$$\underset{\boldsymbol{\phi}^0, z}{\text{Minimize}} \quad z \tag{10a}$$

subject to

$$f_i(\boldsymbol{\phi}^0) \leq z \quad , \quad i \in I_c, \tag{10b}$$

where z is an independent additional variable.

The solution of the optimization problem (10) provides us with theoretically justifiable critical (or active) functions $f_j(\boldsymbol{\phi})$, $j \in I_c^*$, where $I_c^* \subset I_c$ is the index set of these active functions. The active functions are those approximately equal to z at the solution, i.e., the functions that reach the maximum value at the minimax solution.

Normally, each critical function corresponds to a sample frequency, consequently we determine using (10) the frequencies to be monitored during tuning. It is to be noted that the use of a minimax criterion in the nominal design process implies the identification of an active set of frequencies I_c^*, namely, the frequencies where the response typically reaches maxima and minima (equiripple response).

IV. SELECTION OF TUNABLE ELEMENTS

It is required to find the minimum number of tunable parameters to tune all possible manufactured outcomes of the circuit. A manufactured outcome of the circuit would be a point of the region R_ε (2). Worst-case analysis is carried out to identify the critical points of this region [5]. A worst-case point is assumed to occur at a vertex (5) of R_ε. A worst-case algorithm that utilizes first-order sensitivities is employed. The algorithm is similar to the one proposed by Brayton et al. [5].

For every critical function $f_i(\boldsymbol{\phi})$, $i \in I_c^*$, one or more vertices are selected. Let $I_{vi} \subset I_v$ be the index set of worst-case vertices corresponding to the function $f_i(\boldsymbol{\phi})$, $i \in I_c^*$, and let

$$I_v^* \triangleq \bigcup_i I_{vi} , \quad i \in I_c^* , \tag{11}$$

define the index set of critical vertices, $I_v^* \subset I_v$.

Let I_t be the index set of tunable parameter candidates $I_t \subseteq I_\phi$. The tunable parameters are obtained by solving the following optimization problem.

$$\text{Minimize} \sum_{i \in I_t} t_i \tag{12a}$$

w.r.t. t_i, ρ_i^r , $i \in I_t$, $r \in I_v^*$, where

$$t_i \geq 0 \quad , i \in I_t , \tag{12b}$$

$$-1 \leq \rho_i^r \leq 1 \quad , \quad i \in I_t \quad , r \in I_v^* \tag{12c}$$

such that for all $r \in I_v^*$,

$$\boldsymbol{\phi} \in R_c^*, \tag{12d}$$

where

$$\phi_i = \begin{cases} \phi_i^r & , i \notin I_t \\ \phi_i^r + t_i \rho_i^r & , i \in I_t \end{cases} \quad i \in I_\phi \tag{12e}$$

and

$$R_c^* \triangleq \{\boldsymbol{\phi} \mid f_i(\boldsymbol{\phi}) \leq 0,\ i \in I_c^*\}. \tag{12f}$$

The objective function in (12a) is a least-one objective function. In data fitting, the least-one criterion has been extensively applied to eliminate the faulty data. It is utilized here to force as many parameters as possible to have a zero value of t_i. This consequently reduces the number of tunable parameters required to tune all worst-case vertices. At the solution we obtain $I_t^* \subseteq I_t$, where

$$I_t^* \triangleq \{i \mid\ t_i \neq 0,\ i \in I_t\}. \tag{13}$$

V. FUNCTIONAL TUNING ALGORITHMS

After manufacturing and assembling, the circuit performance specifications are checked. If tuning is necessary, a sequence of tunable parameter adjustments is carried out until the specifications are met. Tuning algorithms are devised to automate the tuning assignment problem.

In practice, one of two classes of methods for postproduction tuning is employed, namely, the functional tuning approach and the deterministic tuning approach [6]. In the deterministic tuning approach, all of the parameters of the manufactured circuit and the possible parasitic effects are measured. Then, a matching procedure is carried out, where it is required to match the performance functions at specified frequency points by varying the tunable parameter values. In the functional tuning approach, the actual network element values are generally assumed unknown. For example, it may be difficult to measure or identify the actual circuit element values.

In this section, two functional tuning algorithms are presented. Both algorithms are based on measuring the response of the circuit at a number of critical frequencies and formulating the postproduction tuning problem as an optimization problem.

A. A Linear Approximation Technique for Functional Tuning

The tuning assignment problem could be formulated in a similar way to the design problem, but with the tunable parameters $\boldsymbol{\rho}$ taken as the only variables [7].

