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Abstract

The exact, recently developed adjoint network approach to power network analysis is applied, in this paper, to a variety of test power systems ranging from the simplest 2 bus/3 line system to a 26 bus/32 line system. The full bus and line data as well as a single line diagram for all systems are provided. Results of the load flow analysis for all systems are presented. Detailed load flow sensitivities as well as contingency calculations for some of the systems are also presented. The general analytical aspects of the adjoint network approach are outlined. Some of its computational features are discussed for different test systems. We also illustrate the use of the results in further relevant applications.

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I. INTRODUCTION

This paper concerns itself with the applications of the exact adjoint network approach to power network analysis [1-7] to several test power systems. These applications include the load flow solution, the power network sensitivities and gradient evaluation and the contingency calculations.

A brief description of the main aspects of the adjoint network approach is given in Section III. In each of the subsequent sections, we consider one test power system for which the full data, structure and results are presented.

II. NOTATION

n	number of buses, also index of the slack bus
n_B	number of branches in the network
n_L	number of P, Q-type (load) buses
n_G	number of P, V-type (generator) buses
n_T	number of transmission elements
ℓ	$= 1, 2, \ldots, n_L$ denotes a load bus
g	$= n_L+1, \ldots, n_L+n_G$ denotes a generator bus
t	$= n+1, \ldots, n_B$ denotes transmission elements
m	$= \ell$, g or n, denotes bus number
$S_m=P_m+jQ_m$	complex power at bus m
$V_m=V_{m1}+jV_{m2}=\|V_m\|\angle\delta_m$	complex voltage of bus m
$\underset{\sim}{V}$	vector of branch voltages
I_m	current in branch m
$\underset{\sim}{I}$	vector of branch currents
$Y_t=G_t+jB_t$	admittance of line t

$\underset{\sim}{x}$	real vector of dependent variables
$\underset{\sim}{u}$	real vector of independent variables
j	denotes $\sqrt{-1}$
δ	denotes first-order change
*	denotes the complex conjugate
^	distinguishes adjoint network variables
~	denotes vectors and matrices

III. GENERAL DESCRIPTION OF THE ADJOINT NETWORK APPROACH

The mathematical modelling of a power system implies the definition of several network variables which are properly classified [2] into dependent (state) $\underset{\sim}{x}$ and independent (control) $\underset{\sim}{u}$ variables.

In the adjoint network approach to power network sensitivities, the augmented form of Tellegen's theorem

$$\hat{\underset{\sim}{I}}^T \underset{\sim}{V} + \hat{\underset{\sim}{I}}^{*T} \underset{\sim}{V}^* - \hat{\underset{\sim}{V}}^T \underset{\sim}{I} - \hat{\underset{\sim}{V}}^{*T} \underset{\sim}{I}^* = 0, \qquad (1)$$

is to be transformed into the form

$$\hat{\underset{\sim}{\eta}}_x^T \delta\underset{\sim}{x} + \hat{\underset{\sim}{\eta}}_u^T \delta\underset{\sim}{u} = 0, \qquad (2)$$

where $\underset{\sim}{I}$, $\underset{\sim}{V}$, $\underset{\sim}{x}$ and $\underset{\sim}{u}$ contain, respectively, all branch current, voltage, state and control variables and the vectors $\hat{\underset{\sim}{\eta}}_x$ and $\hat{\underset{\sim}{\eta}}_u$ are, in general, linear functions of the formulated adjoint network current $\hat{\underset{\sim}{I}}$ and voltage $\hat{\underset{\sim}{V}}$ variables. The $\hat{\underset{\sim}{\eta}}_x$ and $\hat{\underset{\sim}{\eta}}_u$ are, hence, related through Kirchhoff's current and voltage laws formulating a set of linear network equations to be solved for the unknown adjoint variables. The adjoint equations employ a simple, mostly constant matrix of coefficients.

Sensitivity Calculations

The transformation from (1) to (2) is performed using standard, tabulated [3] Jacobian matrices. The impact of this transformation on the simplicity of the analysis is evident. The vector $\hat{\eta}_x$ can be freely defined while the solution of the adjoint network provides the corresponding vector $\hat{\eta}_u$. Hence, if we have a general real function $f=f(x,u)$ of which the first-order change is expressed as

$$\delta f = \left(\frac{\partial f}{\partial x}\right)^T \delta x + \left(\frac{\partial f}{\partial u}\right)^T \delta u \tag{3}$$

or, from (2),

$$\delta f = \left(\frac{\partial f}{\partial x} - \hat{\eta}_x\right)^T \delta x + \left(\frac{\partial f}{\partial u} - \hat{\eta}_u\right)^T \delta u, \tag{4}$$

then by setting

$$\hat{\eta}_x = \frac{\partial f}{\partial x} \tag{5}$$

we get

$$\delta f = \left(\frac{\partial f}{\partial u} - \hat{\eta}_u\right)^T \delta u. \tag{6}$$

Hence, the total derivatives of f w.r.t. u are given by

$$\frac{df}{du} = \frac{\partial f}{\partial u} - \hat{\eta}_u. \tag{7}$$

Load Flow Solution

The load flow problem comprises the solution of a set of nonlinear network equations of the form

$$h\ (x) = u, \tag{8}$$

where x and u contain appropriate bus dependent and independent variables. In a gradient-type iterative method, the form (8) is perturbed about a nominal point x^k at iteration k as

$$J^k\ \delta x^k = \delta u^k, \tag{9}$$

where J^k is the Jacobian matrix evaluated at iteration k, $\delta x^k = x^{k+1}-x^k$

and $\delta \underset{\sim}{u}^k = \underset{\sim}{u}_{(scheduled)} - \underset{\sim}{u}^k$ is a mismatch vector.

In the application of the adjoint network approach to the load flow solution [7], the formulation and solution of (9) is replaced by the direct computation of the elements of $(\underset{\sim}{J}^k)^{-1}$ which are simply the sensitivities of $\underset{\sim}{x}^k$ w.r.t. $\underset{\sim}{u}^k$. In this case, the function f of (3)-(7) is defined to represent, successively, the elements of $\underset{\sim}{x}^k$ of (9).

III. A 2-BUS SYSTEM

This simple system is mainly used [5,8,9] to clarify analytical aspects of the approach and to demonstrate the detailed iterative solution of the load flow analysis.

The system consists of a load bus ($m=\ell=1$), the slack bus ($n=2$) and three transmission branches ($t=3,4,5$). The single line diagram for this sample system is shown in Fig. 1 which also shows the input data of the problem. All values are in per unit.

The detailed iterative load flow solution points for this system are shown in Table I. Table II shows the branch currents and voltages at the load flow solution. Table III summarizes the obtained total derivatives of the bus states V_{11} and V_{12}, where $V_1 = V_{11} + jV_{12}$, w.r.t. different control variables.

In terms of the formal (or symbolic) derivatives [2,8], the sensitivities of the complex load bus voltage V_1 can be computed from

$$\frac{dV_1}{d\zeta_k} = \left(\frac{dV_1}{\partial\zeta_{k1}} - j\,\frac{\partial V_1}{\partial\zeta_{k2}}\right)/2 \qquad (10)$$

and

$$\frac{dV_1}{d\zeta_k^*} = \left(\frac{dV_1}{d\zeta_{k1}} + j\,\frac{dV_1}{d\zeta_{k2}}\right)/2, \tag{11}$$

where $\zeta_k = \zeta_{k1} + j\zeta_{k2}$ denotes a complex control variable. Table IV shows the results obtained using (10) and (11). Note that, in terms of the total formal derivatives, the first-order change $\delta V_1 = \delta V_{11} + j\,\delta V_{12}$ can be expressed, directly, as

$$\delta V_1 = \sum_k \left(\frac{dV_1}{d\zeta_k}\ \delta\zeta_k + \frac{dV_1}{d\zeta_k^*}\ \delta\zeta_k^*\right). \tag{12}$$

The results of Tables III and IV may be used to obtain the sensitivities of a general function. For example, consider the real function

$$f = |V_1|^2 = V_{11}^2 + V_{12}^2 \tag{13}$$

for which

$$\frac{df}{du_k} = 2V_{11}\frac{dV_{11}}{du_k} + 2V_{12}\frac{dV_{12}}{du_k} \tag{14}$$

where u_k denotes a control variable. Using the results of Table III and substituting for V_{11} and V_{12} at the load flow solution we get, for u_k = P_1

$$\frac{df}{dP_1} = -0.1123,$$

for $u_k = V_{21}$

$$\frac{df}{dV_{21}} = 3.3577$$

and for $u_k = B_5$

$$\frac{df}{dB_5} = -0.0502$$

which are the same results obtained in [9], for the same system, using the Lagrange multiplier approach.

