INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-266

DESIGN OF TESTS FOR PARAMETER IDENTIFICATION BY VOLTAGE MEASUREMENTS

J.A. Starzyk, R.M. Biernacki and J.W. Bandler

July 1981

FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

No. SOC-266

DESIGN OF TESTS FOR PARAMETER IDENTIFICATION BY VOLTAGE MEASUREMENTS

J.A. Starzyk, R.M. Biernacki and J.W. Bandler

July 1981

DESIGN OF TESTS FOR PARAMETER IDENTIFICATION BY VOLTAGE MEASUREMENTS

J.A. Starzyk, R.M. Biernacki
and J.W. Bandler, Fellow, IEEE

Abstract

This paper presents the theoretical background and resulting algorithm for generating tests which are topologically sufficient for identification of parameter values in linear circuits. Voltage measurements at all the nodes are assumed. The main thrust of this paper is to minimize the number of necessary measurements at different current excitations. Coates flow-graph representation of a network is used.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grant A7239.

J.A Starzyk and J.W. Bandler are with the Group on Simulation, Optimization and Control and the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7. J.A. Starzyk is on leave from the Institute of Electronics Fundamentals, Technical University of Warsaw, Warsaw, Poland.

R.M. Biernacki is with the Institute of Electronics Fundamentals, Technical University of Warsaw, Warsaw, Poland.

I. INTRODUCTION

Fault diagnosis and automatic testing techniques for analog circuits often require parameter identification. Recent papers on the subject [1-11] present different techniques of parameter indentification involving the solution of linear equations. Most of the authors assume voltage measurements, which are more convenient in practice, and consider only current excitations.

A central problem is the formulation of a sufficient number of independent equations subject to a specified number of excitations or voltage measurements. In this paper, we assume that all the nodes in the network are accessible. In particular cases, this assumption may not be necessary. Ozawa [2] has shown how such particular cases can be handled by network transformations.

The principal aim of this paper is to develop general sufficient conditions for the parameter identification problem as well as proposing a strategy for choosing a reasonably small number of excitations and, thereby, a small number of measurements.

The paper extends the results presented in [1] and proposes an efficient algorithm for test generation. The results obtained are of a topological nature. The Coates flow graph representation of network elements is used.

II. GENERAL SUFFICIENT CONDITIONS

Consider a linear network which can be described by nodal equations. Tests under consideration consist of nodal voltage measurements when different current excitations are applied and are assumed to be performed at a single frequency. Mathematically, the <u>ith test</u> is defined as a vector $\underset{\sim}{V}_n^i$ of all measured nodal voltages due to the current excitation vector $\underset{\sim}{J}^i$. Equation

$$\underset{\sim}{Y}_n \underset{\sim}{V}_n^i = \underset{\sim}{J}^i \tag{1}$$

can be written for every test i = 1, 2, ..., each time with the same matrix $\underset{\sim}{Y}_n$. Although we concentrate our discussion on the nodal equations, it is applicable to any other description based on an independent set of cut-sets (see [1]).

Taking N tests, we can write a matrix equation

$$\underset{\sim}{Y}_n \underset{\sim}{V}_t = \underset{\sim}{J}_t \tag{2}$$

where the square matrix

$$\underset{\sim}{V}_t = [\underset{\sim}{V}_n^1 \;\; \underset{\sim}{V}_n^2 \;\; \ldots \;\; \underset{\sim}{V}_n^N] \tag{3}$$

is said to be the matrix of voltage tests, N is the number of rows and columns of $\underset{\sim}{Y}_n$, and the square matrix

$$\underset{\sim}{J}_t = [\underset{\sim}{J}^1 \;\; \underset{\sim}{J}^2 \;\; \ldots \;\; \underset{\sim}{J}^N] \tag{4}$$

is the matrix of consecutive test excitations. From (2), we find the unknown matrix $\underset{\sim}{Y}_n$ as

$$\underset{\sim}{Y}_n = \underset{\sim}{J}_t \underset{\sim}{V}_t^{-1} \tag{5}$$

provided that $\underset{\sim}{V}_t$ is nonsingular. As a consequence of equations (2) and (5), the following result provides sufficient conditions for the identification.

Result 1 [1]

If a given linear network can be described by the nodal equation (1) and the test excitations are chosen in such a way that $\underset{\sim}{J}_t$ is a nonsingular matrix, then $\underset{\sim}{V}_t$ is also nonsingular and the solution (5) exists.

Proof of this result follows from equation (2) since $N = \text{rank } \underset{\sim}{J}_t \leq \text{rank } \underset{\sim}{V}_t \leq N$.

Thus, in order to identify the values of all elements of $\underset{\sim}{Y}_n$, we arrange for N independent current excitations, measure all nodal voltages and then apply equation (5).

One possible choice of the independent excitations is to apply a unit current successively to all N nodes, i.e., we may consider

$$\underset{\sim}{J}_t = \underset{\sim}{1} \, . \tag{6}$$

In such a case, the ith vector $\underset{\sim}{V}_n^i$ is measured with the current excitation applied between the datum and the ith node, which we call the test node. So, on the basis of (5), we obtain the very important relation

$$\underset{\sim}{Y}_n = \underset{\sim}{V}_t^{-1} \, , \tag{7}$$

which imposes constraints on the elements of the matrix $\underset{\sim}{V}_t$. For instance, if $\underset{\sim}{Y}_n$ is symmetrical (as for reciprocal networks), then $\underset{\sim}{V}_t$ is also symmetrical. In general, $\underset{\sim}{Y}_n$ has a particular, usually sparse, form corresponding to the known network topology; hence, there are certain constraints on the elements of the inverse. For example,

$$y_{ij} = 0 \quad (\text{or } \Delta_{ji} = 0) \tag{8}$$

for nonincident nodes (Δ_{ij} denotes the appropriate minor of $\underset{\sim}{V}_t$ and y_{ij} is an element of $\underset{\sim}{Y}_n$),

$$y_{ij} = y_{ji} \quad (\text{or } \Delta_{ji} = \Delta_{ij}) \tag{9}$$

for reciprocal branches.

In order to perform the least number of tests, we must eliminate whole columns of $\underset{\sim}{V}_t$. We propose a systematic way which enables us to identify tests sufficient for component evaluation. The method is based on the assumption that all voltages measured as well as all components have nonzero values.

III. CONDITIONS FOR SUFFICIENT TESTS

Equation (7) can be rewritten in the form

$$\underset{\sim}{V}_t^T \, \underset{\sim}{Y}_n^T = \underset{\sim}{1} \, . \qquad (10)$$

Consider the product of $\underset{\sim}{V}_t^T$ and the jth column of $\underset{\sim}{Y}_n^T$. We have

$$\underset{\sim}{V}_t^T \, \underset{\sim}{y}_j = \begin{bmatrix} {\underset{\sim}{V}_n^1}^T \\ {\underset{\sim}{V}_n^2}^T \\ \cdot \\ \cdot \\ \cdot \\ {\underset{\sim}{V}_n^N}^T \end{bmatrix} \begin{bmatrix} y_{j1} \\ y_{j2} \\ \cdot \\ \cdot \\ \cdot \\ y_{jN} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \\ \\ \\ \leftarrow \text{jth row} \\ \\ \\ \end{matrix} \qquad (11)$$

