INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-254

A NEW METHOD FOR COMPUTERIZED SOLUTION OF POWER FLOW EQUATIONS

J.W. Bandler and M.A. El-Kady

October 1980

FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

A NEW METHOD FOR COMPUTERIZED SOLUTION OF POWER FLOW EQUATIONS

J.W. Bandler, Fellow, IEEE

M.A. El-Kady, Member, IEEE

Group on Simulation, Optimization and Control
Faculty of Engineering, McMaster University
Hamilton, Canada L8S 4L7

SUMMARY

In this paper, we employ the concept of the adjoint network simulation in the context of Tellegen's theorem to describe a new technique for solving the power flow equations and automatically supplying power network sensitivities.

The load flow problem comprises the solution of a set of nonlinear equations in the form

$$f(x) = u, \tag{1}$$

where x denotes a vector of dependent variables (system states) and u represents appropriate independent (control) variables. In a gradient-type iteration method, the form (1) is perturbed about a nominal point x^k at iteration k in the form

$$R^k \, \delta x^k = \delta u^k, \tag{2}$$

where

$$R^k = \begin{bmatrix} H^k & N^k \\ J^k & L^k \end{bmatrix} \tag{3}$$

is the Jacobian matrix of the Newton-Raphson method evaluated at iteration k, $\delta x^k = x^{k+1} - x^k$, $\delta u^k = u_{(\text{scheduled})} - u^k$ is a mismatch vector and δ denotes first-order change.

The adjoint network simulation in the context of Tellegen's theorem exploits the fact that (1) represents electrical network equations rather than general equality constraints. In the sensitivity approach based on Tellegen's theorem, employing the exact steady-state power model, the sensitivities of properly defined network state variables with respect to network control variables, at a base-case point, are supplied via one adjoint simulation and repeat forward and backward substitutions. This adjoint network simulation involves the formulation of a set of linear equations in the form

$$R^k_\tau \, \hat{y}^k_i = \hat{u}^k_i, \tag{4}$$

where i denotes different state variables, $\hat{u}^k_i$ is a simple vector having at most two non-zero elements and

$$R^k_\tau = \begin{bmatrix} H^k_\tau & N^k_\tau \\ J^k_\tau & L^k_\tau \end{bmatrix} \tag{5}$$

is an adjoint matrix of coefficients. Once equations (4) are solved for the adjoint variables $\hat{y}^k_i$, at iteration k, the sensitivities of the ith state variable w.r.t. all the defined control variables can be directly evaluated as linear functions of the corresponding elements of $\hat{y}^k_i$.

Now, if we define the network states to be the elements of x^k of (2) and the network control variables to be the vector u^k, then the sensitivities we obtain from the adjoint network simulation are essentially the elements of the matrix $(R^k)^{-1}$ of (2). Thence, equations (2) are readily solved.

From the above description of the adjoint method, it is evident that we have replaced the formulation and solution of (2) in the Newton-Raphson method by the adjoint formulation and solution of (4). This replacement offers the following far-reaching consequences.

1. The adjoint matrix of coefficients R^k_τ of (5), although of the same size and sparsity as the Jacobian matrix R^k of (3), is much simpler, mostly constant, comprising mainly line susceptances and conductances, applicable to both the rectangular and the polar formulations of the power flow equations, totally free, however, from trigonometric function evaluations and, above all, it permits approximate (and decoupled) versions in <u>both</u> modes of formulation by approximating <u>few</u> elements of R^k_τ (and discarding the matrices N^k_τ and J^k_τ).
2. The exact version of the method proposed enjoys the same rate of convergence as the Newton-Raphson method. The change from an approximate version with strictly constant matrix of coefficients to the exact one and vice versa, during the solution procedure, is accomplished by altering only the voltage-dependent elements of R^k_τ. These elements represent a relatively small portion of the matrix (mainly the diagonal elements of the matrices H^k_τ, N^k_τ, J^k_τ and L^k_τ).
3. Our method automatically supplies the sensitivities of all the dependent variables at the load flow solution without any additional adjoint simulation.

The paper includes the numerical results of applying the exact version of the method and one of its approximate versions to a 6-bus system as well as a 26-bus system (Saskatchewan Power Corporation). The following table summarizes the results of the exact (method I) and the approximate (method II) versions. In the approximate version, a few iterations (3 for the 6-bus system and 2 for the 26-bus system) of the decoupled version are performed and then the exact version is applied, via updating the voltage-dependent elements of the matrix of coefficients, to improve convergence w.r.t. nonsaturated bus voltages. In the table below we list Max{|δP|, |δQ|}, all values are in per unit and k denotes iteration number. Starting flat voltage profile is used.

k	6-Bus System I	6-Bus System II	26-Bus System I	26-Bus System II
1	0.554	1.208	0.837	0.881
2	0.295×10^{-1}	0.171	0.389×10^{-1}	0.558
3	0.166×10^{-3}	0.26×10^{-1}	0.106×10^{-3}	0.490×10^{-2}
4	0.663×10^{-8}	0.226×10^{-3}	0.106×10^{-8}	0.487×10^{-5}
5	0.171×10^{-12}	0.143×10^{-7}	0.134×10^{-11}	0.825×10^{-11}

A NEW METHOD FOR COMPUTERIZED SOLUTION OF POWER FLOW EQUATIONS

J.W. Bandler, Fellow, IEEE M.A. El-Kady, Member, IEEE

Group on Simulation, Optimization and Control
Faculty of Engineering, McMaster University
Hamilton, Canada L8S 4L7

Abstract - We employ an adjoint network concept based on an augmented form of Tellegen's theorem to describe a novel method for solving the load flow problem. The method incorporates successive adjoint network simulations with a sparse, mostly constant matrix of coefficients, the majority of its elements representing basic data of the problem already stored in computer memory. Nevertheless, the exact version of the method enjoys the same rate of convergence as the Newton-Raphson method. Moreover, it automatically supplies the sensitivities of all system states with respect to adjustable variables at the load flow solution without any additional adjoint simulation. An approximate version of the method is also presented. It partly employs very fast repeat forward and backward substitutions with constant LU factors of a reduced matrix of coefficients and is applicable to both the rectangular and the polar formulations of the power flow equations. Numerical examples are presented for illustration and comparison.

