MFNC - A FORTRAN PACKAGE FOR MINIMIZATION WITH GENERAL CONSTRAINTS

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Abstract

MFNC is a package of Fortran subroutines for minimization of a nonlinear objective function subject to nonlinear constraints. It implements the Powell's variable metric method for constrained optimization which is based on the results of Han. The package and documentation have been developed for use on the CDC 170/730 system with level 552 operating system and Fortran Extended (FTN) version 4.8 compiler. This document contains a listing of the MFNC package.

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I. INTRODUCTION

MFNC is a package of Fortran subroutines for minimization of a nonlinear objective function subject to nonlinear constraints. It is an extension and modification of the package VFO2AD of the Harwell Subroutine Library [1]. The package MFNC implements the Powell's variable metric method for constrained optimization [2] which is based on the results of Han [3].

The whole package is written in Fortran IV for the CDC 170/730 system with the NOS 1.4 level 552 operating system. It is available at McMaster University in the form of a library of binary relocatable subroutines in the group indirect file LIBRMFN under the charge RJWBAND.

This document includes a listing of the package MFNC. The user's manual presented together with illustrative examples is found in [4]. The listing contains 1772 lines.

II. REFERENCES

[1] VFO2AD subroutine specification, Harwell Subroutine Library, AERE, Harwell, Oxfordshire, England, February 1978.

[2] M.J.D. Powell, "A fast algorithm for nonlinearly constrained optimization calculations", in Numerical Analysis, Proc. Biennial Conf. (Dundee, Scotland, 1977), Lecture Notes in Mathematics 630, G.A. Watson, Ed. Berlin: Springer-Verlag, 1978, pp. 144-157.

[3] S.H. Han, "Superlinearly convergent variable metric algorithms for general nonlinear programming problem", Mathematical Programming, vol. 11, 1976, pp. 263-282.

[4] J.W. Bandler and W.M. Zuberek, "MFNC - A Fortran package for minimization with general constrains", Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, Report SOS-82-6-U2, 1983.