Similarly to (10), the tuning assignment problem can be formulated as a minimax problem as follows.

$$\underset{\boldsymbol{\rho},\, z}{\text{Minimize}} \quad z \tag{14a}$$

subject to

$$f_i(\boldsymbol{\phi}^0 + \mathbf{E}\,\boldsymbol{\mu}^a + \mathbf{T}\,\boldsymbol{\rho}) \leq z \quad , \; i \in I_c \;, \tag{14b}$$

$$\boldsymbol{\rho}^\ell \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}^u . \tag{14c}$$

The superscript a in (14b) refers to the actual values of a certain manufactured outcome to be tuned. $\boldsymbol{\phi}^0$, $\mathbf{E}$ and $\mathbf{T}$ are all known. $\boldsymbol{\mu}^a$ is unknown and no control could be exerted on it. The optimization is carried out by varying $\boldsymbol{\rho}$. $\boldsymbol{\rho}^\ell$ and $\boldsymbol{\rho}^u$ represent limits on the tuning amounts. In the case of irreversible tuning, where, for example, the elements are allowed to increase, the limits are non-negative.

Since we restrict tuning amounts by (14c), a differentiable approximation can be used to estimate the changes in the functions and the minimax optimization problem, namely (14), can be approximated as follows.

$$\underset{\Delta\boldsymbol{\rho},\, z}{\text{Minimize}} \quad z \tag{15a}$$

subject to

$$f_i(\boldsymbol{\phi}^0 + \mathbf{E}\,\mu^a + \mathbf{T}\,\boldsymbol{\rho}) + \sum_{j \in I_t^*} \frac{\partial f_i}{\partial \rho_j} \Delta\rho_j \leq z \quad , \; i \in I_c \; , \tag{15b}$$

$$\rho_j^\ell \leq \Delta\rho_j \leq \rho_j^u \quad , \; j \in I_t^* . \tag{15c}$$

Initially, $\boldsymbol{\rho}$ in (15b) is equal to $\mathbf{0}$; after each solution of the optimization problem it is updated by $\Delta\boldsymbol{\rho}$. (15) is solved by a linear programming routine. The functions f_i in (15b) are measured directly. The sensitivities $\partial f_i/\partial\rho_j$ should be evaluated at the actual parameter values, which are unknown. As such, we utilize a suitable approximate model $\boldsymbol{\phi}^x$ for simulating these sensitivities (a good initial value could be the nominal parameter values or the parameter values that are predicted using a least-squares estimator). It is to be noted that from (6a) we have

$$\frac{\partial f_i}{\partial \rho_j} = \frac{\partial f_i}{\partial \phi_j} t_j . \tag{16}$$

The network sensitivities could be updated using the Broyden rank-one updating formula [8] after every solution (iteration) of (15). Let $\mathbf{B}$ be the sensitivity matrix whose (i,j) element is given by (16). Using the Broyden formula, $\mathbf{B}$ is updated as follows

$$\mathbf{B}^{i+1} = \mathbf{B}^i + \frac{(\mathbf{f}(\boldsymbol{\rho}^i + \Delta\boldsymbol{\rho}^i) - \mathbf{f}(\boldsymbol{\rho}^i) - \mathbf{B}^i\,\Delta\boldsymbol{\rho}^i)(\Delta\boldsymbol{\rho}^i)^T}{(\Delta\boldsymbol{\rho}^i)^T(\Delta\boldsymbol{\rho}^i)} \; , \tag{17}$$

where superscript i refers to the ith iteration of the algorithm.

The use of the Broyden formula exploits the measurements in improving the initial network model $\boldsymbol{\phi}^x$. A better approximation is obtained after each iteration. Application of this technique is reported in [7].

B. Modelling Technique for Functional Tuning

Let $F(\boldsymbol{\phi}, s)$ represent the response function that is monitored during the tuning process. The independent variable s is the complex frequency. We assume that the actual network response is given by [9]

$$F^a(\boldsymbol{\phi}^a, s) \triangleq F^0(\boldsymbol{\phi}^0, s) + F^d(\boldsymbol{\phi}^a, s) \ , \tag{18}$$

where the superscript a refers to the actual values, superscript 0 refers to the nominal values and F^d gives the deviational effect due to the changes in the circuit parameters, including parasitic effects, from nominal.

We model the deviational effect by a rational transfer function in s. Let

$$F^d(s) = \frac{a_N s^N + a_{N-1} s^{N-1} + \dots + a_0}{s^D + b_{D-1} s^{D-1} + \dots + b_0} \ , \tag{19}$$

where the degree of the numerator and that of the denominator, namely, N and D, are determined according to the characteristics of the original function $F(\boldsymbol{\phi}, s)$, together with the different known parasitic effects that affect the performance of the network. Some of the coefficients of (19) are set to zero as appropriate (e.g., if $F^d(s)$ is a pure real or a pure imaginary function).

