

**A MINIMAX APPROACH TO THE BEST  
MECHANICAL ALIGNMENT PROBLEM:  
DATA AND RESULTS**

J.W. Bandler, M.A. El-Kady and W. Kellermann

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A MINIMAX APPROACH TO THE BEST MECHANICAL  
ALIGNMENT PROBLEM: DATA AND RESULTS

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Abstract

A recently developed, minimax approach to the best mechanical alignment problem and its implementation, the BSTALN program, are applied, in this report, to a variety of test problems. The data resulted from practical problems of part alignment in manufactured mechanical systems and have been collected from inspecting actual parts. The complete data for each problem is provided. Results of using the BSTALN program for all sets of data are presented.

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The authors are with the Simulation Optimization Systems Research Laboratory and the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7.

M.A. El-Kady is also with Ontario Hydro, Toronto, Canada.

## I. INTRODUCTION

This report concerns itself with the application of a recently developed minimax approach to the best mechanical alignment problem [1] and its implementation in the form of the BSTALN program [2] to several test problems. The data for test problems, supplied by the Woodward Governor Company [3], resulted from practical problems of part alignment in manufactured mechanical systems and have been collected from inspecting actual parts, so the order of the function values represents the real life situation.

All data sets are given in Section II. Full results of running the BSTALN program are given in Section III.

## II. DATA SETS FOR TEST PROBLEMS

Seven data sets in the form of formatted computer files are presented. The structure of a data file as well as the codes used are described and explained in detail in [2].

5 0.990 SAMPLE NO.: 1 (NAME:#692)  
0 00.0000 0.0000 0.0000 0.0000 0.0010 0.0000  
12 0-0.8500 1.3682 -0.8780 -0.8750 1.3690 1.3720  
12 00.6589 0.7499 0.6610 0.6630 0.7500 0.7520  
12 00.8990 -0.4414 0.8990 0.9010 -0.4410 -0.4380  
12 0-0.5635 -1.5254 -0.5650 -0.5620 -1.5250 -1.5220

7 0.990 SAMPLE NO.: 2 (NAME:DATAJ)  
0 00.0000 -0.0001 0.0000 0.0000 0.0050 0.0000  
0 0-0.6412 1.1089 -0.6405 1.1094 0.0025 0.0000  
0 0-1.2778 -0.0052 -1.2810 0.0000 0.0025 0.0000  
0 0-0.6295 -1.1101 -0.6405 -1.1094 0.0025 0.0000  
0 00.6499 -1.1055 0.6405 -1.1094 0.0025 0.0000  
0 01.2846 0.0083 1.2810 0.0000 0.0025 0.0000  
0 00.6393 1.1126 0.6405 1.1094 0.0025 0.0000

11 0.990 SAMPLE NO.: 3 (NAME:DATA4)  
12 0-1.6967 -1.9628 -1.6980 -1.6940 -1.9640 -1.9600  
12 0-0.4684 2.5505 -0.4710 -0.4670 2.5490 2.5530  
12 02.3957 -0.9516 2.3930 2.3970 -0.9520 -0.9480  
12 0-0.8767 1.3691 -0.8730 -0.8750 1.3690 1.3720  
12 00.6621 0.7502 0.6610 0.6630 0.7500 0.7520  
12 00.8996 -0.4397 0.8990 0.9010 -0.4410 -0.4380  
23 0-0.5632 -1.5233 -1.5260 -1.5210 1.6225 1.6260  
13 4-0.0647 -1.1337 -0.0660 -0.0640 1.1358 1.1378  
13 50.8593 0.6268 0.8590 0.8610 1.0627 1.0647  
0 60.6650 -0.7859 0.6650 -0.7860 0.0010 0.0000  
23 7-0.9605 1.0265 1.0210 1.0260 1.4053 1.4073

11 0.990 SAMPLE NO.: 4 (NAME:DATA6)

0	02.3959	-0.9511	2.3950	-0.9500	0.0010	0.0000
0	0-1.6948	-1.9640	-1.6960	-1.9620	0.0010	0.0000
12	00.6625	0.7502	0.6610	0.6630	0.7500	0.7520
12	00.8990	-0.4402	0.8990	0.9010	-0.4410	-0.4380
23	0-0.5630	-1.5236	-1.5260	-1.5210	1.6225	1.6260
12	0-0.8762	1.3698	-0.8780	-0.8750	1.3690	1.3720
0	1-2.8634	3.5008	-2.8640	3.5010	0.0010	0.0000
12	1-0.8747	2.3276	-0.8750	-0.8710	2.3250	2.3290
0	40.6648	-0.7854	0.6650	-0.7860	0.0010	0.0000
23	5-0.9657	1.0207	1.0210	1.0260	1.4053	1.4073
13	6-0.0640	-1.1348	-0.0660	-0.0640	1.1358	1.1378

