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LOCATION OF FAULT REGIONS IN ANALOG CIRCUITS

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Abstract

The multiple-fault location problem for analog circuits is treated on the basis of the nodal equations. The availability of voltage measurements due to current excitations is assumed by the method. Topological restrictions on the possibility of fault location for a given set of measurements are formulated. The emphasis in this paper is on locating subnetworks or regions containing all the faults of the network. Two algorithms are presented for this purpose. Coates flow-graph representation of a network is used for topological considerations.

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I. INTRODUCTION

Testing of analog circuits with the aim of fault location is important in network analysis. There are different approaches to the problem depending on the information available from tests conducted on the network. Generally, the network topology is known and we try to identify the faulty elements and evaluate them. If the number of measurements is large enough we can evaluate all elements and single out the faulty ones [1,2]. However, when the number of measurements is limited we can use various methods to predict regions where faults may appear [3,4]. To verify whether a predicted region contains all the faults, the multiple-fault location method based on the multiport description of a network can be used [5].

In this paper, we present a method based on the nodal equations which extends the possibilities of the above-mentioned multiport method. Topological restrictions on multiple-fault location are discussed. These are effectively used to locate faulty regions. Some illustrative examples and practical remarks for effective calculations are given.

II. MULTIPLE-FAULT VERIFICATION BY NODAL EQUATIONS

In this section we discuss the method of multiple fault location on the basis of the nodal equations. The principal difference between the nodal and the multiport approach is that in the multiport approach we aim to find changes in element values whereas in the nodal method we design the changes in nodal currents only. Changes in element values can be computed by the nodal method after the network topology is considered.

Nodal Equations for Faulty Network

Let us assume that the network has n+1 nodes, m of them accessible, and $f < m$ is the number of faulty elements. The nodal equations for the nominal values of the elements have the form

$$\underset{\sim}{Y}\,\underset{\sim}{V} = \underset{\sim}{J}. \tag{1}$$

For the faulty network, assuming the same excitations, we obtain

$$(\underset{\sim}{Y} + \Delta\underset{\sim}{Y})(\underset{\sim}{V} + \Delta\underset{\sim}{V}) = \underset{\sim}{J}. \tag{2}$$

Thus

$$\underset{\sim}{Y}\,\Delta\underset{\sim}{V} = -\,\Delta\underset{\sim}{Y}\,\underset{\sim}{V}', \tag{3}$$

where $\underset{\sim}{V}' = \underset{\sim}{V} + \Delta\underset{\sim}{V}$ is the vector of nodal voltages in the faulty network. We can compute $\Delta\underset{\sim}{V}$ assuming that $\underset{\sim}{Y}$ is nonsingular and obtain

$$\Delta\underset{\sim}{V} = -\,\underset{\sim}{Y}^{-1}\,\Delta\underset{\sim}{Y}\,\underset{\sim}{V}'. \tag{4}$$

Let us denote $\Delta\underset{\sim}{J} = -\,\Delta\underset{\sim}{Y}\,\underset{\sim}{V}'$. $\Delta\underset{\sim}{J}$ represents changes in nodal currents caused by faulty elements. The relation (4) becomes

$$\Delta\underset{\sim}{V} = \underset{\sim}{Y}^{-1}\,\Delta\underset{\sim}{J}. \tag{5}$$

We can assume that a few elements are faulty, in which case $\Delta\underset{\sim}{J}$ has the form

$$\Delta\underset{\sim}{J} = \begin{bmatrix} \underset{\sim}{0} \\ \Delta\underset{\sim}{J}^F \\ \underset{\sim}{0} \end{bmatrix}. \tag{6}$$

Assuming that the first m nodal voltages can be measured we obtain

$$\begin{bmatrix} \Delta\underset{\sim}{V}^M \\ \\ \Delta\underset{\sim}{V}^{N-M} \end{bmatrix} = \underset{\sim}{Y}^{-1} \begin{bmatrix} \underset{\sim}{0} \\ \Delta\underset{\sim}{J}^F \\ \underset{\sim}{0} \end{bmatrix}, \tag{7}$$

where N indicates the set of all nodes and M the set of measurement nodes. Hence,

$$\Delta\underset{\sim}{V}^M = \underset{\sim}{Z}_{MF}\,\Delta\underset{\sim}{J}^F, \tag{8}$$

where

$$\underset{\sim}{Y}^{-1} = \begin{bmatrix} \underset{\sim}{Z}_{MN} \\ \\ \underset{\sim}{Z}_{N-M,N} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_{M1} & \underset{\sim}{Z}_{MF} & \underset{\sim}{Z}_{M2} \\ \\ \underset{\sim}{Z}_{N-M,1} & \underset{\sim}{Z}_{N-M,F} & \underset{\sim}{Z}_{N-M,2} \end{bmatrix} . \tag{9}$$

Relation (8) has to be satisfied when the set F of network nodes includes all nodes associated with faulty elements in the network.

Reduction of the Number of Equations

It is clear from relation (8) that in order to design $\Delta \underset{\sim}{J}^F$ we must have at least 1 + card F measurement nodes. This may cause some redundancies in the case of isolated faults. If there is an isolated fault in the network it causes changes in two elements of the $\Delta \underset{\sim}{J}^F$ vector. In the example shown in Fig. 1 we have $\Delta J_k^F = - \Delta Y_e U^1 = - \Delta J_j^F$. In such a case vector $\Delta \underset{\sim}{J}^F$ will contain variables which are not independent. We can transform the equation (8) to reduce the column rank of the coefficient matrix $\underset{\sim}{Z}_{MF}$. The reduction realized depends on the location of different faults. Let us discuss the following two cases.

1) The case of isolated faults

If an isolated fault appears between nodes k and j (see Fig. 1) then equation (8) can be written in the form

$$\Delta \underset{\sim}{V}^M = [\underset{\sim}{a}_1 \ \dots \ \underset{\sim}{a}_k \ \dots \ \underset{\sim}{a}_j \ \dots \ \underset{\sim}{a}_f] \begin{bmatrix} \Delta J_1^F \\ \vdots \\ \Delta J_k^F \\ \vdots \\ -\Delta J_k^F \\ \vdots \\ \Delta J_f^F \end{bmatrix} \tag{10}$$

or, after summing columns $\underset{\sim}{a}_k$ and $\underset{\sim}{a}_j$ and deleting column $\underset{\sim}{a}_j$,

$$\Delta \underset{\sim}{V}^M = [\underset{\sim}{a}_1 \cdots \underset{\sim}{a}_k - \underset{\sim}{a}_j \cdots \underset{\sim}{a}_{j-1}\ \underset{\sim}{a}_{j+1} \cdots \underset{\sim}{a}_f] \begin{bmatrix} \Delta J_1^F \\ \vdots \\ \Delta J_k^F \\ \vdots \\ \Delta J_{j-1}^F \\ \Delta J_{j+1}^F \\ \vdots \\ \Delta J_f^F \end{bmatrix} . \qquad (11)$$

2) <u>The case of connected faults</u>

If connected faults form a subtree in the network then the number of variables in $\Delta \underset{\sim}{J}^F$ can be reduced by one in similar way to Case 1. The reduction holds for every connected subgraph formed by faulty elements. If the subgraph contains a circuit then the number of variables can not be reduced.

