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A LINEAR PROGRAMMING APPROACH TO FAULT LOCATION IN ANALOG CIRCUITS

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Abstract

This paper deals with fault isolation in linear analog circuits with design tolerances on nonfaulty elements and inaccurate measurements taken into account. Using a single current excitation and corresponding voltage measurements a system of linear equations is constructed. This underdetermined system of linear equations is solved using optimization to find the most likely faulty elements. Then, algebraic invariant equations associated with the estimated faulty set are constructed to verify the results obtained.

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I. INTRODUCTION

Fault diagnosis of analog circuits [1-15] has received much attention in recent years. The question of fault location arises when a circuit, which has already been manufactured, does not satisfy performance specifications.

There are four main approaches to the problem: constructing a fault dictionary [2,3], identifying all component values [4-7], constructing and checking invariants of an assumed faulty set [8-12], and using estimation criteria [14,15]. The first approach requires the simulation of different possible faults and storing the results as a dictionary. It fits in well with the needs of catastrophic fault location. However, it is unsuitable for multiple soft fault detection. The second approach requires most nodes of the circuit to be accessible. It cannot, therefore, be used under the constraint of limited measurements. The last two approaches are suitable for soft fault location with limited measurements, but they require most of the circuit elements to be nonfaulty. This paper addresses itself to these two approaches using a single test with limited measurements, i.e., when an excitation is applied and a number of simultaneous measurements are taken. We consider a current excitation and voltage measurements at a single frequency.

We present a unified formulation for both approaches. We propose the use of linear programming to solve the resulting problems. In particular, for the approach of invariants, we extend the method of Biernacki and Bandler [8]. Their method was based on pre-test construction of certain algebraic invariants, each of them corresponding to different combinations of elements suspected to be faulty and then checking them using the measurements taken. They also provided estimates when the nonfaulty elements vary inside the tolerance region. The estimates, however, may be too wide. Here, we extend their formulation by including tolerances in a more exact way.

For the estimation approach, we have extended and simplified the method of Merrill [14]. We propose the use of the least one (least absolute value) objective function and also, by appropriate choice of error parameters, we obtain a linear relationship for the constraints. We are, consequently, able to formulate a linear programming problem.

II. PROBLEM FORMULATION

We consider a linear circuit in the steady state at a single frequency and having p components. The circuit is excited by a current source I^g at the input port and ℓ corresponding independent voltages

$$\underset{\sim}{V}^m \triangleq [\, V_1^m \;\; V_2^m \; \ldots \; V_\ell^m \,]^T \tag{1}$$

are measured.

A change of a component value w.r.t. the nominal can be represented by a current or voltage source which is in parallel or in series, respectively, with the element as is shown in Fig. 1. Thus, we have n_1 current sources

$$\underset{\sim}{I}^x \triangleq [\, I_1^x \;\; I_2^x \; \ldots \; I_{n_1}^x \,]^T \tag{2}$$

and n_2 voltage sources

$$\underset{\sim}{V}^y \triangleq [\, V_1^y \;\; V_2^y \; \ldots \; V_{n_2}^y \,]^T , \tag{3}$$

where $n \stackrel{\Delta}{=} n_1 + n_2$ is the total number of elements considered as different from nominal. All other p-n elements assume their nominal values. If we treat the additional sources $\underset{\sim}{I}^x$ and $\underset{\sim}{V}^y$ as external to the circuit, as is shown in Fig. 2, then the interior of the circuit is known, since it consists of nominal components only. Therefore, based on the principle of superposition we can write the linear relationship

$$\underset{\sim}{V}^m = [\, \underset{\sim}{H}_{mg} \;\; \underset{\sim}{H}_{mx} \;\; \underset{\sim}{H}_{my} \,] \begin{bmatrix} I^g \\ \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix} \stackrel{\Delta}{=} \underset{\sim}{H}_m \begin{bmatrix} I^g \\ \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix}, \tag{4}$$

where the matrix $\underset{\sim}{H}_m$ is known and can efficiently be calculated by means of the adjoint network approach (see Appendix). This is the basic system of equations which has to be satisfied by the faulty circuit. Dependently on the number ℓ of voltages measured and the number n of additional sources considered, the system (4) may be underdetermined, determined or overdetermined. Because of their essential difference, we have to treat the latter two cases separately from the first one. If (4) is determined or overdetermined we can only check whether it is consistent or not. In particular, we can check if the value of I^g calculated from the equations is the same as that which was applied. If it is inconsistent it means that our selection of n elements is not the proper one. That is why this approach will be called a verification method.

