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GENERALIZED POWER NETWORK SENSITIVITIES

PART I: MODES OF FORMULATION

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Abstract

A unified study of the class of adjoint network approaches to power system sensitivity analysis which exploits the Jacobian matrix of the load flow solution is presented. Generalized sensitivity expressions which are easily derived, compactly described and effectively used for calculating first-order changes and gradients of functions of interest are obtained. These generalized sensitivity expressions are common to all modes of formulation, e.g., polar and cartesian. A first step towards deriving these generalized sensitivity expressions is performed here, in Part I, where we utilize a special complex notation to compactly describe the transformations relating different ways of formulating power network equations. This special notation and the derived transformations are used in Part II to effectively derive the required sensitivity expressions only by matrix manipulations.

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I. INTRODUCTION

Two kinds of analysis can be distinguished in power system operation and planning studies. In the first kind, which implies the load flow solution [1,2] of the power network, the system states are obtained with the control variables fixed at particular values. The solution obtained describes the power system steady state behaviour associated with these particular values of the control variables. The second kind of analysis deals with variations in control variables and the resulting effect on either system states or, in general, on a particular function of interest [3]. This analysis is usually referred to as sensitivity analysis. The importance of sensitivity analysis has been recognized [3-5] in power system operation and planning studies to supply first-order changes of functions of interest and their gradients required for effective optimization techniques.

The class of adjoint network approaches [4,6] incorporating the method of Lagrange multipliers provides the advantage of using the transpose of the Jacobian of the load flow problem as an adjoint matrix of coefficients. When describing adjoint network approaches which exploit the Jacobian of the load flow problem, the sensitivity expressions for different elements are derived according to the mode of formulation used, e.g., polar or cartesian. Different forms of sensitivity expressions have been presented for different studies. A unified sensitivity study for this class of adjoint network approaches has not, however, been previously described.

The impact of the conjugate notation [7], which describes a first-order changes of general complex functions in terms of formal derivatives w.r.t. complex system variables provides a useful tool for describing a generalized adjoint network sensitivity approach, as presented in this paper, where generalized sensitivity expressions are easily derived, compactly described and effectively used subject to any mode of formulation. The adjoint matrix of coefficients is always the transpose of the Jacobian of the original load flow problem and, regardless of the formulation, these generalized sensitivity expressions can be used.

The first part of the paper, namely Part I, presents the modes of formulation while Part II of the paper deals with sensitivity analysis. To illustrate the concepts and justify the relationship between the different formulas derived, two examples of the simplest 2-bus sample power system are employed throughout the paper. The formulas derived, however, are general and can be directly programmed for a general power system of practical size.

II. NOTATION

We denote by $\mathcal{C}$ and $\mathcal{R}$, respectively, the field of complex numbers and the field of real numbers. The vector space over $\mathcal{C}$, of n-tuples $(\zeta_1, \ldots, \zeta_n)$, $\zeta_i \in \mathcal{C}$ is denoted by $\mathcal{C}^n$. Similarly, $\mathcal{R}^n$ stands for the vector space over $\mathcal{R}$, of n-tuples $(\zeta_{1m}, \ldots, \zeta_{nm})$, m = 1, 2 and $\zeta_{im} \in \mathcal{R}$. Also, we write

$$\underset{\sim}{\zeta} = \underset{\sim}{\zeta}_1 + j\, \underset{\sim}{\zeta}_2, \tag{1}$$

where $\underset{\sim}{\zeta}$ is a column vector of components ζ_i given by

$$\zeta_i = \zeta_{i1} + j\, \zeta_{i2}, \tag{2}$$

$\underset{\sim}{\zeta}_1, \underset{\sim}{\zeta}_2 \in \mathcal{R}^n$, $\zeta_{i1}, \zeta_{i2} \in \mathcal{R}$, i = 1, 2, .., n and $j = \sqrt{-1}$.

For continuously differentiable complex valued function f (which possesses partial derivatives w.r.t. all the variables $\underset{\sim}{\zeta}_1$ and $\underset{\sim}{\zeta}_2$) on an open set $\Omega \subset C^n$ [8], we define the formal [9] (or symbolic) partial derivatives

$$\frac{\partial f}{\partial \underset{\sim}{\zeta}} \triangleq \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}_1} - j \frac{\partial f}{\partial \underset{\sim}{\zeta}_2} \right)/2 \tag{3}$$

and

$$\frac{\partial f}{\partial \underset{\sim}{\zeta}^*} \triangleq \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}_1} + j \frac{\partial f}{\partial \underset{\sim}{\zeta}_2} \right)/2, \tag{4}$$

where * denotes the complex conjugate and $\partial f/\partial \underset{\sim}{\zeta}$, $\partial f/\partial \underset{\sim}{\zeta}^*$, $\partial f/\partial \underset{\sim}{\zeta}_1$ and $\partial f/\partial \underset{\sim}{\zeta}_2$ are column vectors. Note that in formal derivatives, the Cauchy-Riemann differential equations may be written [9] as

$$\frac{\partial f}{\partial \underset{\sim}{\zeta}^*} = \underset{\sim}{0}. \tag{5}$$