Let the unknown elements of $\underset{\sim}{y}_j$ be identified by the set of second indices $B_j = \{j_1, \ldots, j_k\}$. We denote the set of elements y_{ji}, $i \in B_j$ a <u>reduced cut-set</u>. Transferring the known terms from the left-hand side to the right-hand side of (11), we rewrite the equation as

$$\underset{\sim}{V}_t^T \underset{\sim}{y}_{B_j} = \begin{bmatrix} \underset{\sim}{V}_n^{1^T} \\ \\ \underset{\sim}{V}_n^{2^T} \\ \cdot \\ \cdot \\ \\ \cdot \\ \cdot \\ \\ \underset{\sim}{V}_n^{N^T} \end{bmatrix} \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ y_{jj_1} \\ \cdot \\ y_{jj_k} \\ \cdot \\ 0 \end{bmatrix} = \begin{bmatrix} J_{j1} \\ \\ J_{j2} \\ \cdot \\ \cdot \\ \cdot \\ \\ \\ J_{Jn} \end{bmatrix} \tag{12}$$

In order to determine the elements y_{jj_1}, ..., y_{jj_k}, we can solve a subsystem of (12) given by

$$\underset{\sim}{V}_t^T[A_j \mid B_j] \begin{bmatrix} y_{jj_1} \\ \cdot \\ \cdot \\ \cdot \\ y_{jj_k} \end{bmatrix} = \begin{bmatrix} J_{ji_1} \\ \cdot \\ \cdot \\ \cdot \\ J_{ji_k} \end{bmatrix} \tag{13}$$

where the k equations are chosen from (12) in such a way that the square submatrix $\underset{\sim}{V}_t^T[A_j \mid B_j]$, is obtained as the intersection of rows $A_j = \{i_1, \ldots, i_k\}$ and columns B_j, is nonsingular. See Fig. 1 for illustration. According to relationship (7), the matrix $\underset{\sim}{V}_t^T[A_j \mid B_j]$ is nonsingular iff the matrix $\underset{\sim}{Y}_n(A_j \mid B_j)$ is also nonsingular where $\underset{\sim}{Y}_n(A_j \mid B_j)$ denotes the submatrix of $\underset{\sim}{Y}_n$ obtained by removing rows A_j and columns B_j (see Fig. 1).

Consider a sequence of sets B_j, $j = j_1. \ldots, j_N$ which corresponds to a sequence of reduced cut-sets of the <u>current graph</u> of the network.

Based on (7) and (13), the following result can be summarized.

Result 2 [1]

Test excitations at the subset of nodes $A \subset \{1, 2, \ldots, N\}$ are sufficient for the identification of all elements of $\underset{\sim}{Y}_n$ if and only if

$$\forall B_j \ \exists A_j \subset A \quad \det \underset{\sim}{Y}_n(A_j \mid B_j) \neq 0, \tag{14}$$

where

$$\text{card } A_j = \text{card } B_j. \tag{15}$$

As a consequence of (15), we have the following corollary.

Corollary 1

$$\text{card } A \geq \max_j \text{ card } B_j . \tag{16}$$

It is seen from (16) that the choice of the sequence of B_j is crucial for the minimization of the number of sufficient tests.

Now, in order to characterize tests A_j feasible for a given B_j, we consider topological equations for the nodal admittance matrix.

$$\underset{\sim}{Y}_n = \underset{\sim}{A}_- \underset{\sim}{Y} \underset{\sim}{A}_+^T , \tag{17}$$

where the element ij of $\underset{\sim}{A}_-$ is equal to 1 if the jth edge is directed towards the ith node, otherwise zero; and the element ij of $\underset{\sim}{A}_+$ is equal to 1 if the jth edge is directed away from the ith node, otherwise zero; $\underset{\sim}{Y}$ is a diagonal matrix of element admittances.

The submatrix of $\underset{\sim}{Y}_n$ obtained by removing columns B_j can be expressed as

$$\underset{\sim}{Y}_n(\cdot \mid B_j) = \underset{\sim}{A}_- \underset{\sim}{Y} \underset{\sim}{A}_+^{'T} , \tag{18}$$

where $\underset{\sim}{A}_+^{'}$ is obtained from $\underset{\sim}{A}_+$ by removing rows B_j. In the Coates graph, this corresponds to deleting all the edges outgoing from nodes B_j.

Similarly,

$$\underset{\sim}{Y}_n(A_j \mid B_j) = \underset{\sim}{A}_-^{'} \underset{\sim}{Y} \underset{\sim}{A}_+^{'T} , \tag{19}$$

where $\underset{\sim}{A}'_-$ is obtained from $\underset{\sim}{A}_-$ by removing rows A_j. In the Coates graph, this corresponds to deleting all the edges incoming to nodes A_j.

Let us consider the Coates graph $G_{(A_j \mid B_j)}$ obtained from the graph of the given network after deleting all the edges incoming to nodes A_j and all the edges outgoing from nodes B_j. The following theorem can be proved on the basis of the Cauchy-Binet theorem [12] and the concept of the k-connection [13].

Theorem 1

If $\det \underset{\sim}{Y}_n(A_j \mid B_j) \neq 0$, there exists in $G_{(A_j \mid B_j)}$ at least one k-connection c_p (see Fig. 2), where

$$P = \{(v_s, v_e) \mid v_s \in A_j \cap (N - B_j),\ v_e \in B_j \cap (N - A_j)\} \tag{20}$$

$$k = \text{card } P = \text{card } (A_j \cap (N - B_j)) = \text{card } (B_j \cap (N - A_j)) \tag{21}$$

(v_s, v_e) represents a path directed from the node v_s to the node v_e, and N denotes all nodes of the graph. The condition stated in the theorem is sufficient almost everywhere.

Proof

According to the Cauchy-Binet theorem and relation (19), we have

$$\det \underset{\sim}{Y}_n(A_j \mid B_j) = \Sigma \det \underset{\sim}{C}^- \cdot \det \underset{\sim}{C}^+ , \tag{22}$$

where $\underset{\sim}{C}^-$ is a major submatrix of $\underset{\sim}{A}'_- \cdot \underset{\sim}{Y}$ with order equal to $(N - \text{card } A_j)$ and $\underset{\sim}{C}^+$ is the corresponding major submatrix of $\underset{\sim}{A}'^T_+$. If $\det Y_n(A_j \mid B_j) \neq 0$, then there exists at least one pair of corresponding determinants, both different from zero. A major determinant of $\underset{\sim}{A}'_- \cdot \underset{\sim}{Y}$ is different from zero if there exists one nonzero element in every row of the chosen submatrix (chosen set of columns). This corresponds to the set of $(N - \text{card } A_j)$ edges, such that every edge has a different endpoint, belonging to the set of nodes $(N - A_j)$. The corresponding submatrix is different from zero if the same edges have different origins, belonging to the same set of nodes ($N - B_j$). Now it is easy to check that these edges form a k-connection, as stated in the theorem. The determinant of $\underset{\sim}{Y}_n(A_j \mid B_j)$ equals zero, in spite of having nonzero components in (22), only when particular values of elements are chosen.

As a consequence of Theorem 1, we have an important corollary.

Corollary 2

To satisfy (14), we should find a set A_j such that, after deleting all the edges outgoing from nodes B_j and after deleting all the edges incoming to nodes A_j, there are no isolated nodes in the set $N - (A_j \cap B_j)$.

Theorem 1 does not guarantee that $\underset{\sim}{Y}_n(A_j \mid B_j)$ is nonsingular. It may, however, be singular for particular element values only. If we know the nominal values of the elements, then we can easily check whether the tests A_j chosen are sufficient for the solution.