11 0.990 SAMPLE NO.: 5 (NAME:DATA7)  
0 02.3965 -0.9522 2.3950 -0.9500 0.0010 0.0000  
0 0-1.6952 -1.9644 -1.6960 -1.9620 0.0010 0.0000  
12 00.6629 0.7498 0.6610 0.6630 0.7500 0.7520  
12 00.8995 -0.4404 0.8990 0.9010 -0.4410 -0.4380  
23 0-0.5627 -1.5241 -1.5260 -1.5210 1.6225 1.6260  
12 0-0.8760 1.3692 -0.8780 -0.8750 1.3690 1.3720  
0 1-2.8645 3.5018 -2.8640 3.5010 0.0010 0.0000  
12 1-0.8750 2.3283 -0.8750 -0.8710 2.3250 2.3290  
0 40.6644 -0.7859 0.6650 -0.7860 0.0010 0.0000  
23 5-0.9648 1.0218 1.0210 1.0260 1.4053 1.4073  
13 6-0.0642 -1.1344 -0.0660 -0.0640 1.1358 1.1378



11 0.990 SAMPLE NO.: 6 (NAME:DATAG)  
0 02.3970 -0.9508 2.3950 -0.9500 0.0010 0.0000  
0 0-1.6955 -1.9621 -1.6960 -1.9620 0.0010 0.0000  
12 00.6620 0.7507 0.6610 0.6630 0.7500 0.7520  
12 00.8998 -0.4393 0.8990 0.9010 -0.4410 -0.4380  
23 0-0.5629 -1.5231 -1.5260 -1.5210 1.6225 1.6260  
12 0-0.8773 1.3700 -0.8780 -0.8750 1.3690 1.3720  
0 1-2.8646 3.5015 -2.8640 3.5010 0.0010 0.0000  
12 1-0.8764 2.3274 -0.8750 -0.8710 2.3250 2.3290  
0 40.6653 -0.7855 0.6650 -0.7860 0.0010 0.0000  
23 5-0.9642 1.0227 1.0210 1.0260 1.4053 1.4073  
13 6-0.0641 -1.1348 -0.0660 -0.0640 1.1358 1.1378

13 0.990 SAMPLE NO.: 7 (NAME:DATA12)  
12 0-1.6960 -1.9619 -1.6980 -1.6940 -1.9640 -1.9600  
12 0-0.4698 2.5496 -0.4710 -0.4670 2.5490 2.5530  
12 02.3954 -0.9517 2.3930 2.3970 -0.9520 -0.9480  
13 0-0.8773 1.3705 -0.8790 -0.8740 1.6247 1.6289  
13 00.6621 0.7503 0.6590 0.6640 0.9997 1.0025  
23 00.9014 -0.4385 -0.4420 -0.4370 1.0000 1.0031  
13 0-0.5628 -1.5233 -0.5660 -0.5610 1.6224 1.6263  
0 4-0.0621 0.4096 -0.0620 0.4095 0.0013 0.0000  
0 4-0.3093 0.9011 -0.3103 0.9012 0.0013 0.0000  
0 4-0.0635 -1.1348 -0.0650 -1.1350 0.0010 0.0000  
0 50.8587 0.6280 0.8600 0.6260 0.0010 0.0000  
0 60.7519 -0.7473 0.7510 -0.7510 0.0010 0.0000  
0 7-0.9645 1.0233 -0.9640 1.0240 0.0010 0.0000

### III. RESULTS FOR TEST PROBLEMS

The results of running the BSTALN program for all sets of data given in Section II are presented in the form of computer outputs. The information contained in the printed output is described and explained in detail in [2].

INPUT DATA FOR THE BEST ALIGNMENT PROBLEM DATE : 83/06/29 TIME : 12.05.56:

TOTAL NUMBER OF HOLES OF THE SAMPLE: 5 HOLES C SAMPLE NO.: 1 (NAME:#692)

I	KTC(I)	KOC(I)	XA(I)	YA(I)	T1(I)	T2(I)	T3(I)	T4(I)
1	0	0	0.000000	0.000000	0.000000	0.000000	0.01000	0.000000
2	12	0	-0.880000	1.368200	-0.878000	-0.875000	1.369000	1.372000
3	12	0	0.653900	0.749900	0.661000	0.663000	0.750000	0.752000
4	12	0	0.899000	-0.441400	0.899000	0.901000	-0.441000	-0.438000
5	12	0	-0.563500	-1.525400	-0.565000	-0.562000	-1.525000	-1.522000

ERROR FUNCTIONS AT THE STARTING POINT

1	-1.0000000E-03
2	2.0000000E-03
3	2.1000000E-03
4	4.0000000E-04
5	4.0000000E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 3.6078837E-04 N.IT: 12 N.SH: 1

