

**A UNIFIED DECOMPOSITION APPROACH  
FOR FAULT LOCATION  
IN LARGE ANALOG NETWORKS**

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A UNIFIED DECOMPOSITION APPROACH FOR  
FAULT LOCATION IN LARGE ANALOG CIRCUITS

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Abstract

This paper deals with the problem of fault location in analog circuits. The circuit under test is decomposed into subnetworks using nodes at which voltages have been measured. We localize the faults to within the smallest possible subnetworks according to the final decomposition. Then, further identification of the faulty elements inside the subnetworks is carried out. The method is applicable to large networks, linear or nonlinear. It requires a limited number of measurement nodes and its on-line computation requirements are minimal. The method is based on checking the consistency of KCL in the decomposed circuit. A measure of the effect of tolerances on the elements is introduced, and a number of examples are considered to illustrate the application of the method in both the linear and the nonlinear cases.

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## I. INTRODUCTION

This paper addresses itself to the problem of fault location in analog circuits. There are two main approaches to the problem [1]: the simulation-after-test approach [2-11] and the simulation-before-test approach [12,13].

The simulation-before-test requires the simulation of different possible faults and storage of the results as a dictionary. The faulty subnetwork responses are compared with the dictionary entries and the closest entry to the responses by a certain measure determines the possible fault. The method is usually suitable for single catastrophic fault location. For multiple soft fault situations the size of the directory becomes very large and the method is impractical. In the simulation-after-test approach, using the faulty network responses, either all network elements are identified (parameter identification) [3], or a search for the faulty set (assumed of small cardinality) is conducted to locate the faulty elements [2,4]. In both cases there is a compromise between the computational effort and the number of accessible nodes.

Recently [2], there has been a real attempt at reducing the number of accessible nodes while keeping the computations within an acceptable limit. For the multiple fault situation, Wu et al. [2] proposed an algorithm which is based on a heuristic that the effect of two analog faults will not cancel each other. They also assumed a maximum bound on the number of possible faults in the circuit.

Here, we present a new simulation-after-test method for fault location with the aim of keeping both the computations and measurements to acceptable bounds. A nodal decomposition [14] of the network into

smaller uncoupled subnetworks is carried out. The measurement nodes must include the nodes of decomposition. The voltage measurements are employed to isolate the faulty subnetworks. Utilizing the incidence relations between subnetworks and KCL we develop necessary and almost sufficient conditions for a subnetwork or a group of subnetworks to be fault free. Logical analysis of the results of these tests is carried out to identify faulty subnetworks.

In analog circuits the good network elements are usually not at their nominal values, but are randomly distributed within specified tolerance intervals. A probabilistic approach is used to check whether the testing conditions can be satisfied under these random changes.

Further analysis can be carried out to find the faulty elements or regions within each faulty subnetwork. This is the second part of our method. Depending on the size of the subnetwork either fault verification is carried out immediately or the testing conditions are applied to find smaller faulty regions inside the faulty subnetwork, then fault verification [4] is carried out in this region if possible.

## II. NETWORK DECOMPOSITION AND LOGICAL ANALYSIS

### Network Decomposition

The topology of the network under test is known. In the pre-test stage we perform a nodal decomposition of the network. This results in subnetworks connected by the nodes of decomposition. There should be no mutual coupling between any two subnetworks and the nodes of decomposition should be chosen from the set where voltage measurements can be performed. The decomposition is either performed by inspection (for networks of relatively small size) or a special algorithm is used

for that purpose, e.g., the heuristic algorithm proposed by Sangiovanni-Vincentelli et al. [15].

#### Example 1: illustration of network decomposition

As an example we consider the video amplifier circuit [16] of Fig. 1. Nodes of decomposition are assumed to be nodes 1, 2, 5, 7 and 10. The circuit is decomposed into eight subnetworks as shown in Fig. 2. In actual testing we perform voltage measurements at the nodes of decomposition indicated. The network is not torn physically.

#### Logical Analysis

Testing conditions are applied to identify the nonfaulty subnetworks. The application of a testing condition is referred to as a test. The outcome of a test is classified simply as pass or fail. The test passes if and only if all subnetworks involved in the test are fault-free. The test fails if and only if at least one of those subnetworks is faulty. A subnetwork is faulty if it contains one or more faulty elements. A test that is applied to check whether subnetwork  $S_i$  is fault-free or not is described as a self-testing condition (STC). A test that is applied to check whether a group of  $k$  subnetworks  $S_{j_1}, S_{j_2}, \dots, S_{j_k}$  are fault-free or not is called a mutual-testing condition (MTC). In practice, we utilize the measurements together with the incidence relation between subnetworks to expedite these tests.

The results of different tests are analyzed to identify the faulty and nonfaulty subnetworks. Logical functions are utilized for this purpose. Every subnetwork has associated with it a logical variable  $\sigma$ , which takes the value 1 if the subnetwork is good and 0 if it is faulty. Every test is associated with a logical test function (LTF), which is equal to the complete product of variables  $\sigma_{j_i}$  if the test is a pass

$$T_{J_t} \triangleq \sigma_{j_1} \wedge \sigma_{j_2} \wedge \dots \wedge \sigma_{j_k}, \quad (1)$$

where

$$J_t \triangleq \{j_1, j_2, \dots, j_k\}, \quad (2)$$

$j_i$  refers to subnetwork  $S_{j_i}$ ,  $k$  is the number of subnetworks involved in the test, or the complete union of complemented variables  $\bar{\sigma}_{j_i}$

$$T_{J_t} \triangleq \bar{\sigma}_{j_1} \vee \bar{\sigma}_{j_2} \vee \dots \vee \bar{\sigma}_{j_k} \quad (3)$$

if the test is a fail.

A logical diagnostic function (LDF) is given by

$$D_\ell \triangleq \left( \bigwedge_{t=1}^g T_{J_t} \right) \wedge \left( \bigwedge_{t=g+1}^{\ell} \bar{T}_{J_t} \right), \quad (4)$$

where the first  $g$  LTFs correspond to successful tests and  $\ell$  is the total number of tests. In the LDF, the subnetworks which are represented by  $\bar{\sigma}_i$  are faulty and those which are represented by  $\sigma_i$  are nonfaulty. If a subnetwork is not represented in the LDF we assume nothing about its status: more tests are necessary. We usually construct the LDF in a sequential manner by combining the results of the current test with previous tests. This usually reduces the number of tests needed since some of the tests could be redundant.

### Example 2: illustration of logical analysis

In the decomposed network of Fig. 2 let  $S_2$  be the faulty subnetwork in an otherwise fault-free network. We will apply MTCs to evaluate  $T_{23}$ ,  $T_{3567}$ ,  $T_{3456}$ ,  $T_{58}$  and  $T_{69}$ . Only the test for  $T_{23}$  is a fail so we have

$$\begin{aligned} D_5 &= (\bar{\sigma}_2 \vee \bar{\sigma}_3) \wedge (\sigma_3 \wedge \sigma_5 \wedge \sigma_6 \wedge \sigma_7) \wedge (\sigma_3 \wedge \sigma_4 \wedge \sigma_5 \wedge \sigma_6) \wedge \\ &\quad (\sigma_5 \wedge \sigma_8) \wedge (\sigma_6 \wedge \sigma_9) \\ &= \bar{\sigma}_2 \wedge \sigma_3 \wedge \sigma_4 \wedge \sigma_5 \wedge \sigma_6 \wedge \sigma_7 \wedge \sigma_8 \wedge \sigma_9, \end{aligned}$$

from which it is evident that  $S_2$  is the only faulty subnetwork.

### III. APPLICATION OF TESTING CONDITIONS TO SUBNETWORKS

In this section we give necessary and almost sufficient conditions for a subnetwork or group of subnetworks to be fault-free. The conditions are based on invoking KCL and topological relations.

For analog circuits the effect of two independent faults is highly unlikely to cancel at the measurement nodes. We adopt this reasonable heuristic [2].

The input-output relation for a subnetwork  $S_i$ , that is connected to the rest of the network by  $m_i+1$  external nodes, as shown in Fig. 3, with one of the nodes taken as the reference, is given by

$$\tilde{i}_i^{M_i}(t) = \tilde{h}_i^{M_i}(\tilde{v}_i^{M_i}(t), \phi_i), \quad (5)$$

where  $\phi_i$  is the vector of the subnetwork parameters and the cardinality of  $\tilde{i}_i^{M_i}(t)$ ,  $\tilde{h}_i^{M_i}$  and  $\tilde{v}_i^{M_i}(t)$  is  $m_i$ . We assume that the subnetwork  $S_i$  is connected, i.e., there exists a path between any two nodes of subnetwork  $S_i$  and the  $m_i+1$  external nodes do not decompose the subnetwork further, i.e., we cannot partition  $S_i$  into smaller uncoupled subnetworks using only the set of  $m_i+1$  external nodes. Let

$$M_i = M_{i\alpha} \cup M_{i\beta} \cup M_{i\gamma} \cup M_{i\delta}, \quad (6)$$

where  $M_{i\alpha}$  is the set of nodes where both voltages and currents are known,  $M_{i\beta}$  is the set of nodes where only voltages are known,  $M_{i\gamma}$  is the set of nodes where only currents are known,  $M_{i\delta}$  is the set of nodes where neither currents nor voltages are known and  $M_i$  is the set of the  $m_i$  nodes. Accordingly, we can rewrite (5) as

$$\tilde{i}_i^{M_i\alpha}(t) = \tilde{h}_i^{M_i\alpha}(\tilde{v}_i^{M_i\alpha}(t), \tilde{v}_i^{M_i\beta}(t), \tilde{v}_i^{M_i\gamma}(t), \tilde{v}_i^{M_i\delta}(t), \phi_i), \quad (7a)$$

$$\tilde{v}_i^{M_{i\beta}}(t) = \tilde{h}_i^{M_{i\beta}}(\tilde{v}_i^{M_{i\alpha}}(t), \tilde{v}_i^{M_{i\beta}}(t), \tilde{v}_i^{M_{i\gamma}}(t), \tilde{v}_i^{M_{i\delta}}(t), \phi_i^0), \quad (7b)$$

$$\tilde{v}_i^{M_{i\gamma}}(t) = \tilde{h}_i^{M_{i\gamma}}(\tilde{v}_i^{M_{i\alpha}}(t), \tilde{v}_i^{M_{i\beta}}(t), \tilde{v}_i^{M_{i\gamma}}(t), \tilde{v}_i^{M_{i\delta}}(t), \phi_i^0), \quad (7c)$$

$$\tilde{v}_i^{M_{i\delta}}(t) = \tilde{h}_i^{M_{i\delta}}(\tilde{v}_i^{M_{i\alpha}}(t), \tilde{v}_i^{M_{i\beta}}(t), \tilde{v}_i^{M_{i\gamma}}(t), \tilde{v}_i^{M_{i\delta}}(t), \phi_i^0). \quad (7d)$$

If the cardinality of the set  $M_{i\alpha}$  is greater than the cardinality of the set  $M_{i\delta}$ , i.e.,  $m_{i\alpha} > m_{i\delta}$ , a necessary condition for the subnetwork  $S_i$  to be fault-free is that

$$\tilde{v}_i^{M_{i\alpha}}(t) = \tilde{h}_i^{M_{i\alpha}}(\tilde{v}_i^{M_{i\alpha}}(t), \tilde{v}_i^{M_{i\beta}}(t), \tilde{v}_i^{M_{i\gamma}}(t), \tilde{v}_i^{M_{i\delta}}(t), \phi_i^0), \quad (8a)$$

$$\tilde{v}_i^{M_{i\gamma}}(t) = \tilde{h}_i^{M_{i\gamma}}(\tilde{v}_i^{M_{i\alpha}}(t), \tilde{v}_i^{M_{i\beta}}(t), \tilde{v}_i^{M_{i\gamma}}(t), \tilde{v}_i^{M_{i\delta}}(t), \phi_i^0) \quad (8b)$$

is a consistent system of overdetermined equations at any instant of time, where  $\phi_i^0$  is the vector of nominal parameter values of the subnetwork. We refer to this condition as the internal-self-testing condition (ISTC). We utilize this condition in locating faulty regions inside faulty subnetworks.

When all the voltages of  $M_i$  are known and  $m_{i\alpha}$  is greater than or equal to one, we can state the following stronger result.