INTRODUCTION

The load flow problem [1], being solved in a wide variety of power system analysis and planning studies, is tackled in this paper in a new and different way. We employ the concept of the adjoint network simulation in the context of Tellegen's theorem [2] to describe a technique for solving the power flow equations.

We devote the first few sections to describing the novel method and to illustrating its analytical features in comparison with the Newton-Raphson method [3]. We then illustrate the implementation of the exact, approximate and decoupled versions of the method.

This paper aims mainly at introducing the new method and illustrating its analytical and general computational aspects. The results of two (6-bus and 26-bus) test power systems are presented which, we feel, amply serve the purpose of the paper.

PRINCIPAL NOTATION

n	number of buses, also index of the slack bus
n_B	number of branches in the network
n_L	number of P, Q-type (load) buses
n_G	number of P, V-type (generator) buses
ℓ	$= 1, 2, \ldots, n_L$ denotes a load bus
g	$= n_L+1, \ldots n_L+n_G$ denotes a generator bus
t	$= n+1, \ldots, n_B$ denotes transmission elements
m	$= \ell$, g or n, denotes bus number
$S_m=P_m+jQ_m$	complex power at bus m
$\tilde{S}_M$	vector of bus powers
$V_m=\lvert V_m\rvert\angle\theta_m$	complex voltage of bus m
I_m	current in branch m
$\tilde{V}_M$	vector of bus voltages
$\tilde{Y}_T$	bus admittance matrix
$\tilde{x}$	real vector of dependent variables
$\tilde{u}$	real vector of independent variables
j	denotes $\sqrt{-1}$
k	iteration count
δ	denotes first-order change
$*$	denotes the complex conjugate
$\hat{}$	distinguishes adjoint network variables
$\tilde{}$	identifies vectors and matrices

BACKGROUND

The load flow problem comprises the solution of a set of nonlinear equations of the form

$$\tilde{E}_M^* \tilde{Y}_T \tilde{V}_M = \tilde{S}_M^*, \tag{1}$$

where $\tilde{E}_M$ is a diagonal matrix of elements of $\tilde{V}_M$ in a corresponding order. The set of equations (1) are normally solved in either the rectangular or the polar forms. When rearranging (1) for the unknown variables, in the appropriate mode of formulation, the resulting set of real equations is written in the general form

$$\tilde{f}(\tilde{x}) = \tilde{u}. \tag{2}$$

The order of (2) is 2n-2 for the rectangular form and $2n-n_G-2$ for the polar form.

In a gradient-type iterative method, the form (2) is perturbed about a nominal point $\tilde{x}^k$ at iteration k as

$$\tilde{R}^k \, \delta\tilde{x}^k = \delta\tilde{u}^k, \tag{3}$$

where

$$\tilde{R}^k = \begin{bmatrix} \tilde{H}^k & \tilde{N}^k \\ \tilde{J}^k & \tilde{L}^k \end{bmatrix} \tag{4}$$

is the Jacobian matrix evaluated at iteration k, $\delta\tilde{x}^k = \tilde{x}^{k+1}-\tilde{x}^k$ and $\delta\tilde{u}^k = \tilde{u}_{(scheduled)}-\tilde{u}^k$ is a mismatch vector.

The Newton-Raphson Method

In the Newton-Raphson method of solving the power flow equations (NRM), the set of linear equations (3) is solved at each iteration for the perturbed variables

$$\delta \underset{\sim}{x}^k = (\underset{\sim}{R}^k)^{-1}\ \delta \underset{\sim}{u}^k. \tag{5}$$

The computational burden per iteration consists mainly of evaluating the elements of the Jacobian matrix $\underset{\sim}{R}^k$, calculating its LU factors and performing the subsequent forward and backward substitutions. The elements of $\underset{\sim}{R}^k$ are voltage-dependent and have to be updated and stored at each iteration. In the polar formulation this task is reduced by discarding the matrices $\underset{\sim}{N}^k$ and $\underset{\sim}{J}^k$ of (4), in the Newton's decoupled load flow [4] and eliminated in the fast decoupled load flow [5] by further approximations to the matrices $\underset{\sim}{H}^k$ and $\underset{\sim}{L}^k$ of (4). This is done at the cost of the rate of convergence.

The Tellegen's Theorem Method

The adjoint network simulation in the context of Tellegen's theorem exploits the fact that (2) represents electrical network equations rather than general equality constraints. In the sensitivity approach based on Tellegen's theorem [6,7] employing the exact a.c. power model, the sensitivities of properly defined network state (dependent) variables with respect to network control (independent) variables at a base-case point are supplied via one adjoint simulation and repeat forward and backward substitutions. In the proposed application of this method to the power flow solution, we define the network states to be the elements of $\underset{\sim}{x}^k$ of (3) and the network control variables to be the vector $\underset{\sim}{u}^k$. Hence, the sensitivities of $\underset{\sim}{x}^k$ w.r.t. $\underset{\sim}{u}^k$ obtained from the adjoint simulation are essentially the elements of the matrix $(\underset{\sim}{R}^k)^{-1}$ of (5). Therefore, the vector $\delta \underset{\sim}{x}^k$ of (3) can be directly calculated knowing the mismatch vector $\delta \underset{\sim}{u}^k$.

