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Abstract

This paper provides an attempt to formulate and to solve the best mechanical alignment problem, which arises in many practical situations when a relatively expensive manufactured product does not meet design specifications and a decision is to be made for partial retreatment of the product. We define and use concepts of regular points, reference points and referenced points for a mechanical design. These points represent important features which must be reproduced subject to tolerances, which are defined w.r.t. various coordinate systems. The algorithm proposed identifies candidates for reworking using minimax optimization. While the concepts introduced and the method presented resulted from a variety of approaches to solving mechanical problems in two dimensions, this class of problem can arise in other areas and further generalization is possible.

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I. INTRODUCTION

An important practical problem is optimal design subject to tolerances [1]. Generally, the problem is to ensure that a design, when manufactured, will satisfy specifications. In many practical situations, however, due to manufacturing errors, a product may not meet the specifications [2]. There are two principal ways of tackling this problem: complete rejection and replacement of the manufactured part, or to align or rework (if possible) the part. In the case of very expensive materials, the latter may be justified. The problem we address in this paper is how to efficiently perform the part alignment process and, if reworking is needed, how to choose the best way to do it. We provide an attempt to formulate and to solve this problem using minimax optimization [3-5].

In Section II, basic definitions and concepts are given and the problem is formulated in terms of minimax optimization. Tolerance regions, error functions and their derivatives are described in Section III, which also contains examples of tolerance regions. The general structure of the computer program is given in Section IV. Section V shows the test results obtained by running the program for several samples [6]. Conclusions and suggestions for further development are given in Section VI.

II. FORMULATION OF THE PROBLEM

Preliminary Concepts

Suppose we have a set of points P in a two-dimensional space

$$P \stackrel{\Delta}{=} \{p_1, p_2, \ldots, p_m\} \quad , \quad m \geq 1, \tag{1}$$

and a system of coordinates $\overline{YOX}$ associated with this set. Let

$$I \triangleq \{1,2,\ldots,m\} \tag{2}$$

be the index set for these points.

The coordinates of a point $p_i \in P$, $i \in I$, may be given either w.r.t. the main origin of the $\overline{Y}O\overline{X}$ system of coordinates or w.r.t. another point of the set P. Let

$$I^0 \triangleq \{1,2,\ldots,n_0\} \quad ; \quad 1 \leq n_0 \leq m, \tag{3}$$

be the index set for points which are referenced to the main origin of the $\overline{Y}O\overline{X}$ system of coordinates. With each point $p_i \in P$, $1 \leq i \leq n_0$, we associate a set of indices I^i such that elements of I^i are indices of points referenced to p_i. The set I^i, $1 \leq i \leq n_0$, may be an empty set or a subset of the set I.

Let $1 \leq \ell \leq n_0$. For $1 \leq i < \ell$ we have

$$I^i \triangleq \emptyset, \tag{4}$$

which means that no points are referenced to $p_i \in P$, $1 \leq i < \ell$. For $\ell \leq i \leq n_0$, we define the following index sets

$$\begin{aligned}
&I^{\ell} \triangleq \{n_0+1, \ldots, n_0+n_\ell\} \quad , \\
&I^{\ell+1} \triangleq \{n_0+n_\ell+1, \ldots, n_0+n_\ell+n_{\ell+1}\}, \\
&I^{\ell+2} \triangleq \{n_0+n_\ell+n_{\ell+1}+1, \ldots, n_0+n_\ell+n_{\ell+1}+n_{\ell+2}\}, \\
&\vdots \\
&I^{i} \triangleq \{n_0+n_\ell+n_{\ell+1}+\ldots+n_{i-1}+1, \ldots, n_0+n_\ell+n_{\ell+1}+\ldots+n_i\}, \ \ell \leq i \leq n_0, \\
&\vdots \\
&I^{n_0} \triangleq \{n_0+\ldots+n_{n_0-1}+1, \ldots, n_0+\ldots+n_{n_0}\}.
\end{aligned} \tag{5}$$

For each point $p_i \in P$, $i \in I$, we introduce a superscript indicating its reference point. For example, $p_i^0(\bar{x}_i^0, \bar{y}_i^0)$, $1 \leq i \leq n_0$, is the ith point of the set P with coordinates $\bar{x}_i^0$, $\bar{y}_i^0$ referenced to the main origin, $p_i^j(\bar{x}_j^0 + \bar{x}_i^j, \bar{y}_j^0 + \bar{y}_i^j)$, $n_0 < i \leq m$, $\ell \leq j \leq n_0$, is the ith point of the set P with coordinates $\bar{x}_i^j$, $\bar{y}_i^j$ referenced to the p_j^0.

Definitions of Subsets of Points

Three disjoint subsets of points can be distinguished in the set P:

- regular points, P_{reg},
- reference points, P_{ref},
- referenced points, P_{refd}.

For each of these subsets there is an associated index set.

Definition 1

A point $p_i^j \in P$ is a regular point if its coordinates are given w.r.t. the main origin of the $\overline{YOX}$ system of coordinates and if it is not a reference point for other points. Formally,

$$j = 0,\ i \in I_{reg} \triangleq \{1,2,\ldots,\ell-1) \implies p_i^j \in P_{reg}. \tag{6}$$

Definition 2

A point $p_i^j \in P$ is a reference point if its coordinates are given w.r.t. the main origin of the $\overline{YOX}$ system of coordinates and if it is treated as an origin for other points. Formally,

$$j = 0,\ i \in I_{ref} \triangleq \{\ell,\ell+1,\ldots,n_0\} \implies p_i^j \in P_{ref}. \tag{7}$$

Definition 3

A point $p_i^j \in P$ is a referenced point if its coordinates are given w.r.t. another point of the subset P_{ref} and if it is not a reference point for other points. Formally,

$$j \in I_{ref},\ i \in I_{refd} \triangleq \{n_0+1,\ldots,m\} \implies p_i^j \in P_{refd}. \tag{8}$$

The concepts and definitions introduced are illustrated in Fig. 1.

Example

Let $P \triangleq \{p_1,p_2,p_3,p_4,p_5,p_6,p_7\}$. From Fig. 1 we can define the following index sets: $I = \{1,2,3,4,5,6,7\}$, the index set for the set P; $I^0 = \{1,2,3\}$, the index set for points referenced to the main origin of the $\overline{YOX}$ system of coordinates; $I^1 = \emptyset$, the index set for points referenced to p_1^0; $I^2 = \{4,5\}$, the index set for points referenced to p_2^0; $I^3 = \{6,7\}$, the index set for points referenced to p_3^0. We can also define the index sets for regular points $I_{reg} = \{1\}$, reference points $I_{ref} = \{2,3\}$ and referenced points $I_{refd} = \{4,5,6,7\}$.

Tolerance Regions

Suppose we have a set R of tolerance regions R_i, $i \in I \triangleq \{1,2,\ldots,m\}$, in the 2-dimensional space.

$$R \triangleq \{R_1,R_2,\ldots,R_m\} \tag{9}$$

and a system of coordinates YOX associated with this set. We can define a one-to-one mapping g which assigns elements $R_i \in R$ to elements $p_i^j \in P$,

$$\{g:P \rightarrow R\}. \tag{10}$$

The sets P, R and the mapping g are shown in the Fig. 2.

The regions $R_i \in R$, $i \in I$, may have different shapes (e.g., circular, rectangular), they may be defined using polar coordinates, rectangular coordinates or combined polar and rectangular coordinates. Dimensions of tolerance regions may be given either w.r.t. the main origin of the YOX system of coordinates (for $R_i = g(p_i)$, $i \in I^0 = I_{reg} \cup I_{ref}$) or w.r.t. the reference point (for $R_i = g(p_i)$, $i \in I_{refd}$).

We can use the same notation indicating the reference points for tolerance regions as for points, e.g., R_i^0, $1 \leq i \leq n_0$, is the ith tolerance region of the set R with dimensions given w.r.t. the main origin of the YOX system of coordinates and R_i^j, $n_0 < i \leq m$, $\ell \leq j \leq n_0$, is the ith tolerance region of the set R with dimensions given w.r.t. the transformed coordinates of p_j^0 from the $\overline{YOX}$ to the YOX system of coordinates.

Transformation of Coordinates

The two systems of coordinates, YOX and $\overline{YOX}$ are related by the following transformation of coordinates

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} \cos\phi_3 & -\sin\phi_3 \\ \sin\phi_3 & \cos\phi_3 \end{bmatrix} \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \end{bmatrix} + \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{11}$$

where

$$\underset{\sim}{\phi}_0 \triangleq [\phi_1 \; \phi_2 \; \phi_3]^T \tag{12}$$

is a set of variables relating the two systems of coordinates (Fig. 3).

Formulation of the Problem

The first step in the solution of the best alignment problem is to find $\underset{\sim}{\phi}_0$ such that the maximum number of points $p_i^j \in P$, $i \in I$, $j \in I_{ref}$ or $j = 0$, are inside or on the boundary of the corresponding $R_i^j \in R$, $R_i^j = g(p_i^j)$. However, the solution to the problem stated above may not be unique and may not be equal to the number of points m.

If it is not possible to find $\underset{\sim}{\phi}_0 = [\phi_1 \; \phi_2 \; \phi_3]^T$ such that all m points are inside or on the boundary of the corresponding tolerance region then it is necessary to delete one or more points in the set P to ensure that all other points satisfy this condition.

In general, the number of variables for the best alignment problem depends on the type of the point (regular, reference or referenced) being a candidate for deletion. The vector of variables $\underset{\sim}{\phi}$ may be extended with new variables, which are the coordinates of reference points if these are the candidates for deleting.

