INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-258

EXACT POWER NETWORK SENSITIVITIES VIA GENERALIZED COMPLEX BRANCH MODELLING

J.W. Bandler and M.A. El-Kady

December 1980

(Revised September 1981)

FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

EXACT POWER NETWORK SENSITIVITIES
VIA GENERALIZED COMPLEX BRANCH MODELLING

J.W. Bandler, Fellow, IEEE, and M.A. El-Kady, Member, IEEE

Abstract

This paper presents an application of the Tellegen's theorem approach to power network sensitivity calculations. Our theory employs an adjoint network concept based upon a novel, generalized complex branch modelling procedure allowing the exact steady-state component models of power networks to be considered without any approximation. Exact formulas for first-order change and reduced gradients are derived and tabulated. The theoretical results are fully verified numerically on a 6-bus system and on a 26-bus, 32 line system. The full bus and line data are provided for the examples to permit independent verification of our results.

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grants A7239 and G0647.

J.W. Bandler and M.A. El-Kady (part-time) are with the Group on Simulation, Optimization and Control and Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada L8S 4L7.

M.A. El-Kady is with Ontario Hydro, Toronto, Canada.

I. INTRODUCTION

Efficient sensitivity analysis and gradient evaluation, essential in power system operation and planning, is the subject of this paper. A number of relevant papers have dealt with appropriate computational techniques in the context of applying the Lagrange multiplier approach [1,2] and the Tellegen's theorem [3] approach [4-7] to sensitivity calculations in electrical power networks.

The term sensitivity calculations is used to indicate the procedure of expressing the first-order change of a function f solely in terms of variations in the independent variables $\tilde{u}$, which are related to the system states by a set of nonlinear power flow equations, and the subsequent formulation of the reduced gradients $df/d\tilde{u}$.

Previous work based on Tellegen's theorem approximates the a.c. power model to permit direct application of the theorem. These approximations have been successively improved from the use of the d.c. load flow model [4] to the use of an improved a.c. approximate model [5]. The use of the exact a.c. power model, which leads to gradients free from errors, has been encountered by the relatively difficult modelling of power network components in terms of the current and voltage variables employed in the Tellegen's theorem formulation.

In this paper, we employ a novel concept of generalized, perturbed complex branch modelling together with a pertinent adjoint technique of derivation to attain general sensitivity formulas based upon the exact a.c. power model without any approximation. In Section II we introduce the notation and illustrate the concept of generalized complex branch modelling. The subsequent technique of adjoint network simulation and analysis is outlined in Section III. In Section IV we present the applications to an important special version of the approach which employs a simple and efficient adjoint analysis. Numerical results are presented in Section V on 6-bus and 26-bus systems.

II. POWER NETWORK STEADY-STATE ELEMENT MODELS

We denote by n and n_B, respectively, the number of buses (nodes) and the number of branches in the network. We shall use $b = 1,2,\ldots, n_B$ to denote a branch index. In general, we denote by ζ_b a complex variable associated with branch b, ζ may represent voltage V, current I, power S, admittance Y, transformer tap ratio a, etc. The complex conjugate of ζ is written as ζ^* and δ will be used to denote the first-order change.

Classification of Branches

The branches of the network are classified into two main types, namely the bus-type branches denoted by $b = m = 1,2,\ldots,n$ and the line-type branches for which $b=t=n+1,\ldots,n_B$. For the power networks, we may further classify the bus-type branches so that $m = \ell = 1,2,\ldots,n_L$ identify load branches associated with P, Q-type buses (Fig. 1a) for which the complex power $S_\ell = P_\ell + jQ_\ell$ is to be specified, $g = n_L + 1, \ldots, n_L + n_G$ identify generator branches associated with P, V-type buses (Fig. 1b) for which the real (active) power P_g and the voltage magnitude $|V_g|$ are to be specified and $n = n_L + n_G + 1$ identifies the slack generator branch for which the bus voltage is to be specified. The line-type branches, on the other hand, may contain the ordinary passive elements (Fig. 1c) of equivalent π-networks representing, for example, the transmission lines and transformers with real turns ratio as well as the elements of equivalent π-networks, derived [8] using the general branch modelling of Fig. 2, for the transformers with complex turns ratio (phase shifting transformers). Note [9] that the phase shifting transformers cannot be modelled by an equivalent π-network using the ordinary passive elements of Fig. 1c.

Complex Perturbed Form of Branch Models

In general, we deal with branch models of the form

$$h_b(I_b, I_b^*, V_b, V_b^*, U_b, U_b^*) = 0, \tag{1}$$

where U_b denotes an independent (control) variable to be specified for branch b. In terms of variations in the complex current and voltage variables and their complex conjugate, we write (1) in the perturbed form

$$h_{bi}\ \delta I_b + \bar{h}_{bi}\ \delta I_b^* = h_{bv}\ \delta V_b + \bar{h}_{bv}\ \delta V_b^* + W_b^S, \tag{2}$$

where the coefficients h_{bi}, $\bar{h}_{bi}$, h_{bv} and $\bar{h}_{bv}$ represent the formal [8] partial derivatives of h_b w.r.t. I_b, I_b^*, V_b and V_b^*, respectively. These formal derivatives may be evaluated using the ordinary differentiation rules. Moreover, it can be shown [8] that, for real h_b, we have $\bar{h}_{bi} = h_{bi}^*$ and $\bar{h}_{bv} = h_{bv}^*$. Observe that the perturbed branch models of Figs. 1a–c are special cases of the general form (2). Note also that in the perturbed branch models of typical electronic circuits we may exclude the coefficients $\bar{h}_{bi}$ and $\bar{h}_{bv}$ of the conjugate variables (e.g., the constant voltage sources are modelled by $h_{bi} = \bar{h}_{bi} = \bar{h}_{bv} = 0$ and $h_{bv} = 1$).

In this paper we manage to handle, directly, the generalized perturbed form (2) and to use the proper theory and techniques of analysis to avoid any approximation. The adjoint network simulation in the context of Tellegen's theorem, hence, represents the basis of a suitable approach to follow where the theory itself is expressed in terms of the fundamental network branch models.

III. THE ADJOINT SIMULATION OF POWER NETWORKS

The Augmented Forms of Tellegen's Theorem

Tellegen's theorem, which depends solely upon Kirchhoff's laws and the topology of the network, states that

$$\sum_b \hat{I}_b V_b = 0 \text{ and } \sum_b \hat{V}_b I_b = 0, \tag{3}$$

where the summation is taken over all branches, the ^ distinguishing the variables associated with the topologically similar adjoint network.

Since the V_b and $\hat{V}_b$ of (3) satisfy Kirchhoff's voltage law (KVL), the V_b^* and $\hat{V}_b^*$ also satisfy KVL. Similarly, since the I_b and $\hat{I}_b$ satisfy Kirchhoff's current law (KCL), the I_b^* and $\hat{I}_b^*$ also satisfy KCL. Hence, including (3), we may consider some [10] or all [11] of the exhaustive valid perturbed forms

$$\sum_b \hat{I}_b \,\delta V_b = 0,\quad \sum_b \hat{V}_b \,\delta I_b = 0,\quad \sum_b \hat{I}_b^* \,\delta V_b^* = 0,\quad \sum_b \hat{V}_b^* \,\delta I_b^* = 0,$$

$$\sum_b \hat{I}_b \,\delta V_b^* = 0,\quad \sum_b \hat{V}_b \,\delta I_b^* = 0,\quad \sum_b \hat{I}_b^* \,\delta V_b = 0 \text{ and } \sum_b \hat{V}_b^* \,\delta I_b = 0,$$

where we have perturbed only the variables of the original power network. The considered terms are then added, subtracted or augmented via arbitrary complex coefficients [6] together or to other valid expressions [8] to formulate an augmented Tellegen sum of the form

$$\sum_b \hat{\underset{\sim}{f}}^T \,\delta \underset{\sim}{w}_b = 0, \tag{4}$$

where T denotes transposition,

$$\underset{\sim}{w}_b = \begin{bmatrix} \underset{\sim}{w}_{bv} \\ --- \\ \underset{\sim}{w}_{bi} \end{bmatrix} \triangleq \begin{bmatrix} V_b \\ V_b^* \\ -- \\ I_b \\ I_b^* \end{bmatrix} \tag{5}$$

and

$$\hat{\underset{\sim}{f}}_b = \begin{bmatrix} \hat{\underset{\sim}{f}}_{bi} \\ \hat{\underset{\sim}{f}}_{bv} \end{bmatrix} \tag{6}$$

is a complex vector the elements of which are, in general, linear functions of the adjoint current and voltage variables and their complex conjugate, $\hat{\underset{\sim}{f}}_{bi}$ and $\hat{\underset{\sim}{f}}_{bv}$ being 2-component vectors.