Now, the adjoint network simulation involves the formulation of the linear equations

$$\underset{\sim}{T}^k\ \hat{\underset{\sim}{x}}_i^k = \hat{\underset{\sim}{b}}_i^k, \tag{6}$$

where i denotes different elements of $\underset{\sim}{x}^k$, $\hat{\underset{\sim}{b}}_i^k$ is a simple vector having at most two non-zero elements and $\underset{\sim}{T}^k$ is an adjoint matrix of coefficients. Equations (6) are to be solved for the adjoint variables $\hat{\underset{\sim}{x}}_i^k$ and then the sensitivities of x_i^k w.r.t. the elements of $\underset{\sim}{u}_i^k$ are, simply, linear functions of the corresponding elements of $\hat{\underset{\sim}{x}}_i^k$.

From the above description of the adjoint method, it is evident that we have replaced the formulation and solution of (3) in the NRM by the adjoint formulation and solution of (6). Hence both methods, when applied without approximations, create the same sequence of iterative solution points. Therefore, they have the same rate of convergence. The evaluation of the two methods must be on the basis of the computational effort and storage requirements involved in (3) and (6).

STRUCTURE OF ADJOINT EQUATIONS

In the following, we summarize the specific structure of (6). A summary of the main derivation steps is presented in the Appendix. For more details, the reader is referred to [6-8].

The system of linear equations (6) has [7] the general, detailed structure

$$\begin{bmatrix} (\underset{\sim}{G}_{LL}+\underset{\sim}{\Psi}_{L1}) & \underset{\sim}{G}_{LG} & (-\underset{\sim}{B}_{LL}+\underset{\sim}{\Psi}_{L2}) & -\underset{\sim}{B}_{LG} \\ \bar{\underset{\sim}{B}}_{GL} & (\bar{\underset{\sim}{B}}_{GG}-\underset{\sim}{\Psi}_{G2}) & \bar{\underset{\sim}{G}}_{GL} & (\bar{\underset{\sim}{G}}_{GG}+\underset{\sim}{\Psi}_{G1}) \\ (\underset{\sim}{B}_{LL}+\underset{\sim}{\Psi}_{L2}) & \underset{\sim}{B}_{LG} & (\underset{\sim}{G}_{LL}-\underset{\sim}{\Psi}_{L1}) & \underset{\sim}{G}_{LG} \\ \underset{\sim}{0} & \mathrm{diag}\{V_{g2}\} & \underset{\sim}{0} & \mathrm{diag}\{V_{g1}\} \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{V}}_{L1} \\ \hat{\underset{\sim}{V}}_{G1} \\ \hat{\underset{\sim}{V}}_{L2} \\ \hat{\underset{\sim}{V}}_{G2} \end{bmatrix} = \begin{bmatrix} \hat{\underset{\sim}{I}}_{L1} \\ \hat{\underset{\sim}{I}}_{G1} \\ \hat{\underset{\sim}{I}}_{L2} \\ \hat{\underset{\sim}{I}}_{G2} \end{bmatrix}, \tag{7}$$

where L and G denote load and generator buses, respectively, subscripts 1 and 2 denote, respectively, the real and imaginary parts of the quantities $\hat{\underset{\sim}{I}}_L$, $\hat{\underset{\sim}{I}}_G$, V_g, $\underset{\sim}{\Psi}_L = \mathrm{diag}\{-S_\ell/V_\ell^2\}$ and $\underset{\sim}{\Psi}_G = \mathrm{diag}\{S_g/V_g\}$. The bus admittance matrix of the network $\underset{\sim}{Y}_T$, with the minor adjustments [8] to include phase-shifting transformers, has been partitioned in the form

$$\underset{\sim}{Y}_T = \underset{\sim}{G}_T + j\underset{\sim}{B}_T = \begin{bmatrix} \underset{\sim}{Y}_{LL} & \underset{\sim}{Y}_{LG} \\ \underset{\sim}{Y}_{GL} & \underset{\sim}{Y}_{GG} \end{bmatrix}. \tag{8}$$

Also, in (7), $\bar{\underset{\sim}{G}}_{GL} + j\bar{\underset{\sim}{B}}_{GL} = \mathrm{diag}\{V_g\}\ \underset{\sim}{Y}_{GL}$ and $\bar{\underset{\sim}{G}}_{GG} + j\bar{\underset{\sim}{B}}_{GG} = \mathrm{diag}\{V_g\}\ \underset{\sim}{Y}_{GG}$.

The form (7) is common to both the rectangular and the polar forms of the power flow equations. The elements of the vectors $\hat{\underset{\sim}{I}}_L$ and $\hat{\underset{\sim}{I}}_G$ which constitute the RHS of (7) are given by Table I. Observe that each of $\hat{\underset{\sim}{I}}_L$ and $\hat{\underset{\sim}{I}}_G$ has at most one non-zero component. The solution of (7) is then substituted to obtain the sensitivities of the dependent variables. The expression for δx_i^k is given by

$$\delta x_i^k = -\hat{\underset{\sim}{\eta}}_u^T\ \delta \underset{\sim}{u}^k, \tag{9}$$

where $\hat{\eta}_u$, which constitutes elements of $(\underset{\sim}{R}^k)^{-1}$ of (5), is given by Table II.