Introducing new variables is necessary when deleting a reference point, because we have to determine the locations of all tolerance regions referenced to it. The general form of the vector of variables for the best alignment problem is

$$\underset{\sim}{\phi}_k^T = [\phi_1\ \phi_2\ \phi_3\ x_{i_1}\ y_{i_1}\ x_{i_2}\ y_{i_2}\ \ldots\ x_{i_k}\ y_{i_k}],\ i_1, i_2, \ldots, i_k \in I_{delref}, \quad (13)$$

where

$$I_{delref} \triangleq I_{ref} \cap I_{del} \quad (14)$$

and k is the cardinality of I_{delref}. The index set I_{del} represents deleted points. For example, if the ith point of the set P ($i \in I_{ref}$) is a candidate for deleting then $\underset{\sim}{\phi}_k$ reduces to the form $\underset{\sim}{\phi}_1^T = [\phi_1\ \phi_2\ \phi_3\ x_i\ y_i]$, $i \in I_{delref}$. If the ith and jth points ($i,j \in I_{ref}$) are candidates for deleting then $\underset{\sim}{\phi}_2^T = [\phi_1\ \phi_2\ \phi_3\ x_i\ y_i\ x_j\ y_j]$, $i,j \in I_{delref}$. If the candidates for deleting are not reference points then $\underset{\sim}{\phi}_k = \underset{\sim}{\phi}_0$.

The best alignment problem can be formulated as

$$\underset{I_{del} \in 2^I}{\text{minimize}}\ n_{del} \triangleq \text{card}\ (I_{del}) \quad (15)$$

subject to the constraint

$$f_i(\underset{\sim}{\phi}_k) \leq 0\ ,\ i \in I' \triangleq (I - I_{del}) \cup I_{delref}, \quad (16)$$

where I is the index set for points p_i which are to be aligned, I_{del} is the index set for points which should be deleted, 2^I is the family of all subsets of the set I, n_{del} is the cardinality of I_{del} and $\underset{\sim}{\phi}_k$ is the vector of optimization variables corresponding to the set I_{del}. Variables ϕ_1, ϕ_2 and ϕ_3 relate the $\overline{YOX}$ and the YOX systems of coordinates and x_{i_k}, y_{i_k} are transformed coordinates (from $\overline{YOX}$ to YOX) of a reference point actually being deleted. The error function $f_i(\Phi_k)$ is associated with the point p_i^j to indicate whether the point p_i^j is in ($f_i(\Phi_k) \leq 0$) or out ($f_i(\Phi_k) > 0$) of the tolerance region $R_i^j = g(p_i^j)$.

Solution to the Problem

The solution to the best alignment problem consists of two stages. The first stage corresponds to a discrete (or combinatorial) minimization of the number of points which should be deleted from the original set of points, and the second stage is an unconstrained minimax optimization of a set of error functions f_i, $i \in I'$, determined by the first stage. The discrete minimization of the first stage is usually implemented as a systematic search of the solution in the family 2^I of all subsets of the set I. It is convenient to represent this search in the form of a multi-level tree in which the root (level 0) corresponds to the set $I_{del} = \emptyset$ ($\emptyset$ denotes the empty set), the level 1 contains all the single element subsets $I^1_{del,1} = \{1\}$, $I^1_{del,2} = \{2\}$, ..., the level 2 all the subsets of I which contain two elements, and so on. The first stage minimization traverses the tree level after level until the solution is found, i.e., until such a subset I_{del} is encountered for which the constraint (16) is satisfied. It can be observed, however, that the minimax optimization of the second stage, which is performed for each step of the first stage search, can be used to eliminate those nodes (and their subtrees) of the search tree which cannot influence the solution. In fact, if the minimax constraint corresponding to the subset I_{del} at a particular level of the search tree is not satisfied then the next level subsets should be derived from the I_{del} of the previous level by adding only the indices of those points which correspond to the active error functions at the solution ϕ_k^* of the minimax optimization since the remaining, nonactive error functions do not affect the solution. This observation is the basis of the implemented combinatorial search algorithm which dynamically creates and traverses the reduced search tree.

Algorithm

The algorithm always starts with the set $I_{del} = \emptyset$ (the root of the tree) and $\phi_k = \phi_0 = 0$. If the minimax objective function

$$F(\phi_0) = \max_{i \varepsilon I} f_i(\phi_0) \tag{17}$$

at the solution ϕ_0^* is non-positive, $F(\phi_0^*) \leq 0$, then ϕ_0^* corresponds to the best alignment solution, and the solution is optimally centered. If $F(\phi_0^*) > 0$, there is no possible alignment of all the points p_i, $i \varepsilon I$, and at least one of the points has to be deleted to allow the alignment of the remaining points. The candidates for deletion are the points for which the corresponding error functions are active at the solution ϕ_k^*, and their indices are attached to the root I_0 of the search tree, creating the level 1 nodes. The search is continued node after node of the created level and the minimax optimization with one less function (except the case of deleting a reference point) is repeated at each node. During the traversal of the level 1 nodes, the new nodes are attached to the search tree creating the next level, and so on, until a subset I_{del} is found for which the minimax constraint is satisfied, $F(\phi_k^*) \leq 0$. It should be noted that corresponding to each node of the search tree there is a unique associated index, and the set I_{del}

determined as the set of indices of the path from the node j to the root of the tree.

III. TOLERANCE REGIONS, ERROR FUNCTIONS AND THEIR DERIVATIVES

To form the error functions for the best alignment problem we have to decide in which system of coordinates these functions will be expressed. It is convenient to choose the system of coordinates associated with the regions, first of all because it is easier to transform points than tolerance regions to the new system of coordinates, and second, because the derivatives of the error functions w.r.t. optimization variables can be easily obtained using transformed coordinates of points and the Jacobian of the transformation.

Preliminary Considerations of Derivatives

For $\underset{\sim}{\phi}_k = \underset{\sim}{\phi}_0$ (no deletions of any points or deletions only of regular or referenced points) the error function is of the form

$$f_i(\underset{\sim}{\phi}_0) = f_i(x_i(\underset{\sim}{\phi}_0), y_i(\underset{\sim}{\phi}_0)), \; i \in I - I_{del} \; . \tag{18}$$

The derivative of f_i w.r.t. $\underset{\sim}{\phi}_0$ can be written as

$$\frac{\partial f_i}{\partial \underset{\sim}{\phi}_0} = \frac{\partial f_i}{\partial x_i}\frac{\partial x_i}{\partial \underset{\sim}{\phi}_0} + \frac{\partial f_i}{\partial y_i}\frac{\partial y_i}{\partial \underset{\sim}{\phi}_0} \; , \tag{19}$$

where

$$\frac{\partial f_i}{\partial \underset{\sim}{\phi}_0} \triangleq \begin{bmatrix} \frac{\partial f_i}{\partial \phi_1} \\ \frac{\partial f_i}{\partial \phi_2} \\ \frac{\partial f_i}{\partial \phi_3} \end{bmatrix}, \quad \frac{\partial x_i}{\partial \underset{\sim}{\phi}_0} \triangleq \begin{bmatrix} \frac{\partial x_i}{\partial \phi_1} \\ \frac{\partial x_i}{\partial \phi_2} \\ \frac{\partial x_i}{\partial \phi_3} \end{bmatrix}, \quad \frac{\partial y_i}{\partial \underset{\sim}{\phi}_0} \triangleq \begin{bmatrix} \frac{\partial y_i}{\partial \phi_1} \\ \frac{\partial y_i}{\partial \phi_2} \\ \frac{\partial y_i}{\partial \phi_3} \end{bmatrix} . \tag{20}$$

The terms of the form $\frac{\partial f_i}{\partial x_i}, \frac{\partial f_i}{\partial y_i}$ depend on the shape of the tolerance region and usually are not very complicated because the function f_i and the coordinates x_i, y_i are expressed in the same system. The terms of the form $\frac{\partial x_i}{\partial \phi_0}, \frac{\partial y_i}{\partial \phi_0}$ depend only on the transformation formula and are the same for the derivatives of all minimax functions. They can be calculated once for the actual point ϕ_0 and used for all functions.

Partial derivatives $\frac{\partial x_i}{\partial \phi_0}$ and $\frac{\partial y_i}{\partial \phi_0}$ can be arranged in a matrix called the Jacobian of the transformation

$$J_0^i \triangleq \begin{bmatrix} \frac{\partial x_i}{\partial \phi_1} & \frac{\partial x_i}{\partial \phi_2} & \frac{\partial x_i}{\partial \phi_3} \\ \frac{\partial y_i}{\partial \phi_1} & \frac{\partial y_i}{\partial \phi_2} & \frac{\partial y_i}{\partial \phi_3} \end{bmatrix}, \qquad (21)$$

which, for the transformation (11) takes the form

$$J_0^i = \begin{bmatrix} 1 & 0 & (-\bar{x}_i \sin\phi_3 - \bar{y}_i \cos\phi_3) \\ 0 & 1 & (\bar{x}_i \cos\phi_3 - \bar{y}_i \sin\phi_3) \end{bmatrix}. \qquad (22)$$

General Formulation of Derivatives

For $\phi_k \neq \phi_0$ (deletion of reference points) depending on the type of a point for which we form the error function we have three cases:

$i \in I - I_{delref} - I^{i_1} - I^{i_2} - \dots - I^{i_k}$ (regular point or reference point not deleted)

The error function is of the form

$$f_i(\phi_k) = f_i(x_i(\phi_0), y_i(\phi_0)). \qquad (23)$$

The derivatives w.r.t. optimization variables ϕ_0 are given by (19) and derivatives w.r.t. additional variables are

$$\frac{\partial f_i}{\partial x_{i_1}} = 0 \ ,$$

$$\vdots$$

$$\frac{\partial f_i}{\partial x_{i_k}} = 0 \ , \qquad (24)$$

$$\frac{\partial f_i}{\partial y_{i_1}} = 0 \ ,$$

$$\vdots$$

$$\frac{\partial f_i}{\partial y_{i_k}} = 0 \ ,$$

where $i_1, \ldots, i_k \in I_{delref}$.