Lemma 1: self-testing condition (STC)

A necessary and almost sufficient condition for a connected subnetwork  $S_i$  with  $m_i+1$  external nodes that do not decompose it further,  $m_{i\alpha} \geq 1$  and  $m_{i\gamma} = m_{i\delta} = 0$  to be fault-free is that

$$\tilde{v}_i^{M_{i\alpha}}(t) - \tilde{h}_i^{M_{i\alpha}}(\tilde{v}_i^{M_i}(t), \phi_i^0) = 0 \quad \forall t. \quad (9)$$

The necessity of (9) is obvious. For the sufficiency part of Lemma 1 the adjoint network concept [17] can be utilized to prove that any



change in the subnetwork should be observable at the  $M_i$  nodes, thus changing  $\tilde{i}_i^{M_i}$ . If no change has occurred in  $\tilde{i}_i^{M_i}$  from that computed using the given  $\tilde{v}_i^{M_i}$  and the nominal parameters of the subnetwork, this implies that the subnetwork is fault-free. It is sufficient to check Lemma 1 using only one external current to the subnetwork.

Normally, the voltages of the  $m_i$  nodes are directly measured. The currents  $\tilde{i}_i^{M_i}$  are not directly measured since it is difficult to do so practically except when they represent the input excitation to the whole network. The application of KCL and topological relations overcomes this difficulty. The currents are not measured: they are computed using the nominal parameter values together with the measured voltages, then KCL is invoked.

Let us assume we have a set of  $k$  subnetworks  $S_i$ ,  $i \in J_t$ , which are incident on common node  $c$  as shown in Fig. 4. Each subnetwork is assumed to be connected and has  $m_i+1$  external nodes that do not decompose the subnetwork further. The input-output relation for every subnetwork is similar to that given in (5). The voltages of the  $m_i$  external nodes are assumed to be measured. The current incident to the common node  $c$  from subnetwork  $S_i$  is given by

$$\tilde{i}_c^{M_i}(t) = h_c^{M_i}(\tilde{v}_i^{M_i}(t), \phi_i) . \quad (10)$$

Lemma 2: mutual-testing condition (MTC)

A necessary and almost sufficient condition for  $S_i$ ,  $i \in J_t$ , to be fault-free is that

$$\sum_{i \in J_t} h_c^{M_i}(\tilde{v}_i^{M_i}(t), \phi_i^0) = 0 \quad \forall t , \quad (11)$$

i.e., the currents incident to the common node  $c$  computed using the

measured voltages and nominal parameter values should satisfy KCL.

If (11) is satisfied, then the current incident with the measurement node  $c$  from subnetwork  $S_i$  is actually given by

$$h_c^{M_i}(\tilde{v}^i(t), \phi_i^0). \quad (12)$$

Invoking Lemma 1, this implies that the subnetwork  $S_i$  is fault-free: thus all subnetworks  $S_i$ ,  $i \in J_t$ , are nonfaulty.

When the number of subnetworks which are incident to a common node  $c$  is two we refer to the test as the bi-testing condition (BTC).

Example 3: illustration of Lemma 2

Consider the decomposed network of Fig. 2. Subnetworks  $S_3$ ,  $S_5$ ,  $S_6$  and  $S_7$  are incident to node 1. So, according to the MTC of (11) they are fault-free if and only if

$$I_1^3 + I_1^5 + I_1^6 + I_1^7 = 0,$$

where all currents are computed using nominal parameter values and the measured voltages of the nodes of decomposition.

Lemma 3: generalized-mutual-testing condition (GMTC)

Let  $E_i$ ,  $i \in J_t$ , denote some external nodes of the subnetwork  $S_i$ . Each subnetwork  $S_i$  is connected and has  $m_i+1$  nodes that do not decompose it further,  $E_i \subseteq M_i$ . If the currents incident to  $E_i$ ,  $i \in J_t$ , form a cut set, then a necessary and almost sufficient condition for these subnetworks to be fault-free is that

$$\sum_{i \in J_t} \sum_{j \in E_i} h_j^{M_i}(\tilde{v}^i(t), \phi_i^0) = 0 \quad \forall t. \quad (13)$$

Example 4: illustration of Lemma 3

The branches that connect  $S_2$ ,  $S_4$ ,  $S_5$ ,  $S_6$  and  $S_7$  with  $S_3$  form a cut set, as shown in Fig. 1. According to the GMTC  $S_2$ ,  $S_4$ ,  $S_5$ ,  $S_6$  and  $S_7$

are fault free if and only if

$$I_2^2 + I_5^4 + I_1^5 + I_5^5 + I_1^6 + I_5^6 + I_1^7 = 0,$$

where the currents are computed using the measured voltages of the nodes of decomposition and nominal design parameters of the subnetworks involved in the test.

#### IV. TOLERANCE CONSIDERATIONS

The actual values of nonfaulty elements can deviate from their nominal values within prescribed tolerance bounds. Thus, in practice, we face the situation that Lemmas 1-3 are not satisfied to the required degree of accuracy. Taking the tolerance changes in the subnetwork elements into consideration we may write condition (9) as

$$\tilde{i}_i^{M_i \alpha}(t) - \tilde{h}_i^{M_i \alpha}(\tilde{v}_i^{M_i}(t), \phi_i^0 + \Delta\phi_i) = 0, \quad (14)$$

where  $\Delta\phi_i \triangleq [\Delta\phi_{i1} \ \Delta\phi_{i2} \ \dots \ \Delta\phi_{ip}]^T$  defines the tolerance changes in the  $p$  elements of the subnetwork under consideration. For small tolerances the first-order approximation can be utilized to describe the changes in the network response. Accordingly, we may write (14) as

$$\tilde{i}_i^{M_i \alpha}(t) - \tilde{h}_i^{M_i \alpha}(\tilde{v}_i^{M_i}(t), \phi_i^0) = \sum_{j=1}^p \frac{\partial \tilde{h}_i^{M_i \alpha}}{\partial \phi_{ij}} \Delta\phi_{ij}, \quad (15)$$

where the partial derivatives are calculated at  $\phi_i^0$ . Let

$$\Delta\tilde{i}_i^{M_i \alpha}(t) \triangleq \tilde{i}_i^{M_i \alpha}(t) - \tilde{h}_i^{M_i \alpha}(\tilde{v}_i^{M_i}(t), \phi_i^0) = \tilde{B}_i \Delta\phi_i, \quad (16)$$

where

$$\tilde{B}_i \triangleq \left[ \frac{\partial \tilde{h}_i^{M_i \alpha}}{\partial \phi_{i1}} \quad \frac{\partial \tilde{h}_i^{M_i \alpha}}{\partial \phi_{i2}} \quad \dots \quad \frac{\partial \tilde{h}_i^{M_i \alpha}}{\partial \phi_{ip}} \right]. \quad (17)$$

At a certain instant  $t_0$  of time equation (16) is an underdetermined system of linear equations in the variable  $\Delta\phi_i$ . The weighted least-squares solution of (16) is given by [9, 10]

$$\Delta\phi_i = B_i^+ \Delta i_{\sim i}^{M_{i\alpha}}(t_0), \quad (18)$$

where

$$B_i^+ \Delta = C_i B_i^T [B_i C_i B_i^T]^{-1} \quad (19)$$

and  $C_i$  is a weighting matrix. For  $\Delta\phi_i$  normally distributed with mean 0 and covariance matrix  $C_i^{-1}$ , the solution given in (18) is the conditional expected value of the parameters  $\Delta\phi_i$  [9], i.e.,

$$\Delta\phi_i = E [\Delta\phi_i | \Delta i_{\sim i}^{M_{i\alpha}}(t_0)], \quad (20)$$

where E denotes the expectation. Moreover, the solution is a minimum in the weighted least-squares sense. So  $\Delta\phi_i$  is the solution of

$$\text{minimize } \Delta\phi_i^T C_i^{-1} \Delta\phi_i \quad (21)$$

subject to

$$B_i \Delta\phi_i = \Delta i_{\sim i}^{M_{i\alpha}}(t_0). \quad (22)$$

Using the probabilistic interpretation of the result, namely (20), we can have a measure of how far (9) is satisfied under the variations caused by the tolerances. If any component of the computed vector  $\Delta\phi_i$  from (18) significantly exceeds its tolerance value we consider that the test is unsuccessful. The consideration of the matrix  $C_i$  in (19) provides the possibility of considering the known correlation between the elements of the subnetworks.

The effect of tolerances on conditions (11) and (13) is treated in

a similar way.

## V. FAULT LOCATION INSIDE FAULTY SUBNETWORKS

Further diagnosis is usually necessary to identify faulty element(s) or at least the faulty region inside a faulty subnetwork. Our approach to this problem depends on the structure and size of the subnetwork. For small subnetworks with few elements a search for the faulty element inside the subnetwork (fault verification) is feasible, since the number of different combinations to be considered is very few. For relatively larger subnetworks we first apply the ISTC to find a smaller region inside the subnetwork that contains the faulty elements. Then, we apply the fault verification technique to this faulty region if possible.

### Fault Verification Based on Nominal Models

For a faulty subnetwork  $S_i$  it is required to verify the existence of  $f$  faults inside the subnetwork. These  $f$  faults may have been predicted using an approximate fault location method as reported in [6,7] or we may try all possible combinations of  $f$  faulty elements [4]. The latter strategy is feasible for subnetworks with few elements. Necessary and almost sufficient conditions for fault verification in linear and nonlinear networks have been developed in [2,4,5,8,18].

For a faulty subnetwork  $S_i$  with  $m_{i\gamma} = m_{i\delta} = 0$  and with  $m_{i\alpha} > 0$ , we may write (7a) as

$$\tilde{y}_i^{M_i\alpha}(t) = \tilde{h}_i^{M_i\alpha} (\tilde{y}_i^{M_i}(t), \tilde{\phi}_i^0, \tilde{\Delta\phi}_i^F), \quad (23)$$

where  $\tilde{\Delta\phi}_i^F$  represents the unknown changes from nominal for  $f$  faulty elements of the subnetwork. For  $m_{i\alpha} \geq f+1$  a necessary condition for  $f$

elements to be the correctly chosen faulty elements is that the overdetermined system of equations (23) is consistent. It is to be noted that the condition that  $m_{i\alpha} \geq f+1$  is needed when testing linear networks using a single excitation and considering (23) at an instant of time  $t_0$ .

For frequency dependent linear networks as well as for nonlinear networks  $m_{i\alpha}$  may be less than  $f$  since, by changing the input excitation (level, frequency, position, ...) further information is revealed about the subnetwork. Following [8] and considering (23) at instant  $t_0$  with  $m_{i\alpha} \geq f+1$  the solution of (23) is locally unique in  $\Delta\phi_i^F$  if

$$\text{Rank} \left[ \begin{array}{ccc} \frac{\partial \tilde{h}^{i\alpha}}{\partial \phi_{i1}} & \frac{\partial \tilde{h}^{i\alpha}}{\partial \phi_{i2}} & \dots & \frac{\partial \tilde{h}^{i\alpha}}{\partial \phi_{if}} \end{array} \right] < \text{Rank} \left[ \begin{array}{ccc} \frac{\partial \tilde{h}^{i\alpha}}{\partial \phi_{i1}} & \frac{\partial \tilde{h}^{i\alpha}}{\partial \phi_{i2}} & \dots & \frac{\partial \tilde{h}^{i\alpha}}{\partial \phi_{if}} & \frac{\partial \tilde{h}^{i\alpha}}{\partial \phi_{ix}} \end{array} \right]$$

$$\forall x \in A-F, t = t_0, \quad (24)$$

where  $A$  is the set of all subnetwork elements  $\phi_{i1}, \phi_{i2}, \dots, \phi_{ip}$ , and  $F$  is the set of assumed faulty elements  $\phi_{i1}, \phi_{i2}, \dots, \phi_{if}$ . For linear networks this condition evolves into an almost sufficient condition with a graph theoretical interpretation [5].