TABLE I

RHS VECTOR OF THE ADJOINT EQUATIONS

Mode of Formulation	Dependent Variable	Element $\hat{I}_\ell$ ℓ=m	$\ell \neq$m	Element $\hat{I}_g$ g=m	g$\neq$m
Rectangular	V_{m1}	-1	0	$-V_{g2}$	0
	V_{m2}	j	0	V_{g1}	0
Polar	$\lvert V_m \rvert$	$-\lvert V_\ell \rvert / V_\ell$	0	0	0
	θ_m	j/V_ℓ	0	1	0

TABLE II

SENSITIVITIES w.r.t. INDEPENDENT VARIABLES

Independent Variable	Corresponding Element of $\hat{\underset{\sim}{\eta}}_u$
P_ℓ	$\mathrm{Re}\{\hat{V}_\ell/V_\ell^*\}$
P_g	$\mathrm{Re}\{\hat{V}_g/V_g^*\}$
Q_ℓ	$\mathrm{Im}\{\hat{V}_\ell/V_\ell^*\}$
$\lvert V_g \rvert$	$-\mathrm{Re}\{\hat{I}_g\ V_g + \hat{V}_g^*\ I_g^*\}/\lvert V_g \rvert$

FEATURES OF LOAD FLOW ANALYSIS USING TELLEGEN'S THEOREM

The set of linear equations to be solved each iteration, in the Tellegen's theorem method of solving the power flow equations (TTM), is of form (7). The adjoint matrix of coefficients of (7) is much simpler than the Jacobian matrix $\underset{\sim}{R}^k$ of equations (3) to be solved each iteration in the NRM. As is clear from (7), the majority of elements of the adjoint matrix are line conductances and susceptances representing basic data of the problem available and already stored in computer memory. Moreover, they are constants and do not have to be updated at each iteration. Observe that, in the case when no voltage-controlled buses are considered, these constant elements represent all the off-diagonal elements of the submatrices in (7). On the other hand, the elements of the Jacobian matrix of the NRM reflects mainly partial derivatives of bus powers w.r.t bus voltages. These elements are voltage-dependent and they have to be recalculated whenever the bus voltages are altered.

It is to be noticed, however, that several forward and backward substitutions are required (at least from the theoretical point of view) in each iteration of the TTM. In the NRM, only one forward and one backward substitution is required.

The overall computational effort in any of the two methods is, hence, evaluated based upon the whole process of updating the matrix of coefficients, factorizing it and performing the forward and backward substitutions. From the preliminary experience we have, we find that the overall computational effort (not the storage) of each method depends on the network size and configuration, the mode of formulation and the number of P,V-type buses considered. The TTM was found superior for medium networks analyzed in the polar coordinates with fewer voltage-controlled buses. For large networks, however, the NRM, in rectangular coordinates, applied with sparsity utilization is superior due to the increasing effort of performing the forward and backward substitutions when applying the exact version of the TTM. It is to be remarked that this general statement is valid only when applying the two methods in an exact way. It is not applicable, for example, to the case when only some of the variables (those which do not reach their saturation values) are to be updated in each iteration of the TTM. It is also not applicable to the use of decoupled versions of the two methods as will be illustrated in subsequent sections.

As stated before, the form of the adjoint matrix of (7) is common to both the rectangular and the polar formulations of the power flow equations. Hence, our formulation eliminates the trigonometric function evaluations in calculating the voltage-dependent elements of the matrix of coefficients when the polar form is used. Observe that the trigonometric functions are computationally more time-consuming than the simple operations involved through the use of (7).

The number of equations of (7) is 2n-2. However, in the polar formulation, the vector $\hat{I}_{G2}$ is zero from Table I. Hence, the set of equations corresponding to $\hat{I}_{G2}$ can be easily omitted by eliminating the variables $\hat{V}_{G2}$. This will reduce the order of (7) to $2n - n_G - 2$ while preserving the sparsity structure. Therefore, we conclude that the adjoint matrix of coefficients has the same size and sparsity as (but is simpler than) the Jacobian matrix of the NRM in any mode of formulation.

It is important to remark that, in the proposed method for solving the power flow equations, the sensitivities of all the dependent variables (system states) in the power flow equations w.r.t. bus control variables are readily available at the load flow solution without further adjoint simulation. The 2n-2 forward and backward substitutions, which would be required to obtain these sensitivities by the Lagrange multiplier approach [9], are already performed in the TTM and the results are readily available. In addition, the sensitivities w.r.t. line variables can be obtained directly by substitution into appropriate formulas [6,7] similar to those of Table II.

APPLICATIONS

In this section, we illustrate the practical implementation of the exact version of the TTM for solving the power flow equations described in the last two sections.

Algorithm

(i) Set $k \leftarrow 0$.

(ii) Calculate $u^k = f(x^k)$, x^0 is assumed.

(iii) Evaluate those elements of the adjoint matrix T^k of (7) required to be updated.

(iv) Using the LU factors of T^k, solve the linear equations (6) for different i.

(v) From the solution of (6), evaluate the vector $\hat{\eta}_u$ of (9) using the expressions of Table II.

(vi) Update the dependent variables using

$$x_i^{k+1} = x_i^k - \hat{\eta}_u^T \; \delta u^k ,$$

where $\delta u^k = u_{(scheduled)} - u^k$.

(vii) If convergence is attained stop, otherwise set $k \leftarrow k+1$ and go to (ii).

Simple Example

Consider the simple 2-bus example [3] shown in Fig. 1, which consists of a load (ℓ=1), a slack generator (n=2) and one transmission line (t=3). Equations (9) have the form

$$\begin{bmatrix} 1.8 + \psi_{\ell 1} & 11.0 + \psi_{\ell 2} \\ -11.0 + \psi_{\ell 2} & 1.8 - \psi_{\ell 1} \end{bmatrix} \begin{bmatrix} \hat{V}_{\ell 1} \\ \hat{V}_{\ell 2} \end{bmatrix} = \begin{bmatrix} \hat{I}_{\ell 1} \\ \hat{I}_{\ell 2} \end{bmatrix} .$$

where $\psi_{\ell 1} + j\psi_{\ell 2} = -S_\ell / V_\ell^2$ and, from Table I,

$$\left.\begin{array}{l} \hat{I}_{\ell 1} = -1, \; \hat{I}_{\ell 2} = 0 \text{ for sensitivities of } V_{\ell 1} \\ \hat{I}_{\ell 1} = \;\; 0, \; \hat{I}_{\ell 2} = 1 \text{ for sensitivities of } V_{\ell 2} \end{array}\right\} ,$$

where the rectangular coordinates have been used, 1 and 2 denoting real and imaginary parts.

