

## THE ALGORITHM

1. Given  $\mathbf{x}_f^{(1)} = \mathbf{x}_c^*$ ,  $\lambda^{(1)} = 1$ ,  $\delta^{(1)}$ ,  $\alpha$ ,  $\mathbf{J}_f^{(1)} = \mathbf{J}_c^*$  and  $i=1$ .

2. Construct  $V^{(i)}$ .

3. Apply the optimization procedure

$$[\mathbf{B}^{(i)}, \mathbf{s}^{(i)}, \mathbf{t}^{(i)}, \sigma^{(i)}, \mathbf{c}^{(i)}, \gamma^{(i)}] = \arg \left\{ \min_{\mathbf{B}, \mathbf{s}, \mathbf{t}, \sigma, \mathbf{c}, \gamma} \left\| \begin{bmatrix} \mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_N^T \end{bmatrix}^T \right\| \right\}$$

$$\mathbf{e}_j = \mathbf{R}_c(\mathbf{P}^{(i)}(\mathbf{x}_f^{(j)}, \psi_k), \mathbf{P}_\psi^{(i)}(\mathbf{x}_f^{(j)}, \psi_k)) - \mathbf{R}_f(\mathbf{x}_f^{(j)}, \psi_k) \quad \forall \mathbf{x}_f^{(j)} \in V^{(i)}, \quad \forall \psi_k, k=1, 2, \dots, N_\psi, N=N_p N_\psi$$

to obtain the mapping parameters.

4. Construct the surrogate model

$$\mathbf{R}_s^{(i)}(\mathbf{x}_f) = \lambda^{(i)} \mathbf{R}_m^{(i)}(\mathbf{x}_f) + (1 - \lambda^{(i)}) (\mathbf{R}_f(\mathbf{x}_f^{(i)}) + \mathbf{J}_f^{(i)} \Delta \mathbf{x}_f), \quad \lambda^{(i)} \in [0, 1]$$

where  $\mathbf{R}_f(\mathbf{x}_f, \psi_j) \approx \mathbf{R}_m^{(i)}(\mathbf{x}_f, \psi_j) = \mathbf{R}_c(\mathbf{P}^{(i)}(\mathbf{x}_f, \psi_j), \mathbf{P}_\psi^{(i)}(\mathbf{x}_f, \psi_j)), j=1, 2, \dots, N_\psi$

and

$$\begin{bmatrix} \mathbf{P}^{(i)}(\mathbf{x}_f, \psi_j) \\ \mathbf{P}_\psi^{(i)}(\mathbf{x}_f, \psi_j) \end{bmatrix} = \begin{bmatrix} \mathbf{B}^{(i)} & \mathbf{s}^{(i)} \\ \mathbf{t}^{(i)T} & \sigma^{(i)} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x}_f \\ \psi_j \end{bmatrix} + \begin{bmatrix} \mathbf{c}^{(i)} \\ \gamma^{(i)} \end{bmatrix}$$

5. Obtain the suggested step  $\mathbf{h}^{(i)}$  by solving

$$\mathbf{h}^{(i)} = \arg \left\{ \min_{\mathbf{h}^{(i)}} U(\mathbf{R}_s^{(i)}(\mathbf{x}_f^{(i)} + \mathbf{h}^{(i)})) \right\}, \quad \|\mathbf{h}^{(i)}\| \leq \delta^{(i)}$$

