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MULTIPORT APPROACH TO MULTIPLE-FAULT LOCATION IN ANALOG CIRCUITS

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Abstract

This paper deals with the multiport method for multiple-fault location in linear analog circuits. A hybrid multiport description of the linear network has been used in the presentation, which generalizes and explains proposals made by Biernacki and Bandler. The problem of consistency of the chosen set of equations used for fault identification is discussed. The restrictions of the method are explained on the basis of network topology.

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I. INTRODUCTION

Multiple-fault location is an important problem in testing analog circuits, particularly when the number of measurements is too small to evaluate all of the network elements. Various researchers have discussed and elaborated upon the problem of fault location [1-7] under the assumption that the network elements assume their nominal values (or are close to them) and only a few elements are faulty.

In a recent paper by Biernacki and Bandler [3], the multiport approach to fault location was discussed and some necessary conditions on the set of equations used for fault identification were formulated. The concept of block dependency of these equations is of special importance in the identification problem and is discussed in particular in this paper for a general hybrid description of the multiport used to relate ports of fault to ports of measurement. Certain conjectures made by Biernacki and Bandler are addressed in a rigorous manner.

II. HYBRID REPRESENTATION FOR MULTIPORT APPROACH TO FAULT LOCATION

Assume for simplicity that the linear network under investigation contains one-port elements and controlled sources only. Assume also that the network has n+1 nodes, e elements with f of them faulty. To identify all the faults we measure m voltages in the network, $m > f$.

Changes in element values w.r.t. the nominal can be represented by current sources in parallel with elements (for one-ports and controlled current sources) or by voltage sources in series with elements (for one-ports and controlled voltage sources). See Fig. 1. Changes w.r.t.

nominal values, i.e., the <u>faults</u> can be represented as loads of the (m+f)-port network consisting of all the elements of the original network which are at their nominal values. See Fig. 2. Assuming that the hybrid matrix $\underset{\sim}{H}$ of the (m+f)-port exists we obtain the relation

$$\begin{bmatrix} \underset{\sim}{V}^M \\ \underset{\sim}{V}^{F_1} \\ \underset{\sim}{I}^{F_2} \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{I}^M \\ \underset{\sim}{I}^{F_1} \\ \underset{\sim}{V}^{F_2} \end{bmatrix}, \tag{1}$$

where

$$\underset{\sim}{V}^M \triangleq [V_1^M \; \dots \; V_m^M]^T, \qquad \underset{\sim}{I}^M \triangleq [I_1^M \; \dots \; I_m^M]^T \tag{2}$$

are measured voltages and currents,

$$\underset{\sim}{V}^{F_1} \triangleq [V_1^F \; \dots \; V_{f_1}^F]^T, \qquad \underset{\sim}{V}^{F_2} \triangleq [V_{f_1+1} \; \dots \; V_f]^T \tag{3}$$

are voltages at fault ports, and

$$\underset{\sim}{I}^{F_1} \triangleq [I_1^F \; \dots \; I_{f_1}^F]^T, \qquad \underset{\sim}{I}^{F_2} \triangleq [I_{f_1+1} \; \dots \; I_f]^T \tag{4}$$

are currents flowing through fault ports.

Equation (1) can be represented in the simpler form

$$\begin{bmatrix} \underset{\sim}{V}^M \\ \underset{\sim}{R}^F \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{MM} & \underset{\sim}{H}_{MF} \\ \underset{\sim}{H}_{FM} & \underset{\sim}{H}_{FF} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}^M \\ \underset{\sim}{S}^F \end{bmatrix}, \tag{5}$$

where

$$\underset{\sim}{S}^F = \begin{bmatrix} \underset{\sim}{I}^{F_1} \\ \underset{\sim}{V}^{F_2} \end{bmatrix}, \qquad \underset{\sim}{R}^F = \begin{bmatrix} \underset{\sim}{V}^{F_1} \\ \underset{\sim}{I}^{F_2} \end{bmatrix} \tag{6}$$

are source and response vectors at fault ports. When there are no faults in the network we obtain the nominal response vector

$$\begin{bmatrix} \underset{\sim}{V}^{MO} \\ \underset{\sim}{R}^{FO} \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{I}^{M} \\ \underset{\sim}{0} \end{bmatrix}. \tag{7}$$