If the system (4) is underdetermined a solution always exists. It is our choice to select a solution which could be reasonable from the point of view of fault location. That is why this approach will be called an approximate method.

III. THE APPROXIMATE METHOD

In the approximate method we typically consider all the circuit elements as different from the nominals, most of them being within their tolerances and some of them far from the assigned tolerances. So we assume n=p. In order to eliminate I^g, which is always possible, we consider the nominal measurements which would appear if the network elements did not deviate from the nominals, i.e.,

$$\underset{\sim}{V}^{m0} = \underset{\sim}{H}_m \begin{bmatrix} I^g \\ \underset{\sim}{0} \\ \underset{\sim}{0} \end{bmatrix} . \tag{5}$$

These nominal voltages can be calculated. Now, subtracting (5) from (4) we get

$$\Delta \underset{\sim}{V}^m = [\underset{\sim}{H}_{mx} \quad \underset{\sim}{H}_{my}] \begin{bmatrix} \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix} \triangleq \underset{\sim}{\overline{H}}_m \begin{bmatrix} \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix} . \tag{6}$$

In order to find a reasonable solution to the system (6) we propose to use an optimization technique where the sum of absolute values of error functions is to be minimized. Such an objective function (least one approximation) tends to select the minimal number of errors different from zero. This, in turn, corresponds to a reasonable assumption that there are few faulty elements in the circuit.

Perhaps, the best choice for the error parameters would be to take direct changes of the components. However, this would create a non-linear relationship in (6) and additional variables in the form of unknown voltages and currents would appear in the problem. Therefore, taking advantage of the dependence of the additional sources $\underset{\sim}{I}^x$ and $\underset{\sim}{V}^y$

on the component value changes we propose to define the error parameters as

$$\underset{\sim}{e} \triangleq \begin{bmatrix} \underset{\sim}{e}_1 \\ \underset{\sim}{e}_2 \end{bmatrix} , \tag{7}$$

where

$$\underset{\sim}{e}_1 \triangleq \mathrm{Re} \begin{bmatrix} \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix} \text{ and } \underset{\sim}{e}_2 \triangleq \mathrm{Im} \begin{bmatrix} \underset{\sim}{I}^x \\ \underset{\sim}{V}^y \end{bmatrix} . \tag{8}$$

Hence, the optimization problem can be stated as follows.

$$\text{Minimize} \sum_{i=1}^{2n} |e_i| \tag{9}$$

subject to the linear equality constraints

$$\Delta \underset{\sim}{V}^m = \underset{\sim}{\overline{H}}_m (\underset{\sim}{e}_1 + j\underset{\sim}{e}_2) . \tag{10}$$

The optimization problem can easily be converted to the regular linear programming form by an apropriate transformation of the variables.

In (4) it is assumed that the actual values of the voltages measured are known exactly. However, due to measurement errors the actual voltage vector which should appear on the left hand side of (4) differs from $\underset{\sim}{V}^m$ by a vector

$$\underset{\sim}{\rho} \triangleq [\rho_1 \; \rho_2 \; \dots \; \rho_\ell]^T \tag{11}$$

if $\underset{\sim}{\rho}$ represents the absolute errors, or by

$$\mathrm{diag}\,(\underset{\sim}{V}^m)\; \underset{\sim}{\rho} \tag{12}$$

if $\underset{\sim}{\rho}$ represents the relative errors. Incorporating the measurement errors, for instance in the form of (11), the constraints (10) become

$$\underset{\sim}{\Delta V}^{m} = \underset{\sim m}{\overline{H}}(\underset{\sim 1}{e} + j\underset{\sim 2}{e}) - \underset{\sim}{\rho} \tag{13}$$

which are again linear w.r.t. the variables $\underset{\sim}{e}$ and $\underset{\sim}{\rho}$.

Typically the measurement errors are small and their bounds are usually known. However, the impact of the exact bounds does not seem to be very important, at least from the point of view of the approximate approach. Therefore, for the purpose of the linear programming formulation we may consider bounds on the real and imaginary parts of $\underset{\sim}{\rho}$ as

$$\begin{aligned} -\underset{\sim}{\xi} \leq \mathrm{Re}\,(\underset{\sim}{\rho}) \leq \underset{\sim}{\xi} \\ -\underset{\sim}{\eta} \leq \mathrm{Im}\,(\underset{\sim}{\rho}) \leq \underset{\sim}{\eta} \end{aligned} \tag{14}$$

and so the optimization problem can be stated as solving (9) subject to the linear equality constraints (13) and the linear inequality constraints (14).

