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PRACTICAL COMPLEX SOLUTION OF POWER FLOW EQUATIONS

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Abstract

This paper applies the compact, complex notation introduced by Bandler and El-Kady to the practical solution of the power flow equations. The solution of the complex linearized power flow equations, which is required by the iterative Newton-Raphson method, is obtained by a direct method. The method, fully and exactly, incorporates generator buses as well as dummy load buses. An elimination scheme is applied to diagonalize the conjugate tableau, which contains the complex coefficients associated with the conjugate of the perturbed bus voltages. This conjugate tableau is then eliminated, simultaneously reducing the basic tableau, which contains the complex coefficients associated with the perturbed bus voltages, to upper triangular form. Alternatively, the conjugate tableau is explicitly eliminated, exposing a set of linear, complex equations in the perturbed complex bus voltages. The theoretical results are illustrated by solving the load flow equations for a 6-bus, a 23-bus and a 26-bus system.

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I. INTRODUCTION

Bandler and El-Kady [1-4] demonstrated the application of a compact, complex notation to power system simulation [2-4] through solution techniques for the ubiquitous power flow equations [5] and sensitivities of system states w.r.t. control or design variables. They presented a new technique for solving the power flow equations, which automatically supplies power network sensitivities, employing the concept of the adjoint network simulation in the context of Tellegen's theorem [6]. This particular method, while using the compact, complex notation, does not employ it in the numerical analysis. It exploits simple network properties, however, of the corresponding exact, approximate and decoupled versions, while the exact version enjoys the same rate of convergence as the Newton Raphson method [7,8].

Bandler and El-Kady have investigated the principle of retaining the compact, complex form of the perturbed (or linearized) load flow equations and have derived a suitable elimination technique, which deals directly with the linear complex equations expressed in terms of a set of complex variables and their complex conjugate [3]. The authors departed from the conventional approach to the Newton-Raphson method, which employs the real mode, and invoked the formal interpretation in terms of first-order changes of problem complex variables [2,4]. They discussed not only the appropriate linearized, complex equations for buses of the load bus type but also showed how their approach could handle generator type buses in a direct manner. While some remarks were made about sparsity considerations, their derivation of the algorithm did not permit a simple exposition of structurally determined sparsity in the complex equations.

The present paper has a number of objectives. Firstly, we explain the steps of the complex elimination scheme in simple, matrix form which exposes the sparsity structure more explicitly. Secondly, we introduce generator-type buses into the tableau from the beginning and handle dummy loads, which may be present in the system in an explicit manner. Thirdly, we elaborate on the elimination of blocks of the conjugate tableau which exploits its sparsity. Finally, we illustrate the results of a computer program, written to implement Newton's method in the complex mode, on a 6-bus, a 23-bus and a 26-bus system.

II. COMPLEX FORMULATION OF LINEARIZED LOAD FLOW EQUATIONS

The power network performance equations [9] are written, using the bus frame of reference in the admittance form

$$\underset{\sim}{Y}_T \underset{\sim}{V}_M = \underset{\sim}{I}_M, \tag{1}$$

where $\underset{\sim}{Y}_T$ is the complex bus admittance matrix of the network, $\underset{\sim}{V}_M$ is a column vector of the complex bus voltages and $\underset{\sim}{I}_M$ is the corresponding column vector of the complex injected bus currents.

The bus loading equations are written in the matrix form

$$\underset{\sim}{E}_M^* \underset{\sim}{I}_M = \underset{\sim}{S}_M^*, \tag{2}$$

where

$$\underset{\sim}{E}_M \triangleq \text{diag } \underset{\sim}{V}_M, \tag{3}$$

$$\underset{\sim}{S}_M \triangleq \underset{\sim}{P}_M + j\underset{\sim}{Q}_M \tag{4}$$

and * denotes the complex conjugate. From (4)

$$\underset{\sim}{E}_M \underset{\sim}{v} = \underset{\sim}{V}_M, \tag{5}$$

where

$$\underset{\sim}{v} \triangleq \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} . \tag{6}$$

Substituting (1) into (2) we obtain the system of complex nonlinear equations

$$\underset{\sim}{E}_M^* \underset{\sim}{Y}_T \underset{\sim}{V}_M = \underset{\sim}{S}_M^*, \tag{7}$$

representing the typical load flow problem.

Load Buses

A typical equation fitting into the system (7) corresponding to a load bus is expressed as

$$S_\ell^* = V_\ell^* \underset{\sim}{y}_\ell^T \underset{\sim}{V}_M, \quad \ell \in I_L, \tag{8}$$

where

$$I_L \triangleq \{1, 2, \ldots, n_L\}, \tag{9}$$

$\underset{\sim}{y}_\ell^T$ is an appropriate row of $\underset{\sim}{Y}_T$ and n_L is the number of such buses. For load buses, the real power P_ℓ and reactive power Q_ℓ are specified, hence the modelling (8) is in an appropriate form, with

$$S_\ell \triangleq P_\ell + jQ_\ell, \quad \ell \in I_L. \tag{10}$$

The perturbed form of (8) corresponding to first-order changes in states and controls is, for a given network,

$$\delta S_\ell^* = V_\ell^* \underset{\sim}{y}_\ell^T \delta \underset{\sim}{V}_M + \underset{\sim}{y}_\ell^T \underset{\sim}{V}_M \delta V_\ell^*, \quad \ell \in I_L. \tag{11}$$

We distinguish, in this paper, dummy load buses which are described by equations corresponding to (8) in the form

$$S_d^* = 0 = V_d^* \underset{\sim}{y}_d^T \underset{\sim}{V}_M, \quad d \in I_D, \tag{12}$$

where

$$I_D \triangleq \{n_L+1, \ldots, n_L+n_D\}, \tag{13}$$

$\underset{\sim}{y}_d^T$ is an appropriate row of $\underset{\sim}{Y}_T$ and n_D is the number of dummy load buses. Therefore, the perturbed form becomes, using (12)

$$\delta S_d^* = V_d^* \underset{\sim}{y}_d^T \delta \underset{\sim}{V}_M, \quad d \in I_D. \tag{14}$$