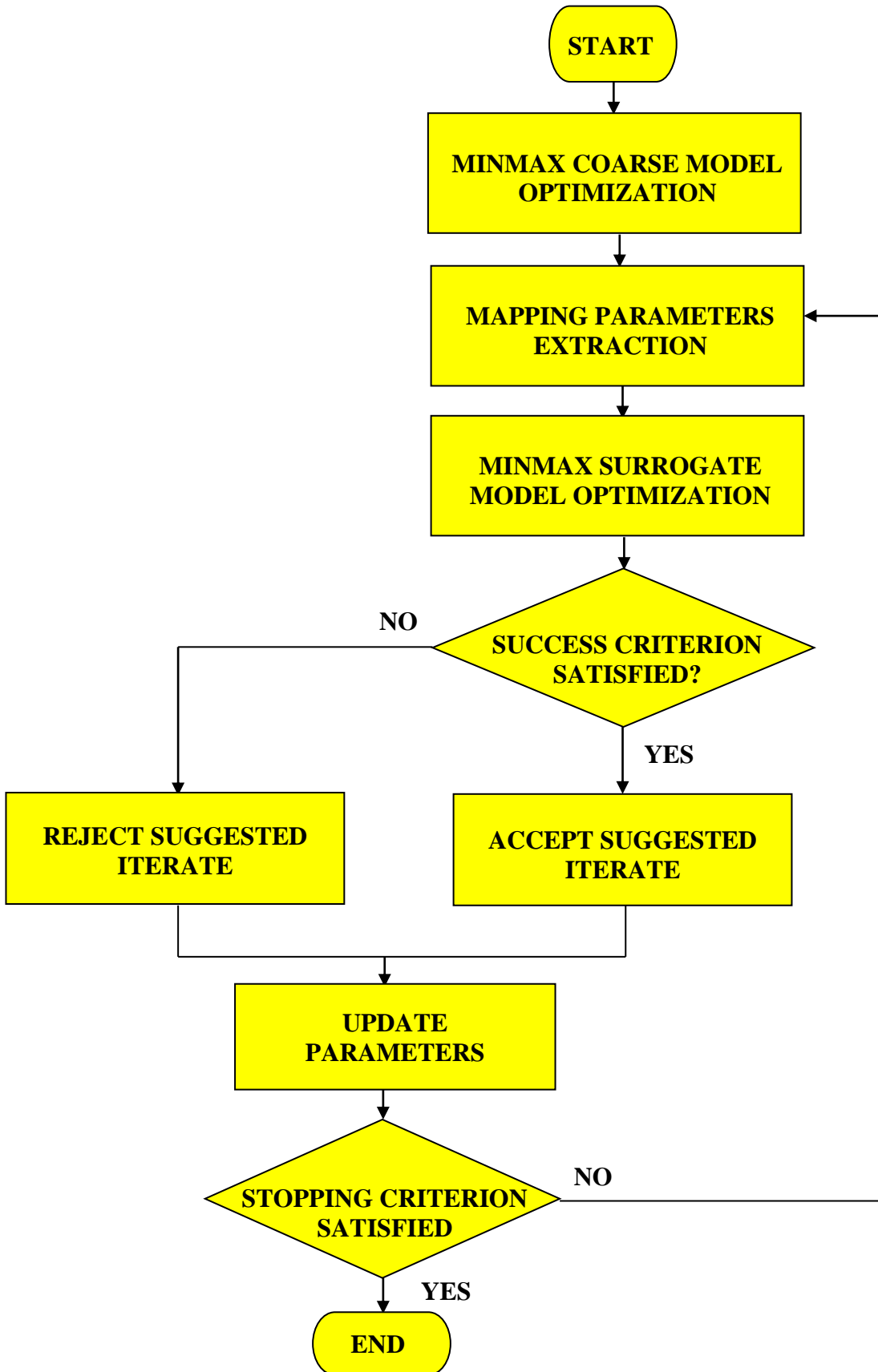
6. If  $U(\mathbf{R}_f(\mathbf{x}_f^{(i)} + \mathbf{h}^{(i)})) < U(\mathbf{R}_f(\mathbf{x}_f^{(i)}))$ , set  $\mathbf{x}_f^{(i+1)} = \mathbf{x}_f^{(i)} + \mathbf{h}^{(i)}$  else  $\mathbf{x}_f^{(i+1)} = \mathbf{x}_f^{(i)}$ .