Hence, the voltage change vector $\Delta \underset{\sim}{V}^{M} \triangleq \underset{\sim}{V}^{M} - \underset{\sim}{V}^{MO}$ can be expressed as

$$\Delta \underset{\sim}{V}^{M} = \underset{\sim}{H}_{MF} \; \underset{\sim}{S}^{F}. \tag{8}$$

Assuming that $\underset{\sim}{H}_{MF}$ is of full column rank, the necessary condition for the set F of network elements to contain all the faults, is given by the relation

$$(\overline{\underset{\sim}{H}}_{MF} - \underset{\sim}{1}) \; \Delta \underset{\sim}{V}^{M} = \underset{\sim}{0}, \tag{9}$$

where

$$\overline{\underset{\sim}{H}}_{MF} \triangleq \underset{\sim}{H}_{MF} (\underset{\sim}{H}^{T}_{MF} \; \underset{\sim}{H}_{MF})^{-1} \; \underset{\sim}{H}^{T}_{MF}. \tag{10}$$

$\underset{\sim}{H}_{MF}$ is called the test matrix. Equation (9) allows us to check whether the assumed set F contains all faults existing in the network on the basis of the measured voltage change vector $\Delta \underset{\sim}{V}^{M}$.

The test matrix $\underset{\sim}{H}_{MF}$ can be designed using the adjoint network analysis. For the adjoint network with $\hat{\underset{\sim}{S}}^{F} = \underset{\sim}{0}$ we obtain (cf. [8])

$$\begin{bmatrix} \hat{\underset{\sim}{V}}^{F_1} \\ -\hat{\underset{\sim}{I}}^{F_2} \end{bmatrix} = \underset{\sim}{H}^{T}_{MF} \; \hat{\underset{\sim}{I}}^{M}. \tag{11}$$

For m linearly independent excitations $\hat{\underset{\sim}{I}}^{M1}, \hat{\underset{\sim}{I}}^{M2}, \ldots, \hat{\underset{\sim}{I}}^{Mm}$ we have

$$\begin{bmatrix} \hat{\underset{\sim}{V}}^{F_1 1} & \hat{\underset{\sim}{V}}^{F_1 2} & \cdots & \hat{\underset{\sim}{V}}^{F_1 m} \\ -\hat{\underset{\sim}{I}}^{F_2 1} & -\hat{\underset{\sim}{I}}^{F_2 2} & \cdots & -\hat{\underset{\sim}{I}}^{F_2 m} \end{bmatrix} = \underset{\sim}{H}^{T}_{MF} \; [\hat{\underset{\sim}{I}}^{M1} \; \hat{\underset{\sim}{I}}^{M2} \; \ldots \; \hat{\underset{\sim}{I}}^{Mm}] \tag{12}$$

and

$$[\hat{\underset{\sim}{I}}^{M1} \; \hat{\underset{\sim}{I}}^{M2} \; \ldots \; \hat{\underset{\sim}{I}}^{Mm}] = \underset{\sim}{1}, \tag{13}$$

giving the test matrix

$$\tilde{H}_{MF}^{T} = \begin{bmatrix} \hat{\tilde{V}}^{F_1 1} & \cdots & \hat{\tilde{V}}^{F_1 m} \\ -\hat{\tilde{I}}^{F_2 1} & \cdots & -\hat{\tilde{I}}^{F_2 m} \end{bmatrix} \tag{14}$$

Having calculated voltages and currents in all adjoint network elements with different current excitations, $\hat{\tilde{I}}^{Mi}$, we can obtain the test matrix $\tilde{H}_{MF}$ corresponding to any set F of faulty elements. Thus the adjoint network analysis does not have to be repeated if the set of predicted faulty elements is changed.