The coefficients of (19) are obtained from (18) since the nominal response $F^0(\boldsymbol{\phi}^0, s_i)$ at a certain frequency s_i is simulated, and the actual response $F^a(\boldsymbol{\phi}^a, s_i)$ is measured directly. Measuring the response at ℓ different frequencies such that $2\ell > N+D+1$, we get an overdetermined linear real system of equations in the coefficients a_k, $k = 0, 1, \dots, N$ and b_j, $j = 0, 1, \dots, D-1$ in the form

$$\begin{bmatrix} -1 & -s_1 & \cdots & -s_1^{\,N} & F^d(s_1) & s_1F^d(s_1) & \cdots & s_1^{\,D-1}F^d(s_1) \\ -1 & -s_2 & & -s_2^{\,N} & F^d(s_2) & s_2F^d(s_2) & \cdots & s_2^{\,D-1}F^d(s_2) \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot & & \cdot \\ -1 & -s_\ell & & -s_\ell^{\,N} & F^d(s_\ell) & s_\ell F^d(s_\ell) & \cdots & s_\ell^{\,D-1}F^d(s_\ell) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_N \\ b_0 \\ b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_{D-1} \end{bmatrix} = \begin{bmatrix} -s_1^{\,D}F^d(s_1) \\ -s_2^{\,D}F^d(s_2) \\ \cdot \\ \cdot \\ \cdot \\ -s_\ell^{\,D}F^d(s_\ell) \end{bmatrix}, \tag{20a}$$

where

$$F^d(s_i) = F^a(\boldsymbol{\phi}^a, s_i) - F^0(\boldsymbol{\phi}^0, s_i). \tag{20b}$$

The number of real equations in (20a) is greater than the number of unknowns, as such the linear least-squares method is used to provide the solution. Let (20a) be represented by

$$\mathbf{C}\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \mathbf{W} \tag{21}$$

and

$$\mathbf{A} \triangleq \begin{bmatrix} \mathrm{Re}(\mathbf{C}) \\ \mathrm{Im}(\mathbf{C}) \end{bmatrix}, \tag{22}$$

then the solution of (21) is given by

$$\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = [\,\mathbf{A}^T\mathbf{A}\,]^{-1}\mathbf{A}^T \begin{bmatrix} \mathrm{Re}(\mathbf{W}) \\ \mathrm{Im}(\mathbf{W}) \end{bmatrix}. \tag{23}$$

Recalling (7), the error functions are defined by

$$\bar{e}_{ui}(\boldsymbol{\rho}) \triangleq w_{ui}\,(F_i(\boldsymbol{\rho}) + F^d(s_i) - S_{ui})\ ,\ i \in I_u, \tag{24a}$$

$$\bar{e}_{\ell i}(\boldsymbol{\rho}) \triangleq w_{\ell i}\,(F_i(\boldsymbol{\rho}) + F^d(s_i) - S_{\ell i})\ ,\ i \in I_\ell, \tag{24b}$$

where $F_i(\boldsymbol{\rho})$ is the response function evaluated at

$$\phi_j = \begin{cases} \phi_j^0 & ,\ j \notin I_t \\ \phi_j^0 + \rho_j t_j & ,\ j \in I_t \end{cases} \quad j \in I_\phi \tag{25}$$

and s_i. Consequently, we define

$$\bar{f}_i \triangleq \begin{cases} \bar{e}_{uj}, & j \in I_u \\ -\bar{e}_{\ell k}, & k \in I_\ell \end{cases} \quad i \in I_c . \tag{26}$$

Similarly to (10), the tuning assignment problem is formulated as

$$\underset{\boldsymbol{\rho},\, z}{\text{Minimize}}\; z \tag{27a}$$

subject to

$$z \geq \bar{f}_i , \quad i \in I_c, \tag{27b}$$

$$\rho_i^\ell \leq \rho_i \leq \rho_i^u , \quad i \in I_t^* . \tag{27c}$$

The solution of the optimization problem (27) provides us with $\boldsymbol{\rho}$. The tunable network parameters are adjusted by the amount indicated and the process is repeated until the circuit meets its specifications.

C. Tuning Algorithm

The two proposed tuning techniques could be applied on-line for the tuning of a microwave circuit as follows.

Step 1 Measure the network response. Check whether the design specifications are satisfied. If they are satisfied, stop.

Step 2 Utilize the performed measurements in constructing the error functions as well as their derivatives as required by the optimization problems (15) or (27).

Step 3 Solve the optimization problems (15) or (27) for the changes in the tunable parameters ($\boldsymbol{\rho}$ or $\Delta\boldsymbol{\rho}$). The upper and lower limits in the optimization problems are defined to ensure the validity of the approximation employed and the type of tuning.

Step 4 If the absolute value of a tunable element is less than the minimum amount of tuning which can be carried out in practice, we assume that it is zero. If all the absolute values of the tunable amounts are less than their corresponding minimum allowable values, stop.

<u>Step 5</u> Adjust the parameters to the extent possible by the amounts obtained from the optimization problem. If the maximum number of iterations has not been exceeded, return to 1.

VI. CONVERGENCE PROPERTIES OF THE ALGORITHMS

The algorithm proposed in (15) usually has a fast rate of convergence. This can be interpreted as follows [5]. The solution of (15) is usually at a point where

$$f_{i_1} = f_{i_2} = \ldots = f_{i_\ell} = z \quad , i_j \in I_c . \tag{28}$$

Usually $\ell \geq k + 1$, where k is equal to the number of tunable parameters. For $\ell = k + 1$ (15) corresponds to a Newton iteration for solving a system of nonlinear equations, namely (28). The Newton algorithm is known to have a quadratic rate of convergence under reasonable assumptions. The type of problem where $\ell \geq k + 1$ is referred to under certain conditions, as a regular type of problem [10]. In the tuning assignment problem, the number of functions considered in (15), m, is larger than the number of the tunable parameters, k. As such, we expect a problem of regular type.