7. Update  $\mathbf{J}_f^{(i)}$ ,  $\delta^{(i)}$  and  $\lambda^{(i)}$ .

8. If the stopping criterion is satisfied stop.

9. Set  $i=i+1$  and go to step 2.

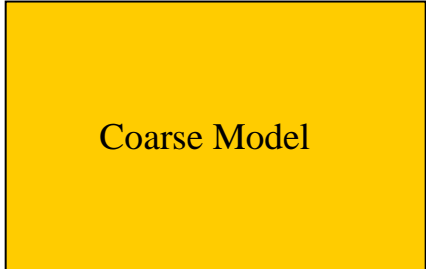
ALGORITHM FLOWCHART



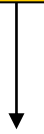
## SOME USEFUL NOTATIONS

- $\psi$ : The independent parameter
- $\mathbf{x}_c$ : Vector of coarse model parameters
- $\mathbf{x}_{c,\psi}$ : Extended vector of coarse model parameters; the value of  $\psi$  is included as the last component of this vector
- $\boldsymbol{\psi}$ : Vector of simulated values of the independent parameter
- $\mathbf{R}_{c,\psi}$ : Coarse model response at a single value of  $\psi$
- $\mathbf{R}_c$ : Coarse model response at all values of  $\psi$
- $\mathbf{x}_f$ : Vector of fine model parameters
- $\mathbf{x}_{f,\psi}$ : Extended vector of fine model parameters;  $\psi$  is included as the last component of this vector
- $\mathbf{R}_{f,\psi}$ : Fine model response at a single value of  $\psi$
- $\mathbf{R}_f$ : Fine model response at all values of  $\psi$
- $\mathbf{x}_s$ : Vector of surrogate model parameters
- $\mathbf{R}_s$ : Surrogate model response at all value of  $\psi$
- $\mathbf{X}_E$ : Vector of mapping parameters
- $\mathbf{R}_E$ : Vector of matched responses
- $V$ : Set of used fine model points