We have used the rectangular formulation for the results of Tables III and IV. Sensitivities of polar quantities can be obtained via suitable transformations [8]. For example, $\delta|V_1|$ may be expressed in terms of formal derivatives as

$$\delta|V_1| = \frac{\partial|V_1|}{\partial V_1}\,\delta V_1 + \frac{\partial|V_1|}{\partial V_1^*}\,\delta V_1^* \tag{15}$$

or

$$\delta|V_1| = 2\mathrm{Re}\{\frac{V_1^*}{2|V_1|}\,\delta V_1\}\,. \tag{16}$$

At the load flow solution we have

$$\delta|V_1| = 2\mathrm{Re}\{(0.4818+j0.1337)\ \delta V_1\}. \tag{17}$$

Considering the complex control variable S_1 and substituting for δV_1 of (17) using the results of Table IV, we get

$$\delta|V_1| = 2\mathrm{Re}\{(-0.0216+j0.0130)\ \delta S_1 + (-0.0152-j0.0454)\ \delta S_1^*\},$$

from which

$$\frac{d|V_1|}{dS_1} = -0.0368+j0.0584.$$

Since

$$\frac{d|V_1|}{dS_1} = (\frac{d|V_1|}{dP_1} - j\,\frac{d|V_1|}{dQ_1})/2, \tag{18}$$

hence

$$\frac{d|V_1|}{dP_1} = -0.0736 \text{ and } \frac{d|V_1|}{dQ_1} = -0.1168,$$

which are identical to the results shown in [9], for the same system, using the Lagrange multiplier approach.

IV. A 6-BUS SYSTEM

This system has been considered in a variety of steady state analysis and planning studies [3,10]. We shall consider the applications of the adjoint network approach for this system in some detail.

The system consists of three specified load buses (ℓ = 1,2,3), two generator buses (g = 4,5), the slack bus (n=6) and eight transmission lines (t = 7, ..., 14). The single line diagram for this system is shown in Fig. 2. The line and bus data are shown, respectively, in Tables V and VI. All values shown are in per unit. The application of the adjoint network approach results in the load flow solution shown in Table VII. The detailed iterative solution, in the polar coordinates, is illustrated in Table VIII. A flat starting voltage profile has been used. The reader is encouraged to verify that the same sequence of iterative solution points can be obtained by applying the Newton-Raphson method [11].

The sensitivities of bus states, namely $|V_1|$, $|V_2|$, $|V_3|$, Q_4, Q_5, δ_1, δ_2, δ_3, δ_4 and δ_5 w.r.t. system bus and line control variables are shown in Tables IX to XVIII. The estimated effects of the line and circuit outages on the different states, based on first-order changes, are also shown. Observe that the sensitivities w.r.t. non-existing elements, e.g., the shunt parameters in Tables IX to XVIII, can be evaluated as well.

Although the sensitivites of a general function can be evaluated [2] using the same adjoint matrix of coefficients at the load flow solution and by defining the RHS of the adjoint equations corresponding to the function considered, these sensitivities can also be obtained, directly, using the results of Tables IX to XVIII by expressing the function in terms of the system bus states. For example, consider the function

$$f = |I_7|^2 = |V_1 - V_4|^2 \cdot |Y_7|^2 \qquad (19)$$

which may denote the loading of line 1,4. The sensitivities of this function w.r.t. a control variable u_k is given by

$$\frac{df}{du_k} = \frac{\partial f}{\partial u_k} + 2|V_1 - V_4||Y_7|^2 \frac{\partial |V_1 - V_4|}{\partial u_k} \qquad (20)$$

which, when substituting values at the load flow solution and noting that $|V_4|$ is constant, reduces to

$$\frac{df}{du_k} = \frac{\partial f}{\partial u_k} - 1.6871 \frac{\partial |V_1|}{\partial u_k} - 4.8588 \frac{\partial \delta_1}{\partial u_k} + 4.8588 \frac{\partial \delta_4}{\partial u_k} .$$

Now, let u_k denote the conductance of line 2,4. Hence, from Tables IX, XIV and XVII, we get (t=10 for line 2,4)

$$\frac{df}{dG_{10}} = - 0.0324.$$

Similarly, if u_k denotes the susceptance of line 2,4, we get

$$\frac{df}{dB_{10}} = - 0.0932.$$

The effect of line 2,4 outage on the function considered can be estimated using the relation

$$\delta f = - \frac{df}{dG_{10}} G_{10} - \frac{df}{dB_{10}} B_{10}, \qquad (21)$$

where we have set the changes in line conductance and susceptance, respectively, to $-G_{10}$ and $-B_{10}$. Substituting the values of G_{10} (=0.5882) and B_{10}(=-2.3529) in (21), we get

$$\delta f = 0.019 - 0.219 = - 0.200$$

which is identical to the result presented in [3], where the function f $= |I_7|^2$ was considered, directly, in the adjoint simulation without state transformations.

The accuracy of prediction based on first-order change may be examined by calculating the exact change of the function considered. Table XIX summarizes the contingency results for the above line outage as well as three other cases for different functions and outages.

The function

$$f = \sum_t |I_t|^2 R_t \tag{22}$$

representing the total system losses is of particular interest in the economic dispatch and other operational planning studies. In Table XX, we show the sensitivities of this function w.r.t. main control variables.

The 6-bus system considered in this section does not contain tap changing or phase shifting transformers. The inclusion of transformers in the adjoint network studies is, however, possible using proper adjoint modelling [6]. Consider, for example, a phase shifter with the arbitrary turns ratio a_t = 0.6+j0.8 and internal impedence Z_t=0.05+j0.20 replacing the line 1,4 of the previous system configuration. Other problem data remains the same. The load flow solution obtained in this case is shown in Table XXI.

V. A 23-BUS SYSTEM

This system has been considered in [12]. The data provided includes realistic lower and upper bounds for the system variables which may be used in a constrained steady-state analysis and optimization. We shall consider the application of the adjoint network approach to the load flows solution for this system with specified bus types.

The single line diagram of this system is shown in Fig. 3. Its line data is shown in Table XXII. Table XXIII shows the specified lower and upper limits on the transformer taps while Table XXIV gives the limits for bus voltages. The real and reactive power generation limits are shown in Table XXV. The nominal power demand at different buses is shown in Table XXVI. All data given is in per unit.

The operating bus data and transformer taps for two schedules considered in the power flow solution are shown in Tables XXVII and XXVIII, respectively ($n_L = 17$, $n_G = 5$, $n_T = 30$). Tables XXIX and XXX summarize the load flows solution obtained for the first (no phase shifting) and the second (with phase shifting) schedules, respectively. The detailed iterative solution, in the polar coordinates, is illustrated in Tables XXXI and XXXII where flat voltage profile has been used for the starting point. Observe that all constraints on variables and parameters are met in the results of Tables XXXI and XXXII.

VI. A 26-BUS SYSTEM

This power system (Saskatchewan Power Corporation System) has been considered [7,13] in some relevant steady-state power system analysis.

The single line diagram of this system is shown in Fig. 4. The line data is shown in Table XXXIII. The operating bus data and transformer taps considered in the load flows analysis are shown, respectively, in Tables XXXIV and XXXV ($n_L = 17$, $n_G = 8$, $n_T = 32$). All values shown are in per unit. The load flow solution obtained is shown in Table XXXVI. Table XXXVII illustrates the detailed iterative solution, in the polar coordinates, with starting flat voltage profile.

VII. CONCLUSIONS

We have considered several test power systems to illustrate the applications of the exact, recently developed adjoint network approach to power network analysis. The full descriptive data for all systems is provided to facilitate further applications in the context of steady-state power system analysis and planning. The applications considered in this paper include the load flow solution, the power flow sensitivities and contingency analysis. We have also illustrated some analytical and computational features of the approach regarding the complex gradient evaluation and complex function sensitivities.