The method described has the following advantages as compared with the multiport methods [5].

1. Fault regions can be located even if fault elements form a circuit or cutset.
2. We do not face the situation of block dependent systems when only one element in a circuit or cutset is not faulty.

It should be noted, however, that the evaluation of faulty elements on the basis of identified changes in current excitations representing the faults is not always possible. For example, when only one element in a circuit is not faulty, then the problem of identification is not solvable, which is a simple consequence of the transformation of current excitations (cf. [6]).

The nodal approach is restricted to two-terminal elements and voltage controlled current sources only, but it can be extended to any linear active network using the modified nodal description [7].

III. TOPOLOGICAL RESTRICTIONS

In this section we will discuss the problem of the placement of measurements in the network to make possible the identification of a certain set of faults on the basis of network topology.

Necessary conditions presented in [8] will be modified to conditions which are almost sufficient for obtaining a unique solution of equation (8). According to Corollary 2.1 and Lemma 2 of Liu, Lin and Huang [9], if equation (8) is consistent, and rank $\tilde{Z}_{MX} = f + 1$ for any set of columns X of $\tilde{Z}_{MN}$ such that card $X = f + 1$, then (8) has unique solution $\Delta\tilde{J}^{F}$ almost surely.

In practice, however, this requirement is too strong, especially if we are interested in a certain set of faults F. We can formulate the following result.

<u>Result 1</u>

If equation (8) is consistent and rank $\tilde{Z}_{MF_x} = f + 1$, where F_x is the set of columns of the matrix $\tilde{Z}_{MN}$,

$$F_x = F \cup x, \qquad \forall\, x \in N-F$$

then (8) has a unique solution almost surely.

The condition stated in Result 1 is equivalent to the existence of a square, nonsingular (f+1) x (f+1) submatrix of $\tilde{Z}_{MF_x}$.

Let $\tilde{Z}_{EF_x}$ denote a square submatrix of $\tilde{Z}_{MF_x}$ and $\tilde{Y}\,(F_x|E)$ denote the submatrix of $\underset{\sim}{Y}$ obtained by removing F_x rows and E columns. Using the equivalence

$$\det \underset{\sim}{Z}_{EF_x} \neq 0 \iff \det \underset{\sim}{Y}(F_x|E) \neq 0 \qquad (12)$$

we can find topological restrictions for the fault location problem. We can use the approach presented in [10]. Let us assume that the topological equations for the nodal admittance matrix and the Coates graph representation of the network are

$$\underset{\sim}{Y} = \underset{\sim}{\lambda}_- \underset{\sim}{Y}_e \underset{\sim}{\lambda}_+, \qquad (13)$$

where the element ij of $\underset{\sim}{\lambda}_-$ is equal to 1 if the jth edge is directed towards the ith vertex, otherwise zero, and the element ij of $\underset{\sim}{\lambda}_+$ is equal to 1 if the jth edge is directed away from the ith vertex, otherwise zero and $\underset{\sim}{Y}_e$ is a diagonal matrix of element admittances.

The submatrix $\underset{\sim}{Y}(F_x|E)$ can be presented in the form [8]

$$\underset{\sim}{Y}(F_x|E) = \underset{\sim}{\lambda}_{-Fx} \underset{\sim}{Y}_e \underset{\sim}{\lambda}_{+E}^T, \qquad (14)$$

where $\underset{\sim}{\lambda}_{-Fx}$ ($\underset{\sim}{\lambda}_{+E}$) is obtained from $\underset{\sim}{\lambda}_-$ ($\underset{\sim}{\lambda}_+$) by removing rows F_x (E), respectively.

Following Starzyk et al. [10] we can formulate the following theorem.

<u>Theorem 1</u>

If $\det \underset{\sim}{Y}(F_x|E) \neq 0$ then there exists at least one k-connection c_S in the graph $G(F_x|E)$ obtained from the Coates graph of the network after deleting all the edges incoming to nodes F_x and all the edges outgoing from nodes E, where

$$S = \{(v_s, v_e);\ v_s \in F_x \cap (N-E),\ v_e \in E \cap (N-F_x)\}, \qquad (15)$$

$$\text{card } S = \text{card } (E \cap (N-F_x)) = \text{card } (F_x \cap (N-E)), \qquad (16)$$

where (v_s, v_e) represents a path directed from the node v_s to the node v_e, and N is the set of all graph nodes.

The condition stated in Theorem 1 is sufficient almost everywhere. As a consequence of Theorem 1 we have an important corollary.

<u>Corollary 1</u>

If det $\underset{\sim}{Y}(F_x|E) \neq 0$ then after deleting all the edges outgoing from nodes E and incoming to nodes F_x there are no isolated nodes in the set $N - (E \cap F_x)$.

To locate the faults of elements incident with nodes F such that after deleting all the edges incoming to nodes F some of them become isolated, we must include all of these isolated nodes in the set E, which means that all of them must be accessible nodes (i.e., the nodes at which voltages can be measured).

Following the method described in [10] we can investigate the problem of two subnetworks the graphs of which have c common nodes when $c \leq$ card F. In this case we can not identify uniquely the faults appearing in one of the subnetworks by measuring the voltages in the second only because the k-connection required by Theorem 1 does not exist. But even in this case we still have the possibility of identifying the faulty region, where we check whether or not the only faults are included in a subgraph isolated from c+1 measurements by a c common nodes. This property is effectively used in Section IV. In the case when f faults appear in a subgraph connected to the rest of the graph through c common nodes, we must have at least f-c+1 measurements inside this subgraph to identify all these faults uniquely (see Fig. 2).

The restriction on the placement of measurement nodes appears also in more complex cases when faults and measurements are in different weakly connected subgraphs.

To ensure that the system of equations (8) is overdetermined we should have at least two nonsingular (card F) x (card F) submatrices of $\underset{\sim}{Z}_{MF}$.

Lemma 1

If $\underset{\sim}{Z}_{EF}$ is a nonsingular full column rank submatrix of $\underset{\sim}{Z}_{MF}$ and $\underset{\sim}{z}_0^T$ is a nonzero row of $\underset{\sim}{Z}_{MF}$ not belonging to $\underset{\sim}{Z}_{EF}$ then there exists a nonsingular submatrix of $\underset{\sim}{Z}_{MF}$ that contains $\underset{\sim}{z}_0^T$.