III. THE BLOCK DEPENDENCY PROBLEM

Fulfilling the relation (9) is a necessary but not sufficient condition on the set F to contain all the faults in the network. One of the important reasons is the concept of block dependency of two overdetermined systems of equations. In this section we will specifically address this problem.

Systems

$$\tilde{A}_1 \tilde{x}_1 = \tilde{b} \quad \text{and} \quad \tilde{A}_2 \tilde{x}_2 = \tilde{b} \tag{15}$$

are <u>block dependent</u> if for any $\tilde{b}$ both are consistent or both are inconsistent. It was shown [3] that two systems are block dependent if and only if

$$\bar{\tilde{A}}_1 \tilde{A}_2 = \tilde{A}_2 \quad \text{or} \quad \bar{\tilde{A}}_2 \tilde{A}_1 = \tilde{A}_1, \tag{16}$$

where

$$\bar{\tilde{A}} \triangleq \tilde{A} (\tilde{A}^T \tilde{A})^{-1} \tilde{A}^T. \tag{17}$$

The reasons for block dependency are very often connected with the particular form of the test matrix $\tilde{H}_{MF}$. We will discuss them first.

Lemma 1

Let us assume that we have the overdetermined system of equations

$$\underset{\sim}{A}\underset{\sim}{x} = \underset{\sim}{b}, \tag{18}$$

where $\underset{\sim}{A} = [\underset{\sim}{a}_1 \ \ldots \ \underset{\sim}{a}_f]$ is an (mxf) matrix, $f < m$, and rank $\underset{\sim}{A} = f$. Every overdetermined system of equations

$$\underset{\sim}{B}\underset{\sim}{x}_2 = \underset{\sim}{b}, \tag{19}$$

where $\underset{\sim}{B} = [\underset{\sim}{b}_1 \ \ldots \ \underset{\sim}{b}_f]$, rank $\underset{\sim}{B} = f$ and $\underset{\sim}{b}_i$, $i = 1, \ldots, f$ are linear combinations of $\underset{\sim}{a}_i \in \underset{\sim}{A}$, is block dependent to the system (18), so according to [3] we can write $\underset{\sim}{A} \sim \underset{\sim}{B}$.

Proof

Assume that $\underset{\sim}{b}_i = \underset{\sim}{a}_i$, $i = 1, \ldots, f-1$, and $\underset{\sim}{b}_f = \underset{\sim}{a}_f + \sum_j k_j \underset{\sim}{a}_j$, where k_j are scalars. We prove first the following equivalence:

$$\bar{\underset{\sim}{A}} \ \underset{\sim}{A}_f = \underset{\sim}{A}_f, \tag{20}$$

where

$$\underset{\sim}{A}_f = [k_1\underset{\sim}{a}_1 \ \ k_2\underset{\sim}{a}_2 \ \ \ldots \ \ k_f\underset{\sim}{a}_f] \tag{21}$$

and $\underset{\sim}{A}$ is as in (18). We have $\bar{\underset{\sim}{A}}\ \underset{\sim}{A} = \underset{\sim}{A}$, so