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TABLE I

LOAD FLOW SOLUTION FOR 2-BUS SYSTEM USING ADJOINT NETWORK APPROACH

Quantity	Iteration			
	1	2	3	4
δP_1	5.0	0.3776	0.0598	0.0032
δQ_1	1.0	1.1328	0.1794	0.0096
dV_{11}/dP_1	-0.0169	-0.0540	-0.0784	-0.0876
dV_{11}/dQ_1	-0.0562	-0.0701	-0.1028	-0.1152
δV_{11}	-0.1404	-0.0998	-0.0231	-0.0014
dV_{12}/dP_1	-0.0449	-0.0438	-0.0431	-0.0428
dV_{12}/dQ_1	0.0169	0.0173	0.0183	0.0186
δV_{12}	-0.2079	0.0030	0.0007	0.0000
V_{11}	0.8596	0.7598	0.7366	0.7352
V_{12}	-0.2079	-0.2048	-0.2041	-0.2041

TABLE II

BRANCH CURRENTS AND VOLTAGES OF 2-BUS SYSTEM

b	I_b	V_b
1	5.2623-j5.5411	0.7352-j0.2041
2	-5.6705+j1.0706	1.0+j0.0
3	0.4082+j1.4705	0.7352-j.2041
4	0.0+j3.0	1.0+j0.0
5	-5.6705+j4.0706	-0.2648-j0.2041

TABLE V

LINE DATA FOR 6-BUS POWER SYSTEM

Branch Index, t	Terminal Buses	Resistance R_t (pu)	Reactance X_t (pu)	Number of Lines
7	1,4	0.05	0.20	1
8	1,5	0.025	0.10	2
9	2,3	0.10	0.40	1
10	2,4	0.10	0.40	1
11	2,5	0.05	0.20	1
12	2,6	0.01875	0.075	4
13	3,4	0.15	0.60	1
14	3,6	0.0375	0.15	2

TABLE VI

BUS DATA FOR 6-BUS POWER SYSTEM

Bus Index, m	Bus Type	P_m (pu)	Q_m (pu)	$\lvert V_m \rvert \angle \delta_m$ (pu)
1	load	-2.40	0	$- \angle -$
2	load	-2.40	0	$- \angle -$
3	load	-1.60	-0.40	$- \angle -$
4	generator	-0.30	-	$1.02 \angle -$
5	generator	1.25	-	$1.04 \angle -$
6	slack	-	-	$1.04 \angle -$

TABLE VII

LOAD FLOW SOLUTION OF 6-BUS POWER SYSTEM

Load Buses

$V_1 = 0.9787 \angle -0.6602$

$V_2 = 0.9633 \angle -0.2978$

$V_3 = 0.9032 \angle -0.3036$

Generator Buses

$Q_4 = 0.7866 \qquad \delta_4 = -0.5566$

$Q_5 = 0.9780 \qquad \delta_5 = -0.4740$

Slack Bus

$P_6 = 6.1298 \qquad Q_6 = 1.3546$

TABLE VIII

DETAILED LOAD FLOW SOLUTION OF 6-BUS SYSTEM

Quantity	Iteration 1	2	3	4	5
Max $\|\delta P_m\|$	0.167	0.127×10^{-1}	0.948×10^{-4}	0.463×10^{-8}	0.171×10^{-12}
Max $\|\delta Q_m\|$	0.554	0.295×10^{-1}	0.166×10^{-3}	0.663×10^{-8}	0.119×10^{-12}
Max $\|e_v\|$	0.463×10^{-1}	0.367×10^{-1}	0.244×10^{-4}	0.109×10^{-8}	0.195×10^{-13}
Max $\|e_\delta\|$	0.698×10^{-1}	0.649×10^{-2}	0.434×10^{-4}	0.185×10^{-8}	0.748×10^{-13}

$e_v \triangleq \delta|V_m|, \qquad e_\delta \triangleq \delta(\delta_m)$

TABLE IX

6-BUS SYSTEM: SENSITIVITIES OF $|V_1|$

Line Quantities

Line	Total Derivatives: Conductance	Total Derivatives: Susceptance	Contingency Effect: Outage of One Line	Contingency Effect: Outage of Circuit
1,4	-0.006326	-0.005283	0.017421	0.017421
1,5	-0.011838	-0.008884	0.027880	0.055760
2,3	0.000027	-0.000012	0.000044	0.000044
2,4	-0.000207	-0.000597	-0.001282	0.001282
2,5	0.000163	0.000294	-0.001192	-0.001192
2,6	-0.000002	0.000039	-0.000123	-0.000494
3,4	-0.000265	-0.000443	0.000591	0.000591
3,6	-0.000017	-0.000120	0.000362	0.000724

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.029522	0.070273	-0.028275	-0.067306
2	-0.000131	-0.000005	0.000122	0.000005
3	0.000378	0.000169	-0.000308	-0.000138

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	0.357365	0.002243	-0.002334	0.0
5	0.732004	-0.001804	0.001951	0.0

TABLE X

6-BUS SYSTEM: SENSITIVITIES OF $|V_2|$

Line Quantities

Line	Total Derivatives: Conductance	Total Derivatives: Susceptance	Contingency Effect: Outage of One Line	Contingency Effect: Outage of Circuit
1,4	-0.000306	-0.000215	0.000650	0.000650
1,5	-0.000986	-0.000505	0.001214	0.002428
2,3	-0.000651	0.002065	-0.005242	-0.005242
2,4	0.010661	-0.000869	0.008316	0.008316
2,5	0.007844	-0.002633	0.021621	0.021621
2,6	-0.013772	-0.009648	0.194670	0.077869
3,4	0.000838	-0.004081	0.006730	0.006730
3,6	-0.003363	-0.005385	0.014256	0.028511

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.028847	0.000393	-0.027629	-0.000376
2	0.027782	0.049673	-0.025778	-0.046089
3	0.014540	0.016153	-0.011861	-0.013177

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	0.135817	0.026902	-0.027989	0.0
5	0.230368	0.026257	-0.028400	0.0

TABLE XI

6-BUS SYSTEM: SENSITIVITIES OF $|V_3|$

Line Quantities

Line	Total Derivatives		Contingency Effect	
	Conductance	Susceptance	Outage of One Line	Outage of Circuit
1,4	-0.000544	0.000329	-0.002190	-0.002190
1,5	-0.000729	-0.000962	0.003672	0.007343
2,3	0.001664	-0.005748	0.014504	0.014504
2,4	0.001407	-0.003853	0.009892	0.009892
2,5	0.001507	-0.001870	0.010575	0.010575
2,6	-0.003937	-0.005161	0.013104	0.052416
3,4	0.027165	-0.002716	0.014913	0.014913
3,6	-0.028570	-0.025622	0.057974	0.115947

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.026681	0.000512	-0.025555	-0.000491
2	0.016034	0.015022	-0.014877	-0.013938
3	0.057311	0.118208	-0.046752	-0.096428

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	0.194811	0.030046	-0.031260	0.0
5	0.079778	0.021688	-0.023457	0.0

TABLE XII

6-BUS SYSTEM: SENSITIVITIES OF Q_4

Line Quantities

Line	Total Derivatives: Conductance	Total Derivatives: Susceptance	Contingency Effect: Outage of One Line	Contingency Effect: Outage of Circuit
1,4	-0.056140	-0.044515	0.143437	0.143437
1,5	0.065943	0.060168	-0.205565	-0.411130
2,3	0.000236	0.004289	-0.009954	-0.009954
2,4	0.256340	0.022413	0.098051	0.098051
2,5	-0.015503	0.028010	-0.150048	-0.150048
2,6	0.046139	0.039093	-0.086459	-0.345835
3,4	0.243148	-0.031249	0.144371	0.144371
3,6	0.062174	0.056610	-0.128837	-0.257674

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	-0.457852	-0.358531	0.438519	0.343391
2	-0.115872	-0.168723	0.107512	0.156551
3	-0.127525	-0.258052	0.104029	0.210506