Proof

Since rank $\underset{\sim}{Z}_{MF}$ = rank $\underset{\sim}{Z}_{EF}$ the row $\underset{\sim}{z}_0^T$ is a linear combination of rows $\underset{\sim}{z}_i^T \in \underset{\sim}{Z}_{EF}$, $i \in I$. If we remove row $\underset{\sim}{z}_k^T$, $k \in I$, then $\underset{\sim}{z}_0^T$ will be linearly independent from the rows z_i^T, $i \in I - \{k\}$, and because of the linear independency of rows $\underset{\sim}{z}_i^T$ will form a new set of linearly independent rows $\{\underset{\sim}{z}_0^T, \underset{\sim}{z}_i^T : i \in I, i \neq k\}$.

Corollary 2

If $\underset{\sim}{Z}_{MF}$ contains a zero row and the corresponding voltage $\Delta V^M \in \Delta \underset{\sim}{V}^M$ is nonzero then $\Delta \underset{\sim}{J}^F$ does not represent all the faults in the network, therefore, other candidates for faults should be considered.

A simple topological interpretation can be given to illustrate Lemma 1 and Corollary 2. Element $z_{ij} \in \underset{\sim}{Z}_{MF}$ is nonzero if and only if det $\underset{\sim}{Y}(j|i) \neq 0$ $(i \in M,\ j \in F)$. This condition is topologically equivalent to the existence of the 1-connection which contains the path directed from node j to node i in the graph $G(j|i)$.

Graph $G(j|i)$ is obtained from the Coates graph of the network after deleting all the edges incoming to the node j and all the edges outgoing from the node i.

To fulfill the condition stated in the Lemma 1 it is sufficient that there exists a node $i \in M-E$ which is the end of a path outgoing from one of the F nodes, and if after deleting the edges incident to this path the remaining graph contains at least one 0-connection.

Element $z_{ij} \in \underset{\sim}{Z}_{MF}$ is zero when there is no path directed from the node j to i or for every such path if I denotes the set of nodes belonging to the path det $\underset{\sim}{Y}(I|I) = 0$. The latter case is rare in electronic circuits.

IV. NETWORK PARTITIONING INTO FAULT REGIONS

The main problem in multiple-fault location is to guess the set F that contains all faulty elements but has a number of elements $f < m$. We discuss how to choose this proper set of elements. The aim is to improve efficiency of computations when no additional information about possible faults exist.

Any set of w elements which contains all f faults $(w \geq f)$ we call a <u>fault region</u> and denote it by $F_{w,f}$. The fault region can be predicted or designed initially by the approximate fault isolation method described in [11]. If we have no initial information about the system we can try to guess the proper set F but then the probability of being correct is low because the number of different combinations is equal to

$\binom{p}{f}$, where p denotes the number of elements (cf. [5]). Below we describe two algorithms which can be used to detect the fault region and are very effective if the number of measurements is large.

It is evident that if we have m measurement ports then the maximum number of faulty elements we can identify uniquely is equal to $w = m-1$. The first algorithm we present is based on logical and combinatorial operations and does not take advantage of network topology.

Combinatorial Algorithm

Step 1

Divide arbitrarily the set of all elements on k distinct subsets $S_1, \ldots, S_k$, each of them of cardinality equal to $E(\frac{m-1}{f})$.

Comment

Of course, if $p/E(\frac{m-1}{f})$ is not integer the last subset has less than $E(\frac{m-1}{f})$ elements. So $k \geq p/E(\frac{m-1}{f})$.

Step 2

Examine all combinations of f subsets out of the k subsets using Result 1 and Theorem 1. If all combinations fail increase f and go to Step 1.

Comment

The number of these combinations $\binom{k}{f}$ is usually much less than $\binom{p}{f}$ when $(m-1) \geq 2f$.

If the number of faulty elements is really f (or less) there always exists such a combination of f subsets for which the relation (8) is fulfilled. The sum of the subsets of this combination is the first fault region

$$F_{w,f} = \bigcup_{i \in I} S_i, \tag{17}$$

where $w = f.E(\frac{m-1}{f})$, and $I = \{i_1, i_2, \ldots, i_k\}$, $i_j \neq i_\ell$ for $j \neq \ell$.

Step 3

Divide the set $F_{w,f}$ arbitrarily on f+1 subsets and then check which combination of f subsets contains all faults. If there is no such combination then stop. Eliminate at least $E(\frac{w}{f+1})$ elements as not being faulty. Define a new fault region $F_{w',f}$ with $w' < w$.

Step 4

If $w < f$ go to Step 3.

Step 5

Set $f = f-1$ and go to Step 3.

Example 1

Let us assume that a network under consideration has $p = 76$, $m = 39$, $f = 2$. Then realizing the algorithm we design:

1. $k \geq \frac{p}{E(\frac{m-1}{f})} = \frac{76}{19} = 4$, for example, $k = 4$.

2. We check $\binom{k}{f} = \binom{4}{2} = 6$ different combinations of elements, to find the first fault region $F_{38,2}$.

3. In every step of this stage of the algorithm we check 3 combinations obtaining successively the following fault regions: $F_{26,2}$, $F_{18,2}$, $F_{12,2}$, $F_{8,2}$, $F_{6,2}$, $F_{4,2}$, $F_{3,2}$, $F_{2,2}$.

In this example, therefore, we have to check not more than 30 combinations instead of $\binom{76}{2} = 2850$, which is much easier in spite of the higher ranks of the matrices to be computed.

It is evident from Theorem 1 that for two subgraphs separated by c common nodes rank $\underset{\sim}{Z}_{MX} \leq c$ for all X representing nodes in one subgraph when all measurements are in the second one. This can be used in fault location, when we take X = {c common nodes} to represent $f \geq c$ faults in the second subgraph. If $f >> c$ then this strategy allows us to locate the faulty region even when $m << f$. Let us decompose the graph G into subgraphs $G_1, \ldots, G_k$. Let c_i denote the cardinality of the set of G_i nodes incident to the other graphs

$$c_i \triangleq \text{card} \left(\bigcup_{\substack{j=1 \\ j \neq i}}^{k} N_j \cap N_i \right) , \qquad (18)$$

where N_j, N_i represent sets of nodes of subgraphs G_j and G_i respectively. Let M_i denote the set of measurement nodes belonging to G_i and m_i = card M_i. We can formulate an important result.

<u>Result 2</u>

If subgraph G_i contains f_i faults and

$$m_i > c_i + f_i$$

we can locate this fault taking $M = M_i$ and

$$F_x = F \cup x \triangleq \bigcup_{\substack{j=1 \\ j\neq i}}^{k} N_j \cap N_i \cup F_i \cup x \qquad \forall\, x \in N_i - F,$$

where F_i represents all the faults in subgraph G_i, and checking the consistency of equation (8) and the condition stated in Result 1.

Now we can formulate an algorithm which can be used to locate fault regions on the basis of topological restrictions.

Topological Algorithm

Step 1

Decompose the Coates signal-flow graph G of the network into subgraphs $G_1, \ldots, G_k$ separated by a small number of nodes.

Comment

It is better if the common nodes contain measurement points.