The optimization problem proposed does not account for chain-faults, fault correlation or knowledge of the most likely areas of faults. However, this can be done by using appropriate weighting factors based on the experience and knowledge of the particular circuit under test.

IV. THE VERIFICATION METHOD

In the verification method we select n elements, $n<\ell$, in order to obtain a determined or overdetermined system of equations (4) and then we check whether the system is consistent or not.

Assume, for the time being, $n=\ell-1$ and $\underset{\sim m}{H}$ to be a square nonsingular matrix. Calculating I^g from (4) we get

$$I^g = \sum_{i=1}^{\ell} V_i^m \Delta_{i1}/\Delta = \sum_{i=1}^{\ell} V_i^m d_{i1} , \tag{15}$$

where Δ_{i1} is the cofactor of the element (i,1) of $\underset{\sim}{H}_m$ and Δ denotes its determinant. The coefficients $d_{i1} \overset{\Delta}{=} \Delta_{i1}/\Delta$ can efficiently be calculated by means of the adjoint network approach (see Appendix). On the other hand, the applied value of the excitation I^g can be expressed in terms of nominal "measurements" as

$$I^g = \sum_{i=1}^{\ell} V_i^{m0} d_{i1} \tag{16}$$

and the consistency of both the expressions (15) and (16) should be checked.

The exact consistency, however, can appear only if the measurements $\underset{\sim}{V}^m$ are exact and if all the other p-n elements assume exact nominal values. Thus, we have to incorporate the measurement error $\underset{\sim}{\rho}$ and the actual changes δ_{i1} of the coefficients d_{i1} which are due to the variation of the p-n elements within their tolerances. So, taking for instance (11), we have

$$I^g = \sum_{i=1}^{\ell} (V_i^m + \rho_i)(d_{i1} + \delta_{i1}) , \tag{17}$$

instead of (15), with unknown values of ρ_i and δ_{i1}. The tolerance region of the elements supposed to be nonfaulty is defined by

$$- \varepsilon_i \leq \Delta\phi_i \leq \varepsilon_i , \ i \in I_\varepsilon , \tag{18}$$

where $\Delta\phi_i \overset{\Delta}{=} \phi_i - \phi_i^0$ is the change of the ith element value w.r.t. its nominal, ε_i is its associated tolerance and I_ε denotes the set of indices of the p-n nonfaulty elements.

Now the consistency of the expressions (16) and (17) requires ρ_i and δ_{i1}, $i=1,2,\ldots,\ell$, satisfying (14) and (18) such that both the expressions are equal. It should be noted that although (17) gives the exact relationship it is a nonlinear function in the unknown variables. However, for a small tolerance region and reasonably good measurement accuracy we can use the first-order approximation and substantially reduce the computational effort required. We obtain

$$I^g \cong \sum_{i=1}^{\ell} \left(V_i^m d_{i1} + d_{i1}\rho_i + V_i^m \sum_{j\in I_\varepsilon} \frac{\partial d_{i1}}{\partial \phi_j} \Delta\phi_j\right) . \tag{19}$$

Comparing (16) and (19), after some manipulations, we get

$$\sum_{i=1}^{\ell} d_{i1}\rho_i + \sum_{j\in I_\varepsilon} a_j \Delta\phi_j = b , \tag{20}$$

where

$$a_j = \sum_{i=1}^{\ell} V_i^m \frac{\partial d_{i1}}{\partial \phi_j} \tag{21}$$

and

$$b = \sum_{i=1}^{\ell} (V_i^{m0} - V_i^m) d_{i1} . \tag{22}$$

The sensitivities which appear in (21) can also be calculated using the adjoint network approach (see Appendix). After separating real and imaginary parts of (20) we obtain

$$\begin{aligned} \underset{\sim}{c}_1^T \underset{\sim}{x} &= b_1 \\ \underset{\sim}{c}_2^T \underset{\sim}{x} &= b_2 \end{aligned} , \tag{23}$$

where the vector $\underset{\sim}{x}$ comprises real and imaginary parts of $\underset{\sim}{\rho}$ and all $\Delta\phi_j$ and consists of n+p+2 components.

If the system (4) is overdetermined, i.e., $\ell > n+1$, we proceed similarly for all independent combinations of n+1 equations to formulate equations of the form of (23). Putting all the equations into one matrix equation we get

$$\underset{\sim}{C}\,\underset{\sim}{x} = \underset{\sim}{B} \,. \tag{24}$$

The equation (24) together with (14) and (18) determine the consistency in question. In other words, if a feasible solution exists we consider (4) as consistent and simultaneously we verify the n selected elements as a possibly faulty set. If no feasible solution exists we can be sure that one or more faulty elements exist besides those selected.