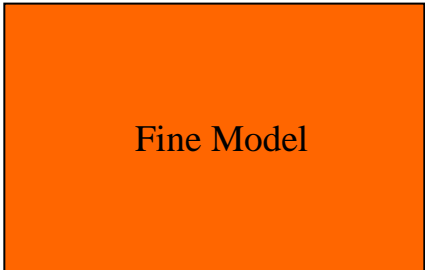
**COARSE AND FINE MODELS**



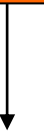
$\mathbf{x}_{c,\psi}$



$\mathbf{R}_{c,\psi}$

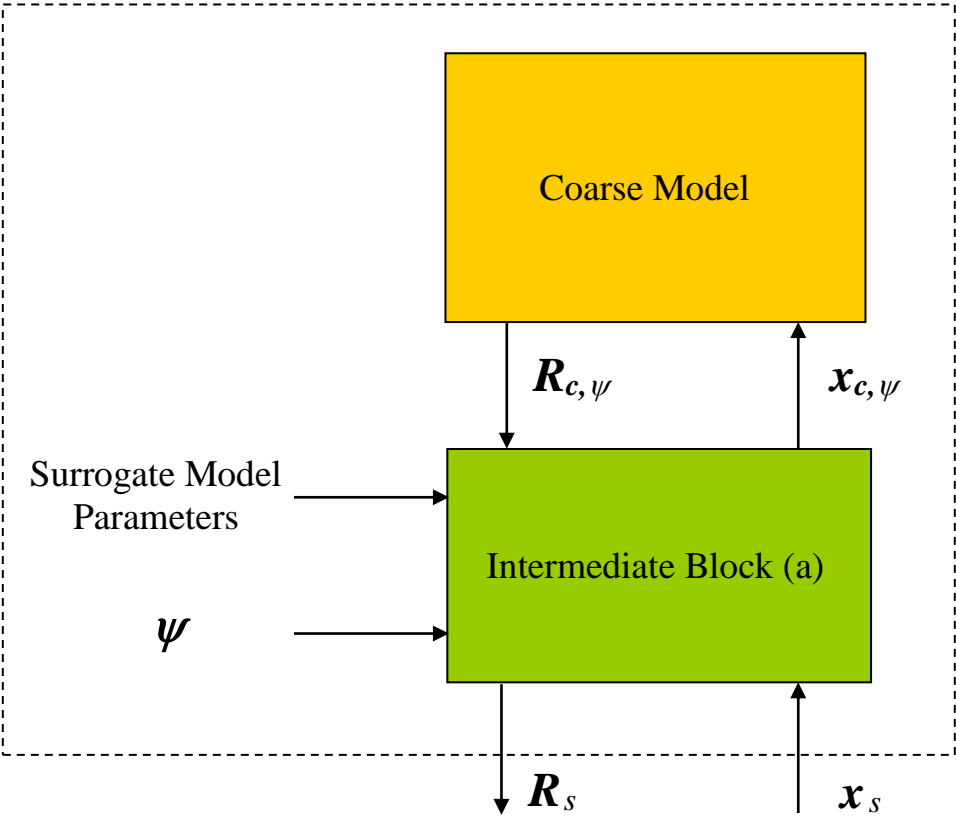


$\mathbf{x}_{f,\psi}$

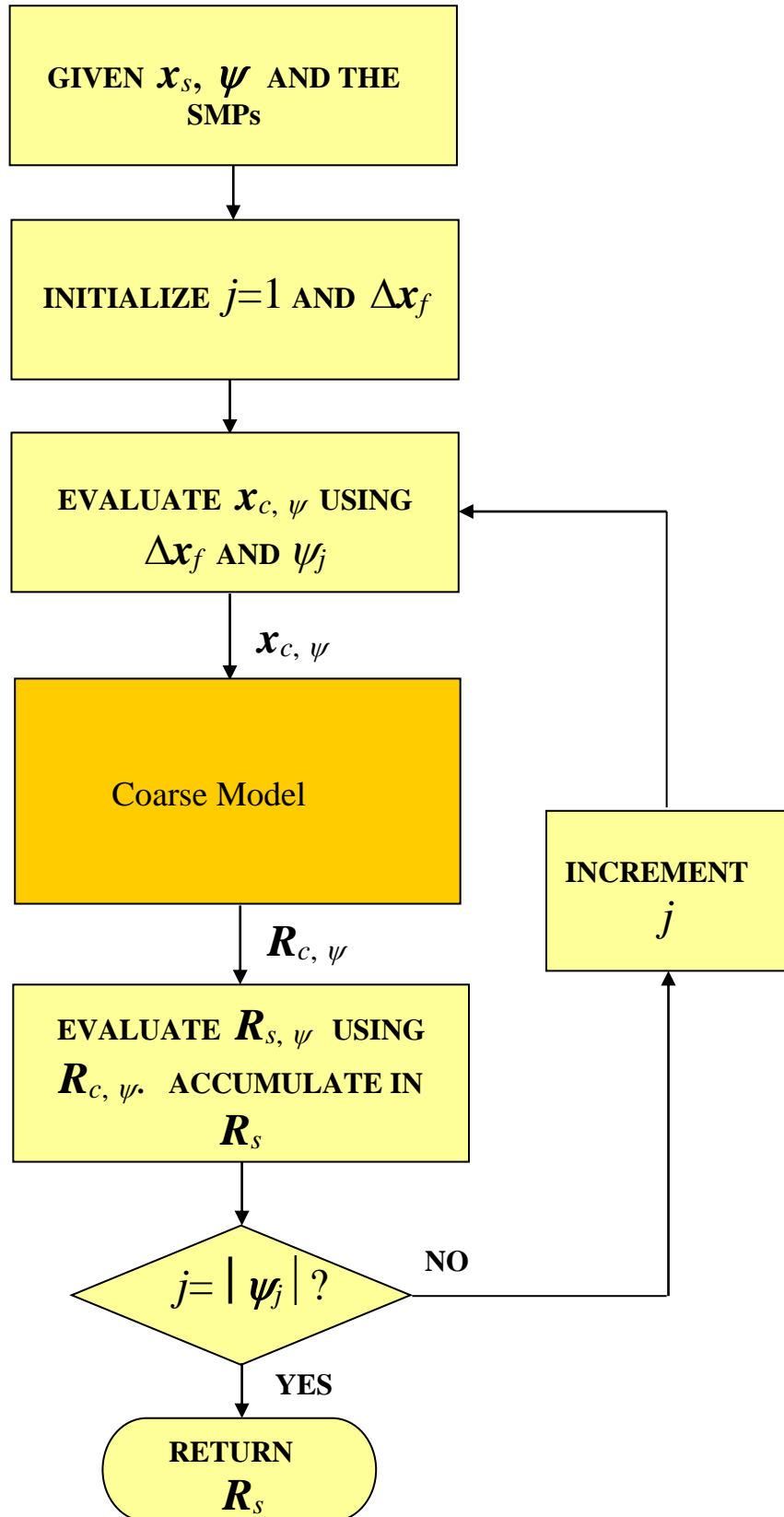


$\mathbf{R}_{f,\psi}$