<u>$i \in I_{delref}$ (reference point deleted)</u>

The error function is of the form

$$f_i(\underset{\sim}{\phi}_k) = f_i(x_{i_1}, y_{i_1}, \ldots, x_{i_j}, y_{i_j}, \ldots, x_{i_k}, y_{i_k}) \qquad (25)$$

and the derivatives are

$$\frac{\partial f_i}{\partial \underset{\sim}{\phi}_0} = \underset{\sim}{0} \ ,$$

$$\frac{\partial f_i}{\partial x_{i_j}} = \begin{cases} (\ldots) & \text{for } i = i_j \\ 0 & \text{for } i \neq i_j \end{cases} \qquad (26)$$

$$i_j \in I_{delref}, \ j = 1, \ldots, k.$$

$$\frac{\partial f_i}{\partial y_{i_j}} = \begin{cases} (\ldots) & \text{for } i = i_j \\ 0 & \text{for } i \neq i_j \end{cases}$$

$i \varepsilon I^{i_1} \cup I^{i_2} \cup \ldots \cup I^{i_j} \cup \ldots \cup I^{i_k}$ (referenced point)

The error function is of the form

$$f_i(\underset{\sim}{\phi}_k) = f_i(x_i(\underset{\sim}{\phi}_0), y_i(\underset{\sim}{\phi}_0), x_{i_1}, y_{i_1}, \ldots, x_{i_j}, y_{i_j}, \ldots, x_{i_k}, y_{i_k}) \quad (27)$$

and the derivatives are given by (19) for $\underset{\sim}{\phi}_0$ and w.r.t. additional variables by

$$\frac{\partial f_i}{\partial x_{i_j}} = \begin{cases} (\ldots) & \text{for } i \varepsilon I^{i_j}, \\ 0 & \text{for } i \not\varepsilon I^{i_j}, \end{cases}$$
$$\frac{\partial f_i}{\partial y_{i_j}} = \begin{cases} (\ldots) & \text{for } i \varepsilon I^{i_j}, \\ 0 & \text{for } i \not\varepsilon I^{i_j}. \end{cases} \quad (28)$$

Tables of Error Functions and Derivatives

For the general form of the vector of variables, given by (13), we form error functions and derivatives for three cases:

1) regular point or reference point not deleted;

2) reference point deleted;

3) referenced point.

The general form of derivatives of f_i w.r.t. $\underset{\sim}{\phi}_0$ is given by (19), where the terms of the form $\frac{\partial x_i}{\partial \underset{\sim}{\phi}_0}$, $\frac{\partial y_i}{\partial \underset{\sim}{\phi}_0}$ may be calculated as in (22), and the terms of the form $\frac{\partial f_i}{\partial x_i}$, $\frac{\partial f_i}{\partial y_i}$ are tabulated in Tables I-III for each type of tolerance region. The coordinates x_i, y_i are transformed coordinates of points using the transformation (11). The derivatives of error functions w.r.t. additional variables are also given in these tables.

For the circular tolerance region error, functions and derivatives are given in Table I. As an example, consider three points with circular tolerance regions shown in Fig. 4. Assume that x_2^0, y_2^0 are additional variables, so

$$\underline{\phi}_k = \underline{\phi}_1 = [\phi_1 \quad \phi_2 \quad \phi_3 \quad x_2^0 \quad y_2^0]^T .$$

The error functions and derivatives for p_1^0 (regular point), p_2^0 (reference point deleted), and p_3^2 (referenced point) can be calculated using formulas given in Table I.

For other types of tolerance region the location of a point w.r.t. corresponding tolerance region can be characterized by a system of four linear or nonlinear functions. For a regular point and rectangular tolerance region (Fig. 5), these functions result from the inequalities

$$x_{iL}^0 \leq x_i^0 \leq x_{iU}^0 , \tag{29}$$

$$y_{iL}^0 \leq y_i^0 \leq y_{iU}^0 , \tag{30}$$

and have the form

$$f_i^1 = x_{iL}^0 - x_i^0 , \tag{31}$$

$$f_i^2 = x_i^0 - x_{iU}^0 , \tag{32}$$

$$f_i^3 = y_{iL}^0 - y_i^0 , \tag{33}$$

$$f_i^4 = y_i^0 - y_{iU}^0 . \tag{34}$$

For a regular point and the X-R tolerance region (Fig. 6), the error functions result from (29) and from

$$(R_{iL}^0)^2 \leq (x_i^0)^2 + (y_i^0)^2 \leq (R_{iU}^0)^2 , \tag{35}$$

and f_i^1, f_i^2 have the form of (31), (32), respectively, while f_i^3 and f_i^4 can be expressed as

$$f_i^3 = R_{iL}^0 - \sqrt{(x_i^0)^2 + (y_i^0)^2} , \tag{36}$$

$$f_i^4 = \sqrt{(x_i^0)^2 + (y_i^0)^2} - R_{iU}^0 \ . \qquad (37)$$

Finally, for a regular point and the Y-R tolerance region (Fig. 7), the error functions result from (30) and (35), and f_i^1, f_i^2, f_i^3, f_i^4 are given by (33), (34), (36) and (37), respectively.

For each of these tolerance regions, we represent a point using only one error function. The four error functions may be combined into one using the following function [7-8]

$$f_i = \begin{cases} M \left[\sum_{s \epsilon S} \left[\frac{f_i^s(\underset{\sim}{\phi})}{M} \right]^q \right]^{1/q} & , \text{ for } M \neq 0, \\ 0 & , \text{ for } M = 0, \end{cases} \qquad (38)$$

where

$$M \triangleq \max_{s \epsilon S} (f_i^s) \ , \ S \triangleq \{1,2,3,4\}, \qquad (39)$$

$$q \triangleq \frac{M}{|M|} \ p \begin{cases} 1 < p < \infty \ , \text{ for } M > 0 \ , \\ 1 \leq p < \infty \ , \ \text{ for } M < 0. \end{cases} \qquad (40)$$

The gradient vector of the combined error function is given by

$$\underset{\sim}{\nabla} f_i(\underset{\sim}{\phi}) = \left[\sum_{s \epsilon S} \left[\frac{f_i^s(\underset{\sim}{\phi})}{M} \right]^q \right]^{(1/q)-1} \cdot \sum_{s \epsilon S} \left[\frac{f_i^s(\underset{\sim}{\phi})}{M} \right]^{q-1} \underset{\sim}{\nabla} f_i^s(\underset{\sim}{\phi}), \text{ for } M \neq 0. \qquad (41)$$

From (38) and (41), it can be seen that if $f_i^s(\underset{\sim}{\phi})$, s = 1,2,3,4, are continuous with continous first partial derivatives, then, under the stated conditions, the function f_i is continuous everywhere with continuous first partial derivatives (except possibly when both M = 0 and two or more maxima are equal). For $p \to \infty$, practically $f_i = \max_{s \epsilon S}(f_i^s)$.

The elements of the gradient vector $\underset{\sim}{\nabla} f_i^s$, $s \epsilon S$, for the rectangular and the X-R tolerance regions are given in Tables II and III, respectively. For the Y-R tolerance region error functions and their derivatives are the corresponding entries of tables for the rectangular and the X-R tolerance regions.

IV. COMPUTER IMPLEMENTATION OF THE ALGORITHM

In this section, the FORTRAN program for solving the best alignment problem is briefly described. It has some limitations, resulting from the fact that it was designed for solving particular practical problems (e.g., the number of different shapes of tolerance regions is limited to 4). The program employs a package for linearly constrained minimax optimization [4] available in the form of a library of subroutines.

The structure of the program is shown in Fig. 8. The main segment is BSTALN. It reads the data from the input file SAMPLE, prints the data, calls subroutine FDF at the starting point, calls subroutine PRSRCH and prints the final results. The subroutine PRSRCH organizes the workspace memory for SEARCH and calls SEARCH. The subroutine SEARCH implements the decision-tree structure described in Section II. It calls SOLVER and INSRCH. The subroutine SOLVER prepares parameters and calls the minimax optimization routine MMLA1Q. The subroutine INSRCH eliminates identical entries in the decision-tree structure. The subroutine FDF performs the transformation of coordinates, evaluates error functions and calculates final derivatives. It calls TOLCIR, TOLXY, TOLXR and TOLYR. Subroutines TOLCIR, TOLXY, TOLXR and TOLYR calculate the error function and its derivatives for the circular, rectangular, X-R and Y-R tolerance regions, respectively, using $p = \infty$.

For the purpose of illustration an artificial simple example has been constructed.

Example

Suppose we have a set of points $P \stackrel{\Delta}{=} \{p_1, p_2, p_3, p_4, p_5\}$ and a set of tolerance regions, $R \stackrel{\Delta}{=} \{R_1, R_2, R_3, R_4, R_5\}$. Fig. 9 illustrates the situation before the alignment. Error functions at the starting point $\underset{\sim}{\phi}_0^T = [0.0 \quad 0.0 \quad 0.0]$ are the following

$$f_1 = 2.071 \times 10^{-1} ,$$
$$f_2 = -5.000 \times 10^{-1} ,$$
$$f_3 = 5.000 \times 10^{-1} ,$$
$$f_4 = -5.000 \times 10^{-1} ,$$
$$f_5 = 5.000 \times 10^{-1} .$$

Fig. 10 shows the situation after running the alignment program. The best alignment was found at $\underset{\sim}{\phi}_0^T = [-2.316 \times 10^{-1} \quad -2.792 \times 10^{-1} \quad 4.758 \times 10^{-2}]$ with point 5 deleted. Remaining error functions at the solution are

$$f_1 = -1.540 \times 10^{-1} ,$$
$$f_2 = -1.206 \times 10^{-1} ,$$
$$f_3 = -1.204 \times 10^{-2} ,$$
$$f_4 = -1.204 \times 10^{-2} .$$

V. TEST RESULTS ON PRACTICAL PROBLEMS

The program described in the previous section has been extensively tested. It has been run for seven sets of data [6] supplied by the Woodward Governor Company. The data resulted from practical problems of part alignment in manufactured mechanical systems and have been collected from inspecting actual parts, so the order of error function values represents the real life situation. The points represent holes in one part which have to meet certain specifications when coupled together with another part. Test samples have different numbers of points, varying from 5 to 13 and specified tolerance regions of different shapes. To give an idea of what the samples are like, we describe briefly two simple samples and one interesting sample in more detail.

Sample 1 (Table IV)

This sample has 5 points, 1 with circular and 4 with the rectangular tolerance regions. It has no reference points. Originally, the number of points out-of-tolerance was 4. After 12 iterations of stage 0, the minimum value of the maximum error function was 3.6078×10^{-4}. Three points (1, 3 and 4) have been selected as potential candidates for deleting. It turned out that deleting point number 1 gives the solution for which the remaining error functions are negative and the maximum error at the solution was -6.45668×10^{-4} (after 25 additional minimax iterations).