Standard Branch Jacobian Matrices

The augmented Tellegen sum (4) has been written in terms of variations in V_b, V_b^*, I_b and I_b^*. We shall call these variables the basic variables since the theory is expressed in terms of them. Now, for each branch, and according to its type, another set of variables called the element variables is of practical interest. The element variables will be denoted by the vector $\underset{\sim}{z}_b$ of four components describing the practical state $\underset{\sim}{x}_b$ and control $\underset{\sim}{u}_b$ variables associated with branch b,

$$\underset{\sim}{z}_b = \begin{bmatrix} \underset{\sim}{x}_b \\ \underset{\sim}{u}_b \end{bmatrix} , \tag{7}$$

$\underset{\sim}{x}_b$ and $\underset{\sim}{u}_b$ being 2-component real and/or complex vectors.

The element variables associated with a given branch can be chosen appropriately in different ways [8]. Table I illustrates the conventional selection of the element variables for different power network branch types where real variables are considered. It should be noticed that other element variables can be defined for a particular branch [6] which may constitute complex variables. For example, we may define $\underset{\sim}{u}_\ell = [S_\ell \; S_\ell^*]^T$, $\underset{\sim}{x}_t = [I_t \; I_t^*]^T$, etc.

We relate the variations of the element variables $\underset{\sim}{z}_b$ to those of the basic variables $\underset{\sim}{w}_b$ by

$$\delta \underset{\sim}{z}_b = \underset{\sim}{J}_b \, \delta \underset{\sim}{w}_b, \tag{8}$$

where $\underset{\sim}{J}_b = (\partial \underset{\sim}{z}_b^T / \partial \underset{\sim}{w}_b)^T$ is a transformation matrix containing the conventional and/or formal derivatives of $\underset{\sim}{z}_b$ w.r.t. $\underset{\sim}{w}_b$. The inverse transpose of $\underset{\sim}{J}_b$ is of major interest in our derivations and we shall denote it by the partitioned form

$$(\underset{\sim}{J}_b^{-1})^T = \begin{bmatrix} \underset{\sim}{M}_{11}^b & \underset{\sim}{M}_{12}^b \\ \underset{\sim}{M}_{21}^b & \underset{\sim}{M}_{22}^b \end{bmatrix}, \tag{9}$$

where the submatrices $\underset{\sim}{M}_{11}^b$, $\underset{\sim}{M}_{12}^b$, $\underset{\sim}{M}_{21}^b$ and $\underset{\sim}{M}_{22}^b$ are 2x2 Jacobian matrices which are standard for a branch type of a network. The branch Jacobian matrices for different branch types of power networks are shown in Table I.

We remark that other branch types can be modelled, by defining appropriate element variables according to the physical nature of the corresponding element, and analyzed in a similar straightforward way [8]. In this respect, we also remark that the same argument applies equally well to typical electronic circuits liable to be modelled in an analogous way. Recalling the interesting discussion of [5] with regard to the modelling difficulties imposed by the source elements of power networks in comparison with those of typical electronic circuits, we observe that the element variables for the constant voltage and constant current sources constitute directly the basic variables $\underset{\sim}{w}_b$. Hence, the corresponding branch Jacobian matrices of Table I are simply $\underset{\sim}{0}$ or $\underset{\sim}{1}$ (the identity matrix).

Transformed Adjoint Variables and Network Sensitivities

Let

$$\hat{\underset{\sim}{\eta}}_{bx} = \underset{\sim}{M}^b_{11} \hat{\underset{\sim}{f}}_{bi} + \underset{\sim}{M}^b_{12} \hat{\underset{\sim}{f}}_{bv} \tag{10}$$

and

$$\hat{\underset{\sim}{\eta}}_{bu} = \underset{\sim}{M}^b_{21} \hat{\underset{\sim}{f}}_{bi} + \underset{\sim}{M}^b_{22} \hat{\underset{\sim}{f}}_{bv} \tag{11}$$

be transformed adjoint variables associated with the bth branch, where $\hat{\underset{\sim}{\eta}}_{bx}$ and $\hat{\underset{\sim}{\eta}}_{bu}$ are 2-component vectors, the elements of which are linear functions of the adjoint current and voltage variables and their complex conjugate. Hence, using (6)-(9), the augmented Tellegen sum (4) is written in terms of variations in the element variables as

$$\sum_b (\hat{\underset{\sim}{\eta}}^T_{bx} \delta \underset{\sim}{x}_b + \hat{\underset{\sim}{\eta}}^T_{bu} \delta \underset{\sim}{u}_b) = 0. \tag{12}$$

The first-order change of a general real or complex function f of all the state vectors $\underset{\sim}{x}_b$ and the control vectors $\underset{\sim}{u}_b$ is expressed in the form

$$\delta f = \sum_b [(\frac{\partial f}{\partial \underset{\sim}{x}_b})^T \delta \underset{\sim}{x}_b + (\frac{\partial f}{\partial \underset{\sim}{u}_b})^T \delta \underset{\sim}{u}_b]. \tag{13}$$

Assuming a possible consistent modelling [12] of the adjoint system, we set

$$\hat{\underset{\sim}{\eta}}_{bx} = \frac{\partial f}{\partial \underset{\sim}{x}_b} , \tag{14}$$

hence, from (12) and (13)

$$\delta f = \sum_b \left[\left(\frac{\partial f}{\partial \underset{\sim}{u}_b} \right) - \hat{\underset{\sim}{\eta}}_{bu}^T \right] \delta \underset{\sim}{u}_b , \qquad (15)$$

which expresses the first-order change of f solely in terms of variations in the control variables so that the total derivatives (the reduced gradients) of f are obtained as

$$\frac{df}{d\underset{\sim}{u}_b} = \frac{\partial f}{\partial \underset{\sim}{u}_b} - \hat{\underset{\sim}{\eta}}_{bu} . \qquad (16)$$

Consistent Modelling of Adjoint System

The adjoint network is defined, for a given function, by (14) which in general requires two complex relationships to be satisfied for each branch. The argument of the consistent modelling of the adjoint system is directly related to the possibility of satisfying these two relationships simultaneously which, in turn, depends [12] on the form and the mode (i.e., real or complex) of the function f as well as on the form of the augmented Tellegen sum considered in the analysis. In this respect, we shall only mention two examples and, for more analytical and theoretical details, the reader is referred to [8]. The first example is the case of typical electronic circuits where f may represent, e.g., a complex node voltage and the Tellegen sum is formulated as a difference form of (3). The second example is the case of a power network represented as in Table I, f is a general real function and the augmented Tellegen sum used is of the real form

$$\sum_b (\hat{I}_b V_b + \hat{I}_b^* V_b^* - \hat{V}_b I_b - \hat{V}_b^* I_b^*) = 0. \qquad (17)$$

In the above two examples, the relation (14) is satisfied. The form (17) will be used in Section IV to derive exact sensitivity formulas for a general real function.

The Adjoint Analysis

For a given function f, the adjoint network branches are modelled by (14), where, as shown before, the elements of $\hat{\underset{\sim}{n}}_{bx}$ are linear functions of $\hat{V}_b$, $\hat{V}_b^*$, $\hat{I}_b$ and $\hat{I}_b^*$. This adjoint modelling defines the first set of relationships which the adjoint currents and voltages must satisfy. The second set of relationships is simply the Kirchhoff's current and voltage laws implied in the Tellegen's theorem formulation. The application of Kirchhoff's laws results in a set of adjoint linear equations to be solved for the unknown adjoint current and voltage variables. A generalized form of the adjoint network equations which is common to all forms of augmented Tellegen sum and for general complex functions has been derived [8]. The solution of the adjoint system is then substituted in (15) and (16) to obtain the first-order change and the total derivatives of f. Note that in the case of complex control variables $\underset{\sim}{u}_b$ the formula (16) represents the formal total derivatives which are related to the conventional total derivatives w.r.t. real and imaginary parts of $\underset{\sim}{u}_b$ by simple relations [8].

IV. AN IMPORTANT SPECIAL VERSION

In this section, we consider the adjoint network simulation based on the special augmented Tellegen sum of the form (17) which provides a consistent adjoint network modelling for all real functions. We shall consider the set of element variables of Table I and state the specific structure of the adjoint equations to be solved. Hence, our exact formulas for first-order change and reduced gradients may be directly implemented and programmed.