Since the derivatives used in (15) are not the exact derivatives, it has been proved that if the iterative procedure (15) produces a converging sequence, then the rate of convergence is superlinear provided that every k x k submatrix of the sensitivity matrix **B** is nonsingular. This is usually referred to as the Haar condition [11].

When $\ell < k + 1$, we will have a problem of the singular type [10]. In this case, constraints (15c) will control the step size and we will have a steepest descent type algorithm. A linear rate of convergence is to be expected. The bounds in (15c) could be updated after every iteration to ensure that the difference between the error functions and linear approximations is small. This will ensure a faster rate of convergence and no overshooting of the optimum point. The scheme proposed in [11] could be adopted for this situation.

The optimization problem (27) is solved using a recent algorithm by Hald and Madsen [12]. The optimization algorithm has two stages and always starts in Stage 1. In Stage 1, the error functions $\bar{f}_j$, $j \in I_c$, are approximated by linear functions using the first-order derivatives. The Stage 2 iteration is introduced in order to speed up the final rate of convergence for problems which are singular at the solution. The Stage 2 algorithm is a modified quasi-Newton method, i.e., approximate second-order information is utilized. The convergence is usually fast since a quadratic rate of convergence is expected for this type of algorithm [12].

The modelling utilized in (18) and (19) is reminiscent of the system identification techniques, where the F^d function is identified from input-output measurements. F^d is expressed as a function in the independent variable s only. This is an approximation since the coefficients of the rational function F^d depend on the network parameters and they should vary with the variation in the tunable parameters. Under the assumption that the variation in the tunable parameters as constrained by (27c) is small, F^d could be assumed constant during the solution of (27). Then, F^d is updated after every iteration. Thus, (27c) has an important role in justifying the approximation taken in (19) and guaranteeing convergence by continuous adjustment of these bounds as in the linear approximation algorithm.

VII. TUNING OF A MICROWAVE MATCHING AMPLIFIER

As an example, we consider a broadband amplifier with a complex antenna load as shown in Fig. 2. The object is to match the antenna load over the frequency band 150 MHz to 300 MHz. The power given at a certain frequency is given by

$$4\, R_S G_L \frac{|V_L|^2}{|V_S|^2}\,, \tag{29}$$

where R_S is the source resistance, G_L is the real part of the admittance of the load, $|V_L|$ is the absolute value of the voltage across the load, and $|V_S|$ is the absolute value of the input voltage which we assumed to be unity. The response was assumed to be measured at sixteen uniformly distributed frequencies over the given frequency band. At each frequency, an error function was defined as the absolute difference between the measured response and the 10 dB specified power gain value. The source resistance was assumed to be 50 ohms. The transistor scattering parameter values and the antenna impedances at the sixteen frequencies were obtained from [13].

First, we applied optimization problem (10) to get the nominal design parameters using a minimax design criterion. The nominal parameters given in [13] were used as the initial design parameters for (10). We utilized the optimization package MMLC [14] for linearly constrained minimax optimization, as described in [12]. The MMLC package is an adaptation of the MMLA1Q package [15]. In the optimization, an upper practical bound of 200 ohms was assumed for the characteristic impedances. The reoptimized nominal response is superior to that obtained in [13]. This is partly because we relaxed the bounds on the design parameters. The new and previous nominal design parameters are given in Table I. The nominal response at the sixteen chosen frequencies is listed in Table II. The response alternated between maxima and minima at the critical frequency points. These frequencies, namely, 150, 160, 170, 220, 250, 280 and 300 MHz are identified by an asterisk in Table II. This set of frequencies constitutes the required I_c^*.

Then, worst-case analysis is performed using $\pm$ 5% tolerances and no parasitics are assumed. The number of vertices is equal to $2^8 = 256$. We assume that the design specifications tolerate $\pm$ 1 dB deviation from a specified value of 10 dB. At every critical frequency, $i \in I_c^*$, the worst-case vertices are obtained, as well as the corresponding worst-case responses. Four worst- case responses violate the design specifications, as is shown in Table II. Consequently, the set I_v^* consists of vertices {123, 134, 153}, as indicated in Table II.

Third, we performed the optimization problem (12), using the three critical vertices to determine the tunable parameters. The results of this optimization problem are given in Table I. It is clear that Z_1 and ℓ_4 are the tunable parameters. The optimization package MFNC [16], which implements the Han-Powell algorithm described in [17], is utilized in solving this problem. The MFNC package is an adaptation of the VF02AD subroutine of the Harwell Subroutine Library [18].