Sample 2 (Table V)

This sample has 7 points, all with the circular tolerance regions and all referenced to the main origin. Originally, the number of points out-of-tolerance was 5. After 15 iterations, the solution was found with no deletions and the maximum error at the solution was -7.73563×10^{-4}.

Sample 6 (Table VI)

This sample is very interesting: it has 11 points, 4 with circular, 4 with rectangular, 1 with the X-R and 2 with the Y-R tolerance regions.

Five points are referenced to points other than the main origin. Previous work on the best alignment program [6] do not permit a reference point to be deleted (translated). In our approach, any point can be deleted. Originally, there were 2 points out-of-tolerance, and one of them is a reference point. When a point which is an origin for one or more points is found to be out-of-tolerance, there is a good chance that any point referenced to it will also appear to be off location. In this sample, points 7 and 8 are referenced to point 1. Points 1 and 8 were both found to be out-of-tolerance. However, if point 1 was shifted by the amount specified (in other words, if hole number 1 was plugged and re-drilled in the proper location), point 8 would be in-tolerance without any rework needed. Thus, in a practical mechanical sense, there is only one point out-of-tolerance, that being point 1 [2]. Results of running the program for Sample 6 show that indeed deleting reference point 1 (plugging and redrilling hole) implies that all other points will be in-tolerance and the maximum error at the solution is -1.9911×10^{-4}.

We can observe how point 1 was selected for deleting from the details of the solution, given in Table VII. From the results of minimax optimization at stage 0, points 1, 7 and 8 are selected as candidates for deleting. Results of minimax optimization with point 1 deleted (translated) show that a solution can be obtained with only one point deleted.

The results of running the program for all test samples are summarized in Table VIII.

VI. CONCLUSIONS

This paper provides an attempt to formulate and to solve the best mechanical alignment problem using minimax optimization. Results of running the best alignment program for practical problems (Table VIII) confirm the efficiency of our approach. The concepts introduced and the algorithm proposed are described in this paper by tackling a particular mechanical engineering problem. However, this class of problem may come from different sources and further generalization is possible. One natural extension of this approach, which may be very useful from the practical point of view, is considering alignment problems in three dimensions. Another suggestion for further exploration is the investigation of the least pth formulation to reduce the number of minimax functions.

The problem which originated from aligning mechanical designs is here formulated as a general optimization problem and we feel that this approach should prove useful in many other areas where problems of a similar nature may exist.

REFERENCES

[1] J.W. Bandler, P.C. Liu and H. Tromp, "A nonlinear programming approach to optimal design centering, tolerancing and tuning", IEEE Trans. Circuits and Systems, vol. CAS-23, 1976, pp. 155-165.

[2] P. Petersen and R. Johnson, Corporate Engineering, Woodward Governor Company, Rockford, Illinois 61101, 1979, 1980, 1982, private communications.

[3] J. Hald and K. Madsen, "Combined LP and quasi-Newton methods for minimax optimization", Mathematical Programming, vol. 20, 1981, pp. 49-62.

[4] J. Hald (adapted and edited by J.W. Bandler and W.M. Zuberek), "MMLA1Q - a Fortran package for linearly constrained minimax optimization", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-281, 1981.

[5] J.W. Bandler and W.M. Zuberek, "MMLC - a Fortran package for linearly constrained minimax optimization", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-292, 1982.

[6] Woodward Governor Company, Rockford, Illinois 61101. Sample data sent to McMaster University, Feb. 23, 1982.

[7] J.W. Bandler and C. Charalambous, "Theory of generalized least pth approximation", IEEE Trans. Circuit Theory, vol. CT-19, 1972, pp. 287-289.

[8] J.W. Bandler and C. Charalambous, "Practical least pth optimization of networks", IEEE Trans. Microwave Theory Tech., vol. MTT-20, 1972, pp. 834-840.

TABLE I

DERIVATIVES OF ERROR FUNCTIONS FOR CIRCULAR TOLERANCE REGION

	Regular Point or Reference Point Not Deleted $f_i = D - r_i$	Reference Point Deleted $f_i = D - r_i$	Referenced Point $f_i = D_1 - r_i$
$\partial f_i/\partial x_i$	$-A/D$	*	$-A_1/D_1$
$\partial f_i/\partial y_i$	$-B/D$	*	$-B_1/D_1$
$\partial f_i/\partial x_{i_j}$	0	$-A/D, \quad i = i_j$ $0, \quad i \neq i_j$	$A_1/D_1, \quad i \in I^{i_j}$ $0, \quad i \notin I^{i_j}$
$\partial f_i/\partial y_{i_j}$	0	$-B/D, \quad i = i_j$ $0, \quad i \neq i_j$	$B_1/D_1, \quad i \in I^{i_j}$ $0, \quad i \notin I^{i_j}$

$$A = x_{n_i}^0 - x_i^0 \qquad A_1 = x_{n_i}^{i_j} + x_{i_j}^0 - x_i$$

$$B = y_{n_i}^0 - y_i^0 \qquad B_1 = y_{n_i}^{i_j} + y_{i_j}^0 - y_i$$

$$D = (A^2 + B^2)^{1/2} \qquad D_1 = (A_1^2 + B_1^2)^{1/2}$$

* For this case, terms $\frac{\partial x_i}{\partial \underset{\sim}{\phi}_0}$, $\frac{\partial y_i}{\partial \underset{\sim}{\phi}_0}$ are equal to zero, and consequently, $\frac{\partial f_i}{\partial \underset{\sim}{\phi}_0} = \underset{\sim}{0}$.

TABLE II

DERIVATIVES OF ERROR FUNCTIONS FOR RECTANGULAR TOLERANCE REGION

Regular Point or Reference Point not Deleted

f_i^s	$f_i^1 = x_{iL}^0 - x_i^0$	$f_i^2 = x_i^0 - x_{iU}^0$	$f_i^3 = y_{iL}^0 - y_i^0$	$f_i^4 = y_i^0 - y_{iU}^0$
$\partial f_i^s/\partial x_i^0$	-1	1	0	0
$\partial f_i^s/\partial y_i^0$	0	0	-1	1
$\partial f_i^s/\partial x_{i_j}$	0	0	0	0
$\partial f_i^s/\partial y_{i_j}$	0	0	0	0

Reference Point Deleted

f_i^s	$f_i^1 = x_{iL}^0 - x_i^0$	$f_i^2 = x_i^0 - x_{iU}^0$	$f_i^3 = y_{iL}^0 - y_i^0$	$f_i^4 = y_i^0 - y_{iU}^0$
$\partial f_i^s/\partial x_{i_j}$	$-1,\ i = i_j$ $0,\ i \neq ij$	$1,\ i = i_j$ $0,\ i \neq i_j$	0	0
$\partial f_i^s/\partial y_{i_j}$	0	0	$-1,\ i = i_j$ $0,\ i \neq i_j$	$1,\ i = i_j$ $0,\ i \neq i_j$

Referenced Point

f_i^s	$f_i^1 = x_{i_j}^0 + x_{iL}^{i_j} - x_i$	$f_i^2 = x_i - (x_{iU}^{i_j} + x_{i_j}^0)$	$f_i^3 = y_{iL}^{i_j} + y_i^0 - y_i$	$f_i^4 = y_i - (y_{iU}^{i_j} + y_{i_j}^0)$
$\partial f_i^s/\partial x_i$	-1	1	0	0
$\partial f_i^s/\partial y_i$	0	0	-1	1
$\partial f_i^s/\partial x_{i_j}$	$1,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$	$-1,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$	0	0
$\partial f_i^s/\partial y_{i_j}$	0	0	$1,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$	$-1,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$

TABLE III
DERIVATIVES OF ERROR FUNCTIONS FOR X-R TOLERANCE REGION

Regular Point or Reference Point not Deleted

f_i^s	$f_i^1 = x_{iL}^0 - x_i^0$	$f_i^2 = x_i^0 - x_{iU}^0$	$f_i^3 = R_{iL} - E$	$f_i^4 = E - R_{iU}$
$\partial f_i^s/\partial x_i^0$	-1	1	$-x_i^0/E$	x_i^0/E
$\partial f_i^s/\partial y_i^0$	0	0	$-y_i^0/E$	y_i^0/E
$\partial f_i^s/\partial x_{i_j}$	0	0	0	0
$\partial f_i^s/\partial y_{i_j}$	0	0	0	0

Reference Point Deleted

f_i^s	$f_i^1 = x_{iL}^0 - x_i^0$	$f_i^2 = x_i^0 - x_{iU}^0$	$f_i^3 = R_{iL} - E$	$f_i^4 = E - R_{iU}$
$\partial f_i^s/\partial x_{i_j}$	$-1,\ i = i_j$ $0,\ i \neq ij$	$1,\ i = i_j$ $0,\ i \neq i_j$	$-x_i^0/E,\ i = i_j$ $0,\ i \neq i_j$	$x_i^0/E,\ i = i_j$ $0,\ i \neq i_j$
$\partial f_i^s/\partial y_{i_j}$	0	0	$-y_i^0/E,\ i = i_j$ $0,\ i \neq i_j$	$y_i^0/E,\ i = i_j$ $0,\ i \neq i_j$

Referenced Point

f_i^s	$f_i^1 = x_{i_j}^0 + x_{iL}^{i_j} - x_i$	$f_i^2 = x_i - (x_{iU}^{i_j} + x_{i_j}^0)$	$f_i^3 = R_{iL} - D$	$f_i^4 = D - R_{iU}$
$\partial f_i^s/\partial x_i$	-1	1	$-A/D$	A/D
$\partial f_i^s/\partial y_i$	0	0	$-B/D$	B/D
$\partial f_i^s/\partial x_{i_j}$	$1,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$	$-1,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$	$A/D,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$	$-A/D,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$
$\partial f_i^s/\partial y_{i_j}$	0	0	$B/D,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$	$-B/D,\ i \in I^{i_j}$ $0,\ i \notin I^{i_j}$

$A = x_i - x_{i_j}^0$, $B = y_i - y_{i_j}^0$, $D = (A^2 + B^2)^{1/2}$, $E = ((x_i^0)^2 + (y_i^0)^2)^{1/2}$

TABLE IV

DATA FOR SAMPLE 1 [6]

Point	Tolerance Code +	Origin Code *	Actual $\bar{x}$	Actual $\bar{y}$	Tolerances			
					x_N	y_N	r_N	
1	0	0	0.0000	0.0000	0.0000	0.0000	0.0010	
					x_L	x_U	y_L	y_U
2	12	0	-0.8800	1.3682	-0.8780	-0.8750	1.3690	1.3720
3	12	0	0.6589	0.7499	0.6610	0.6630	0.7500	0.7520
4	12	0	0.8990	-0.4414	0.8990	0.9010	-0.4410	-0.4380
5	12	0	-0.5635	-1.5254	-0.5650	-0.5620	-1.5250	-1.5220

+ The tolerance code is one of four (0, 12, 13, 23), where

0 - the code for the circular tolerance region,
12 - the code for the rectangular tolerance region,
13 - the code for the X-R tolerance region,
23 - the code for the Y-R tolerance region.