In order to find a meaningful feasible solution, we propose to use a linear programming formulation of the form

$$\text{minimize } f(\underset{\sim}{x}) \tag{25}$$

subject to the linear equality constraints (24) and linear inequality constraints (14) and (18), where $f(\underset{\sim}{x})$ is a suitable linear function of $\underset{\sim}{x}$.

V. CALCULATION OF FAULTY ELEMENT VALUES

After solving (9) by the approximate method or (25) by the verification method we may be interested in calculating the component values corresponding to the solution obtained.

The solution of (9) gives us the vectors $\underset{\sim}{I}^x$ and $\underset{\sim}{V}^y$ of the additional sources. Simulating the nominal circuit with $\underset{\sim}{I}^x$ and $\underset{\sim}{V}^y$ together with I^g we calculate all the voltages and currents which appear in $\underset{\sim}{I}^x$ and $\underset{\sim}{V}^y$ (see Fig. 1). Then, we can easily calculate the change $\Delta\phi_j$ of every component and check it against its tolerance. If the change violates the tolerance, we consider the corresponding element as faulty. If the value calculated is within the tolerance then the element is treated as nonfaulty. However, if the tolerance is slightly violated, we decide whether or not to apply the verification method. If the decision is sufficiently sharp we do not need to verify the combination selected.

The verification method does not supply the vectors $\tilde{I}^x$ and $\tilde{V}^y$. They can be calculated from (4) after correcting $\Delta\tilde{V}^m$ by $\tilde{\rho}$ and recalculating $\tilde{H}_m$ taking into account the changes $\Delta\phi_j$ obtained from (25).

VI. EXAMPLES

Example 1

Consider the ladder network shown in Fig. 3 with nominal values of elements $G_i = 1$ and tolerances $\varepsilon_i = \pm 0.05$, i = 1, 2,... ,5. Assume that the network is excited at the port 11' and voltage measurements are taken at the ports 11', 22' and 33'. For $I_g = 1A$ the nominal responses are $V^0_{11'} = 0.625V$, $V^0_{22'} = 0.25V$ and $V^0_{33'} = 0.125V$.

1. The Approximate Method

Using the three available measurements we have three equations in five unknowns. We solve this underdetermined system by the optimization formulation proposed in (9) and (10). We have taken the network elements to be $G_1 = 1.02$, $G_2 = 0.5$, $G_3 = 0.98$, $G_4 = 0.98$ and $G_5 = 0.95$. It is clear that element 2 is faulty and all other elements are within tolerances. Exciting the network at port 11' with $I_g = 1A$, we obtain the measured values $V_{11'} = 0.718V$, $V_{22'} = 0.183V$ and $V_{33'} = 0.093V$. The linear program gives the changes $\Delta G_1 = 0.0$, $\Delta G_2 = -0.473$, $\Delta G_3 = 0.0331$, $\Delta G_4 = 0.03$ and $\Delta G_5 = 0.0$. It is clear that the change in the second element violates the corresponding tolerance and we consider it faulty, which is a correct conclusion.

2. The Verification Method

The approximate method detects G_2 as faulty. We have applied the verification algorithm with a single fault hypothesis. From the three measurements, two equality constraints are constructed. Using the optimization formulation (24) and (25) with $\underset{\sim}{\rho} = \underset{\sim}{0}$ and $f(\underset{\sim}{x}) = \sum_{j \in I_\varepsilon} |\Delta\phi_j|$, we first check whether G_2 is faulty. The linear program gives a feasible solution. We have also checked the single fault hypothesis for the remaining 4 elements. No feasible solution was found. This confirms that the second element as detected by the approximate method is really faulty.

Carrying out the proposed calculations for obtaining the faulty element value, we find that $G_2 = 0.532$, which is very close to the actual value. The values differ because the feasible tolerance vector found by the linear program is not the actual one.

Example 2

Consider the active filter shown in Fig. 4 with the nominal element values $G_1 = G_2 = 1$, $C_1 = C_2 = 1$ and $K = 1$. Also, we assume that the amplifier has an output conductance $G_{out} = 1$. All elements are assumed to have design tolerances of ±5%. We excite the network by a current source $I_g = 1A$ and its source resistance is assumed to be fault free, $R_g = 1$. We take ports 11', 22' and 33' as our ports of measurement.