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	7.51274	-0.550625	0.572870	0.0
5	-4.66462	-0.219233	0.237122	0.0

TABLE XIII

6-BUS SYSTEM: SENSITIVITIES OF Q_5

Line Quantities

Line	Total Derivatives: Conductance	Total Derivatives: Susceptance	Contingency Effect: Outage of One Line	Contingency Effect: Outage of Circuit
1,4	0.063610	0.065954	-0.235535	-0.235535
1,5	-0.046596	-0.004421	-0.034014	-0.068028
2,3	0.001612	-0.010535	-0.025736	-0.025736
2,4	-0.043764	0.048459	-0.139764	-0.139764
2,5	0.163045	-0.023595	0.302851	0.302851
2,6	0.076306	0.050501	-0.098586	-0.394346
3,4	0.014771	0.054970	-0.080435	-0.080435
3,6	0.019837	0.038517	-0.105279	-0.210558

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	-0.709069	-0.713165	0.679128	0.683051
2	-0.143975	-0.274202	0.133588	0.254420
3	-0.107990	-0.101658	0.088093	0.082927

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	-4.51866	-0.312777	0.325413	0.0
5	7.58088	-0.461238	0.498875	0.0

TABLE XIV

6-BUS SYSTEM: SENSITIVITIES OF δ_1

Line Quantities

Line	Total Derivatives		Contingency Effect	
	Conductance	Susceptance	Outage of One Line	Outage of Circuit
1,4	0.001197	-0.010358	0.050152	0.050152
1,5	-0.004594	-0.016180	0.070737	0.141473
2,3	-0.001609	0.000178	-0.001366	-0.001366
2,4	-0.010354	-0.031650	0.068981	0.068981
2,5	-0.011653	-0.025839	0.107885	0.107885
2,6	-0.005283	-0.025867	0.077008	0.308030
3,4	-0.020029	-0.036084	0.054530	0.054530
3,6	-0.002723	-0.019449	0.058881	0.117762

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.309969	-0.002339	-0.296880	0.002240
2	0.085296	0.026631	-0.079143	-0.024709
3	0.061420	0.027332	-0.050104	-0.022297

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	0.192793	0.208858	-0.217296	0.0
5	0.271949	0.223549	-0.241790	0.0

TABLE XV

6-BUS SYSTEM: SENSITIVITIES OF δ_2

Line Quantities

Line	Total Derivatives		Contingency Effect	
	Conductance	Susceptance	Outage of One Line	Outage of Circuit
1,4	-0.000741	-0.001026	0.003955	0.003955
1,5	-0.003114	-0.001172	0.001854	0.003708
2,3	-0.002744	-0.000146	-0.001270	-0.001270
2,4	-0.004340	0.000328	-0.003324	-0.003324
2,5	-0.002447	-0.000565	-0.000221	-0.000221
2,6	0.001248	-0.023429	0.074481	0.297925
3,4	-0.006384	-0.010744	0.014350	0.014350
3,6	-0.000257	-0.010616	0.033103	0.066206

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.087248	0.001083	-0.083564	-0.001037
2	0.079319	0.003805	-0.073596	-0.003530
3	0.034686	0.010875	-0.028295	-0.008871

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	0.020059	0.077683	-0.080822	0.0
5	0.002719	0.081268	-0.087900	0.0

TABLE XVI

6-BUS SYSTEM: SENSITIVITIES OF δ_3

Line Quantities

Line	Total Derivatives: Conductance	Total Derivatives: Susceptance	Contingency Effect: Outage of One Line	Contingency Effect: Outage of Circuit
1,4	-0.001205	0.000743	-0.004915	-0.004915
1,5	-0.001595	-0.002133	0.008162	0.016323
2,3	0.005312	-0.000166	0.003515	0.003515
2,4	-0.003359	-0.008465	0.017941	0.017941
2,5	-0.001260	-0.002965	0.012469	0.012469
2,6	-0.001242	-0.009986	0.030353	0.121413
3,4	0.000744	0.015220	-0.023583	-0.023583
3,6	0.010158	-0.037461	0.125492	0.250984

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.058622	0.001131	-0.056146	-0.001084
2	0.033200	0.007596	-0.030805	-0.007048
3	0.132854	0.001969	-0.108375	-0.001606

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	-0.008082	0.066205	-0.068879	0.0
5	0.056708	0.047554	-0.051434	0.0

TABLE XVII

6-BUS SYSTEM: SENSITIVITIES OF δ_4

Line Quantities

Line	Total Derivatives: Conductance	Total Derivatives: Susceptance	Contingency Effect: Outage of One Line	Contingency Effect: Outage of Circuit
1,4	-0.006119	0.005953	-0.035213	-0.035213
1,5	-0.004959	-0.011033	0.046087	0.092174
2,3	-0.000725	-0.000212	0.000073	0.000073
2,4	-0.017094	-0.051045	0.110050	0.110050
2,5	-0.006343	-0.016282	0.069157	0.069157
2,6	-0.005360	-0.024608	0.072997	0.291989
3,4	-0.028650	-0.050482	0.067952	0.067952
3,6	-0.003276	-0.023336	0.017661	0.035321

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.222333	0.005176	-0.212945	-0.004957
2	0.081031	0.026460	-0.075185	-0.024551
3	0.073688	0.032826	-0.060111	-0.026778

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	-0.047087	0.281747	-0.293130	0.0
5	0.272518	0.164929	-0.178387	0.0

TABLE XVIII

6-BUS SYSTEM: SENSITIVITIES OF δ_5

Line Quantities

Line	Total Derivatives		Contingency Effect	
	Conductance	Susceptance	Outage of One Line	Outage of Circuit
1,4	0.000289	-0.007712	0.036634	0.036634
1,5	-0.010462	0.001216	-0.018030	-0.036060
2,3	-0.002054	0.000375	-0.002091	-0.002091
2,4	-0.006958	-0.021878	0.047385	0.047385
2,5	-0.014328	-0.030654	0.155106	0.155106
2,6	-0.005245	-0.026501	0.079028	0.316113
3,4	-0.015685	-0.028828	0.039070	0.039070
3,6	-0.002444	-0.017490	0.052955	0.105905

Load Bus Quantities - Total Derivatives

Bus	Real Power	Reactive Power	Shunt Conductance	Shunt Susceptance
1	0.246249	0.001696	-0.235851	-0.001625
2	0.087446	0.026717	-0.081137	-0.024789
3	0.055239	0.024564	-0.045061	-0.020038

Generator Bus Quantities - Total Derivatives

Bus	Voltage Magnitude	Real Power	Shunt Conductance	Shunt Susceptance
4	0.173629	0.172131	-0.179086	0.0
5	-0.088893	0.253086	-0.273737	0.0

TABLE XIX

6-BUS SYSTEM CONTINGENCY RESULTS FOR $f = |I_t|^2$

Function Line Index	Removed Line Index	Calculated Function Change	Exact Function Change
1,4	2,4	-0.200	-0.224
2,3	1,5*	0.002	0.005
2,3	2,3	-0.029	-0.021
2,4	2,4	-0.470	-0.404

* Only one line of branch 1,5 is removed.