* Any point with an origin code of 0 is referenced to the main origin of $\bar{x}$ = 0.0, $\bar{y}$ = 0.0. Any other origin code refers to the point by that number on the same sample. For instance, for an origin code of 4, the actual $\bar{x}$ and $\bar{y}$ dimensions are measured from the actual $\bar{x}$ and $\bar{y}$ dimensions of point number 4.

TABLE V

DATA FOR SAMPLE 2 [6]

Point	Tolerance Code +	Origin Code *	Actual $\bar{x}$	Actual $\bar{y}$	Tolerances		
					x_N	y_N	r_N
1	0	0	0.0000	-0.0001	0.0000	0.0000	0.0050
2	0	0	-0.6412	1.1080	-0.6405	1.1094	0.0025
3	0	0	-1.2778	-0.0052	-1.2810	0.0000	0.0025
4	0	0	-0.6295	-1.1101	-0.6405	-1.1094	0.0025
5	0	0	0.6499	-1.1055	0.6405	-1.1094	0.0025
6	0	0	1.2846	0.0083	1.2810	0.0000	0.0025
7	0	0	0.6393	1.1126	0.6405	1.1094	0.0025

+ see p. 25 for code explanations.

* see p. 25 for code explanations.

TABLE VI

DATA FOR SAMPLE 6 [6]

Point	Tolerance Code +	Origin Code *	Actual $\bar{x}$	Actual $\bar{y}$	Tolerances			
					x_N	y_N	r_N	
1	0	0	2.3970	-0.9508	2.3950	-0.9500	0.0010	
2	0	0	-1.6955	-1.9621	-1.6960	-1.9620	0.0010	
					x_L	x_U	y_L	y_U
3	12	0	0.6620	0.7507	0.6610	0.6630	0.7500	0.7520
4	12	0	0.8998	-0.4393	0.8990	0.9010	-0.4410	-0.4380
					y_L	y_U	R_L	R_U
5	23	0	-0.5629	-1.5231	-1.5260	-1.5210	1.6225	1.6260
					x_L	x_U	y_L	y_U
6	12	0	-0.8773	1.3700	-0.8780	-0.8750	1.3690	1.3720
					x_N	y_N	r_N	
7	0	1	-2.8646	3.5015	-2.8640	3.5010	0.0010	
					x_L	x_U	y_L	y_U
8	12	1	-0.8764	2.3274	-0.8750	-0.8710	2.3250	2.3290
					x_N	y_N	r_N	
9	0	4	0.6653	-0.7855	0.6650	-0.7860	0.0010	
					y_L	y_U	R_L	R_U
10	23	5	-0.9642	1.0227	1.0210	1.0260	1.4053	1.4073
					x_L	x_U	R_L	R_U
11	13	6	-0.0641	-1.1348	-0.0660	-0.0640	1.1358	1.1378

+, * See p. 25 for code explanations.

TABLE VII

RESULTS OF BEST MINIMAX ALIGNMENT FOR SAMPLE 6 [6]

Error Function	Values of Error Functions+		
	Starting Point	Stage 0 Optimization (no deletions)	Optimization with Point 1 Deleted (translated)
1	1.1540659×10^{-3}	<u>7.8766877×10^{-4}</u>	$-6.0836163 \times 10^{-4}$*
2	$-4.9009805 \times 10^{-4}$	7.8054088×10^{-4}	$-3.1859860 \times 10^{-4}$
3	$-7.0000000 \times 10^{-4}$	$-6.7451522 \times 10^{-4}$	$-6.0366698 \times 10^{-4}$
4	$-8.0000000 \times 10^{-4}$	$-5.1145712 \times 10^{-4}$	$-6.0460585 \times 10^{-4}$
5	$-1.2887855 \times 10^{-3}$	$-4.1859431 \times 10^{-4}$	$-1.3816043 \times 10^{-3}$
6	$-7.0000000 \times 10^{-4}$	$-6.1087476 \times 10^{-4}$	<u>$-1.9911453 \times 10^{-4}$</u>
7	$-2.1897503 \times 10^{-4}$	<u>7.8766877×10^{-4}</u>	<u>$-1.9911453 \times 10^{-4}$</u>
8	1.4000000×10^{-3}	<u>7.8766877×10^{-4}</u>	<u>$-1.9911453 \times 10^{-4}$</u>
9	$-4.1690481 \times 10^{-4}$	$-2.2387620 \times 10^{-4}$	<u>$-1.9911453 \times 10^{-4}$</u>
10	$-2.5929437 \times 10^{-4}$	$-2.7637365 \times 10^{-4}$	<u>$-1.9911453 \times 10^{-4}$</u>
11	$-1.0000000 \times 10^{-4}$	6.1249301×10^{-4}	$-4.0926333 \times 10^{-4}$

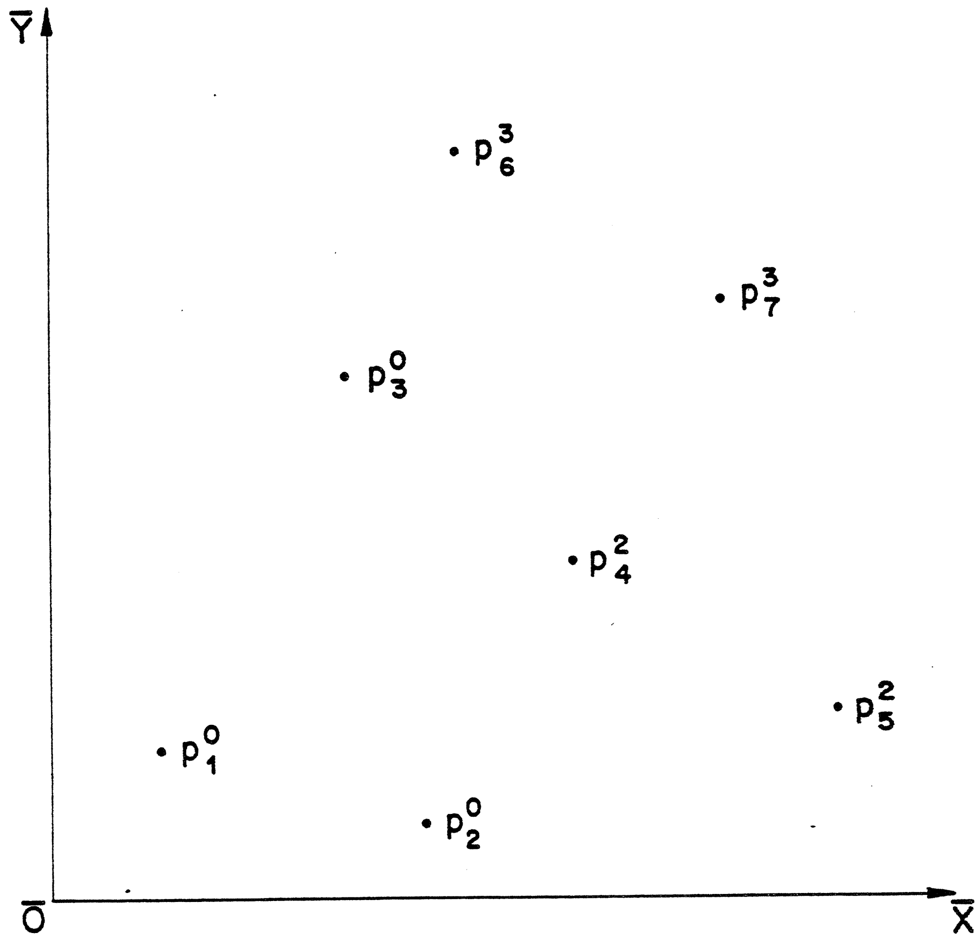
+ Maximum error functions are underlined

* This error function value corresponds to the new location of point 1.

TABLE VIII

RESULTS OF RUNNING THE BEST ALIGNMENT PROGRAM
ON DATA SUPPLIED BY THE WOODWARD GOVERNOR COMPANY [6]

Sample No.	Total No. of Points	No. of Points Originally Out of Tolerance	Results (Points Deleted)	Comments	CYBER 170/730 Execution Time in Seconds
1	5	4	1	Reg. Point Deleted	0.7
2	7	5	0	No Deletions	0.4
3	11	2	1	Ref. Point Deleted	0.9
4	11	3	2	Reference and Reg. Points Deleted	2.8
5	11	3	2	Reference and Reg. Points Deleted	1.5
6	11	2	1	Reference Point Deleted	1.2
7	13	3	3	Regular Points Deleted	3.6



$p_1^0(\bar{x}_1^0, \bar{y}_1^0)$ $\quad$ $p_4^2(\bar{x}_4^2 + \bar{x}_2^0, \bar{y}_4^2 + \bar{y}_2^0)$

$p_2^0(\bar{x}_2^0, \bar{y}_2^0)$ $\quad$ $p_5^2(\bar{x}_5^2 + \bar{x}_2^0, \bar{y}_5^2 + \bar{y}_2^0)$

$p_3^0(\bar{x}_3^0, \bar{y}_3^0)$ $\quad$ $p_6^3(\bar{x}_6^3 + \bar{x}_3^0, \bar{y}_6^3 + \bar{y}_3^0)$

$p_7^3(\bar{x}_7^3 + \bar{x}_3^0, \bar{y}_7^3 + \bar{y}_3^0)$

Fig. 1 The set of points P and the $\overline{YOX}$ system of coordinates associated with it.