Definition

A node is said to be a corner if there exists a complete subgraph containing all the edges incoming to the vertex and all the edges having the same weight.

It follows that there may exist edges outgoing from a corner to other parts of the graph. Also, the order of the complete graph is not defined. In particular, it may be a complete graph of zero order (see Fig. 3a).

Based on Corollary 2, the following theorem can be proved.

Theorem 2

Sufficient tests must include excitations at all the corners.

Proof

Assume that there is no excitation in the corner. If we identify an edge incident to the corner, then every reduced cut-set containing the edge must contain all the nodes of the complete subgraph. After deleting all the edges outgoing from the nodes of this reduced cut-set, the corner will be an isolated node, and if it is not a test node, we obtain an isolated node in the set $\mathcal{N} - (A_j \cap B_j)$ and a contradiction to Corollary 2.

Thus, the number of corners estimates the minimal cardinality of A. In order to further estimate this, the following remarks may be helpful.

Remark 1 card A $\geq$ order of the maximal complete subgraph.

Remark 2 card A $\geq$ minimal incoming degree in the remaining graph after deleting all corners with incident edges.

The incoming degree of a vertex is the number of edges incoming to this vertex.

The number of voltages measured in every test is not necessarily equal to the number of nodes. We can suggest a simple strategy for reducing the number of voltage measurements.

Let

$$\mathcal{B}_v = \{B_j \mid v \in B_j\} \tag{23}$$

and

$$\mathcal{A}_{bj} = \{A_j \mid \det \underset{\sim}{Y}_n(A_j \mid B_j) \neq 0\} \quad . \tag{24}$$

For every $B_j \in \mathcal{B}_v$, we should choose $A_{jo} \in \mathcal{A}_{bj}$ to minimize the set

$$V_v = \bigcup_{B_j \in \mathcal{B}_v} A_{jo} \quad . \tag{25}$$

V_v represents the set of excitations that require the voltage at node v to be measured. It is evident from (25) that the minimum number of voltage measurements at the node v is not less than the cardinality of the maximum reduced cut-set which is incident to that node. Minimization of sets V_v can easily be done during the algorithm described.

IV. THE ALGORITHM STRATEGY

An optimal selection of tests could be done in a combinatorial way, where different sets of reduced cut-sets are considered and then different combinations of test nodes are checked. However, for large networks, it may be quite tedious to check the conditions of Theorem 1, even if reduced cut-sets and a set A are known.

A reasonable approach should select tests likely to be sufficient for the identification and consisting of the necessary number of test nodes for a given set of reduced cut-sets. Such a number can be determined by some necessary conditions. Then, at the end, the algorithm should check the conditions of Theorem 1 and, if necessary, add some extra test nodes. The algorithm should simultaneously select a set of cut-sets.

The algorithm proposed is heuristic and is based on the so-called greedy strategy described, for example, in [14]. It does not pretend to provide the best possible solution. However, our goal is to find a set of tests of a reasonably small cardinality. Simultaneously, the reduced cut-sets and test nodes are designed.

Main Concepts of the Algorithms

1. The Coates signal-flow graph representation is used to describe the network topology. An efficient way of representing any linear active network is discussed, for example, in [15].

One of the corners, or if there are no corners, a node j of maximum distance $d_j(G)$ from the graph nodes, is taken as the starting point of the algorithm, where

$$d_j(G) = \max_{i \in N} \text{card } (p(i \rightarrow j)) \tag{26}$$

and $p(i \rightarrow j)$ is the shortest path directed from i to j.

2. We realize the greedy strategy for contour nodes of minimum incoming degree. At step i, contour C(i) is defined w.r.t. C(i - 1) by substituting for the node of minimum incoming degree, which is placed into the set of internal nodes, In(i), the initial nodes of the incoming edge(s). These incoming edges form the ith reduced cut-set and are deleted from the graph. Also deleted are edges having the same weight. Contour C(0) contains only the starting point. See Fig. 4.

If there are no contour nodes with nonzero incoming degree and $C(i) \cup In(i)$ does not contain all the nodes, then the flow-graph contains so-called weakly connected components, i.e., the signal flows from one part to another one in one direction only (see Fig. 5). In such a case, we choose another starting point and continue the procedure described. This greedy strategy allows us to find reduced cut-sets of reasonably small cardinality.

3. To explain the way of searching for the set of test nodes T, we need some additional definitions.

Every internal node is called open if there exists an edge outgoing from it. The node is called active if it is the terminal node of a deleted edge. Fig. 6 illustrates the sets of nodes being defined and possible edge connections between them.

In every step of the algorithm, we check whether the node moved from the contour C(i) into the set of internal nodes In(i) is active. If not, we must add this node to test nodes. So, in order to have a small number of test nodes, we prefer to choose active nodes from the actual contour unless their incoming degrees are greater than the number of tests required.

The nodes added to the contour influence the strategy of test searching. If some of the nodes are not active, then we should have at least the same number of test nodes placed in the set $\mathcal{N} - [In(i)-Op(i)]$, where Op(i) denotes the set of open nodes.

If the maximum number of nodes $n_c(i)$ in the reduced cut-set is greater than the cardinality of the actual contour card (C(i)), then we should have at least $n_c(i)$ - card (C(i)) tests included in the actual set In(i).

Fig. 7 illustrates the possible paths from the test nodes to nodes of the reduced cut-set in such a case. Not all the test nodes included in In(i) can be accessible from the nodes of the given reduced cut-set. The number of these nodes is bounded by cardinality of the set of active nodes A(b) in bottleneck positions, as illustrated in Fig. 8.

All of the foregoing remarks influence the algorithm strategy and should be considered in choosing the test nodes. For the sake of compactness we will let |A| denote card (A) in the algorithm description. The following steps set the algorithm out in sufficient detail.

Algorithm

Step 1 Set $LS \leftarrow 1$, $I \leftarrow \emptyset$, $t_o(1) \leftarrow 0$.

Remark LS denotes the index of a weakly connected subgraph, I denotes the set of all internal nodes in all subgraphs considered, and t_o is the number of test nodes which should be placed outside.

Step 2 Choose a starting point. Set $Max \leftarrow max(1) \leftarrow b \leftarrow 0$, $i \leftarrow 1$, $T_I(1) \leftarrow T(LS) \leftarrow A(0) \leftarrow \emptyset$.

Remark T_I denotes the set of test nodes within internal nodes, Max is the number of nodes in the maximum reduced cut-set in the network and max(i) since last bottleneck position, respectively, i denotes iteration number, b is the last bottleneck position.

Step 3 Design In(i), C(i), A(i), Op(i), $n_c(i)$. If the node placed in In(i) was not active or it was a corner, add it to the set $T_I(i)$ and set $t_o(i) \leftarrow t_o(i) - 1$.

If $max(i) < n_c(i)$, then set $max(i) \leftarrow n_c(i)$.

If $Max < n_c(i)$, then set $Max \leftarrow n_c(i)$.

If $t_o(i) > |C(i)|$, then go to Step 4,

otherwise, set $t_o(i + 1) \leftarrow t_o(i)$ and go to Step 5.

Step 4 Add $\Delta t = t_o(i) - |C(i)|$ nodes from the set $In(i) - A(b) - T_I(i)$ to the set $T_I(i)$, and set $t_o(i + 1) \leftarrow |C(i)|$.

Step 5 If $max(i) \leq |T_I(i)| + |C(i)|$, then go to Step 7.