III. LISTING OF THE MFNC PACKAGE

Subroutine	Number of lines	Number of words (source text)(compiled code)	Listing from page
MFNC1A	93	766	4
MFNC2A	9	77	5
MFNC9A	42	365	5
MFNOOZ	8	23	6
MFNOOQ	35	210	6
MFNOOV	26	245	7
MFNOOG	36	261	7
MFNOOH	58	460	8
MFNOOA	28	150	9
MMXPSZ	12	42	9
MMXPLM	11	37	9
MMXLLM	11	36	9
MMXHDR	16	47	10
MMXGLM	13	44	10
MMXGVL	11	41	10
VF02A	71	261	10
VF02B	261	1216	11
VF02C	122	462	16
VE02A	373	2512	17
LA02A	360	2115	23
FM02AS	25	60	29
MB01C	103	623	29

```
      SUBROUTINE MFNC1A (FCD,N,L,LEQ,X,EPS,MAXF,W,IW,LCH,IPR,IFLAG)          000001
      EXTERNAL FCD,MFN00Q                                                    000002
C                                                                            000003
C           LEVEL 1 INTERFACE (STANDARD ENTRY)                               000004
C                                                                            000005
      DIMENSION X(1), W(1)                                                   000006
      COMMON /MMX000/ NCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG     000007
     1H,DAT,TIM,LHT,H(8)                                                     000008
      COMMON /VF02E/ DUMMY(6),IDUMMY,ITERQ,JDUMMY(3)                         000009
      NCH=LCH                                                                000010
      IF (LCH.LE.0) GO TO 40                                                 000011
      I=IABS(IPR)                                                            000012
      J=I/10                                                                 000013
      LG2=MOD(I,10)                                                          000014
      I=J/10                                                                 000015
      LG1=MOD(J,10)                                                          000016
      J=I/10                                                                 000017
      LV2=MOD(I,10)                                                          000018
      LV1=J                                                                  000019
      LG1=LG1*LV1                                                            000020
      NRP=0                                                                  000021
      CALL MMXPSZ (-1)                                                       000022
      CALL MMXPLM (-1)                                                       000023
      CALL MMXLLM (-1)                                                       000024
      CALL MMXHDR (-1,H)                                                     000025
      CALL MMXGLM (-1,-1)                                                    000026
      CALL MMXGVL (-1)                                                       000027
      IF (MXL.NE.0) LML=MXL*LMP+100                                          000028
      IF (MXL.EQ.0) MXL=LML+100                                              000029
      CALL DATE (DAT)                                                        000030
      CALL TIME (TIM)                                                        000031
      CALL MFN00A                                                            000032
      WRITE (LCH,10) N,LEQ,L,EPS,MAXF,IW,IPR                                 000033
   10 FORMAT (11H0INPUT DATA/11H ----------//                                000034
     1 27H   NUMBER OF VARIABLES (N)  ,25(2H. ),I4//                         000035
     2 39H   NUMBER OF EQUALITY CONSTRAINTS (LEQ),19(2H. ),I4//              000036
     3 35H   TOTAL NUMBER OF CONSTRAINTS (L)  ,21(2H. ),I4//                 000037
     4 19H   ACCURACY (EPS)   ,26(2H. ),1PE10.3//                           000038
     5 45H   MAX NUMBER OF FUNCTION EVALUATIONS (MAXF)  ,16(2H. ),I4//      000039
     6 22H   WORKING SPACE (IW)  ,26(2H. ),1H.,I6//                         000040
     7 26H   PRINTOUT CONTROL (IPR)  ,24(2H. ),1H.,I6/)                     000041
      NRL=NRL-18                                                             000042
      LML=LML-18                                                             000043
      JF=1                                                                   000044
      JC=JF+1                                                                000045
      JG=JC+L                                                                000046
      JD=JG+N                                                                000047
      IF (LV2.NE.0.OR.LV1.EQ.1) GO TO 30                                     000048
      CALL FCD (N,L,X,W(JF),W(JG),W(JC),W(JD),N)                             000049
      WRITE (LCH,20) W(JF)                                                   000050
   20 FORMAT (19H   STARTING POINT. ,20X,21HOBJECTIVE FUNCTION : ,           000051
     1 1PE19.12)                                                             000052
      NRL=NRL-1                                                              000053
      LML=LML-1                                                              000054
      CALL MFN00V (X,N,W(JG),W(JC),L)                                        000055
      IF (LG2.NE.0) CALL MFN00G (W(JD),N,L,N)                                000056
   30 IF (IPR.GE.0) GO TO 40                                                 000057
      L1=L+1                                                                 000058
      JF1=JD+N*L                                                             000059
      JG1=JF1+L1                                                             000060
      JG2=JG1+L1                                                             000061
      LW=2*(N*L+N+L+1)                                                       000062
      IF (LW.GT.IW) GO TO 80                                                 000063
      CALL MFN00H (FCD,N,L,X,W(JF),W(JG),W(JF1),W(JG1),W(JG2),L1)            000064
   40 CALL SECOND (TBEG)                                                     000065
```

```
      CALL MFNC9A (MFN00Q,FCD,N,L,LEQ,X,EPS,MAXF,W,IW,IFLAG)                000066
      CALL SECOND (TEND)                                                    000067
      IF (LCH.LE.0) RETURN                                                  000068
      IF (IFLAG.EQ.-1) GO TO 100                                            000069
      IF (NRL.LT.9) CALL MFN00A                                             000070
      WRITE (LCH,50)                                                        000071
   50 FORMAT (//9H SOLUTION/9H --------)                                    000072
      WRITE (LCH,60) W(1)                                                   000073
   60 FORMAT (39X,21HOBJECTIVE FUNCTION : ,1PE19.12/)                       000074
      NRL=NRL-6                                                             000075
      LML=LML-6                                                             000076
      CALL MFN00V (X,N,W(L+2),W(2),L)                                       000077
      CPU=TEND-TBEG                                                         000078
      IF (NRL.LT.9) CALL MFN00A                                             000079
      WRITE (LCH,70) IFLAG,MAXF,ITERQ,CPU                                   000080
   70 FORMAT (//29H   TYPE OF SOLUTION (IFLAG)  ,24(2H. ),I4//              000081
     1 35H   NUMBER OF FUNCTION EVALUATIONS  ,21(2H. ),I4//                 000082
     2 35H   NUMBER OF QUADRATIC ITERATIONS  ,21(2H. ),I4//                 000083
     3 31H   EXECUTION TIME (IN SECONDS) ,21(2H. ),1H.,F7.3/)               000084
      RETURN                                                                000085
   80 IFLAG=-1                                                              000086
      WRITE (LCH,90) LW                                                     000087
   90 FORMAT(/55H0INSUFFICIENT WORKSPACE. IT SHOULD HAVE LENGTH AT LEAST    000088
     1 ,I6/)                                                                000089
  100 WRITE (LCH,110)                                                       000090
  110 FORMAT (///40H I N C O R R E C T   P A R A M E T E R S/)              000091
      RETURN                                                                000092
      END                                                                   000093
C                                                                           000094
C                                                                           000095
      SUBROUTINE MFNC2A (FCD,N,L,LEQ,X,EPS,MAXF,W,IW,IFLAG)                 000096
      EXTERNAL FCD,MFN00Z                                                   000097
C                                                                           000098
C          LEVEL 2 INTERFACE (BASIC ENTRY)                                  000099
C                                                                           000100
      DIMENSION X(1), W(1)                                                  000101
      CALL MFNC9A (MFN00Z,FCD,N,L,LEQ,X,EPS,MAXF,W,IW,IFLAG)                000102
      RETURN                                                                000103
      END                                                                   000104
C                                                                           000105
C                                                                           000106
      SUBROUTINE MFNC9A (FQQ,FCD,N,M,LEQ,X,EPS,MAXF,W,LW,IFLAG)             000107
      DIMENSION X(1), W(1)                                                  000108
      EXTERNAL FQQ,FCD                                                      000109
      DIMENSION IEE(8)                                                      000110
      COMMON /LA02B/ LP1,IE1                                                000111
      COMMON /MB01D/ LP2,IE2                                                000112
      COMMON /VF02D/ VLN,LP3                                                000113
      COMMON /VE02X/ LP4                                                    000114
      COMMON /MFN000/ MARK                                                  000115
      COMMON /MFN111/ LP0                                                   000116
      DATA LP0/0/,IEE/0,2,-3,1,-2,-4,-1,-1/                                 000117
      DATA XZERO/0.0/                                                       000118
      MARK=1                                                                000119
      LP1=LP0                                                               000120
      LP2=LP0                                                               000121
      LP3=LP0                                                               000122
      LP4=LP0                                                               000123
      LWR=5*N*N+24*N+6*M+N*M+19+MAX0(M,3*N+3)                               000124
      IF (N.LE.0.OR.LEQ.GT.N.OR.LEQ.GT.M.OR.M.LT.0.OR.LEQ.LT.0.OR.EPS.LT    000125
     1.XZERO.OR.MAXF.LE.0) GO TO 20                                         000126
      IF (LWR.LE.LW) GO TO 30                                               000127
      IF (LP0.GT.0) WRITE (LP0,10) LWR                                      000128
   10 FORMAT(/55H0INSUFFICIENT WORKSPACE. IT SHOULD HAVE LENGTH AT LEAST    000129
     1 ,I6/)                                                                000130
```

```
   20 IFLAG=-1                                                          000131
      RETURN                                                            000132
   30 IF=1                                                              000133
      IC=2                                                              000134
      IG=IC+M                                                           000135
      ID=IG+N                                                           000136
      IW=ID+N*M+M                                                       000137
      LL=LW-IW+1                                                        000138
      IE=-1                                                             000139
      NCALL=0                                                           000140
   40 CALL FCD (N,M,X,W(IF),W(IG),W(IC),W(ID),N+1)                      000141
      NCALL=NCALL+1                                                     000142
      CALL FQQ (N,M,X,W(IF),W(IG),W(IC),W(ID),N+1,NCALL)                000143
      IF (MARK.EQ.0) GO TO 50                                           000144
      CALL VF02A (N,M,LEQ,X,W(IF),W(IG),W(IC),W(ID),N+1,MAXF,EPS,0,IE,W( 000145
     1IW),LL)                                                           000146
      IF (IE.EQ.0) GO TO 40                                             000147
      IFLAG=IEE(IE)                                                     000148
      MAXF=NCALL                                                        000149
      RETURN                                                            000150
   50 IFLAG=3                                                           000151
      MAXF=NCALL                                                        000152
      RETURN                                                            000153
      END                                                               000154
C                                                                       000155
C                                                                       000156
      SUBROUTINE MFN00Z (N,M,X,F,G,C,D,L,K)                             000157
C                                                                       000158
C     DUMMY SUBROUTINE WHICH FOR BASIC ENTRY SUBSTITUTES SUBROUTINE     000159
C     MFN00Q/11Q.                                                       000160
C                                                                       000161
      DIMENSION X(N), G(N), C(M), D(L,M)                                000162
      RETURN                                                            000163
      END                                                               000164
C                                                                       000165
C                                                                       000166
      SUBROUTINE MFN00Q (N,M,X,F,G,C,D,L,K)                             000167
C                                                                       000168
C     PRINT RESULTS OF FUNCTION EVALUATION.                             000169
C                                                                       000170
      DIMENSION X(N), G(N), C(M), D(L,M)                                000171
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG 000172
     1H,DAT,TIM,LHT,H(8)                                                000173
      IF (LCH.LE.0) RETURN                                              000174
      IF (LV1+LV2.EQ.0) RETURN                                          000175
      IF (K.LE.LV2) GO TO 10                                            000176
      IF (LV1.EQ.0) RETURN                                              000177
      IF (MOD(K,LV1).NE.0) RETURN                                       000178
   10 IF (NRP.LE.LMP.AND.LML.GE.0) GO TO 30                             000179
      LV1=0                                                             000180
      LV2=0                                                             000181
      WRITE (LCH,20)                                                    000182
   20 FORMAT (//26H ( LISTING LIMIT REACHED )//)                        000183
      NRL=NRL-5                                                         000184
      LML=LML-5                                                         000185
      RETURN                                                            000186
   30 IF (NRL.LT.7) CALL MFN00A                                         000187
      WRITE (LCH,40) K,F                                                000188
   40 FORMAT (22H0FUNCTION EVALUATION :,I4,13X,21HOBJECTIVE FUNCTION : , 000189
     1 1PE19.12)                                                        000190
      NRL=NRL-2                                                         000191
      LML=LML-2                                                         000192
      CALL MFN00V (X,N,G,C,M)                                           000193
      IF (LG1+LG2.EQ.0) RETURN                                          000194
      IF (K.LE.LG2) GO TO 50                                            000195
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      IF (K.LE.LV2) RETURN                                                    000196
      IF (LG1.EQ.0) RETURN                                                    000197
      IF (MOD(K,LG1).NE.0) RETURN                                             000198
   50 CALL MFN00G (D,N,M,L)                                                   000199
      RETURN                                                                  000200
      END                                                                     000201
C                                                                             000202
C                                                                             000203
      SUBROUTINE MFN00V (X,N,G,C,M)                                           000204
C                                                                             000205
C     PRINT VALUES OF VARIABLES AND CONSTRAINTS.                              000206
C                                                                             000207
      DIMENSION X(1), G(1), C(1)                                              000208
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG      000209
     1H,DAT,TIM,LHT,H(8)                                                      000210
      IF (LCH.LE.0) RETURN                                                    000211
      K=MAX0(N,M)                                                             000212
      IF (NRL.LT.5) CALL MFN00A                                               000213
      WRITE (LCH,10)                                                          000214
   10 FORMAT (/17X,9HVARIABLES,11X,8HGRADIENT,20X,11HCONSTRAINTS/)            000215
      NRL=NRL-3                                                               000216
      LML=LML-3                                                               000217
      DO 40 I=1,K                                                             000218
      IF (NRL.LE.0) CALL MFN00A                                               000219
      IF (I.LE.N.AND.I.LE.M) WRITE (LCH,20) I,X(I),G(I),I,C(I)                000220
      IF (I.LE.N.AND.I.GT.M) WRITE (LCH,20) I,X(I),G(I)                       000221
      IF (I.GT.N.AND.I.LE.M) WRITE (LCH,30) I,C(I)                            000222
   20 FORMAT (5X,I4,2X,1PE19.12,2X,1PE17.10,5X,I4,2X,1PE19.12)                000223
   30 FORMAT (54X,I4,2X,1PE19.12)                                             000224
      NRL=NRL-1                                                               000225
      LML=LML-1                                                               000226
   40 CONTINUE                                                                000227
      RETURN                                                                  000228
      END                                                                     000229
C                                                                             000230
C                                                                             000231
      SUBROUTINE MFN00G (D,N,M,L)                                             000232
C                                                                             000233
C     PRINT PARTIAL DERIVATIVES OF CONSTRAINTS.                               000234
C                                                                             000235
      DIMENSION D(L,M)                                                        000236
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG      000237
     1H,DAT,TIM,LHT,H(8)                                                      000238
      IF (LCH.LE.0) RETURN                                                    000239
      IF (M.LE.0) RETURN                                                      000240
      IF (NRL.LT.7) CALL MFN00A                                               000241
      MM=MIN0(M,LMF)                                                          000242
      NN=MIN0(N,LMV)                                                          000243
      WRITE (LCH,10)                                                          000244
   10 FORMAT (43H0  CONSTRAINT DERIVATIVES ( DF.I / DX.J ) :)                 000245
      NRL=NRL-2                                                               000246
      LML=LML-2                                                               000247
      DO 60 K=1,NN,LCH                                                        000248
      IF (NRL.LT.5) CALL MFN00A                                               000249
      J1=K                                                                    000250
      J2=MIN0(NN,K+LCH-1)                                                     000251
      WRITE (LCH,20) (J,J=J1,J2)                                              000252
   20 FORMAT (1H0,9X,12HVARIABLES(J),10(I5,5X))                               000253
      WRITE (LCH,30)                                                          000254
   30 FORMAT (8X,14HCONSTRAINTS(I))                                           000255
      NRL=NRL-3                                                               000256
      LML=LML-3                                                               000257
      DO 50 I=1,MM                                                            000258
      IF (NRL.LE.0) CALL MFN00A                                               000259
      WRITE (LCH,40) I,(D(J,I),J=J1,J2)                                       000260
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```
   40 FORMAT (10X,I6,4X,10(1PE10.2))                                    000261
      NRL=NRL-1                                                         000262
      LML=LML-1                                                         000263
   50 CONTINUE                                                          000264
   60 CONTINUE                                                          000265
      RETURN                                                            000266
      END                                                               000267
C                                                                       000268
C                                                                       000269
      SUBROUTINE MFN00H (FDF,N,M,X,F,DF,G,DG,DH,M1)                     000270
C                                                                       000271
C     NUMERICAL VERIFICATION OF USER-DEFINED PARTIAL DERIVATIVES        000272
C     (VARIABLES ARE DISTURBED ONE BY ONE).                             000273
C                                                                       000274
      DIMENSION X(N), F(M1), DF(N,M1), G(M1), DG(M1), DH(N,M1)          000275
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG 000276
     1H,DAT,TIM,LHT,H(8)                                                000277
      IF (LCH.LE.0) RETURN                                              000278
      K=0                                                               000279
      CALL FDF (N,M,X,F(1),DF(1,1),F(2),DF(1,2),N)                      000280
      DO 60 I=1,N                                                       000281
      Z=X(I)                                                            000282
      DX=1.E-6*Z                                                        000283
      IF (ABS(DX).LT.1.E-10) DX=1.E-10                                  000284
      DX2=DX+DX                                                         000285
      X(I)=Z+DX                                                         000286
      CALL FDF (N,M,X,F(1),DH(1,1),F(2),DH(1,2),N)                      000287
      DO 10 J=1,M1                                                      000288
      DG(J)=DH(I,J)                                                     000289
   10 CONTINUE                                                          000290
      X(I)=Z-DX                                                         000291
      CALL FDF (N,M,X,G(1),DH(1,1),G(2),DH(1,2),N)                      000292
      X(I)=Z                                                            000293
      DO 50 J=1,M1                                                      000294
      Y=DF(I,J)                                                         000295
      Z=F(J)-G(J)                                                       000296
      IF (ABS(Z).LE.0.5E-13*(F(J)+G(J))) Z=0.0                          000297
      Z=Z/DX2                                                           000298
      IF (ABS(Y).LE.1.E-20.AND.ABS(Z).LE.1.E-20) GO TO 50               000299
      IF (ABS(Z).LT.1.E-20) Z=SIGN(1.E-20,Z)                            000300
      R=100.0*ABS((Z-Y)/Z)                                              000301
      IF (R.LE.1.0) GO TO 50                                            000302
      IF (SIGN(1.0,DG(J))+SIGN(1.0,DH(I,J)).EQ.0.0) GO TO 50            000303
      IF (K.NE.0) GO TO 30                                              000304
      IF (NRL.LT.5) CALL MFN00A                                         000305
      WRITE (LCH,20)                                                    000306
   20 FORMAT(38H0VERIFICATION OF PARTIAL DERIVATIVES :/                 000307
     1 1H0,18X,52H DF.I / DX.J  :  USER DEFINED  NUMERICAL  DIFFERENCE) 000308
      NRL=NRL-4                                                         000309
      LML=LML-4                                                         000310
   30 K=K+1                                                             000311
      IF (NRL.LE.0) CALL MFN00A                                         000312
      L=J-1                                                             000313
      WRITE (LCH,40) L,I,Y,Z,R                                          000314
   40 FORMAT (19X,I5,3X,I4,5X,1PE10.3,2X,1PE10.3,4X,0PF6.1,2H %)        000315
      NRL=NRL-1                                                         000316
      LML=LML-1                                                         000317
   50 CONTINUE                                                          000318
   60 CONTINUE                                                          000319
      IF (K.NE.0) GO TO 80                                              000320
      IF (NRL.LT.2) CALL MFN00A                                         000321
      WRITE (LCH,70)                                                    000322
   70 FORMAT (47H0VERIFICATION OF PARTIAL DERIVATIVES PERFORMED.)       000323
      NRL=NRL-2                                                         000324
      LML=LML-2                                                         000325
```

```
   80 RETURN                                                              000326
      END                                                                 000327
C                                                                         000328
C                                                                         000329
      SUBROUTINE MFN00A                                                   000330
C                                                                         000331
C     CHANGE PAGE AND PRINT PAGE HEADER.                                  000332
C                                                                         000333
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG  000334
     1H,DAT,TIM,LHT,H(8)                                                  000335
      IF (LCH.LE.0) RETURN                                                000336
      IF (NRP.LT.LMP) GO TO 20                                            000337
      LV1=0                                                               000338
      LV2=0                                                               000339
      WRITE (LCH,10)                                                      000340
   10 FORMAT (//27H ( LIMIT OF PAGES REACHED ))                           000341
   20 NRP=NRP+1                                                           000342
      NRL=MXL-5                                                           000343
      LML=LML-5                                                           000344
      WRITE (LCH,30) DAT,TIM,NRP                                          000345
   30 FORMAT (1H1/7H DATE :,A10,19X,6HTIME :,A10,20X,6HPAGE :,I3/         000346
     1 55H MINIMIZATION WITH NONLINEAR CONSTRAINTS (MFNC PACKAGE),17X,    000347
     2 9H(V:82.05))                                                       000348
      IF (LHT.LE.0) GO TO 50                                              000349
      WRITE (LCH,40) (H(J),J=1,LHT)                                       000350
   40 FORMAT (1H0,8A10)                                                   000351
      NRL=NRL-2                                                           000352
      LML=LML-2                                                           000353
   50 WRITE (LCH,60)                                                      000354
   60 FORMAT (/1X)                                                        000355
      RETURN                                                              000356
      END                                                                 000357
C                                                                         000358
C                                                                         000359
      SUBROUTINE MMXPSZ (L)                                               000360
C                                                                         000361
C     DEFINE THE PAGE SIZE (I.E. THE NUMBER OF LINES PER PAGE).           000362
C                                                                         000363
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG  000364
     1H,DAT,TIM,LHT,H(8)                                                  000365
      DATA LL/65/                                                         000366
      IF (L.GT.0) LL=MAX0(25,L)                                           000367
      IF (L.EQ.0) LL=0                                                    000368
      MXL=LL                                                              000369
      RETURN                                                              000370
      END                                                                 000371
C                                                                         000372
C                                                                         000373
      SUBROUTINE MMXPLM (L)                                               000374
C                                                                         000375
C     DEFINE THE LIMIT OF PRINTED PAGES.                                  000376
C                                                                         000377
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG  000378
     1H,DAT,TIM,LHT,H(8)                                                  000379
      DATA LL/10/                                                         000380
      IF (L.GT.0) LL=MIN0(50,L)                                           000381
      LMP=LL                                                              000382
      RETURN                                                              000383
      END                                                                 000384
C                                                                         000385
C                                                                         000386
      SUBROUTINE MMXLLM (L)                                               000387
C                                                                         000388
C     DEFINE THE LIMIT OF PRINTED LINES.                                  000389
C                                                                         000390
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      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG  000391
     1H,DAT,TIM,LHT,H(8)                                                  000392
      DATA LL/750/                                                        000393
      IF (L.GT.0) LL=L                                                    000394
      LML=LL                                                              000395
      RETURN                                                              000396
      END                                                                 000397
C                                                                         000398
C                                                                         000399
      SUBROUTINE MMXHDR (L,T)                                             000400
C                                                                         000401
C     DEFINE THE HEADER LINE.                                             000402
C                                                                         000403
      DIMENSION T(1)                                                      000404
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG  000405
     1H,DAT,TIM,LHT,H(8)                                                  000406
      DATA LL/0/                                                          000407
      IF (L.GE.0) LL=MIN0(8,L)                                            000408
      LHT=LL                                                              000409
      IF (L.LE.0) RETURN                                                  000410
      DO 10 I=1,LL                                                        000411
      H(I)=T(I)                                                           000412
   10 CONTINUE                                                            000413
      RETURN                                                              000414
      END                                                                 000415
C                                                                         000416
C                                                                         000417
      SUBROUTINE MMXGLM (K,L)                                             000418
C                                                                         000419
C     DEFINE THE SIZE OF PRINTED JACOBIAN.                                000420
C                                                                         000421
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG  000422
     1H,DAT,TIM,LHT,H(8)                                                  000423
      DATA KK/25/,LL/10/                                                  000424
      IF (K.GT.0) KK=K                                                    000425
      IF (L.GT.0) LL=L                                                    000426
      LMF=KK                                                              000427
      LMV=LL                                                              000428
      RETURN                                                              000429
      END                                                                 000430
C                                                                         000431
C                                                                         000432
      SUBROUTINE MMXGVL (L)                                               000433
C                                                                         000434
C     DEFINE THE NUMBER OF JACOBIAN COLUMNS PRINTED IN ONE LINE.          000435
C                                                                         000436
      COMMON /MMX000/ LCH,LV1,LV2,LG1,LG2,LMF,LMV,NRP,NRL,MXL,LMP,LML,LG  000437
     1H,DAT,TIM,LHT,H(8)                                                  000438
      DATA LL/10/                                                         000439
      IF (L.GT.0) LL=MAX0(MIN0(10,L),5)                                   000440
      LGH=LL                                                              000441
      RETURN                                                              000442
      END                                                                 000443
C                                                                         000444
C                                                                         000445
      SUBROUTINE VF02A  (N,M,MEQ,X,F,G,C,CN,LCN,MAXFUN,ACC,IPRINT,INF,W,  000446
     1LW)                                                                 000447
      REAL X(N),F,G(N),C(M),CN(LCN,M),ACC,W(LW)                           000448
C                                                                         000449
C     N       IS THE NUMBER OF VARIABLES                                  000450
C     M       IS THE TOTAL NUMBER OF CONSTRAINTS                          000451
C     MEQ     IS THE NUMBER OF EQUALITY CONSTRAINTS                       000452
C     X       IS THE VECTOR OF VARIABLES                                  000453
C             IT MUST BE SET BY THE USER BEFORE THE INITIAL CALL AND LEFT  000454
C             UNCHANGED THEREAFTER.                                       000455
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C     F       IS THE VALUE OF THE OBJECTIVE FUNCTION                          000456
C     G       IS THE GRADIENT OF THE OBJECTIVE FUNCTION                       000457
C     C       IS THE VECTOR OF CONSTRAINT FUNCTIONS                           000458
C     CN      IS THE MATRIX OF CONSTRAINT NORMALS                             000459
C     LCN     IS THE FIRST DIMENSION OF CN                                    000460
C                                                                             000461
C     F,G,C,CN MUST ALL BE SET BY THE USER BEFORE EACH CALL.                  000462
C                                                                             000463
C     MAXFUN BOUNDS THE NUMBER OF CALLS OF VF02A/AD                           000464
C     ACC     CONTROLS THE FINAL ACCURACY - THE CALCULATION ENDS WHEN THE     000465
C             OBJECTIVE FUNCTION PLUS SUITABLY WEIGHTED MULTIPLES OF THE      000466
C             CONSTRAINT FUNCTIONS ARE PREDICTED TO DIFFER FROM THEIR         000467
C             OPTIMAL VALUES BY AT MOST ACC                                   000468
C     IPRINT CONTROLS THE AMOUNT OF PRINTING                                  000469
C          .LT.0  NO PRINTING                                                 000470
C          .EQ.0  DIAGNOSTICS ONLY                                            000471
C          .GT.0  X, F AND C AT START OF EVERY IPRINT ITERATIONS              000472
C     INF CONTROLS THE CALCULATION                                            000473
C          =-1 ON INITIAL CALL                                                000474
C          =0 DURING CALCULATION                                              000475
C          =1 WHEN REQUIRED ACCURACY IS ACHIEVED                              000476
C          =2 WHEN VF02A/AD IS CALLED MAXFUN TIMES                            000477
C          =3 WHEN A LINE SEARCH REQUIRES 5 CALLS OF VF02A/AD                 000478
C          =4 WHEN AN UPHILL SEARCH DIRECTION IS CALCULATED                   000479
C          =5 WHEN NO FEASIBLE POINT IS FOUND BY VF02C/CD                     000480
C          =6 WHEN AN ARTIFICIAL BOUND RESTRICTS VF02C/CD                     000481
C          =7 WHEN LW IS TOO SMALL                                            000482
C          =8 WHEN N,M OR MEQ HAS A SILLY VALUE                               000483
C     W       IS A WORKSPACE ARRAY OF LENGTH LW                               000484
C     LW      IS THE LENGTH OF ARRAY W                                        000485
C                                                                             000486
      COMMON /VF02D/ VLARGE,LP                                                000487
      DATA VLARGE/1.0E6/,LP/0/                                                000488
C                                                                             000489
C     LP IS THE UNIT NUMBER FOR PRINTING MESSAGES                             000490
C                                                                             000491
      IW=5*N*N+23*N+18+4*M+MAX0(M,3*N+3)                                      000492
      IF (IW.LE.LW) GO TO 20                                                  000493
      IF (IPRINT.GE.0.AND.LP.GT.0) WRITE (LP,10) IW                           000494
   10 FORMAT (49H ERROR RETURN FROM VF02A/AD BECAUSE W SHOULD HAVE,           000495
     1 16H LENGTH AT LEAST,I8)                                                000496
      INF=7                                                                   000497
      GO TO 50                                                                000498
   20 IF (N.GT.0.AND.M.GE.MEQ.AND.MEQ.GE.0) GO TO 40                          000499
      IF (IPRINT.GE.0.AND.LP.GT.0) WRITE (LP,30) N,M,MEQ                      000500
   30 FORMAT (38H ERROR RETURN FROM VF02A/AD BECAUSE N=,I6,5H    M=,I6,       000501
     1 7H   MEQ=,I6)                                                          000502
      INF=8                                                                   000503
      GO TO 50                                                                000504
   40 IVLAM=1                                                                 000505
      IVMU=IVLAM+M                                                            000506
      IB=IVMU+M                                                               000507
      IDELTA=IB+(N+1)*(N+1)                                                   000508
      IGLAG=IDELTA+N+1                                                        000509
      IGLAGA=IGLAG+N                                                          000510
      IXA=IGLAGA+N                                                            000511
      CALL VF02B  (N,M,MEQ,X,F,G,C,CN,LCN,MAXFUN,ACC,IPRINT,INF,W,LW,N+1      000512
     1,W(IVLAM),W(IVMU),W(IB),W(IDELTA),W(IGLAG),W(IGLAGA),W(IGLAGA),W(I      000513
     2GLAGA),W(IXA),W(IXA))                                                   000514
   50 RETURN                                                                  000515
      END                                                                     000516
C                                                                             000517
C                                                                             000518
      SUBROUTINE VF02B  (N,M,MEQ,X,F,G,C,CN,LCN,MAXFUN,ACC,IPRINT,INF,W,      000519
     1LW,NP,VLAM,VMU,B,DELTA,GLAG,GLAGA,GAMMA,ETA,XA,BDELTA)                  000520
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      REAL X(N),F,G(N),C(MD),CN(LCN,MD),ACC,W(LW)                        000521
      DIMENSION VLAM(MD),VMU(MD),DELTA(NP),GLAG(N),GLAGA(N),XA(N),GAMMA(N) 000522
     1,BDELTA(N),ETA(N),B(NP,NP)                                         000523
C                                                                        000524
C     VLAM   IS THE VECTOR OF LAGRANGE MULTIPLIERS                       000525
C     VMU    HOLDS THE PARAMETERS FOR THE LINE SEARCH FUNCTION           000526
C     DELTA  IS THE SEARCH DIRECTION TIMES THE STEP-LENGTH               000527
C     GLAG   IS THE GRADIENT OF THE LAGRANGIAN FUNCTION                  000528
C     GLAGA  IS THIS GRADIENT AT THE START OF AN ITERATION               000529
C     XA     IS THE VECTOR OF VARIABLES AT THE START OF AN ITERATION     000530
C     GAMMA  IS THE CHANGE IN GRADIENT OF THE LAGRANGIAN FUNCTION        000531
C     BDELTA IS B TIMES DELTA                                            000532
C     ETA    REPLACES GAMMA IN THE B-F-G-S FORMULA FOR REVISING B        000533
C     B      IS THE VARIABLE METRIC MATRIX                               000534
C                                                                        000535
C     (GLAGA,GAMMA,ETA) AND (XA,BDELTA) ARE ESSENTIALLY EQUIVALENT,      000536
C          SINCE THEY ARE ASSOCIATED WITH THE SAME PART OF ARRAY W.      000537
C                                                                        000538
      COMMON /VF02D/ VLARGE,LP                                           000539
      COMMON /VF02E/  FLS,SUM,FLSA,ALPHA,DFLSA,SPGDEL,NF,ITER,ITERP,NFIN 000540
     1IT,MACT                                                            000541
      DATA XZERO,XONE,XHALF,XZONE,XZTWO/0.0E0,1.0E0,0.5E0,0.1E0,0.2E0/   000542
C                                                                        000543
C       SET SOME PARAMETERS FOR THE CALCULATION                          000544
C         NFLINE  CONTROLS THE ERROR RETURN FROM THE LINE SEARCH         000545
C         PARACC  IS THE SLOPE OF THE ARMIJO CHORD                       000546
C         PARSTP  LIMITS THE REDUCTION IN THE LINE SEARCH STEP-LENGTH    000547
C         PARB    BOUNDS THE REDUCTION IN THE DETERMINANT OF B           000548
C                                                                        000549
      NFLINE=5                                                           000550
      PARACC=XZONE                                                       000551
      PARSTP=XZONE                                                       000552
      PARB=XZTWO                                                         000553
      IF (INF.EQ.0) GO TO 270                                            000554
C                                                                        000555
C       SET INITIAL VALUES OF SOME VARIABLES                             000556
C         NF     IS THE NUMBER OF CALLS OF VF02A/AD                      000557
C         ITER   IS THE ITERATION NUMBER                                 000558
C         ITERP  IS THE NEXT ITERATION ON WHICH PRINTING OCCURS          000559
C                                                                        000560
      NF=1                                                               000561
      ITER=0                                                             000562
      ITERP=MIN0(1,IPRINT)                                               000563
C                                                                        000564
C       SET THE INITIAL ELEMENTS OF B AND VMU                            000565
C                                                                        000566
      DO 20 I=1,N                                                        000567
      DO 10 J=1,N                                                        000568
      B(I,J)=XZERO                                                       000569
   10 CONTINUE                                                           000570
      B(I,I)=XONE                                                        000571
   20 CONTINUE                                                           000572
      IF (M.EQ.0) GO TO 40                                               000573
      DO 30 K=1,M                                                        000574
      VMU(K)=XZERO                                                       000575