#### Fault Verification Based on Fault Models

Practically, in any subnetwork there are some elements that are fault prone. Fault models of these elements are usually known and in the directory approach they are used to construct the dictionary. We exploit this to our advantage by computing using the measured voltages and the fault models

$$\tilde{v}_{iF_j}^{i\alpha}(t) = \tilde{h}^{i\alpha}(\tilde{v}^i(t), \phi_i^j), \quad j=1,2,\dots,k, \quad (25)$$

where  $\phi_{i}^{F_j}$  refers to the subnetwork parameters that model the  $j$ th faulty case and  $k$  different faulty cases are considered. Utilizing the nearest neighbour rule [13], the exact faulty case is the one that has the minimum distance  $d_j$  from the actual  $i^{M_i\alpha}(t)$ , where

$$d_j \triangleq \int_0^{t_f} \|\tilde{i}_{F_j}^{M_i\alpha}(t) - i^{M_i\alpha}(t)\| dt, \quad j=1,2,\dots,k. \quad (26)$$

Alternatively, we consider  $d_j$  at just a single instant  $t_o$  as

$$d_j = \|\tilde{i}_{F_j}^{M_i\alpha}(t_o) - i^{M_i\alpha}(t_o)\|, \quad (27)$$

e.g., in dc testing.

#### Identification of Faulty Regions

The application of the ISTC starts by partitioning the faulty subnetwork  $S_i$  into two smaller subnetworks  $S_j, S_k$  such that  $S_i = S_j \cup S_k$ , as shown in Fig. 5. See also [5]. For at least one of these subnetworks and preferably for both of them,  $m_{l\alpha} > m_{l\delta}$ , where  $l = j$  or  $k$ , as appropriate. Utilizing condition (8) we can identify whether  $S_j$  or  $S_k$  are fault-free or not. We continue the binary partitioning process in the identified faulty region until we cannot find a partition that satisfies the cardinality condition, namely  $m_{l\alpha} > m_{l\delta}$ . At this stage we apply the verification technique to identify the faulty elements inside a faulty region that in many cases is much smaller than the subnetwork  $S_i$ .

## VI. FAULT LOCATION IN LINEAR NETWORKS

For linear networks, the matrix description of the subnetworks greatly simplifies the computational effort needed for checking the

testing conditions. Without loss of generality we assume sinusoidal excitations are applied. Whence, we represent the voltages and currents by their phasor variables.

### General Description

Consider a subnetwork  $S_i$  which has  $m_i+1$  external nodes, one of which is the reference node, and  $n_i$  internal nodes. The nodal equations are given by

$$\begin{bmatrix} \tilde{Y}_{M_i M_i} & \tilde{Y}_{M_i N_i} \\ \tilde{Y}_{N_i M_i} & \tilde{Y}_{N_i N_i} \end{bmatrix} \begin{bmatrix} \tilde{V}^{M_i} \\ \tilde{V}^{N_i} \end{bmatrix} = \begin{bmatrix} \tilde{I}^{M_i} + \tilde{I}_g^{M_i} \\ \tilde{I}_g^{N_i} \end{bmatrix}, \quad (28)$$

where

$$\tilde{I}_g^i \triangleq \begin{bmatrix} M_i \\ \tilde{I}_g \\ N_i \\ \tilde{I}_g \end{bmatrix}, \quad (29)$$

defines the current sources associated with the subnetwork,  $\tilde{V}^{M_i}$  is the voltage vector of the external nodes,  $\tilde{V}^{N_i}$  is the voltage vector of the internal nodes and  $\tilde{I}^{M_i}$  is the current input vector to the subnetwork from outside through  $m_i$  external nodes. Eliminating the  $n_i$  internal nodes we get

$$\tilde{I}^{M_i} = - \left[ \tilde{I}_g^{M_i} - \tilde{Y}_{M_i N_i} \tilde{Y}_{N_i N_i}^{-1} \tilde{I}_g^{N_i} \right] + \left[ \tilde{Y}_{M_i M_i} - \tilde{Y}_{M_i N_i} \tilde{Y}_{N_i N_i}^{-1} \tilde{Y}_{N_i M_i} \right] \tilde{V}^{M_i}, \quad (30)$$

or more compactly

$$\tilde{I}^{M_i} = \tilde{H}_{M_i} \tilde{I}_g^i + \tilde{Y}_{M_i} \tilde{V}^{M_i}, \quad (31)$$

where

$$\tilde{H}_{M_i} \triangleq - \left[ 1 - \tilde{Y}_{M_i N_i} \tilde{Y}_{N_i N_i}^{-1} \right], \quad (32)$$



$$\tilde{Y}_{M_i} \triangleq \begin{bmatrix} Y_{M_i M_i} & -Y_{M_i N_i} Y_{N_i N_i}^{-1} Y_{N_i M_i} \\ \tilde{Y}_{M_i M_i} & \tilde{Y}_{M_i N_i} \tilde{Y}_{N_i N_i}^{-1} \tilde{Y}_{N_i M_i} \end{bmatrix} \quad (33)$$

and  $\tilde{1}$  is a unit matrix of order  $m_i$ .

Equation (31) describes the input-output relation of the subnetwork. This relation is the one we are interested in to verify Lemmas 1-3.

During testing we follow a hierarchical decomposition approach [19]. This is illustrated in Fig. 6 and is represented by the so-called tree of decomposition. The subnetworks at the final level of decomposition are called blocks. We begin by considering suitable STCs and MTCs for the subnetworks at the first level of decomposition. If a subnetwork is declared nonfaulty no further partitioning of it need be carried out. Faulty subnetworks and those which we are not sure about are decomposed further using measurement nodes, if possible.

Since the hierarchical decomposition could be obtained prior to actual testing, the characteristics of the resulting subnetworks, namely,  $\tilde{Y}_{M_i}$  and  $\tilde{H}_{M_i} \tilde{I}_g^i$  are known and could be computed off-line using nominal values and stored before conducting the actual testing. At the time of testing, the only on-line computation required is the matrix by vector multiplication, namely  $\tilde{Y}_{M_i} \tilde{V}_1^i$ . Let  $E$  define the set of faulty subnetworks or the ones which we are uncertain about. The procedure in carrying out the tests can be summarized as follows.

#### Procedure for Locating Faulty Subnetworks

Step 0  $j = 0$ .

$$E_j = \{S_1\}. \quad (S_1 \text{ is the network under test.})$$

Step 1 Partition, using the least number of test nodes every  $S_i \in E_j$ , if possible, into smaller uncoupled subnetworks to constitute

$E_{j+1}$ . Otherwise go to 8.

Comment Only decomposed parts of every  $S_i \in E_j$  will be contained in  $E_{j+1}$  or  $S_i$  itself if it is not decomposable.

Step 2 For every  $S_i \in E_{j+1}$ , find the sets  $M_{i\alpha}$  and  $M_{i\beta}$ .

Step 3 Check the testing conditions of Lemmas 1-3.

Step 4 Identify faulty subnetworks using a logical analysis of the tests.

Step 5 Utilize the nonfaulty subnetworks to determine the external currents of the faulty subnetworks.

Step 6 Update the set  $E_{j+1}$  by removing nonfaulty subnetworks.

Step 7  $j = j+1$ . Go to 1.

Step 8 Print out the components of the set  $E = E_j$ .

#### Computational Effort

The number of nodes where measurements are performed and the computational effort depend on the size of the blocks and the number of levels of decomposition. If we assume that we have  $L$  levels of decomposition and the resulting hierarchical decomposition is binary and symmetric, the number of subnetworks (blocks) at the final level of decomposition will be equal to  $2^L$ . If each block has  $n$  nodes and  $b$  is the number of interconnection nodes between any two blocks, then the total number of network nodes is approximately given by

$$N_T \approx 2^L (n-b) + b. \quad (34)$$

Assume that all the interconnection nodes are measurement nodes. Then their number  $N_m$  can be estimated from

$$N_m \leq 2^L b - b. \quad (35)$$

Accordingly, a measure of the needed degree of accessibility is given by the ratio

$$r = \frac{N_m}{N_T} \leq \frac{b}{n-b} . \quad (36)$$

For a smaller  $r$ ,  $n$  should be much greater than  $b$ . On the other hand, we wish to have  $n$  as small as possible to obtain better diagnosis and decreased computational effort. There is clearly a compromise between the degree of accessibility and the size of the block.

If the faulty elements are in one block, following the hierarchical decomposition strategy and assuming binary partition, we check the testing conditions for just two subnetworks at each level. The total number of subnetworks to be considered is consequently  $2L$ . In a number of steps proportional to  $\log N_T$  we isolate the faulty subnetwork. Obviously, we do not need to measure all the voltages of the test nodes. Less than  $bL$  measurements are actually required.

#### Location of Faulty Elements

Representing the change from nominal in a faulty element by a current source across that element, we may write (31) as

$$\tilde{I}^{M_i} = \tilde{H}_{M_i} \tilde{I}_g^i + \tilde{Y}_{M_i} \tilde{V}^{M_i} + \tilde{H}_{M_i F} \tilde{I}^F , \quad (37)$$

where  $\tilde{I}^F$  represents the faulty current sources and  $\tilde{H}_{M_i F}$  is computed using the nominal parameter values of the subnetwork and defines the transfer relation between  $\tilde{I}^{M_i}$  and  $\tilde{I}^F$ . It is normally computed using the adjoint network concept as in [6]. Considering (37) for the  $m_{i\alpha}$  known currents we have

$$\tilde{H}_{M_{i\alpha} F} \tilde{I}^F = \tilde{I}^{M_{i\alpha}} - \tilde{H}_{M_{i\alpha}} \tilde{I}_g^i - \tilde{Y}_{M_{i\alpha}} \tilde{V}^{M_i} , \quad (38)$$

where only rows  $M_{i\alpha}$  are considered in the matrices  $\tilde{H}_{M_{i\alpha} F}$ ,  $\tilde{H}_{M_{i\alpha}}$  and

$\tilde{Y}_{M_{i\alpha}}$ . If  $m_{i\alpha}$  is greater than or equal  $f+1$ , then the system of equations is overdetermined. A necessary condition for  $F$  to contain the faulty set is that (38) is a consistent system of equations. The set  $F$  is unique if [5]

$$\text{Rank} \begin{bmatrix} \tilde{H}_{M_{i\alpha}F} & \tilde{H}_{M_{i\alpha}x} \end{bmatrix} = f + 1, \quad \forall x \notin F, \quad (39)$$

where  $\tilde{H}_{M_{i\alpha}x}$  represents a transfer vector from a current source across an element  $x$  in the subnetwork to the measurement nodes  $M_{i\alpha}$ , and (39) is considered for all elements  $x$  in the subnetwork other than the elements in the faulty set  $F$ .

Internal-self-testing and fault verification in the faulty subnetworks can be applied as discussed in Section V using (38). All matrices used in (38) are computed using nominal element values and can be stored before performing the test. The computational effort will be only that of verifying the consistency of (38), which is usually performed using elementary operations on the matrix  $\tilde{H}_{M_{i\alpha}F}$  as in [5].

#### Example 5: linear network example

The network under test is composed of two identical low-pass filter sections in cascade. The low-pass filter section is shown in Fig. 7 and its nominal elements values are given in Table I [20]. The operational amplifier is modeled by a controlled source and output resistance, as shown in Fig. 8. This, in general, facilitates the use of nodal analysis programs and guarantees the existence of  $\tilde{Y}_{N_i N_i}^{-1}$  in (30). The network has 52 resistors and capacitors and 16 operational amplifiers. In the first section, nodes 1, 3, 5, 6, 8, 10, 12, 14, 15, 17 and 19 are taken to be the measurement nodes. The corresponding nodes in the second section are chosen as measurement nodes. We simulated the network with a

sinusoidal current source  $i_g(t) = 0.01 \cos 2000t$  A. A number of faulty elements were randomly chosen in the first section and they are identified by an asterisk in Table I. The procedure follows.

Stage 0  $E_0 = \{S_1\}$ . ( $S_1$  is the network under test).