Step 6 Add $\Delta t = max(i) - |T_I(i)| - |C(i)|$ nodes from the set $In(i) - A(b) - T_I(i)$ to the set $T_I(i)$.

Step 7 Set $T_I(i + 1) \leftarrow T_I(i)$, $t_o(i + 1) \leftarrow max(i) - |T_I(i + 1)|$, $max(i + 1) \leftarrow max(i)$, $T(LS) \leftarrow T(LS) \cup T_I(i)$.

If $|T_I(i + 1)| \leq |A(i) - In(i)|$, then go to Step 9.

Step 8 Set $b \leftarrow i$, $max(i + 1) \leftarrow 0$, remove $\Delta t = |T_I(i + 1)| - |A(i) - In(i)|$ nodes from $T_I(i + 1)$.

Step 9 If $C(i) - Op(i) \neq 0$, then choose the next node from $C(i) - Op(i)$, set $i \leftarrow i + 1$, and go to Step 3.

Step 10 Add $\Delta t = \text{Max} - |T(LS)|$ nodes from the last available $C(j) - T(LS)$, $j = n-1, n-2, \ldots,$ nodes to the set $T(LS)$, set $I \leftarrow I \cup In(i)$.

If $I = N$, then set $T \leftarrow \bigcup_{i=1}^{LS} t(i)$, stop.

Otherwise, set $LS \leftarrow LS + 1$ and go to Step 2.

Remark Each reduced cut-set B_j has $n_c(j) = |B_j|$ nodes. The subset A_j can be chosen by taking $t_o(j)$ test nodes from the last available $T_I(i) - T_I(j)$, $i = j+1, j+2, \ldots$ nodes and $\Delta t = n_c(j) - t_o(j)$ nodes from $T_I(j)$.

Discussion

In particular cases, when the number of paths between two subnetworks is low, a small modification of the algorithm described would be very helpful. We can simply add some known branches to the existing network to increase the number of necessary paths and so decrease the number of necessary tests. This is also applicable when we have too many corners in the network. If the number of corners is greater than the maximal order of complete subgraph in the network, it can be reduced by adding new branches; consequently, we can reduce the number of sufficient tests.

V. EXAMPLES

The following examples explain how to use the results obtained from the test finding algorithm to identify all network elements.

Example 1

The network whose parameters we want to design and its Coates graph are shown in Fig. 9 (node 0 is chosen as the reference node). There are 3 corners in this network - nodes 1, 6 and 7. We choose node 1 as a starting point. The algorithm is illustrated step by step in Table I. The Σ in a column denotes that to obtain the set in the ith step we add all the elements of the column up to the ith row.

TABLE I

EXAMPLE ILLUSTRATING THE ALGORITHM

Itera- tion i	In Σ	C(i)	A Σ	Op(i)	Nodes in reduced cut-set	$n_c(i)$	$T_I(i)$	T Σ	$t_o(i)$	max(i)	b
0	$\emptyset$	1	$\emptyset$	$\emptyset$	$\emptyset$	0	$\emptyset$	$\emptyset$	0	0	0
1	1	2	1,2	$\emptyset$	1,2	2	1	1	-1	2	
2	2	3,4	3,4	$\emptyset$	2,3,4	3	1		1	3	
3	4	3,6	6	$\emptyset$	3,4,6	3	1		2	3	
4	6	3		6	3,6	2	1,6	6	1	3	4
5	3	5	5	6	3,5	2	6		1	2	
6	5	7,8	7,8	6	5,7,8	3	6		1	3	
7	7	8		6	7,8	2	6,7	7	1	3	7
8	8	6		6	6,8	2	7		1	2	8

So, for identification of network elements, we should apply excitations at nodes 1, 6 and 7. The nodal voltages measured with unit excitations at different nodes are shown in Table II.

TABLE II

NODAL VOLTAGES FOR EXAMPLE 1

Excitation at the Node No.	Voltage at the Node No.							
	1	2	3	4	5	6	7	8
1	.7764	.3293	- .006648	.1426	-.5715	.03842	-.9163	-.02094
6	-.3878	-1.163	-4.657	-1.47	-15.75	.8996	-22.76	-50.34
7	.01617	.04852	.1753	.07647	.4752	.09138	4.531	-2.126

Now we formulate equations (13) for successive reduced cut-sets and compute element values. The first equation is as follows:

$$\begin{bmatrix} V_{11} & V_{12} \\ V_{61} & V_{62} \end{bmatrix} \begin{bmatrix} Y_1 + Y_2 \\ - Y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0.7764 & 0.3293 \\ -0.3878 & -1.163 \end{bmatrix} \begin{bmatrix} Y_1 + Y_2 \\ - Y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and we obtain $Y_1 = 1$, $Y_2 = 0.5$.

The second equation

$$\begin{bmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{61} & V_{62} & V_{63} & V_{64} \\ V_{71} & V_{72} & V_{73} & V_{74} \end{bmatrix} \begin{bmatrix} - Y_2 \\ Y_2 + Y_3 + Y_4 + Y_5 \\ - Y_5 \\ - Y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

can be transformed, because Y_2 is now known, to

$$\begin{bmatrix} V_{12} & V_{13} & V_{14} \\ V_{62} & V_{63} & V_{64} \\ V_{72} & V_{73} & V_{74} \end{bmatrix} \begin{bmatrix} Y_3 + Y_4 + Y_5 \\ - Y_5 \\ - Y_4 \end{bmatrix} = \begin{bmatrix} (V_{11} - V_{12})\, Y_2 \\ (V_{61} - V_{62})\, Y_2 \\ (V_{71} - V_{72})\, Y_2 \end{bmatrix}$$

or

$$\begin{bmatrix} 0.3293 & -0.00648 & 0.1426 \\ -1.163 & -4.657 & -1.47 \\ 0.04852 & -1.753 & 0.07647 \end{bmatrix} \begin{bmatrix} Y_3 + Y_4 + Y_5 \\ - Y_5 \\ - Y_4 \end{bmatrix} = \begin{bmatrix} 0.2236 \\ 0.3876 \\ -0.01618 \end{bmatrix}$$

and we obtain $Y_3 = 0.3333$, $Y_4 = 0.25$, $Y_5 = 0.2$.

Continuing the procedure we design all the other network elements as

$$Y_6 = 0.1667,\ Y_7 = 0.1429,\ Y_8 = 0.125,\ Y_9 = 0.111,\ Y_{10} = 0.1,$$
$$Y_{11} = 0.0909,\ Y_{12} = 0.0833,\ Y_{13} = 0.07692,\ Y_{14} = 0.07143,$$
$$Y_{15} = 0.06667,\ Y_{16} = 0.0625,\ Y_{17} = 0.05882,\ g_m = 8.5.$$

Example 2

We apply the algorithm proposed to the passive grid circuit shown in Fig. 10. In such circuits, the number of nodes $n = k^2$ and number of passive elements $e = 2k^2 - 2k$, where $k = 2, 3, \ldots$ We assume that the voltage at each node can be measured. Using the algorithm described we find that no matter what the size of the grids three tests are sufficient for determining all the element values from voltage measurements at a single frequency.

VI. CONCLUSIONS

The method presented enables us to find a reasonably small number of tests which are topologically sufficient for the identification of all component values of linear analog circuits. This has been achieved due to searching for a "good" sequence of reduced cut-sets, whose elements are consecutively determined from (13). The notion of corner is particularly important, since it determines necessary tests independently of a sequence of cut-sets. The method is easy to program and gives a linear dependence of computational effort on the size of the network.