   30 CONTINUE                                                           000576
C                                                                        000577
C       START THE ITERATION BY PROVIDING PRINTING                        000578
C                                                                        000579
   40 ITER=ITER+1                                                        000580
      IF (ITER.NE.ITERP) GO TO 100                                       000581
   50 IF (LP.GT.0) WRITE (LP,60) ITER,NF                                 000582
   60 FORMAT (/5X,12HITERATIONS =,I5,5X,19HCALLS OF VF02A/AD =,I5)       000583
      IF (LP.GT.0) WRITE (LP,70) (X(I),I=1,N)                            000584
   70 FORMAT (/5H  X =,5E20.10/(5X,5E20.10))                             000585
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      IF (LP.GT.0) WRITE (LP,80) F                                          000586
   80 FORMAT (/5H  F =,E20.10)                                              000587
      IF (M.GT.0.AND.LP.GT.0) WRITE (LP,90) (C(K),K=1,M)                    000588
   90 FORMAT (/5H  C =,5E20.10/(5X,5E20.10))                                000589
      IF (ITER.NE.ITERP) GO TO 440                                          000590
      ITERP=ITER+IPRINT                                                     000591
C                                                                           000592
C       CALL THE QUADRATIC PROGRAMMING SUBROUTINE                           000593
C                                                                           000594
  100 IDELTA=1+2*M+NP*NP                                                    000595
C                                                                           000596
C       THIS CHOICE OF IDELTA MEANS THAT ARRAYS DELTA OF THIS SUBROUTINE    000597
C       AND OF VF02C/CD ARE DYNAMICALLY EQUIVALENT                          000598
C                                                                           000599
      LDELTA=4*NP+MAX0(M,3*NP)                                              000600
      IGM=IDELTA+LDELTA                                                     000601
      ICM=IGM+NP                                                            000602
      IBDL=ICM+M                                                            000603
      IBDU=IBDL+N+1                                                         000604
      IH=IBDU+NP                                                            000605
      ILT=IH+NP*NP*4                                                        000606
      LLT=M+NP*6                                                            000607
      CALL VF02C  (N,M,MEQ,G,C,CN,LCN,IPRINT,INF,NP,2*NP,VLAM,B,W(IDELTA    000608
     1),LDELTA,W(IGM),W(ICM),W(IBDL),W(IBDU),W(IH),W(ILT),LLT)              000609
      IF (INF.LE.1) GO TO 120                                               000610
      IF (IPRINT.LT.0) GO TO 460                                            000611
      IF (LP.GT.0) WRITE (LP,110)                                           000612
  110 FORMAT (/5X,24HERROR CONDITION FOUND IN,                              000613
     1 34H QUADRATIC PROGRAMMING CALCULATION)                               000614
      GO TO 430                                                             000615
C                                                                           000616
C       CALCULATE THE GRADIENT OF THE LAGRANGIAN FUNCTION                   000617
C       NFINIT IS THE VALUE OF NF AT THE START OF AN ITERATION              000618
C                                                                           000619
  120 NFINIT=NF                                                             000620
  130 DO 140 I=1,N                                                          000621
      GLAG(I)=G(I)                                                          000622
  140 CONTINUE                                                              000623
      IF (M.EQ.0) GO TO 170                                                 000624
      DO 160 K=1,M                                                          000625
      IF (VLAM(K).EQ.XZERO) GO TO 160                                       000626
      DO 150 I=1,N                                                          000627
      GLAG(I)=GLAG(I)-CN(I,K)*VLAM(K)                                       000628
  150 CONTINUE                                                              000629
  160 CONTINUE                                                              000630
  170 IF (NF.NE.NFINIT) GO TO 350                                           000631
C                                                                           000632
C       STORE THE ELEMENTS OF GLAG AND X                                    000633
C       SET SPGDEL TO THE SCALAR PRODUCT OF G AND DELTA                     000634
C                                                                           000635
      SPGDEL=XZERO                                                          000636
      DO 180 I=1,N                                                          000637
      SPGDEL=SPGDEL+G(I)*DELTA(I)                                           000638
      GLAGA(I)=GLAG(I)                                                      000639
      XA(I)=X(I)                                                            000640
  180 CONTINUE                                                              000641
C                                                                           000642
C       REVISE THE VECTOR VMU                                               000643
C       TEST FOR CONVERGENCE                                                000644
C                                                                           000645
      SUM=ABS(SPGDEL)                                                       000646
      IF (M.LE.0) GO TO 200                                                 000647
      DO 190 K=1,M                                                          000648
      AUX=ABS(VLAM(K))                                                      000649
      VMU(K)=AMAX1(AUX,XHALF*(AUX+VMU(K)))                                  000650
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      SUM=SUM+ABS(VLAM(K)*C(K))                                          000651
  190 CONTINUE                                                           000652
  200 IF (SUM.GT.ACC) GO TO 320                                          000653
      INF=1                                                              000654
      GO TO 430                                                          000655
C                                                                        000656
C       SET THE INITIAL CONDITIONS FOR THE LINE SEARCH                   000657
C         FLSA   IS THE INITIAL VALUE OF THE LINE SEARCH FUNCTION        000658
C         DFLSA  IS USUALLY ITS FIRST DERIVATIVE                         000659
C         ALPHA  IS THE NEXT REDUCTION IN THE STEP-LENGTH                000660
C                                                                        000661
  210 FLSA=FLS                                                           000662
      DFLSA=SPGDEL-DELTA(N+1)*SUM                                        000663
      IF (DFLSA.GE.XZERO) GO TO 410                                      000664
      ALPHA=XONE                                                         000665
C                                                                        000666
C       MULTIPLY DELTA BY ALPHA AND CALCULATE THE NEW X                  000667
C                                                                        000668
  220 DO 230 I=1,N                                                       000669
      DELTA(I)=ALPHA*DELTA(I)                                            000670
      X(I)=XA(I)+DELTA(I)                                                000671
  230 CONTINUE                                                           000672
      DFLSA=ALPHA*DFLSA                                                  000673
C                                                                        000674
C       TEST NF AGAINST MAXFUN AND RETURN FOR MORE VALUES OF F,G,C,CN    000675
C                                                                        000676
      IF (NF.LT.MAXFUN) GO TO 260                                        000677
  240 DO 250 I=1,N                                                       000678
      X(I)=XA(I)                                                         000679
  250 CONTINUE                                                           000680
      IF (NF.EQ.NFINIT) GO TO 280                                        000681
  260 NF=NF+1                                                            000682
      GO TO 460                                                          000683
  270 IF (NF.LE.MAXFUN) GO TO 300                                        000684
  280 INF=2                                                              000685
      IF (IPRINT.LT.0) GO TO 460                                         000686
      IF (LP.GT.0) WRITE (LP,290) NF                                     000687
  290 FORMAT (/5X,33HERROR RETURN FROM VF02A/AD DUE TO,I5,               000688
     1 21H FUNCTION EVALUATIONS)                                         000689
      GO TO 430                                                          000690
C                                                                        000691
C       TEST FOR ERROR RETURN FROM LINE SEARCH                           000692
C                                                                        000693
  300 IF (NF.LE.NFINIT+NFLINE) GO TO 320                                 000694
      INF=3                                                              000695
      IF (IPRINT.LT.0) GO TO 460                                         000696
      IF (LP.GT.0) WRITE (LP,310) NFLINE                                 000697
  310 FORMAT (/5X,49HRETURN FROM VF02A/AD BECAUSE LINE SEARCH REQUIRES,  000698
     1 10H MORE THAN,I3,6H STEPS)                                        000699
      GO TO 430                                                          000700
C                                                                        000701
C       SET SUM TO THE WEIGHTED SUM OF INFEASIBILITIES                   000702
C       SET FLS TO THE LINE SEARCH OBJECTIVE FUNCTION                    000703
C                                                                        000704
  320 SUM=XZERO                                                          000705
      IF (M.LE.0) GO TO 340                                              000706
      DO 330 K=1,M                                                       000707
      AUX=C(K)                                                           000708
      IF (K.GT.MEQ) AUX=XZERO                                            000709
      SUM=SUM+VMU(K)*AMAX1(AUX,-C(K))                                    000710
  330 CONTINUE                                                           000711
  340 FLS=F+SUM                                                          000712
      IF (NF.EQ.NFINIT) GO TO 210                                        000713
C                                                                        000714
C       CALCULATE THE GRADIENT OF THE LAGRANGIAN FUNCTION                000715
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C       IF THE LINE SEARCH IS COMPLETE                                  000716
C                                                                       000717
      AUX=FLS-FLSA                                                      000718
      IF (AUX.LE.PARACC*DFLSA) GO TO 130                                000719
      IF (NF.GE.NFINIT+NFLINE) GO TO 240                                000720
C                                                                       000721
C       CALCULATE THE NEXT REDUCTION IN THE STEP-LENGTH                 000722
C                                                                       000723
      ALPHA=AMAX1(PARSTP,XHALF*DFLSA/(DFLSA-AUX))                       000724
      GO TO 220                                                         000725
C                                                                       000726
C       CALCULATE GAMMA AND BDELTA IN ORDER TO REVISE B                 000727
C       SET DG TO THE SCALAR PRODUCT OF DELTA AND GAMMA                 000728
C       SET DBD TO THE SCALAR PRODUCT OF DELTA AND BDELTA               000729
C                                                                       000730
  350 DG=XZERO                                                          000731
      DBD=XZERO                                                         000732
      DO 370 I=1,N                                                      000733
      GAMMA(I)=GLAG(I)-GLAGA(I)                                         000734
      BDELTA(I)=XZERO                                                   000735
      DO 360 J=1,N                                                      000736
      BDELTA(I)=BDELTA(I)+B(I,J)*DELTA(J)                               000737
  360 CONTINUE                                                          000738
      DG=DG+DELTA(I)*GAMMA(I)                                           000739
      DBD=DBD+DELTA(I)*BDELTA(I)                                        000740
  370 CONTINUE                                                          000741
C                                                                       000742
C       CALCULATE THE VECTOR ETA FOR THE B-F-G-S FORMULA                000743
C       REPLACE DG BY THE SCALAR PRODUCT OF DELTA AND ETA               000744
C                                                                       000745
      AUX=PARB*DBD                                                      000746
      IF (DG.GE.AUX) GO TO 390                                          000747
      THETA=(DBD-AUX)/(DBD-DG)                                          000748
      THCOMP=XONE-THETA                                                 000749
      DO 380 I=1,N                                                      000750
      ETA(I)=THETA*GAMMA(I)+THCOMP*BDELTA(I)                            000751
  380 CONTINUE                                                          000752
      DG=AUX                                                            000753
C                                                                       000754
C       REVISE THE MATRIX B                                             000755
C                                                                       000756
  390 DO 400 I=1,N                                                      000757
      AUX=BDELTA(I)/DBD                                                 000758
      AUXA=ETA(I)/DG                                                    000759
      DO 400 J=I,N                                                      000760
      B(I,J)=B(I,J)-AUX*BDELTA(J)+AUXA*ETA(J)                           000761
  400 B(J,I)=B(I,J)                                                     000762
      GO TO 40                                                          000763
C                                                                       000764
C       TEST IF ANY PRINTING IS REQUIRED BEFORE THE RETURN              000765
C                                                                       000766
  410 INF=4                                                             000767
      IF (IPRINT.LT.0) GO TO 460                                        000768
      IF (LP.GT.0) WRITE (LP,420)                                       000769
  420 FORMAT (/5X,43HERROR RETURN FROM VF02A/AD DUE TO AN UPHILL,       000770
     1 17H SEARCH DIRECTION/5X,30HUSUALLY THIS IS DUE TO LOSS OF,       000771
     2 50H ACCURACY IN THE QUADRATIC PROGRAMMING CALCULATION)           000772
  430 IF (IPRINT.LE.0) GO TO 460                                        000773
      IF (ITERP.NE.ITER+IPRINT) GO TO 50                                000774
  440 IF (LP.GT.0) WRITE (LP,450)                                       000775
  450 FORMAT (/5X,44HTHE PRINTING OF THE LAST ITERATION GIVES THE,      000776
     1 48H VALUES THAT ARE RETURNED BY SUBROUTINE VF02A/AD)             000777
  460 RETURN                                                            000778
      END                                                               000779
C                                                                       000780
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C                                                                       000781
      SUBROUTINE VF02C  (N,M,MEQ,G,C,CN,LCN,IPRINT,INF,NP,NPP,VLAM,B,DEL  000782
     1TA,LDELTA,GM,CM,BDL,BDU,H,LT,LLT)                                 000783
      DIMENSION G(N),C(M),CN(LCN,M),VLAM(M),B(NP,NP),DELTA(LDELTA),GM(NP  000784
     1),CM(M),BDL(NP),BDU(NP),H(NPP,NPP)                                000785
      INTEGER LT(LLT)                                                   000786
      COMMON /VF02D/ VLARGE,LP                                          000787
      COMMON /VF02E/ DUMMY(6),IDUMMY(4),MACT                            000788
      DATA XZERO,XONE,XONEM6,XZNINE/0.0E0,1.0E0,1.0E-6,0.9E0/           000789
C                                                                       000790
C       NP = N + 1                                                      000791
C       NPP = NP + NP                                                   000792
C                                                                       000793
C       GM IS SET TO MINUS THE VECTOR G                                 000794
C       CM IS SET TO MINUS THE VECTOR C                                 000795
C       BDL AND BDU GIVE LOWER AND UPPER BOUNDS ON DELTA                000796
C       H AND LT ARE USED AS WORKING SPACE BY VE02A/AD                  000797
C                                                                       000798
C       SET SOME PARAMETERS THAT ARE USED BY VF02C/CD AND VE02A/AD      000799
C         VLARGE IS ASSUMED TO BE A LARGE NUMBER                        000800
C         VSMALL IS ASSUMED TO BE A SMALL POSITIVE NUMBER               000801
C         FEASP IS A SCALING FACTOR THAT IS USED TO ACHIEVE FEASIBILITY  000802
C                                                                       000803
      MODE=1                                                            000804
      VSMALL=XONEM6                                                     000805
      FEASP=XZNINE                                                      000806
      MTOTAL=M+NPP                                                      000807
      NSIX=6*NP                                                         000808
      IF (INF.GE.0) GO TO 50                                            000809
C                                                                       000810
C       SET INITIAL VALUES OF SOME VARIABLES                            000811
C                                                                       000812
      INF=0                                                             000813
      MACT=MEQ+1                                                        000814
C                                                                       000815
C       SET THE INITIAL ELEMENTS OF BDL, BDU, DELTA AND LT              000816
C                                                                       000817
      DO 10 I=1,N                                                       000818
      BDL(I)=-VLARGE                                                    000819
      BDU(I)=VLARGE                                                     000820
      DELTA(I)=XZERO                                                    000821
   10 CONTINUE                                                          000822
      BDL(NP)=XZERO                                                     000823
      DELTA(NP)=XONE                                                    000824
      IF (MEQ.LE.0) GO TO 30                                            000825
      DO 20 K=1,MEQ                                                     000826
      LT(K)=K+NPP                                                       000827
   20 CONTINUE                                                          000828
   30 LT(MACT)=NPP                                                      000829
C                                                                       000830
C       EXTEND GM AND B BECAUSE OF THE EXTRA VARIABLE THAT IS           000831
C       INTRODUCED TO ALLOW FOR INFEASIBILITY                           000832
C                                                                       000833
      GM(NP)=VLARGE                                                     000834
      DO 40 I=1,NP                                                      000835
      B(I,NP)=XZERO                                                     000836
      B(NP,I)=XZERO                                                     000837
   40 CONTINUE                                                          000838
C                                                                       000839
C       SET THE ELEMENTS OF GM, CM AND CN(NP,*)                         000840
C                                                                       000841
   50 DO 60 I=1,N                                                       000842
      GM(I)=-G(I)                                                       000843
   60 CONTINUE                                                          000844
      IF (M.LE.0) GO TO 90                                              000845
```

```
      DO 80 K=1,M                                                       000846
      IF (K.LE.MEQ) GO TO 70                                            000847
      IF (C(K).LT.XZERO) GO TO 70                                       000848
      CM(K)=-C(K)                                                       000849
      CN(NP,K)=XZERO                                                    000850
      GO TO 80                                                          000851
   70 CM(K)=XZERO                                                       000852
      CN(NP,K)=C(K)                                                     000853
   80 VLAM(K)=XZERO                                                     000854
C                                                                       000855
C       CALL SUBROUTINE VE02A/AD                                        000856
C                                                                       000857
   90 BDU(NP)=XONE                                                      000858
      IFLAG=-1                                                          000859
  100 CALL VE02A  (NP,MTOTAL,B,NP,GM,CN,LCN,CM,BDL,BDU,DELTA,MACT,MEQ,H, 000860
     1NPP,LT,MODE)                                                      000861
C                                                                       000862
C       CHECK WHETHER THE REQUIRED FEASIBILITY CONDITIONS HOLD          000863
C                                                                       000864
      IF (DELTA(NP).LE.VSMALL) GO TO 150                                000865
      DO 110 J=1,MACT                                                   000866
      IF (LT(J).GT.NPP) GO TO 110                                       000867
      IF (LT(J).LT.NPP) GO TO 170                                       000868
      IFLAG=1                                                           000869
  110 CONTINUE                                                          000870
      IF (IFLAG.GE.1) GO TO 120                                         000871
      IF (IFLAG.GE.0) GO TO 150                                         000872
      BDU(NP)=FEASP*DELTA(NP)                                           000873
      IFLAG=0                                                           000874
      GO TO 100                                                         000875
C                                                                       000876
C       CALCULATE THE LAGRANGE MULTIPLIERS                              000877
C                                                                       000878
  120 DO 140 J=1,MACT                                                   000879
      K=LT(J)-NPP                                                       000880
      NPJ=NP+J                                                          000881
      IF (K.LE.0) GO TO 140                                             000882
      DO 130 I=1,N                                                      000883
      NSIXI=NSIX+I                                                      000884
      VLAM(K)=VLAM(K)+H(NPJ,I)*DELTA(NSIXI)                             000885
  130 CONTINUE                                                          000886
  140 CONTINUE                                                          000887
      GO TO 190                                                         000888
C                                                                       000889
C       RETURN FROM THE SUBROUTINE                                      000890
C                                                                       000891
  150 INF=5                                                             000892
      IF (IPRINT.LT.0) GO TO 190                                        000893
      IF (LP.GT.0) WRITE (LP,160)                                       000894
  160 FORMAT (/5X,45HTHE GIVEN CONSTRAINTS SEEM TO BE INCONSISTENT)     000895
      GO TO 190                                                         000896
  170 INF=6                                                             000897
      IF (IPRINT.LT.0) GO TO 190                                        000898
      IF (LP.GT.0) WRITE (LP,180) VLARGE                                000899
  180 FORMAT (/5X,46HVE02A FINDS THAT AN ARTIFICIAL BOUND IS ACTIVE     000900
     1 /5X,45HTHE PREDICTED CHANGE IN THE VARIABLES EXCEEDS,1PE12.4)    000901
  190 RETURN                                                            000902
      END                                                               000903
C                                                                       000904
C                                                                       000905
      SUBROUTINE VE02A  (N,M,A,IA,B,C,IC,D,BDL,BDU,X,K,KE,H,IH,LT,MODE) 000906
      DIMENSION A(IA,1), B(1), C(IC,1), D(1), BDL(1), BDU(1), X(1), H(IH 000907
     1,1), LT(1)                                                        000908
      LOGICAL RETEST,PASSIV,POSTIV                                      000909
      COMMON /VE02X/ LPR                                                000910
```

```
      DATA LPR/0/                                                       000911
      DATA XZERO,XONE,XONE75/0.0E0,1.0E0,1.0E75/                        000912
      RETEST=.FALSE.                                                    000913
      NN=N+N                                                            000914
      N3=NN+N                                                           000915
      N4=NN+NN                                                          000916
      N5=N4+N                                                           000917
      N6=N5+N                                                           000918
      NNM=NN+M+1                                                        000919
      IF (MODE.GE.3) GO TO 30                                           000920
C                                                                       000921
C       CALL FEASIBLE VERTEX ROUTINE                                    000922
C                                                                       000923
   10 CALL LA02A  (N,M,C,IC,D,BDL,BDU,X,K,KE,H,IH,LT)                   000924
      IF (K.EQ.0) RETURN                                                000925
      IF (MODE.EQ.2.AND..NOT.RETEST) GO TO 60                           000926
C                                                                       000927
C       INITIAL OPERATORS H=0 AND CSTAR=C(-1)                           000928
C                                                                       000929
      DO 20 I=1,N                                                       000930
      DO 20 J=1,N                                                       000931
      H(N+I,J)=H(I,J)                                                   000932
   20 H(I,J)=XZERO                                                      000933
      GO TO 260                                                         000934
C                                                                       000935
   30 DO 40 I=1,M                                                       000936
      LT(NN+I)=1                                                        000937
   40 CONTINUE                                                          000938
C                                                                       000939
C       CONSTRAINTS INDEXED AS  -1=EQUALITY, 0=ACTIVE, 1=INACTIVE       000940
C                                                                       000941
      IF (K.EQ.0) GO TO 60                                              000942
      DO 50 I=1,K                                                       000943
      J=0                                                               000944
      IF (I.LE.KE) J=-1                                                 000945
      LT(NN+LT(I))=J                                                    000946
   50 CONTINUE                                                          000947
   60 IF (MODE.EQ.5.AND..NOT.RETEST) GO TO 150                          000948
C                                                                       000949
C       SET UP MATRIX AND RHS OF EQUATIONS GOVERNING EQUALITY PROBLEM   000950
C                                                                       000951
      DO 70 I=1,N                                                       000952
      X(N+I)=B(I)                                                       000953
      DO 70 J=1,N                                                       000954
   70 H(I,J)=A(I,J)                                                     000955
      IF ((MODE.EQ.2.OR.MODE.EQ.3).AND..NOT.RETEST) GO TO 820           000956
      IF (K.EQ.0) GO TO 140                                             000957
      DO 130 I=1,K                                                      000958
      LI=LT(I)                                                          000959
      IF (LI.GT.NN) GO TO 100                                           000960
      DO 80 J=1,N                                                       000961
      H(J,N+I)=XZERO                                                    000962
      H(N+I,J)=XZERO                                                    000963
   80 CONTINUE                                                          000964
      IF (LI.GT.N) GO TO 90                                             000965
      H(N+I,LI)=XONE                                                    000966
      H(LI,N+I)=XONE                                                    000967
      X(NN+I)=BDL(LI)                                                   000968
      GO TO 120                                                         000969
   90 LI=LI-N                                                           000970
      H(N+I,LI)=-XONE                                                   000971
      H(LI,N+I)=-XONE                                                   000972
      X(NN+I)=-BDU(LI)                                                  000973
      GO TO 120                                                         000974
  100 LI=LI-NN                                                          000975
```

```
      DO 110 J=1,N                                                      000976
      H(N+I,J)=C(J,LI)                                                  000977
      H(J,N+I)=C(J,LI)                                                  000978
  110 CONTINUE                                                          000979
      X(NN+I)=D(LI)                                                     000980
  120 DO 130 J=1,K                                                      000981
  130 H(N+I,N+J)=XZERO                                                  000982
  140 NK=N+K                                                            000983
C                                                                       000984
C       INVERT MATRIX GIVING OPERATORS H AND CSTAR                      000985
C                                                                       000986
      CALL MB01C  (H,NK,IH,LT(NNM),X(N3+1))                             000987
      GO TO 200                                                         000988
C                                                                       000989
C       SET UP RHS ONLY                                                 000990
C                                                                       000991
  150 DO 160 I=1,N                                                      000992
      X(N+I)=B(I)                                                       000993
  160 CONTINUE                                                          000994
      DO 190 I=1,K                                                      000995
      LI=LT(I)                                                          000996
      IF (LI.GT.NN) GO TO 180                                           000997
      IF (LI.GT.N) GO TO 170                                            000998
      X(NN+I)=BDL(LI)                                                   000999
      GO TO 190                                                         001000
  170 X(NN+I)=-BDU(LI-N)                                                001001
      GO TO 190                                                         001002
  180 X(NN+I)=D(LI-NN)                                                  001003
  190 CONTINUE                                                          001004
C                                                                       001005
C       SOLVE FOR SOLUTION POINT X                                      001006
C                                                                       001007
      NK=N+K                                                            001008
  200 DO 210 I=1,N                                                      001009
      X(I)=FM02AS(NK,H(1,I),1,X(N+1),1)                                 001010
  210 CONTINUE                                                          001011
C                                                                       001012
C       CHECK FEASIBILITY, IF NOT EXIT TO 10                            001013
C                                                                       001014
      DO 250 I=1,M                                                      001015
      IF (LT(NN+I).LE.0) GO TO 250                                      001016
      IF (I.GT.N) GO TO 220                                             001017
      Z=X(I)-BDL(I)                                                     001018
      GO TO 240                                                         001019
  220 IF (I.GT.NN) GO TO 230                                            001020
      Z=BDU(I-N)-X(I-N)                                                 001021
      GO TO 240                                                         001022
  230 J=I-NN                                                            001023
      Z=FM02AS(N,C(1,J),1,X(1),1)-D(J)                                  001024
  240 IF (Z.LT.XZERO) GO TO 10                                          001025
  250 CONTINUE                                                          001026
  260 CONTINUE                                                          001027
C                                                                       001028
C       CALCULATE GRADIENT G AND LAGRANGE MULTIPLIERS -CSTAR.G,         001029
C       FIND LARGEST MULTIPLIER,  EXIT IF NOT POSITIVE                  001030
C                                                                       001031
      DO 270 I=1,N                                                      001032
      X(N6+I)=FM02AS(N,A(I,1),IA,X(1),1)-B(I)                           001033
  270 CONTINUE                                                          001034
      IF (K.EQ.0) RETURN                                                001035
      Z=-XONE75                                                         001036
      DO 280 I=1,K                                                      001037
      IF (LT(NN+LT(I)).EQ.-1) GO TO 280                                 001038
      ZZ=-FM02AS(N,H(N+I,1),IH,X(N6+1),1)                               001039
      IF (ZZ.LE.Z) GO TO 280                                            001040
```

```
      Z=ZZ                                                              001041
      II=I                                                              001042
  280 CONTINUE                                                          001043
      IF (Z.GT.XZERO) GO TO 310                                         001044
      IF (RETEST.OR.MODE.GE.4) GO TO 290                                001045
      RETEST=.TRUE.                                                     001046
      GO TO 60                                                          001047
  290 IF (Z.NE.XZERO) RETURN                                            001048
      IF (LPR.GT.0) WRITE (LPR,300)                                     001049
  300 FORMAT (43H0SOLUTION MAY BE A DEGENERATE LOCAL MINIMUMD           001050
      RETURN                                                            001051
C                                                                       001052
C       SET DIRECTION OF SEARCH AS CORRESPONDING ROW OF CSTAR           001053
C                                                                       001054
  310 DO 320 I=1,N                                                      001055
      X(NN+I)=H(N+II,I)                                                 001056
  320 CONTINUE                                                          001057
  330 DO 340 I=1,N                                                      001058
      X(N+I)=FM02AS(N,A(I,1),IA,X(NN+1),1)                              001059
  340 CONTINUE                                                          001060
      CAC=FM02AS(N,X(NN+1),1,X(N+1),1)                                  001061
      IF (CAC.GT.XZERO) GO TO 350                                       001062
      POSTIV=.FALSE.                                                    001063
      Y=XONE                                                            001064
      GO TO 360                                                         001065
  350 POSTIV=.TRUE.                                                     001066
      Y=Z/CAC                                                           001067
  360 DO 370 I=1,N                                                      001068
      X(N5+I)=X(NN+I)*Y                                                 001069
  370 CONTINUE                                                          001070
      PASSIV=.TRUE.                                                     001071
  380 ALPHA=XONE75                                                      001072
      NK=N+K                                                            001073
C                                                                       001074
C       LINEAR SEARCH ALONG DIRECTION OF SEARCH,  PASSIV INDICATES      001075
C       A CONSTRAINT HAS BEEN REMOVED TO GET SEARCH DIRECTION,          001076
C       POSTIV INDICATES POSITIVE CURVATURE ALONG THE DIRECTION         001077
C                                                                       001078
      DO 420 I=1,M                                                      001079
      IF (LT(NN+I).LE.0) GO TO 420                                      001080
      IF (I.GT.N) GO TO 390                                             001081
      IF (X(N5+I).GE.XZERO) GO TO 420                                   001082
      CC=(BDL(I)-X(I))/X(N5+I)                                          001083
      GO TO 410                                                         001084
  390 IF (I.GT.NN) GO TO 400                                            001085
      IF (X(N4+I).LE.XZERO) GO TO 420                                   001086
      CC=(BDU(I-N)-X(I-N))/X(N4+I)                                      001087
      GO TO 410                                                         001088
  400 J=I-NN                                                            001089
      ZZ=FM02AS(N,C(1,J),1,X(N5+1),1)                                   001090
      IF (ZZ.GE.XZERO) GO TO 420                                        001091
      CC=D(J)-FM02AS(N,C(1,J),1,X(1),1)                                 001092
      CC=CC/ZZ                                                          001093
  410 IF (CC.GE.ALPHA) GO TO 420                                        001094
      ALPHA=CC                                                          001095
      IAL=I                                                             001096
  420 CONTINUE                                                          001097
      IF (PASSIV) LT(NN+LT(II))=1                                       001098
C                                                                       001099
C       IF MINIMUM FOUND, GO TO 680                                     001100
C                                                                       001101
      IF (POSTIV.AND.ALPHA.GE.XONE) GO TO 680                           001102
C                                                                       001103
C       CALCULATE H.C AND CSTAR.C                                       001104
C                                                                       001105
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      DO 430 I=1,N                                                      001106
      X(I)=X(I)+ALPHA*X(N5+I)                                           001107
  430 CONTINUE                                                          001108
      ALPHA=ALPHA*Y                                                     001109
      J=1                                                               001110
      IF (K.EQ.N) J=N+1                                                 001111
      IF (IAL.GT.N) GO TO 450                                           001112
      DO 440 I=J,NK                                                     001113
      X(N3+I)=H(I,IAL)                                                  001114
  440 CONTINUE                                                          001115
      CHC=X(N3+IAL)                                                     001116
      GO TO 500                                                         001117
  450 IB=IAL-N                                                          001118
      IF (IB.GT.N) GO TO 470                                            001119
      DO 460 I=J,NK                                                     001120
      X(N3+I)=-H(I,IB)                                                  001121
  460 CONTINUE                                                          001122
      CHC=-X(N3+IB)                                                     001123
      GO TO 500                                                         001124
  470 IB=IB-N                                                           001125
      DO 480 I=1,N                                                      001126
      X(N5+I)=C(I,IB)                                                   001127
  480 CONTINUE                                                          001128
      DO 490 I=J,NK                                                     001129
      X(N3+I)=FM02AS(N,H(I,1),IH,X(N5+1),1)                             001130
  490 CONTINUE                                                          001131
      IF (K.NE.N) CHC=FM02AS(N,X(N5+1),1,X(N3+1),1)                     001132
  500 LT(NN+IAL)=0                                                      001133
      IF (K.EQ.N) GO TO 770                                             001134
      IF (PASSIV) GO TO 600                                             001135
C                                                                       001136
C       APPLY FORMULA FOR ADDING A CONSTRAINT                           001137
C                                                                       001138
  510 IF (K.EQ.0) GO TO 530                                             001139
      DO 520 I=1,K                                                      001140
      ALPHA=X(N4+I)/CHC                                                 001141
      NI=N+I                                                            001142
      DO 520 J=1,N                                                      001143
  520 H(NI,J)=H(NI,J)-ALPHA*X(N3+J)                                     001144
  530 K=K+1                                                             001145
      LT(K)=IAL                                                         001146
      DO 540 J=1,N                                                      001147
      H(N+K,J)=X(N3+J)/CHC                                              001148
  540 CONTINUE                                                          001149
      IF (K.LT.N) GO TO 560                                             001150
      DO 550 I=1,N                                                      001151
      DO 550 J=1,N                                                      001152
  550 H(I,J)=XZERO                                                      001153
      GO TO 580                                                         001154
  560 DO 570 I=1,N                                                      001155
      ALPHA=X(N3+I)/CHC                                                 001156
      DO 570 J=1,I                                                      001157
      H(I,J)=H(I,J)-ALPHA*X(N3+J)                                       001158
  570 H(J,I)=H(I,J)                                                     001159
  580 IF (.NOT.PASSIV) GO TO 650                                        001160
C                                                                       001161
C       REMOVAL OF A CONSTRAINT HAS BEEN DEFERRED,  SET UP AS IF        001162
C       THE CONSTRAINT IS BEING REMOVED FROM AUGMENTED BASIS            001163
C                                                                       001164
      DO 590 I=1,N                                                      001165
      X(N6+I)=FM02AS(N,A(I,1),IA,X(1),1)-B(I)                           001166
      X(NN+I)=H(N+II,I)                                                 001167
  590 CONTINUE                                                          001168
      Z=-FM02AS(N,X(N6+1),1,X(NN+1),1)                                  001169
      IF (Z.EQ.XZERO) GO TO 700                                         001170
```