1. The Approximate Method

In this example we have five sources of error. We have available, in general, three complex measurements. Following our formulation, we have 6 equations in 10 variables. We considered $G_1 = 0.5$, $G_2 = 1.02$, $C_1 = 0.98$, $C_2 = 0.5$ and for the amplifier $K = 1.02$ and $G_{out} = 0.98$. The following changes in the elements are given by the linear program: $|\Delta G_1| = 0.5$, $|\Delta G_2| = 0.0185$, $|\Delta C_1| = 0.039$, $|\Delta C_2| = 0.487$, $|\Delta G_{out}| = 0.081$ and $|\Delta K| = 0.0$.

The changes in C_2 and G_1 substantially violate the design tolerances so we assume them faulty. Also, the change in G_{out} is slightly larger than the assigned tolerance so we are not certain whether or not it is faulty.

2. The Verification Method

We have applied the verification method to check whether or not G_1 and C_2 are faulty. We have three measurements (complex), therefore we have four equality constraints. Using the same approach as in Example 1, the program gives a feasible solution corresponding to the combination of G_1 and C_2. This confirms that they are faulty and we can exclude G_{out} from being faulty. We have checked the remaining 9 combinations for the double fault hypothesis and no feasible solution was detected. The calculated element values are $G_1 = 0.5$ and $C_2 = 0.488$, which are very close to the exact values.

VII. CONCLUSION

We have presented a method for fault location in linear analog circuits taking into consideration the tolerances on the nonfaulty elements and the inaccuracy in the performed measurements.

An underdetermined system of linear equations is constructed. This system of equations relates the change in the performed voltage measurements to an appropriate set of error parameters. Each error parameter is related to the deviation of a corresponding network element. Under the assumption that only few network elements are faulty, we utilize linear programming for solving the underdetermined system of equations. The objective function is the sum of the absolute values of the error parameters. This least-one objective function tends to single out very few error parameters with larger values compared to the other parameters, which agrees with our physically realistic assumption.

The results obtained from the linear program indicate whether or not we need to consider the verification phase of the method, where we construct algebraic equations invariant on the faulty set. We extended the formulation of Biernacki and Bandler [8] to consider the effects of nonzero tolerances on the nonfaculty elements and imprecise values of the performed measurements. We compensate for the uncertainty caused by the aforementioned effects by constructing as many independent algebraic invariant equations as possible from the available voltage measurements. First-order changes caused by these effects are considered in the algebraic invariants. This yields a system of linear equations. Linear programming is utilized to solve this system of equations subject to feasibility conditions on the tolerances and the measurement errors.

Utilizing the approximate method to locate the most likely faulty elements and then verifying the results when they are not sharp enough, we have, in general, enhanced the reliability of the approximate method and reduced the computational burden of the verification approach.

APPENDIX

The adjoint network concept is utilized to compute the required coefficients to construct equations (4), (15) and (19). With $\hat{I}^g=0$, $\hat{\underset{\sim}{I}}^x=\underset{\sim}{0}$ and $\hat{\underset{\sim}{V}}^y=\underset{\sim}{0}$ the adjoint equations to (4) are given by

$$\begin{bmatrix} \hat{V}^g \\ \hat{\underset{\sim}{V}}^x \\ -\hat{\underset{\sim}{I}}^y \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}^T_{mg} \\ \underset{\sim}{H}^T_{mx} \\ \underset{\sim}{H}^T_{my} \end{bmatrix} \hat{\underset{\sim}{I}}^m = \hat{\underset{\sim}{H}}_m \hat{\underset{\sim}{I}}^m \, . \tag{A1}$$

Connecting a current source to the ith measurement port and calculating the voltages $\hat{V}^g$, $\hat{\underset{\sim}{V}}^x$ and the currents $\hat{\underset{\sim}{I}}^y$, we get the elements of the ith column of the matrices $\underset{\sim}{H}^T_{mg}$, $\underset{\sim}{H}^T_{mx}$ and $\underset{\sim}{H}^T_{my}$. Repeating for all measurement ports, all coefficients of (4) are directly obtained.

In general, $\hat{\underset{\sim}{H}}_m$ is a nonsingular square matrix. Solving (A1) for the ith measurement current $\hat{I}^m_i$ we get

$$\hat{I}^m_i = \hat{d}_{1i} \hat{V}^g + \sum_{j=2}^{n_1+1} \hat{d}_{ji} (\hat{V}^x_{j-1}) + \sum_{j=n_1+2}^{n+1} \hat{d}_{ji} (-\hat{I}^y_{j-1}) \, . \tag{A2}$$