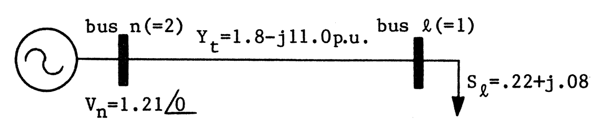

Fig. 1 2-bus sample power system

Table III shows the results obtained at successive iterations. The initial value of V_ℓ is 1 + j0. It can be shown that these results are identical to those obtained by applying the NRM. The value of V_ℓ at the solution is 1.2013 - j0.0151.

Applications to Test Power Systems

In this paper we consider two power systems (6-bus and 26-bus) to illustrate the analytical and general computational aspects of the method introduced. The detailed data of the 6-bus system can be found in [7]. For the structure and the line data of the 26-bus system (Saskatchewan Power Corporation System), the reader is referred to [10]. Table IV shows the operating bus data used. The net injected powers are shown.

Table V shows the principal results obtained by applying the exact version of the TTM to both power systems. Polar coordinates with starting flat voltage profile have been used. All values shown are in per unit. The computations have been performed on a CYBER 170 computer. As illustrated before, the results of Table V are identical to those obtained by applying the NRM.

TABLE III

EXAMPLE OF LOAD FLOW SOLUTION USING TTM

Quantity	Iteration			
	1	2	3	4
δP_ℓ	-0.1580	0.1155	0.0044	0.0000
δQ_ℓ	-2.2300	0.7056	0.0270	0.0000
$dV_{\ell 1}/dP_\ell$	-0.0183	-0.0129	-0.0140	-0.0140
$dV_{\ell 1}/dQ_\ell$	-0.1121	-0.0681	-0.0737	-0.0739
$\delta V_{\ell 1}$	0.2528	-0.0495	-0.0020	-0.0000
$dV_{\ell 2}/dP_\ell$	-0.0732	-0.0732	-0.0732	-0.0732
$dV_{\ell 2}/dQ_\ell$	0.0120	0.0120	0.0120	0.0120
$\delta V_{\ell 2}$	-0.0151	0.0000	0.0000	0.0000

TABLE IV

BUS DATA FOR 26-BUS SYSTEM

Bus m	$\lvert V_m \rvert$	θ_m	P_m	Q_m
1	-	-	-0.82	-0.21
2	-	-	0.0	0.0
3	-	-	-0.57	-0.17
4	-	-	-0.48	-0.21
5	-	-	-0.43	-0.11
6	-	-	-0.40	-0.10
7	-	-	-1.11	-0.27
8	-	-	-0.23	-0.06
9	-	-	-0.67	-0.21
10	-	-	-1.02	-0.27
11	-	-	-0.43	-0.14
12	-	-	-0.43	-0.12
13	-	-	0.0	0.0
14	-	-	0.0	0.0
15	-	-	0.0	0.0
16	-	-	-1.31	-0.30
17	-	-	-0.03	-0.01
18	1.07	-	2.80	0.0
19	1.05	-	1.45	0.0
20	1.0	-	2.80	0.0
21	1.02	-	1.10	0.0
22	0.89	-	-0.56	0.0
23	1.0	-	-0.04	0.0
24	1.0	-	-0.05	0.0
25	1.0	-	0.63	0.0
26	1.01	0.0	0.0	0.0

Transformer tap ($a_{mm'}$) between buses m and m'

$a_{13,26} = 1.03$, $a_{26,16} = 0.96$, $a_{2,10} = 1.03$

$a_{20,21} = 0.97$, $a_{15,1} = 0.89$, $a_{1,3} = 0.98$

$a_{24,3} = 0.98$, $a_{5,21} = 0.99$, $a_{5,25} = 1.03$

Bus Types

$n_L = 17$, $n_G = 8$

TABLE V

RESULTS OF EXACT TELLEGEN'S THEOREM METHOD

Quantity		6-Bus System	26-Bus System
Largest Initial Mismatches	$\lvert \delta P \rvert$	2.2824	2.80000
	$\lvert \delta Q \rvert$	0.7373	8.9999
Largest Final Mismatches	$\lvert \delta P \rvert$	0.463×10^{-8}	0.106×10^{-8}
	$\lvert \delta Q \rvert$	0.663×10^{-8}	0.985×10^{-9}
No. of Iterations		4	4

THE DECOUPLED VERSIONS OF TELLEGEN'S LOAD FLOW

The implementation of the exact version of the TTM, although possessing the quadratic rate of convergence of the NRM with a simpler, mostly constant matrix of coefficients, may not be practically justified for very large power networks due to the increasing computational burden per iteration. As in the NRM, where efficient decoupled verisons have been successively developed [4,5], the TTM can also be applied in decoupled and approximate forms as illustrated in this section.