Generator Buses

For generator buses, being voltage-controlled, the modulus of the voltage and the active power are specified. These constraints do not fit conveniently into (7), so they have to be manipulated accordingly. We let [1-4]

$$S_g \triangleq P_g + j|V_g|$$

$$= \frac{1}{2}(V_g I_g^* + V_g^* I_g) + j(V_g V_g^*)^{1/2}$$

$$= \frac{1}{2}(V_g \underset{\sim}{y}_g^{*T} \underset{\sim}{V}_M^* + V_g^* \underset{\sim}{y}_g^T \underset{\sim}{V}_M) + j(V_g V_g^*)^{1/2}, \; g \in I_G, \tag{15}$$

where

$$I_G \triangleq \{n_L + n_D + 1, \ldots, n-1\}, \tag{16}$$

$\underset{\sim}{y}_g^T$ is an appropriate row of $\underset{\sim}{Y}_T$ and n is the number of buses including the slack bus. We write the perturbed form for generator buses as

$$2\delta S_g = V_g \underset{\sim}{y}_g^{*T} \delta \underset{\sim}{V}_M^* + \underset{\sim}{y}_g^{*T} \underset{\sim}{V}_M^* \delta V_g + V_g^* \underset{\sim}{y}_g^T \delta \underset{\sim}{V}_M + \underset{\sim}{y}_g^T \underset{\sim}{V}_M \delta V_g^*$$

$$+ j(V_g \delta V_g^* + V_g^* \delta V_g)/(|V_g|), \quad g \in I_G. \tag{17}$$

We group the terms of (17) as

$$2\delta S_g = V_g^* \underset{\sim}{y}_g^T \delta \underset{\sim}{V}_M + (\underset{\sim}{y}_g^{*T} \underset{\sim}{V}_M^* + jV_g^*/|V_g|)\delta V_g$$

$$+ V_g \underset{\sim}{y}_g^{*T} \delta \underset{\sim}{V}_M^* + (\underset{\sim}{y}_g^T \underset{\sim}{V}_M + jV_g/|V_g|)\delta V_g^*, \quad i \in I_G. \qquad (18)$$

Power Flow Equations: General Form

It is clear from the foregoing discussion that the power flow equations can be expressed as

$$\underset{\sim}{S} = \underset{\sim}{f}(\underset{\sim}{V}_M, \underset{\sim}{V}_M^*), \qquad (19)$$

where $\underset{\sim}{S}$ includes S_ℓ^*, S_d^* and S_g as already defined. Thus, the complex perturbed form can be written as

$$\underset{\sim}{K}_{SM}\delta\underset{\sim}{V}_M + \overline{\underset{\sim}{K}}_{SM}\delta\underset{\sim}{V}_M^* = \underset{\sim}{b}_M, \qquad (20)$$

where the complex matrices of coefficients $\underset{\sim}{K}_{SM}$ and $\overline{\underset{\sim}{K}}_{SM}$ are explicitly available from (11), (14) and (18), and

$$b_\ell \triangleq \delta S_\ell^*, \quad \ell \in I_L, \qquad (21)$$

$$b_d \triangleq \delta S_d^*, \quad d \in I_D, \qquad (22)$$

$$b_g \triangleq \delta S_g, \quad g \in I_G. \qquad (23)$$

Newton-Raphson: jth Iteration

In the jth iteration of the complex mode Newton-Raphson method we solve the system (20) for $\delta\underset{\sim}{V}_M^j$ given

$$\underset{\sim}{b}_M^j = \underset{\sim}{S}_{(\text{scheduled})} - \underset{\sim}{f}(\underset{\sim}{V}_M^j, \underset{\sim}{V}_M^{j*}) \qquad (24)$$

using (19). We let

$$\underset{\sim}{V}_M^{j+1} = \underset{\sim}{V}_M^j + \delta\underset{\sim}{V}_M^j \qquad (25)$$

and continue in this manner until an appropriate criterion for $\delta\underset{\sim}{V}_M^j$ and $\underset{\sim}{b}_M^j$ has been satisfied.

III. COMPLEX TABLEAU FOR LINEARIZED POWER FLOW EQUATIONS

While properly accounting for the equation for the slack bus in the power flow equations (7), and deleting it from the rest of the perturbed equations (20) we can write

$$\underset{\sim}{K}\,\underset{\sim}{x} + \underset{\sim}{\overline{K}}\,\underset{\sim}{x}^{*} = \underset{\sim}{b}, \tag{26}$$

where

$$x_i \overset{\Delta}{=} \delta V_i, \quad i \in \{1, 2, \ldots, n-1\} \tag{27}$$

and where an equation of (20) has the form

$$\underset{\sim}{k}_i^T\,\underset{\sim}{x} + \underset{\sim}{\overline{k}}_i^T\,\underset{\sim}{x}^{*} = b_i. \tag{28}$$

This is equivalent to

$$\underset{\sim}{\overline{k}}_i^{*T}\,\underset{\sim}{x} + \underset{\sim}{k}_i^{*T}\,\underset{\sim}{x}^{*} = b_i^{*}. \tag{29}$$

We define coefficients of $\underset{\sim}{x}$ to comprise the <u>basic tableau</u> and coefficients of $\underset{\sim}{x}^{*}$ to comprise the <u>conjugate tableau</u>. Equations (28) and (29) can be represented in tableau form as follows.

<u>Original Tableau: ith Row</u>

$$\begin{array}{ccc} \underset{\sim}{x}^T\text{(basic)} & \underset{\sim}{x}^{*T}\text{(conjugate)} & \underset{\sim}{b}\text{(RHS)} \\ [\quad \underset{\sim}{k}_i^T & \vdots \quad \underset{\sim}{\overline{k}}_i^T \quad] & [\quad b_i \quad] \end{array} \tag{30}$$

Taking the complex conjugate and interchanging storage locations between the basic and conjugate tableaus, we obtain the following.

<u>Consistent Tableau: ith Row</u>

$$[\quad \underset{\sim}{\overline{k}}_i^{*T} \quad \vdots \quad \underset{\sim}{k}_i^{*T} \quad]\;[\quad b_i^{*} \quad] \tag{31}$$

The complex tableau as it stands at this stage in our presentation can be set out as follows, where the conjugate coefficients associated with the load buses have been normalized (the ℓth equation (11) is multiplied by $1/(\underset{\sim}{y}_{\ell}^{T} \underset{\sim}{V}_{M})$, $\ell \in I_L$.

Initial Complex Tableau

$$\begin{array}{c} \\ n_L \\ \\ n_D \\ \\ n_G \end{array} \left[\begin{array}{ccc|ccc|c} \leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow & \leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow & \\ \underset{\sim}{K}_{LL}^{(0)} & \underset{\sim}{K}_{LD}^{(0)} & \underset{\sim}{K}_{LG}^{(0)} & \underset{\sim}{1} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{b}_{L}^{(0)} \\ \hline \underset{\sim}{K}_{DL}^{(0)} & \underset{\sim}{K}_{DD}^{(0)} & \underset{\sim}{K}_{DG}^{(0)} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{b}_{D}^{(0)} \\ \hline \underset{\sim}{K}_{GL}^{(0)} & \underset{\sim}{K}_{GD}^{(0)} & \underset{\sim}{K}_{GG}^{(0)} & \overline{\underset{\sim}{K}}_{GL}^{(0)} & \overline{\underset{\sim}{K}}_{GD}^{(0)} & \overline{\underset{\sim}{K}}_{GG}^{(0)} & \underset{\sim}{b}_{G}^{(0)} \end{array} \right] . \quad (32)$$

In the tableau, $\underset{\sim}{1}$ denotes the square unit matrix of appropriate dimensions and $\underset{\sim}{0}$ is the null matrix. We remark here that, in practice, n_D is expected to be quite small as is n_G, in comparison with n_L. Obviously, retaining the complex form of these linearized equations results in immediate savings in computer storage [10]. To prepare the tableau for subsequent conjugate elimination we diagonalize the conjugate tableau in the following way, noting that the storage locations reserved for $\underset{\sim}{K}_{LL}$, $\underset{\sim}{K}_{LD}$, $\underset{\sim}{K}_{LG}$, $\underset{\sim}{K}_{DL}$, $\underset{\sim}{K}_{DD}$, $\underset{\sim}{K}_{DG}$, $\underset{\sim}{K}_{GL}$, $\underset{\sim}{K}_{GD}$, $\underset{\sim}{K}_{GG}$, $\overline{\underset{\sim}{K}}_{GL}$, $\overline{\underset{\sim}{K}}_{GD}$, $\overline{\underset{\sim}{K}}_{GG}$, $\underset{\sim}{b}_{L}$, $\underset{\sim}{b}_{D}$, and $\underset{\sim}{b}_{G}$ are used for intermediate computations.