TABLE XX

SENSITIVITIES OF TOTAL LOSSES FUNCTION OF 6-BUS SYSTEM

Line Quantities

Line	Derivatives w.r.t. G_t	Derivatives w.r.t. B_t
1,4	0.016462	0.008741
1,5	0.048977	0.027370
2,3	0.003490	0.002102
2,4	0.084665	0.044962
2,5	0.045468	0.022680
2,6	0.103966	0.060904
3,4	0.089397	0.042758
3,6	0.113314	0.069869

Load Bus Quantities

Bus	Derivatives w.r.t. P_ℓ	Derivatives w.r.t. Q_ℓ
1	-0.453538	-0.020390
2	-0.201703	-0.054098
3	-0.221666	-0.094646

Generator Bus Quantities

Bus	Derivatives w.r.t. $\lvert V_g \rvert$	Derivatives w.r.t. P_g
4	-0.373561	-0.375812
5	-0.184047	-0.312838

TABLE XXI

LOAD FLOW SOLUTION OF 6-BUS SYSTEM WITH A PHASE SHIFTER

Load Buses		
	$V_1 = 0.9729 \underline{/-0.3543}$	
	$V_2 = 0.9405 \underline{/-0.3012}$	
	$V_3 = 0.8563 \underline{/-0.3736}$	
Generator Buses		
	$Q_4 = 0.1800$	$\delta_4 = -0.9657$
	$Q_5 = 0.9419$	$\delta_5 = -0.2693$
Slack Bus		
	$P_6 = 6.5393$	$Q_6 = 2.0156$

TABLE XXII

LINE DATA FOR 23-BUS POWER SYSTEM

No.	Terminal Buses	Resistance R_t(pu)	Reactance X_t(pu)	1/2 Shunt Susceptance
1	23,1	0.0242	0.0540	0.00590
2	23,2	0.0309	0.0693	0.00755
3	18,3	0.0404	0.0888	0.00985
4	6,3	0.0325	0.0709	0.00785
5	18,5	0.0615	0.1620	0.01710
6	1,4	0.0576	0.1520	0.01600
7	2,7	0.0266	0.0700	0.00740
8	7,5	0.0229	0.0504	0.00560
9	6,4	0.0446	0.1003	0.01090
10	19,8	0.0233	0.0514	0.02280
11	6,8	0.0597	0.1315	0.01455
12	7,8	0.0597	0.1315	0.01455
13	10,20	0.0043	0.0351	0.11865
14	20,9	0.0043	0.0351	0.11865
15	11,9	0.0038	0.0307	0.10390
16	14,11	0.0035	0.0288	0.09755
17	22,10	0.0089	0.0726	0.24355
18	12,13	0.0010	0.0080	0.02715
19	13,14	0.0021	0.0167	0.05665
20	15,14	0.0016	0.0127	0.04310
21	21,15	0.0045	0.0362	0.12255
22	17,14	0.0024	0.0192	0.06490
23	21,16	0.0019	0.0156	0.05280
24	16,17	0.0014	0.0114	0.03850
25	22,12	0.0020	0.0164	0.05545
26	9,6	0.0023	0.0839	0.0
27	10,6	0.0023	0.0839	0.0
28	9,7	0.0019	0.1300	0.0
29	10,7	0.0023	0.0839	0.0
30	23,18	0.0025	0.2000	0.0

TABLE XXIII

TRANSFORMER TAP LIMITS FOR 23-BUS SYSTEM

No.	Terminal Buses	Lower Limit Real	Lower Limit Imaginary	Upper Limit Real	Upper Limit Imaginary
1	9,6	0.95	-0.10	1.05	0.10
2	10,6	0.94	-0.15	1.06	0.15
3	9,7	0.95	-0.13	1.05	0.13
4	10,7	0.93	-0.08	1.07	0.08

TABLE XXIV

BUS VOLTAGE LIMITS FOR 23-BUS SYSTEM

Bus	Lower Limit	Upper Limit
1	0.94	1.06
2	0.95	1.05
3	0.95	1.05
4	0.95	1.05
5	0.95	1.05
6	0.95	1.05
7	0.95	1.05
8	0.95	1.05
9	0.93	1.07
10	0.93	1.07
11	0.95	1.05
12	0.95	1.05
13	0.95	1.05
14	0.95	1.05
15	0.95	1.05
16	0.94	1.06
17	0.94	1.06
18	0.95	1.05
19	0.95	1.05
20	0.95	1.05
21	0.95	1.05
22	0.95	1.05
23	0.95	1.05

TABLE XXV

GENERATION LIMITS FOR 23-SYSTEM

Bus	Real Power		Reactive Power	
	Lower	Upper	Lower	Upper
18	0.75	1.83	-0.30	1.947
19	1.72	2.33	-0.40	1.891
20	5.00	5.00	-1.20	4.303
21	3.00	8.03	-1.55	5.411
22	3.44	10.52	-2.05	7.750
23	0.43	1.83	-0.30	3.143

TABLE XXVI

NOMINAL DEMAND DATA FOR 23-BUS SYSTEM

Bus	Real Power	Reactive Power
1	0.00	0.00
2	0.47	0.12
3	0.51	0.13
4	0.41	0.10
5	0.48	0.12
6	0.01	0.00
7	1.50	0.38
8	1.77	0.44
9	0.06	0.00
10	-0.04	0.00
11	2.01	0.50
12	1.32	0.33
13	3.44	0.86
14	1.04	0.26
15	3.76	0.94
16	3.75	0.94
17	-2.10	-0.52
18	1.01	0.25
19	1.30	0.32
20	4.80	1.20
21	-1.00	-0.25
22	1.29	0.32
23	0.64	0.16

TABLE XXVII

BUS DATA FOR LOAD FLOW ANALYSIS OF 23-BUS SYSTEM

Bus	Injected Power		Bus Voltage	
	P_m	Q_m	$\|V_m\|$	δ_m
1	0.00	0.00	-	-
2	-0.47	-0.12	-	-
3	-0.51	-0.13	-	-
4	-0.41	-0.10	-	-
5	-0.48	0.12	-	-
6	-0.01	0.00	-	-
7	-1.50	-0.38	-	-
8	-1.77	-0.44	-	-
9	-0.06	0.00	-	-
10	0.04	0.00	-	-
11	-2.01	-0.50	-	-
12	-1.32	-0.33	-	-
13	-3.44	-0.86	-	-
14	-1.04	-0.26	-	-
15	-3.76	-0.94	-	-
16	-3.75	-0.94	-	-
17	2.10	0.52	-	-
18	0.26	-	1.03	-
19	0.89	-	1.05	-
20	0.20	-	1.05	-
21	9.03	-	1.05	-
22	9.23	-	1.05	-
23	-	-	1.04	0.0

TABLE XXVIII

TRANSFORMER SCHEDULES FOR LOAD FLOW SOLUTION OF 23-BUS SYSTEM

No.	Terminal Buses	First Schedule		Second Schedule	
		Real	Imaginary	Real	Imaginary
1	9,6	1.04	0.0	1.03	-0.09
2	10,6	1.03	0.0	1.02	0.14
3	9,7	1.05	0.0	1.04	-0.11
4	10,7	1.06	0.0	1.05	0.07

TABLE XXIX

LOAD FLOW SOLUTION OF 23-BUS SYSTEM FOR THE FIRST TRANSFORMER SCHEDULE

Load Buses

$\|V_1\|$	= 1.0320	δ_1	= 0.0124
$\|V_2\|$	= 1.0063	δ_2	= 0.0137
$\|V_3\|$	= 1.0074	δ_3	= 0.0463
$\|V_4\|$	= 1.0052	δ_4	= 0.0477
$\|V_5\|$	= 0.9984	δ_5	= 0.0226
$\|V_6\|$	= 1.0164	δ_6	= 0.1100
$\|V_7\|$	= 0.9930	δ_7	= 0.0561
$\|V_8\|$	= 0.9941	δ_8	= 0.0147
$\|V_9\|$	= 1.0337	δ_9	= 0.1807
$\|V_{10}\|$	= 1.0421	δ_{10}	= 0.2125
$\|V_{11}\|$	= 1.0118	δ_{11}	= 0.2204
$\|V_{12}\|$	= 1.0195	δ_{12}	= 0.3614
$\|V_{13}\|$	= 1.0101	δ_{13}	= 0.3286
$\|V_{14}\|$	= 1.0116	δ_{14}	= 0.3142
$\|V_{15}\|$	= 1.0080	δ_{15}	= 0.3181
$\|V_{16}\|$	= 1.0273	δ_{16}	= 0.3852
$\|V_{17}\|$	= 1.0270	δ_{17}	= 0.3728

Generator Buses

Q_{18}	= 0.5268	δ_{18}	= 0.0070
Q_{19}	= 0.7203	δ_{19}	= 0.0419
Q_{20}	= 0.4451	δ_{20}	= 0.1985
Q_{21}	= 1.8990	δ_{21}	= 0.4546
Q_{22}	= 1.2508	δ_{22}	= 0.4461