Primal Formulation

In order to facilitate the subsequent derivation and illustration of the decoupled forms of the TTM, we first rearrange (7) to be in the form

$$\begin{bmatrix} \underset{\sim}{H}^k_\tau & \underset{\sim}{N}^k_\tau \\ \underset{\sim}{J}^k_\tau & \underset{\sim}{L}^k_\tau \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{e}}^k \\ \hat{\underset{\sim}{f}}^k \end{bmatrix} = \begin{bmatrix} \hat{\underset{\sim}{c}}^k \\ \hat{\underset{\sim}{d}}^k \end{bmatrix}, \qquad (10)$$

where the subscript τ is to distinguish the formulation of the TTM from that of (4) of the NRM. The vectors $\hat{\underset{\sim}{e}}^k$, $\hat{\underset{\sim}{f}}^k$, $\hat{\underset{\sim}{c}}^k$ and $\hat{\underset{\sim}{d}}^k$ are related to corresponding vectors of (7) by

$$\hat{\underset{\sim}{e}}^k = \begin{bmatrix} \hat{\underset{\sim}{V}}_{L2} \\ \hat{\underset{\sim}{V}}_{G2} \end{bmatrix}, \quad \hat{\underset{\sim}{f}}^k = \begin{bmatrix} \hat{\underset{\sim}{V}}_{L1} \\ \hat{\underset{\sim}{V}}_{G1} \end{bmatrix}, \quad \hat{\underset{\sim}{c}}^k = \begin{bmatrix} \hat{\underset{\sim}{I}}_{L1} \\ \hat{\underset{\sim}{I}}_{G2} \end{bmatrix} \text{ and } \hat{\underset{\sim}{d}}^k = \begin{bmatrix} \hat{\underset{\sim}{I}}_{L2} \\ \hat{\underset{\sim}{I}}_{G1} \end{bmatrix}, \qquad (11)$$

hence, the submatrices of (10) are given by

$$\underset{\sim}{H}^k_\tau = \begin{bmatrix} (-\underset{\sim}{B}_{LL} + \underset{\sim}{\Psi}_{L2}) & -\underset{\sim}{B}_{LG} \\ \underset{\sim}{0} & \text{diag}\{V_{g1}\} \end{bmatrix}, \quad \underset{\sim}{N}^k_\tau = \begin{bmatrix} (\underset{\sim}{G}_{LL} + \underset{\sim}{\Psi}_{L1}) & \underset{\sim}{G}_{LG} \\ \underset{\sim}{0} & \text{diag}\{V_{g2}\} \end{bmatrix}, \qquad (12)$$

$$\underset{\sim}{J}^k_\tau = \begin{bmatrix} (\underset{\sim}{G}_{LL} - \underset{\sim}{\Psi}_{L1}) & \underset{\sim}{G}_{LG} \\ \bar{\underset{\sim}{G}}_{GL} & (\bar{\underset{\sim}{G}}_{GG} + \underset{\sim}{\Psi}_{G1}) \end{bmatrix} \text{ and } \underset{\sim}{L}^k_\tau = \begin{bmatrix} (\underset{\sim}{B}_{LL} + \underset{\sim}{\Psi}_{L2}) & \underset{\sim}{B}_{LG} \\ \bar{\underset{\sim}{B}}_{GL} & (\bar{\underset{\sim}{B}}_{GG} - \underset{\sim}{\Psi}_{G2}) \end{bmatrix}. \qquad (13)$$

Observe that, in the formulation above, and under the assumption of flat voltage profile, the off-diagonal elements of the matrices $\underset{\sim}{H}^k_\tau$ and $\underset{\sim}{L}^k_\tau$ comprise line susceptances while those of the matrices $\underset{\sim}{N}^k_\tau$ and $\underset{\sim}{J}^k_\tau$ comprise line conductances. Upon neglecting the matrices $\underset{\sim}{N}^k_\tau$ and $\underset{\sim}{J}^k_\tau$ w.r.t. $\underset{\sim}{H}^k_\tau$ and $\underset{\sim}{L}^k_\tau$, a decoupled structure of the TTM similar to that of NRM is obtained.

Features of Decoupled Versions of TTM

We state some of the pioneering features of the decoupled versions of the TTM and the main aspects which may be exploited in developing improved decoupled versions.

(i) The decoupling principle is applicable in the TTM to both the polar and the rectangular formulations of the power flow equations. The exploitation of this fact can lead to the construction of more efficient decoupled versions in the rectangular formulation where the trigonometric function evaluations are totally eliminated. Note that, in the NRM, the decoupling principle is valid in the polar formulation only.

(ii) An approximate version with strictly constant matrix of coefficients can be reached in the TTM by approximating few elements. This is clear from the structure of the matrix of coefficients in (10) where most of its elements are already constant.

(iii) The symmetry of the matrices in the decoupled TTM can be attained via some approximations regarding the modelling of the phase shifters [8]. This usually leads to more efficient computations [5].

(iv) The structure of (10)-(12) developed in the context of Tellegen's theorem provides valuable, explicit information about the degree of approximation in the decoupled versions of both NRM and TTM. The voltage-dependent elements (the $\underset{\sim}{\Psi}$ matrices) in (10) are mainly diagonally added to the constant $\underset{\sim}{G}$ and $\underset{\sim}{B}$ matrices. This may be exploited in constructing a hybrid exact/decoupled version in which the exact version is to be applied at the final iterations to improve the convergence w.r.t. certain bus voltages. This procedure will be followed in one of the approximate versions presented in the next section. It also has the advantage of providing more accurate sensitivity information at the load flow solution.