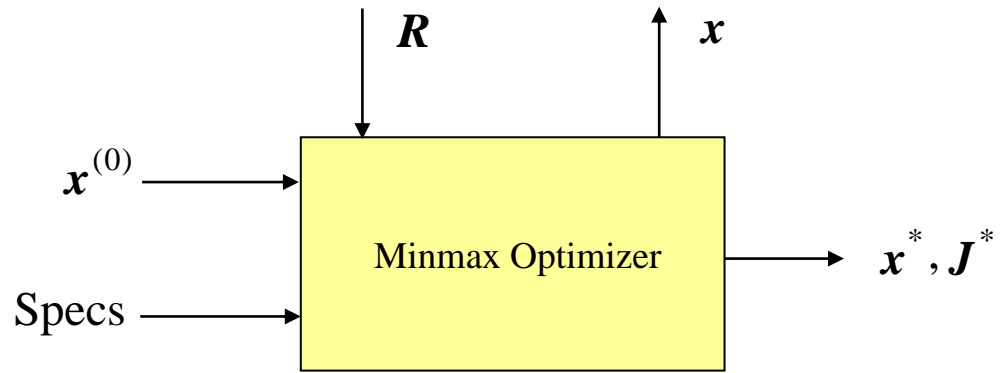
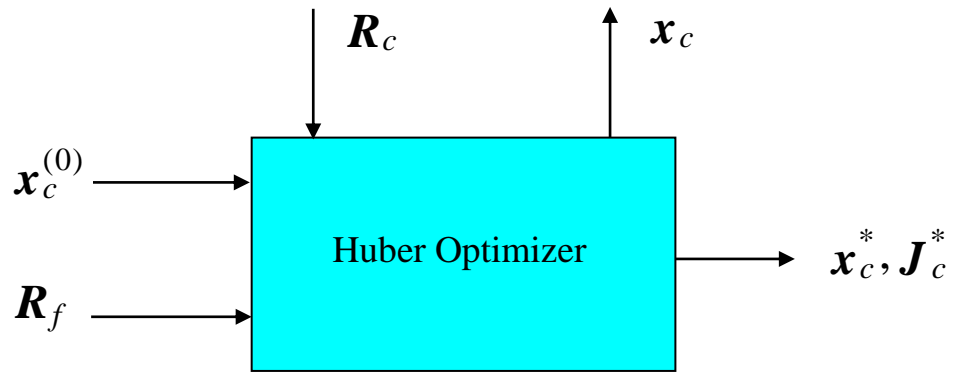
THE SURROGATE MODEL



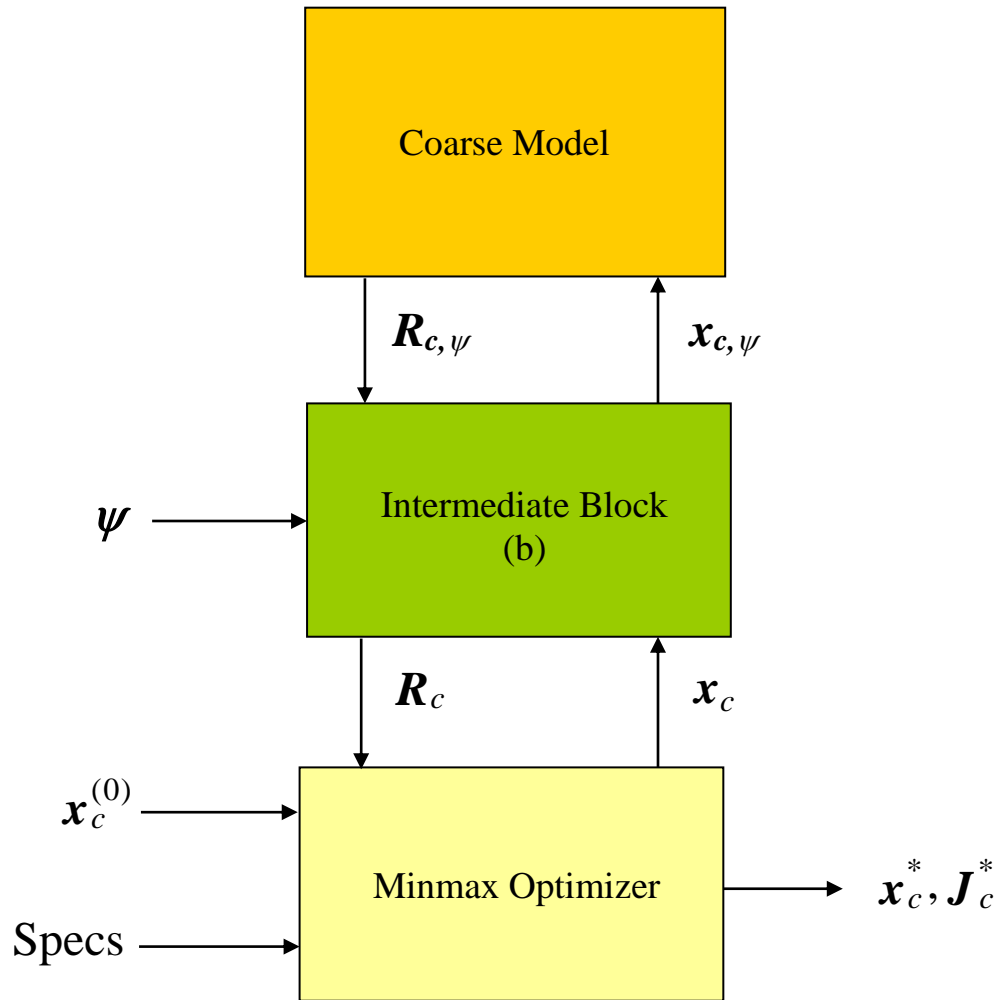
INTERMEDIATE BLOCK (a)