The method could be used to isolate the faulty subnetworks of the network when there are inaccessible nodes in the subnetworks. Further studies of the application of the method described to fault diagnosis are currently being concentrated on development of an efficient strategy for fault analysis of large networks with a reasonable number of voltage measurements and excitation nodes.

REFERENCES

[1] R.M. Biernacki and J.A. Starzyk, "Sufficient test conditions for parameter identification of analog circuits based on voltage measurements", Proc. European Conf. Circuit Theory and Design (Warsaw, Poland, 1980), pp. 233-241.

[2] T. Ozawa, "Fault detection of linear active networks by solving linear equations only", Proc. 14th Asilomar Conf. on Circuits, Systems and Computers (Pacific Grove, CA, 1980).

[3] T. Ozawa and M. Yamada, "Conditions for determining parameter values in a linear active network from node voltage measurements", Proc. IEEE Int. Symp. Circuits and Systems (Chicago, 1981), pp. 274-277.

[4] R.S. Berkowitz, "Conditions for network-element-value solvability", IRE Trans. Circuit Theory, vol. CT-9, 1962, pp. 24-29.

[5] W. Mayeda and G. Peponides, "Determination of component values in passive networks under limited measurements", Proc. 12th Asilomar Conf. on Circuits, Systems and Computers, Western Periodicals, North Hollywood, Nov. 1978, pp. 761-764.

[6] T.N. Trick and A.A. Sakla, "A new algorithm for the fault analysis and tuning of analog circuits", Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978), pp. 156-160.

[7] T.N. Trick, W. Mayeda and A.A. Sakla, "Calculation of parameter values from node voltage measurements", IEEE Trans. Circuits and Systems, vol. CAS-26, 1979, pp. 466-474.

[8] R.M. Biernacki and J.W. Bandler, "Postproduction parameter identification of analog circuits", Proc. IEEE Int. Symp. Circuits and Systems, (Houston, TX, 1980), pp. 1082-1086.

[9] N. Navid and A.N. Wilson, Jr., "A theory and an algorithm for analog circuit fault diagnosis", IEEE Trans. Circuits and Systems, vol. CAS-26, 1979, pp. 440-457.

[10] N. Sen and R. Saeks, "Fault diagnosis for linear systems via multifrequency measurements", IEEE Trans. Circuits and Systems, vol. CAS-26, 1979, pp. 457-465.

[11] T. Ozawa and Y. Kajitani, "Diagnosability of linear active networks", IEEE Trans. Circuits and Systems, vol. CAS-26, 1979, pp. 485-489.

[12] N. Deo, Graph Theory with Applications to Engineering and Computer Sciences. Englewood Cliffs, N.J.: Prentice-Hall, 1974.

[13] J.A. Starzyk, "Signal flow-graph analysis by decomposition method", IEE Proc. vol. 127, part G, 1980, pp. 81-86.

[14] A. Sangiovanni-Vincentelli, L.K. Chen and L.O. Chua, "An efficient heuristic cluster algorithm for tearing large-scale networks", IEEE Trans. Circuits and Systems, vol. CAS-24, 1977, pp. 707-717.

[15] J.A. Starzyk and A. Kończykowska, "Signal flow-graph analysis of large electronic networks", in preparation.

FIGURE CAPTIONS

Fig. 1	Illustrations of equation (13) and Result 2.
Fig. 2	Example of required 3-connections.
Fig. 3	Examples of corners. Corners are denoted by v.
Fig. 4	Formation of contours.
Fig. 5	Partition of a graph on weakly connected subgraphs.
Fig. 6	Set of nodes and connections.
Fig. 7	Illustration of the contour restriction case.
Fig. 8	Illustration of the bottleneck restriction case.
Fig. 9	9(a) Network. 9(b) Coates graph.
Fig. 10	Grid circuit.