APPLICATIONS OF THE APPROXIMATE TTM

Neglecting the $\underset{\sim}{\Psi}$ matrices in (12) and (13) and assuming flat voltage profile the matrices $\underset{\sim}{H}_{\tau}^{k}$, $\underset{\sim}{N}_{\tau}^{k}$, $\underset{\sim}{J}_{\tau}^{k}$ and $\underset{\sim}{L}_{\tau}^{k}$ are reduced to constant matrices. Hence, we reach a constant matrix of coefficients of (10) in the form

$$\underset{\sim}{R}_{\tau} = \begin{bmatrix} -\underset{\sim}{B}_{LL} & -\underset{\sim}{B}_{LG} & \underset{\sim}{G}_{LL} & \underset{\sim}{G}_{LG} \\ \underset{\sim}{0} & \underset{\sim}{1} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{G}_{LL} & \underset{\sim}{G}_{LG} & \underset{\sim}{B}_{LL} & \underset{\sim}{B}_{LG} \\ \underset{\sim}{G}_{GL} & \underset{\sim}{G}_{GG} & \underset{\sim}{B}_{GL} & \underset{\sim}{B}_{GG} \end{bmatrix}. \qquad (14)$$

As pointed out before, the off-diagonal block matrices of $\underset{\sim}{R}_{\tau}$ may be discarded. This leads to a decoupled version with two sets of equations to be solved at each iteration.

Applications to Test Power Systems

In our paper, the results of one approximate version are presented. A few iterations (3 for the first system and 2 for the second system) of the decoupled version are performed and then the exact version is applied, via updating the voltage-dependent elements of the matrix of coefficients, to improve convergence w.r.t. nonsaturated bus voltages.

In Tables VI and VII, we list the results of this approximate version (method C) as well as the results of a fast decoupled version (method B) of the NRM with no adjustments to the matrix of coefficients. The corresponding results of the exact TTM are also shown (method A). All values are, again, in per unit.

TABLE VI

RESULTS OF DIFFERENT VERSIONS FOR 6-BUS SYSTEM

		Iteration No. 1	2	3	4
MAX $\|\delta P\|$	A	0.167	$0.127\text{x}10^{-1}$	$0.948\text{x}10^{-4}$	$0.463\text{x}10^{-8}$
	B	0.205	0.167	$0.116\text{x}10^{-1}$	$0.251\text{x}10^{-2}$
	C	0.201	0.171	$0.220\text{x}10^{-1}$	$0.105\text{x}10^{-3}$
MAX $\|\delta Q\|$	A	0.554	$0.295\text{x}10^{-1}$	$0.166\text{x}10^{-3}$	$0.663\text{x}10^{-8}$
	B	1.221	$0.843\text{x}10^{-1}$	$0.287\text{x}10^{-1}$	$0.133\text{x}10^{-1}$
	C	1.208	$0.601\text{x}10^{-1}$	$0.260\text{x}10^{-1}$	$0.226\text{x}10^{-3}$
MAX $\|e_V\|$	A	$0.463\text{x}10^{-1}$	$0.367\text{x}10^{-1}$	$0.244\text{x}10^{-4}$	$0.109\text{x}10^{-8}$
	B	$0.836\text{x}10^{-1}$	$0.583\text{x}10^{-2}$	$0.233\text{x}10^{-2}$	$0.835\text{x}10^{-3}$
	C	$0.833\text{x}10^{-1}$	$0.558\text{x}10^{-2}$	$0.397\text{x}10^{-2}$	$0.312\text{x}10^{-4}$
MAX $\|e_\theta\|$	A	$0.698\text{x}10^{-1}$	$0.649\text{x}10^{-2}$	$0.434\text{x}10^{-4}$	$0.185\text{x}10^{-8}$
	B	$0.118\text{x}10^{-1}$	$0.413\text{x}10^{-1}$	$0.480\text{x}10^{-2}$	$0.140\text{x}10^{-2}$
	C	$0.156\text{x}10^{-1}$	$0.388\text{x}10^{-1}$	$0.946\text{x}10^{-2}$	$0.681\text{x}10^{-4}$

Very Accurate Solution

	A	B	C
Max{$\|\delta P\|$, $\|\delta Q\|$}	$0.171\text{x}10^{-12}$	$0.476\text{x}10^{-9}$	$0.171\text{x}10^{-12}$
No. of Iterations	5	18	3+3

Method Code

A Exact TTM $\qquad e_V \triangleq \delta|V|$

B Fast decoupled version $\qquad e_\theta \triangleq \delta\theta$

C Approximate version of TTM

CONCLUSIONS

We have presented a method for solving the power flow equations. The method utilizes an adjoint network concept in the context of applying Tellegen's theorem to the power model. Our approach, hence, is novel since it does not belong to any of the existing techniques of load flow analysis.

The exact version of our method enjoys the same rate of convergence as well as the size and the sparsity of equations as the Newton-Raphson method, while employing a much simpler, mostly constant matrix

TABLE VII

RESULTS OF DIFFERENT VERSIONS FOR 26-BUS SYSTEM

		Iteration No. 1	2	3	4
MAX $\lvert\delta P\rvert$	A	0.271	0.116×10^{-1}	0.591×10^{-4}	0.106×10^{-8}
	B	0.546	0.584	0.554×10^{-1}	0.673×10^{-1}
	C	0.881	0.558	0.329×10^{-2}	0.487×10^{-5}
MAX $\lvert\delta Q\rvert$	A	0.837	0.389×10^{-1}	0.106×10^{-3}	0.985×10^{-9}
	B	0.548	0.657×10^{-1}	0.500×10^{-1}	0.828×10^{-2}
	C	0.566	0.191	0.490×10^{-2}	0.367×10^{-5}
MAX $\lvert e_V\rvert$	A	0.528×10^{-1}	0.272×10^{-2}	0.818×10^{-5}	0.103×10^{-9}
	B	0.931×10^{-1}	0.340×10^{-2}	0.112×10^{-1}	0.653×10^{-3}
	C	0.932×10^{-1}	0.140×10^{-1}	0.805×10^{-3}	0.922×10^{-6}
MAX $\lvert e_\theta\rvert$	A	0.596×10^{-1}	0.462×10^{-2}	0.181×10^{-4}	0.256×10^{-9}
	B	0.615×10^{-1}	0.216×10^{-1}	0.915×10^{-2}	0.512×10^{-2}
	C	0.826×10^{-1}	0.744×10^{-1}	0.116×10^{-2}	0.131×10^{-5}