Slack Bus

P_{23}	= -0.1683	Q_{23}	= 0.7605

TABLE XXX

LOAD FLOW SOLUTION OF 23-BUS
SYSTEM FOR THE SECOND TRANSFORMER SCHEDULE

Load Buses

$\lvert V_1 \rvert$	= 1.0318	δ_1	= 0.0100
$\lvert V_2 \rvert$	= 1.0080	δ_2	= 0.0163
$\lvert V_3 \rvert$	= 1.0068	δ_3	= 0.0372
$\lvert V_4 \rvert$	= 1.0047	δ_4	= 0.0385
$\lvert V_5 \rvert$	= 1.0007	δ_5	= 0.0259
$\lvert V_6 \rvert$	= 1.0150	δ_6	= 0.0961
$\lvert V_7 \rvert$	= 0.9963	δ_7	= 0.0614
$\lvert V_8 \rvert$	= 0.9947	δ_8	= 0.0107
$\lvert V_9 \rvert$	= 1.0253	δ_9	= 0.1521
$\lvert V_{10} \rvert$	= 1.0458	δ_{10}	= 0.2499
$\lvert V_{11} \rvert$	= 1.0052	δ_{11}	= 0.2035
$\lvert V_{12} \rvert$	= 1.0177	δ_{12}	= 0.3657
$\lvert V_{13} \rvert$	= 1.0077	δ_{13}	= 0.3297
$\lvert V_{14} \rvert$	= 1.0085	δ_{14}	= 0.3088
$\lvert V_{15} \rvert$	= 1.0052	δ_{15}	= 0.3126
$\lvert V_{16} \rvert$	= 1.0262	δ_{16}	= 0.3798
$\lvert V_{17} \rvert$	= 1.0252	δ_{17}	= 0.3675

Generator Buses

Q_{18}	= 0.5056	δ_{18}	= 0.0038
Q_{19}	= 0.7095	δ_{19}	= 0.0381
Q_{20}	= 0.6539	δ_{20}	= 0.2029
Q_{21}	= 2.0371	δ_{21}	= 0.4492
Q_{22}	= 1.2641	δ_{22}	= 0.4565

Slack bus

P_{23}	= -0.1529	Q_{23}	= 0.7391

TABLE XXXI

DETAILED LOAD FLOW SOLUTION OF 23-BUS SYSTEM FOR THE FIRST TRANSFORMER SCHEDULE

Quantity	Iteration 1	2	3	4
$Max\|\delta P_m\|$	0.421	0.968×10^{-2}	0.119×10^{-4}	0.132×10^{-10}
$Max\|\delta Q_m\|$	0.965	0.219×10^{-1}	0.157×10^{-4}	0.218×10^{-10}
$Max\|e_v\|$	0.181×10^{-1}	0.630×10^{-3}	0.795×10^{-6}	0.119×10^{-11}
$Max\|e_\delta\|$	0.714×10^{-1}	0.229×10^{-2}	0.232×10^{-5}	0.356×10^{-11}

$e_v \triangleq \delta|V_m|$, $e_\delta \triangleq \delta(\delta_m)$

TABLE XXXII

DETAILED LOAD FLOW SOLUTION OF 23-BUS SYSTEM FOR THE SECOND TRANSFORMER SCHEDULE

Quantity	Iteration 1	2	3	4
$Max\|\delta P_m\|$	0.429	0.175×10^{-1}	0.243×10^{-4}	0.412×10^{-10}
$Max\|\delta Q_m\|$	0.440	0.281×10^{-1}	0.265×10^{-4}	0.509×10^{-10}
$Max\|e_v\|$	0.211×10^{-1}	0.784×10^{-3}	0.102×10^{-5}	0.272×10^{-11}
$Max\|e_\delta\|$	0.734×10^{-1}	0.271×10^{-2}	0.339×10^{-5}	0.820×10^{-11}

$e_v \triangleq \delta|V_m|$, $e_\delta \triangleq \delta(\delta_m)$

TABLE XXXIII

LINE DATA FOR 26-BUS POWER SYSTEM

No.	Terminal Buses	Resistance R_t (p.u.)	Reactance X_t (p.u.)	1/2 Shunt Susceptance
1	13,26	0.0	0.0131	0.0
2	26,16	0.0	0.0392	0.0
3	16,23	0.0	0.4320	0.0
4	23,26	0.0	0.3140	0.0
5	2,10	0.0	0.0150	0.0
6	9,10	0.1494	0.3392	0.4120
7	9,12	0.0658	0.1494	0.0182
8	12,26	0.0533	0.1210	0.0147
9	9,14	0.0618	0.2397	0.0319
10	11,14	0.0676	0.2620	0.0349
11	19,26	0.0610	0.2521	0.0295
12	6,26	0.0513	0.1986	0.0265
13	6,19	0.0129	0.0532	0.0074
14	7,19	0.0906	0.3742	0.0437
15	6,7	0.0921	0.3569	0.0475
16	11,22	0.0513	0.2118	0.0248
17	8,11	0.0865	0.3355	0.0447
18	17,22	0.0281	0.1869	0.0237
19	8,21	0.0735	0.2847	0.0379
20	17,21	0.0459	0.3055	0.0387
21	1,4	0.0619	0.2401	0.0319
22	4,21	0.0610	0.2365	0.0315
23	20,21	0.0	0.0305	0.0
24	15,1	0.0	0.0147	0.0
25	2,13	0.0086	0.0707	0.3017
26	1,7	0.0199	0.0785	0.0404
27	15,20	0.0107	0.0617	0.4471
28	2,18	0.0074	0.0608	0.2593
29	1,3	0.0	0.0392	0.0
30	24,3	0.0	0.1450	0.0
31	5,21	0.0	0.1750	0.0
32	5,25	0.0	0.154	0.0

TABLE XXXIV

BUS DATA FOR LOAD FLOW ANALYSIS OF 26-BUS SYSTEM

Bus	Injected Power		Bus Voltage	
	P_m	Q_m	$\|V_m\|$	δ_m
1	-0.82	-0.21	-	-
2	0.0	0.0	-	-
3	-0.57	-0.17	-	-
4	-0.48	-0.21	-	-
5	-0.43	-0.11	-	-
6	-0.40	-0.10	-	-
7	-1.11	-0.27	-	-
8	-0.23	-0.06	-	-
9	-0.67	-0.21	-	-
10	-1.02	-0.27	-	-
11	-0.43	-0.14	-	-
12	-0.43	-0.12	-	-
13	0.0	0.0	-	-
14	0.0	0.0	-	-
15	0.0	0.0	-	-
16	-1.31	-0.30	-	-
17	-0.03	-0.01	-	-
18	2.80	-	1.07	-
19	1.45	-	1.05	-
20	2.80	-	1.00	-
21	1.10	-	1.02	-
22	-0.56	-	0.89	-
23	-0.04	-	1.00	-
24	-0.05	-	1.00	-
25	0.63	-	1.00	-
26	0.0	-	1.01	0.0

TABLE XXXV

TRANSFORMER TAPS FOR LOAD FLOW
SOLUTION OF 26-BUS SYSTEM

No.	Terminal Buses	Real	Imaginary
1	13,26	1.03	0.0
2	20,21	0.97	0.0
3	24,3	0.98	0.0
4	26,16	0.96	0.0
5	15,1	0.89	0.0
6	5,21	0.99	0.0
7	2,10	1.03	0.0
8	1,3	0.98	0.0
9	5,25	1.03	0.0

TABLE XXXVI

LOAD FLOW SOLUTION OF 26-BUS SYSTEM

Load Buses

$\lvert V_1\rvert$	= 1.0357	δ_1	= 0.0747
$\lvert V_2\rvert$	= 1.0685	δ_2	= 0.0884
$\lvert V_3\rvert$	= 1.0438	δ_3	= 0.0527
$\lvert V_4\rvert$	= 0.9908	δ_4	= 0.0989
$\lvert V_5\rvert$	= 1.0081	δ_5	= 0.2607
$\lvert V_6\rvert$	= 1.0339	δ_6	= 0.0536
$\lvert V_7\rvert$	= 1.0133	δ_7	= 0.0178
$\lvert V_8\rvert$	= 0.9450	δ_8	= 0.0426
$\lvert V_9\rvert$	= 0.9675	δ_9	= -0.1127
$\lvert V_{10}\rvert$	= 1.0393	δ_{10}	= 0.0667
$\lvert V_{11}\rvert$	= 0.9037	δ_{11}	= -0.1100
$\lvert V_{12}\rvert$	= 0.9699	δ_{12}	= -0.0764
$\lvert V_{13}\rvert$	= 1.0465	δ_{13}	= 0.0150
$\lvert V_{14}\rvert$	= 0.9449	δ_{14}	= -0.1136
$\lvert V_{15}\rvert$	= 0.9324	δ_{15}	= 0.1042
$\lvert V_{16}\rvert$	= 1.0363	δ_{16}	= -0.0455
$\lvert V_{17}\rvert$	= 0.9322	δ_{17}	= 0.0298