STAGE: 0	FMAX: 3.6078837E-04
1	3.6078837E-04
2	-3.1240099E-04
3	3.6078837E-04
4	3.6078837E-04
5	-9.9390625E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 3) -6.4566765E-04 N.IT: 25 N.SH: 2

1	1.8703916E-03	5	-6.4568441E-04
2	2.0952197E-03	2	-6.7804852E-04
3	-1.1678011E-03	3	-6.4567620E-04
		4	-6.4566765E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.8124698E-04 N.IT: 10 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 3) 3.1002078E-04 N.IT: 26 N.SH: 2

SOLUTION (TYPE: 0)

1.8703916E-03  
2.0952197E-03  
-1.1678011E-03

MAX ERROR AT THE SOLUTION: -6.4566765E-04  
5 -6.4568441E-04  
2 -6.7804852E-04  
3 -6.4567620E-04  
4 -6.4566765E-04

NUMBER OF DELETIONS: 1  
1 1.0000000E+99

TOTAL EXECUTION TIME : .603 SECONDS

INPUT DATA FOR THE BEST ALIGNMENT PROBLEM DATE : 83/06/28 TIME : 14.34.19.

TOTAL NUMBER OF HOLES OF THE SAMPLE: 7 HOLES C SAMPLE NO.: 2 (NAME:DATAJ)

I	KTC(I)	KOC(I)	XA(I)	YA(I)	T1(I)	T2(I)	T3(I)	T4(I)
1	0	0	0.000000	-.000100	0.000000	0.000000	.005000	0.000000
2	0	0	-.641200	1.108000	-.640500	1.109400	.002500	0.000000
3	0	0	-1.277800	-.005200	-1.281000	0.000000	.002500	0.000000
4	0	0	-.629500	-1.110100	-.640500	-1.109400	.002500	0.000000
5	0	0	.649900	-1.105500	.640500	-1.109400	.002500	0.000000
6	0	0	1.284600	.008300	1.281000	0.000000	.002500	0.000000
7	0	0	.639300	1.112600	.640500	1.109400	.002500	0.000000

ERROR FUNCTIONS AT THE STARTING POINT

1	-4.900000E-03
2	-9.347524E-04
3	3.605735E-03
4	8.522250E-03
5	7.676934E-03
6	6.547099E-03
7	9.176015E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -7.735630E-04 N.IT: 15 N.SH: 1

1	-3.938434E-03
2	-1.432119E-03
3	-5.229598E-03
4	-7.735630E-04
5	-1.527117E-03
6	-2.153341E-03
7	-7.735630E-04

SOLUTION (TYPE: 0)

-3.9384343E-03  
-1.4321191E-03  
-5.2295988E-03

MAX ERROR AT THE SOLUTION: -7.7356303E-04  
1 -7.7356303E-04  
2 -1.2301572E-03  
3 -1.7501557E-03  
4 -7.7356303E-04  
5 -1.5271174E-03  
6 -2.1533414E-03  
7 -7.7356303E-04

NUMBER OF DELETIONS: 0

TOTAL EXECUTION TIME : .348 SECONDS

INPUT DATA FOR THE BEST ALIGNMENT PROBLEM DATE : 83/06/28 TIME : 14.40.13.

TOTAL NUMBER OF HOLES OF THE SAMPLE: 11 HOLES C SAMPLE NO.: 3 (NAME:DATA4)

I	KTC(I)	KOC(I)	XA(I)	YA(I)	T1(I)	T2(I)	T3(I)	T4(I)
1	12	0	-1.696700	-1.962300	-1.698000	-1.694000	-1.964000	-1.960000
2	12	0	-.468400	2.550500	-.471000	-.467000	2.549000	2.553000
3	12	0	2.395700	-.951600	2.393000	2.397000	-.952000	-.948000
4	12	0	-.876700	1.369100	-.878000	-.875000	1.369000	1.372000
5	12	0	.662100	.750200	.661000	.663000	.750000	.752000
6	12	0	.899600	-.439700	.899000	.901000	-.441000	-.438000
7	23	0	-.563200	-1.523300	-1.526000	-1.521000	1.622500	1.626000
8	13	4	-.064700	-1.133700	-.066000	-.064000	1.135800	1.137800
9	13	5	.859300	.626800	.859000	.861000	1.062700	1.064700
10	0	6	.665000	-.785900	.665000	-.786000	.001000	0.000000
11	23	7	-.960500	1.026500	1.021000	1.026000	1.405300	1.407300

ERROR FUNCTIONS AT THE STARTING POINT

1	-1.2000000E-03
2	-1.4000000E-03
3	-4.0000000E-04
4	-1.0000000E-04
5	-2.0000000E-04
6	-6.0000000E-04
7	-1.5803952E-03
8	2.5529370E-04
9	-3.0000000E-04
10	-9.0000000E-04
11	5.0000000E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 7.4930085E-05 N.IT: 9 N.SH: 0