$$\begin{aligned}\bar{\underset{\sim}{A}}\ \underset{\sim}{A}_f &= \bar{\underset{\sim}{A}}[k_1\underset{\sim}{a}_1 \ \ \underset{\sim}{0} \ \ldots \ \underset{\sim}{0}] + \bar{\underset{\sim}{A}}[\underset{\sim}{0} \ \ k_2\underset{\sim}{a}_2 \ \ \underset{\sim}{0} \ \ldots \ \underset{\sim}{0}] + \ldots \\ &\quad + \bar{\underset{\sim}{A}}[\underset{\sim}{0} \ \ldots \ \underset{\sim}{0} \ \ k_f\underset{\sim}{a}_f] \\ &= k_1[\underset{\sim}{a}_1 \ \ \underset{\sim}{0} \ \ldots \ \underset{\sim}{0}] + k_2[\underset{\sim}{0} \ \ \underset{\sim}{a}_2 \ \ \underset{\sim}{0} \ \ldots \ \underset{\sim}{0}] \\ &\quad + k_f[\underset{\sim}{0} \ \ldots \ \underset{\sim}{0} \ \ \underset{\sim}{a}_f] = \underset{\sim}{A}_f.\end{aligned}$$

Now we can easily check that

$$\bar{\underset{\sim}{A}}\ \underset{\sim}{B} = \bar{\underset{\sim}{A}}(\underset{\sim}{A} + \underset{\sim}{A}_f) = \underset{\sim}{A} + \underset{\sim}{A}_f = \underset{\sim}{B}. \tag{22}$$

We can extend the proof to the case when every column of $\underset{\sim}{B}$ is a linear combination of columns of the $\underset{\sim}{A}$ matrix when rank $\underset{\sim}{B} = f$.

Now we can formulate the following important result.

Result 1

If the set of faulty elements contains a subset consisting of either

(a) a circuit formed by one-ports, controlled voltage sources or voltages that control faulty voltage or current sources;

(b) a cutset formed by one-ports, controlled current sources or currents that control faulty voltage or current sources;

then the test matrix $\underset{\sim}{H}_{MF}$ of (8) contains linearly dependent columns.

Proof

The proof of Result 1 is connected with the adjoint network interpretation of the test matrix as described in (11). The jth column of the $\underset{\sim}{H}_{MF}$ matrix ($j \leq f_1$) corresponds to voltages on the element e_j calculated for all independent current excitations and nominal values of elements. So if faulty elements (and/or controlled voltages) form a circuit then the voltages are dependent. The same dependency holds for all excitations and we find columns of $\underset{\sim}{H}_{MF}$ linearly dependent. A similar argument can be used for the cutset formed by faulty elements. The jth column of the $\underset{\sim}{H}_{MF}$ matrix ($j > f_1$) corresponds to currents flowing through the element e_j calculated for all independent excitations. Thus, if faulty elements (and/or controlled currents) form a cutset then currents are dependent. This leads to the linear dependency of columns of $\underset{\sim}{H}_{MF}$.

A corollary follows immediately from Result 1.

Corollary 1

If the set of faulty elements contains any subset defined in Result 1 then the multiport method cannot be used for fault location.

Although the Corollary 1 is of a negative nature it provides precise information about the topological restriction on the multiport method, thus being a constructive extension of Theorem 2 given by Biernacki and Bandler [2].

From Lemma 1 and the proof of Result 1 it is clear that even if one of the elements from subsets defined in Result 1 (i.e., from a circuit or a cutset) is not faulty the multiport method will not provide unique results. Assume that the set of faulty elements in the network $F = \{e_1, e_2, \ldots, e_f\}$, and that elements $e_{\ell+1}, e_{\ell+2}, \ldots, e_f$ together with e_{f+1} form a circuit or cutset (e_{f+1} may be a controlled variable as well as an element). Under this assumption we can prove the following lemma.