Applying ℓ=n+1 independent excitations to the measurement ports of the adjoint network we get

$$\begin{bmatrix} \hat{I}^{m1}_i \\ \cdot \\ \cdot \\ \cdot \\ \hat{I}^{m\ell}_i \end{bmatrix} = \begin{bmatrix} \hat{V}^{g1} & [\hat{\underset{\sim}{V}}^{x1}]^T & [-\hat{\underset{\sim}{I}}^{y1}]^T \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \hat{V}^{g\ell} & [\hat{\underset{\sim}{V}}^{x\ell}]^T & [-\hat{\underset{\sim}{I}}^{y\ell}]^T \end{bmatrix} \begin{bmatrix} \hat{d}_{1i} \\ \cdot \\ \cdot \\ \cdot \\ \hat{d}_{\ell i} \end{bmatrix} \tag{A3}$$

or, more compactly,

$$\hat{\underset{\sim}{I}}^m_i = \underset{\sim}{R} \, \hat{\underset{\sim}{d}}_i \, , \tag{A4}$$

where the superscripts 1 to ℓ stand for the ℓ independent excitations. Since $d_{i1} = \hat{d}_{1i}$, d_{i1} can be computed by solving (A3). For different values of i, the $\tilde{R}$ matrix will be the same and only the LHS of (A3) will change. Using the appropriate LHS in (A3) all the coefficients of (15) can be computed.

The sensitivities of d_{i1} , $i=1,2,\ldots,\ell$, relative to the nonfaulty circuit elements are evaluated by computing the sensitivities of the elements of the matrix $\tilde{R}$ w.r.t. the nonfaulty elements. These sensitivities are directly obtained using the adjoint network concept.

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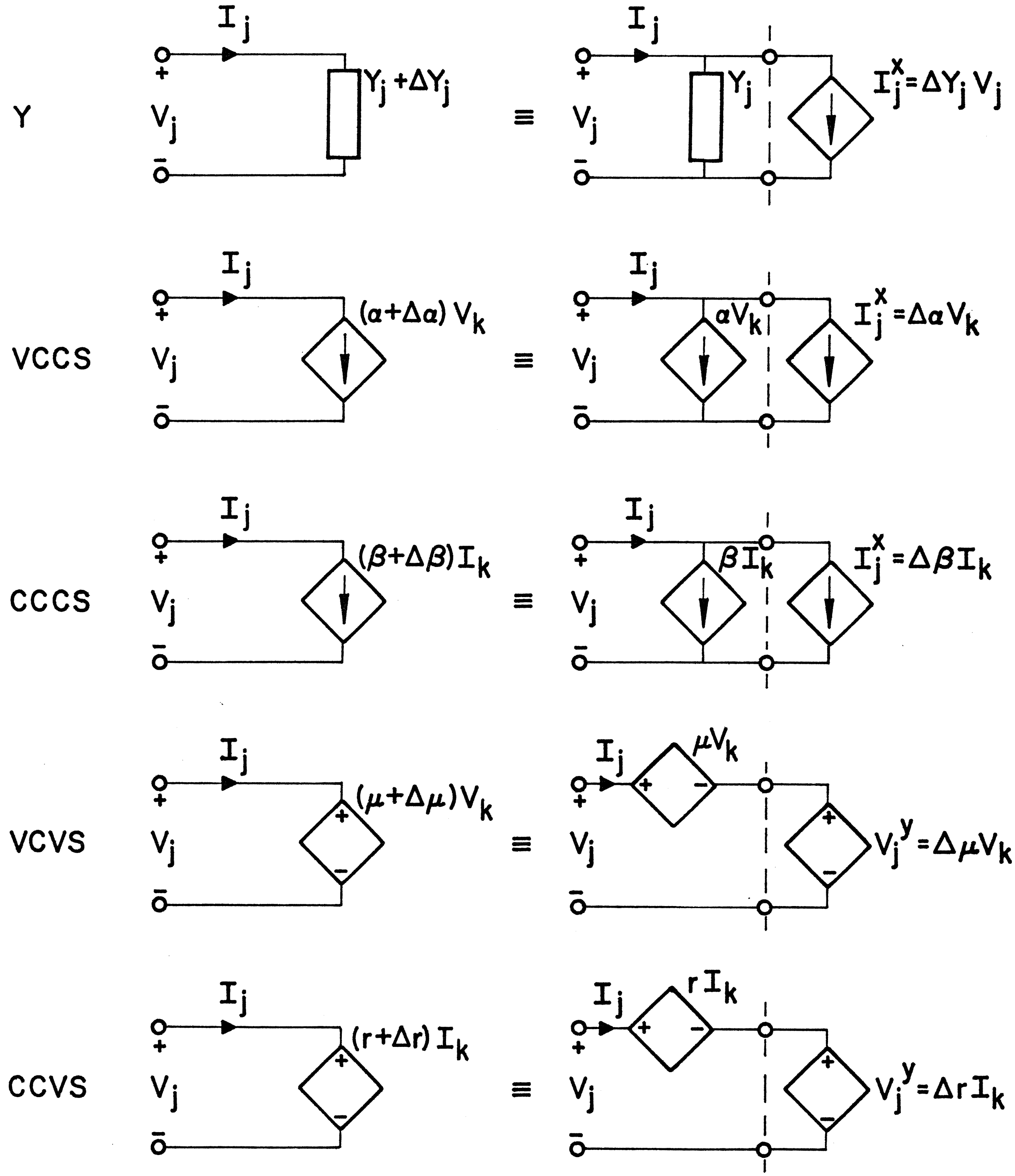