Stage 1  $S_1$  is decomposed as shown in Fig. 9 into  $S_2$  and  $S_3$ .

$$E_1 = \{S_2, S_3\}.$$

$$M_{2\alpha} = \{1\}, \quad M_{2\beta} = \{19\}$$

$$M_{3\alpha} = \{37\}, \quad M_{3\beta} = \{19\}$$

From the results of Table II,  $S_2$  is faulty and  $S_3$  is non-faulty.

$$E_1 = \{S_2\}.$$

Stage 2  $S_2$  is decomposed as shown in Fig. 9 into  $S_4$  and  $S_5$ .

$$E_2 = \{S_4, S_5\}.$$

$$M_{4\alpha} = \{1\}, \quad M_{4\beta} = \{10\}$$

$$M_{5\alpha} = \{19\}, \quad M_{5\beta} = \{10\}$$

From the results of Table III,  $S_4$  and  $S_5$  are both faulty.

$$E_2 = \{S_4, S_5\}.$$

Stage 3  $S_4$  and  $S_5$  are decomposed as shown in Fig. 9.  $S_4$  is decomposed

into  $S_6, S_7$  and  $S_8$ .  $S_5$  is decomposed into  $S_9, S_{10}$  and  $S_{11}$ .

$$E_3 = \{S_6, S_7, S_8, S_9, S_{10}, S_{11}\}.$$

$$M_{6\alpha} = \{\emptyset\}, \quad M_{6\beta} = \{1, 3, 6\}$$

$$M_{7\alpha} = \{\emptyset\}, \quad M_{7\beta} = \{3, 6\}$$

$$M_{8\alpha} = \{\emptyset\}, \quad M_{8\beta} = \{1, 3, 6, 10\}$$

$$M_{9\alpha} = \{\emptyset\}, \quad M_{9\beta} = \{10, 12, 15\}$$

$$M_{10\alpha} = \{\emptyset\}, \quad M_{10\beta} = \{12, 15\}$$

$$M_{11\alpha} = \{19\}, \quad M_{11\beta} = \{10, 12, 15\}$$

From the results of Table IV, subnetwork  $S_{11}$  is faulty and

further tests are needed for all other subnetworks.

$$E_3 = \{S_6, S_7, S_8, S_9, S_{10}, S_{11}\}.$$

Stage 4 Subnetworks  $S_7, S_8, S_{10}$  and  $S_{11}$  are decomposed into  $S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}, S_{18}, S_{19}, S_{20}$  and  $S_{21}$  as shown in Fig. 9. No further decomposition of  $S_6$  and  $S_9$  is possible using only the measurement nodes.

$$E_4 = \{S_6, S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_9, S_{17}, S_{18}, S_{19}, S_{20}, S_{21}\}$$

$$M_{6\alpha} = \{\emptyset\}, \quad M_{6\beta} = \{1, 3, 6\}$$

$$M_{12\alpha} = \{\emptyset\}, \quad M_{12\beta} = \{3, 5\}$$

$$M_{13\alpha} = \{\emptyset\}, \quad M_{13\beta} = \{5, 6\}$$

$$M_{14\alpha} = \{\emptyset\}, \quad M_{14\beta} = \{3, 8\}$$

$$M_{15\alpha} = \{\emptyset\}, \quad M_{15\beta} = \{1, 8\}$$

$$M_{16\alpha} = \{\emptyset\}, \quad M_{16\beta} = \{6, 8, 10\}$$

$$M_{9\alpha} = \{\emptyset\}, \quad M_{9\beta} = \{10, 12, 15\}$$

$$M_{17\alpha} = \{\emptyset\}, \quad M_{17\beta} = \{12, 14\}$$

$$M_{18\alpha} = \{\emptyset\}, \quad M_{18\beta} = \{14, 15\}$$

$$M_{19\alpha} = \{\emptyset\}, \quad M_{19\beta} = \{10, 17\}$$

$$M_{20\alpha} = \{\emptyset\}, \quad M_{20\beta} = \{12, 17\}$$

$$M_{21\alpha} = \{19\}, \quad M_{21\beta} = \{15, 17\}$$

From the results of Table V subnetworks  $S_6, S_{17}$  and  $S_{20}$  are faulty and all other subnetworks are nonfaulty.

$$E_4 = \{S_6, S_{17}, S_{20}\}.$$

No further decomposition is possible for subnetworks  $S_6, S_{17}$  and  $S_{20}$  using the measurement nodes. So we have  $E = E_4 = \{S_6, S_{17}, S_{20}\}$ .

For subnetwork  $S_6$ ,  $m_{6\alpha} = 3$  and we are able to verify the existence of at most double faults. For subnetwork  $S_{17}$ ,  $m_{17\alpha} = 2$  and we are able to verify the existence of a single fault, and for subnetwork  $S_{20}$ ,  $m_{20\alpha}$

= 1 and, since it contains single element, we can immediately find its value. The fault verification procedure identified  $R_1$ ,  $C_2$  in subnetwork  $S_6$  as faulty elements and  $R_{23}$  in  $S_{17}$  as faulty. Using the computational procedure outlined in [6] we computed the changes in these parameters and they are given by  $|\Delta R_1| = 0.082$ ,  $|\Delta C_2| = 0.01$ ,  $|\Delta R_{23}| = 4.0$  and  $|\Delta R_{25}| = 400.0$ , which are the exact changes.

It is to be noted that in four steps (levels of decomposition) we were able to identify the faults to within very small subnetworks. Also since  $S_3$  is fault-free after Stage 1, no further decomposition is carried out and, accordingly, we do not need to measure the accessible nodes inside  $S_3$ .

#### VII. TESTING OF NONLINEAR NETWORKS

In typical nonlinear networks, the network is dominantly linear with a few nonlinear elements. The nodes of decomposition are chosen such that the part of the network that contains the nonlinear elements is decomposed into subnetworks, each of them having very few nonlinear elements or being completely linear. The part of the network that contains only linear elements is treated exactly as in the linear case. We decompose the network into blocks that contain the nonlinear elements and a number of subnetworks that contain only linear elements. The latter could be decomposed further. In applying Lemmas 1-3 for nonlinear networks we need a nonlinear network solver. Intuitively, by having very few nonlinear elements in each subnetwork the nonlinear network solver converges rapidly in just one or two iterations starting from the nonfaulty state. Also, analyzing several subnetworks simultaneously is possible utilizing the parallel processing

capabilities of modern ATE. Any general circuit analysis program, such as SPICE, can perform the subnetwork analysis.

For locating faulty elements within faulty blocks that contain nonlinear elements we adopt the fault model approach of fault verification. We analyze the faulty block using assumed fault models of its elements, then we compare the different cases using the nearest neighbour rule, as outlined in Section V.

Example 6: nonlinear network example

We considered the video amplifier circuit shown in Fig. 1. Every subnetwork contains at most one nonlinear element (transistor), which agrees with our requirements on the decomposition.

We considered dc testing of the circuit. All capacitors are, therefore, open circuits. To investigate faulty capacitors ac testing is needed. The nominal values of circuit elements are given in Table VI. We have considered the well known Ebers-Moll model of the transistor. The nominal operating conditions for the circuit are given in Table VII. All transistors are operating in their active regions.

Different faulty situations have been simulated. The results for four different cases with the nonfaulty parameters assumed at nominal values are summarized in Tables VIII-XI. In Case 1 (Table VIII), we considered  $Q_1$  faulty, namely its collector-base junction is shorted. A shorted junction is simulated by connecting a very small resistance across the junction. The logical diagnosis function of Case 1 is that which we considered earlier in Example 2. In Case 2 (Table IX), the base-emitter junction of  $Q_3$  is shorted, and in Case 3 (Table X), the resistor  $R_{10}$  is increased to 7.8 k $\Omega$ . In Case 4 (Table XI), transistor  $Q_2$  has a shorted base-emitter junction. We considered also Case 2 when



all resistors are allowed to change within  $\pm 10\%$  of their nominal values and the transistor gain,  $\beta = (\alpha_N / 1 - \alpha_N)$ , is allowed to change  $\pm 10\%$  of its nominal values or equivalently  $\alpha_N$  to change within  $\pm 0.1\%$  of its nominal value. The predicted changes in the subnetworks using equation (18) for the different tests is summarized in Table XII (Case 5). It is clear that the diagnosis of the different tests will be exactly as in the non-tolerance case (Case 2). The matrix  $\tilde{C}_i$  in (18) has been taken to be

$$\tilde{C}_i = \text{diag} \{w_1 \phi_{i1}^2, w_2 \phi_{i2}^2, \dots, w_p \phi_{ip}^2\},$$

where  $p$  is the number of elements in the subnetworks considered in the test that are subjected to tolerance changes and  $w_i$  is an appropriate weighting function.

In all the cases considered we were quite successful in identifying the faulty subnetworks. In Case 3 further diagnosis may be needed after repairing the faulty element  $R_{10}$  since, due to abnormal operating conditions subnetworks  $S_4$ ,  $S_8$  and  $S_9$  are short circuited, and any fault in them will not show up until  $R_{10}$  is repaired. Also, knowing that  $S_7$  is faulty we compute

$$I_1^7 = -I_1^3 - I_1^5 - I_1^6 = 2.8379 \text{ mA}$$

and

$$R_{10} = \frac{V_1 - V_{cc}}{I_1^7} = 7.8 \text{ k}\Omega,$$

which is the exact fault value.

## VIII. CONCLUSIONS

We have described a novel and unified method for fault location in analog circuits. The method has the following characteristics:

(1) Due to the decomposition of the whole network into smaller uncoupled subnetworks, the method is directly applicable to large networks.

(2) The testing conditions are a result of network topology and KCL: they do not depend on network type, so the method is applicable to both linear and nonlinear networks. Also, depending on the type of circuits the network could be tested using different types of excitations.

(3) The measurement nodes are chosen as the nodes of decomposition. Their number can consequently be limited for practical implementation.

(4) For linear networks the on-line computational requirements are minimal (matrix by vector multiplications) and the off-line computation involves the analysis of the nominal network only. For nonlinear networks the on-line computation is reduced by performing the computation in a parallel processing mode.

(5) The decomposition of the network into subnetworks allowed us to deal with the tolerance problem at the subnetwork level, thus we have localized its effects.

(6) The method is initially modular, where nominal circuit models are used for the subnetworks. Subsequently, it is element oriented at which time faulty elements are located inside subnetworks. Typical faulty models may be utilized at this stage.

(7) General circuit analysis programs, e.g. SPICE, ECAP, etc, can be effectively utilized to locate and identify faulty elements.

A computer program realizing this method has been written and other

practical examples [12] were tested yielding positive results.

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TABLE I

NOMINAL ELEMENT VALUES FOR THE LINEAR NETWORK EXAMPLE

Resistors ( $k\Omega$ )		Capacitors ( $\mu F$ )	
R <sub>1</sub> *	0.182/0.1	C <sub>2</sub> *	0.01/0.02
R <sub>3</sub>	1.57	C <sub>12</sub>	0.01
R <sub>5</sub>	2.64	C <sub>18</sub>	0.01
R <sub>6</sub>	10.00	C <sub>29</sub>	0.01
R <sub>7</sub>	10.00		
R <sub>9</sub>	100.0		
R <sub>10</sub>	11.1		
R <sub>11</sub>	2.64		
R <sub>14</sub>	5.41		
R <sub>15</sub>	1.0		
R <sub>17</sub>	1.0		
R <sub>19</sub>	4.84		
R <sub>21</sub>	2.32		
R <sub>22</sub>	10.0		
R <sub>23</sub> *	10.0/6.0		
R <sub>25</sub> *	500.0/100.0		
R <sub>26</sub>	111.1		
R <sub>27</sub>	1.14		
R <sub>28</sub>	2.32		
R <sub>31</sub>	72.4		
R <sub>32</sub>	10.0		
R <sub>34</sub>	10.0		

\* refers to a faulty element: its faulty value follows the slash.

TABLE II  
DIAGNOSIS FOR THE FIRST LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents	Diagnosis	Test
<u>Measured voltages</u>	$I_1^2 = 5.53 - j0.0039 \text{ mA}$	$I_1 - I_1^2 \neq 0$	$T_2$
$V_1 = 0.956 + j0.0044 \text{ V}$	$I_{19}^2 = 7.05 - j1.52 \text{ A}$	$I_{19}^2 + I_{19}^3 \neq 0$	$T_{23}$
$V_{19} = 1.67 - j0.265 \text{ V}$	$I_{19}^3 = 9.66 - j1.57 \text{ mA}$		
$V_{37} = 1.53 - j0.421 \text{ V}$	$I_{37}^3 = 0$	$I_{37} - I_{37}^3 = 0$	$T_3$
<u>Designed Currents</u>			
$I_1 = 10.0 \text{ mA}$			
$I_{37} = 0$			
Logical diagnostic function: $D_3 = \bar{\sigma}_2 n(\bar{\sigma}_2 \cup \bar{\sigma}_3) n \sigma_3 = \bar{\sigma}_2 n \sigma_3$ .			
Result: $S_3$ is declared nonfaulty and $I_{19}^3$ is known in Table III.			