Transformation of $\overline{\underset{\sim}{K}}_{DD}$

Diagonalizing $\underset{\sim}{K}_{DD}^{(0)}$ and transferring it to the conjugate tableau by invoking the consistent form (31) we have

$$\left[\begin{array}{ccc|ccc|c}
\mathbf{K}_{LL}^{(0)} & \mathbf{K}_{LD}^{(0)} & \mathbf{K}_{LG}^{(0)} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{b}_{L}^{(0)} \\
\hline
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{DL}^{*(1)} & \mathbf{1} & \mathbf{K}_{DG}^{*(1)} & \mathbf{b}_{D}^{*(1)} \\
\hline
\mathbf{K}_{GL}^{(0)} & \mathbf{K}_{GD}^{(0)} & \mathbf{K}_{GG}^{(0)} & \overline{\mathbf{K}}_{GL}^{(0)} & \overline{\mathbf{K}}_{GD}^{(0)} & \overline{\mathbf{K}}_{GG}^{(0)} & \mathbf{b}_{G}^{(0)}
\end{array}\right], \quad (33)$$

where superscripts 0 identify no changes in the tableau from the initial form, while $\mathbf{K}_{DL}^{*(1)}$, $\mathbf{K}_{DG}^{*(1)}$ and $\mathbf{b}_{D}^{*(1)}$ represent appropriate changes in the consistent tableau associated with dummy loads and are stored in $\mathbf{K}_{DL}$, $\mathbf{K}_{DG}$ and $\mathbf{b}_{D}$ locations without confusion.

Reduction of $\overline{\mathbf{K}}_{GD}$

Note that in the forward reduction process for $\overline{\mathbf{K}}_{GD}$, rows $n_L+1, \ldots, n_L+n_D$ of the basic tableau are zero, therefore the basic tableau for rows $n_L + n_D + 1, \ldots, n-1$ is unchanged. The new tableau is, using the unit $\overline{\mathbf{K}}_{DD}$ matrix,

$$\left[\begin{array}{ccc|ccc|c}
\mathbf{K}_{LL}^{(0)} & \mathbf{K}_{LD}^{(0)} & \mathbf{K}_{LG}^{(0)} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{b}_{L}^{(0)} \\
\hline
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{DL}^{*(1)} & \mathbf{1} & \mathbf{K}_{DG}^{*(1)} & \mathbf{b}_{D}^{*(1)} \\
\hline
\mathbf{K}_{GL}^{(0)} & \mathbf{K}_{GD}^{(0)} & \mathbf{K}_{GG}^{(0)} & \overline{\mathbf{K}}_{GL}^{(1)} & \mathbf{0} & \overline{\mathbf{K}}_{GG}^{(1)} & \mathbf{b}_{G}^{(1)}
\end{array}\right]. \quad (34)$$

<u>Reduction of $\bar{\underset{\sim}{K}}_{GL}$</u>

Using the unit $\bar{\underset{\sim}{K}}_{LL}$ matrix we have

$$
\begin{array}{c}
\\ n_L \\ \\ n_D \\ \\ n_G
\end{array}
\left[\begin{array}{ccc|ccc|c}
\leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow & \leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow & \\
\underset{\sim}{K}_{LL}^{(0)} & \underset{\sim}{K}_{LD}^{(0)} & \underset{\sim}{K}_{LG}^{(0)} & \underset{\sim}{1} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{b}_L^{(0)} \\
\hline
\underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{K}_{DL}^{*(1)} & \underset{\sim}{1} & \underset{\sim}{K}_{DG}^{*(1)} & \underset{\sim}{b}_D^{*(1)} \\
\hline
\underset{\sim}{K}_{GL}^{(1)} & \underset{\sim}{K}_{GD}^{(1)} & \underset{\sim}{K}_{GG}^{(1)} & \underset{\sim}{0} & \underset{\sim}{0} & \bar{\underset{\sim}{K}}_{GG}^{(1)} & \underset{\sim}{b}_G^{(2)}
\end{array}\right], \quad (35)
$$

where we note no changes to the conjugate tableau in its columns n_L+1, ..., $n-1$.

<u>Diagonalization of $\bar{\underset{\sim}{K}}_{GG}$</u>

We diagonalize $\bar{\underset{\sim}{K}}_{GG}^{(1)}$, affecting only rows n_L+n_D+1, ..., $n-1$ and obtain

$$
\begin{array}{c}
\\ n_L \\ \\ n_D \\ \\ n_G
\end{array}
\left[\begin{array}{ccc|ccc|c}
\leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow & \leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow & \\
\underset{\sim}{K}_{LL}^{(0)} & \underset{\sim}{K}_{LD}^{(0)} & \underset{\sim}{K}_{LG}^{(0)} & \underset{\sim}{1} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{b}_L^{(0)} \\
\hline
\underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{K}_{DL}^{*(1)} & \underset{\sim}{1} & \underset{\sim}{K}_{DG}^{*(1)} & \underset{\sim}{b}_D^{*(1)} \\
\hline
\underset{\sim}{K}_{GL}^{(2)} & \underset{\sim}{K}_{GD}^{(2)} & \underset{\sim}{K}_{GG}^{(2)} & \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{1} & \underset{\sim}{b}_G^{(3)}
\end{array}\right]. \quad (36)
$$

<u>Reduction of $\bar{\underset{\sim}{K}}_{DL}$ and $\bar{\underset{\sim}{K}}_{DG}$</u>

Using the unit matrices $\bar{\underset{\sim}{K}}_{LL}$ and $\bar{\underset{\sim}{K}}_{GG}$ we carry out a simultaneous reduction process which affects only rows n_L+1, ..., n_L+n_D. This gives

$$\begin{array}{c} \\ \uparrow \\ n_L \\ \downarrow \\ \uparrow \\ n_D \\ \downarrow \\ \uparrow \\ n_G \\ \downarrow \end{array} \begin{array}{c} \begin{array}{cccccc} \leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow & \leftarrow n_L \rightarrow & \leftarrow n_D \rightarrow & \leftarrow n_G \rightarrow \end{array} \\ \left[\begin{array}{ccc|ccc|c} \tilde{K}_{LL}^{(0)} & \tilde{K}_{LD}^{(0)} & \tilde{K}_{LG}^{(0)} & \tilde{1} & \tilde{0} & \tilde{0} & \tilde{b}_L^{(0)} \\ \hline \tilde{K}_{DL}^{(2)} & \tilde{K}_{DD}^{(1)} & \tilde{K}_{DG}^{(2)} & \tilde{0} & \tilde{1} & \tilde{0} & \tilde{b}_D^{*(2)} \\ \hline \tilde{K}_{GL}^{(2)} & \tilde{K}_{GD}^{(2)} & \tilde{K}_{GG}^{(2)} & \tilde{0} & \tilde{0} & \tilde{1} & \tilde{b}_G^{(3)} \end{array}\right] \end{array}, \quad (37)$$

where we see the second change to the storage locations for $\tilde{K}_{DL}$ and $\tilde{K}_{DG}$ and the first to $\tilde{K}_{DD}$.