Finally, we applied the modelling tuning algorithm to illustrate its utilization in tuning any possible outcome. We assumed that the actual power gain is given by

$$F^{a}(s) = F^{0}(s) + F^{d}(s) \ , \tag{30a}$$

where

$$F^{d}(s) = \frac{a_4 s^4 + a_2 s^2 + a_0}{s^4 + b_2 s^2 + b_0} \ . \tag{30b}$$

The lower and upper bounds in (27c) are taken to be $\pm$ 1. The results of tuning for the critical vertices I_v^* are given in Table III. The lower and upper bounds of (27c) were not active at the solution and only one iteration is needed to satisfy the specifications. The responses before and after tuning are shown in Figs. 3, 4 and 5. The solution of optimization problem (27) is obtained by the optimization package MMLC [14].

VIII. CONCLUSIONS

We have presented a unified integrated approach to the postproduction tuning problem. The approach optimally utilizes the information obtained at the design stage in specifying both the tunable parameters and the essential tuning frequencies.

Two new functional tuning techniques are presented. The techniques optimally use available response measurements and eliminate completely the experimental trial-and-error and one-at-a-time approach. They are quite general and can be applied to any network for both reversible and irreversible tuning.

The linear approximation technique will perform quite reasonably as long as the linear approximation is valid. Carrying out the tuning procedure in stages and updating the sensitivity matrix by the Broyden formula will ensure the validity of the linear approximation.

The modelling technique usually needs fewer response measurements than the linear approximation technique, but requires much more on-line computational capabilities. For reasonably small deviations of the network elements from nominal, the technique converges in one or two iterations.

The different techniques have been implemented using well-documented computer optimization packages.

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TABLE I

NOMINAL ELEMENT VALUES AND TUNABLE AMOUNTS

Element	Original Nominal Values	New Nominal Values	Relative Tunable Amounts
ℓ_1	2.012	1.741	0.0
Z_1	86.76	68.778	0.0088
ℓ_2	0.976	1.534	0.0
Z_2	97.57	200.0	0.0
ℓ_3	0.833	1.140	0.0
Z_3	125.	181.252	0.0
ℓ_4	0.925	1.280	0.079
Z_4	132.	105.105	0.0

ℓ is the normalized length. The actual length equals $\ell\lambda_n/2\pi$, where λ_n is the wavelength at 230 MHz. Z is the characteristic impedance in ohms.

TABLE II

THE OPTIMAL NOMINAL RESPONSE AND WORST-CASE RESPONSE FOR ± 5% TOLERANCE

Frequency (MHz)	Power Gain (dB)	Worst-Case Vertex	Worst-Case Response (dB)
150	10.058 *	123	11.318
160	9.926 *	134	8.559
170	10.072 *	123	11.274
180	10.043	107	11.155
190	10.053	107	11.189
200	10.006	107	11.095
210	10.028	104	11.053
220	9.926 *	153	8.794
230	10.031	104	10.894
240	10.028	112	10.765
250	10.072 *	80	10.726
260	10.031	80	10.640
270	9.965	189	9.313
280	9.926 *	189	9.302
290	9.983	61	9.392
300	10.072 *	212	10.657

* identifies critical frequencies

TABLE III

RESULTS OF TUNING

	Case 1	Case 2	Case 3
Vertex No.	123	134	153
No. of Iterations of Functional Tuning Algorithm	1	1	1
Tunable Element Values	$Z_1 = 66.66$ $\ell_4 = 1.209$	$Z_1 = 70.616$ $\ell_4 = 1.331$	$Z_1 = 66.66$ $\ell_4 = 1.154$