Step 2

Choose the subgraph G_i containing m_i measurement nodes M_i, such that

$$m_i > c_i \quad .$$

If there is no such subgraph use the combinatorial algorithm.

Step 3

If the chosen subgraph G_i contains f_i faults such that

$$m_i > c_i + f_i$$

locate them using Result 1 and Theorem 1 and taking F_x as in Result 2.

Comment

Realization of this step allows us to find all the faults in a certain region and to calculate the actual values of the faulty elements. Then all the currents and voltages in subgraph G_i can be calculated and G_i may be represented as the set of external excitations with known voltages for incident subnetworks.

Step 4

If no subgraph chosen in Step 2 satisfies the condition stated in Step 3 then try different combinations of two or more subgraphs and check if Step 3 may be realized for the combination considered. If not, we return to the combinatorial algorithm.

Example 2

Assume two subgraphs G_1, G_2 separated by $c_1 = c_2 = 2$ common nodes. Let subgraph G_1 have $m_1 = 5$ measurements and no faults. The conditions stated in Steps 2 and 3 are obviously satisfied because $c_1 + f_1 = 2 < m_1$. So no matter how many faults are in subgraph G_2 we find G_1 as nonfaulty and can use two measurements from G_1 together with measurements placed in G_2 to locate the faults in G_2.

V. SOME PRACTICAL REMARKS

Biernacki and Bandler [5] stated that condition (8) is satisfied if and only if the following relation holds

$$(\bar{\underset{\sim}{Z}}_{MF} - \underset{\sim}{1}) \, \Delta \underset{\sim}{V}^M = \underset{\sim}{0}, \tag{19}$$

where

$$\bar{\underset{\sim}{Z}}_{MF} \triangleq \underset{\sim}{Z}_{MF} \, (\underset{\sim}{Z}_{MF}^T \, \underset{\sim}{Z}_{MF})^{-1} \, \underset{\sim}{Z}_{MF}^T. \tag{20}$$

We propose a simpler method which can be used to verify the condition (8).

One can prove that the solution of the equation

$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}, \tag{21}$$

where $\underset{\sim}{A}$ is an mxf full column rank matrix $f < m$, exists if and only if it can be transformed to the form

$$\left[\begin{array}{cccc} x & x & & x \\ 0 & x & & x \\ \cdot & & \cdot & \cdot \\ \cdot & & & \cdot\;\cdot \\ 0 & & & x \\ \hline & & & \\ & \underset{\sim}{0} & & \end{array}\right] \underset{\sim}{x} = \left[\begin{array}{c} \underset{\sim}{b}_1 \\ \\ \hline \\ \underset{\sim}{0} \end{array}\right] \tag{22}$$

after row manipulation, where $\underset{\sim}{b}_1$ is a column vector having f elements. The form (21) is also more convenient to obtain the solution of the set of equations.

Example 3

To compare the two methods let us solve the overdetermined system of equations

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 4 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -6 \\ -4 \end{bmatrix}. \tag{23}$$

We have

$$\underset{\sim}{A}^T \underset{\sim}{A} = \begin{bmatrix} 30 & 28 \\ 28 & 30 \end{bmatrix}, (\underset{\sim}{A}^T \underset{\sim}{A})^{-1} = \frac{1}{116} \begin{bmatrix} 30 & -28 \\ -28 & 30 \end{bmatrix}.$$

$$\bar{\underset{\sim}{A}} = \underset{\sim}{A}(\underset{\sim}{A}^T \underset{\sim}{A})^{-1} \underset{\sim}{A}^T = \frac{1}{116} \begin{bmatrix} 38 & 50 & -8 & -20 \\ 50 & 78 & 20 & -8 \\ -8 & 20 & 78 & 50 \\ -20 & -8 & 50 & 38 \end{bmatrix}$$

$$(\bar{\underset{\sim}{A}} - \underset{\sim}{1}) \begin{bmatrix} 1 \\ -1 \\ -6 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} .$$

Consequently, we have checked consistency using 83 multiplications and divisions and still do not know the solution, while after transforming the system to the form

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ \hline 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ \hline 0 \\ 0 \end{bmatrix}$$

we can easily compute x_1, x_2 and we used 11 multiplications and divisions only.

For ill-conditioned systems the method of Householder orthogonal transformations can be used to reduce to zero the subdiagonal elements of $\underset{\sim}{A}$ [12].

For practical situations when both measurement errors and effects of tolerances appear, the technique proposed by Bandler, Biernacki and Salama [11] can be used. In the first stage of computation we solve optimization problem that can be stated as

$$\text{minimize} \sum_{i=1}^{n} (|\text{Re}(\Delta J_i^F)| + |\text{Im}(\Delta J_i^F)|) \tag{24}$$

subject to linear equality constraints (8). Solution of this problem gives us the most likely faulty elements. Then the verification technique in the presence of tolerances can be used to check (8) in the way described in [11].

VI. EXAMPLES

To see how the method works in practical networks let us discuss two examples where the number of measurements was less than the number of faults.

Example 4

The resistive network shown in Fig. 3 has 20 nodes and 39 resistors with nominal values $R_i = 1$ for $i \neq 12$ and $R_{12} = 2\Omega$. We take measurements at nodes number 9, 11, 13 and 19. These measurements are simulated for the faulty network where we increase the value of resistors R_k $(k = 5,6,\dots,12)$ and R_{37} from the nominal by 1 Ω and $J = 1A$. They are equal to

$$V_9 = 0.1316V,\ V_{11} = 0.06177V,\ V_{13} = 0.03797V,\ V_{19} = 0.05351V.$$

The graph of the network can be decomposed into three subgraphs as shown in Fig. 4.

All the measurements are placed in subgraph G_2. Following the topological algorithm we find that subgraph G_2 has a number of measurements m_2 larger than c_2. In this case $m_2 > c_2 + f_2$ only for $f_2 = 0$, so we can only check if subgraph G_2 represents a nonfaulty subgraph. We take F_x as in Result 2 having $F_2 = \emptyset$. So, in this case $F = N_1 \cap N_2 \cup N_3 \cap N_2 = \{9,13,18\}$. We check that the rank of $\underset{\sim}{Z}_{MFx}$ is 4 for all $x \in N_2 - F = \{7,11,12,19,20\}$, so on the basis of Result 1, if equation (8) is consistent, then the solution $\Delta \underset{\sim}{J}^F$ represents all the faulty elements. The equation (8) now has the form

$$\begin{bmatrix} \Delta V_9^M \\ \Delta V_{11}^M \\ \Delta V_{13}^M \\ \Delta V_{19}^M \end{bmatrix} = \begin{bmatrix} 0.0676 \\ 0.0319 \\ 0.0197 \\ 0.0277 \end{bmatrix} = \begin{bmatrix} 0.506 & 0.144 & 0.170 \\ 0.237 & 0.384 & 0.572 \\ 0.144 & 0.536 & 0.381 \\ 0.205 & 0.396 & 0.698 \end{bmatrix} \begin{bmatrix} \Delta J_9^F \\ \Delta J_{13}^F \\ \Delta J_{18}^F \end{bmatrix}. \quad (25)$$