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      GO TO 330                                                          001171
  600 CC=X(N4+II)                                                        001172
      Y=CHC*CAC+CC**2                                                    001173
      GHC=FM02AS(N,X(N6+1),1,X(N3+1),1)                                  001174
      IF (ALPHA*Y.LT.CHC*(Z-ALPHA*CAC)+GHC*CC) GO TO 510                 001175
C                                                                        001176
C       APPLY FORMULA FOR EXCHANGING NEW CONSTRAINT                      001177
C       WITH PASSIVE CONSTRAINT                                          001178
C                                                                        001179
      DO 610 I=1,K                                                       001180
      NI=N+I                                                             001181
      X(N5+I)=FM02AS(N,H(NI,1),IH,X(N+1),1)                              001182
  610 CONTINUE                                                           001183
      DO 620 I=1,N                                                       001184
      X(N+I)=(CHC*X(NN+I)-CC*X(N3+I))/Y                                  001185
      X(N6+I)=(CAC*X(N3+I)+CC*X(NN+I))/Y                                 001186
  620 CONTINUE                                                           001187
      DO 630 I=1,N                                                       001188
      DO 630 J=1,I                                                       001189
      H(I,J)=H(I,J)+X(N+I)*X(NN+J)-X(N6+I)*X(N3+J)                       001190
  630 H(J,I)=H(I,J)                                                      001191
      X(N4+II)=X(N4+II)-XONE                                             001192
      DO 640 I=1,K                                                       001193
      NI=N+I                                                             001194
      DO 640 J=1,N                                                       001195
  640 H(NI,J)=H(NI,J)-X(N4+I)*X(N6+J)-X(N5+I)*X(N+J)                     001196
      LT(II)=IAL                                                         001197
  650 IF (K.EQ.N) GO TO 260                                              001198
C                                                                        001199
C       CALCULATE G,  NEW SEARCH DIRECTION IS -H.G                       001200
C                                                                        001201
      DO 660 I=1,N                                                       001202
      X(N+I)=FM02AS(N,A(I,1),IA,X(1),1)-B(I)                             001203
  660 CONTINUE                                                           001204
      Z=XZERO                                                            001205
      DO 670 I=1,N                                                       001206
      X(N5+I)=-FM02AS(N,H(I,1),IH,X(N+1),1)                              001207
      IF (X(N5+I).NE.XZERO) Z=XONE                                       001208
  670 CONTINUE                                                           001209
      PASSIV=.FALSE.                                                     001210
      IF (Z.EQ.XZERO) GO TO 260                                          001211
      POSTIV=.TRUE.                                                      001212
      GO TO 380                                                          001213
  680 DO 690 I=1,N                                                       001214
      X(I)=X(I)+X(N5+I)                                                  001215
  690 CONTINUE                                                           001216
C                                                                        001217
C       X IS NOW THE MINIMUM POINT IN THE BASIS                          001218
C       UPDATE THE OPERATORS IF A CONSTRAINT HAD BEEN REMOVED            001219
C                                                                        001220
      IF (.NOT.PASSIV) GO TO 260                                         001221
  700 DO 710 I=1,N                                                       001222
      ALPHA=X(NN+I)/CAC                                                  001223
      DO 710 J=1,I                                                       001224
      H(I,J)=H(I,J)+ALPHA*X(NN+J)                                        001225
  710 H(J,I)=H(I,J)                                                      001226
      IF (K.GT.1) GO TO 720                                              001227
      K=0                                                                001228
      GO TO 260                                                          001229
  720 IF (II.EQ.K) GO TO 740                                             001230
      DO 730 I=1,N                                                       001231
      H(N+II,I)=H(N+K,I)                                                 001232
  730 CONTINUE                                                           001233
      LT(II)=LT(K)                                                       001234
  740 K=K-1                                                              001235
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      DO 750 I=1,K                                                      001236
      NI=N+I                                                            001237
      X(N3+I)=FM02AS(N,H(NI,1),IH,X(N+1),1)                             001238
  750 CONTINUE                                                          001239
      DO 760 I=1,K                                                      001240
      ALPHA=X(N3+I)/CAC                                                 001241
      NI=N+I                                                            001242
      DO 760 J=1,N                                                      001243
  760 H(NI,J)=H(NI,J)-ALPHA*X(NN+J)                                     001244
      GO TO 260                                                         001245
  770 Z=XONE/X(N4+II)                                                   001246
C                                                                       001247
C       APPLY SIMPLEX FORMULA TO EXCHANGE CONSTRAINTS                   001248
C                                                                       001249
      DO 810 I=1,N                                                      001250
      NI=N+I                                                            001251
      IF (I.NE.II) GO TO 790                                            001252
      DO 780 J=1,N                                                      001253
      H(NI,J)=H(NI,J)*Z                                                 001254
  780 CONTINUE                                                          001255
      GO TO 810                                                         001256
  790 ZZ=Z*X(N4+I)                                                      001257
      DO 800 J=1,N                                                      001258
      H(NI,J)=H(NI,J)-ZZ*X(NN+J)                                        001259
  800 CONTINUE                                                          001260
  810 CONTINUE                                                          001261
      LT(II)=IAL                                                        001262
      GO TO 260                                                         001263
C                                                                       001264
  820 K=0                                                               001265
      IF (KE.NE.0.AND.LPR.GT.0) WRITE (LPR,830)                         001266
  830 FORMAT (30H0KE MUST BE 0 IN MODES 2 AND 3)                        001267
      KE=0                                                              001268
      DO 840 I=1,M                                                      001269
      LT(NN+I)=1                                                        001270
  840 CONTINUE                                                          001271
      CALL MB01C  (H,N,IH,LT(NNM),X(N6+1))                              001272
C                                                                       001273
C       START WITH EMPTY BASIS FROM FEASIBLE POINT                      001274
C       SEARCH DIRECTION IS -A(-1).B                                    001275
C                                                                       001276
      GO TO 650                                                         001277
      END                                                               001278
C                                                                       001279
C                                                                       001280
      SUBROUTINE LA02A (N,M,C,IC,D,BDL,BDU,X,K,KE,H,IH,LT)              001281
      COMMON /LA02B/ LP,IFLAG                                           001282
      REAL C(IC,1),D(1),BDL(N),BDU(N),X(1),H(IH,1)                      001283
      INTEGER LT(1)                                                     001284
      COMMON /MB01D/ LPMB01,IFMB01                                      001285
      REAL ZERO,ONE,RANGE                                               001286
      DATA ZERO,ONE,RANGE/0.0,1.0,1.0E75/                               001287
      DATA LP/0/                                                        001288
C                                                                       001289
C       SUPPRESS ERROR MESSAGES FROM MB01C/CD                           001290
C                                                                       001291
      LPMB1=LPMB01                                                      001292
      LPMB01=0                                                          001293
   10 IFLAG=0                                                           001294
      NN=N+N                                                            001295
      N3=NN+N                                                           001296
      DO 20 I=1,M                                                       001297
      N1=NN+I                                                           001298
      LT(N1)=1                                                          001299
   20 CONTINUE                                                          001300
```