Lemma 2

If $\underset{\sim}{H}_{MF_i}$ denotes the test matrix constructed for the set F_i of predicted faulty elements then

$$\underset{\sim}{H}_{MF_1} \sim \underset{\sim}{H}_{MF_2} \sim \cdots \sim \underset{\sim}{H}_{MF_{f-\ell+1}}. \tag{23}$$

where

$$\begin{aligned} &F_1 = \{e_1, \ldots, e_f\}, \quad F_2 = \{e_1, \ldots, e_{f-1}, e_{f+1}\}, \\ &F_{f-\ell+1} = \{e_1, \ldots, e_\ell, \ e_{\ell+2}, \ldots, e_{f+1}\}. \end{aligned} \tag{24}$$

Proof

The column of the $\underset{\sim}{H}_{MF_i}$ $(i > 1)$ matrix which corresponds to the element e_{f+1} is a linear combination of columns which correspond to the elements $e_{\ell+1}, \ldots, e_f$ (compare with the proof of Result 1). So, according to Lemma 1, we have the relation (23).⌋ This case occurred in Example 2 of Biernacki and Bandler [3], where the matrices $\underset{\sim}{Z}_{mx}^{13}$, $\underset{\sim}{Z}_{mx}^{23}$ and

$\underset{\sim}{Z}_{mx}^{12}$ were block dependent.

Not only does linear dependency of columns influence the possibility of multiple fault location, but linear dependency of rows does as well. It is evident that the number of independent voltage measurements should at least be equal to the number of columns of the test matrix to obtain full column rank. The following lemma is helpful to a better understanding of the influence of row dependency on the solvability of the fault location problem.

Lemma 3

If the number of independent adjoint current excitations is less than or equal to the number of columns of the test matrix we cannot locate the faults using the multiport approach.

Proof

Assume for simplicity that we have the overdetermined system of equations

$$\underset{\sim}{H}_{MF} \underset{\sim}{S}^{F} = \Delta \underset{\sim}{V}^{M}, \tag{25}$$

where $\underset{\sim}{H}_{MF}$ is an (f+1)xf matrix with rank $\underset{\sim}{H}_{MF}$ = f. Then $\underset{\sim}{H}_{MF}$ can be presented in the form

$$\underset{\sim}{H}_{MF} = \begin{bmatrix} \underset{\sim}{a}_1^T \\ \cdot \\ \cdot \\ \cdot \\ \underset{\sim}{a}_f^T \\ \sum_{j=1}^{f} k_j \underset{\sim}{a}_j^T \end{bmatrix} . \tag{26}$$

Condition (9) is then equivalent to

$$\Delta V^M_{f+1} = \sum_{j=1}^{f} k_j \, \Delta V^M_j, \tag{27}$$

where

$$\Delta \underset{\sim}{V}^M = \begin{bmatrix} \Delta V^M_1 \\ \cdot \\ \cdot \\ \cdot \\ \Delta V^M_{f+1} \end{bmatrix}. \tag{28}$$

But relation (27) is always fulfilled if the current excitations are linearly dependent, because each row of the $\underset{\sim}{H}_{MF}$ matrix corresponds to voltages or currents on elements e_1, ..., e_f calculated for one adjoint current excitation and nominal values of elements (cf (14)). So if the current in, say, the (f+1)th excitation is a linear combination of the first f excitations then the (f+1)th row of the $\underset{\sim}{H}_{MF}$ matrix is the same combination of the first f rows. According to the structure of $\underset{\sim}{H}_{MF}$ we can see that the same combination of measurement voltages will appear in the (f+1)th measurement, because excitations in the adjoint network are imposed at the same ports as measurements in the faulty network (see (12) and (8)). Thus, condition (27) is fulfilled independently from e_1, e_2, ..., e_f.

<u>Corollary 2</u>

The maximum number of faults which can be located by the multiport method is equal to the number of nodes in the network minus two.

This corollary is the simple result of Lemma 3 because the maximum number of independent excitations is equal to number of nodes minus one.

IV. CONCLUSIONS

The hybrid multiport approach to fault location has been presented together with a detailed and rigorous discussion of the implications of block dependency. Topological restrictions on the method have been derived from the analysis. The theory is applicable to multiple fault location in linear networks due to current excitations at a single frequency. We feel that the present paper permits a deeper understanding of the limitations of using the multiport method for fault location. The influence of tolerances can, in practice, be handled for this approach in a similar manner to the treatment of Biernacki and Bandler [3].