The Adjoint Equations

For the augmented Tellegen sum (17) the vectors $\hat{\underset{\sim}{f}}_{bi}$ and $\hat{\underset{\sim}{f}}_{bv}$ of (6) are simply given by

$$\hat{\underset{\sim}{f}}_{bi} = \begin{bmatrix} \hat{I}_b \\ \hat{I}_b^* \end{bmatrix} \quad \text{and} \quad \hat{\underset{\sim}{f}}_{bv} = - \begin{bmatrix} \hat{V}_b \\ \hat{V}_b^* \end{bmatrix} . \tag{18}$$

Using the expressions of Table I, the adjoint branch modelling is given, from (10) and (14), for a load branch by

$$\hat{I}_\ell = -(S_\ell/V_\ell^2)\hat{V}_\ell^* + [|V_\ell| \frac{\partial f}{\partial |V_\ell|} - j \frac{\partial f}{\partial \delta_\ell}]/(2V_\ell), \tag{19}$$

for a generator branch by

$$V_g \hat{I}_g - V_g^* \hat{I}_g^* = (S_g^*/V_g^*) \hat{V}_g - (S_g/V_g) \hat{V}_g^* - j \frac{\partial f}{\partial \delta_g} \tag{20a}$$

and

$$V_g \hat{V}_g - V_g^* \hat{V}_g^* = - j V_g V_g^* \frac{\partial f}{\partial Q_g} , \tag{20b}$$

for the slack generator branch by

$$\hat{V}_n = -V_n^* (\frac{\partial f}{\partial P_n} + j \frac{\partial f}{\partial Q_n})/2 \tag{21}$$

and for a transmission branch by

$$\hat{I}_t = Y_t \hat{V}_t + Y_t [\frac{\partial f}{\partial Re\{I_t\}} - j \frac{\partial f}{\partial Im\{I_t\}}]/2. \tag{22}$$

Observe that the adjoint branch models are also of the general form (1) in terms of the adjoint currents and voltages and their complex conjugate. The application of KCL and KVL to the adjoint network of the branch models (19)-(22) results in [7] the following real structure of the adjoint equations.

$$\begin{bmatrix} \hat{\underset{\sim}{G}} & \hat{\underset{\sim}{B}} \\ \hat{\underset{\sim}{B}}' & \hat{\underset{\sim}{G}}' \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{V}}_1 \\ \hat{\underset{\sim}{V}}_2 \end{bmatrix} = \begin{bmatrix} \hat{\underset{\sim}{I}}_1 \\ \hat{\underset{\sim}{I}}_2 \end{bmatrix}, \tag{23}$$

where subscripts 1 and 2 denote, respectively, the real and imaginary parts of complex quantities, $\hat{\underset{\sim}{V}}$ $(= \hat{\underset{\sim}{V}}_1 + j\hat{\underset{\sim}{V}}_2)$ is a vector of n-1 components representing the unknown adjoint load and generator bus voltages and $\hat{\underset{\sim}{I}}$ is a corresponding RHS vector,

$$\hat{\underset{\sim}{V}} = \begin{bmatrix} \hat{\underset{\sim}{V}}_L \\ \hat{\underset{\sim}{V}}_G \end{bmatrix} \quad \text{and} \quad \hat{\underset{\sim}{I}} = \begin{bmatrix} \hat{\underset{\sim}{I}}_L \\ \hat{\underset{\sim}{I}}_G \end{bmatrix}, \tag{24}$$

L and G denoting, respectively load and generator buses. The elements $\hat{\underset{\sim}{I}}_\ell$ and $\hat{\underset{\sim}{I}}_g$ of the vectors $\hat{\underset{\sim}{I}}_L$ and $\hat{\underset{\sim}{I}}_G$ are given by the formulas

$$\begin{aligned} \hat{I}_\ell \triangleq & - \frac{1}{2V_\ell} (|V_\ell| \frac{\partial f}{\partial |V_\ell|} - j \frac{\partial f}{\partial \delta_\ell}) \\ & + \frac{1}{2} y_{\ell n} V_n^* (\frac{\partial f}{\partial P_n} + j \frac{\partial f}{\partial Q_n}) \\ & - \frac{1}{2} \sum_t [\lambda_{\ell t} Y_t (\frac{\partial f}{\partial I_{t1}} - j \frac{\partial f}{\partial I_{t2}})] \end{aligned} \tag{25a}$$

and

$$\begin{aligned} \hat{I}_g \triangleq & \frac{\partial f}{\partial \delta_g} - j |V_g|^2 \frac{\partial f}{\partial Q_g} \\ & - \mathrm{Im}\{V_g \sum_t [\lambda_{gt} Y_t (\frac{\partial f}{\partial I_{t1}} - j \frac{\partial f}{\partial I_{t2}})]\} \\ & + \mathrm{Im}\{V_g y_{gn} V_n^* (\frac{\partial f}{\partial P_n} + j \frac{\partial f}{\partial Q_n})\}, \end{aligned} \tag{25b}$$

where λ_{mt} denote elements of the bus incidence matrix of the network and y_{mm}, m = ℓ or g, are elements of the symmetric bus admittance matrix. The submatrices in (23) are given by

$$\hat{\underset{\sim}{G}} + j\,\hat{\underset{\sim}{B}} = \begin{bmatrix} (\underset{\sim}{Y}_{LL}^{*} + \underset{\sim}{\Psi}_{L}) & \underset{\sim}{Y}_{LG}^{*} \\ \underset{\sim}{\overline{Y}}_{GL}^{*} & (\underset{\sim}{\overline{Y}}_{GG}^{*} + \underset{\sim}{\Psi}_{G}) \end{bmatrix} \qquad (26a)$$

and

$$\hat{\underset{\sim}{G}}` + j\,\hat{\underset{\sim}{B}}` = \begin{bmatrix} (\underset{\sim}{Y}_{LL} - \underset{\sim}{\Psi}_{L}^{*}) & \underset{\sim}{Y}_{LG} \\ \underset{\sim}{0} & 2\,\mathrm{diag}\{V_g\} \end{bmatrix}, \qquad (26b)$$

where the bus admittance matrix $\underset{\sim}{Y}$, excluding the column and row corresponding to the slack bus, has been partitioned in the form

$$\underset{\sim}{Y} = \underset{\sim}{G} + j\,\underset{\sim}{B} = \begin{bmatrix} \underset{\sim}{Y}_{LL} & \underset{\sim}{Y}_{LG} \\ \underset{\sim}{Y}_{GL} & \underset{\sim}{Y}_{GG} \end{bmatrix}, \qquad (27)$$

$$[\underset{\sim}{\overline{Y}}_{GL}\ \underset{\sim}{\overline{Y}}_{GG}] \overset{\Delta}{=} -j2\,\mathrm{diag}\{V_g\}[\underset{\sim}{Y}_{GL}\ \underset{\sim}{Y}_{GG}], \qquad (28)$$

$$\underset{\sim}{\Psi}_L \overset{\Delta}{=} -\mathrm{diag}\{S_\ell/V_\ell^2\} \text{ and } \underset{\sim}{\Psi}_G \overset{\Delta}{=} j2\,\mathrm{diag}\{S_g/V_g\}. \qquad (29)$$

In practice, the 2n-2 real adjoint equations (23) are to be solved for the adjoint bus voltages $\hat{\underset{\sim}{V}}$. Knowing $\hat{\underset{\sim}{V}}$, the adjoint branch currents and voltages, which constitute the vectors $\hat{\underset{\sim}{f}}_{bi}$ and $\hat{\underset{\sim}{f}}_{bv}$ of (18), are easily obtained. Using the standard expressions of Table I, the vector $\hat{\underset{\sim}{\eta}}_{bu}$ of (11) is evaluated and, then, substituted in (15) and (16).

Remarks

We remark on the simplicity of formulation and efficiency of computations involved in the adjoint analysis using the form (23) of the adjoint equations and leading to exact formulas for first-order change and reduced gradients of general real functions. The adjoint matrix of coefficients of (23) is at least as sparse as the bus admittance matrix of the power network. It is simple, mostly constant, the majority of its elements are line conductances and susceptances representing basic data of the problem available and already stored in the computer memory. Moreover, it is independent of the function f which is represented only in the RHS of the adjoint equations. Hence, several functions can be handled by repeat forward and backward substitutions using the LU factors of the adjoint matrix at a base-case point. These functions, whether expressed in terms of bus or line variables, are treated directly without transformations (the RHS of (23) contains partial derivatives w.r.t. all network state variables).