```
C                                                                       001301
C       CONSTRAINTS INDEXED AS                                          001302
C       -1=EQUALITY,  0=ACTIVE,  1=INACTIVE,  2=VIOLATED                001303
C                                                                       001304
      IF (K.NE.0) GO TO 70                                              001305
C                                                                       001306
C       NO DESIGNATED CONSTRAINTS,  VERTEX CHOSEN FROM UPPER AND        001307
C       LOWER BOUNDS,  INVERSE MATRIX TRIVIAL.                          001308
C                                                                       001309
      DO 60 I=1,N                                                       001310
      DO 30 J=1,N                                                       001311
      H(I,J)=ZERO                                                       001312
   30 CONTINUE                                                          001313
      IF (X(I)-BDL(I).GT.BDU(I)-X(I)) GO TO 40                          001314
      LT(I)=I                                                           001315
      H(I,I)=ONE                                                        001316
      GO TO 50                                                          001317
   40 LT(I)=N+I                                                         001318
      H(I,I)=-ONE                                                       001319
   50 N1=NN+LT(I)                                                       001320
      LT(N1)=0                                                          001321
   60 CONTINUE                                                          001322
      K=N                                                               001323
      GO TO 330                                                         001324
C                                                                       001325
C       SET UP NORMALS V OF THE K DESIGNATED CONSTRAINTS IN BASIS       001326
C                                                                       001327
   70 DO 120 I=1,K                                                      001328
      J=0                                                               001329
      IF (I.LE.KE) J=-1                                                 001330
      N1=NN+LT(I)                                                       001331
      LT(N1)=J                                                          001332
      LI=LT(I)                                                          001333
      NI=N+I                                                            001334
      IF (LI.GT.NN) GO TO 100                                           001335
      DO 80 J=1,N                                                       001336
      H(J,NI)=ZERO                                                      001337
   80 CONTINUE                                                          001338
      IF (LI.GT.N) GO TO 90                                             001339
      H(LI,NI)=ONE                                                      001340
      GO TO 120                                                         001341
   90 L=LI-N                                                            001342
      H(L,NI)=-ONE                                                      001343
      GO TO 120                                                         001344
  100 LI=LI-NN                                                          001345
      DO 110 J=1,N                                                      001346
      H(J,NI)=C(J,LI)                                                   001347
  110 CONTINUE                                                          001348
  120 CONTINUE                                                          001349
      IF (K.NE.N) GO TO 140                                             001350
      DO 130 J=1,N                                                      001351
      NJ=N+J                                                            001352
      DO 130 I=1,N                                                      001353
  130 H(I,J)=H(I,NJ)                                                    001354
      CALL MB01C (H,N,IH,LT(N+1),X(N+1))                                001355
      IF (IFMB01.NE.0) GO TO 160                                        001356
      GO TO 330                                                         001357
  140 CONTINUE                                                          001358
C                                                                       001359
C       FORM M=(VTRANSPOSE.V)(-1)                                       001360
C                                                                       001361
      DO 150 I=1,K                                                      001362
      N1=N+I                                                            001363
      DO 150 J=I,K                                                      001364
      N2=N+J                                                            001365
```