STAGE: 0 FMAX: 7.4930085E-05

1	-7.0811807E-04
2	-1.4829245E-03
3	-7.7422163E-04
4	7.4930085E-05
5	-6.2087123E-04
6	-9.2787169E-04
7	-1.3715829E-03
8	5.9551994E-05
9	7.4930085E-05
10	7.4930085E-05
11	7.4930085E-05

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -3.8573753E-05 N.IT: 20 N.SH: 2

1	-9.1060760E-07	1	-6.2872590E-04
2	-2.5034284E-04	2	-1.1610038E-03
3	1.8917097E-04	3	-6.0287106E-04
4	-8.7670000E-01	11	-3.8573753E-05
5	1.3690899E+00	5	-7.4893829E-05
		6	-6.8225177E-04
		7	-1.6844624E-03
		8	-1.6493327E-04
		9	-3.8573753E-05
		10	-7.2698941E-04
		4	-8.9935654E-05 *

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 5.3453434E-05 N.IT: 9 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 3) 7.6379485E-05 N.IT: 25 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 6.7265600E-05 N.IT: 4 N.SH: 0

SOLUTION (TYPE: 0)

-9.1060760E-07
-2.5034284E-04
1.8917097E-04

MAX ERROR AT THE SOLUTION: -3.8573753E-05

1 -6.2872590E-04  
2 -1.1610038E-03  
3 -6.0287106E-04  
11 -3.8573753E-05  
5 -7.4893829E-05  
6 -6.8225177E-04  
7 -1.6844624E-03  
8 -1.6498327E-04  
9 -3.8573753E-05  
10 -7.2698941E-04

NUMBER OF DELETIONS:

1

4 -8.9935654E-05 -8.7670000E-01 1.3690899E+00

TOTAL EXECUTION TIME : .915 SECONDS

INPUT DATA FOR THE BEST ALIGNMENT PROBLEM DATE : 88/06/28 TIME : 14.41.54.

TOTAL NUMBER OF HOLES OF THE SAMPLE: 11 HOLES C SAMPLE NO.: 4 (NAME:DATA6)

I	KTC(I)	KOC(I)	XA(I)	YA(I)	T1(I)	T2(I)	T3(I)	T4(I)
1	0	0	2.395900	-.951100	2.395000	-.950000	.001000	0.000000
2	0	0	-1.694800	-1.964000	-1.696000	-1.962000	.001000	0.000000
3	12	0	.662500	.750200	.661000	.663000	.750000	.752000
4	12	0	.899000	-.440200	.899000	.901000	-.441000	-.438000
5	23	0	-.563000	-1.523600	-1.526000	-1.521000	1.622500	1.626000
6	12	0	-.876200	1.369800	-.873000	-.875000	1.369000	1.372000
7	0	1	-2.863400	3.500800	-2.864000	3.501000	.001000	0.000000
8	12	1	-.874700	2.327600	-.875000	-.871000	2.325000	2.329000
9	0	4	.664800	-.785400	.665000	-.786000	.001000	0.000000
10	23	5	-.965700	1.020700	1.021000	1.026000	1.405300	1.407300
11	13	6	-.064000	-1.134300	-.066000	-.064000	1.135800	1.137800

ERROR FUNCTIONS AT THE STARTING POINT

1	4.2126704E-04
2	1.3323808E-03
3	-2.0000000E-04
4	0.
5	-1.7075510E-03
6	-8.0000000E-04
7	-3.6754447E-04
8	-3.0000000E-04
9	-3.6754447E-04
10	3.0000000E-04
11	0.

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.7923925E-04 N.IT: 5 N.SH: 0

STAGE: 0 FMAX: 1.7923925E-04

1	1	1.7714141E-05
2	2	1.7923925E-04
3	3	-4.5260439E-04
4	4	1.7923925E-04
5	5	-1.1204907E-03
6	6	-1.0346159E-03
7	7	1.7923925E-04
8	8	-4.6661621E-04
9	9	1.7923925E-04
10	10	-7.2417939E-04
11	11	1.2310508E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 6.1421466E-05 N.IT: 5 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.0833212E-05 N.IT: 11 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.3571875E-04 N.IT: 5 N.SH: 0

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.7293332E-04 N.IT: 6 N.SH: 1

STAGE: 1 FMAX: 1.0833212E-05

1	1	1.0833173E-05
2	2	1.0833173E-05
3	3	-6.8573319E-04
4	11	1.0833173E-05
5	5	-1.3883871E-03
6	6	-1.3708130E-03
7	7	-1.7884179E-04
8	8	-5.9168074E-05
9	9	1.0833212E-05
10	10	-6.7487757E-04
11	4	1.0833173E-05