We check consistency using the transformation to the form of (22) and obtain the transformed equation

$$\begin{bmatrix} 0.506 & 0.144 & 0.170 \\ 0 & 0.317 & 0.493 \\ 0 & 3 \cdot 10^{-29} & -0.435 \\ 0 & 4 \cdot 10^{-29} & 3 \cdot 10^{-30} \end{bmatrix} \begin{bmatrix} \Delta J_9^F \\ \Delta J_{13}^F \\ \Delta J_{18}^F \end{bmatrix} = \begin{bmatrix} 0.0676 \\ 0.0003 \\ 10^{-15} \\ 8.6 \cdot 10^{-16} \end{bmatrix}. \quad (26)$$

The last element of the right-hand side is almost zero so we recognize (8) as consistent and solving for $\Delta \underset{\sim}{J}^F$, we obtain

$$\Delta J_9^F = 0.1333A, \; \Delta J_{13}^F = 0.000949A, \; \Delta J_{18}^F = -3 \cdot 10^{-15} A.$$

From the solution obtained, we can be sure that subgraph G_2 is non-faulty, while G_1 and G_3 represent faulty subnetworks. If G_3 contains only one faulty element, as in this example, we can locate them exactly, using the reduction technique described in Section II. We check the combination of every two nodes in G_3 which are connected by a resistor. After subtracting column 16 from column 14 in matrix $\underset{\sim}{Z}_{MN}$, we may write equation (8) as

$$\begin{bmatrix} 0.506 & -0.0289 \\ 0.237 & -0.0769 \\ 0.144 & -0.107 \\ 0.205 & -0.0792 \end{bmatrix} \begin{bmatrix} \Delta J_9^F \\ \\ \Delta J_{14-16}^F \end{bmatrix} = \begin{bmatrix} 0.0676 \\ 0.0319 \\ 0.0197 \\ 0.0277 \end{bmatrix}. \quad (27)$$

Next, we transform it to the form (22) and obtain

$$\begin{bmatrix} 0.506 & -0.0289 \\ 0 & -0.0634 \\ 0 & -6\cdot 10^{-30} \\ 0 & -6\cdot 10^{-30} \end{bmatrix} \begin{bmatrix} \Delta J_9^F \\ \\ \Delta J_{14-16}^F \end{bmatrix} = \begin{bmatrix} 0.0676 \\ 0.0003 \\ 10^{-15} \\ 6\cdot 10^{-16} \end{bmatrix} \tag{28}$$

We can solve this obtaining

$$\Delta J_9^F = 0.1333\text{A}, \ \Delta J_{14-16}^F = -0.00475\text{A} \ .$$

This proves that the faulty element is between nodes 14 and 16. We can find its value by exciting the nominal network with the original excitation plus current sources $\underset{\sim}{\Delta J}^F$ as excitations at faulty nodes. The solution gives us the values of voltages $\underset{\sim}{V}'$ at all the nodes of G_2 and G_3 (not G_1). Now, ΔY_{37} can be calculated as:

$$\Delta Y_{37} = \frac{\Delta J^F_{14-16}}{V'_{16} - V'_{14}} = \frac{-0.00475}{0.0095} = -\frac{1}{2} \ , \tag{29}$$

which is the exact change from the nominal value.

Example 5

The active lowpass filter as shown in Fig. 5 has nominal values of elements equal (cf. [13]) to

$R_1 = 0.182$, $C_2 = 0.01$, $R_3 = 1.57$, $R_5 = 2.64$, $R_6 = 10$, $R_7 = 10$, $R_9 = 100$, $R_{10} = 11.1$, $R_{11} = 2.64$, $C_{12} = 0.01$, $R_{14} = 5.41$, $R_{15} = 1$, $R_{17} = 1$, $C_{18} = 0.01$, $R_{19} = 4.84$, $R_{21} = 2.32$, $R_{22} = 10$, $R_{23} = 10$, $R_{25} = 500$, $R_{26} = 111.1$, $R_{27} = 1.14$, $R_{28} = 2.32$, $R_{29} = 0.01$, $R_{31} = 72.4$, $R_{32} = 10$, $R_{34} = 10$ (all resistors in kΩ and capacitors in μF).

Operational amplifiers are modelled by the circuit shown in Fig. 6.

The input current is equal to $j(t) = 10^{-2}\cos(2000t)$A. Measurements taken at nodes 10, 12, 15 and 17 are equal to

$V_{10} = -9.866+j0.6264$ V, $V_{12} = 0.0822+j0.932$ V, $V_{15} = 20.08-j1.726$ V, $V_{17} = 2.437-j0.2094$ V.

These measurements were simulated in the faulty network with the faults

$R_1 = 0.1$, $C_2 = 0.02$, $R_6 = 20$, $R_7 = 20$, $R_{11} = 2$, $R_{14} = 4$, $R_{15} = 2$, $R_{17} = 2$, $R_{32} = 40$

and the gain of the amplifier A_8 was reduced to 50. Again, we decompose the graph of the network into three subgraphs, as shown in Fig. 7. We repeat the steps from Example 4 testing the middle subnetwork. We obtain equation (8) for the given set of measurements and $F = \{10, 15, 17\}$ as

$$\begin{bmatrix} \Delta V^M_{10} \\ \Delta V^M_{17} \\ \Delta V^M_{12} \\ \Delta V^M_{15} \end{bmatrix} = \begin{bmatrix} -5.43+j0.246 \\ 1.34-j0.0908 \\ 0.0359+j0.513 \\ 11.1-j0.749 \end{bmatrix} = \tag{30}$$

$$\begin{bmatrix} 1.06\cdot10^{-2}+j3.43\cdot10^{-5} & 6.52\cdot10^{-11}-j7.19\cdot10^{-12} & 9.64\cdot10^{-8}+j3.11\cdot10^{-10} \\ -2.62\cdot10^{-3}+j5\cdot10^{-5} & 5.82\cdot10^{-4}+j7.05\cdot10^{-5} & 1.01+j3.76\cdot10^{-10} \\ -2.14\cdot10^{-5}-j10^{-3} & -10^{-2}-j2.41\cdot10^{-4} & -1.57\cdot10^{-10}-j5.82\cdot10^{-9} \\ -2.16\cdot10^{-2}+j4.12\cdot10^{-4} & 4.8\cdot10^{-3}+j5.81\cdot10^{-4} & -1.25\cdot10^{-7}+j3.1\cdot10^{-9} \end{bmatrix} \begin{bmatrix} \Delta J^F_{10} \\ \Delta J^F_{15} \\ \Delta J^F_{17} \end{bmatrix}$$