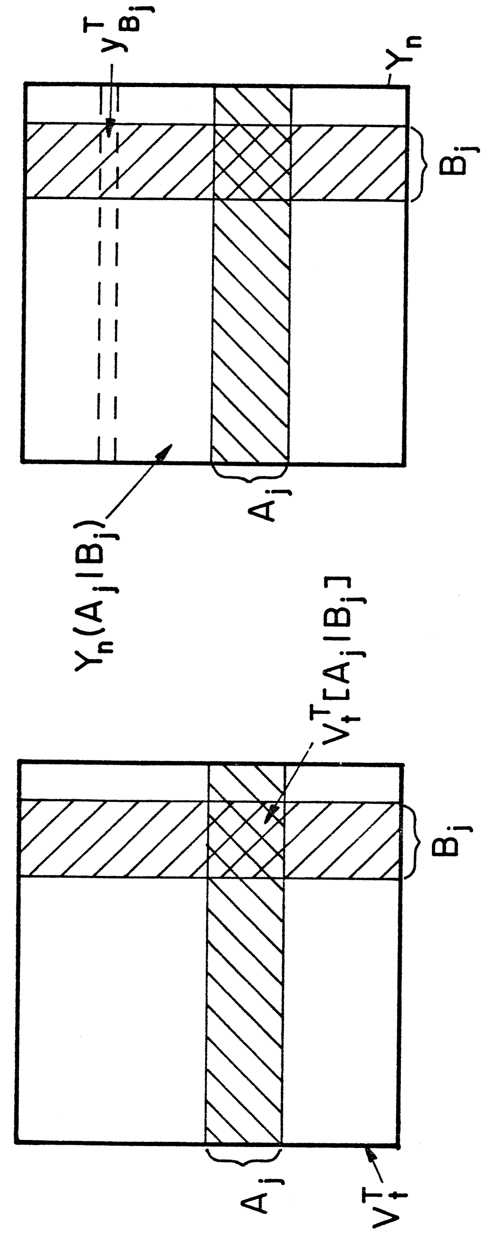

Fig. 1

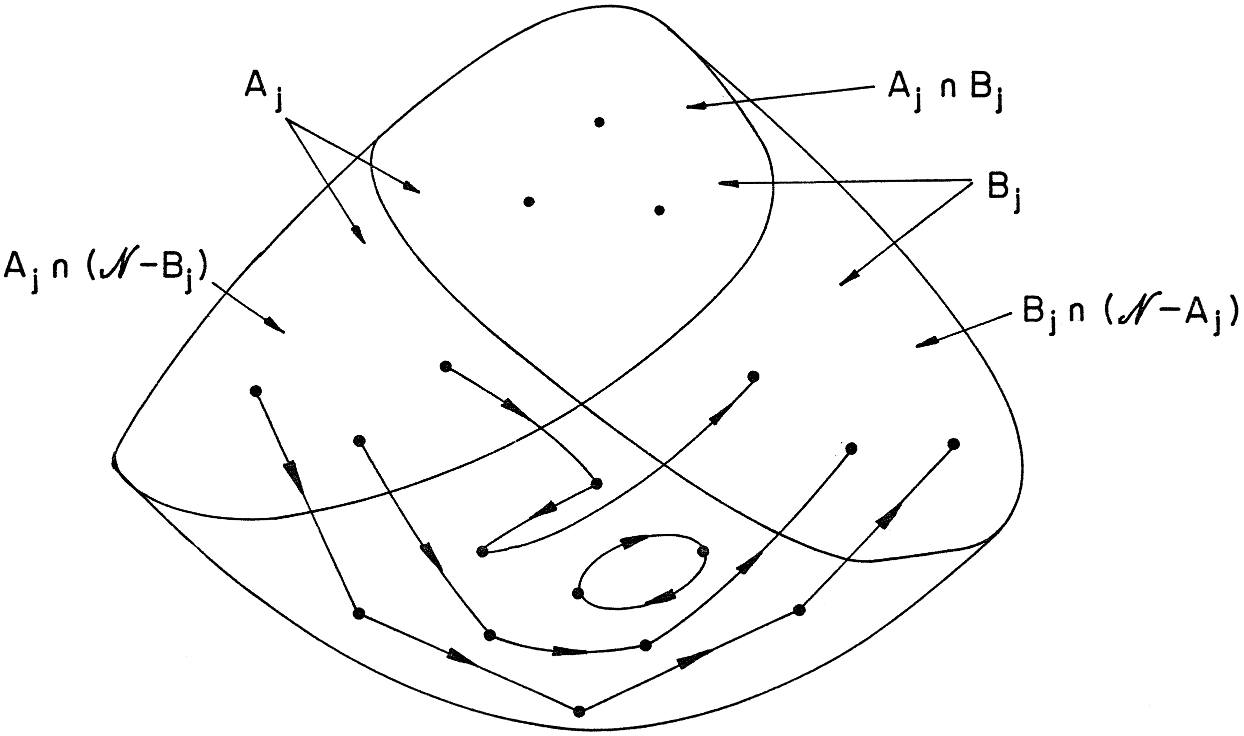

Fig. 2

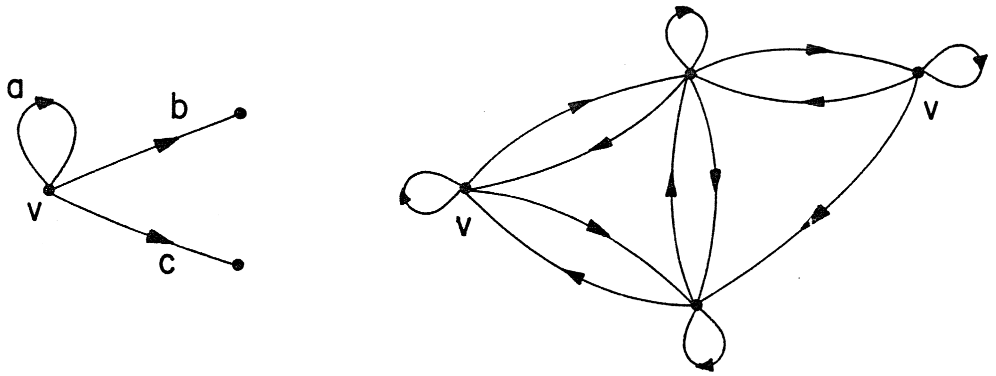

Fig. 3

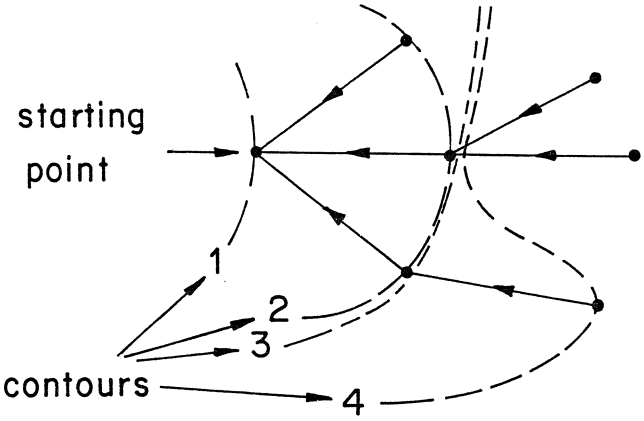

Fig. 4

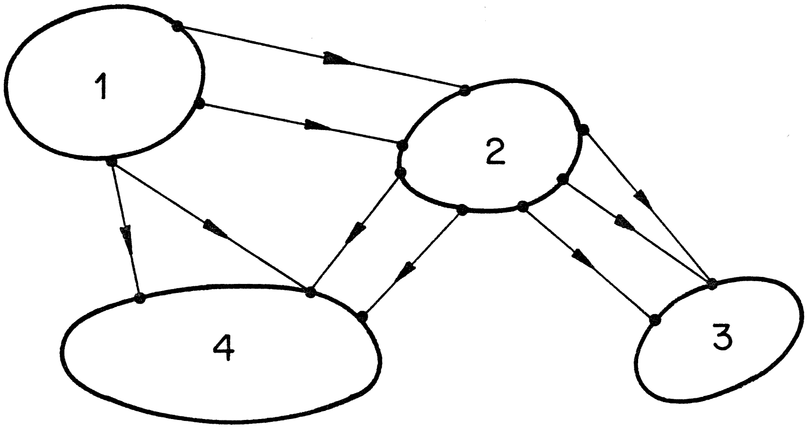

Fig. 5

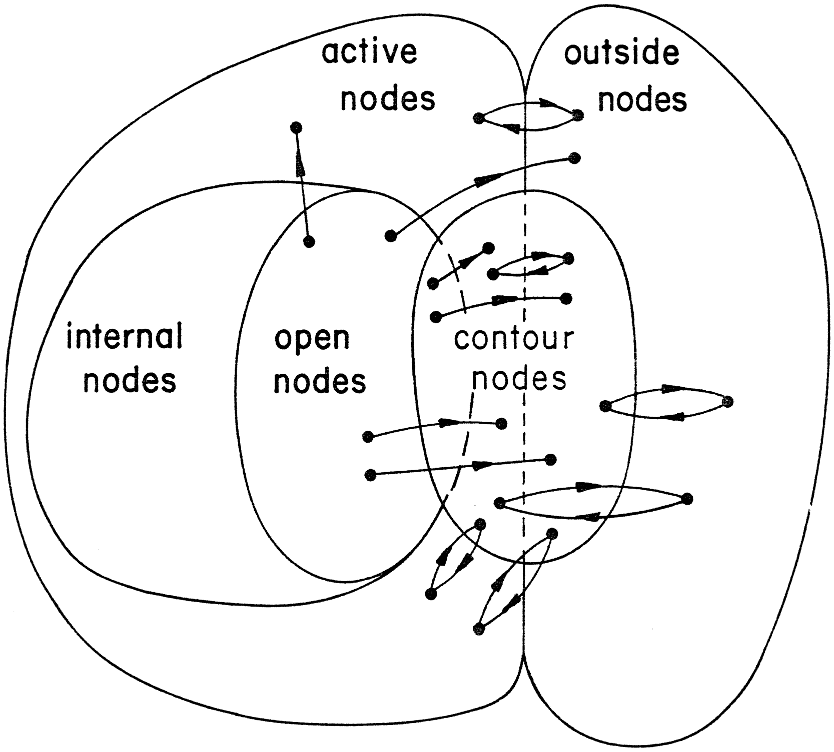
active
nodes
outside
nodes
internal
nodes
open
nodes
contour
nodes