V. NUMERICAL RESULTS

We present some numerical results to illustrate the use of the exact sensitivity formulas derived. So as not to be restricted to any particular application, we consider two systems demonstrating first-order change and gradient evaluation for some of the network bus and line states. The results presented are exact and have been verified by small perturbations about the base-case point.

6-Bus System

A 6-bus sample power system [7] shown in Fig. 3 is considered. The required data is given by Table II. Powers injected into buses are shown. The corresponding a.c. load flow solution is shown in Table III. Table IV shows the adjoint matrix of coefficients at the load flow solution. This matrix is common to all the sensitivity calculations performed at the base-case point.

We first consider the derivatives of the load bus state δ_3 and the generator bus state Q_5. Table V shows the RHS vector of the adjoint equations for both states and the adjoint voltages resulting from the solution of (23). Table VI shows the derivatives calculated using our formulas.

We investigate the effect of line removals by considering line current loading functions of the form

$$f = |I_t|^2.$$

The control variables associated with generator and load buses are maintained at their base-case values. The results of different contingencies based on first-order estimation are shown in Table VII which also shows, for the purpose of comparison, the exact function changes.

Other numerical results for the same system have been presented in [7,8].

26-Bus System

This power system (Saskatchewan Power Corporation System) has been considered [13-15] in some relevant studies on steady-state power system analysis.

The single line diagram of this system is shown in Fig. 4. The line data is shown by Table VIII. The operating bus data and transformer taps considered in the load flow analysis are shown, respectively, in Tables IX and X (n_L = 17, n_G = 8, n_T = 32). All values shown are in per unit. The a.c. load flow solution obtained is shown by Table XI.

We present here the derivatives of the load bus state $|V_6|$ shown in Table XII, the load bus state δ_4 shown in Table XIII, the generator bus state Q_{20} shown in Table XIV and the generator bus state δ_{20} shown in

Table XV, as calculated by our formulas and verified by perturbation.

Finally, we list in Table XVI the effects of certain circuit removals on the states $|V_6|$, δ_4, Q_{20} and δ_{20}. Control variables associated with generator and load buses are maintained at their base-case values. Exact changes as calculated by new load flow solutions are compared with those predicted by first-order estimates. Two cases are considered: an important line 2,13 and a normal line 6,7.

VI. CONCLUSIONS

Instead of approximating the a.c. power flow model to cope with the conventional form and technique of analysis of Tellegen's theorem, we have employed a suitable augmented form of the theorem applicable to the generalized complex branch models of power networks. The proper adjoint network technique followed has led to simple derivation and elegant formulation of exact sensitivity formulas based on the a.c. power model without any approximation. Moreover, it offers the flexibility of working with any set of real and/or complex state and control variables of practical interest. The important special version described in Section IV provides exact sensitivity formulas for general real functions while employing a simple and efficient adjoint analysis. The work presented in this paper claims general concepts in network modelling and analysis applicable to systems of general complex branch models.

REFERENCES

[1] H.W. Dommel and W.F. Tinney, "Optimal power flow solutions", IEEE Trans. Power Apparatus and Systems, vol. PAS-87, 1968, pp. 1866-1876.

[2] S.W. Director and R.L. Sullivan, "A tableau approach to power system analysis and design", Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978), pp. 605-609.

[3] B.D.H. Tellegen, "A general network theorem, with applications", Philips Res. Repts, 7, 1952, pp. 259-269.

[4] R. Fischl and W.R. Puntel, "Computer aided design of electric power transmission networks", IEEE Winter Power Meeting, Paper No. C72-168-8, 1972.

[5] H.B. Püttgen and R.L. Sullivan, "A novel comprehensive approach to power systems sensitivity analysis", IEEE Summer Power Meeting, Paper No. A78-525-8, 1978.

[6] J.W. Bandler and M.A. El-Kady, "A unified approach to power system sensitivity analysis and planning, Part I: family of adjoint systems", Proc. IEEE Int. Symp. Circuits and Systems (Houston, TX, 1980), pp. 681-687.

[7] J.W. Bandler and M.A. El-Kady, "A unified approach to power system sensitivity analysis and planning, Part II: special class of adjoint systems", Proc. IEEE Int. Symp. Circuits and Systems (Houston, TX, 1980), pp. 688-692.

[8] M.A. El-Kady, "A unified approach to generalized network sensitivities with applications to power system analysis and planning", Ph.D. Thesis, McMaster University, Hamilton, Canada, 1980.

[9] R.L. Sullivan, Power System Planning. New York: McGraw-Hill, 1977.

[10] S.W. Director and R.A. Rohrer, "Generalized adjoint network and network sensitivities", IEEE Trans. Circuit Theory, vol. CT-16, 1969, pp. 318-323.

[11] P. Penfield, Jr., R. Spence and S. Duinker, Tellegen's Theorem and Electrical Networks. Cambridge, MA: M.I.T. Press, 1970.

[12] J.W. Bandler and M.A. El-Kady, "A unified approach to power system sensitivity analysis and planning, Part III: consistent selection of adjoining coefficients", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-241, 1980.

[13] J.W. Bandler and M.A. El-Kady, "A new method for computerized solution of power flow equations", 12th Power Industry Computer Application Conf. (Philadelphia, PA, 1981), pp. 27-35.

[14] M.S. Sachdev and S.A. Ibrahim, "A fast approximate technique for outage studies in power system planning and operation", IEEE Trans. Power Apparatus and Systems, vol. PAS-93, 1974, pp. 1133-1142.

[15] J.W. Bandler, M.A. El-Kady and H. Gupta, "Practical complex solution of power flow equations", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-270, 1981.

TABLE I

A REPRESENTATION OF A POWER SYSTEM VIA ELEMENT VARIABLES

Branch Type	Element Variables: b	Element Variables: $\underset{\sim}{x}_b$	Element Variables: $\underset{\sim}{u}_b$	Branch Jacobian Matrices: $\underset{\sim}{M}^b_{11}$	Branch Jacobian Matrices: $\underset{\sim}{M}^b_{12}$	Branch Jacobian Matrices: $\underset{\sim}{M}^b_{21}$	Branch Jacobian Matrices: $\underset{\sim}{M}^b_{22}$
Load	ℓ	$\begin{bmatrix} \lvert V_\ell \rvert \\ \delta_\ell \end{bmatrix}$	$\begin{bmatrix} P_\ell \\ Q_\ell \end{bmatrix}$	$\begin{bmatrix} V_\ell/\lvert V_\ell\rvert & V^*_\ell/\lvert V_\ell\rvert \\ jV_\ell & -jV^*_\ell \end{bmatrix}$	$\begin{bmatrix} -I_\ell/\lvert V_\ell\rvert & -I^*_\ell/\lvert V_\ell\rvert \\ jI_\ell & -jI^*_\ell \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1/V^*_\ell & 1/V_\ell \\ -j/V^*_\ell & j/V_\ell \end{bmatrix}$
Generator	g	$\begin{bmatrix} \delta_g \\ Q_g \end{bmatrix}$	$\begin{bmatrix} \lvert V_g \rvert \\ P_g \end{bmatrix}$	$\begin{bmatrix} jV_g & -jV^*_g \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} jI_g & -jI^*_g \\ -j/V^*_g & j/V_g \end{bmatrix}$	$\begin{bmatrix} V_g/\lvert V_g\rvert & V^*_g/\lvert V_g\rvert \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} -I_g/\lvert V_g\rvert & -I^*_g/\lvert V_g\rvert \\ 1/V^*_g & 1/V_g \end{bmatrix}$
Slack Generator	n	$\begin{bmatrix} P_n \\ Q_n \end{bmatrix}$	$\begin{bmatrix} \lvert V_n \rvert \\ \delta_n \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1/V^*_n & 1/V_n \\ -j/V^*_n & j/V_n \end{bmatrix}$	$\begin{bmatrix} V_n/\lvert V_n\rvert & V^*_n/\lvert V_n\rvert \\ jV_n & -jV^*_n \end{bmatrix}$	$\begin{bmatrix} -I_n/\lvert V_n\rvert & -I^*_n/\lvert V_n\rvert \\ jI_n & -jI^*_n \end{bmatrix}$
Line	t	$\begin{bmatrix} I_{t1} \\ I_{t2} \end{bmatrix}$	$\begin{bmatrix} G_t \\ B_t \end{bmatrix}$	$\begin{bmatrix} 1/Y_t & 1/Y^*_t \\ j/Y_t & -j/Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$	$\begin{bmatrix} -V_t/Y_t & -V^*_t/Y^*_t \\ -jV_t/Y_t & jV^*_t/Y^*_t \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$S_m = P_m + jQ_m$, $V_m = \lvert V_m \rvert \underline{/\ \delta_m}$; m can be ℓ, g or n