TABLE III  
DIAGNOSIS FOR THE SECOND LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents	Diagnosis	Test
<u>Measured Voltages</u>	$I_1^4 = 5.69 \times 10^{-3} - j1.05 \times 10^{-5} \text{ A}$	$I_1 - I_1^4 \neq 0$	$T_4$
$V_1$ known	$I_{10}^4 = -17.72 + j1.61 \text{ A}$	$I_{10}^4 + I_{10}^5 \neq 0$	$T_{45}$
$V_{10} = -4.39 + j0.386 \text{ V}$	$I_{10}^5 = -3.91 \times 10^{-3} + 3.44 \times 10^{-4} \text{ A}$		
$V_{19}$ known	$I_{19}^5 = 0.67 - 0.81 \text{ A}$	$I_{19}^3 + I_{19}^5 \neq 0$	$T_5$
<u>Designed Currents</u>			
$I_1 = 10.0 \text{ mA}$			
$I_{19} = -I_{19}^3$			
Logical diagnostic function $D_3 = \bar{\sigma}_4 \wedge (\bar{\sigma}_4 \vee \bar{\sigma}_5) \wedge \bar{\sigma}_5 = \bar{\sigma}_4 \wedge \bar{\sigma}_5$ .			
Result: no new currents are designable.			

TABLE IV  
DIAGNOSIS FOR THE THIRD LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents	Diagnosis	Test
<u>Measured Voltages</u>	$I_1^6 = 8.911+j0.0143$ mA	$I_1^6 + I_1^8 \neq I_1$	$T_{68}$
$V_1$ known	$I_3^6 = -6657.25+j4.126$ mA	$I_3^6 + I_3^7 + I_3^8 \neq 0$	$T_{678}$
$V_3 = -0.142-j1.33$ V	$I_6^6 = -9.271+j0.823$ mA		
$V_6 = -25.1+j2.17$ V	$I_3^7 = -0.0142-j0.1328$ mA	$I_6^6 + I_6^7 + I_6^8 \neq 0$	$T_{678}$
$V_{10}$ known	$I_6^7 = 13.446-j1.1625$ mA	$I_{10}^8 + I_{10}^9 + I_{10}^{11} \sim 0$	$T_{8,9,11}$
$V_{12} = 0.103+j0.69$ V	$I_1^8 = 0.439-j0.0301$ mA		
$V_{15} = 8.93-j1.12$ V	$I_3^8 = 0.0378-j0.0167$ mA	$I_{12}^9 + I_{12}^{10} + I_{12}^{11} \neq 0$	$T_{9,10,11}$
$V_{19}$ known	$I_6^8 = -3.9224+j0.3392$ mA	$I_{15}^9 + I_{15}^{10} + I_{15}^{11} \neq 0$	$T_{9,10,11}$
<u>Designed Currents</u>	$I_{10}^8 = 3.9039-j0.3436$ mA		
$I_1 = 10.0$ mA	$I_{10}^9 = -3.8546+j0.3389$ mA	$I_{19}^{11} \neq -I_{19}^3$	$T_{11}$
$I_{19} = -I_{19}^3$	$I_{12}^9 = -0.0005-j0.0772$ mA		
	$I_{15}^9 = 3.8473-j0.4837$ mA		
	$I_{12}^{10} = 0.0103+j0.0689$ mA		
	$I_{15}^{10} = -540.09-j2728.18$ mA		
	$I_{10}^{11} = -0.0499+j0.0052$ mA		
	$I_{12}^{11} = -0.0021+j0.0018$ mA		
	$I_{15}^{11} = 0.1083-j0.0136$ mA		
	$I_{19}^{11} = 695.12-j593.77$ mA		

Logical diagnostic function  $D_6 = (\bar{\sigma}_6 \cup \bar{\sigma}_8) \cap \bar{\sigma}_{11}$ .

Although  $T_{8,9,11}$  is almost 0 it contradicts  $T_{11}$ , hence we do not consider it in the LDF, otherwise the LDF =  $\emptyset$ .

Result: no new currents are designable.



TABLE V  
DIAGNOSIS FOR THE FOURTH LEVEL OF DECOMPOSITION

Voltage Measurements and Designed Currents	Computed Currents*	Diagnosis	Test
<u>Measured Voltages</u>	$I_3^{12} = -0.0142 - j0.1328 \text{ mA}$	$I_1^6 + I_1^{15} \neq I_1$	$T_{6,15}$
$V_1$ known	$I_5^{12} = -0.0435 - j0.5034 \text{ mA}$	$I_3^6 + I_3^{12} + I_3^{14} \neq 0$	$T_{6,12,14}$
$V_3$ known	$I_5^{13} = 0.0435 + j0.5034 \text{ mA}$	$I_5^{12} + I_5^{13} = 0$	$T_{12,13}$
$V_5 = 0.141 + j1.33 \text{ V}$	$I_6^{13} = 13.4462 - j1.1625 \text{ mA}$	$I_6^6 + I_6^{13} + I_6^{16} \neq 0$	$T_{6,13,16}$
$V_6$ known	$I_3^{14} = -I_8^{14} = 0.0378 - j0.0167 \text{ mA}$	$I_8^{14} + I_8^{15} + I_8^{16} = 0$	$T_{14,15,16}$
$V_8 = -0.392 + j0.339 \text{ V}$	$I_1^{15} = -I_8^{15} = 0.4391 - j0.0301 \text{ mA}$	$I_{10}^9 + I_{10}^{16} + I_{10}^{19} = 0$	$T_{9,16,19}$
$V_{10}$ known	$I_6^{16} = -3.9224 + j0.3392 \text{ mA}$	$I_{12}^9 + I_{12}^{17} + I_{12}^{20} \neq 0$	$T_{9,17,20}$
$V_{12}$ known	$I_8^{16} = 0.4768 - j0.0468 \text{ mA}$	$I_{14}^{17} + I_{14}^{18} \neq 0$	$T_{17,18}$
$V_{14} = -0.0615 - j0.413 \text{ V}$	$I_{10}^{16} = 3.9039 - j0.3436 \text{ mA}$	$I_{15}^9 + I_{15}^{18} + I_{15}^{21} = 0$	$T_{9,18,21}$
$V_{15}$ known	$I_{12}^{17} = 0.0082 + j0.0552 \text{ mA}$	$I_{17}^{19} + I_{17}^{20} + I_{17}^{21} \neq 0$	$T_{19,20,21}$
$V_{17} = 1.08 - j0.136 \text{ V}$	$I_{14}^{17} = 205.185 + j1379.726 \text{ mA}$	$I_{19}^{21} + I_{19}^3 = 0$	$T_{21}$
$V_{19}$ known	$I_{14}^{18} = -0.0225 - j0.1787 \text{ mA}$	$I_{17}^{21} = -0.0590 + j0.0130 \text{ mA}$	
<u>Designed Currents</u>	$I_{15}^{18} = -3.9556 + j0.4974 \text{ mA}$	$I_{19}^{21} = -9.66 + j1.57 \text{ mA}$	
$I_1 = 10.0 \text{ mA}$	$I_{10}^{19} = -I_{17}^{19} = -0.0493 + j0.0047 \text{ mA}$		
$I_{19} = -I_{19}^3$	$I_{12}^{20} = -I_{17}^{20} = -0.0020 + j0.0017 \text{ mA}$		

Logical diagnostic function  $D_{11} = \bar{\sigma}_6 \wedge \sigma_9 \wedge \sigma_{12} \wedge \sigma_{13} \wedge \sigma_{14} \wedge \sigma_{15} \wedge \sigma_{16} \wedge \bar{\sigma}_{17} \wedge \sigma_{18} \wedge \sigma_{19} \wedge \bar{\sigma}_{20} \wedge \sigma_{21}$ .

Result:  $S_6$ ,  $S_{17}$  and  $S_{20}$  are faulty.

\* for computed currents of  $S_6$  and  $S_9$  see Table IV.

TABLE VI  
NOMINAL VALUES OF NETWORK ELEMENTS

Element	Value
$R_1$	1.2 k $\Omega$
$R_2$	3.0 k $\Omega$
$R_3$	5.672 k $\Omega$
$R_4$	1.2 k $\Omega$
$R_5$	0.33 k $\Omega$
$R_6$	0.33 k $\Omega$
$R_7$	1.0 k $\Omega$
$R_8$	1.7 k $\Omega$
$R_9$	3.3 k $\Omega$
$R_{10}$	0.078 k $\Omega$
$R_{11}$	0.5 k $\Omega$
$R_{12}$	1.0 k $\Omega$
$R_{13}$	1.0 k $\Omega$
$C_1$	1.0 $\mu$ F
$C_2$	3.3 $\mu$ F
$C_3$	1.0 $\mu$ F
$C_4$	1.0 $\mu$ F
$V_{CC}$	28.0 V
$V_{EE}$	28.0 V

TABLE VII  
 NOMINAL OPERATING POINT AND PARAMETERS OF TRANSISTORS

	$Q_1$	$Q_2$	$Q_3$	$Q_4$
$I_C$	4.744 mA	9.091 mA	5.891 mA	3.048 mA
$I_E$	-4.791 mA	-9.183 mA	-5.951 mA	-3.079 mA
$V_{BE}$	0.764 V	0.798 V	0.776 V	0.741 V
$V_{BC}$	-11.767 V	-14.506 V	-10.208 V	-15.315 V
$\alpha_N$	0.99	0.99	0.99	0.99
$\alpha_I$	0.5	0.5	0.5	0.5
$I_{EO}$	$10^{-6}$ mA	$10^{-6}$ mA	$10^{-6}$ mA	$10^{-6}$ mA

TABLE VIII  
CASE 1 - Q<sub>1</sub> FAULTY

Measurements	Computed Currents	Diagnosis	Test
$V_1 = 27.1008 \text{ V}$	$I_2^2 = 4.7437 \text{ mA}$	$I_2^2 + I_2^3 \neq 0$	$T_{23}$
$V_2 = 3.1321 \text{ V}$	$I_1^3 = 9.9952 \text{ mA}$	$I_1^3 + I_1^5 + I_1^6 + I_1^7 = 0$	$T_{3567}$
$V_5 = 2.4126 \text{ V}$	$I_2^3 = -7.9692 \text{ mA}$	$I_5^3 + I_5^4 + I_5^5 + I_5^6 \approx 0$	$T_{3456}$
$V_7 = 1.7256 \text{ V}$	$I_5^3 = -1.9949 \text{ mA}$	$I_7^5 + I_7^8 = 0$	$T_{58}$
$V_{10} = 1.7606 \text{ V}$	$I_5^4 = 2.0104 \text{ mA}$	$I_{10}^6 + I_{10}^9 = 0$	$T_{69}$
	$I_1^5 = 1.0049 \text{ mA}$		
	$I_5^5 = 0.0101 \text{ mA}$		
	$I_7^5 = -1.050 \text{ mA}$		
	$I_1^6 = 0.5282 \text{ mA}$		
	$I_5^6 = 0.0053 \text{ mA}$		
	$I_{10}^6 = -0.5335 \text{ mA}$		
	$I_1^7 = -11.5283 \text{ mA}$		
	$I_7^8 = 1.0150 \text{ mA}$		
	$I_{10}^9 = 0.5335 \text{ mA}$		

Logical diagnostic function  $D_5 = (\bar{\sigma}_2 \cup \bar{\sigma}_3) \cap (\sigma_3 \cap \sigma_5 \cap \sigma_6 \cap \sigma_7) \cap (\sigma_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6) \cap (\sigma_5 \cap \sigma_8) \cap (\sigma_6 \cap \sigma_9) = \bar{\sigma}_2 \cap \sigma_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$ .

Result:  $S_2$  is the only faulty subnetwork.