Fig. 6

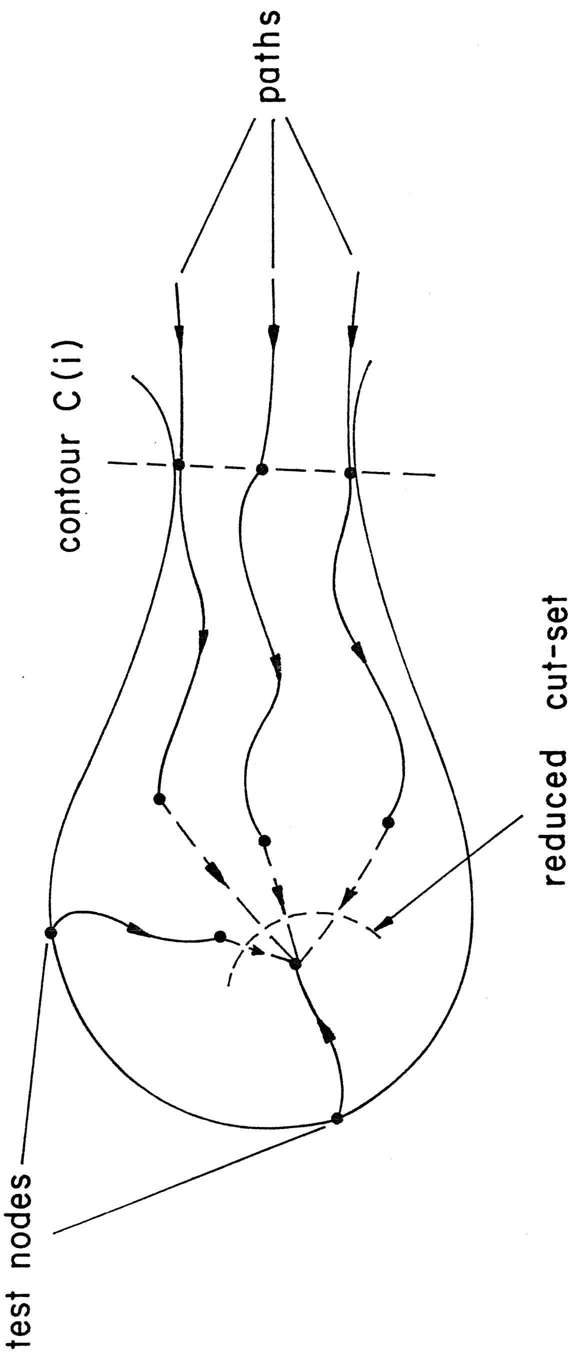

Fig. 7

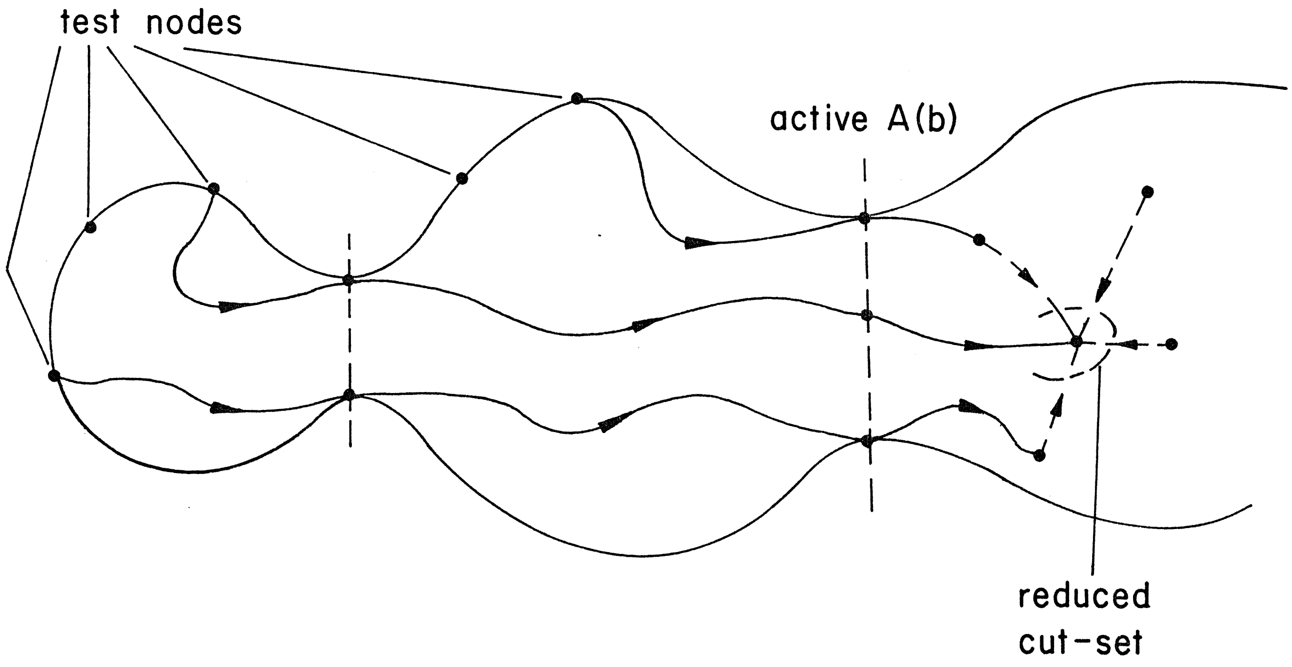

Fig. 8

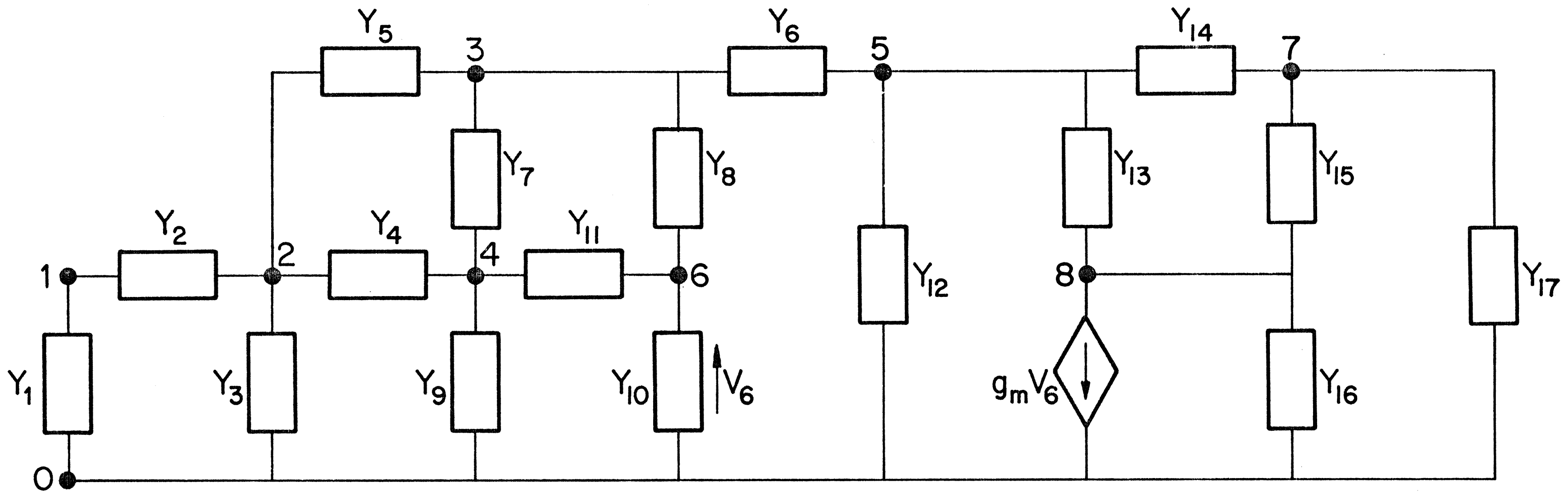
Y5
3
Y6
5
Y14
7
Y7
Y8
Y13
Y15
Y2
Y4
Y11
1
2
4
6
Y12
8
Y17
Y1
Y3
Y9
Y10
V6
gmV6
Y16
0