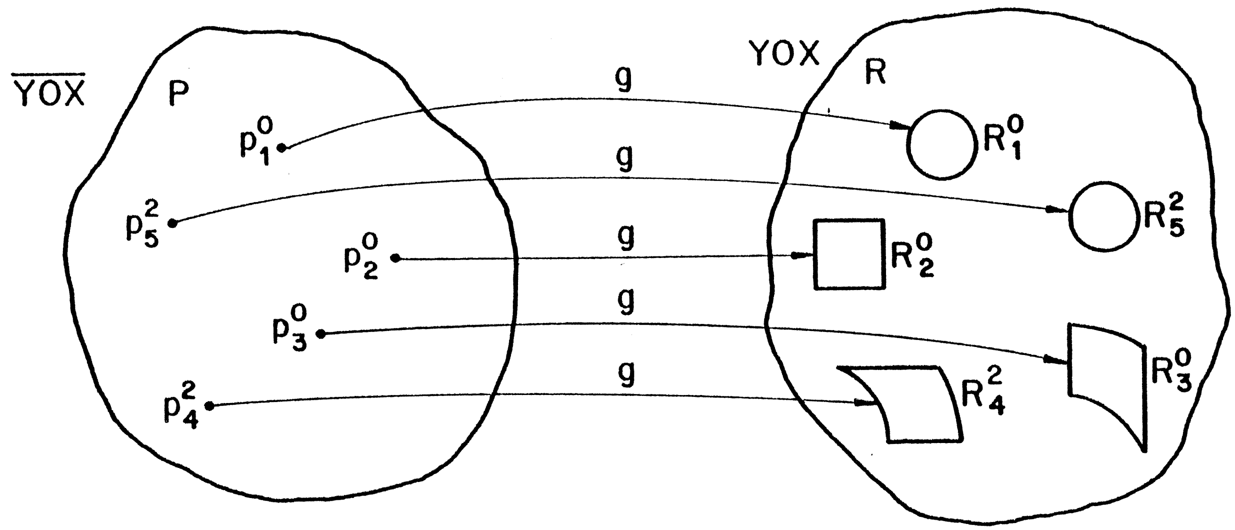


Fig. 2 The mapping g: P → R.

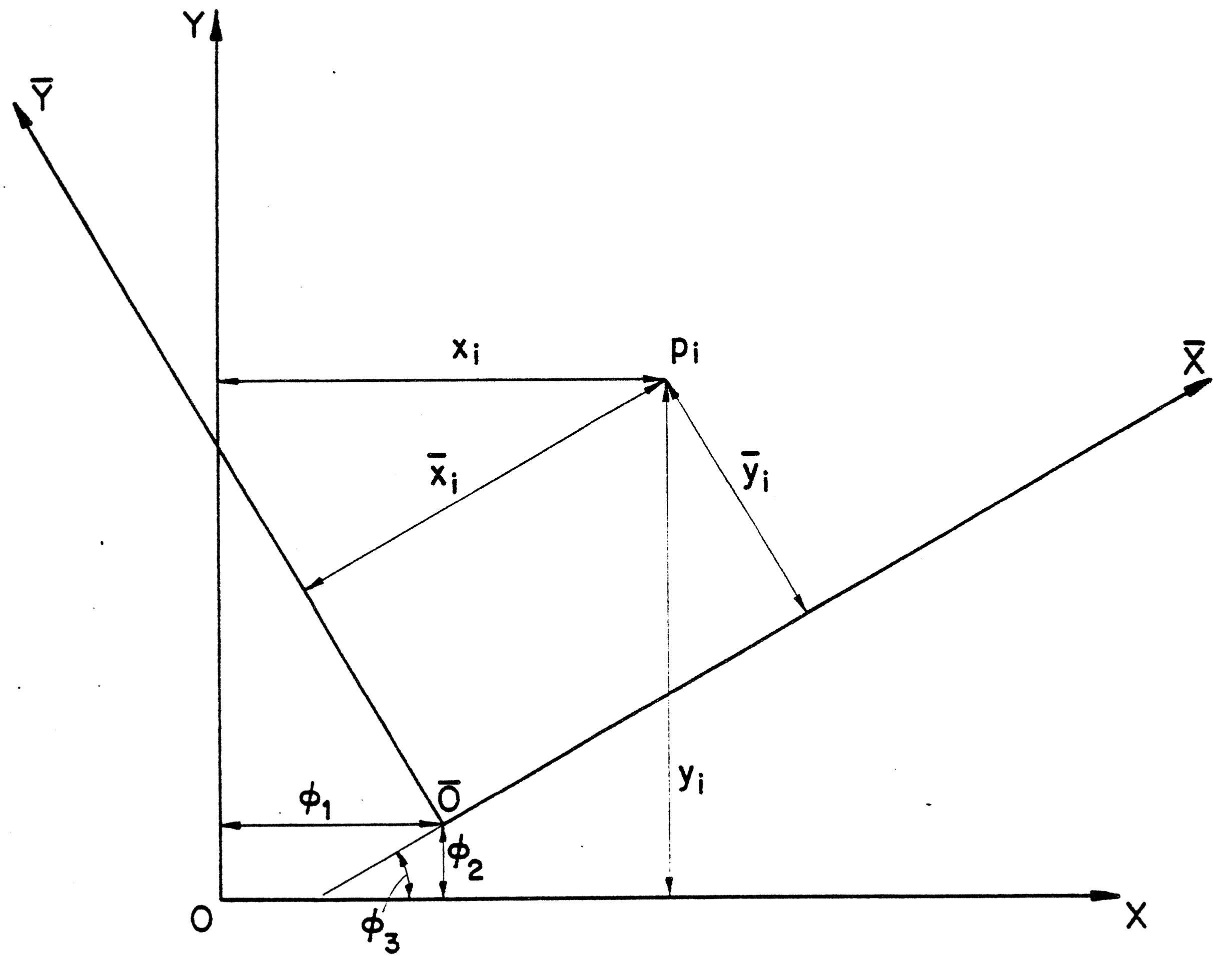


Fig. 3 Transformation of coordinates relating the two systems of coordinates.

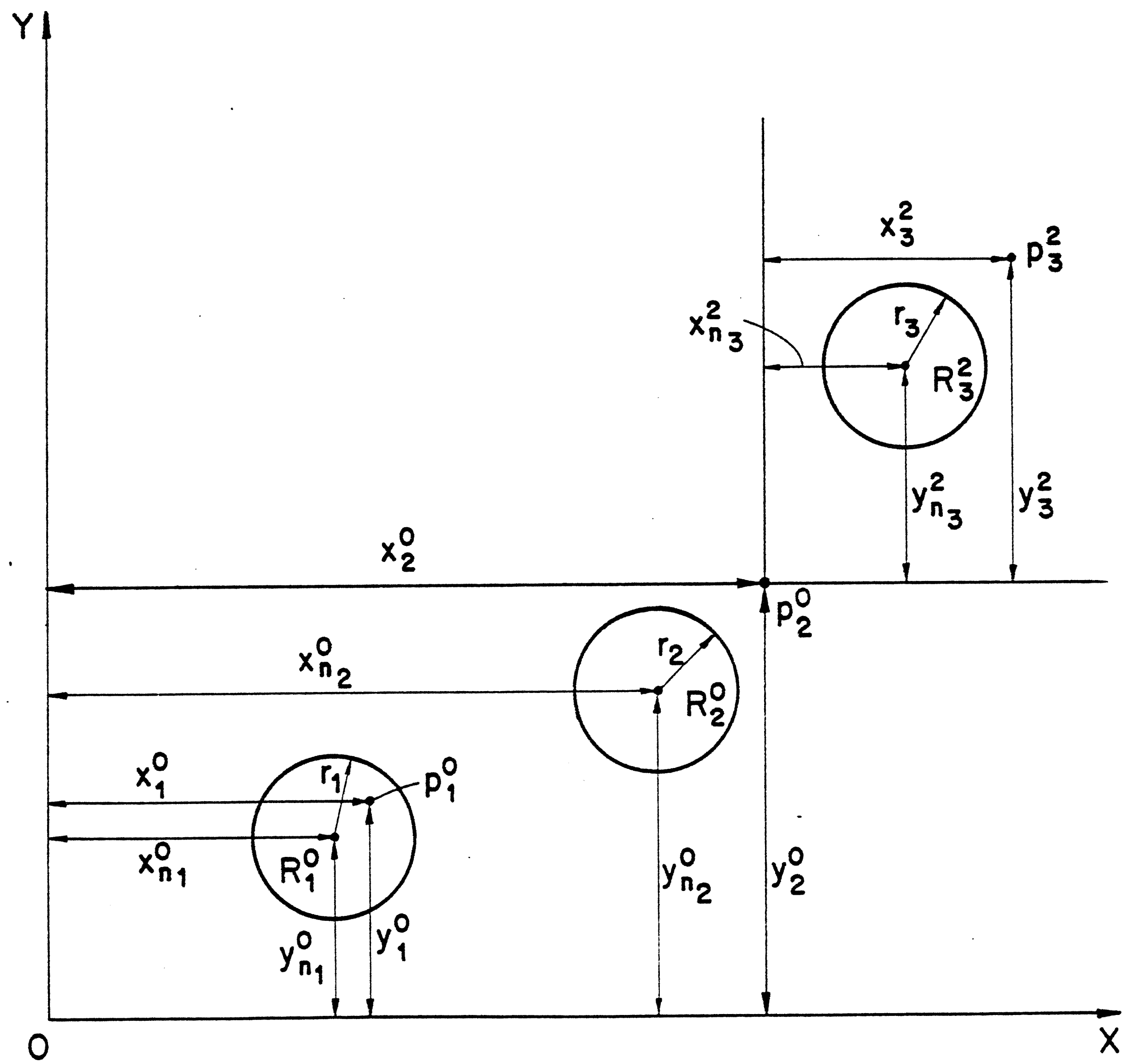


Fig. 4 Points with circular tolerance regions.