```
      H(I,J)=FM02AS(N,H(1,N1),1,H(1,N2),1)                              001366
  150 H(J,I)=H(I,J)                                                     001367
      CALL MB01C (H,K,IH,LT(N+1),X(N+1))                                001368
      IF (IFMB01.EQ.0) GO TO 170                                        001369
  160 IF (K.EQ.KE) GO TO 690                                            001370
      K=KE                                                              001371
      GO TO 10                                                          001372
C                                                                       001373
C       CALCULATE GENERALIZED INVERSE OF V,  VPLUS=M.VTRANSPOSE         001374
C                                                                       001375
  170 DO 190 I=1,K                                                      001376
      DO 180 J=1,K                                                      001377
      N1=N+J                                                            001378
      X(N1)=H(I,J)                                                      001379
  180 CONTINUE                                                          001380
      DO 190 J=1,N                                                      001381
  190 H(I,J)=FM02AS(K,X(N+1),1,H(J,N+1),IH)                             001382
C                                                                       001383
C       SET UP DIAGONAL ELEMENTS OF THE PROJECTION MATRIX  P=V.VPLUS    001384
C                                                                       001385
      DO 200 I=1,N                                                      001386
      N1=N+I                                                            001387
      X(N1)=FM02AS(K,H(1,I),1,H(I,N+1),IH)                              001388
  200 CONTINUE                                                          001389
      DO 210 I=1,N                                                      001390
      N1=N+I                                                            001391
      LT(N1)=0                                                          001392
  210 CONTINUE                                                          001393
      KV=K                                                              001394
C                                                                       001395
C       ADD BOUND E(I) CORRESPONDING TO THE SMALLEST DIAG(P)            001396
C                                                                       001397
  220 Z=ONE                                                             001398
      DO 230 I=1,N                                                      001399
      N1=N+I                                                            001400
      IF (LT(N1).EQ.1) GO TO 230                                        001401
      IF (X(N1).GE.Z) GO TO 230                                         001402
      Z=X(N1)                                                           001403
      II=I                                                              001404
  230 CONTINUE                                                          001405
      Y=ONE                                                             001406
      IF (X(II)-BDL(II).GT.BDU(II)-X(II)) Y=-ONE                        001407
C                                                                       001408
C       CALCULATE VECTORS VPLUS.E(I) AND  U=E(I)-V.VPLUS.E(I)           001409
C                                                                       001410
      IF (Y.NE.ONE) GO TO 250                                           001411
      DO 240 I=1,K                                                      001412
      N1=NN+I                                                           001413
      X(N1)=H(I,II)                                                     001414
  240 CONTINUE                                                          001415
      GO TO 270                                                         001416
  250 DO 260 I=1,K                                                      001417
      N1=NN+I                                                           001418
      X(N1)=-H(I,II)                                                    001419
  260 CONTINUE                                                          001420
  270 CONTINUE                                                          001421
      DO 280 I=1,N                                                      001422
      N1=N+I                                                            001423
      IF (LT(N1).EQ.1) GO TO 280                                        001424
      N1=N3+I                                                           001425
      X(N1)=-FM02AS(KV,H(I,N+1),IH,X(NN+1),1)                           001426
  280 CONTINUE                                                          001427
      DO 290 I=1,N                                                      001428
      H(I,II)=ZERO                                                      001429
  290 CONTINUE                                                          001430
```