$I_t = I_{t1} + jI_{t2}$, $Y_t = G_t + jB_t$

TABLE IIa

BUS DATA FOR 6-BUS POWER SYSTEM

Bus Index, i	Bus Type	P_i (pu)	Q_i (pu)	$\lvert V_i \rvert \angle \delta_i$ (pu)
1	load	-2.40	0	- $\angle$-
2	load	-2.40	0	- $\angle$-
3	load	-1.60	-0.40	- $\angle$-
4	generator	-0.30	-	1.02 $\angle$-
5	generator	1.25	-	1.04 $\angle$-
6	slack	-	-	1.04 $\angle$-

TABLE IIb

LINE DATA FOR 6-BUS POWER SYSTEM

Branch Index, t	Terminal Buses	Resistance R_t (pu)	Reactance X_t (pu)	Number of Lines
7	1,4	0.05	0.20	1
8	1,5	0.025	0.10	2
9	2,3	0.10	0.40	1
10	2,4	0.10	0.40	1
11	2,5	0.05	0.20	1
12	2,6	0.01875	0.075	4
13	3,4	0.15	0.60	1
14	3,6	0.0375	0.15	2

TABLE III

LOAD FLOW SOLUTION OF 6-BUS POWER SYSTEM

Load Buses		
	$\|V_1\| = 0.9787$	$\delta_1 = -0.6602$
	$\|V_2\| = 0.9633$	$\delta_2 = -0.2978$
	$\|V_3\| = 0.9032$	$\delta_3 = -0.3036$
Generator Buses		
	$Q_4 = 0.7866$	$\delta_4 = -0.5566$
	$Q_5 = 0.9780$	$\delta_5 = -0.4740$
Slack Bus		
	$P_6 = 6.1298$	$Q_6 = 1.3546$

TABLE IV

ADJOINT MATRIX OF COEFFICIENTS

2.9085	0	0	- 1.1765	- 2.3529	11.6900	0	0	-4.7059	-9.4118
0	3.3490	- 0.5882	- 0.5882	- 1.1765	0	20.5097	-2.3529	-2.3529	-4.7059
0	- 0.5882	1.2179	- 0.3922	0	0	- 2.3529	8.6744	-1.5686	0
9.4189	4.7095	3.1396	-16.2693	0	3.0338	1.5169	1.0113	-4.2477	0
19.6518	9.8259	0	0	-26.7071	4.5821	2.2910	0	0	-8.1534
-16.5453	0	0	4.7059	9.4118	4.1503	0	0	-1.1765	-2.3529
0	-23.4119	2.3529	2.3529	4.7059	0	7.6314	-0.5882	-0.5882	-1.1765
0	2.3529	-11.7178	1.5686	0	0	- 0.5882	3.8802	-0.3922	0
0	0	0	- 1.0777	0	0	0	0	1.7321	0
0	0	0	0	- 0.9495	0	0	0	0	1.8506

TABLE V

RHS AND SOLUTION VECTORS OF THE ADJOINT NETWORKS FOR δ_3 AND Q_5

Element No.	$f = \delta_3$		$f = Q_5$	
	RHS Vector	Solution Vector	RHS Vector	Solution Vector
1	0	-0.0223	0	0.0600
2	0	-0.0142	0	0.0275
3	-0.1655	-0.0570	0	0.0328
4	0	-0.0287	0	0.1354
5	0	-0.0220	0	-0.0240
6	0	-0.0180	0	0.4884
7	0	-0.0082	0	0.1466
8	0.5283	-0.0188	0	0.0548
9	0	-0.0178	0	0.0843
10	0	-0.0113	1.0816	0.5721

TABLE VI

SENSITIVITIES OF δ_3 AND Q_5 W.R.T CONTROL VARIABLES

Line Quantities

Line	Derivatives w.r.t. G_t		Derivatives w.r.t. B_t	
	$f = \delta_3$	$f = Q_5$	$f = \delta_3$	$f = Q_5$
1,4	-0.001205	0.063610	0.000743	0.065953
1,5	-0.001595	-0.046596	-0.002133	-0.004421
2,3	0.005312	0.001612	-0.000166	-0.010535
2,4	-0.003359	-0.043764	-0.008465	0.048458
2,5	-0.001260	0.163046	-0.002965	-0.023595
2,6	-0.001242	0.076305	-0.009986	0.050501
3,4	0.000744	0.014771	0.015220	0.054970
3,6	0.010158	0.019837	-0.037461	0.038517

Load Bus Quantities

Bus	Derivatives w.r.t. P_ℓ		Derivatives w.r.t. Q_ℓ	
	$f = \delta_3$	$f = Q_5$	$f = \delta_3$	$f = Q_5$
1	0.058622	-0.709070	0.001132	-0.713165
2	0.033200	-0.143975	0.007596	-0.274202
3	0.132854	-0.107990	0.001969	-0.101658

Generator Bus Quantities

Bus	Derivatives w.r.t $\|V_g\|$		Derivatives w.r.t. P_g	
	$f = \delta_3$	$f = Q_5$	$f = \delta_3$	$f = Q_5$
4	-0.008082	-4.51867	0.066205	-0.312777
5	0.056708	7.58088	0.047554	-0.461239

TABLE VII

CONTINGENCY RESULTS FOR 6-BUS POWER SYSTEM

Function Line Index	Removed Line Index	Calculated Function Change	Exact Function Change
1,4	2,4	-0.200	-0.224
2,3	1,5*	0.002	0.005
2,3	2,3	-0.029	-0.021
2,4	2,4	-0.470	-0.404

* Only one line of branch 1,5 is removed.

TABLE VIII

LINE DATA FOR 26-BUS POWER SYSTEM

Line	Terminal Buses	Resistance R_t (pu)	Reactance X_t (pu)	1/2 Shunt Susceptance
1	13,26	0.0	0.0131	0.0
2	26,16	0.0	0.0392	0.0
3	16,23	0.0	0.4320	0.0
4	23,26	0.0	0.3140	0.0
5	2,10	0.0	0.0150	0.0
6	9,10	0.1494	0.3392	0.4120
7	9,12	0.0658	0.1494	0.0182
8	12,26	0.0533	0.1210	0.0147
9	9,14	0.0618	0.2397	0.0319
10	11,14	0.0676	0.2620	0.0349
11	19,26	0.0610	0.2521	0.0295
12	6,26	0.0513	0.1986	0.0265
13	6,19	0.0129	0.0532	0.0074
14	7,19	0.0906	0.3742	0.0437
15	6,7	0.0921	0.3569	0.0475
16	11,22	0.0513	0.2118	0.0248
17	8,11	0.0865	0.3355	0.0447
18	17,22	0.0281	0.1869	0.0237
19	8,21	0.0735	0.2847	0.0379
20	17,21	0.0459	0.3055	0.0387
21	1,4	0.0619	0.2401	0.0319
22	4,21	0.0610	0.2365	0.0315
23	20,21	0.0	0.0305	0.0
24	15,1	0.0	0.0147	0.0
25	2,13	0.0086	0.0707	0.3017
26	1,7	0.0199	0.0785	0.0404
27	15,20	0.0107	0.0617	0.4471
28	2,18	0.0074	0.0608	0.2593
29	1,3	0.0	0.0392	0.0
30	24,3	0.0	0.1450	0.0
31	5,21	0.0	0.1750	0.0
32	5,25	0.0	0.1540	0.0

TABLE IX

BUS DATA FOR 26-BUS POWER SYSTEM

Bus	Injected Power		Bus Voltage	
	P_m	Q_m	$\|V_m\|$	δ_m
1	-0.82	-0.21	-	-
2	0.0	0.0	-	-
3	-0.57	-0.17	-	-
4	-0.48	-0.21	-	-
5	-0.43	-0.11	-	-
6	-0.40	-0.10	-	-
7	-1.11	-0.27	-	-
8	-0.23	-0.06	-	-
9	-0.67	-0.21	-	-
10	-1.02	-0.27	-	-
11	-0.43	-0.14	-	-
12	-0.43	-0.12	-	-
13	0.0	0.0	-	-
14	0.0	0.0	-	-
15	0.0	0.0	-	-
16	-1.31	-0.30	-	-
17	-0.03	-0.01	-	-
18	2.80	-	1.07	-
19	1.45	-	1.05	-
20	2.80	-	1.00	-
21	1.10	-	1.02	-
22	-0.56	-	0.89	-
23	-0.04	-	1.00	-
24	-0.05	-	1.00	-
25	0.63	-	1.00	-
26	0.0	-	1.01	0.0