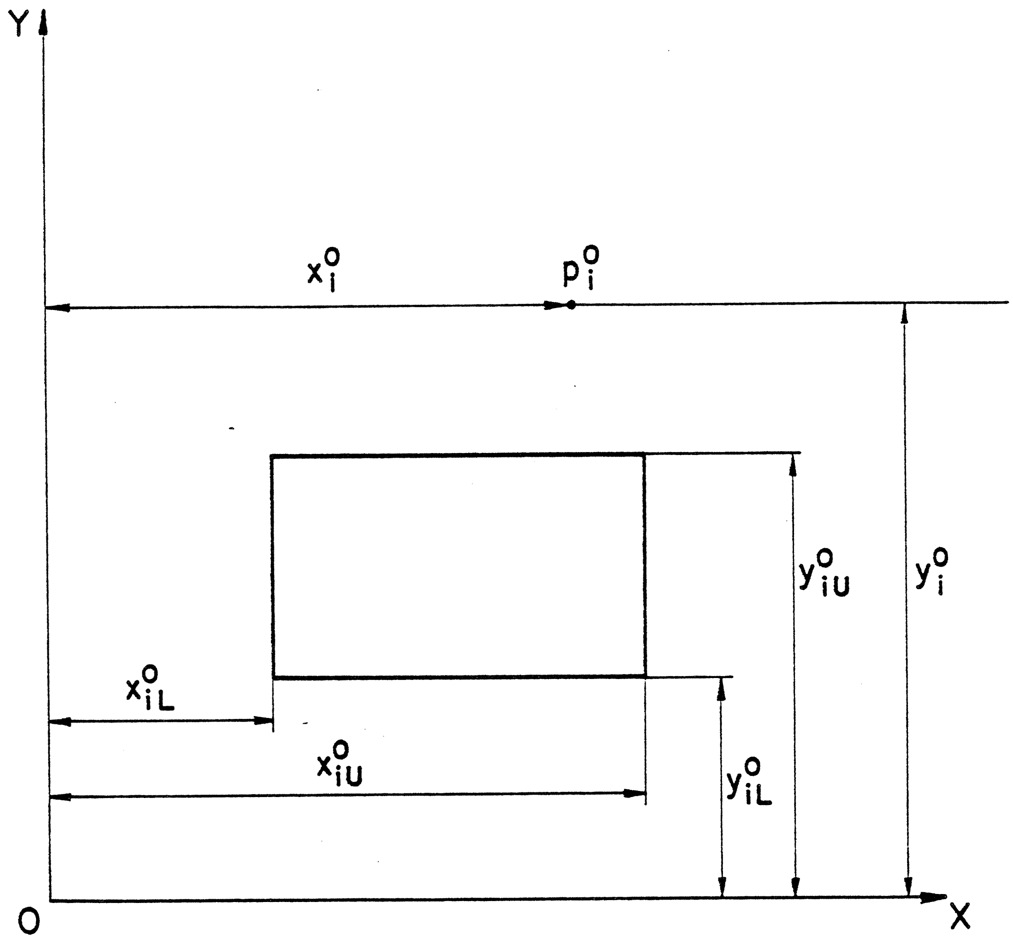


Fig. 5 Regular point with the rectangular tolerance region.

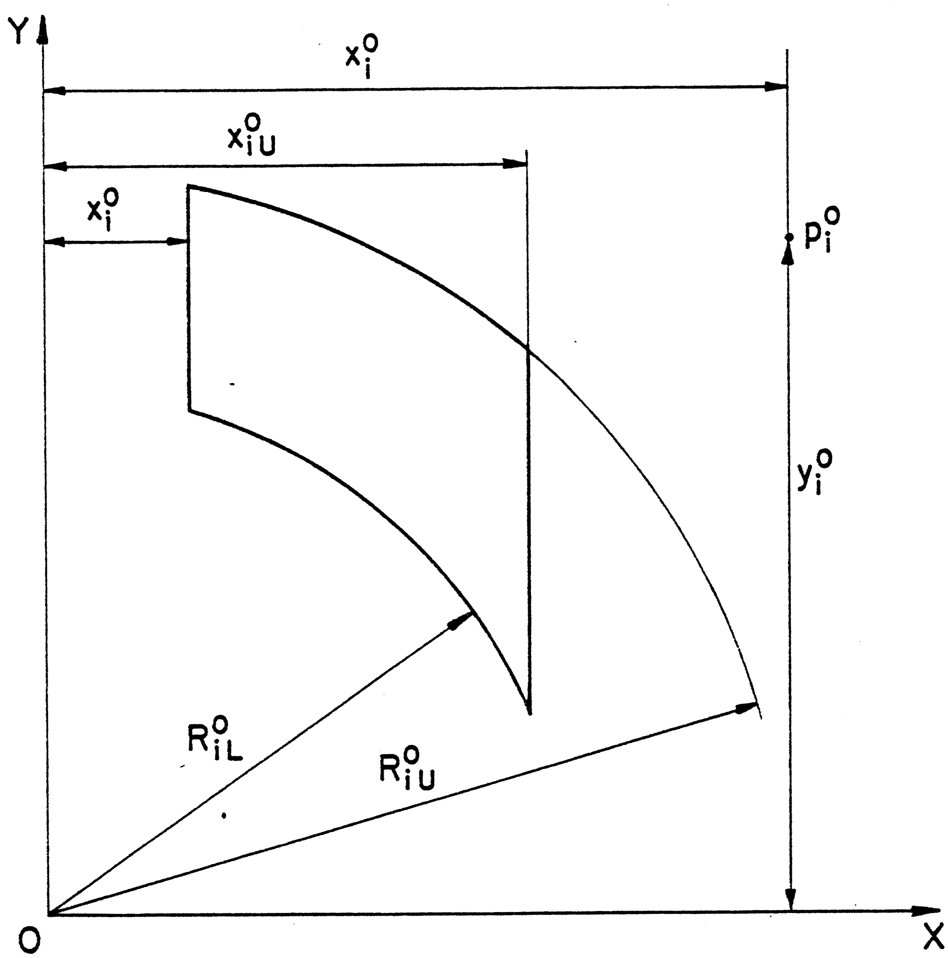


Fig. 6 Regular point and the X-R tolerance region.

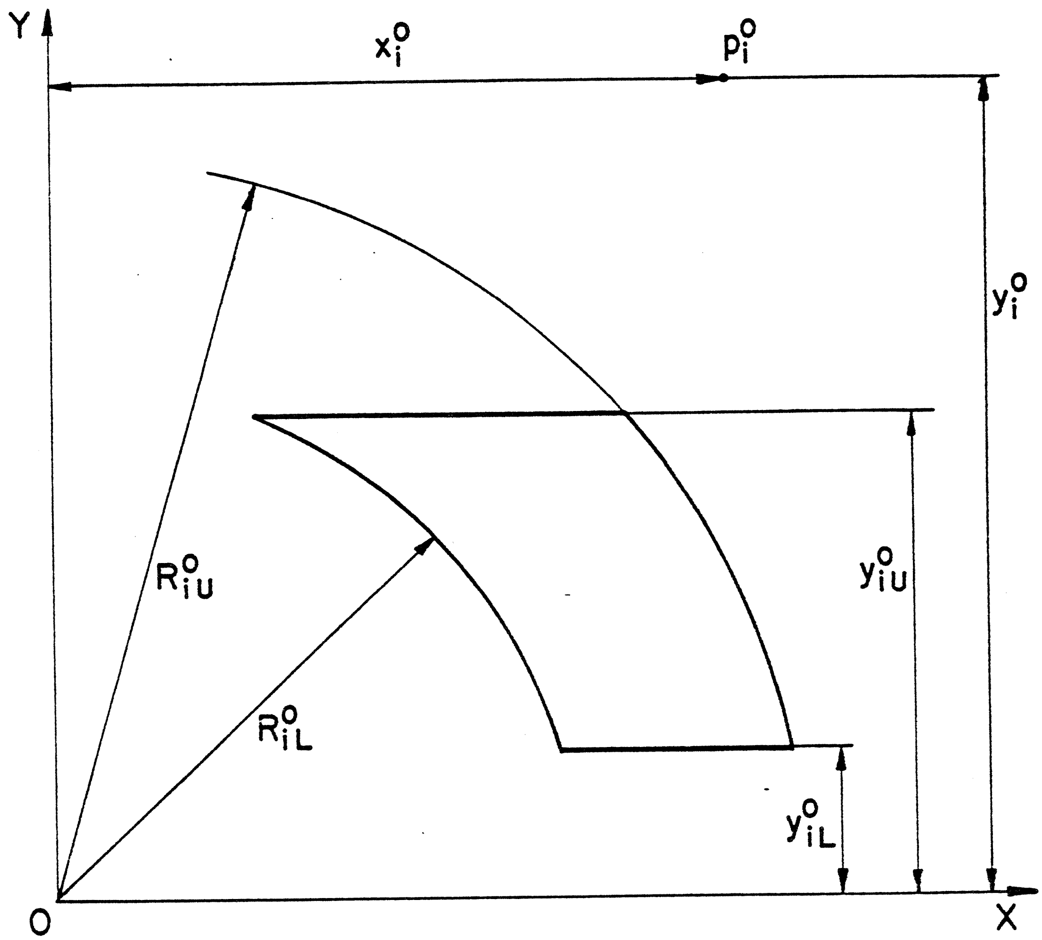


Fig. 7 Regular point and the Y-R tolerance region.