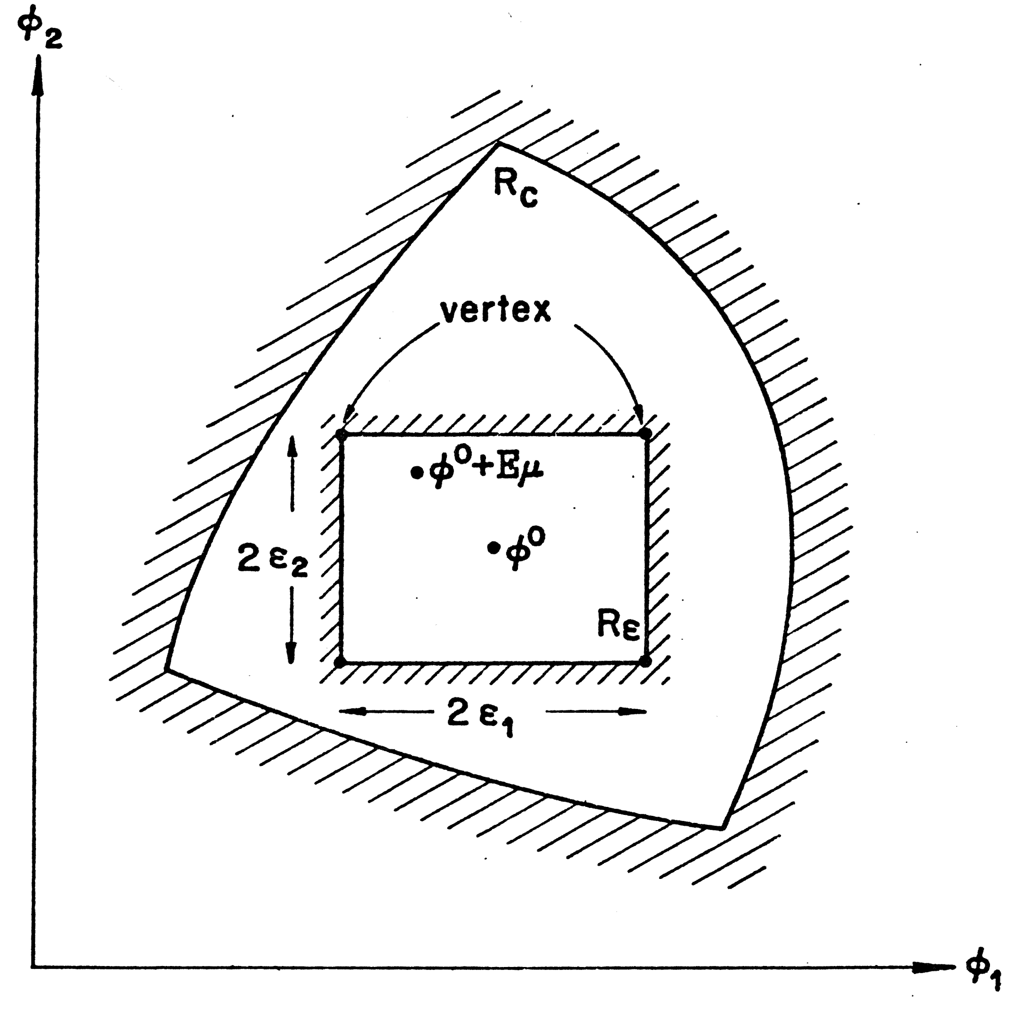

Fig. 1 A tolerance region R_ϵ inscribed in the feasible region R_c.

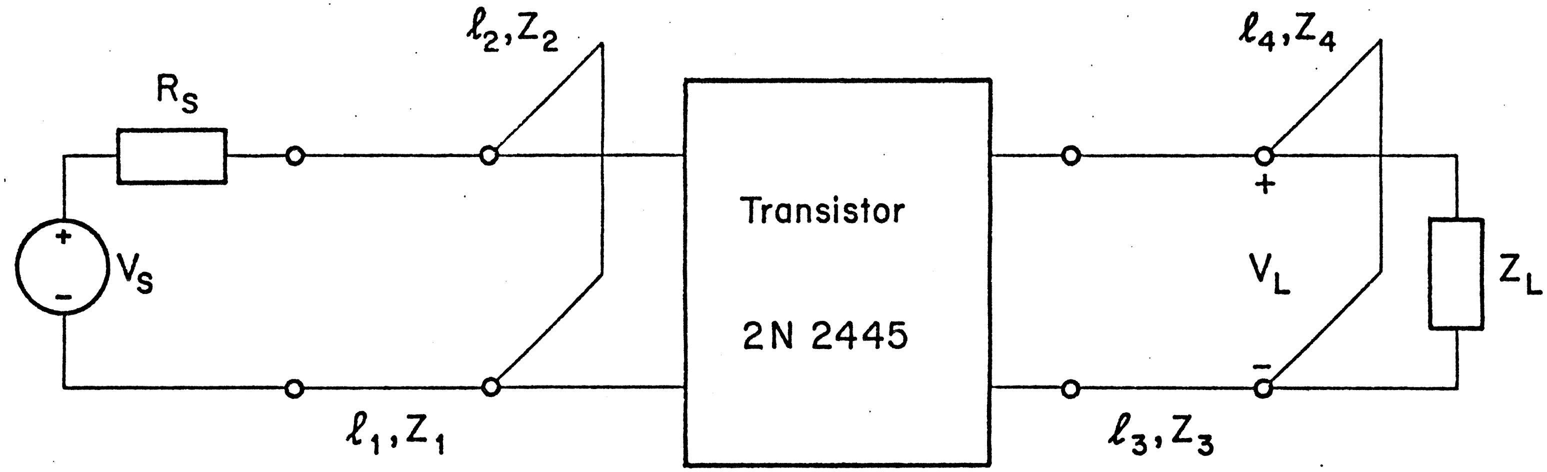

Fig. 2 The broadband microwave amplifier.