<u>Final Complex Tableau</u>

The final complex tableau, in which the conjugate tableau has been diagonalized and exhibiting explicitly the changes by computation necessary to achieve this is

$$\left[\begin{array}{ccc|ccc|c} \tilde{K}_{LL}^{(0)} & \tilde{K}_{LD}^{(0)} & \tilde{K}_{LG}^{(0)} & \tilde{1} & \tilde{0} & \tilde{0} & \tilde{b}_L^{(0)} \\ \hline \tilde{K}_{DL}^{(2)} & \tilde{K}_{DD}^{(1)} & \tilde{K}_{DG}^{(2)} & \tilde{0} & \tilde{1} & \tilde{0} & \tilde{b}_D^{*(2)} \\ \hline \tilde{K}_{GL}^{(2)} & \tilde{K}_{GD}^{(2)} & \tilde{K}_{GG}^{(2)} & \tilde{0} & \tilde{0} & \tilde{1} & \tilde{b}_G^{(3)} \end{array}\right]. \quad (38)$$

It is important to note that no changes have been made in the tableau associated with the load buses, which are usually in the majority, while relatively few rows, namely those associated with dummy loads and generators have undergone changes and, in general, fill-ins in the sparseness.

We summarize the tableau at the present stage as

$$[\ \underset{\sim}{K}^{(0)} \ \vdots \ \underset{\sim}{1} \ \vdots \ \underset{\sim}{b}^{(0)}\ ]. \tag{39}$$

Conjugate reduction combined with forward Gaussian elimination is employed in the manner presented by Bandler and El-Kady for a power system consisting only of load buses [3]. The ith step of the process is illustrated as follows. We write the consistent form (31) for the present situation as

$$[\ \underset{\sim}{u}_i^T \ \vdots \ \underset{\sim}{k}_i^{*T(i-1)} \ \vdots \ \underset{\sim}{b}_i^{*(i-1)}], \tag{40}$$

where $\underset{\sim}{u}_i$ is a unit vector with ith element of unity. Now the remaining rows i, i+1, ..., n-1 are used from the current original tableau to first eliminate the conjugate part of this consistent form resulting in the ith row

$$[\ \underset{\sim}{k}_i^{T(i)} \ \vdots \ \underset{\sim}{0} \ \vdots \ b_i^{(i)}\ ]. \tag{41}$$

The ith row is now used in a Gaussian forward reduction on the ith column, the result of which is

$$\left[\begin{array}{c|c|c} \underset{\sim}{k}_1^{T(1)} & \underset{\sim}{0} & b_1^{(1)} \\ \underset{\sim}{k}_2^{T(2)} & \underset{\sim}{0} & b_2^{(2)} \\ \vdots & \vdots & \vdots \\ \underset{\sim}{k}_i^{T(i)} & \underset{\sim}{0} & b_i^{(i)} \\ \hline \underset{\sim}{k}_{i+1}^{T(i)} & \underset{\sim}{u}_{i+1}^T & b_{i+1}^{(i)} \\ \vdots & \vdots & \vdots \\ \underset{\sim}{k}_{n-1}^{T(i)} & \underset{\sim}{u}_{n-1}^T & b_{n-1}^{(i)} \end{array}\right], \tag{42}$$

where we have created elements

$$k_{rs}^{(i)} = 0, \quad r > s, \quad s < i. \tag{43}$$

Backward substitution gives the desired solution.

Succinctly, the equations may be written as

$$\underset{\sim}{K}\,\underset{\sim}{x} + \underset{\sim}{x}^{*} = \underset{\sim}{b} \tag{44}$$

with accompanying consistent form

$$\underset{\sim}{x} + \underset{\sim}{K}^{*}\underset{\sim}{x}^{*} = \underset{\sim}{b}^{*} \tag{45}$$

yielding

$$(\underset{\sim}{1} - \underset{\sim}{K}^{*}\,\underset{\sim}{K})\underset{\sim}{x} = \underset{\sim}{b}^{*} - \underset{\sim}{K}^{*}\underset{\sim}{b}, \tag{46}$$

which can alternatively be solved by standard techniques.

IV. APPLICATION TO TEST POWER SYSTEMS

In this paper we consider three power systems (6-bus [11,12], 23-bus [13] and 26-bus [6,14] to illustrate general computational aspects of the algorithm presented. The detailed data of the 6-bus, 23-bus and 26-bus power systems are tabulated by Tables I-VI. The structure and line diagrams of these power systems are shown in Figs. 1-3, respectively. All the values shown are in per unit. The algorithm is programmed using rectangular coordinates. For determining the solution of the load flow problems of the systems, flat voltage profiles have been used as starting points. The computations have been performed on a CYBER 170 computer.

Tables VII, VIII and IX show the solutions of the load flow problems for the 6-bus, the 23-bus and the 26-bus power systems, respectively. Solutions are obtained in 5 iterations. The detailed iterative solutions in the rectangular coordinates, with starting flat voltage profiles for the 6-bus, the 23-bus and the 26-bus power systems are tabulated in Tables X, XI and XII, respectively.

V. CONCLUSIONS

The direct solution of the complex linearized power flow equations, required at the jth iteration of a complex Netwon-Raphson method, has been described in this paper. By synthesis of complex variables consisting of the adjustable variables associated with voltage-controlled buses, we fully incorporate generators. The practical solution of large power systems has been emphasized. We have presented our algorithm in a tableau form which exposes the sparsity structure of the matrix of coefficients, and preserves the sparsity of the coefficients associated with the load buses until the conjugate tableau has been diagonalized. Subsequent elimination schemes to solve for the perturbed complex bus voltages are described.