RESULT OF MINIMAX OPTIMIZATION (RETURN: 2) -7.5696174E-06 N.IT: 4 N.SH: 0

1	-2.3005575E-06	10	-7.5696174E-06
2	3.4184682E-04	11	-7.5696174E-06

3	2.2422537E-05	3	-5.1912211E-04
4	2.3956596E+00	4	-7.5696174E-06
5	-9.5074119E-01	5	-1.4725930E-03
		6	-1.1221999E-03
		7	-1.8664904E-04
		8	-5.0720697E-04
		9	-7.5696174E-06
		1	-7.7965523E-06 *

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -1.4400270E-04 N.IT: 12 N.SH: 1

1	-1.8351242E-04	1	-1.4400270E-04
2	6.5164678E-04	11	-1.4400272E-04
3	-1.9939429E-05	3	-6.6851649E-04
4	3.9918322E-01	10	-3.8220472E-04
5	-4.3958773E-01	5	-1.2448633E-03
		6	-1.3561307E-03
		7	-3.4347360E-04
		8	-1.4400272E-04
		9	-1.5185167E-04
		4	-1.8321757E-04 *

RESULT OF MINIMAX OPTIMIZATION (RETURN: 3) 4.6015648E-06 N.IT: 25 N.SH: 3

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 5.8269344E-05 N.IT: 9 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -1.4428908E-05 N.IT: 20 N.SH: 2

1	-4.0177999E-04	10	-6.8619587E-04
2	5.2555536E-04	2	-1.4430492E-05
3	-3.0131333E-04	3	-5.2590122E-04
4	2.3959333E+00	11	-1.4430492E-05
5	-9.5031684E-01	5	-1.4390004E-03
6	8.9901443E-01	6	-1.1890012E-03
7	-4.3954558E-01	7	-1.4430020E-05
		8	-2.7965601E-04
		9	-1.4428908E-05
		1	-1.4430025E-05 *
		4	-1.4430492E-05 *

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -6.3865852E-05 N.IT: 11 N.SH: 1

1	-4.6682560E-04	1	-3.1299951E-04
2	8.7144637E-04	2	-6.3865852E-05
3	-1.6760830E-04	3	-8.4109516E-04
4	8.9906387E-01	10	-8.2767623E-04
5	-4.3899064E-01	5	-1.1371253E-03
		6	-1.1817145E-03

7 -6.3865852E-05  
 8 -6.3865852E-05  
 9 -6.3865852E-05  
 4 -6.3865852E-05 \*

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 4.0060363E-06 N.IT: 6 N.SH: 1  
 RESULT OF MINIMAX OPTIMIZATION (RETURN: 3) 1.0321444E-04 N.IT: 25 N.SH: 3  
 RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 6.1421466E-05 N.IT: 5 N.SH: 1  
 RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.0833212E-05 N.IT: 11 N.SH: 1  
 RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.1564696E-04 N.IT: 5 N.SH: 1  
 RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.3571875E-04 N.IT: 6 N.SH: 1  
 RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.6613247E-04 N.IT: 11 N.SH: 1

SOLUTION (TYPE: 0)

-1.8351242E-04  
 6.5164678E-04  
 -1.9989429E-05

MAX ERROR AT THE SOLUTION: -1.4400270E-04

1 -1.4400270E-04  
 11 -1.4400272E-04  
 3 -6.6851649E-04  
 10 -3.8220472E-04  
 5 -1.2443633E-03  
 6 -1.3561307E-03  
 7 -3.4347360E-04  
 8 -1.4400272E-04  
 9 -1.5185167E-04

NUMBER OF DELETIONS:

2  
 4 -1.8321757E-04  
 2 1.0000000E+99  
 8.9918322E-01 -4.3958773E-01

TOTAL EXECUTION TIME : 2.754 SECONDS

INPUT DATA FOR THE BEST ALIGNMENT PROBLEM DATE : 83/06/28 TIME : 14.44.00.

TOTAL NUMBER OF HOLES OF THE SAMPLE: 11 HOLES C SAMPLE NO.: 5 (NAME:DATA7)

I	KTC(I)	KOC(I)	XA(I)	YA(I)	T1(I)	T2(I)	T3(I)	T4(I)
1	0	0	2.396500	-0.952200	2.395000	-0.950000	0.001000	0.000000
2	0	0	-1.695200	-1.964400	-1.696000	-1.962000	0.001000	0.000000
3	12	0	0.662900	0.749800	0.661000	0.663000	0.750000	0.752000
4	12	0	0.899500	-0.440400	0.899000	0.901000	-0.441000	-0.433000
5	23	0	-0.562700	-1.524100	-1.526000	-1.521000	1.622500	1.626000
6	12	0	-0.876000	1.369200	-0.878000	-0.875000	1.369000	1.372000
7	0	1	-2.864500	3.501800	-2.864000	3.501000	0.001000	0.000000
8	12	1	-0.875000	2.328300	-0.875000	-0.871000	2.325000	2.329000
9	0	4	0.664400	-0.785900	0.665000	-0.786000	0.001000	0.000000
10	23	5	-0.964800	1.021800	1.021000	1.026000	1.405300	1.407300
11	13	6	-0.064200	-1.134400	-0.066000	-0.064000	1.135800	1.137800