# OPTIMIZERS

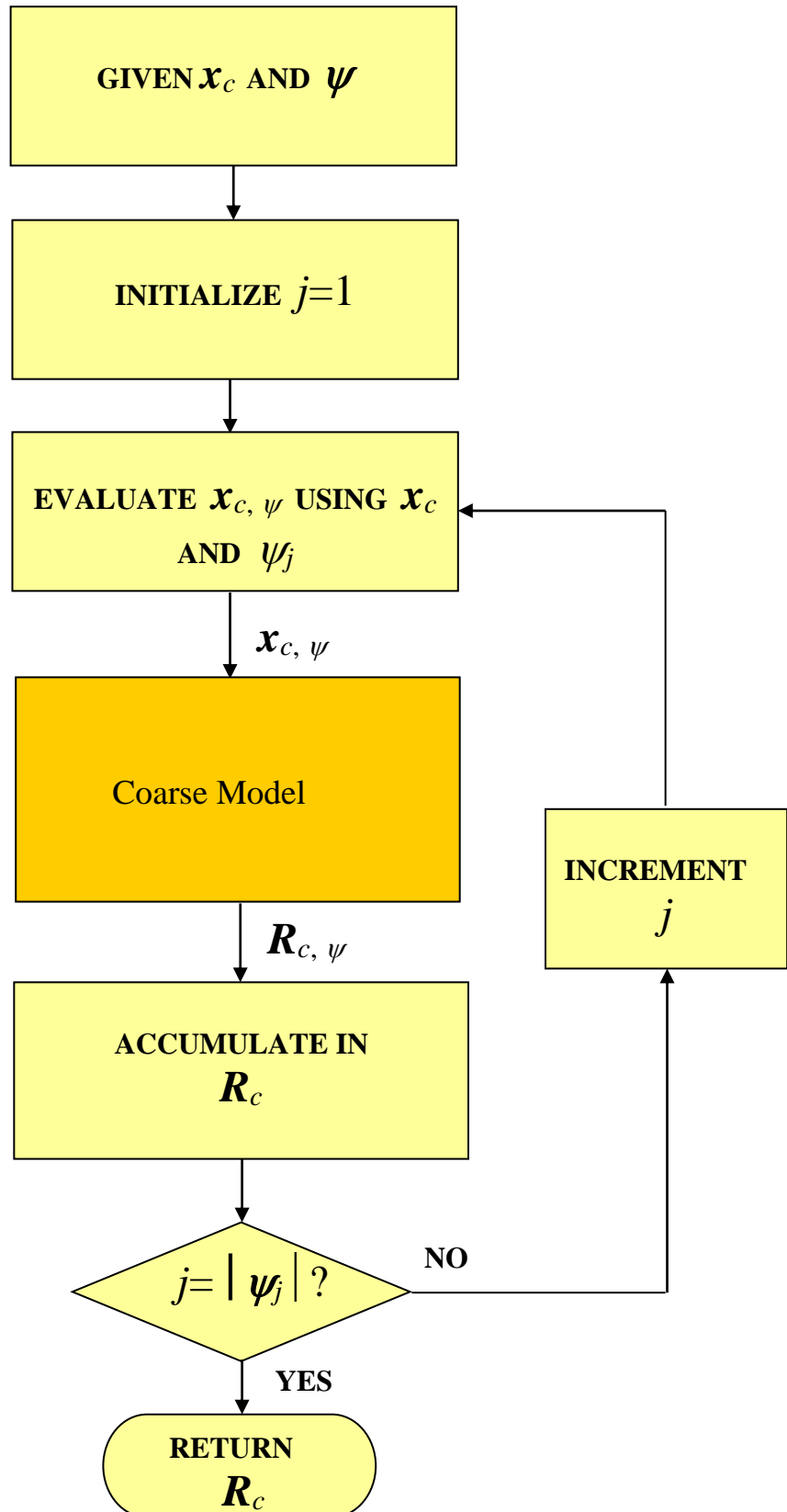


# MINMAX OPTIMIZATION OF THE COARSE MODEL

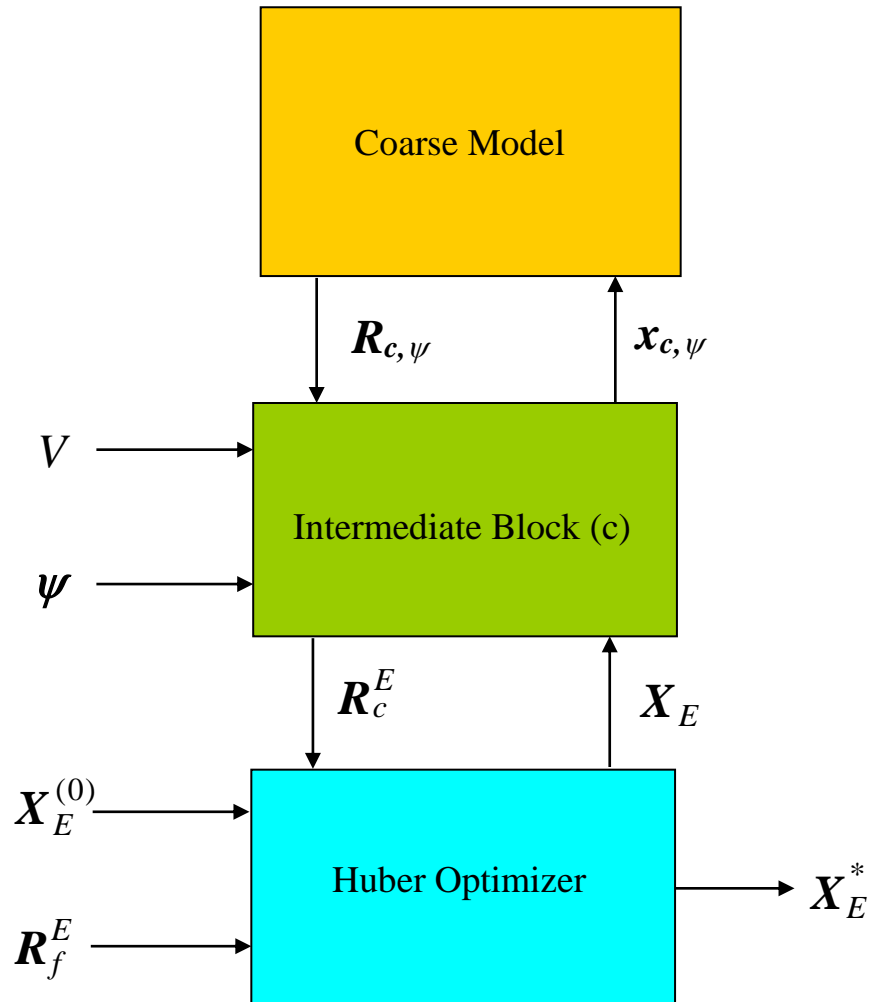