Fig.1 An equivalent representation for changes in element values.

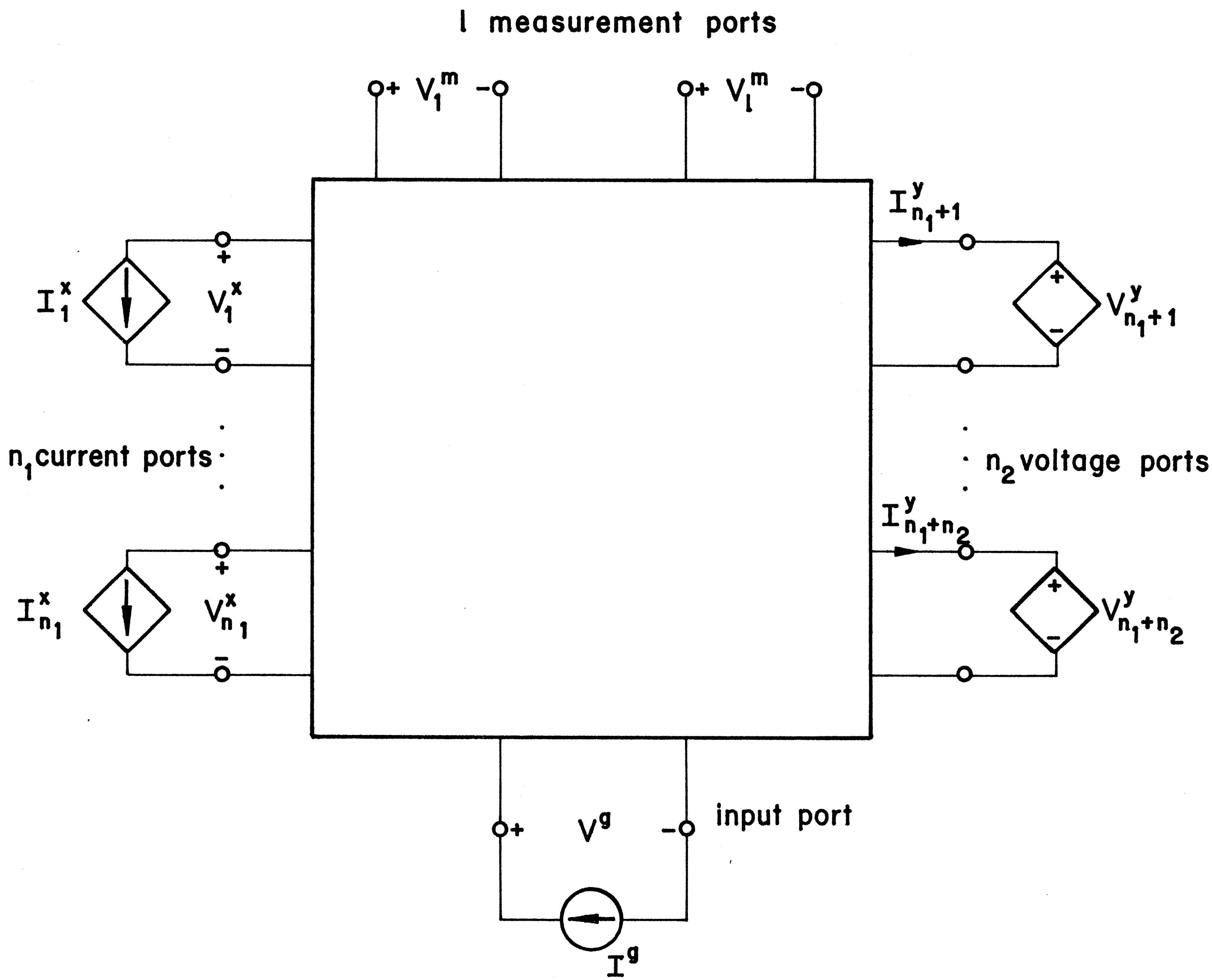

Fig.2 The faulty network after extracting ℓ measurement ports, n_1+n_2 fault ports and an excitation port.