Very Accurate Solution

	A	B	C
Max{$\lvert\delta P\rvert$, $\lvert\delta Q\rvert$}	0.892×10^{-12}	0.565×10^{-8}	0.825×10^{-11}
No. of Iterations	5	29	2+3

Method Code

A Exact TTM $\qquad e_V \triangleq \delta\lvert V\rvert$

B Fast decoupled version $\qquad e_\theta \triangleq \delta\theta$

C Approximate version of TTM

of coefficients. With minor, valid approximations, this matrix of coefficients reduces to a constant matrix that has to be factorized only once for several iterations.

The novel method and its approximate and decoupled versions are all applicable, directly, to both the polar and the rectangular forms of the power flow equations. The matrix of coefficients, which is totally free from the trigonometric functions, is common to both forms.

Our method automatically supplies the sensitivities of all the dependent variables at the load flow solution without any additional adjoint simulation.

The method presented, whether applied in the exact, approximate, decoupled forms or combined with other versions of the Newton's load flow, is believed to provide a novel, very promising phase of power network analysis.

APPENDIX

DERIVATION OF TELLEGEN'S THEOREM SENSITIVITY VERSION

Here, we summarize the main derivation steps incorporated in the sensitivity approach based on Tellegen's theorem.

The application of Tellegen's theorem to the power model results in the identity

$$\hat{\eta}_x^T \, \delta\bar{x} + \hat{\eta}_u^T \, \delta\bar{u} = 0, \qquad \text{(A1)}$$

where $\bar{x}$ and $\bar{u}$ are general $2n_B$ - vectors of all branch state and control variables, respectively, and the vectors $\hat{\eta}_x$ and $\hat{\eta}_u$ are, in general, linear functions of the formulated adjoint network current and voltage variables. Hence, the $\hat{\eta}_x$ and $\hat{\eta}_u$ are related through Kirchhoff's current and voltage laws formulating a set of linear network equations to be solved for the unknown adjoint variables.

The adjoint network is defined for a particular element x_i of x^k of (3) by setting the corresponding component of $\hat{\eta}_x$ to unity and all other components to zero. This results in the form (9). Observe that the vectors x^k, u^k and $\hat{\eta}_u$ of (9) are, respectively, subvectors of $\bar{x}$, $\bar{u}$ and $\hat{\eta}_u$ of (A1).

REFERENCES

[1] B. Stott, "Review of load-flow calculation methods", Proc. IEEE, vol. 62, 1974, pp. 916-929.

[2] B.D.H. Tellegen, "A general network theorem, with applications", Philips Res. Repts., 7, 1952, pp. 259-269.

[3] W.F. Tinney and C.E. Hart, "Power flow solution by Newton's method", IEEE Trans. Power Apparatus and Systems, vol. PAS-86, 1967, pp. 1449-1456.

[4] B. Stott, "Decoupled Newton load flow", IEEE Trans. Power Apparatus and Systems, vol. PAS-91, 1972, pp. 1955-1959.

[5] B. Stott and O. Alsac, "Fast decoupled load flow", IEEE Trans. Power Apparatus and Systems, vol. PAS-93, 1974, pp. 859-869.

[6] J.W. Bandler and M.A. El-Kady, "A unified approach to power system sensitivity analysis and planning, Part I: family of adjoint systems", Proc. IEEE Int. Symp. Circuits and Systems (Houston, TX, 1980), pp. 681-687.

[7] J.W. Bandler and M.A. El-Kady, "A unified approach to power system sensitivity analysis and planning, Part II: special class of adjoint systems", Proc. IEEE Int. Symp. Circuits and Systems (Houston, TX, 1980), pp. 688-692.

[8] M.A. El-Kady, "A unified approach to generalized network sensitivities with applications to power system analysis and planning", Ph.D. Thesis, McMaster University, Hamilton, Canada, 1980.

[9] H.W. Dommel and W.F. Tinney, "Optimal power flow solutions", IEEE Trans. Power Apparatus and Systems, vol. PAS-87, 1968, pp. 1866-1876.

[10] M.S. Sachdev and S.A. Ibrahim, "A fast approximate technique for outage studies in power system planning and operation", IEEE Trans. Power Apparatus and Systems, vol. PAS-93, 1974, pp. 1133-1142.

SOC-254

A NEW METHOD FOR COMPUTERIZED SOLUTION OF POWER FLOW EQUATIONS

J.W. Bandler and M.A. El-Kady

Oct. 1980,   No. of Pages: 7

Revised:

Key Words: Load flow analysis, adjoint network, decoupled load flow, power network sensitivities

Abstract: We employ an adjoint network concept based on an augmented form of Tellegen's theorem to describe a novel method for solving the load flow problem. The method incorporates successive adjoint network simulations with a sparse, mostly constant matrix of coefficients, the majority of its elements representing basic data of the problem already stored in computer memory. Nevertheless, the exact version of the method enjoys the same rate of convergence as the Newton-Raphson method. Moreover, it automatically supplies the sensitivities of all system states with respect to adjustable variables at the load flow solution without any additional adjoint simulation. An approximate version of the method is also presented. It partly employs very fast repeat forward and backward substitutions with constant LU factors of a reduced matrix of coefficients and is applicable to both the rectangular and the polar formulations of the power flow equations. Numerical examples are presented for illustration and comparison.

Description:

Related Work: SOC-237, SOC-238, SOC-253.

Price: $3.00.