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TABLE I

LINE DATA FOR 6-BUS POWER SYSTEM

Line No.	Terminal Buses	Resistance R_t (pu)	Reactance X_t (pu)	Number of Lines
1	1,4	0.05	0.20	1
2	1,5	0.025	0.10	2
3	2,3	0.10	0.40	1
4	2,4	0.10	0.40	1
5	2,5	0.05	0.20	1
6	2,6	0.01875	0.075	4
7	3,4	0.15	0.60	1
8	3,6	0.0375	0.15	2

TABLE II

BUS DATA FOR 6-BUS POWER SYSTEM

Bus No.	Bus Type	P_m (pu)	Q_m (pu)	$\|V_m\| \angle \delta_m$ (pu)
1	load	-2.40	0	- $\angle$-
2	load	-2.40	0	- $\angle$-
3	load	-1.60	-0.40	- $\angle$-
4	generator	-0.30	-	1.02 $\angle$-
5	generator	1.25	-	1.04 $\angle$-
6	slack	-	-	1.04 $\angle$-

TABLE III

LINE DATA FOR 23-BUS POWER SYSTEM

Line No.	Terminal Buses	Resistance R_t (pu)	Reactance X_t (pu)	1/2 Shunt Susceptance
1	23,1	0.0242	0.0540	0.00590
2	23,2	0.0309	0.0693	0.00755
3	18,3	0.0404	0.0888	0.00985
4	6,3	0.0325	0.0709	0.00785
5	18,5	0.0615	0.1620	0.01710
6	1,4	0.0576	0.1520	0.01600
7	2,7	0.0266	0.0700	0.00740
8	7,5	0.0229	0.0504	0.00560
9	6,4	0.0446	0.1003	0.01090
10	19,8	0.0233	0.0514	0.02280
11	6,8	0.0597	0.1315	0.01455
12	7,8	0.0597	0.1315	0.01455
13	10,20	0.0043	0.0351	0.11865
14	20,9	0.0043	0.0351	0.11865
15	11,9	0.0038	0.0307	0.10390
16	14,11	0.0035	0.0288	0.09755
17	22,10	0.0089	0.0726	0.24355
18	12,13	0.0010	0.0080	0.02715
19	13,14	0.0021	0.0167	0.05665
20	15,14	0.0016	0.0127	0.04310
21	21,15	0.0045	0.0362	0.12255
22	17,14	0.0024	0.0192	0.06490
23	21,16	0.0019	0.0156	0.05280
24	16,17	0.0014	0.0114	0.03850
25	22,12	0.0020	0.0164	0.05545
26	9,6	0.0023	0.0839	0.0
27	10,6	0.0023	0.0839	0.0
28	9,7	0.0019	0.1300	0.0
29	10,7	0.0023	0.0839	0.0
30	23,18	0.0025	0.2000	0.0

TABLE IV

BUS DATA FOR 23-BUS POWER SYSTEM

Bus No.	Injected Power		Bus Voltage	
	P_m	Q_m	$\|V_m\|$	δ_m
1	0.00	0.00	-	-
2	-0.47	-0.12	-	-
3	-0.51	-0.13	-	-
4	-0.41	-0.10	-	-
5	-0.48	0.12	-	-
6	-0.01	0.00	-	-
7	-1.50	-0.38	-	-
8	-1.77	-0.44	-	-
9	-0.06	0.00	-	-
10	0.04	0.00	-	-
11	-2.01	-0.50	-	-
12	-1.32	-0.33	-	-
13	-3.44	-0.86	-	-
14	-1.04	-0.26	-	-
15	-3.76	-0.94	-	-
16	-3.75	-0.94	-	-
17	2.10	0.52	-	-
18	0.26	-	1.03	-
19	0.89	-	1.05	-
20	0.20	-	1.05	-
21	9.03	-	1.05	-
22	9.23	-	1.05	-
23	-	-	1.04	0.0

Transformer tap ($a_{mm'}$) between buses m and m'

$a_{9,6} = 1.04$, $a_{10,6} = 1.03$, $a_{9,7} = 1.05$, $a_{10,7} = 1.06$

Bus Type

$n_L = 17$, $n_G = 5$

TABLE V

LINE DATA FOR 26-BUS POWER SYSTEM

Line No.	Terminal Buses	Resistance R_t (pu)	Reactance X_t (pu)	1/2 Shunt Susceptance
1	13,26	0.0	0.0131	0.0
2	26,16	0.0	0.0392	0.0
3	16,23	0.0	0.4320	0.0
4	23,26	0.0	0.3140	0.0
5	2,10	0.0	0.0150	0.0
6	9,10	0.1494	0.3392	0.4120
7	9,12	0.0658	0.1494	0.0182
8	12,26	0.0533	0.1210	0.0147
9	9,14	0.0618	0.2397	0.0319
10	11,14	0.0676	0.2620	0.0349
11	19,26	0.0610	0.2521	0.0295
12	6,26	0.0513	0.1986	0.0265
13	6,19	0.0129	0.0532	0.0074
14	7,19	0.0906	0.3742	0.0437
15	6,7	0.0921	0.3569	0.0475
16	11,22	0.0513	0.2118	0.0248
17	8,11	0.0865	0.3355	0.0447
18	17,22	0.0281	0.1869	0.0237
19	8,21	0.0735	0.2847	0.0379
20	17,21	0.0459	0.3055	0.0387
21	1,4	0.0619	0.2401	0.0319
22	4,21	0.0610	0.2365	0.0315
23	20,21	0.0	0.0305	0.0
24	15,1	0.0	0.0147	0.0
25	2,13	0.0086	0.0707	0.3017
26	1,7	0.0199	0.0785	0.0404
27	15,20	0.0107	0.0617	0.4471
28	2,18	0.0074	0.0608	0.2593
29	1,3	0.0	0.0392	0.0
30	24,3	0.0	0.1450	0.0
31	5,21	0.0	0.1750	0.0
32	5,25	0.0	0.154	0.0

TABLE VI

BUS DATA FOR 26-BUS POWER SYSTEM

Bus No.	Injected Power		Bus Voltage	
	P_m	Q_m	$\|V_m\|$	δ_m
1	-0.82	-0.21	-	-
2	0.0	0.0	-	-
3	-0.57	-0.17	-	-
4	-0.48	-0.21	-	-
5	-0.43	-0.11	-	-
6	-0.40	-0.10	-	-
7	-1.11	-0.27	-	-
8	-0.23	-0.06	-	-
9	-0.67	-0.21	-	-
10	-1.02	-0.27	-	-
11	-0.43	-0.14	-	-
12	-0.43	-0.12	-	-
13	0.0	0.0	-	-
14	0.0	0.0	-	-
15	0.0	0.0	-	-
16	-1.31	-0.30	-	-
17	-0.03	-0.01	-	-
18	2.80	-	1.07	-
19	1.45	-	1.05	-
20	2.80	-	1.00	-
21	1.10	-	1.02	-
22	-0.56	-	0.89	-
23	-0.04	-	1.00	-
24	-0.05	-	1.00	-
25	0.63	-	1.00	-
26	0.0	-	1.01	0.0

Transformer tap ($a_{mm'}$) between buses m and m'