We transform (30) to the form (22) and obtain

$$\begin{bmatrix} 1.06\cdot10^{-2}+j3.43\cdot10^{-5} & 6.52\cdot10^{-11}+j7.19\cdot10^{-12} & 9.64\cdot10^{-8}+j3.11\cdot10^{-10} \\ -8\cdot10^{-32}+j10^{-33} & 5.82\cdot10^{-4}+j7.05\cdot10^{-5} & 1.01-j7.86\cdot10^{-11} \\ -3\cdot10^{-34} & -2\cdot10^{-31}-j2\cdot10^{-32} & 17.2-j1.66 \\ -2\cdot10^{-30}-j10^{-32} & 4\cdot10^{-31}+j6\cdot10^{-32} & -8\cdot10^{-28}+j8\cdot10^{-29} \end{bmatrix} \begin{bmatrix} \Delta J^F_{10} \\ \Delta J^F_{15} \\ \Delta J^F_{17} \end{bmatrix}$$

$$= \begin{bmatrix} -5.43+j0.246 \\ -3.46\cdot 10^{-4}+j2.94\cdot 10^{-5} \\ -5.85\cdot 10^{-3}+j1.07\cdot 10^{-3} \\ -1.74\cdot 10^{-14}+j3.42\cdot 10^{-15} \end{bmatrix}.$$

Again, we can recognize equation (8) for these faults as consistent and solving for $\Delta \underset{\sim}{J}^F$ we obtain

$\Delta J^F_{10} = -512+j24.9A$, $\Delta J^F_{15} = -1.65\cdot 10^{-14} + j7.33\cdot 10^{-14}A$

$\Delta J^F_{17} = -3.43\cdot 10^{-4} + j2.91\cdot 10^{-5}A$.

Once again, we may try to locate the fault in the right-hand side subgraph. After subtracting column 19 from 17 and solving the resulting set of equations, we obtain

$\Delta J^F_{10} = -512+j24.9A$,

$\Delta J^F_{17-19} = -3.46\cdot 10^{-4} + j2.94\cdot 10^{-5}A$.

We can see that the change in faulty currents is very small, which indicates weak sensitivity of the given measurements w.r.t. the faulty current at node 19. This result is easy to predict because the faulty current source of value $3.46\cdot 10^{-4}-j2.94\cdot 10^{-5}A$ is connected in parallel with a controlled current source having a current equal to $J_{19} \approx 0.07-j0.006A$. Again, we may excite the nominal network adding faulty currents and calculate voltages $\underset{\sim}{V}'$. We calculate

$$\Delta Y_{32} = \frac{\Delta J^F_{17-19}}{V'_{19} - V'_{17}} = \frac{-3.46\cdot 10^{-4}+j2.94\cdot 10^{-5}}{4.62-j0.392} \approx -0.749\cdot 10^{-4}.$$

This is quite accurate, since the actual change was $-0.75\cdot 10^{-4}$.

VII. CONCLUSIONS

We have extended the possibilities of multiport methods for multiple-fault location in analog networks. Necessary and sufficient conditions for uniquely evaluating faults have been discussed. Together with topological constraints inherent in a particular network these conditions indicate whether or not the measurements which have been made can be used to evaluate all the faults. Even in cases where the faults can not be evaluated, our analysis can be applied to identify and isolate faulty and nonfaulty subnetworks. Our recommended strategy then would be to subject the subnetworks containing the faults to further analysis. Our ability to evaluate the faults within a subnetwork depends upon the actual number of faults, the number of measurements within the subnetwork and the information which can be used from outside the subnetwork as seen through the nodes common with the rest of the network. Thus, our approach permits us to use effectively all methods which have been proposed for fault evaluation of networks at the subnetwork level. This partitioning into subnetworks not only increases the efficiency of existing algorithms, especially when we have a large network to analyze, but also permits detailed investigation of specific subnetworks.

REFERENCES

[1] T.N. Trick, W. Mayeda and A.A. Sakla, "Calculation of parameter values from node voltage measurements", IEEE Trans. Circuits and Systems, vol. CAS-26, 1979, pp. 466-474.

[2] N. Navid and A.N. Wilson, Jr., "A theory and an algorithm for analog circuit fault diagnosis", IEEE Trans. Circuits and Systems, vol. CAS-26, 1979, pp. 440-457.

[3] H.M. Merrill, "Failure diagnosis using quadratic programming", IEEE Trans. Reliability, vol. R-22, 1973, pp. 207-213.

[4] M.N. Ransom and R. Saeks, "Fault isolation with insufficient measurements", IEEE Trans. Circuit Theory, vol. CT-20, 1973, pp. 416-417.

[5] R.M. Biernacki and J.W. Bandler, "Multiple-fault location of analog circuits", IEEE Trans. Circuits Systems, vol. CAS-28, 1981, pp. 361-367.

[6] L.O. Chua and P.M. Lin, Computer-aided Analysis of Electronic Circuits, Algorithms and Computational Techniques. Englewood Cliffs, NJ: Prentice Hall, 1975.

[7] C.W. Ho, A.E. Ruehli and P.A. Brennan, "The modified nodal approach to network analysis", IEEE Trans. Circuits and Systems, vol. CAS-22, 1975, pp. 504-509.

[8] J.A. Starzyk and J.W. Bandler, "Nodal approach to multiple-fault location in analog circuits", IEEE Int. Symp. Circuits and Systems (Rome, Italy, May 1982).

[9] R. Liu, C. Lin and Z.F. Huang, "Node-fault diagnosis and a design of testability", University of Notre Dame, Notre Dame, IL, Technical Report 811, July 27, 1981.

[10] J.A. Starzyk, R.M. Biernacki and J.W. Bandler, "Design of tests for parameter identification by voltage measurements", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-266, 1981.

[11] J.W. Bandler, R.M. Biernacki and A.E. Salama, "A linear programming approach to fault location in analog circuits", Proc. IEEE Int. Symp. Circuits and Systems (Chicago, IL, 1981), pp. 256-260.

[12] J.K. Reid, Ed., Large Sparse Sets of Linear Equations. New York: Academic Press, 1971.

[13] C.L. Semmelman, E.D. Walsh and G.T. Daryanani, "Linear circuits and statistical design", Bell Syst. Tech. J., vol. 50, 1971, pp. 1149-1171.

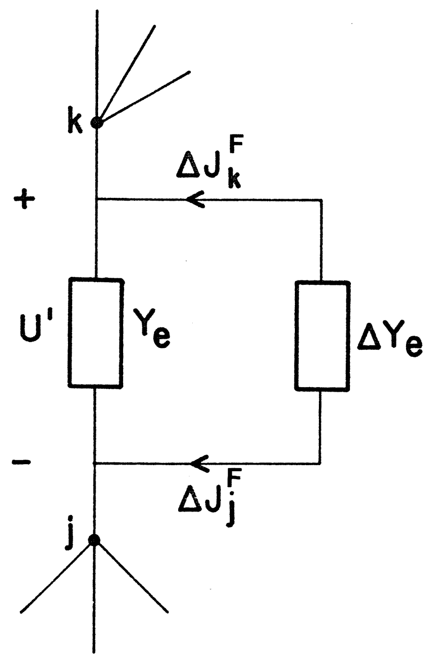

Fig. 1 Changes in nodal current caused by a single fault.