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FIGURE CAPTIONS

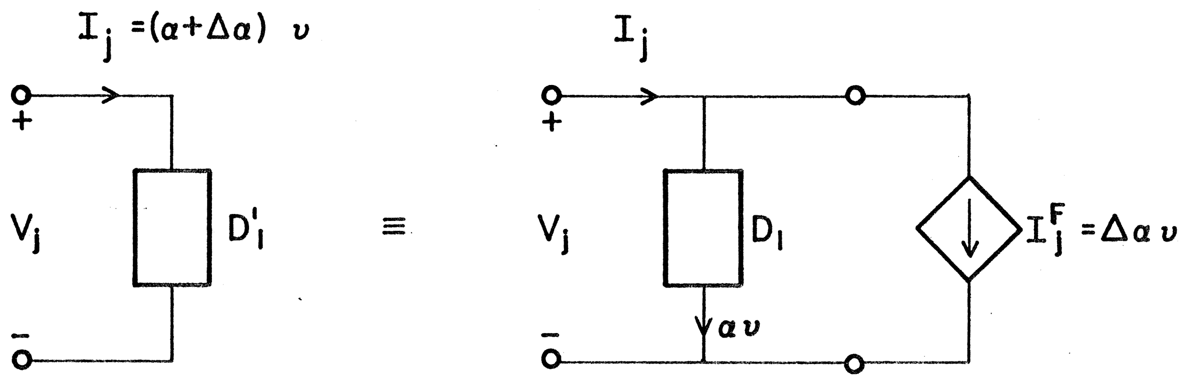
Fig. 1 Representation for changes in element values

(a) D_1 denotes a one-port or controlled current source,

(b) D_2 denotes a one-port or controlled voltage source,

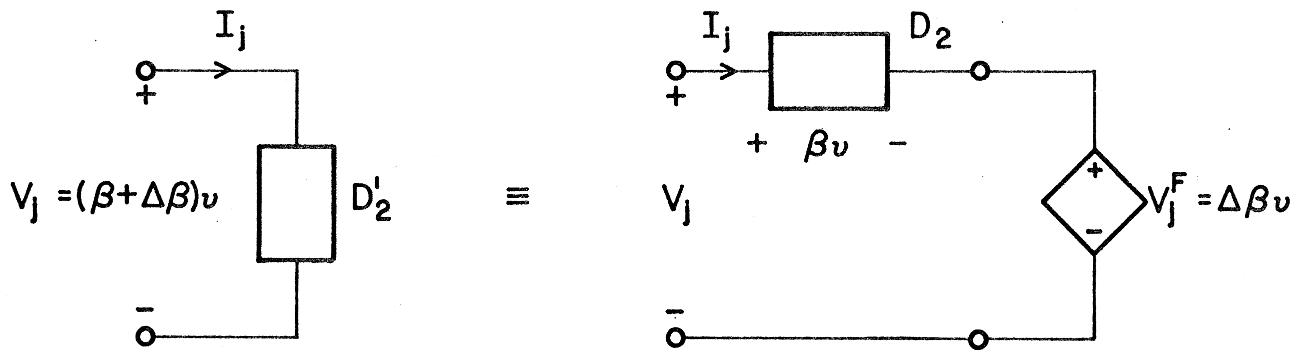
ν denotes the controlling voltage or current.

Fig. 2 Network with faults represented as controlled sources.