$a_{13,26} = 1.03$, $a_{26,16} = 0.96$, $a_{2,10} = 1.03$,

$a_{20,21} = 0.97$, $a_{15,1} = 0.98$, $a_{1,3} = 0.98$,

$a_{24,3} = 0.98$, $a_{5,21} = 0.99$, $a_{5,25} = 1.03$

Bus Type

$n_L = 17$, $n_G = 8$

TABLE VII

LOAD FLOW SOLUTION OF 6-BUS POWER SYSTEM

Load Buses

$V_1 = 0.7730 - j0.6002$

$V_2 = 0.9208 - j0.2826$

$V_3 = 0.8619 - j0.2700$

Generator Buses

$Q_4 = 0.7866$ $V_4 = 0.8660 - j0.5388$

$Q_5 = 0.9780$ $V_5 = 0.9253 - j0.4747$

Slack Bus

$P_6 = 6.1298$ $Q_6 = 1.3546$

TABLE VIII

LOAD FLOW SOLUTION OF 23-BUS POWER SYSTEM

Load Buses

$V_1 = 1.0314 + j0.0179$	$V_{10} = 1.0123 + j0.2473$
$V_2 = 1.0059 + j0.0263$	$V_{11} = 0.9806 + j0.2486$
$V_3 = 1.0040 + j0.0864$	$V_{12} = 0.9430 + j0.3873$
$V_4 = 1.0015 + j0.0670$	$V_{13} = 0.9465 + j0.3528$
$V_5 = 0.9974 + j0.0546$	$V_{14} = 0.9529 + j0.3395$
$V_6 = 1.0061 + j0.1406$	$V_{15} = 0.9477 + j0.3418$
$V_7 = 0.9897 + j0.0807$	$V_{16} = 0.9408 + j0.4125$
$V_8 = 0.9931 + j0.0414$	$V_{17} = 0.9455 + j0.4007$
$V_9 = 1.0112 + j0.2137$	

Generator Buses

$Q_{18} = 0.4204$	$V_{18} = 1.0282 + j0.0615$
$Q_{19} = 0.7228$	$V_{19} = 1.0475 + j0.0722$
$Q_{20} = 0.4510$	$V_{20} = 1.0233 + j0.2351$
$Q_{21} = 1.9016$	$V_{21} = 0.9301 + j0.4873$
$Q_{22} = 1.2589$	$V_{22} = 0.9340 + j0.4797$

Slack Bus

$P_{23} = -0.6839$	$Q_{23} = 0.8913$

TABLE IX

LOAD FLOW SOLUTION OF 26-BUS POWER SYSTEM

Load Buses	$V_1 = 1.0328 + j0.0773$	$V_{10} = 1.0370 + j0.0692$
	$V_2 = 1.0644 + j0.0943$	$V_{11} = 0.8982 - j0.0992$
	$V_3 = 1.0424 + j0.0549$	$V_{12} = 0.9670 - j0.0741$
	$V_4 = 0.9859 + j0.0979$	$V_{13} = 1.0463 + j0.0157$
	$V_5 = 0.9741 + j0.2598$	$V_{14} = 0.9388 - j0.1071$
	$V_6 = 1.0324 + j0.0554$	$V_{15} = 0.9273 + j0.0970$
	$V_7 = 1.0132 + j0.0181$	$V_{16} = 1.0353 - j0.0471$
	$V_8 = 0.9441 + j0.0403$	$V_{17} = 0.9318 + j0.0278$
	$V_9 = 0.9614 - j0.1088$	
Generator Buses	$Q_{18} = -0.4004$	$V_{18} = 1.0397 + j0.2528$
	$Q_{19} = 0.1872$	$V_{19} = 1.0455 + j0.0966$
	$Q_{20} = 0.7795$	$V_{20} = 0.9706 + j0.2408$
	$Q_{21} = -0.0294$	$V_{21} = 0.9938 + j0.2295$
	$Q_{22} = -0.1775$	$V_{22} = 0.8856 - j0.0885$
	$Q_{23} = -0.1144$	$V_{23} = 0.9996 - j0.0265$
	$Q_{24} = -0.1645$	$V_{24} = 0.9989 + j0.0458$
	$Q_{25} = 0.1691$	$V_{25} = 0.9359 + j0.3522$
Slack Bus	$P_{26} = 0.1334$	$Q_{26} = -0.0513$

TABLE X

DETAILED LOAD FLOW SOLUTION OF 6-BUS POWER SYSTEM

Iteration No.	Quantity	
	b_i: $e_b = \max\{\lvert b_{i1}\rvert, \lvert b_{i2}\rvert\}$	δV_i: $e_v = \max\{\lvert \delta V_{i1}\rvert, \lvert \delta V_{i2}\rvert\}$
1	$-0.247 + j0.117\text{x}10^{1}$	$-0.690\text{x}10^{-2} - j0.584$
2	$0.295\text{x}10^{-1} + j0.237$	$-0.197 - j0.178\text{x}10^{-1}$
3	$0.287\text{x}10^{-2} + j0.521\text{x}10^{-2}$	$-0.225\text{x}10^{-1} + j0.157\text{x}10^{-2}$
4	$-0.199\text{x}10^{-5} - j0.831\text{x}10^{-7}$	$-0.436\text{x}10^{-3} + j0.886\text{x}10^{-4}$
5	$-0.490\text{x}10^{-12} - j0.142\text{x}10^{-13}$	$-0.181\text{x}10^{-6} + j0.389\text{x}10^{-7}$

$e_b \triangleq \max_{i,j} \{\lvert b_{i1}\rvert, \lvert b_{j2}\rvert\}$ $\qquad$ $e_v \triangleq \max_{i,j} \{\lvert \delta V_{i1}\rvert, \lvert \delta V_{j2}\rvert\}$

where subscript 1 denotes real part and subscript 2 denotes imaginary part

TABLE XI

DETAILED LOAD FLOW SOLUTION OF 23-BUS POWER SYSTEM

Iteration No.	Quantity	
	b_i: $e_b = \max\{\lvert b_{i1}\rvert, \lvert b_{i2}\rvert\}$	δV_i: $e_v = \max\{\lvert \delta V_{i1}\rvert, \lvert \delta V_{i2}\rvert\}$
1	$-0.241\text{x}10^{1} - j0.152$	$-0.575\text{x}10^{-11} - j0.584$
2	$0.681\text{x}10^{-1} - j0.210$	$-0.123 - j0.902\text{x}10^{-1}$
3	$0.168\text{x}10^{-2} - j0.249\text{x}10^{-4}$	$0.325\text{x}10^{-2} - j0.647\text{x}10^{-2}$
4	$-0.285\text{x}10^{-7} + j0.765\text{x}10^{-7}$	$-0.181\text{x}10^{-4} - j0.627\text{x}10^{-4}$
5	$0.156\text{x}10^{-12} - j0.750\text{x}10^{-12}$	$-0.745\text{x}10^{-9} - j0.238\text{x}10^{-8}$