ERROR FUNCTIONS AT THE STARTING POINT

1	1.6627054E-03
2	1.5298221E-03
3	2.0000000E-04
4	-5.0000000E-04
5	-1.3424669E-03
6	-2.0000000E-04
7	-5.6601887E-05
8	0.
9	-3.9172375E-04
10	-1.6434117E-05
11	-2.0000000E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 7.9814401E-04 N.IT: 11 N.SH: 1



STAGE: 0 FMAX: 7.9814401E-04

1	7.9814401E-04
2	7.9814401E-04
3	-5.1036784E-04
4	-5.9116440E-05
5	-1.6647822E-03
6	-8.6897155E-04
7	7.9814401E-04
8	4.8974722E-04
9	3.1774134E-04
10	-7.9164306E-04
11	2.2492457E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 3.0855461E-04 N.IT: 11 N.SH: 0

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 7.9418523E-04 N.IT: 16 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 5.4007725E-04 N.IT: 10 N.SH: 1

STAGE: 1 FMAX: 3.0855461E-04

1	11	3.0826072E-04
2	2	3.0826283E-04
3	3	3.0855461E-04
4	4	-2.9571915E-04
5	5	-1.5685104E-03
6	6	-2.7277491E-04
7	7	2.9659841E-04
8	8	-1.1844951E-03
9	9	2.9768714E-04
10	10	-9.1923214E-04
11	1	3.0697439E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 9.2411069E-05 N.IT: 10 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 2) -1.8494308E-04 N.IT: 7 N.SH: 0

1	-9.7527513E-05	11	-1.8494308E-04
2	3.2392799E-04	10	-1.8494308E-04

3	9.2047470E-05	-1.8494308E-04
4	2.3954484E+00	-4.4300638E-04
5	-9.5065770E-01	-1.6125728E-03
		-4.4328861E-04
		-4.8905497E-04
		-8.2736794E-04
		-1.8494308E-04
		-2.0397254E-04 *

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 3.0199773E-04 N.IT: 12 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 5.3309191E-04 N.IT: 14 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 3.0860001E-04 N.IT: 8 N.SH: 0

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 5.2600409E-04 N.IT: 6 N.SH: 1

SOLUTION (TYPE: 0)

-9.7527513E-05  
 3.2392799E-04  
 9.2047470E-05

MAX ERROR AT THE SOLUTION: -1.8494308E-04

11	-1.8494308E-04
10	-1.8494308E-04
3	-1.8494308E-04
4	-4.4300638E-04
5	-1.6125728E-03
6	-4.4328861E-04
7	-4.8905497E-04
8	-8.2736794E-04
9	-1.8494308E-04

NUMBER OF DELETIONS: 2

2	1.0000000E+99
1	-2.0397254E-04
	2.3954484E+00 -9.5065770E-01

TOTAL EXECUTION TIME : 1.499 SECONDS

INPUT DATA FOR THE BEST ALIGNMENT PROBLEM DATE : 83/06/28 TIME : 14.45.43.

TOTAL NUMBER OF HOLES OF THE SAMPLE: 11 HOLES C SAMPLE NO.: 6 (NAME: DATAG)

I	KIV(I)	KOC(I)	XA(I)	YA(I)	T1(I)	T2(I)	T3(I)	T4(I)
1	0	0	2.397000	-.950800	2.395000	-.950000	.001000	0.000000
2	0	0	-1.695500	-1.962100	-1.696000	-1.962000	.001000	0.000000
3	12	0	.662000	.750700	.661000	.663000	.750000	.752000
4	12	0	.899800	-.439300	.899000	.901000	-.441000	-.438000
5	23	0	-.562900	-1.523100	-1.526000	-1.521000	1.622500	1.626000
6	12	0	-.877300	1.370000	-.878000	-.875000	1.369000	1.372000
7	0	1	-2.864600	3.501500	-2.864000	3.501000	.001000	0.000000
8	12	1	-.876400	2.327400	-.875000	-.871000	2.325000	2.329000
9	0	4	.665300	-.785500	.665000	-.786000	.001000	0.000000
10	23	5	-.964200	1.022700	1.021000	1.026000	1.405300	1.407300
11	13	6	-.064100	-1.134800	-.066000	-.064000	1.135800	1.137800