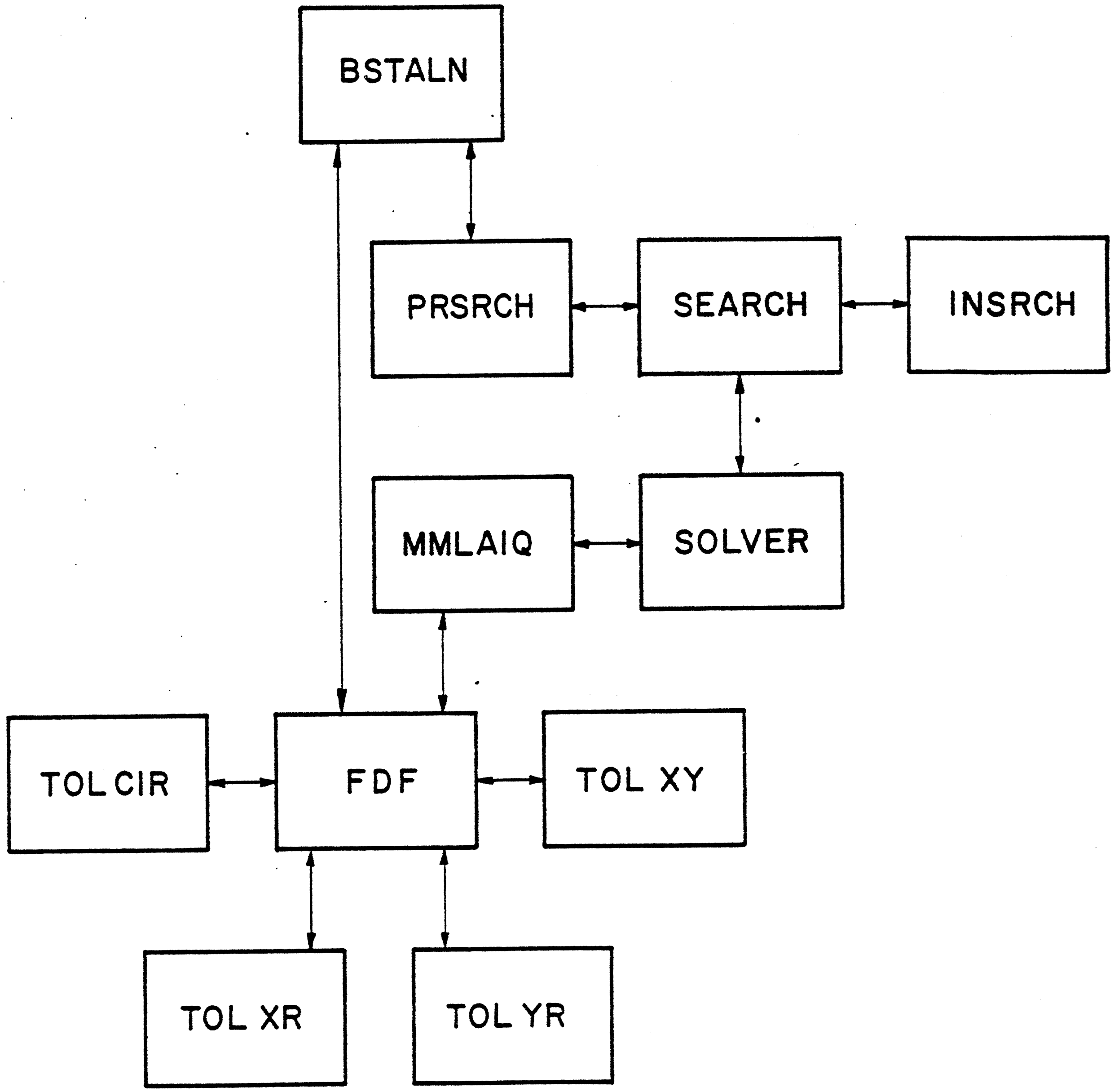


Fig. 8 Structure of the program for the best alignment problem.

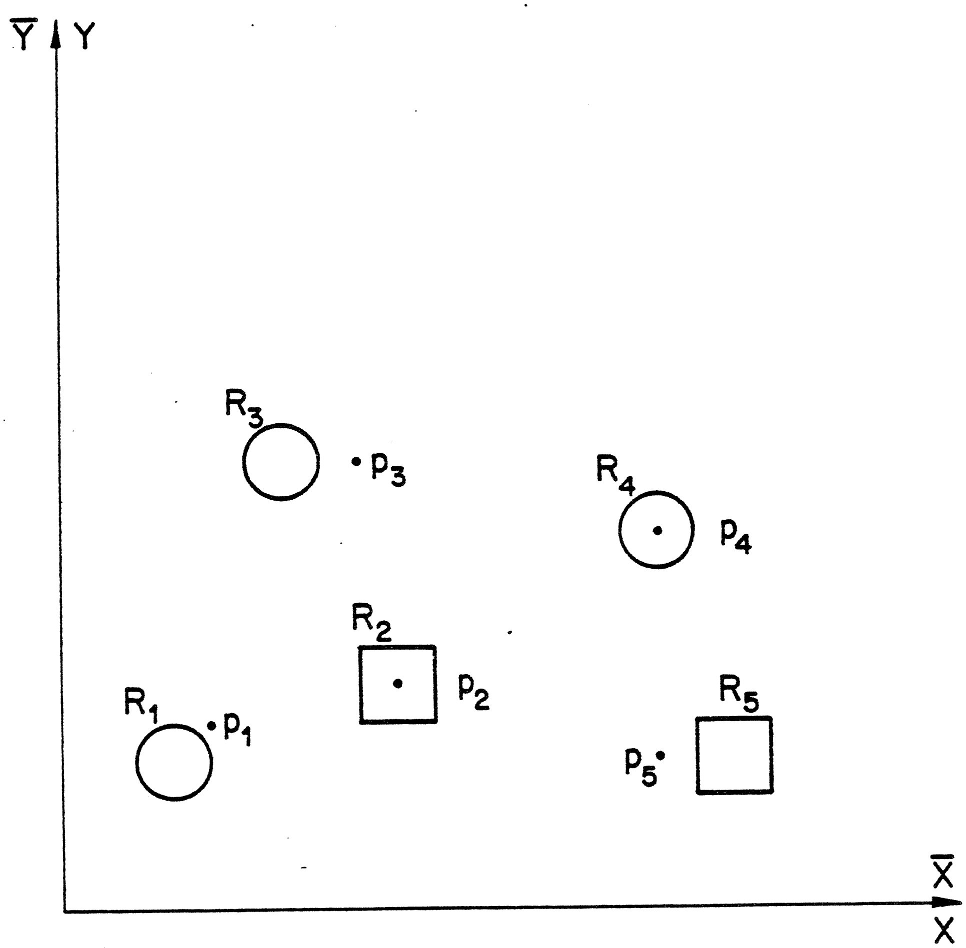


Fig. 9 Points and tolerance regions before alignment.

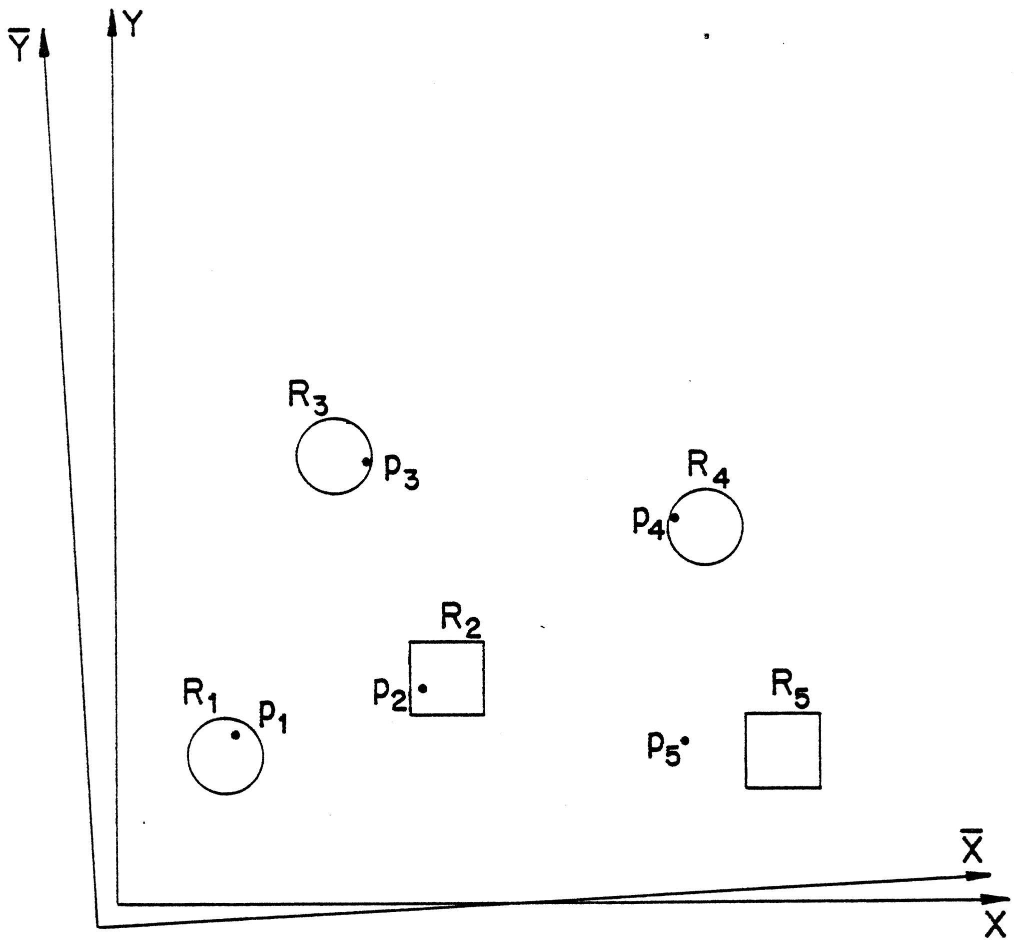


Fig. 10 Results of running the alignment program.

SOC-301

A MINIMAX APPROACH TO THE BEST MECHANICAL ALIGNMENT PROBLEM

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Revised:

Key Words: Mechanical design, computer-aided design, tuning, alignment, minimax optimization

Abstract: This paper provides an attempt to formulate and to solve the best mechanical alignment problem, which arises in many practical situations when a relatively expensive manufactured product does not meet design specifications and a decision is to be made for partial retreatment of the product. We define and use concepts of regular points, reference points and referenced points for a mechanical design. These points represent important features which must be reproduced subject to tolerances, which are defined w.r.t. various coordinate systems. The algorithm proposed identifies candidates for reworking using minimax optimization. While the concepts introduced and the method presented resulted from a variety of approaches to solving mechanical problems in two dimensions, this class of problem can arise in other areas and further generalization is possible.

Description:

Related Work:

Price: $10.00.