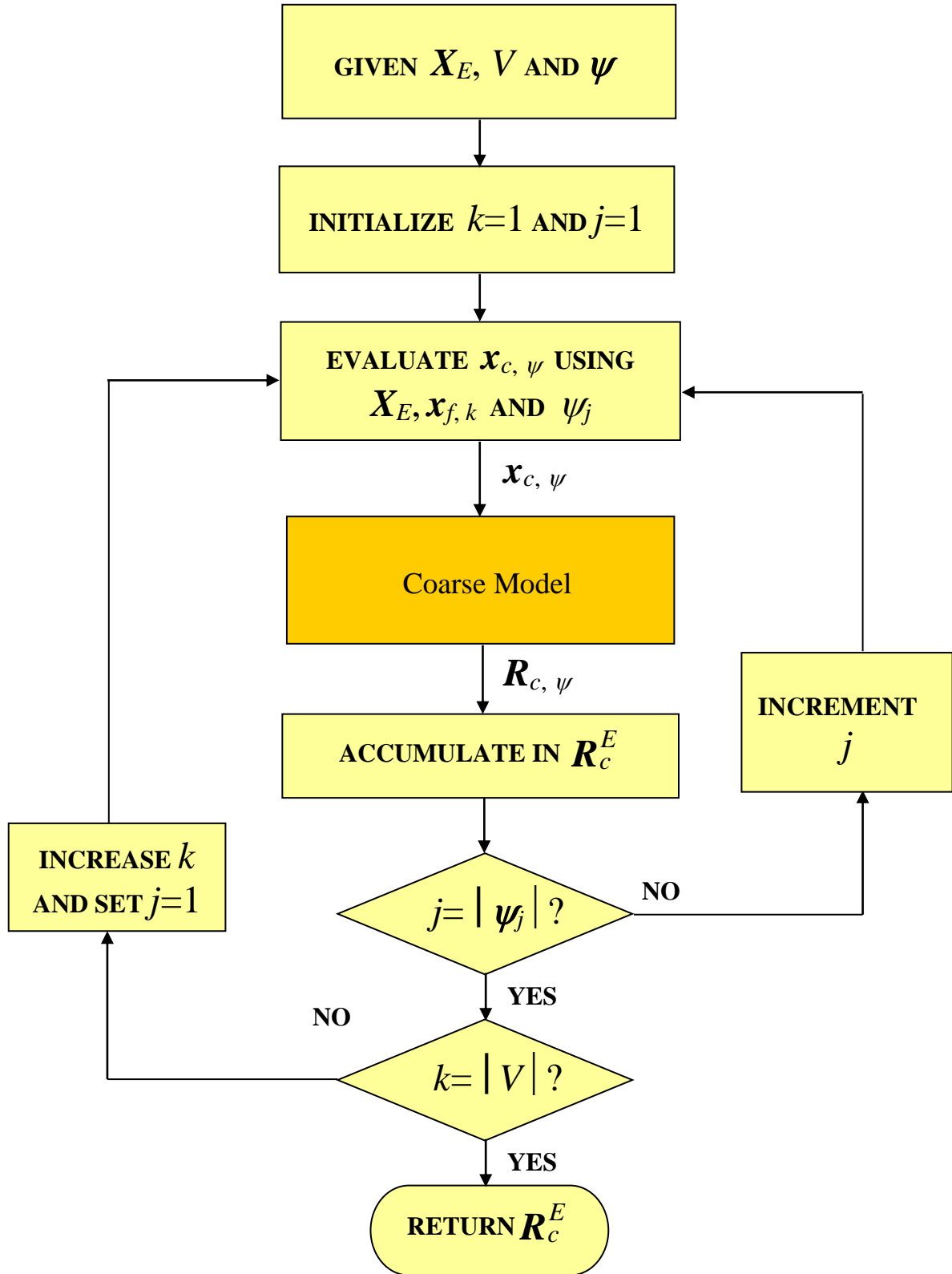
INTERMEDIATE BLOCK (b)



# EXTRACTING THE MAPPING PARAMETERS



INTERMEDIATE BLOCK (c)



# MINMAX OPTIMIZATION OF THE SURROGATE MODEL