```
      I1=N+II                                                          001431
      LT(I1)=1                                                         001432
      I3=N3+II                                                         001433
      Z=ONE+X(I3)*Y                                                    001434
C                                                                      001435
C       UPDATE VPLUS AND DIAG(P)                                       001436
C                                                                      001437
      DO 310 I=1,N                                                     001438
      N1=N+I                                                           001439
      IF (LT(N1).EQ.1) GO TO 310                                       001440
      L=N3+I                                                           001441
      ALPHA=X(L)/Z                                                     001442
      H(K+1,I)=ALPHA                                                   001443
      DO 300 J=1,K                                                     001444
      N2=NN+J                                                          001445
      H(J,I)=H(J,I)-X(N2)*ALPHA                                        001446
  300 CONTINUE                                                         001447
  310 CONTINUE                                                         001448
      DO 320 I=1,N                                                     001449
      N1=N+I                                                           001450
      IF (LT(N1).EQ.1) GO TO 320                                       001451
      N2=N3+I                                                          001452
      X(N1)=X(N1)+X(N2)**2/Z                                           001453
  320 CONTINUE                                                         001454
      K=K+1                                                            001455
      H(K,II)=Y                                                        001456
      IF (Y.NE.ONE) II=II+N                                            001457
      I2=NN+II                                                         001458
      LT(I2)=0                                                         001459
      LT(K)=II                                                         001460
      IF (K.NE.N) GO TO 220                                            001461
C                                                                      001462
C       SET UP RHS OF CONSTRAINTS IN BASIS                             001463
C                                                                      001464
  330 DO 360 I=1,N                                                     001465
      LI=LT(I)                                                         001466
      N1=N+I                                                           001467
      IF (LI.GT.N) GO TO 340                                           001468
      X(N1)=BDL(LI)                                                    001469
      GO TO 360                                                        001470
  340 IF (LI.GT.NN) GO TO 350                                          001471
      L=LI-N                                                           001472
      X(N1)=-BDU(L)                                                    001473
      GO TO 360                                                        001474
  350 LL=LI-NN                                                         001475
      X(N1)=D(LL)                                                      001476
  360 CONTINUE                                                         001477
C                                                                      001478
C       CALCULATE POSITION OF VERTEX                                   001479
C                                                                      001480
      DO 370 I=1,N                                                     001481
      X(I)=FM02AS(N,H(1,I),1,X(N+1),1)                                 001482
  370 CONTINUE                                                         001483
C                                                                      001484
C       CALCULATE THE CONSTRAINT RESIDUALS, THE NUMBER OF VIOLATED     001485
C       CONSTRAINTS, AND THE SUM OF THEIR NORMALS                      001486
C                                                                      001487
  380 KV=0                                                             001488
      DO 390 I=1,N                                                     001489
      N1=N+I                                                           001490
      X(N1)=ZERO                                                       001491
  390 CONTINUE                                                         001492
      DO 460 I=1,M                                                     001493
      N1=NN+I                                                          001494
      IF (LT(N1).LE.0) GO TO 460                                       001495
```