Generator Buses

Q_{18}	= -0.4004	δ_{18}	= 0.2385
Q_{19}	= 0.1872	δ_{19}	= 0.0921
Q_{20}	= 0.7795	δ_{20}	= 0.2432
Q_{21}	= -0.0294	δ_{21}	= 0.2270
Q_{22}	= -0.1775	δ_{22}	= -0.0996
Q_{23}	= -0.1144	δ_{23}	= -0.0266
Q_{24}	= -0.1645	δ_{24}	= 0.0459
Q_{25}	= 0.1691	δ_{25}	= 0.3599

Slack Bus

P_{26}	= 0.1334	Q_{26}	= -0.0513

TABLE XXXVII

DETAILED LOAD FLOW SOLUTION OF 26-BUS SYSTEM

Quantity	Iteration 1	2	3	4	5
$Max\|\delta P_m\|$	0.271	$0.116x10^{-1}$	$0.591x10^{-4}$	$0.106x10^{-8}$	$0.689x10^{-12}$
$Max\|\delta Q_m\|$	0.837	$0.389x10^{-1}$	$0.106x10^{-3}$	$0.985x10^{-9}$	$0.133x10^{-11}$
$Max\|e_v\|$	$0.528x10^{-1}$	$0.272x10^{-2}$	$0.828x10^{-5}$	$0.103x10^{-9}$	$0.662x10^{-13}$
$Max\|e_\delta\|$	$0.596x10^{-1}$	$0.462x10^{-2}$	$0.181x10^{-4}$	$0.256x10^{-9}$	$0.341x10^{-12}$