$e_b \triangleq \max_{i,j} \{\lvert b_{i1}\rvert, \lvert b_{j2}\rvert\}$ $\qquad$ $e_v \triangleq \max_{i,j} \{\lvert \delta V_{i1}\rvert, \lvert \delta V_{j2}\rvert\}$

where subscript 1 denotes real part and subscript 2 denotes imaginary part

TABLE XII

DETAILED LOAD FLOW SOLUTION OF 26-BUS POWER SYSTEM

Iteration No.	Quantity	
	b_i: $e_b = \max\{\lvert b_{i1}\rvert, \lvert b_{i2}\rvert\}$	δV_i: $e_v = \max\{\lvert \delta V_{i1}\rvert, \lvert \delta V_{i2}\rvert\}$
1	$-0.122\text{x}10^{+1} + j0.373$	$-0.299\text{x}10^{-13} + j0.402$
2	$0.680\text{x}10^{-1} + j0.796\text{x}10^{-2}$	$-0.641\text{x}10^{-1} - j0.494\text{x}10^{-1}$
3	$0.179\text{x}10^{-3} - j0.305\text{x}10^{-5}$	$-0.195\text{x}10^{-2} - j0.360\text{x}10^{-2}$
4	$-0.216\text{x}10^{-4} - j0.124\text{x}10^{-4}$	$0.131\text{x}10^{-4} - j0.665\text{x}10^{-5}$
5	$-0.395\text{x}10^{-7} - j0.425\text{x}10^{-7}$	$-0.115\text{x}10^{-4} - j0.519\text{x}10^{-5}$

$e_b \triangleq \max_{i,j} \{\lvert b_{i1}\rvert, \lvert b_{j2}\rvert\}$ $\quad$ $e_v \triangleq \max_{i,j} \{\lvert \delta V_{i1}\rvert, \lvert \delta V_{j2}\rvert\}$