TABLE X

TRANSFORMER TAPS FOR 26-BUS POWER SYSTEM

No.	Terminal Buses	Real	Imaginary
1	13,26	1.03	0.0
2	20,21	0.97	0.0
3	24,3	0.98	0.0
4	26,16	0.96	0.0
5	15,1	0.89	0.0
6	5,21	0.99	0.0
7	2,10	1.03	0.0
8	1,3	0.98	0.0
9	5,25	1.03	0.0

TABLE XI

LOAD FLOW SOLUTION OF 26-BUS POWER SYSTEM

Load Buses

$\|V_1\|$	= 1.0357	δ_1	= 0.0747
$\|V_2\|$	= 1.0685	δ_2	= 0.0884
$\|V_3\|$	= 1.0438	δ_3	= 0.0527
$\|V_4\|$	= 0.9908	δ_4	= 0.0989
$\|V_5\|$	= 1.0081	δ_5	= 0.2607
$\|V_6\|$	= 1.0339	δ_6	= 0.0536
$\|V_7\|$	= 1.0133	δ_7	= 0.0178
$\|V_8\|$	= 0.9450	δ_8	= 0.0426
$\|V_9\|$	= 0.9675	δ_9	= -0.1127
$\|V_{10}\|$	= 1.0393	δ_{10}	= 0.0667
$\|V_{11}\|$	= 0.9037	δ_{11}	= -0.1100
$\|V_{12}\|$	= 0.9699	δ_{12}	= -0.0764
$\|V_{13}\|$	= 1.0465	δ_{13}	= 0.0150
$\|V_{14}\|$	= 0.9449	δ_{14}	= -0.1136
$\|V_{15}\|$	= 0.9324	δ_{15}	= 0.1042
$\|V_{16}\|$	= 1.0363	δ_{16}	= -0.0455
$\|V_{17}\|$	= 0.9322	δ_{17}	= 0.0298

Generator Buses

Q_{18}	= -0.4004	δ_{18}	= 0.2385
Q_{19}	= 0.1872	δ_{19}	= 0.0921
Q_{20}	= 0.7795	δ_{20}	= 0.2432
Q_{21}	= -0.0294	δ_{21}	= 0.2270
Q_{22}	= -0.1775	δ_{22}	= -0.0996
Q_{23}	= -0.1144	δ_{23}	= -0.0266
Q_{24}	= -0.1645	δ_{24}	= 0.0459
Q_{25}	= 0.1691	δ_{25}	= 0.3599

Slack Bus

P_{26}	= 0.1334	Q_{26}	= -0.0513

TABLE XIIa

SENSITIVITIES OF $|V_6|$ W.R.T. LINE CONTROL VARIABLES

Line	Total Derivatives		Line	Total Derivatives	
	Conductance	Susceptance		Conductance	Susceptance
1,3	0.000016	-0.000008	8,21	0.000026	-0.000012
1,4	-0.000063	0.000071	9,10	-0.000007	0.000019
1,7	-0.000259	-0.000207	9,12	0	0.000002
1,15	-0.000040	0.000092	9,14	-0.000004	0
2,10	0	0	11,14	-0.000007	-0.000001
2,13	0	-0.000001	11,22	-0.000001	0
2,18	0.000001	0	12,26	-0.000002	0.000004
3,24	0.000020	0.000132	13,26	0	0
4,21	-0.000227	-0.000123	15,20	-0.003259	-0.000283
5,21	0.000001	0	16,23	0	0
5,25	0.000070	0	16,26	0	0
6,7	0.000938	0.000891	17,21	0.000033	0.000011
6,19	-0.001403	-0.001018	17,22	0.000012	-0.000004
6,26	0.001868	0.001477	19,26	0.000064	-0.000134
7,19	-0.000542	-0.000590	20,21	0	0
8,11	0.000018	-0.000014	23,26	0	0

TABLE XIIb

SENSITIVITIES OF $|V_6|$ W.R.T. LOAD BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Real Power	Reactive Power		Real Power	Reactive Power
1	0.000416	0.003612	10	-0.000023	-0.000005
2	-0.000019	-0.000003	11	-0.000516	0.000014
3	-0.000560	0.002894	12	-0.000060	-0.000001
4	-0.000053	0.001956	13	-0.000003	-0.000001
5	-0.000689	0	14	-0.000309	-0.000012
6	0.008807	0.037495	15	0.000224	0.003030
7	0.002370	0.008613	16	0	0
8	-0.000627	0.000021	17	-0.000633	0.000009
9	-0.000132	0.000007			

TABLE XIIc

SENSITIVITIES OF $|V_6|$ W.R.T. GENERATOR BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Voltage Magnitude	Real Power		Voltage Magnitude	Real Power
18	-0.000060	-0.000018	22	0.000124	-0.000594
19	0.760458	-0.001371	23	0	0
20	0.047441	-0.000697	24	0.021261	0.000579
21	0.008785	-0.000689	25	0	-0.000689

TABLE XIIIa

SENSITIVITIES OF δ_4 W.R.T. LINE CONTROL VARIABLES

Line	Total Derivatives		Line	Total Derivatives	
	Conductance	Susceptance		Conductance	Susceptance
1,3	-0.000192	0.000015	8,21	-0.010632	0.004918
1,4	0.004666	0.004063	9,10	0.002908	-0.007684
1,7	-0.002326	0.002995	9,12	0.000129	-0.001003
1,15	-0.002815	-0.000193	9,14	0.001515	-0.000023
2,10	0.000015	-0.000059	11,14	0.002930	0.000285
2,13	-0.000097	0.000564	11,22	0.000373	0.000195
2,18	-0.000413	-0.000093	12,26	0.000844	-0.001837
3,24	-0.000578	-0.000259	13,26	-0.000046	0.000030
4,21	0.000909	-0.015409	15,20	-0.005599	0.000341
5,21	-0.000368	0	16,23	0	0
5,25	-0.002813	0	16,26	0	0
6,7	0.002793	-0.005164	17,21	-0.013307	0.004458
6,19	-0.000126	0.000021	17,22	-0.004930	0.001574
6,26	-0.002185	0.004555	19,26	-0.003811	0.007994
7,19	0.004486	-0.010855	20,21	-0.000291	-0.000086
8,11	-0.007151	0.005629	23,26	0	0

TABLE XIIIb

SENSITIVITIES OF δ_4 W.R.T. LOAD BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Real Power	Reactive Power		Real Power	Reactive Power
1	0.027288	-0.007064	10	0.009323	0.002098
2	0.007825	0.001294	11	0.210780	-0.005619
3	0.272595	-0.005661	12	0.024345	0.000451
4	0.404849	-0.025155	13	0.001304	0.000253
5	0.281369	0	14	0.126244	-0.004725
6	0.081724	-0.000725	15	0.275659	-0.005067
7	0.221138	-0.004001	16	0	0
8	0.256180	-0.857789	17	0.258648	-0.003580
9	0.054047	-0.002763			

TABLE XIIIc

SENSITIVITIES OF δ_4 W.R.T. GENERATOR BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Voltage Magnitude	Real Power		Voltage Magnitude	Real Power
18	0.002431	0.007195	22	-0.050600	0.242404
19	-0.037700	0.081957	23	0	0
20	-0.186725	0.276178	24	-0.041581	0.272557
21	-0.092270	0.281369	25	0	0.281369

TABLE XIVa

SENSITIVITIES OF Q_{20} W.R.T. LINE CONTROL VARIABLES

Line	Total Derivatives		Line	Total Derivatives	
	Conductance	Susceptance		Conductance	Susceptance
1,3	-0.002376	0.001173	8,21	-0.002223	0.001028
1,4	0.010104	-0.010808	9,10	0.000608	-0.001607
1,7	-0.008648	-0.004200	9,12	0.000027	-0.000210
1,15	0.002555	0.002178	9,14	0.000317	-0.000005
2,10	0.000003	-0.000012	11,14	0.000613	0.000060
2,13	-0.000020	0.000118	11,22	0.000078	0.000041
2,18	-0.000086	-0.000019	12,26	0.000176	-0.000384
3,24	-0.003026	-0.020474	13,26	-0.000010	0.000006
4,21	0.035965	0.020076	15,20	-0.057363	-0.020671
5,21	-0.000077	0	16,23	0	0
5,25	-0.000588	0	16,26	0	0
6,7	0.011300	0.012125	17,21	-0.002783	0.000932
6,19	0.001619	0.001199	17,22	-0.001031	0.000329
6,26	-0.001991	-0.002051	19,26	0.000187	-0.000392
7,19	0.027808	0.025084	20,21	-0.016253	0.020141
8,11	-0.001495	0.001177	23,26	0	0