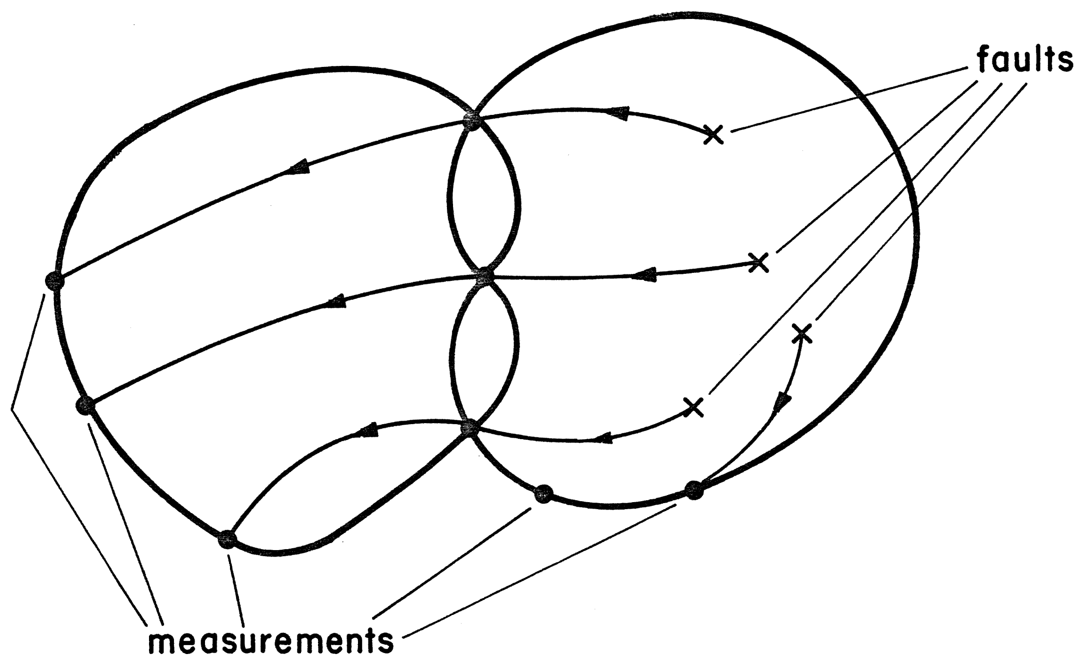

Fig. 2 Illustration of necessary measurements.

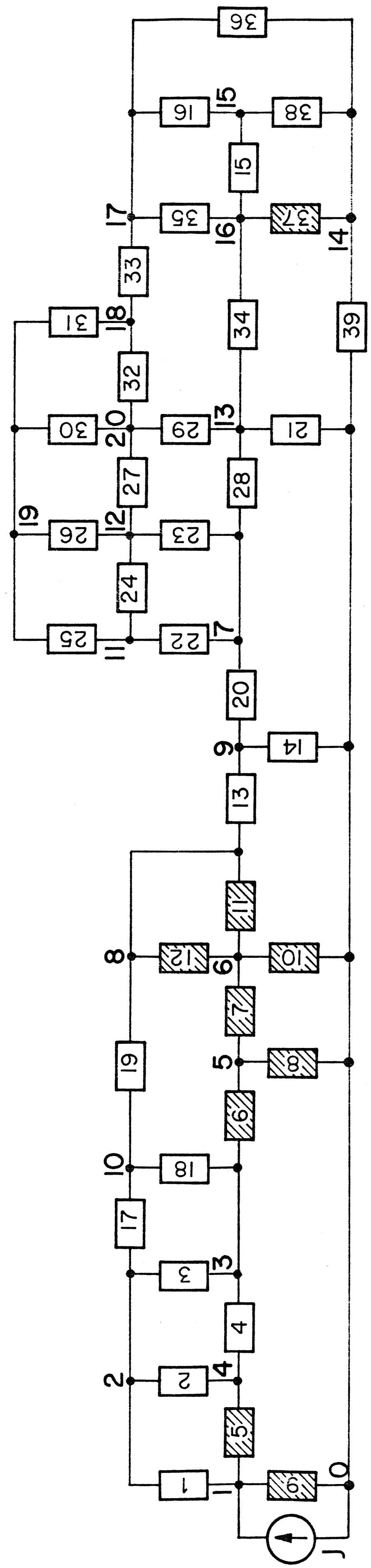

Fig. 3 Resistive network example.

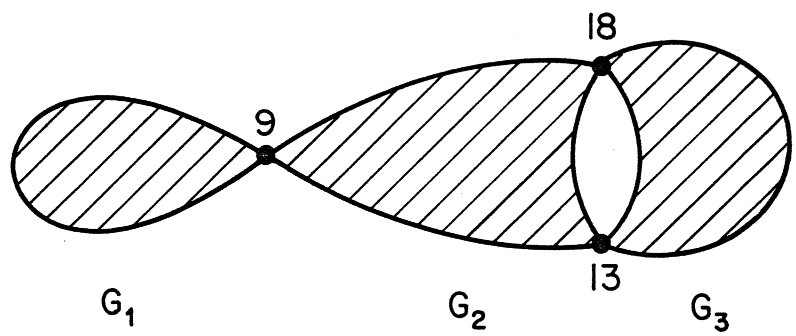

Fig. 4 Decomposition of the graph of the resistive network into subgraphs.