```
      IF (I.GT.N) GO TO 400                                              001496
      Z=X(I)-BDL(I)                                                      001497
      GO TO 420                                                          001498
  400 IF (I.GT.NN) GO TO 410                                             001499
      L=I-N                                                              001500
      Z=BDU(L)-X(L)                                                      001501
      GO TO 420                                                          001502
  410 J=I-NN                                                             001503
      Z=-D(J)+FM02AS(N,C(1,J),1,X(1),1)                                  001504
  420 X(N1)=Z                                                            001505
      IF (Z.GE.ZERO) GO TO 460                                           001506
      KV=KV+1                                                            001507
      LT(N1)=2                                                           001508
      IF (I.GT.N) GO TO 430                                              001509
      N2=N+I                                                             001510
      X(N2)=X(N2)+ONE                                                    001511
      GO TO 460                                                          001512
  430 IF (I.GT.NN) GO TO 440                                             001513
      X(I)=X(I)-ONE                                                      001514
      GO TO 460                                                          001515
  440 DO 450 II=1,N                                                      001516
      NII=N+II                                                           001517
      X(NII)=X(NII)+C(II,J)                                              001518
  450 CONTINUE                                                           001519
  460 CONTINUE                                                           001520
      IF (KV.NE.0) GO TO 470                                             001521
      GO TO 710                                                          001522
C                                                                        001523
C       POSSIBLE DIRECTIONS OF SEARCH OBTAINABLE BY REMOVING A           001524
C       CONSTRAINT ARE ROWS OF H,  CALCULATE THE OPTIMUM DIRECTION       001525
C                                                                        001526
  470 Z=ZERO                                                             001527
      DO 480 I=1,N                                                       001528
      N1=NN+LT(I)                                                        001529
      IF (LT(N1).EQ.-1) GO TO 480                                        001530
      Y=FM02AS(N,H(I,1),IH,X(N+1),1)                                     001531
      IF (Y.LE.Z) GO TO 480                                              001532
      Z=Y                                                                001533
      II=I                                                               001534
  480 CONTINUE                                                           001535
      IF (Z.LE.ZERO) GO TO 670                                           001536
C                                                                        001537
C       SEARCH FOR THE NEAREST OF THE FURTHEST VIOLATED CONSTRAINT       001538
C       AND THE NEAREST NONVIOLATED NONBASIC CONSTRAINT                  001539
C                                                                        001540
      ALPHA=RANGE                                                        001541
      BETA=ZERO                                                          001542
      DO 490 I=1,N                                                       001543
      N1=N+I                                                             001544
      X(N1)=H(II,I)                                                      001545
  490 CONTINUE                                                           001546
      DO 540 I=1,M                                                       001547
      N1=NN+I                                                            001548
      IF (LT(N1).LE.0) GO TO 540                                         001549
      IF (I.GT.N) GO TO 500                                              001550
      N2=N+I                                                             001551
      Z=-X(N2)                                                           001552
      GO TO 520                                                          001553
  500 IF (I.GT.NN) GO TO 510                                             001554
      Z=X(I)                                                             001555
      GO TO 520                                                          001556
  510 JJ=I-NN                                                            001557
      Z=-FM02AS(N,X(N+1),1,C(1,JJ),1)                                    001558
  520 IF (LT(N1).EQ.2) GO TO 530                                         001559
      IF (Z.LE.ZERO) GO TO 540                                           001560
```

```
      Z=X(N1)/Z                                                          001561
      IF (Z.GE.ALPHA) GO TO 540                                          001562
      ALPHA=Z                                                            001563
      IAL=I                                                              001564
      GO TO 540                                                          001565
  530 LT(N1)=1                                                           001566
      IF (Z.GE.ZERO) GO TO 540                                           001567
      I1=NN+I                                                            001568
      Z=X(I1)/Z                                                          001569
      IF (Z.LE.BETA) GO TO 540                                           001570
      BETA=Z                                                             001571
      IB=I                                                               001572
  540 CONTINUE                                                           001573
      IF (ALPHA.GT.BETA) GO TO 550                                       001574
      IB=IAL                                                             001575
      BETA=ALPHA                                                         001576
C                                                                        001577
C       EXCHANGE WITH THE CONSTRAINT BEING REMOVED FROM THE BASIS,       001578
C       USING SIMPLEX FORMULA FOR NEW H                                  001579
C                                                                        001580
  550 I1=NN+LT(II)                                                       001581
      LT(I1)=1                                                           001582
      I2=NN+IB                                                           001583
      LT(I2)=0                                                           001584
      LT(II)=IB                                                          001585
      IF (IB.GT.N) GO TO 570                                             001586
      DO 560 I=1,N                                                       001587
      N1=NN+I                                                            001588
      X(N1)=H(I,IB)                                                      001589
  560 CONTINUE                                                           001590
      GO TO 620                                                          001591
  570 IB=IB-N                                                            001592
      IF (IB.GT.N) GO TO 590                                             001593
      DO 580 I=1,N                                                       001594
      N1=NN+I                                                            001595
      X(N1)=-H(I,IB)                                                     001596
  580 CONTINUE                                                           001597
      GO TO 620                                                          001598
  590 IB=IB-N                                                            001599
      DO 600 I=1,N                                                       001600
      N1=N3+I                                                            001601
      X(N1)=C(I,IB)                                                      001602
  600 CONTINUE                                                           001603
      DO 610 I=1,N                                                       001604
      N1=NN+I                                                            001605
      X(N1)=FM02AS(N,H(I,1),IH,X(N3+1),1)                                001606
  610 CONTINUE                                                           001607
  620 I2=NN+II                                                           001608
      Z=ONE/X(I2)                                                        001609
      DO 660 I=1,N                                                       001610
      N1=N+I                                                             001611
      X(I)=X(I)+BETA*X(N1)                                               001612
      IF (I.NE.II) GO TO 640                                             001613
      DO 630 J=1,N                                                       001614
      H(I,J)=H(I,J)*Z                                                    001615
  630 CONTINUE                                                           001616
      GO TO 660                                                          001617
  640 L=NN+I                                                             001618
      ZZ=Z*X(L)                                                          001619
      DO 650 J=1,N                                                       001620
      N2=N+J                                                             001621
      H(I,J)=H(I,J)-ZZ*X(N2)                                             001622
  650 CONTINUE                                                           001623
  660 CONTINUE                                                           001624
      GO TO 380                                                          001625
```

```
  670 K=0                                                                  001626
      IFLAG=1                                                              001627
      IF (LP.GT.0) WRITE (LP,680)                                          001628
  680 FORMAT (50H ERROR RETURN FROM LA02A/AD ; THERE IS NO SOLUTION)       001629
      GO TO 710                                                            001630
  690 IFLAG=2                                                              001631
      IF (LP.GT.0) WRITE (LP,700)                                          001632
  700 FORMAT (50H ERROR RETURN FROM LA02A/AD BECAUSE GIVEN EQUALITY,       001633
     1 41H CONSTRAINTS ARE NOT LINEARLY INDEPENDENT)                       001634
C                                                                          001635
C       RESTORE UNIT NUMBER FOR MESSAGES FROM MB01C                        001636
C                                                                          001637
  710 LPMB01=LPMB1                                                         001638
      RETURN                                                               001639
      END                                                                  001640
C                                                                          001641
C                                                                          001642
      REAL FUNCTION FM02AS (N,A,IA,B,IB)                                   001643
      DIMENSION A(N), B(N)                                                 001644
C                                                                          001645
C       N    IS THE LENGTH OF THE VECTORS (IF N <= 0  FM02AS/AD = 0)       001646
C       A    IS THE FIRST VECTOR                                           001647
C       IA   IS SUBSCRIPT DISPLACEMENT BETWEEN ELEMENTS OF A               001648
C       B    IS THE SECOND VECTOR                                          001649
C       IB   IS SUBSCRIPT DISPLACEMENT BETWEEN ELEMENTS OF B               001650
C       FM02AS/AD  IS THE RESULT                                           001651
C                                                                          001652
      R1=0D0                                                               001653
      IF (N.LE.0) GO TO 20                                                 001654
      JA=1                                                                 001655
      IF (IA.LT.0) JA=1-(N-1)*IA                                           001656
      JB=1                                                                 001657
      IF (IB.LT.0) JB=1-(N-1)*IB                                           001658
      I=0                                                                  001659
   10 I=I+1                                                                001660
      R1=R1+A(JA)*B(JB)                                                    001661
      JA=JA+IA                                                             001662
      JB=JB+IB                                                             001663
      IF (I.LT.N) GO TO 10                                                 001664
   20 FM02AS=R1                                                            001665
      RETURN                                                               001666
      END                                                                  001667
C                                                                          001668
C                                                                          001669
      SUBROUTINE MB01C (A,M,IA,IND,C)                                      001670
      REAL A,AMAX,C,DIV,STO,W,W1,FM02AS,ZERO,ONE                           001671
      DIMENSION A(IA,1), IND(1), C(1)                                      001672
      COMMON /MB01D/ LP,IFLAG                                              001673
      DATA ZERO,ONE/0.0,1.0/                                               001674
      DATA LP/0/                                                           001675
      IFLAG=0                                                              001676
      IF (M-1) 310,10,20                                                   001677
   10 IF (A(1,1).EQ.ZERO) GO TO 330                                        001678
      A(1,1)=ONE/A(1,1)                                                    001679
      GO TO 350                                                            001680
   20 M1=M-1                                                               001681
      AMAX=ZERO                                                            001682
      DO 40 I=1,M                                                          001683
      IND(I)=I                                                             001684
      IF (ABS(A(I,1))-ABS(AMAX)) 40,40,30                                  001685
   30 AMAX=A(I,1)                                                          001686
      IMAX=I                                                               001687
   40 CONTINUE                                                             001688
      IF (AMAX.EQ.ZERO) GO TO 330                                          001689
      DO 120 J=1,M1                                                        001690
```

```
      IF (IMAX-J) 70,70,50                                              001691
   50 IW=IND(IMAX)                                                      001692
      IND(IMAX)=IND(J)                                                  001693
      IND(J)=IW                                                         001694
      DO 60 K=1,M                                                       001695
      W=A(IMAX,K)                                                       001696
      A(IMAX,K)=A(J,K)                                                  001697
      A(J,K)=W                                                          001698
   60 CONTINUE                                                          001699
   70 J1=J+1                                                            001700
      IF (J.EQ.1) GO TO 90                                              001701
      DO 80 I=J1,M                                                      001702
      A(J,I)=A(J,I)-FM02AS(J-1,A(J,1),IA,A(1,I),1)                      001703
   80 CONTINUE                                                          001704
   90 DIV=AMAX                                                          001705
      AMAX=ZERO                                                         001706
      DO 110 I=J1,M                                                     001707
      A(I,J)=A(I,J)/DIV                                                 001708
      A(I,J+1)=A(I,J+1)-FM02AS(J,A(I,1),IA,A(1,J+1),1)                  001709
      IF (ABS(A(I,J1))-ABS(AMAX)) 110,110,100                           001710
  100 AMAX=A(I,J1)                                                      001711
      IMAX=I                                                            001712
  110 CONTINUE                                                          001713
      IF (AMAX.EQ.ZERO) GO TO 330                                       001714
  120 CONTINUE                                                          001715
      DO 170 I1=1,M1                                                    001716
      I=M+1-I1                                                          001717
      I2=I-1                                                            001718
      DO 150 J1=1,I2                                                    001719
      J=I2+1-J1                                                         001720
      J2=J+1                                                            001721
      W1=-A(I,J)                                                        001722
      IF (I2-J2) 140,130,130                                            001723
  130 W1=W1-FM02AS(I2-J2+1,A(J2,J),1,C(J2),1)                           001724
  140 C(J)=W1                                                           001725
  150 CONTINUE                                                          001726
      DO 160 K=1,I2                                                     001727
      A(I,K)=C(K)                                                       001728
  160 CONTINUE                                                          001729
  170 CONTINUE                                                          001730
      DO 260 I1=1,M                                                     001731
      I=M+1-I1                                                          001732
      I2=I+1                                                            001733
      W=A(I,I)                                                          001734
      DO 240 J=1,M                                                      001735
      IF (I-J) 180,190,200                                              001736
  180 W1=ZERO                                                           001737
      GO TO 210                                                         001738
  190 W1=ONE                                                            001739
      GO TO 210                                                         001740
  200 W1=A(I,J)                                                         001741
  210 IF (I1-1) 230,230,220                                             001742
  220 W1=W1-FM02AS(M-I2+1,A(I,I2),IA,A(I2,J),1)                         001743
  230 C(J)=W1                                                           001744
  240 CONTINUE                                                          001745
      DO 250 J=1,M                                                      001746
      A(I,J)=C(J)/W                                                     001747
  250 CONTINUE                                                          001748
  260 CONTINUE                                                          001749
      DO 300 I=1,M                                                      001750
  270 IF (IND(I)-I) 280,300,280                                         001751
  280 J=IND(I)                                                          001752
      DO 290 K=1,M                                                      001753
      STO=A(K,I)                                                        001754
      A(K,I)=A(K,J)                                                     001755
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      A(K,J)=STO                                                                001756
  290 CONTINUE                                                                  001757
      ISTO=IND(J)                                                               001758
      IND(J)=J                                                                  001759
      IND(I)=ISTO                                                               001760
      GO TO 270                                                                 001761
  300 CONTINUE                                                                  001762
      GO TO 350                                                                 001763
  310 IF (LP.GT.0) WRITE (LP,320)                                               001764
  320 FORMAT (53H ERROR RETURN FROM MB01C/CD BECAUSE M IS NOT POSITIVE)         001765
      IFLAG=1                                                                   001766
      GO TO 350                                                                 001767
  330 IF (LP.GT.0) WRITE (LP,340)                                               001768
  340 FORMAT (54H ERROR RETURN FROM MB01C/CD BECAUSE MATRIX IS SINGULAR)        001769
      IFLAG=2                                                                   001770
  350 RETURN                                                                    001771
      END                                                                       001772
```