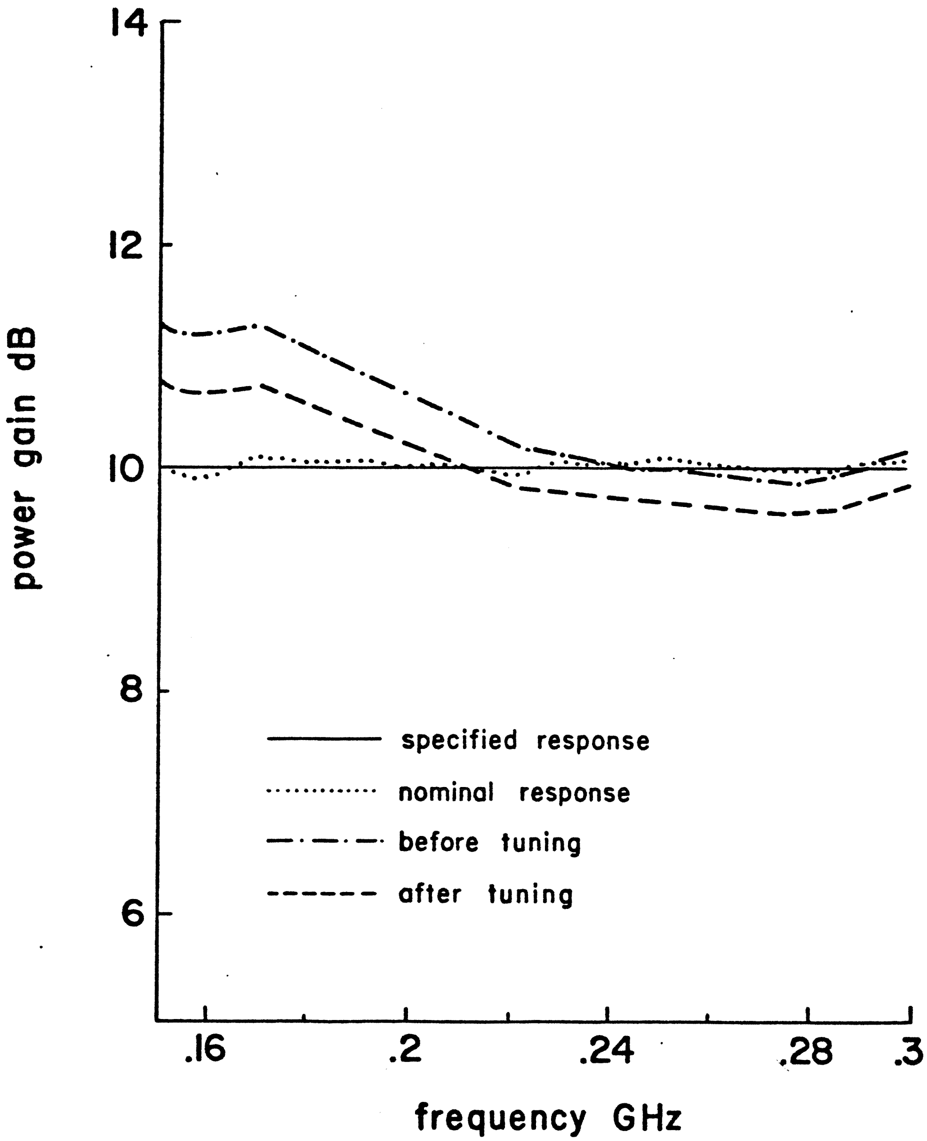

Fig. 3 The results of tuning an outcome corresponding to vertex number 123.