TABLE IX

CASE 2 - Q<sub>3</sub> FAULTY

Measurements	Computed Currents	Diagnosis	Test
$V_1 = 26.1850$ V	$I_2^2 = 4.7437$ mA	$I_2^2 + I_2^3 = 0$	$T_{23}$
$V_2 = 11.6790$ V	$I_1^3 = 13.9057$ mA	$I_1^3 + I_1^5 + I_1^6 + I_1^7 \neq 0$	$T_{3567}$
$V_5 = 10.8809$ V	$I_2^3 = -4.7437$ mA	$I_5^3 + I_5^4 + I_5^5 + I_5^6 \neq 0$	$T_{3456}$
$V_7 = 10.8599$ V	$I_5^3 = -9.1620$ mA	$I_7^5 + I_7^8 \neq 0$	$T_{58}$
$V_{10} = 10.1296$ V	$I_5^4 = 9.0675$ mA	$I_{10}^6 + I_{10}^9 = 0$	$T_{69}$
	$I_1^5 = 5.8736$ mA	$I_2^2 + I_1^7 + I_7^8 + I_5^4$	
	$I_5^5 = 0.0593$ mA	$+ I_{10}^9 \cong 0$	$T_{24789}$
	$I_7^5 = -5.9329$ mA		
	$I_1^6 = 3.0389$ mA		
	$I_5^6 = 0.0307$ mA		
	$I_{10}^6 = -3.0696$ mA		
	$I_1^7 = -23.2685$ mA		
	$I_7^8 = 6.3882$ mA		
	$I_{10}^9 = 3.0696$ mA		

Logical diagnostic function  $D_6 = (\sigma_2 \wedge \sigma_3) \wedge (\bar{\sigma}_3 \vee \bar{\sigma}_5 \vee \bar{\sigma}_6 \vee \bar{\sigma}_7) \wedge (\bar{\sigma}_3 \vee \bar{\sigma}_4 \vee \bar{\sigma}_5 \vee \bar{\sigma}_6) \wedge (\bar{\sigma}_5 \vee \bar{\sigma}_8) \wedge (\sigma_6 \wedge \sigma_9) \wedge (\sigma_2 \wedge \sigma_7 \wedge \sigma_8 \wedge \sigma_4 \wedge \sigma_9) = \sigma_2 \wedge \sigma_3 \wedge \sigma_4 \wedge \bar{\sigma}_5 \wedge \sigma_6 \wedge \sigma_7 \wedge \sigma_8 \wedge \sigma_9$ .

Result:  $S_5$  is the only faulty subnetwork.

TABLE X  
CASE 3 - R<sub>10</sub> FAULTY

Measurements	Computed Currents	Diagnosis	Test
$V_1 = 5.8645 \text{ V}$	$I_2^2 = 2.8379 \text{ mA}$	$I_2^2 + I_2^3 = 0$	$T_{23}$
$V_2 = -2.6491 \text{ V}$	$I_1^3 = 2.8379 \text{ mA}$	$I_1^3 + I_1^5 + I_1^6 + I_1^7 \neq 0$	$T_{3567}$
$V_5 = 0.0$	$I_2^3 = -2.8379 \text{ mA}$	$I_5^3 + I_5^4 + I_5^5 + I_5^6 = 0$	$T_{3456}$
$V_7 = 0.0$	$I_5^3 = 0.0000 \text{ mA}$		
$V_{10} = 0.0$	$I_4^5 = 0.0000 \text{ mA}$	$I_7^5 + I_7^8 = 0$	$T_{58}$
	$I_1^5 = 0.0000 \text{ mA}$	$I_{10}^6 + I_{10}^9 = 0$	$T_{69}$
	$I_5^5 = 0.0000 \text{ mA}$		
	$I_7^5 = 0.0000 \text{ mA}$		
	$I_1^6 = 0.0000 \text{ mA}$		
	$I_5^6 = 0.0000 \text{ mA}$		
	$I_{10}^6 = 0.0000 \text{ mA}$		
	$I_1^7 = -283.7879 \text{ mA}$		
	$I_7^8 = 0.0000 \text{ mA}$		
	$I_{10}^9 = 0.0000 \text{ mA}$		

Logical diagnostic function  $D_5 = (\sigma_5 \wedge \sigma_8) \wedge (\sigma_6 \wedge \sigma_9) \wedge (\sigma_2 \wedge \sigma_3) \wedge (\bar{\sigma}_3 \vee \bar{\sigma}_5 \vee \bar{\sigma}_6 \vee \bar{\sigma}_7) \wedge (\sigma_3 \wedge \sigma_4 \wedge \sigma_5 \wedge \sigma_6) = \sigma_2 \wedge \sigma_3 \wedge \sigma_4 \wedge \sigma_5 \wedge \sigma_6 \wedge \bar{\sigma}_7 \wedge \sigma_8 \wedge \sigma_9$ .

Result:  $S_7$  is the only faulty subnetwork.

TABLE XI

CASE 4 - Q<sub>2</sub> FAULTY

Measurements	Computed Currents	Diagnosis	Test
$V_1 = 26.1241 \text{ V}$	$I_2^2 = 4.7437 \text{ mA}$	$I_2^2 + I_2^3 \neq 0$	$T_{23}$
$V_2 = 11.6001 \text{ V}$	$I_1^3 = 4.8414 \text{ mA}$	$I_1^3 + I_1^5 + I_1^6 + I_1^7 \neq 0$	$T_{3567}$
$V_5 = 11.6001 \text{ V}$	$I_2^3 = -4.8414 \text{ mA}$	$I_5^3 + I_5^4 + I_5^5 + I_5^6 \neq 0$	$T_{3456}$
$V_7 = 10.8001 \text{ V}$	$I_5^3 = 0.000 \text{ mA}$	$I_7^5 + I_7^8 = 0$	$T_{58}$
$V_{10} = 10.8445 \text{ V}$	$I_5^4 = 9.6667 \text{ mA}$	$I_{10}^6 + I_{10}^9 = 0$	$T_{69}$
	$I_1^5 = 6.2895 \text{ mA}$	$I_2^5 + I_1^7 + I_7^8 + I_5^4$	
	$I_5^5 = 0.0635 \text{ mA}$	$+ I_{10}^9 \cong 0$	$T_{24789}$
	$I_7^5 = -6.3530 \text{ mA}$		
	$I_1^6 = 3.2533 \text{ mA}$		
	$I_5^6 = 0.0329 \text{ mA}$		
	$I_{10}^6 = -3.2862 \text{ mA}$		
	$I_1^7 = -24.0497 \text{ mA}$		
	$I_7^8 = 6.3530 \text{ mA}$		
	$I_{10}^9 = 3.2862 \text{ mA}$		

Logical diagnostic function  $D_6 = (\bar{\sigma}_2 \cup \bar{\sigma}_3) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6 \cup \bar{\sigma}_7) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_4 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6) \cap (\sigma_5 \cap \sigma_8) \cap (\sigma_6 \cap \sigma_9) \cap (\sigma_2 \cap \sigma_4 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9) = \sigma_2 \cap \bar{\sigma}_3 \cap \sigma_4 \cap \sigma_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$ .

Result:  $S_3$  is the only faulty subnetwork.

TABLE XII

CASE 5 - Q<sub>3</sub> FAULTY WITH TOLERANCES ON THE NONFAULTY ELEMENTS

Percentage Deviation	Test 1 T <sub>23</sub>	Test 2 T <sub>3567</sub>	Test 3 T <sub>3456</sub>	Test 4 T <sub>58</sub>	Test 5 T <sub>69</sub>	Test 6 T <sub>24789</sub>
$ \Delta R_1/R_1 $	0.0					0.0
$ \Delta R_2/R_2 $	1.983	13.54	0.0			
$ \Delta R_3/R_3 $	1.774					2.11
$ \Delta R_4/R_4 $			26.99*			4.076
$ \Delta R_5/R_5 $		0.088	6.71	9.09		
$ \Delta R_6/R_6 $		0.012	0.0		0.136	
$ \Delta R_7/R_7 $		46.47*	0.31	70.82*		
$ \Delta R_8/R_8 $				27.74*		2.45
$ \Delta R_9/R_9 $					0.878	1.382
$ \Delta R_{10}/R_{10} $		51.44*				8.96
$ \Delta \alpha_{N1}/\alpha_{N1} $	0.0224					0.026
$ \Delta \alpha_{N2}/\alpha_{N2} $	0.0513	0.71*	0.408*			
$ \Delta \alpha_{N3}/\alpha_{N3} $		0.0	0.001	0.194*		
$ \Delta \alpha_{N4}/\alpha_{N4} $		1.27*	0.072		0.14	
	Pass	Fail	Fail	Fail	Pass	Pass

Logical diagnostic function  $D_6 = T_{23} \wedge T_{3567} \wedge T_{3456} \wedge T_{58} \wedge T_{69} \wedge T_{24789}$

$= \sigma_2 \wedge \sigma_3 \wedge \sigma_4 \wedge \bar{\sigma}_5 \wedge \sigma_6 \wedge \sigma_7 \wedge \sigma_8 \wedge \sigma_9$ .

Result: S<sub>5</sub> is the only faulty subnetwork (see Table IX).

\* deviation significantly exceeds tolerance.



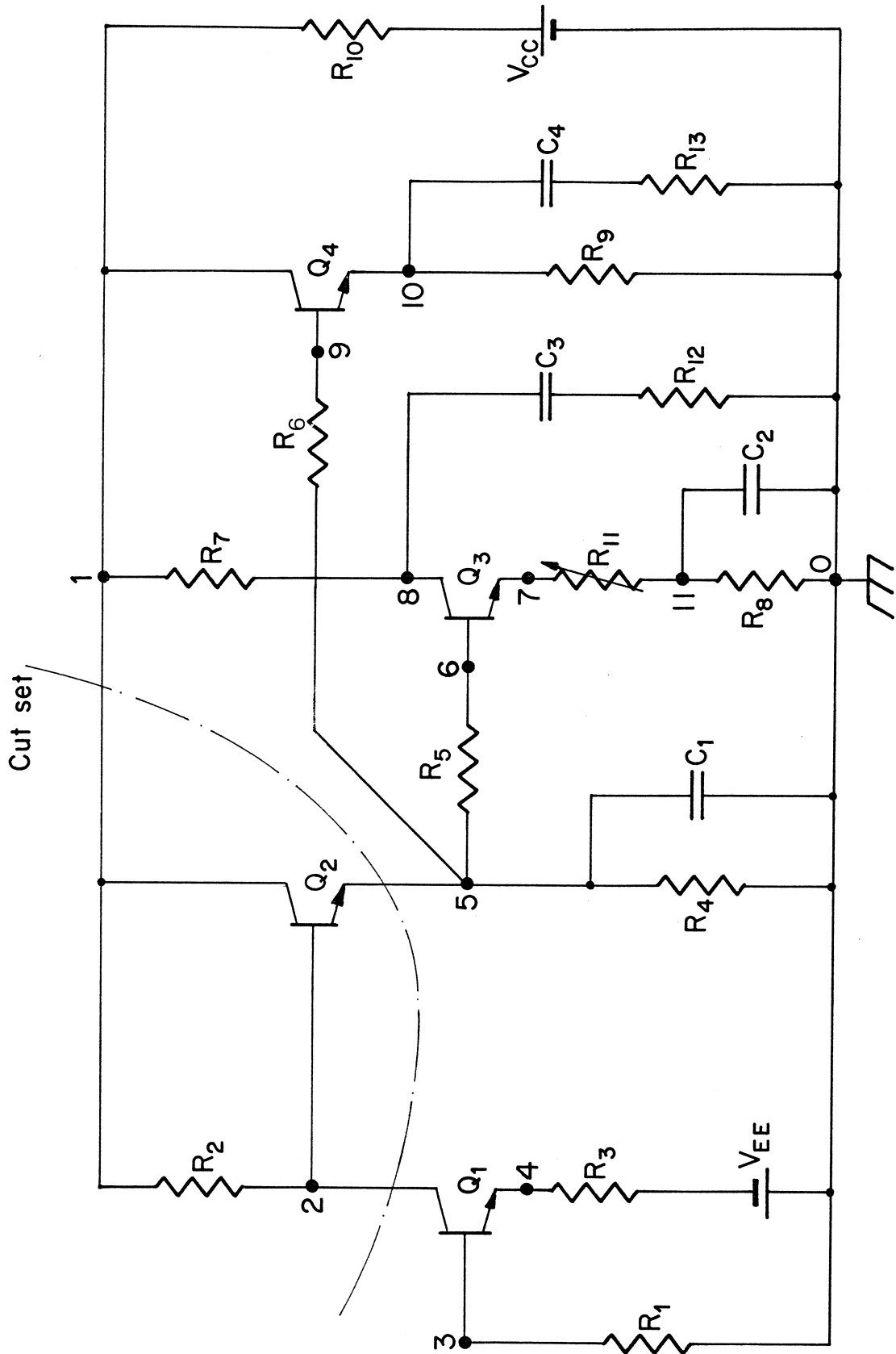


Fig. 1 Video amplifier circuit.