$I_j = (\alpha + \Delta\alpha)\ v$
+
V_j
−
D_1'
≡
I_j
+
V_j
−
D_1
αv
$I_j^F = \Delta\alpha\, v$

(a)



I_j
+
$V_j = (\beta + \Delta\beta) v$
−
D_2'
≡
I_j
+
V_j
−
D_2
+ βv −
$V_j^F = \Delta\beta\, v$

(b)

Fig. 1

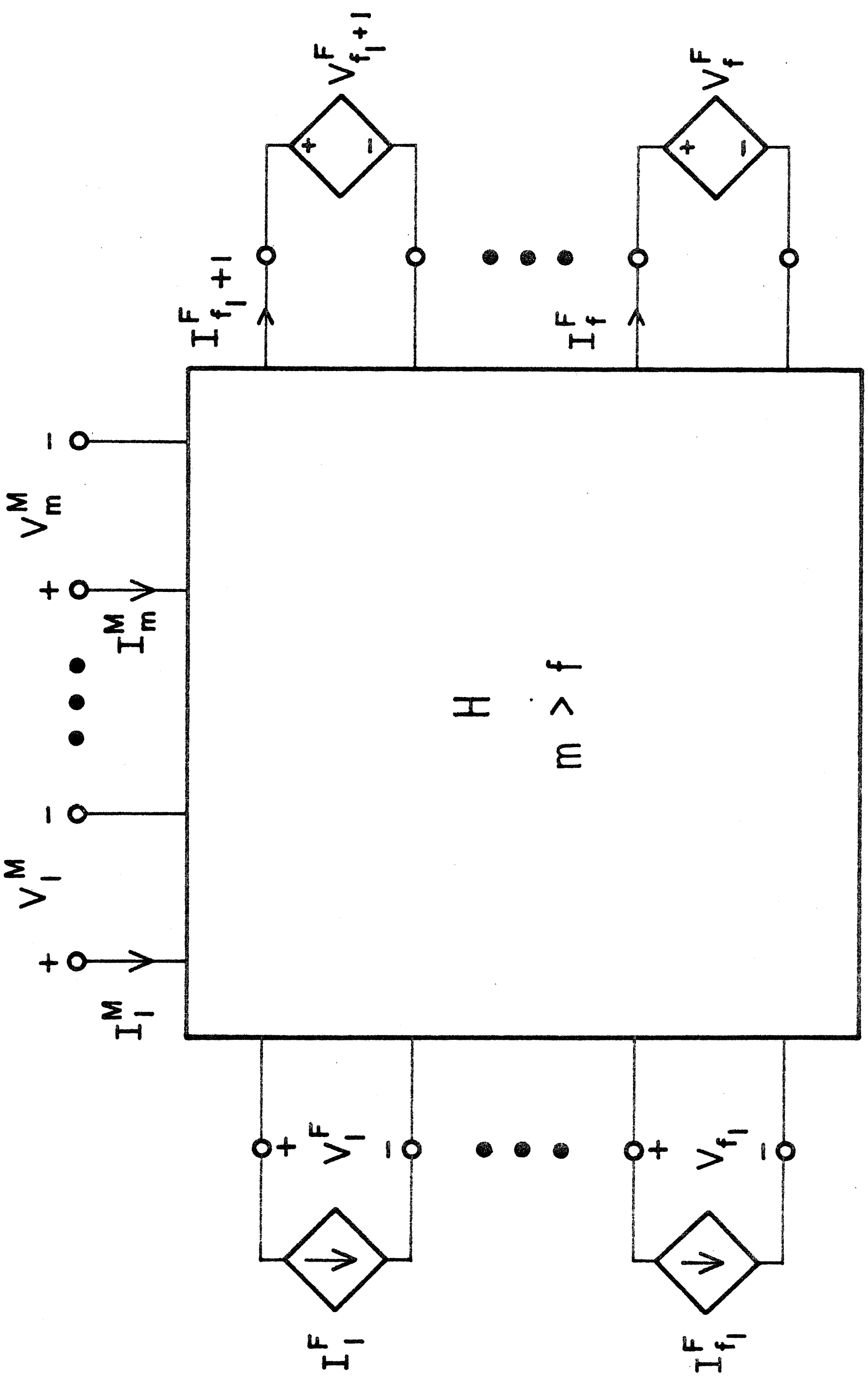


Fig. 2

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