ERROR FUNCTIONS AT THE STARTING POINT

1	1.1540659E-03
2	-4.9009805E-04
3	-7.0000000E-04
4	-8.0000000E-04
5	-1.2887855E-03
6	-7.0000000E-04
7	-2.1897503E-04
8	1.4000000E-03
9	-4.1690481E-04
10	-2.5929437E-04
11	-1.0000000E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 7.8766877E-04 N.IT: 18 N.SH: 2

STAGE: 0 FMAX: 7.8766877E-04

1	7.8766877E-04
2	7.8054088E-04
3	-6.7451522E-04
4	-5.1145712E-04
5	-4.1859431E-04
6	-6.1087476E-04
7	7.8766877E-04
8	7.8766877E-04
9	-2.2387620E-04
10	-2.7637365E-04
11	6.1249301E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -1.9911453E-04 N.IT: 15 N.SH: 2

1	-2.6956110E-04	11	-4.0926333E-04
2	6.4376405E-07	2	-3.1859860E-04
3	1.6885904E-04	3	-6.036698E-04
4	2.3943989E+00	4	-6.0460585E-04
5	-9.5037835E-01	5	-1.3815043E-03
		6	-1.9911453E-04
		7	-1.9911453E-04
		8	-1.9911453E-04
		9	-1.9911453E-04
		10	-1.9911453E-04
		1	-6.0836163E-04 *

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 7.8766877E-04 N.IT: 14 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 7.7354889E-04 N.IT: 14 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 3.2617636E-04 N.IT: 17 N.SH: 2

SOLUTION (TYPE: 0)

-2.6956110E-04
6.4376405E-07
1.6885904E-04

MAX ERROR AT THE SOLUTION: -1.9911453E-04

11 -4.0926883E-04  
12 -3.1859860E-04  
13 -6.0366698E-04  
14 -6.0460585E-04  
15 -1.3816043E-03  
16 -1.9911453E-04  
17 -1.9911453E-04  
18 -1.9911453E-04  
19 -1.9911453E-04  
10 -1.9911453E-04

NUMBER OF DELETIONS:

1

1 -6.0836163E-04

2.3948989E+00 -9.5037835E-01

TOTAL EXECUTION TIME : 1.206 SECONDS

INPUT DATA FOR THE BEST ALIGNMENT PROBLEM DATE : 88/06/28 TIME : 14.47.12.

TOTAL NUMBER OF HOLES OF THE SAMPLE: 13 HOLES C SAMPLE NO.: 7 (NAME:DATA12)

I	KTC(I)	KOC(I)	XA(I)	YA(I)	T1(I)	T2(I)	T3(I)	T4(I)
1	12	0	-1.696000	-1.961900	-1.698000	-1.694000	-1.964000	-1.960000
2	12	0	-.469800	2.549600	-.471000	-.467000	2.549000	2.553000
3	12	0	2.395400	-.951700	2.393000	2.397000	-.952000	-.948000
4	13	0	-.877300	1.370500	-.879000	-.874000	1.624700	1.623900
5	13	0	.662100	.750300	.659000	.664000	.999700	1.002500
6	23	0	.901400	-.438500	-.442000	-.437000	1.000000	1.003100
7	13	0	-.562800	-1.523300	-.566000	-.561000	1.622400	1.626300
8	0	4	-.062100	.409600	-.062000	.409500	.001300	0.000000
9	0	4	-.309300	.901100	-.310300	.901200	.001300	0.000000
10	0	4	-.063500	-1.134800	-.065000	-1.135000	.001000	0.000000
11	0	5	.853700	.628000	.860000	.626000	.001000	0.000000
12	0	6	.751900	-.747300	.751000	-.751000	.001000	0.000000
13	0	7	-.964500	1.023300	-.964000	1.024000	.001000	0.000000

ERROR FUNCTIONS AT THE STARTING POINT

1	-1.9000000E-03
2	-6.0000000E-04
3	-3.0000000E-04
4	-1.6552305E-03
5	-9.6303020E-04
6	-7.0077314E-04
7	-1.5417262E-03
8	-1.1585786E-03
9	-2.9501244E-04

10 5.1327460E-04  
 11 1.3853721E-03  
 12 2.8078866E-03  
 13 -1.3976747E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.2342676E-03 N.IT: 14 N.SH: 2

STAGE: 0 FMAX: 1.2342676E-03

1	1	-8.8243692E-04
2	2	7.1249120E-04
3	3	1.2342676E-03
4	4	-1.2202249E-03
5	5	4.3584672E-04
6	6	-6.3636196E-04
7	7	-8.8996969E-04
8	8	-2.4825008E-06
9	9	1.7970798E-04
10	10	4.2378715E-04
11	11	8.5742274E-04
12	12	1.2342676E-03
13	13	1.2342676E-03