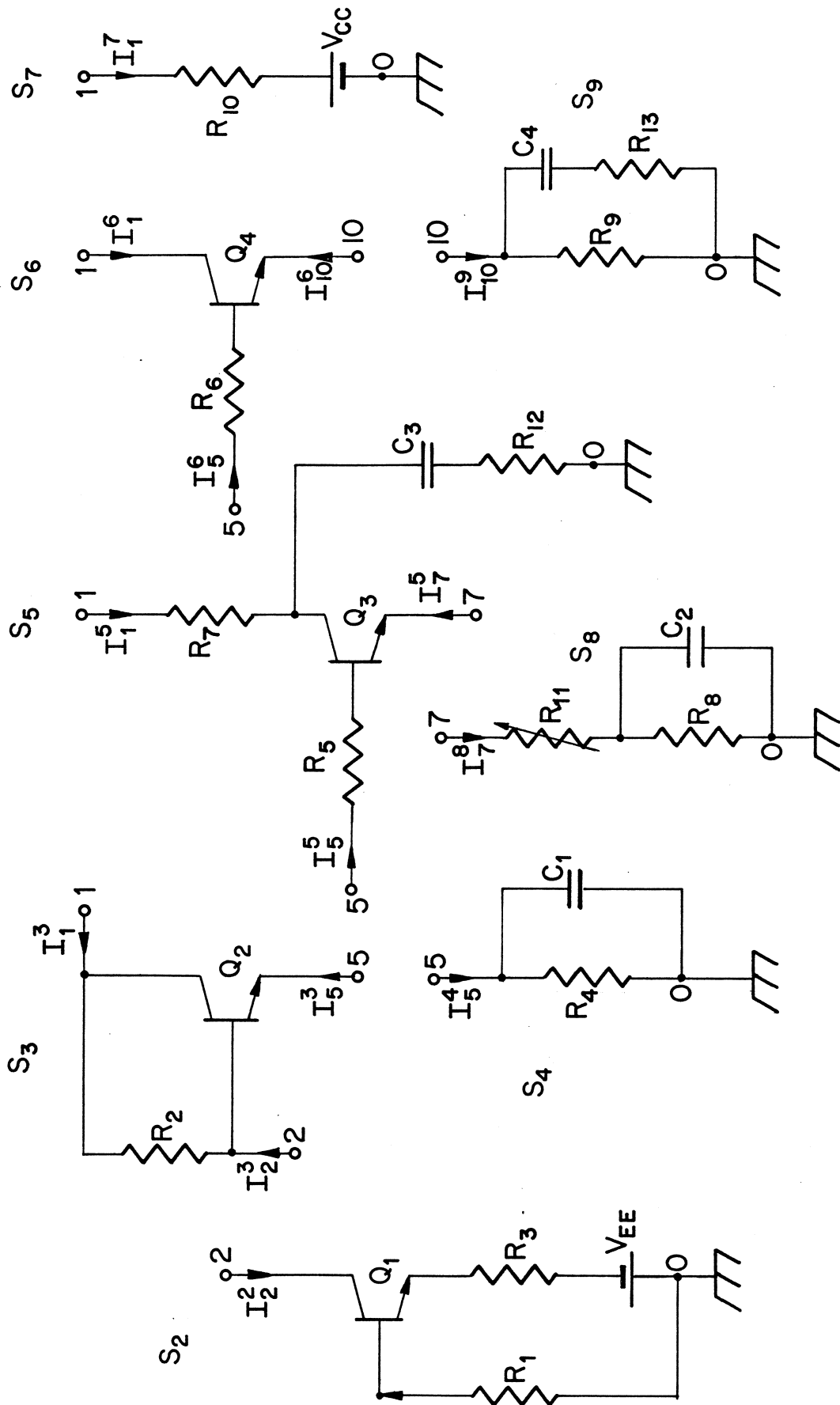


Fig. 2 Network decomposition of the video amplifier.

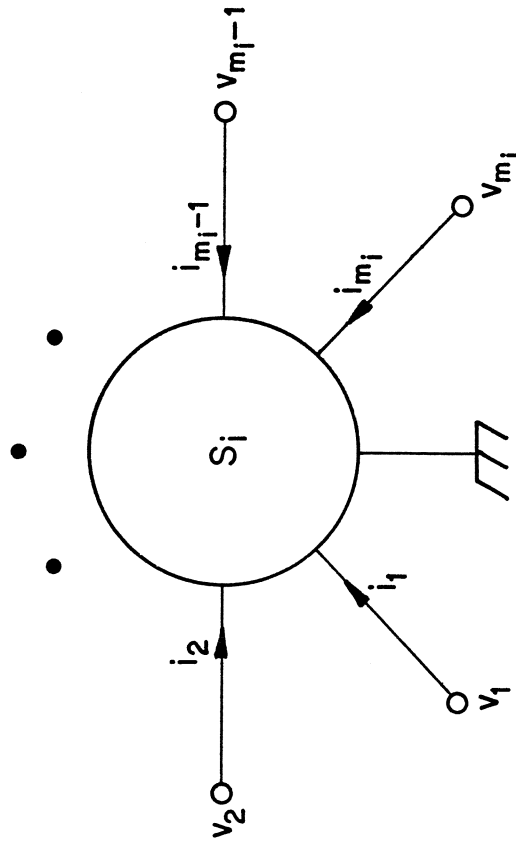


Fig. 3 Subnetwork  $S_i$  with  $m_i+1$  external nodes.

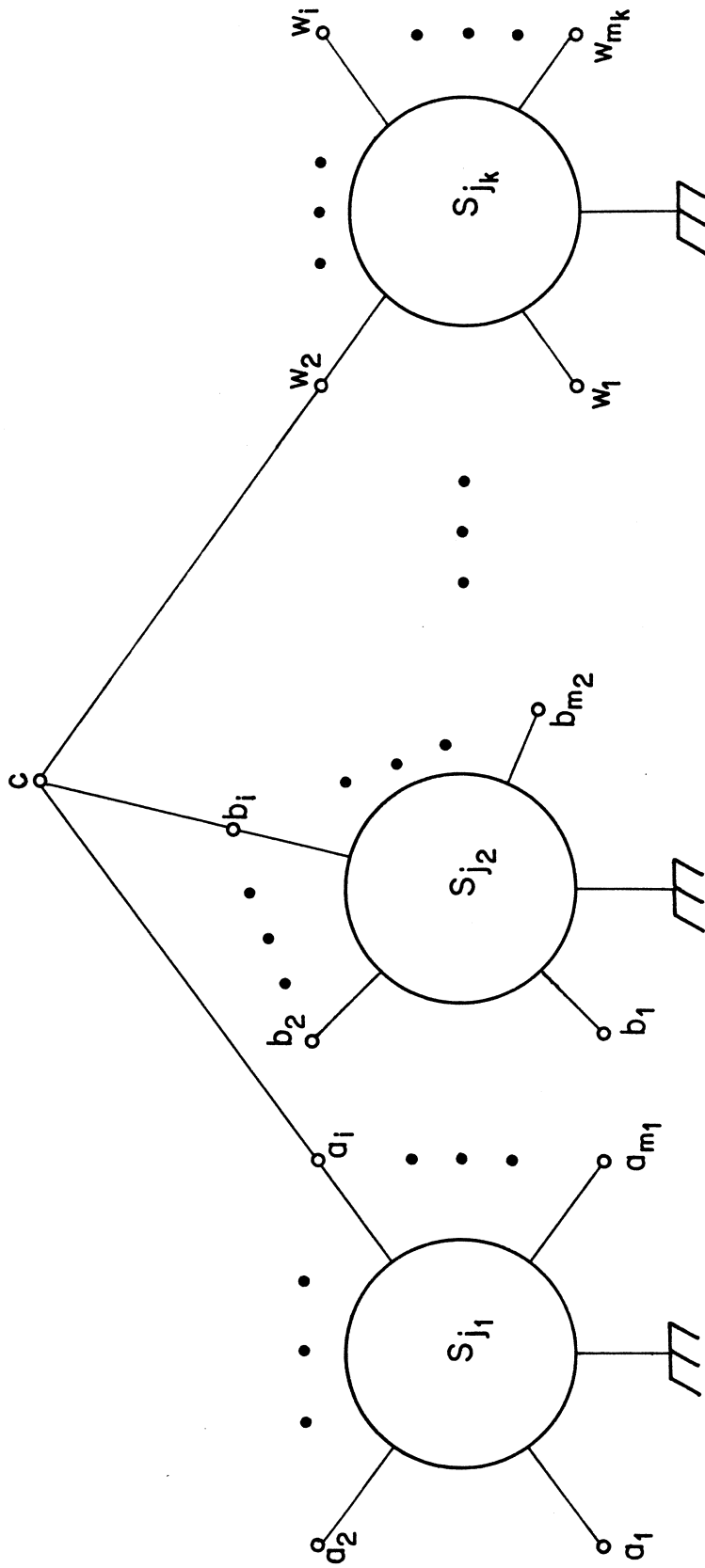


Fig. 4  $k$  subnetworks incident at node  $c$ .

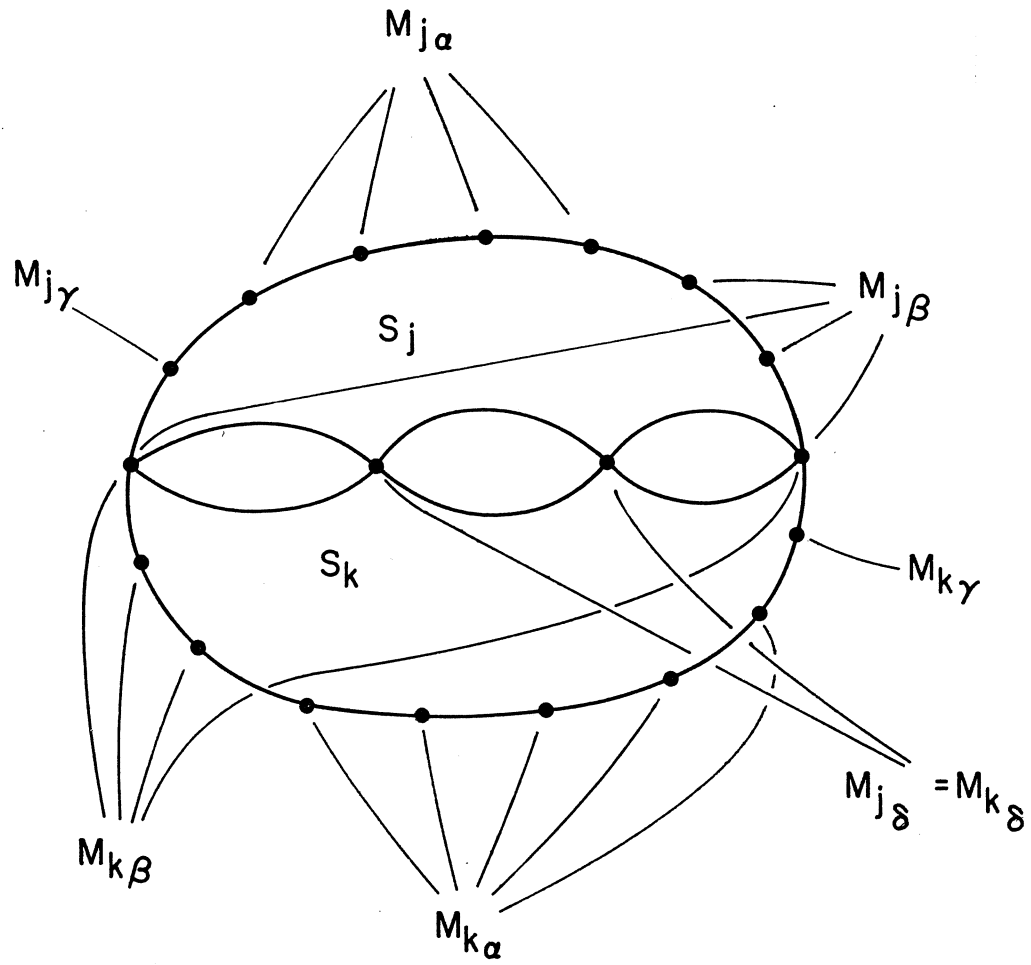


Fig. 5 Decomposition of  $S_i$  into two subnetworks  $S_j$  and  $S_k$ .