TABLE XIVb

SENSITIVITIES OF Q_{20} W.R.T. LOAD BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Real Power	Reactive Power		Real Power	Reactive Power
1	-0.125585	-0.559026	10	0.001950	0.000439
2	0.001636	0.000271	11	0.044075	-0.001175
3	-0.147778	-0.447971	12	0.005091	0.000094
4	-0.047999	-0.303855	13	0.000273	0.000053
5	0.058835	0	14	0.026398	-0.000988
6	-0.016361	-0.043250	15	-0.090397	-0.638373
7	-0.143159	-0.412030	16	0	0
8	0.053568	-0.001794	17	0.054084	-0.000749
9	0.011301	-0.000578			

TABLE XIVc

SENSITIVITIES OF Q_{20} W.R.T. GENERATOR BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Voltage Magnitude	Real Power		Voltage Magnitude	Real Power
18	0.005083	0.001504	22	-0.010581	0.050687
19	-1.198785	-0.004018	23	0	0
20	41.0672	0.075446	24	-1.329069	-0.150827
21	-35.1499	0.058835	25	0	0.058835

TABLE XVa

SENSITIVITIES OF δ_{20} W.R.T. LINE CONTROL VARIABLES

Line	Total Derivatives		Line	Total Derivatives	
	Conductance	Susceptance		Conductance	Susceptance
1,3	-0.000192	0.000016	8,21	-0.011203	0.005182
1,4	0.000448	0.000296	9,10	0.003064	-0.008097
1,7	-0.002295	0.002933	9,12	0.000136	-0.001057
1,15	-0.002164	-0.000038	9,14	0.001597	-0.000024
2,10	0.000016	-0.000062	11,14	0.003089	0.000301
2,13	-0.000102	0.000594	11,22	0.000393	0.000206
2,18	-0.000436	-0.000098	12,26	0.000889	-0.001936
3,24	-0.000572	-0.000283	13,26	-0.000048	0.000032
4,21	-0.005019	0.000829	15,20	-0.007868	0.004280
5,21	-0.000388	0	16,23	0	0
5,25	-0.002964	0	16,26	0	0
6,7	0.002757	-0.005050	17,21	-0.014022	0.004697
6,19	-0.000122	0.000023	17,22	-0.005194	0.001658
6,26	-0.002147	0.004467	19,26	-0.003739	0.007844
7,19	0.004441	-0.010616	20,21	0.000016	0.000179
8,11	-0.007536	0.005931	23,26	0	0

TABLE XVb

SENSITIVITIES OF δ_{20} W.R.T. LOAD BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Real Power	Reactive Power		Real Power	Reactive Power
1	0.267581	-0.007722	10	0.009824	0.002211
2	0.008245	0.001363	11	0.222100	-0.005921
3	0.267275	-0.006188	12	0.025652	0.000475
4	0.290383	-0.002045	13	0.001374	0.000267
5	0.296479	0	14	0.133023	-0.004979
6	0.080168	-0.000773	15	0.276482	-0.005600
7	0.216789	-0.004509	16	0	0
8	0.269938	-0.009039	17	0.272539	-0.003772
9	0.056949	-0.002912			

TABLE XVc

SENSITIVITIES OF δ_{20} W.R.T. GENERATOR BUS CONTROL VARIABLES

Bus	Total Derivatives		Bus	Total Derivatives	
	Voltage Magnitude	Real Power		Voltage Magnitude	Real Power
18	0.025614	0.007581	22	-0.053318	0.255422
19	-0.039806	0.080415	23	0	0
20	-0.354038	0.307280	24	-0.045456	0.267232
21	-0.219182	0.296479	25	0	0.296479

TABLE XVI

CONTINGENCY RESULTS FOR STATES OF 26-BUS POWER SYSTEM

Line Outage	State Variable	Calculated Change	Exact Change
Important Line			
2,13	$\lvert V_6 \rvert$	-0.000	-0.000
	δ_4	0.008	0.067
	Q_{20}	0.002	0.018
	δ_{20}	0.009	0.070
Normal Line			
6,7	$\lvert V_6 \rvert$	0.002	0.003
	δ_4	-0.016	-0.031
	Q_{20}	0.024	0.037
	δ_{20}	-0.015	-0.030

$$S_\ell = V_\ell \, I_\ell^*$$

$$V_\ell \, \delta I_\ell^* = I_\ell^* \, \delta V_\ell + \delta S_\ell$$

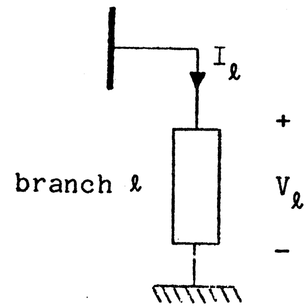


Fig. 1a Load branch modelling

$$V_g \, \delta I_g^* + V_g^* \, \delta I_g = - I_g^* \, \delta V_g - I_g \, \delta V_g^* + \delta P_g$$

$$0 = V_g^* \, \delta V_g + V_g \, \delta V_g^* - \delta |V_g|$$

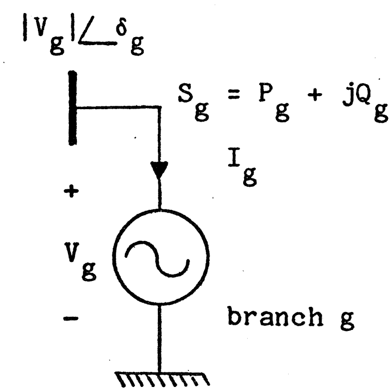


Fig. 1b Modelling of generator branch connected to P, V-type bus

$$I_t = Y_t V_t$$

$$\delta I_t = Y_t \delta V_t + V_t \delta Y_t$$

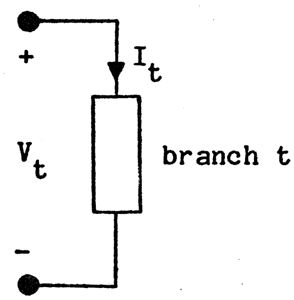
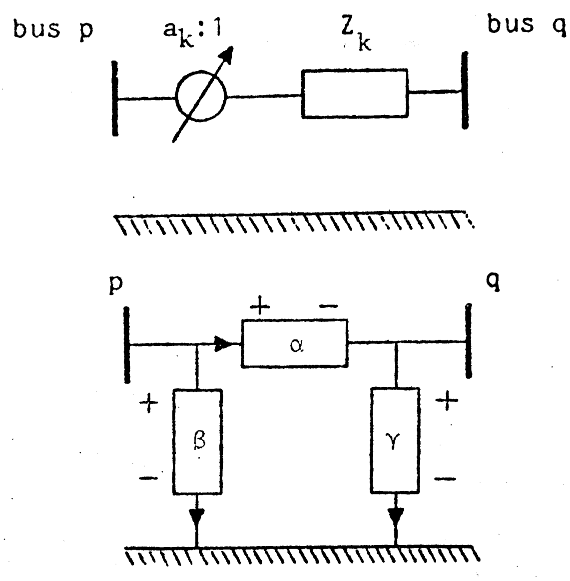


Fig. 1c Modelling of typical passive elements



$$I_\alpha - a_k^* I_\alpha^* = V_\alpha/(Z_k a_k^*) - a_k^* V_\alpha^*/(Z_k^* a_k)$$

$$I_\beta - a_k^* I_\beta^* = (1-a_k)V_\beta/(Z_k a_k a_k^*) - (1-a_k^*)V_\beta^*/(Z_k^* a_k)$$

$$I_\gamma + a_k I_\gamma^* = (a_k-1)V_\gamma/(Z_k a_k) - a_k(1-a_k^*)V_\gamma^*/(Z_k^* a_k^*)$$