$e_v \triangleq \delta|V_m|$, $e_\delta \triangleq \delta(\delta_m)$

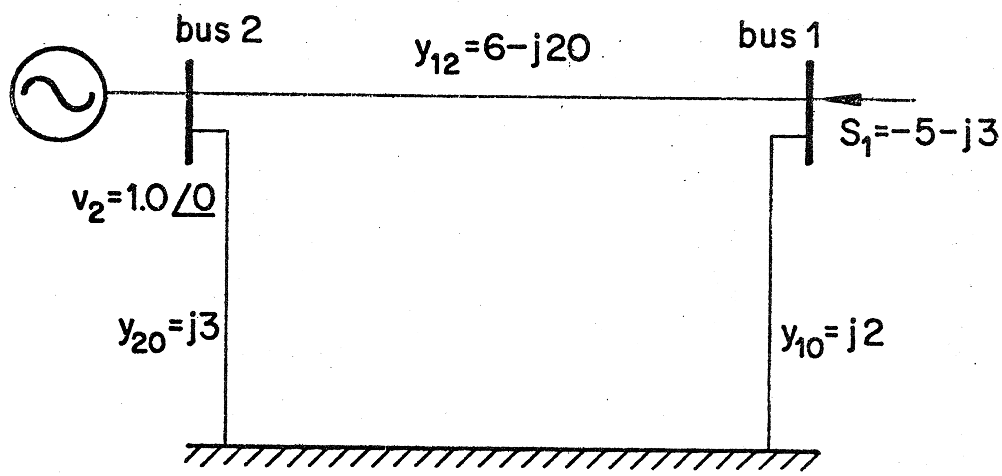

Fig. 1 2-bus power system

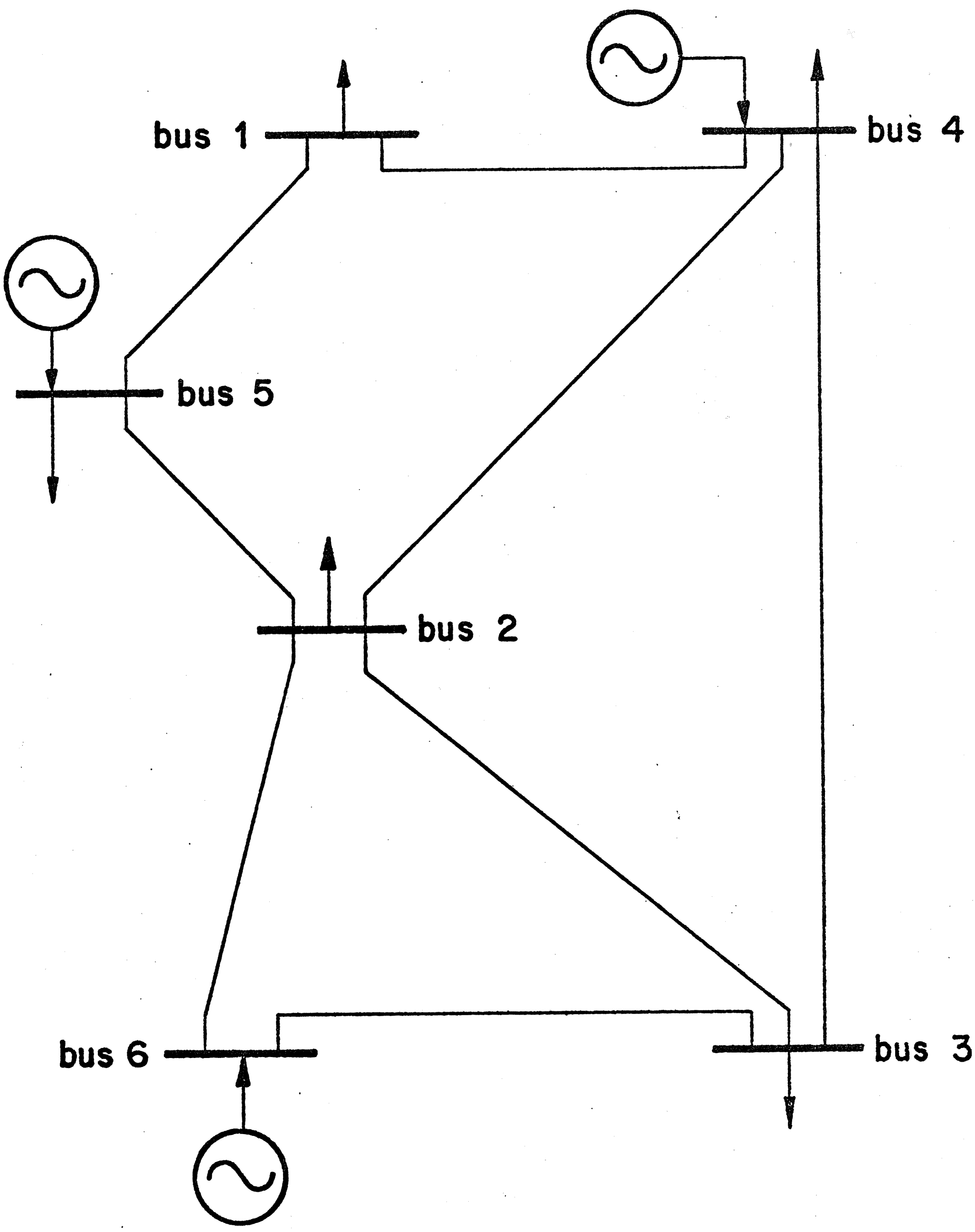

Fig. 2 6-bus power system

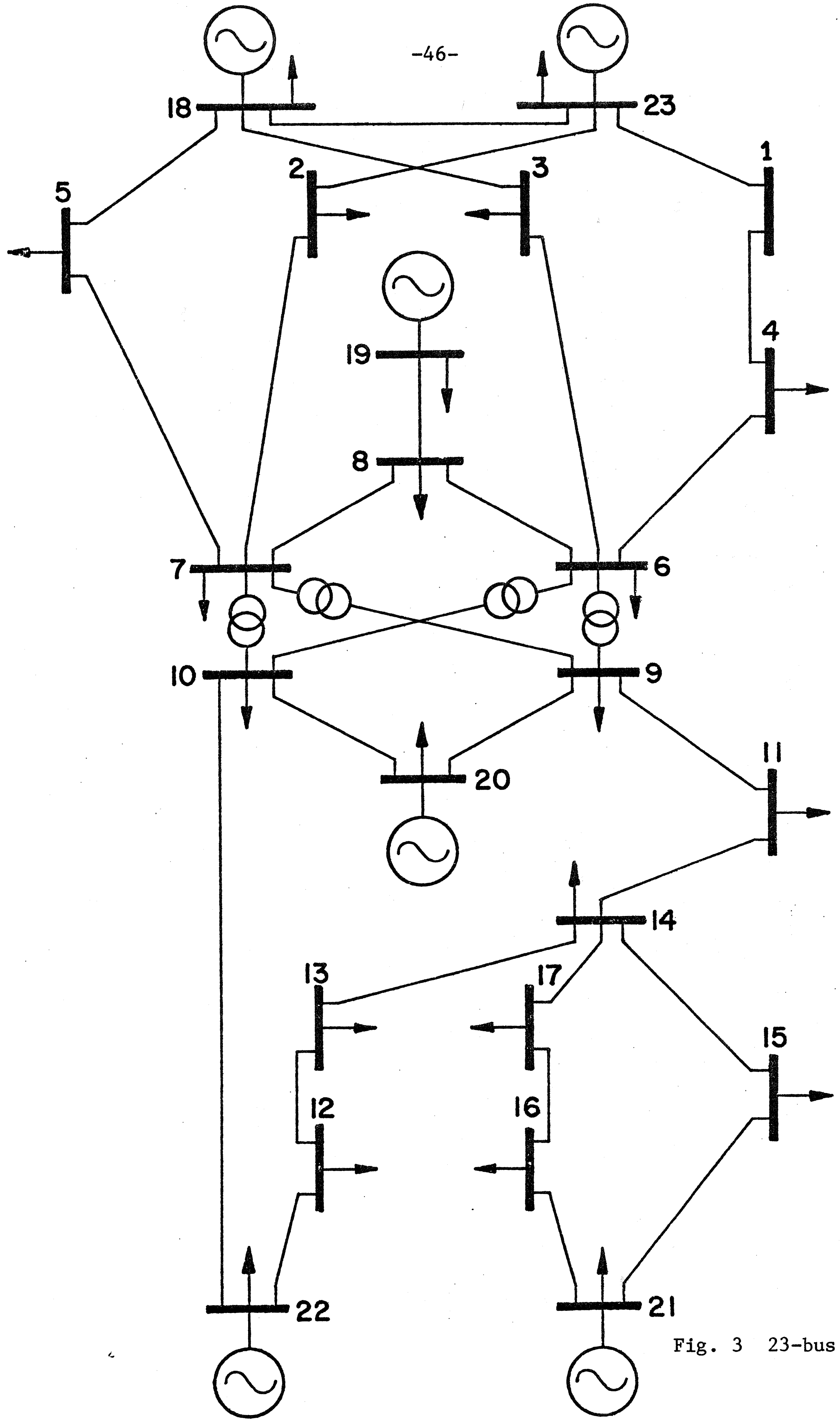

Fig. 3 23-bus power system

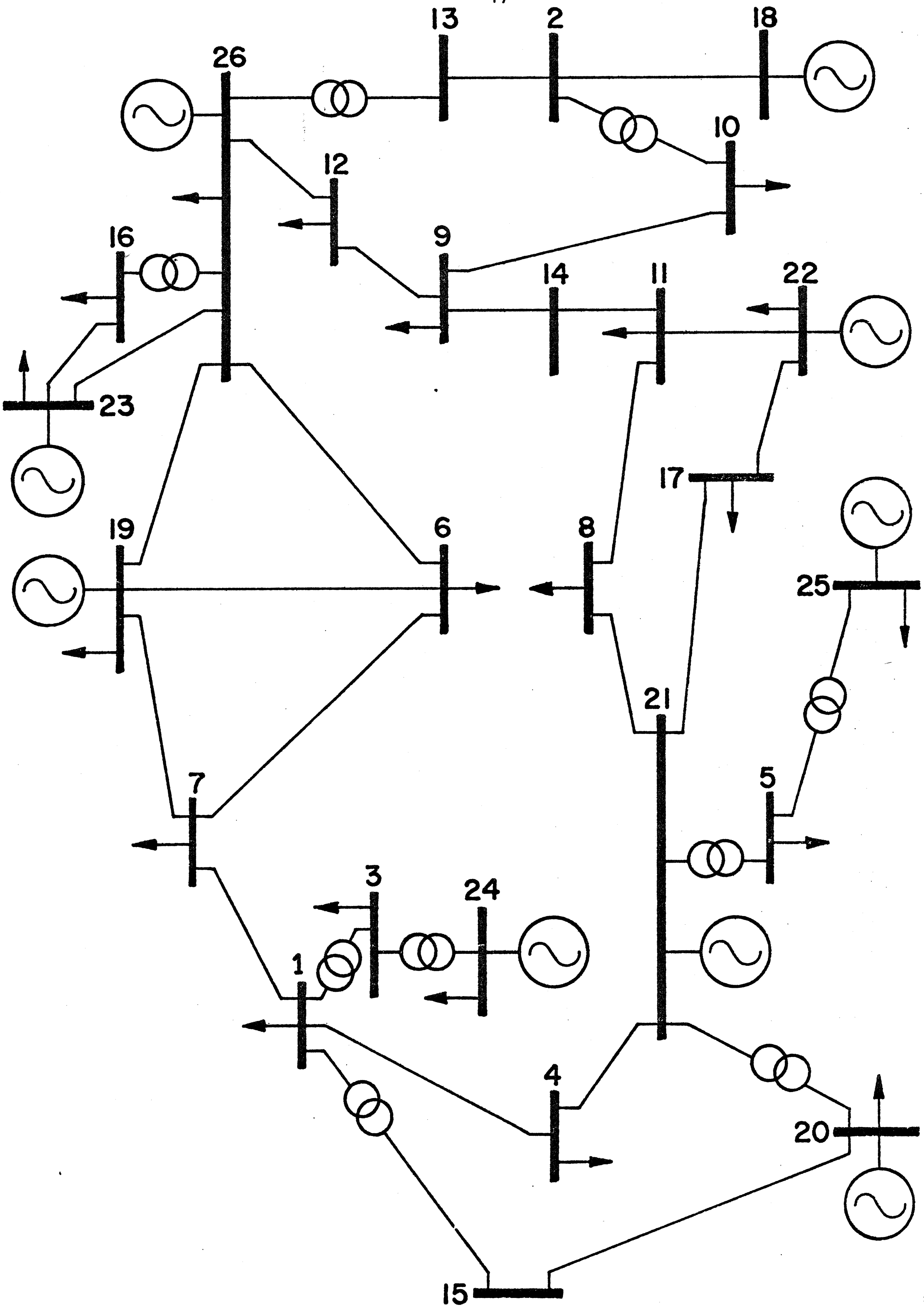

Fig. 4 26-bus power system

SOC-255

THE ADJOINT NETWORK APPROACH TO POWER FLOW SOLUTION AND SENSITIVITIES OF TEST POWER SYSTEMS: DATA AND RESULTS

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Revised:

Key Words: Power system analysis, adjoint network simulation, gradient-type optimization, contingency analysis, sensitivity calculations

Abstract: The exact, recently developed adjoint network approach to power network analysis is applied, in this paper, to a variety of test power systems ranging from the simplest 2 bus/3 line system to a 26 bus/32 line system. The full bus and line data as well as a single line diagram for all systems are provided. Results of the load flow analysis for all systems are presented. Detailed load flow sensitivities as well as contingency calculations for some of the systems are also presented. The general analytical aspects of the adjoint network approach are outlined. Some of its computational features are discussed for different test systems. We also illustrate the use of the results in further relevant applications.

Description:

Related Work: SOC-234, SOC-237, SOC-238, SOC-241, SOC-253.

Price: $30.00.