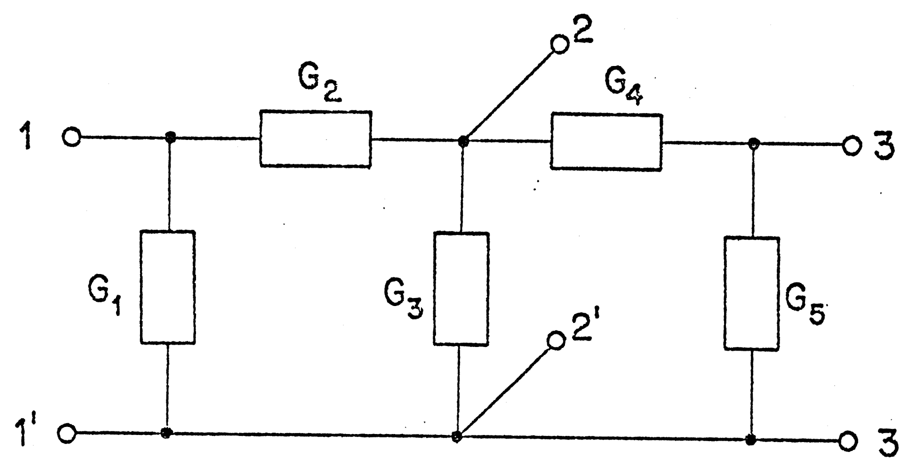

Fig.3 The ladder network example.

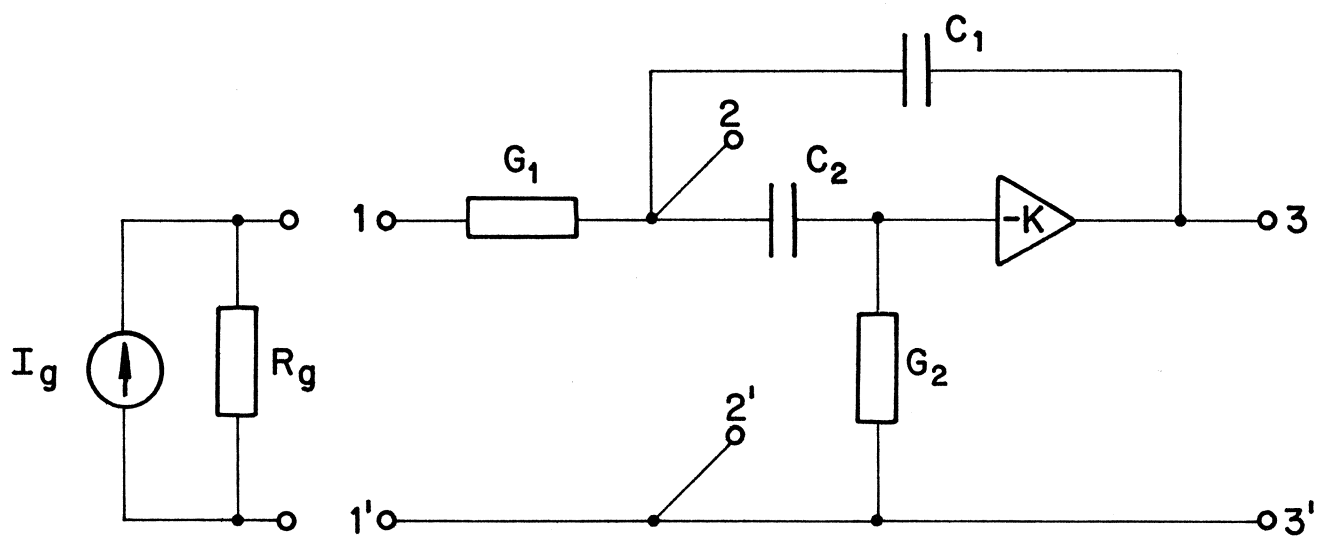

Fig.4 The RC active filter example.

SOC-259

A LINEAR PROGRAMMING APPROACH TO FAULT LOCATION IN ANALOG CIRCUITS

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Abstract: This paper deals with fault isolation in linear analog circuits with design tolerances on nonfaulty elements and inaccurate measurements taken into account. Using a single current excitation and corresponding voltage measurements a system of linear equations is constructed. This underdetermined system of linear equations is solved using optimization to find the most likely faulty elements. Then, algebraic invariant equations associated with the estimated faulty set are constructed to verify the results obtained.

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