Fig. 9(a)

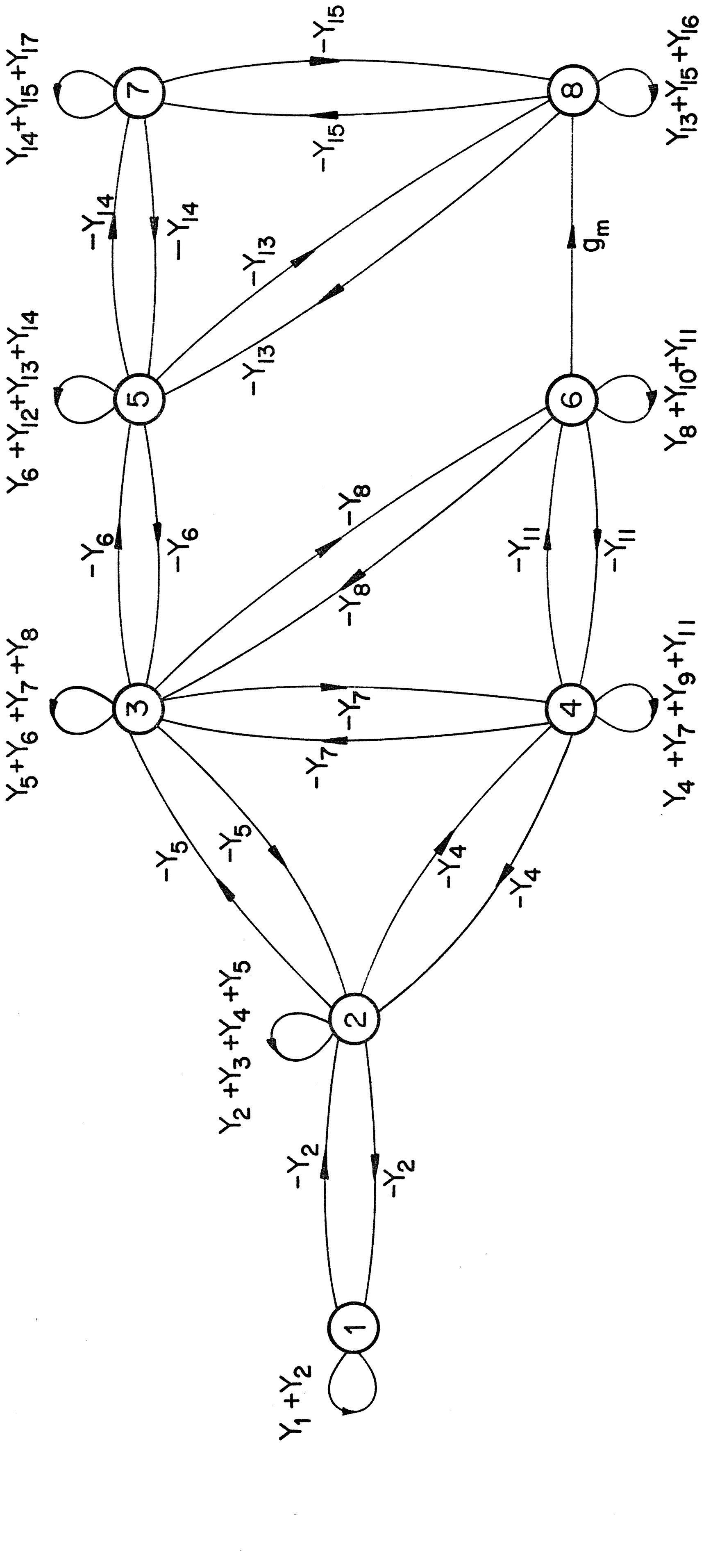
1
2
3
4
5
6
7
8
Y_1+Y_2
$-Y_2$
$-Y_2$
$Y_2+Y_3+Y_4+Y_5$
$-Y_5$
$-Y_5$
$-Y_4$
$-Y_4$
$Y_5+Y_6+Y_7+Y_8$
$Y_4+Y_7+Y_9+Y_{11}$
$-Y_7$
$-Y_7$
$-Y_6$
$-Y_6$
$-Y_8$
$-Y_8$
$-Y_{11}$
$-Y_{11}$
$Y_6+Y_{12}+Y_{13}+Y_{14}$
$Y_8+Y_{10}+Y_{11}$
$-Y_{13}$
$-Y_{13}$
g_m
$-Y_{14}$
$-Y_{14}$
$Y_{14}+Y_{15}+Y_{17}$
$Y_{13}+Y_{15}+Y_{16}$
$-Y_{15}$
$-Y_{15}$

Fig. 9(b)

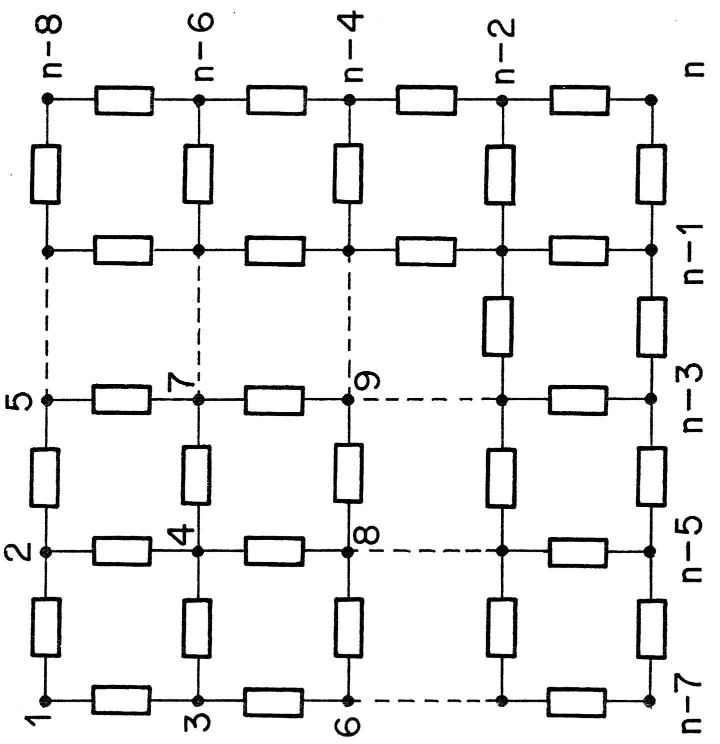

Fig. 10

SOC-266

DESIGN OF TESTS FOR PARAMETER IDENTIFICATION BY VOLTAGE MEASUREMENTS

J.A. Starzyk, R.M. Biernacki and J.W. Bandler

July 1981, No. of Pages: 32

Revised:

Key Words: Fault diagnosis, analog testing, test generation, topological methods, computer-aided design, network analysis

Abstract: This paper presents the theoretical background and resulting algorithm for generating tests which are topologically sufficient for identification of parameter values in linear circuits. Voltage measurements at all the nodes are assumed. The main thrust of this paper is to minimize the number of necessary measurements at different current excitations. Coates flow-graph representation of a network is used.

Description:

Related Work: SOC-233, SOC-235, SOC-236, SOC-244, SOC-251.

Price: $6.00.