Fig. 2 Modelling of transformers with complex turns ratio

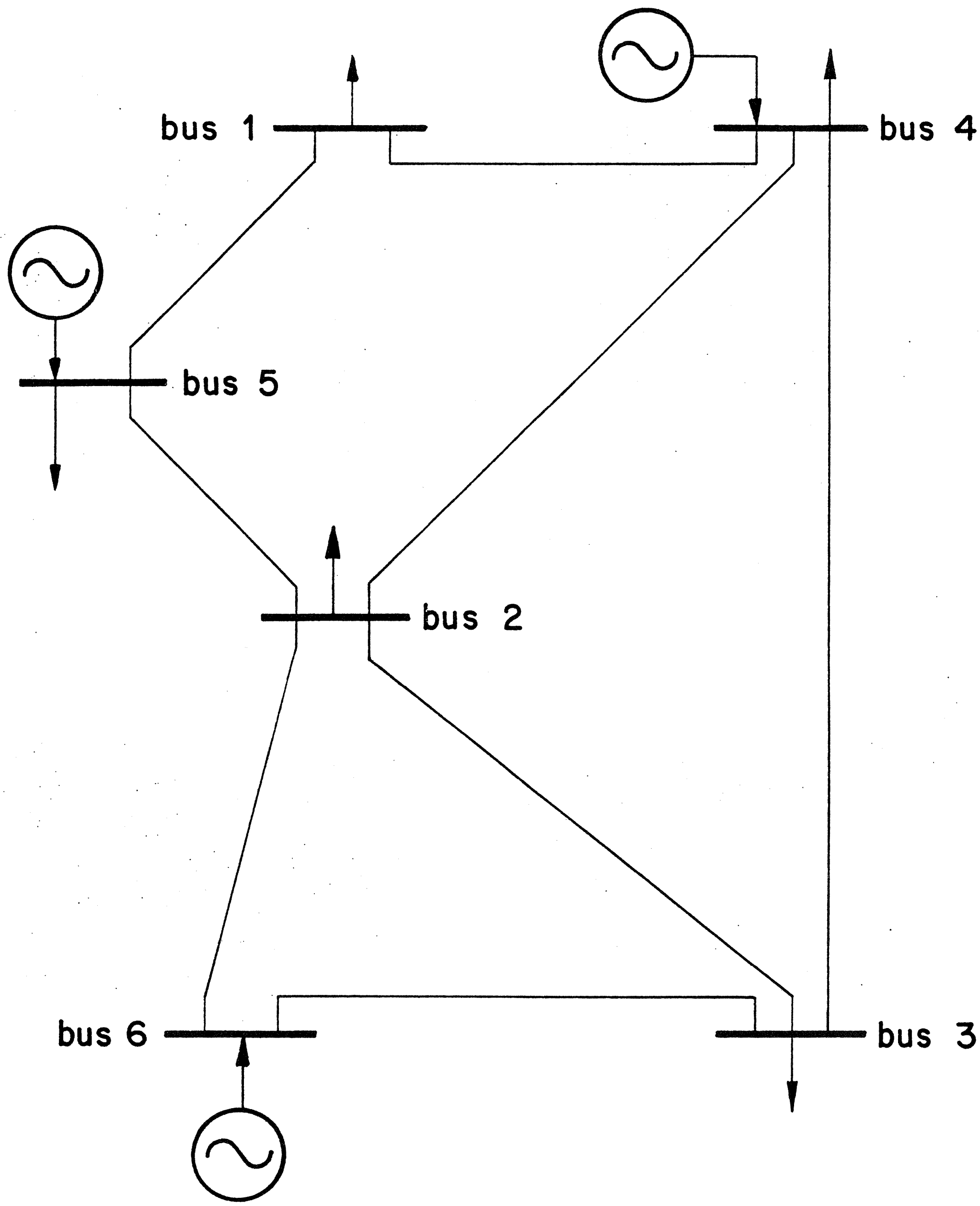


Fig. 3 6-bus power system

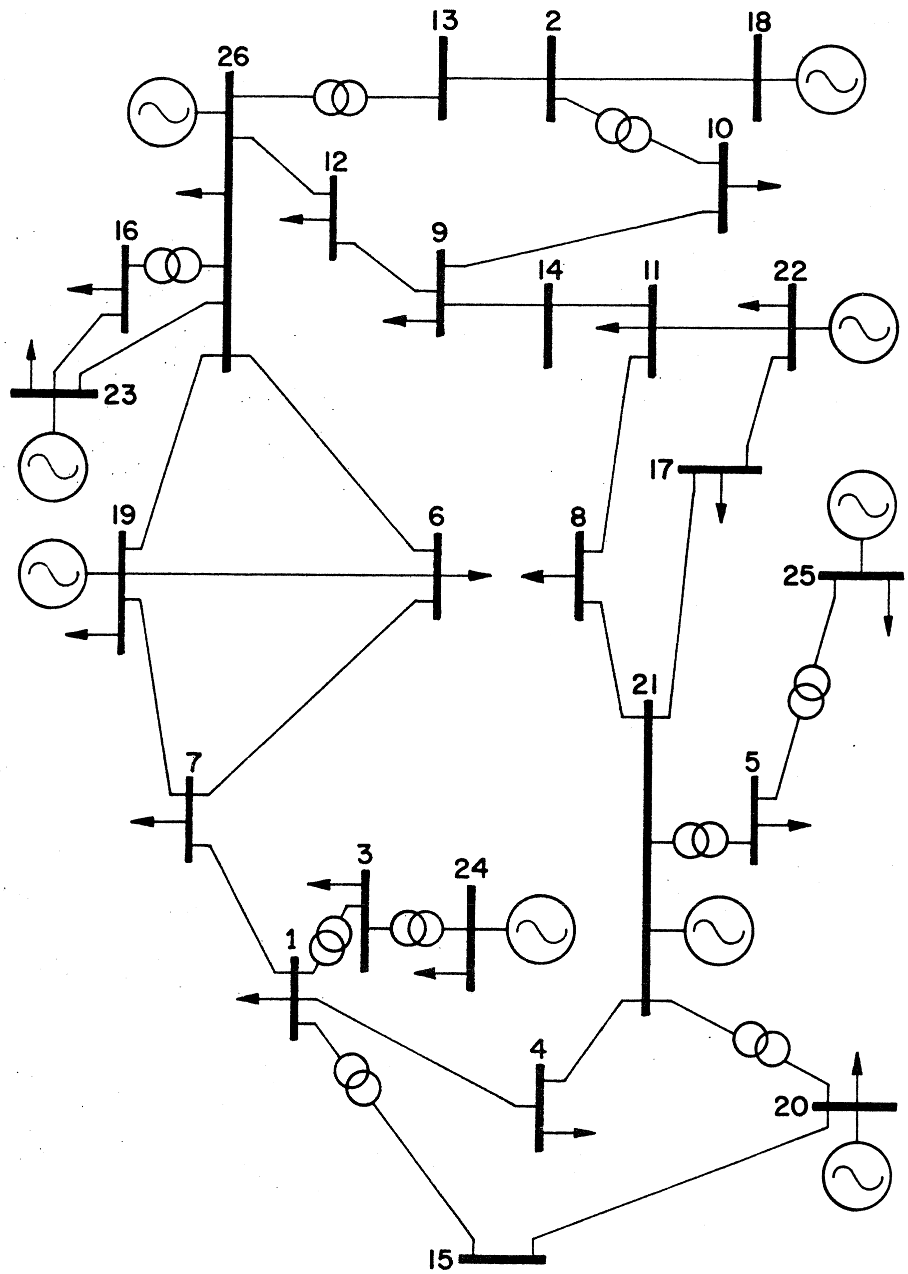


Fig. 4 26-bus power system

SOC-258

EXACT POWER NETWORK SENSITIVITIES VIA GENERALIZED COMPLEX BRANCH MODELLING

J.W. Bandler and M.A. El-Kady

December 1980, No. of Pages: 39

Revised: September 1981

Key Words: Tellegen's theorem, complex branch modelling, adjoint system, power system simulation, power network sensitivities, reduced gradients

Abstract: This paper presents an application of the Tellegen's theorem approach to power network sensitivity calculations. Our theory employs an adjoint network concept based upon a novel, generalized complex branch modelling procedure allowing the exact steady-state component models of power networks to be considered without any approximation. Exact formulas for first-order change and reduced gradients are derived and tabulated. The theoretical results are fully verified numerically on a 6-bus system and on a 26-bus, 32 line system. The full bus and line data are provided for the examples to permit independent verification of our results.

Description: Reformed and simplified version of SOC-237, SOC-238 and SOC-241. Contains new material.

Related Work: SOC-237, SOC-238, SOC-241, SOC-253, SOC-255.

Price: $6.00.