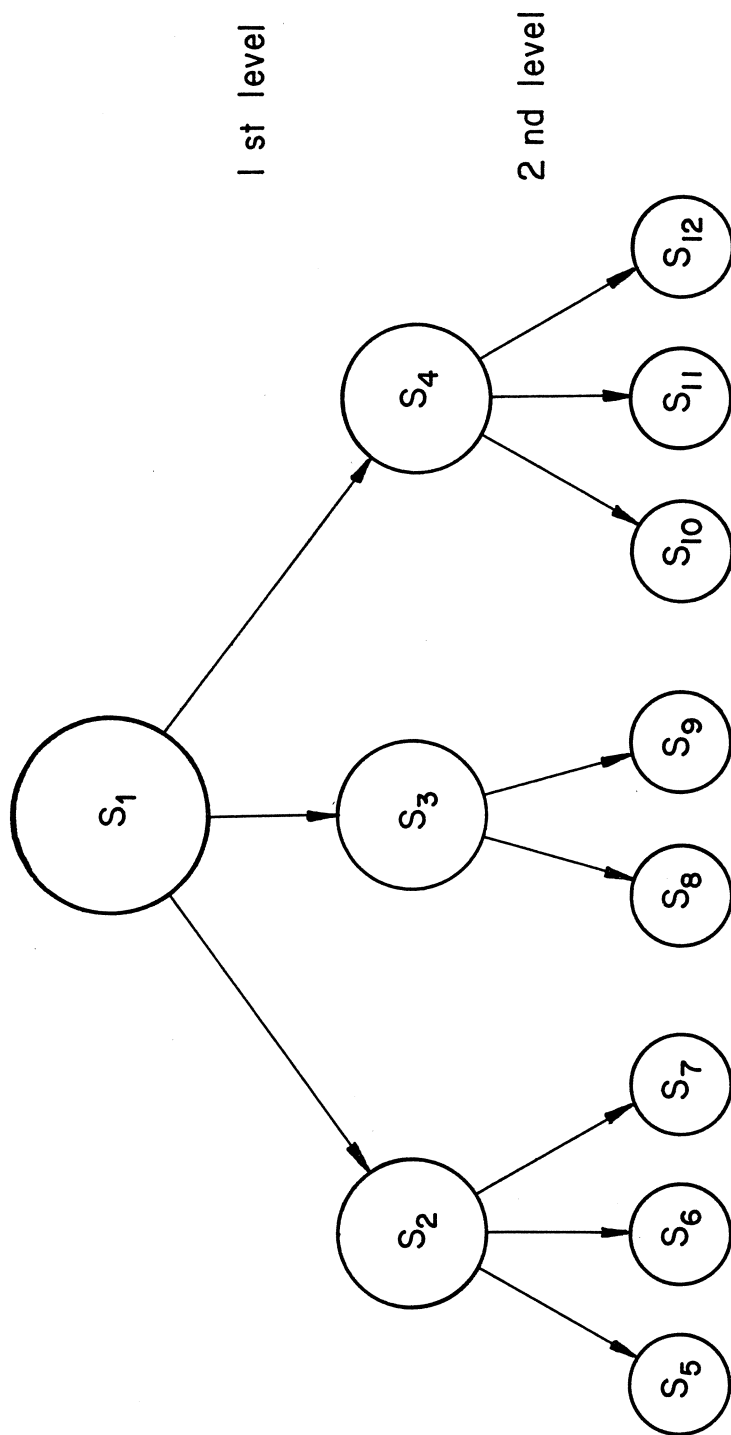


Fig. 6 Illustration of the hierarchical decomposition of a network.

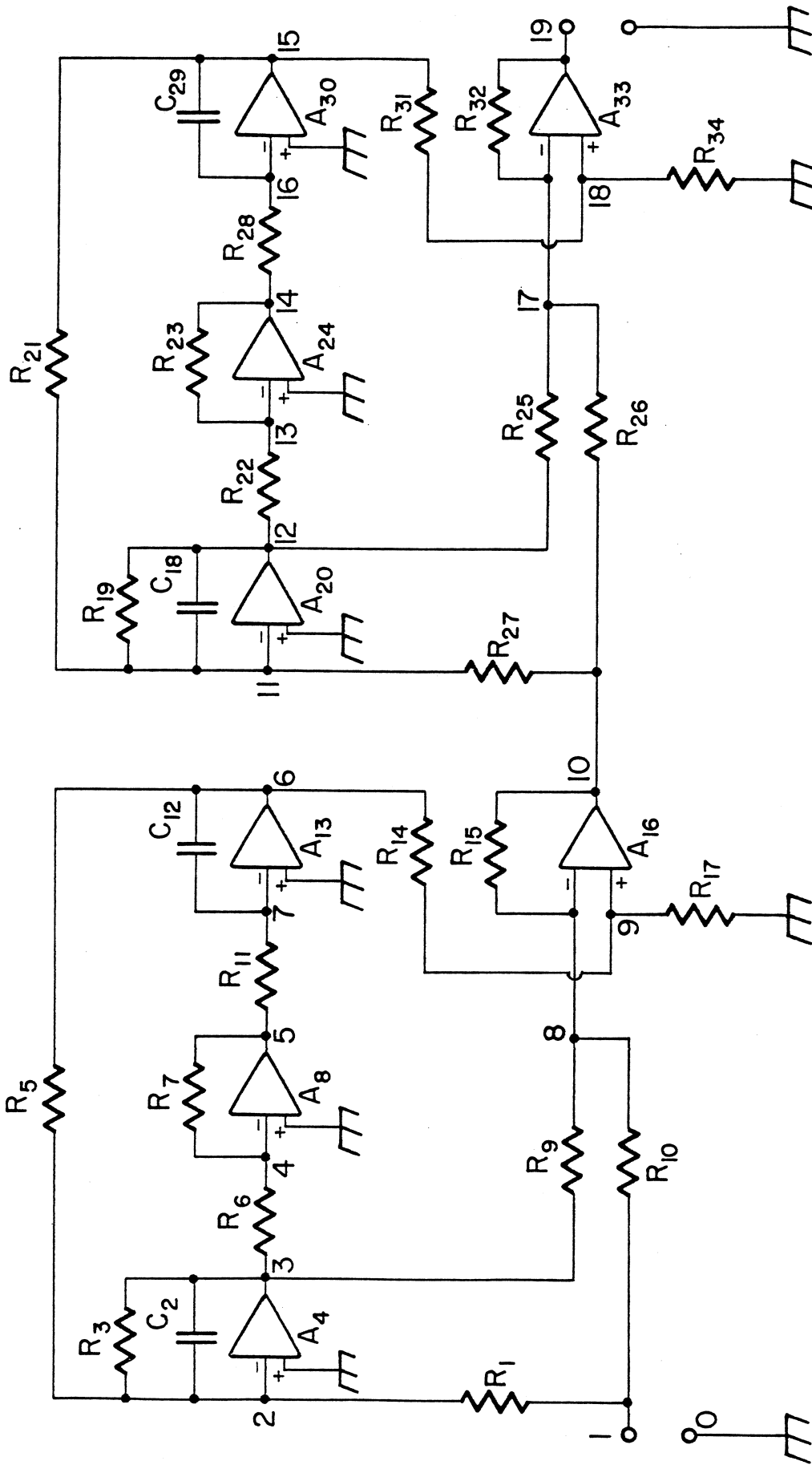


Fig. 7 First section of the active filter example.

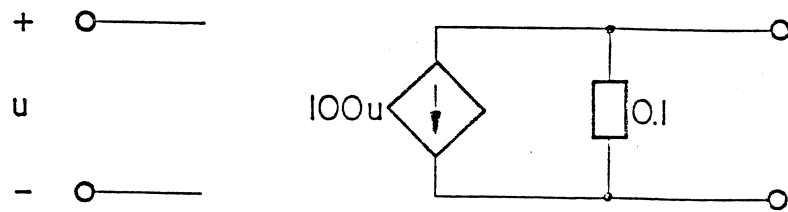


Fig. 8 Equivalent circuit for the Op-Amp.



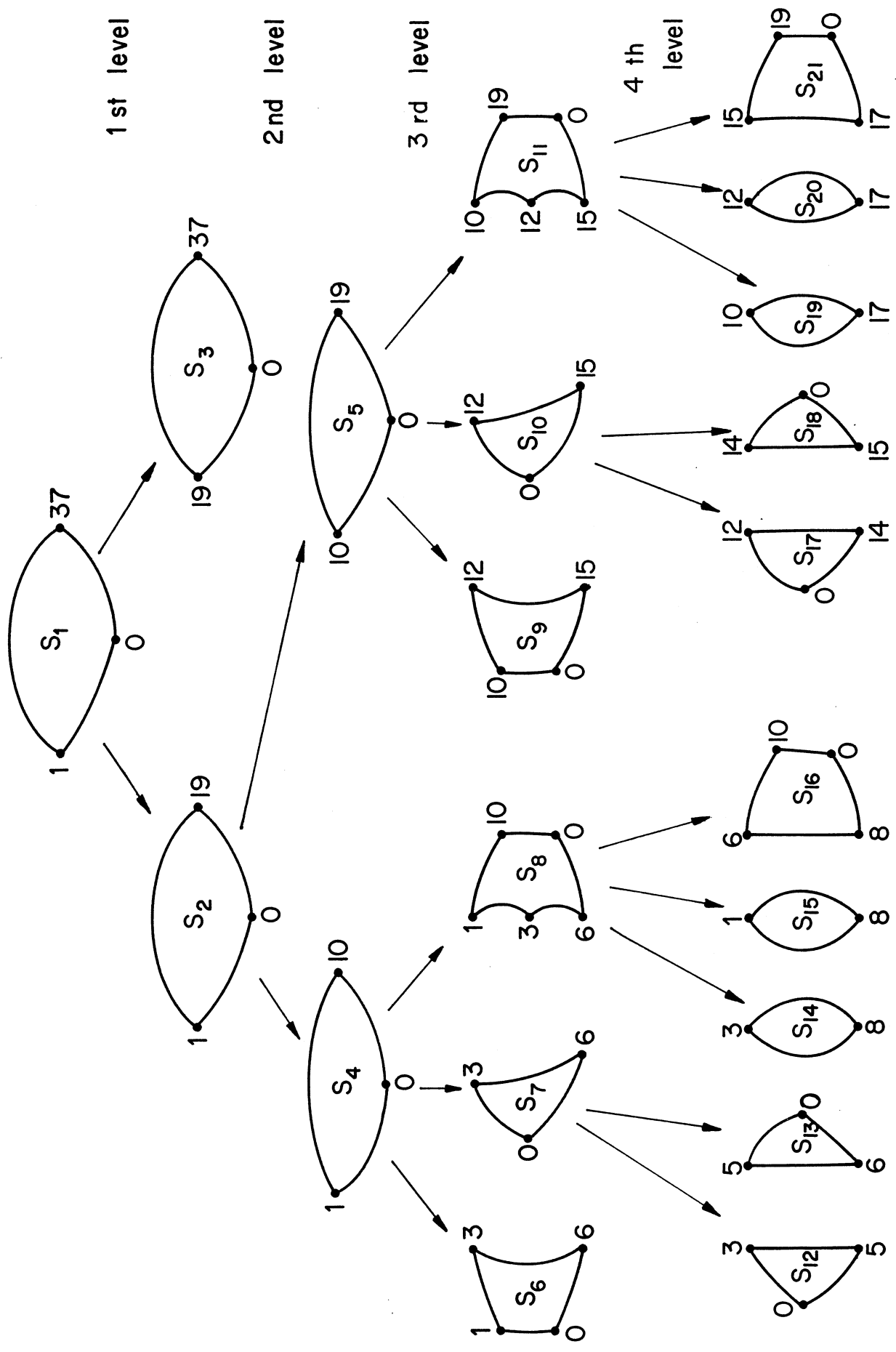


Fig. 9 Hierarchical decomposition of the linear network during testing.