where subscript 1 denotes real part and subscript 2 denotes imaginary part

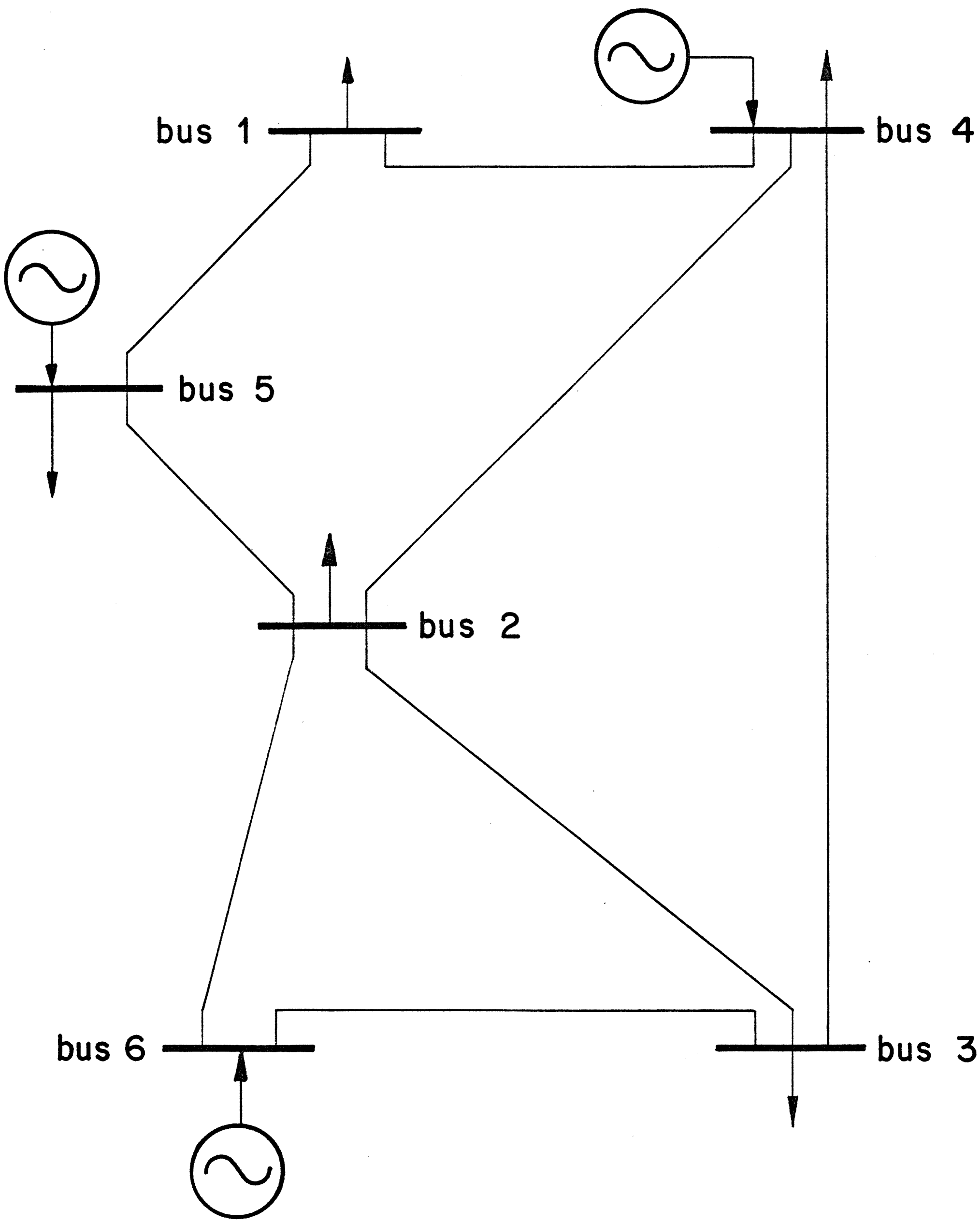

Fig. 1 6-bus power system

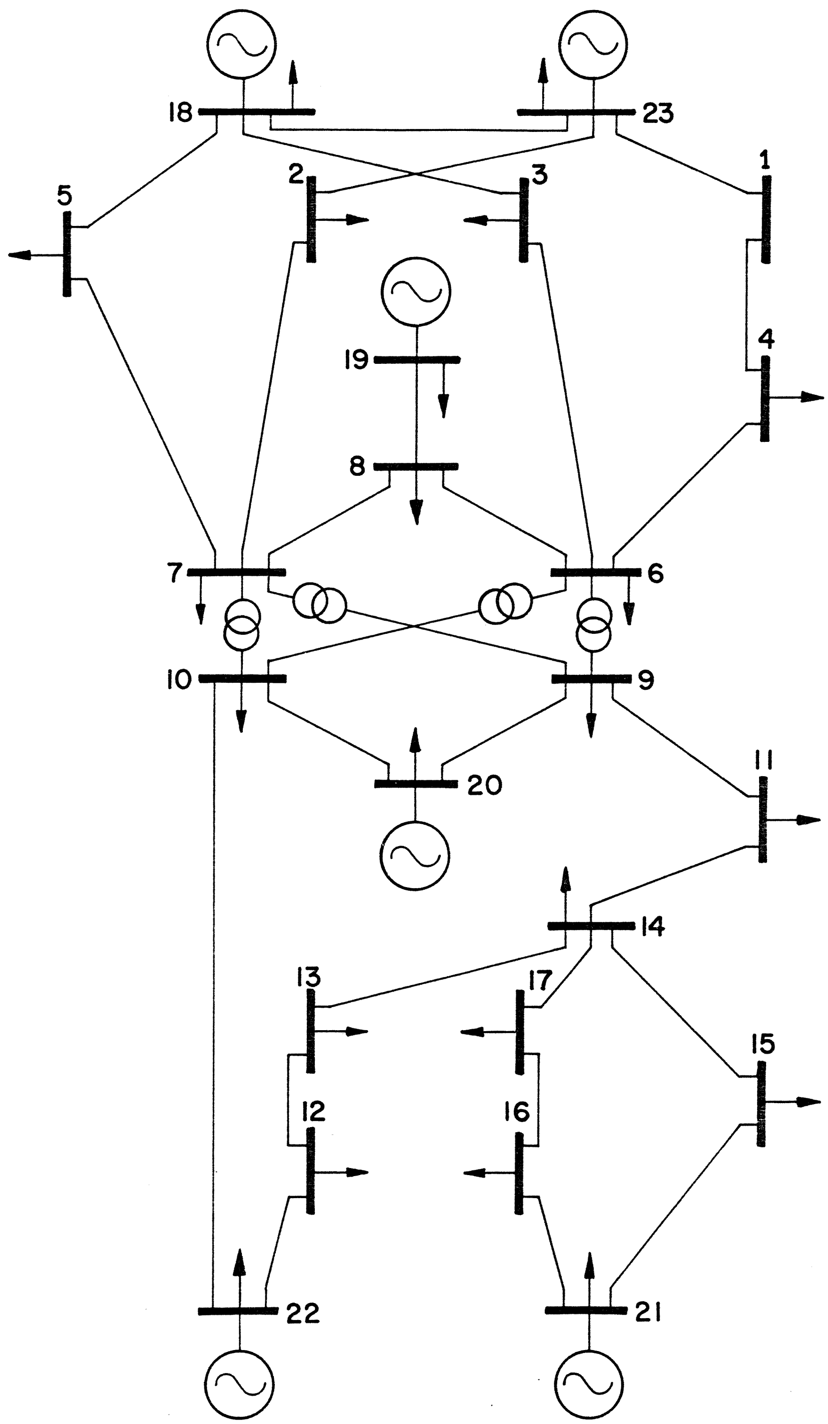

Fig. 2 23-bus power system

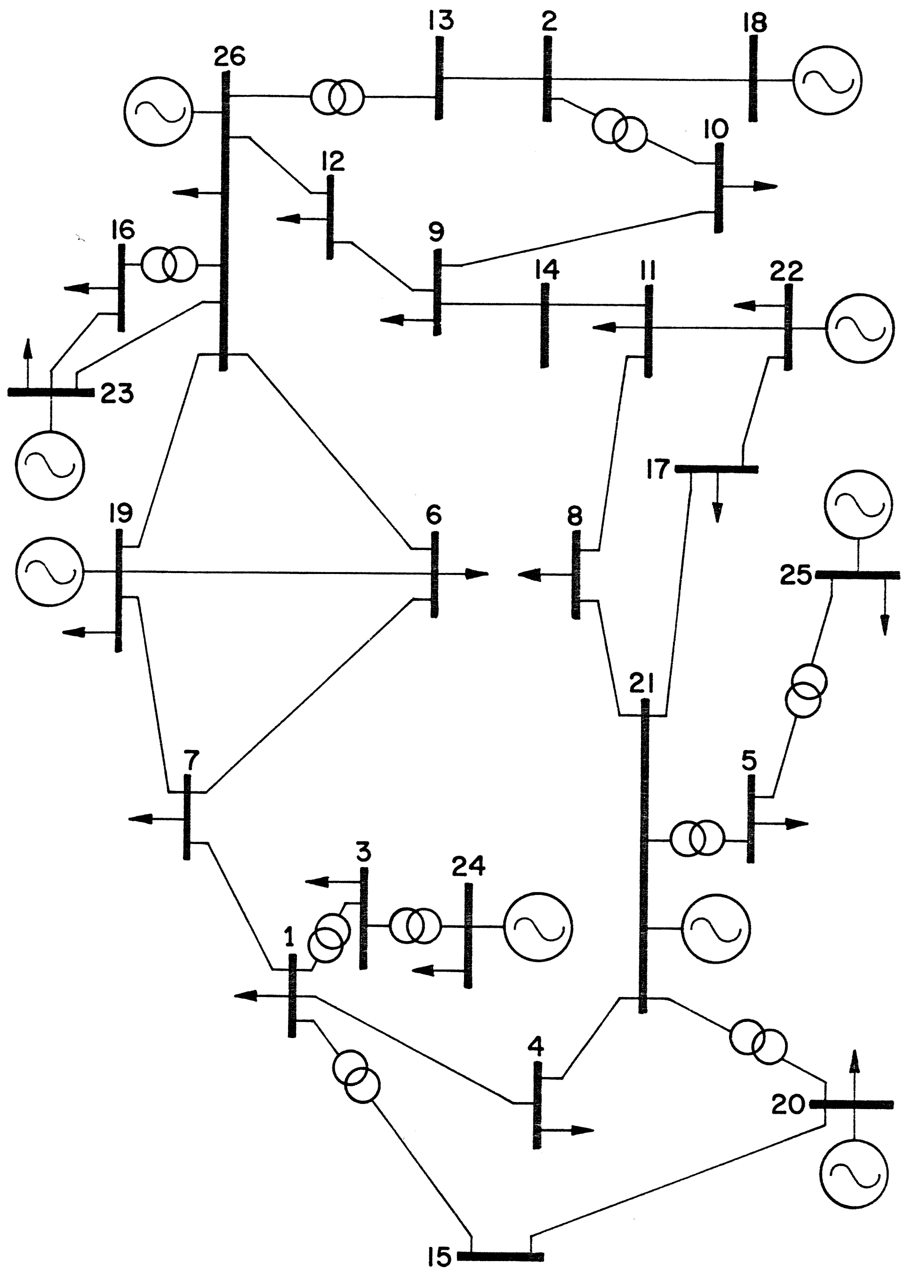

Fig. 3 26-bus power system

SOC-270

PRACTICAL COMPLEX SOLUTION OF POWER FLOW EQUATIONS

J.W. Bandler, M.A. El-Kady and H. Gupta

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Key Words: Load flow analysis, conjugate notation, complex analysis, elimination techniques, power systems analysis

Abstract: This paper applies the compact, complex notation introduced by Bandler and El-Kady to the practical solution of the power flow equations. The solution of the complex linearized power flow equations, which is required by the iterative Newton-Raphson method, is obtained by a direct method. The method, fully and exactly, incorporates generator buses as well as dummy load buses. An elimination scheme is applied to diagonalize the conjugate tableau, which contains the complex coefficients associated with the conjugate of the perturbed bus voltages. This conjugate tableau is then eliminated, simultaneously reducing the basic tableau, which contains the complex coefficients associated with the perturbed bus voltages, to upper triangular form. Alternatively, the conjugate tableau is explicitly eliminated, exposing a set of linear, complex equations in the perturbed complex bus voltages. The theoretical results are illustrated by solving the load flow equations for a 6-bus, 23-bus and a 26-bus system.

Description:

Related Work: SOC-242, SOC-243, SOC-253, SOC-254, SOC-255, SOC-256, SOC-257.

Price: $ 6.00.