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 6.0979363E-04 N.IT: 11 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 6.0122374E-04 N.IT: 12 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.1743574E-03 N.IT: 9 N.SH: 1

STAGE: 1 FMAX: 6.0122374E-04

1	1	-1.6458607E-03
2	2	-2.9237758E-04
3	3	6.0122374E-04
4	4	-2.0244976E-03
5	5	-6.9416522E-04
6	6	-4.7645403E-04
7	7	-1.9033781E-03
8	8	-9.6039168E-04
9	9	2.4492222E-04
10	10	6.0122374E-04
11	11	6.0122374E-04

12 13 -3.7972612E-05  
13 12 1.0000000E+99

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 4.8844439E-04 N.IT: 7 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 3.4708189E-04 N.IT: 12 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 2.9675368E-04 N.IT: 9 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 4.5847390E-04 N.IT: 6 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 4.2908671E-05 N.IT: 16 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.1648191E-03 N.IT: 10 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.1611524E-03 N.IT: 14 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 6.0122374E-04 N.IT: 11 N.SH: 1

STAGE: 2 FMAX: 4.2908671E-05

1	1	-1.8093438E-03
2	2	4.2908671E-05
3	3	-1.5498846E-03
4	4	-7.9119724E-04
5	5	-1.0473696E-03
6	6	-6.7345022E-04
7	7	-1.1863371E-03
8	8	-1.0449391E-04
9	9	-1.0876112E-03
10	10	4.2908671E-05
11	11	4.2908671E-05
12	11	1.0000000E+99
13	12	-1.0000000E+99

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 4.8820960E-04 N.IT: 15 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 2.7123125E-04 N.IT: 16 N.SH: 2



RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 4.5723575E-04 N. IT: 9 N. SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 2.3428008E-04 N. IT: 6 N. SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 2.3123338E-04 N. IT: 15 N. SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 1.2173243E-04 N. IT: 7 N. SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 2.9663909E-04 N. IT: 11 N. SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 4.2968671E-05 N. IT: 13 N. SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 3.8424571E-04 N. IT: 9 N. SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 4.5764901E-04 N. IT: 12 N. SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -5.8561344E-04 N. IT: 14 N. SH: 1

1 1.6270884E-04 1 -1.3808703E-03

2 7.0469206E-04 2 -7.6963651E-04

3 2.3261886E-04 3 -1.2159726E-03

4 -1.1494836E-03

5 -1.2007923E-03

6 -5.8561344E-04

7 -8.2429874E-04

8 -6.1663631E-04

9 -5.8561344E-04

13 -5.8561344E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 3.1037531E-05 N. IT: 14 N. SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) -3.2635347E-04 N. IT: 13 N. SH: 1

1 -3.6184436E-04 1 -1.1433075E-03

2 2.0063244E-05 2 -3.3146049E-04

3 2.6259593E-06 3 -3.2635347E-04

4 -8.3455676E-04

5 -4.0804378E-04

6 -1.4775697E-03

7 -1.8217932E-03

8 -3.2635347E-04  
9 -1.1433344E-03  
10 -3.2635347E-04

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 6.0122374E-04 N.IT: 11 N.SH: 1

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.1647164E-03 N.IT: 13 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 1) 1.1308936E-03 N.IT: 15 N.SH: 2

RESULT OF MINIMAX OPTIMIZATION (RETURN: 0) 4.8844439E-04 N.IT: 7 N.SH: 1

SOLUTION (TYPE: 0)

1.6270884E-04  
7.0469206E-04  
2.3261886E-04

MAX ERROR AT THE SOLUTION: -5.8561344E-04

1 -1.3808703E-03  
2 -7.6963651E-04  
3 -1.2159726E-03  
4 -1.1494836E-03  
5 -1.2007923E-03  
6 -5.8561344E-04  
7 -8.242874E-04  
8 -6.1663631E-04  
9 -5.8561344E-04  
13 -5.8561344E-04

NUMBER OF DELETIONS: 3

11 1.0000000E+99  
10 1.0000000E+99  
12 1.0000000E+99

TOTAL EXECUTION TIME : 3.517 SECONDS

#### IV. REFERENCES

- [1] J.W. Bandler, M.A. El-Kady, W. Kellermann and W.M. Zuberek, "A minimax approach to the best mechanical alignment problem", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-82-10-R, 1982. Also to appear in ASME J. of Mechanisms, Transmissions, and Automation in Design.
- [2] J.W. Bandler, M.A. El-Kady, W. Kellermann and W.M. Zuberek, "BSTALN - a computer implementation of an algorithm for the best mechanical alignment problem", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-83-13-U/L, 1983.
- [3] Woodward Governor Company, Rockford, IL, 61101, Sample data sent to McMaster University, February 23, 1982.