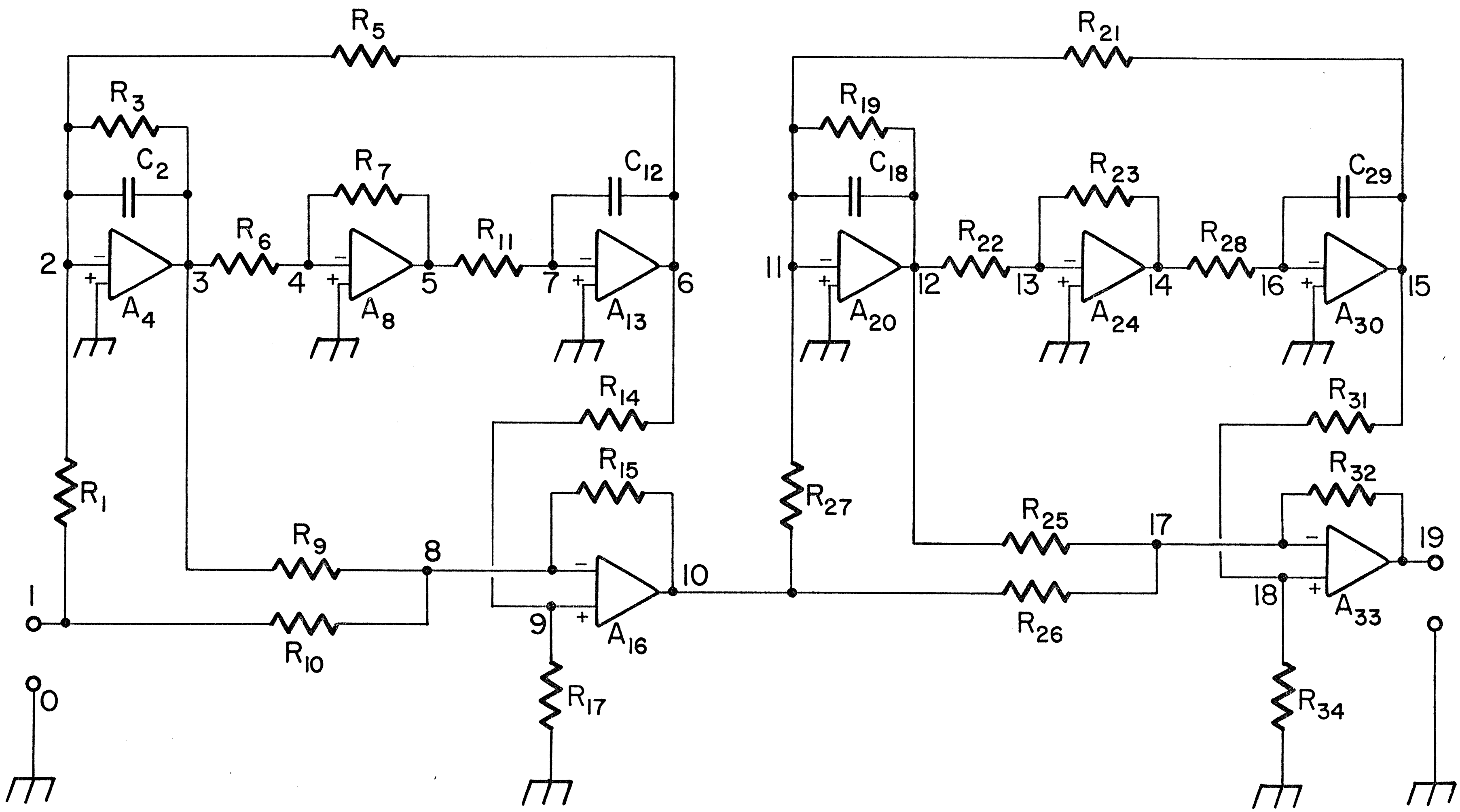

Fig. 5 Active lowpass filter example.

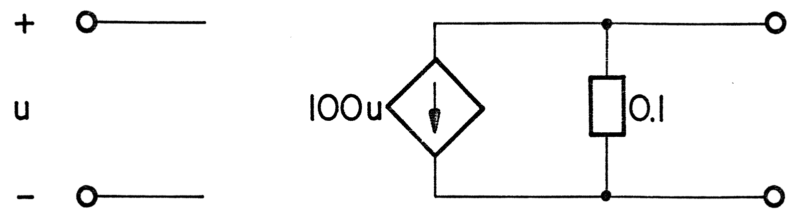

Fig. 6 Operational amplifier model used for the active filter.

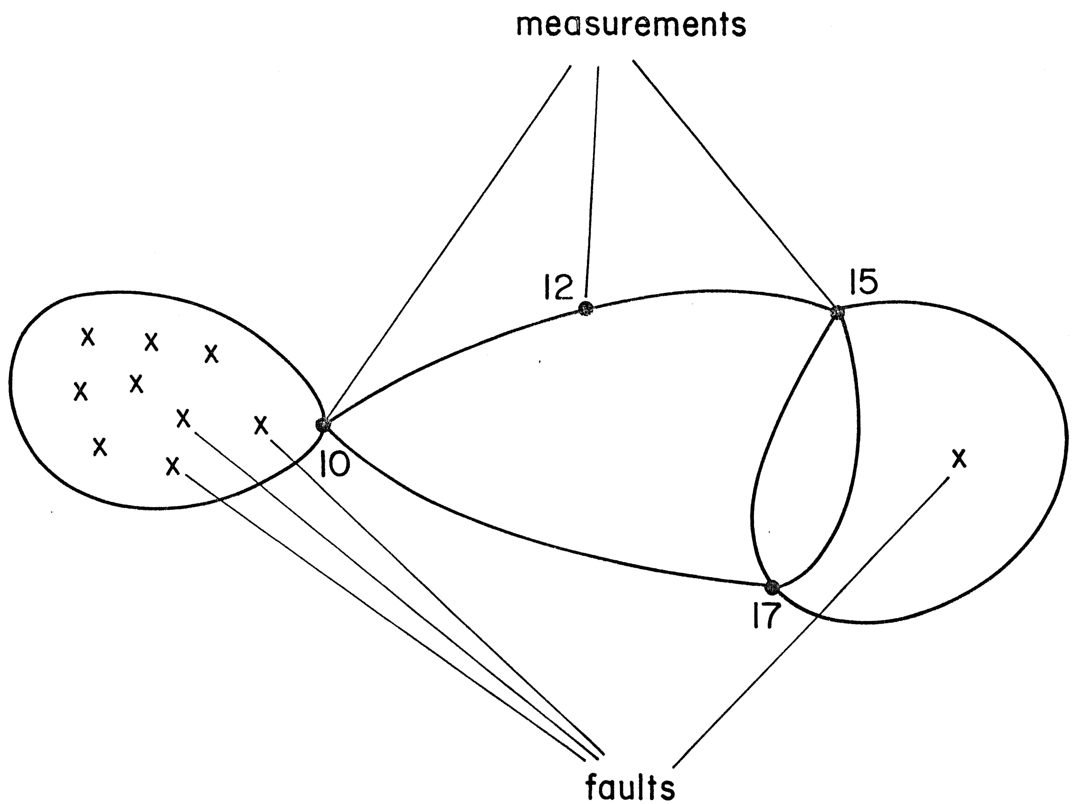

Fig. 7 Decomposition of the graph of the active filter into subgraphs.

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LOCATION OF FAULT REGIONS IN ANALOG CIRCUITS

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Abstract: The multiple-fault location problem for analog circuits is treated on the basis of the nodal equations. The availability of voltage measurements due to current excitations is assumed by the method. Topological restrictions on the possibility of fault location for a given set of measurements are formulated. The emphasis in this paper is on locating subnetworks or regions containing all the faults of the network. Two algorithms are presented for this purpose. Coates flow-graph representation of a network is used for topological considerations.

Description:

Related Work: SOC-233, SOC-235, SOC-236, SOC-244, SOC-251, SOC-259, SOC-266, SOC-267, SOC-268, SOC-271.

Price: $ 6.00.