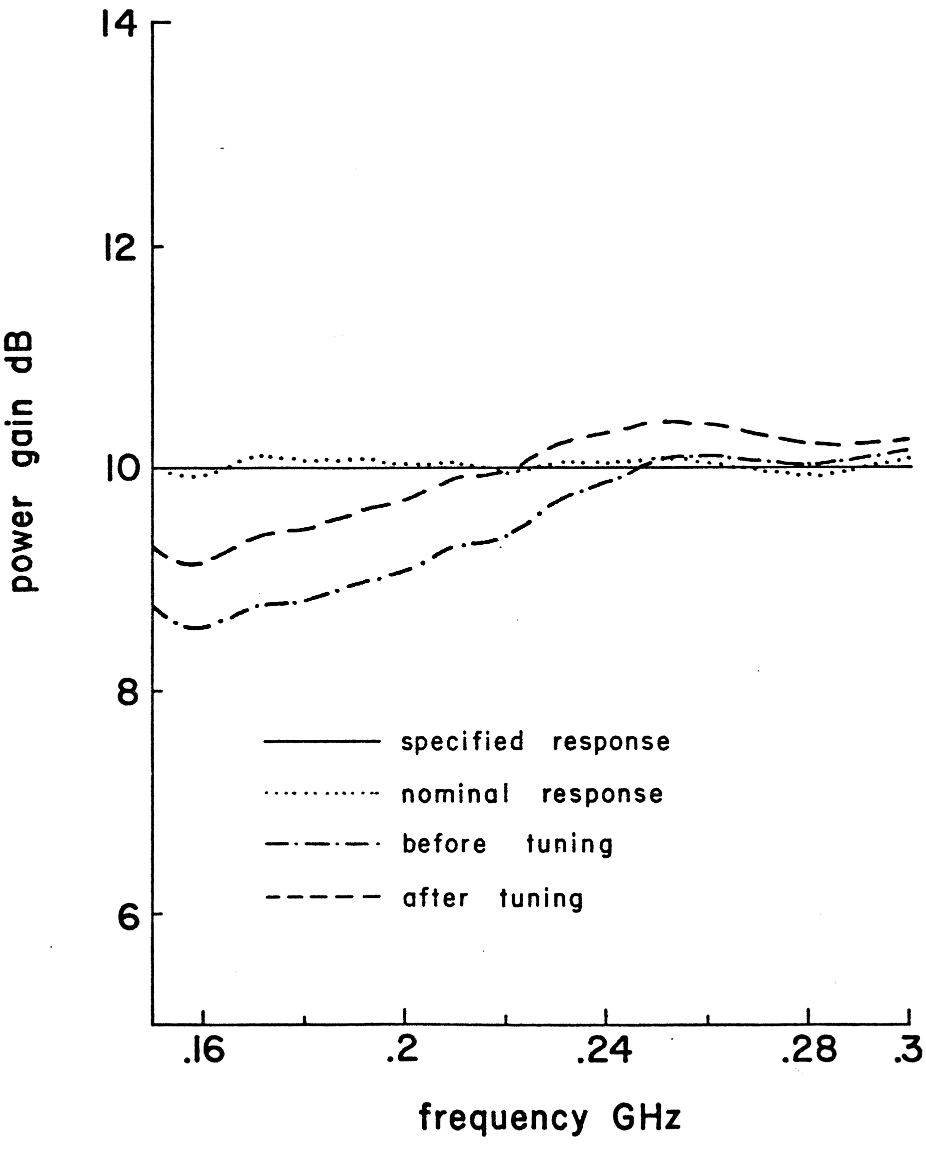

Fig. 4 The results of tuning an outcome corresponding to vertex number 134.

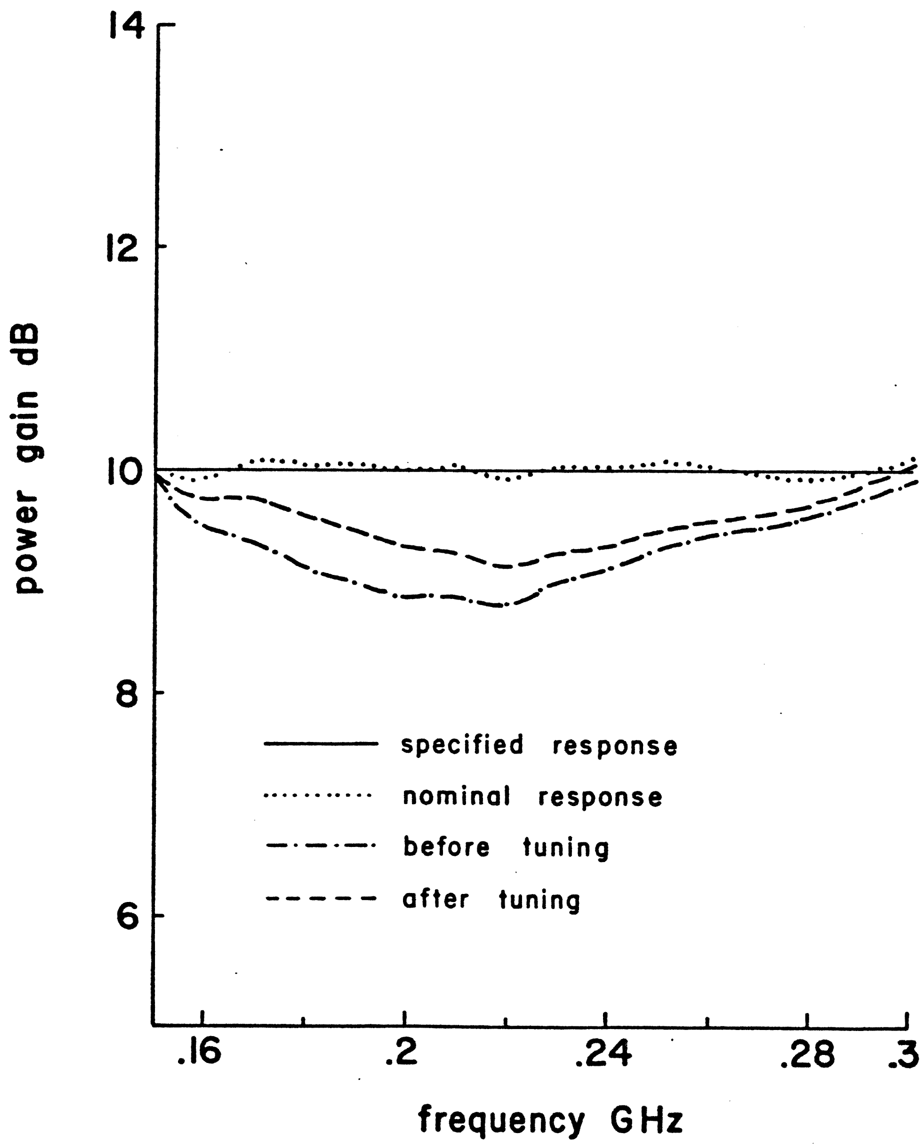

Fig. 5 The results of tuning an outcome corresponding to vertex number 153.