We consider the nonsingular transformation

$$\begin{bmatrix} \underset{\sim}{\zeta}_1 \\ \underset{\sim}{\zeta}_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \underset{\sim}{1}^n & \underset{\sim}{1}^n \\ -\underset{\sim}{j}^n & \underset{\sim}{j}^n \end{bmatrix} \begin{bmatrix} \underset{\sim}{\zeta} \\ \underset{\sim}{\zeta}^* \end{bmatrix}, \tag{6}$$

where $\underset{\sim}{1}^n$ is the identity matrix of order n and

$$\underset{\sim}{j}^n \triangleq j\, \underset{\sim}{1}^n. \tag{7}$$

Equation (6) may be written in the perturbed form

$$\begin{bmatrix} \delta\underset{\sim}{\zeta}_1 \\ \delta\underset{\sim}{\zeta}_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \underset{\sim}{1}^n & \underset{\sim}{1}^n \\ -\underset{\sim}{j}^n & \underset{\sim}{j}^n \end{bmatrix} \begin{bmatrix} \delta\underset{\sim}{\zeta} \\ \delta\underset{\sim}{\zeta}^* \end{bmatrix}. \tag{8}$$

Note that

$$\delta\underset{\sim}{\zeta}^* = (\delta\underset{\sim}{\zeta})^*. \tag{9}$$

The first-order change of f is given by

$$\delta f = \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}_1} \right)^T \delta\underset{\sim}{\zeta}_1 + \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}_2} \right)^T \delta\underset{\sim}{\zeta}_2 \tag{10}$$

or, using (8),

$$\delta f = \frac{1}{2}[(\frac{\partial f}{\partial \boldsymbol{\zeta}_1})^T - (\frac{\partial f}{\partial \boldsymbol{\zeta}_2})^T \mathbf{j}^n]\, \delta\boldsymbol{\zeta} + \frac{1}{2}[(\frac{\partial f}{\partial \boldsymbol{\zeta}_1})^T + (\frac{\partial f}{\partial \boldsymbol{\zeta}_2})^T \mathbf{j}^n]\, \delta\boldsymbol{\zeta}^*, \quad (11)$$

T denoting transposition. Hence, from (3) and (4),

$$\delta f = (\frac{\partial f}{\partial \boldsymbol{\zeta}})^T \delta\boldsymbol{\zeta} + (\frac{\partial f}{\partial \boldsymbol{\zeta}^*})^T \delta\boldsymbol{\zeta}^*. \quad (12)$$

Equation (12) expresses δf in terms of the variations in $\boldsymbol{\zeta}$ and $\boldsymbol{\zeta}^*$ using the formal derivatives $\partial f/\partial\boldsymbol{\zeta}$ and $\partial f/\partial\boldsymbol{\zeta}^*$ of (3) and (4), respectively.

We remark [9] that the terminology of formal derivatives arises because of the possibility of obtaining them formally using the ordinary differentiation rules. The use of the above notation, called [7] conjugate notation, facilitates the required derivations and provides compact formulation of equations and sensitivity expressions.

III. BASIC FORMULATION

The electric power network can be represented by a system of node equations in the form

$$\mathbf{Y}_T \mathbf{V}_M = \mathbf{I}_M, \quad (13)$$

where

$$\mathbf{Y}_T = \mathbf{Y}_{T1} + j\, \mathbf{Y}_{T2} \quad (14)$$

is the bus admittance matrix of the power network,

$$\mathbf{V}_M = \mathbf{V}_{M1} + j\, \mathbf{V}_{M2} \quad (15)$$

is a column vector of the bus voltages, and

$$\mathbf{I}_M = \mathbf{I}_{M1} + j\, \mathbf{I}_{M2} \quad (16)$$

is a vector of bus currents.

We write the bus loading equations in the matrix form

$$\mathbf{E}_M^* \mathbf{I}_M = \mathbf{S}_M^*, \quad (17)$$

where $\underset{\sim}{E}_M$ is a diagonal matrix of components of $\underset{\sim}{V}_M$ in corresponding order, i.e.,

$$\underset{\sim}{E}_M \underset{\sim}{v} = \underset{\sim}{V}_M, \tag{18}$$

where $\underset{\sim}{v}$ is given by

$$\underset{\sim}{v} \triangleq \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \tag{19}$$

and $\underset{\sim}{S}_M$ is a vector of the injected bus powers given by

$$\underset{\sim}{S}_M \triangleq \underset{\sim}{P}_M + j\,\underset{\sim}{Q}_M. \tag{20}$$

Substituting (13) into (17), we get

$$\underset{\sim}{E}_M^* \underset{\sim}{Y}_T \underset{\sim}{V}_M = \underset{\sim}{S}_M^*. \tag{21}$$

The system of nonlinear equations (21) represents the typical load flow problem, whose solution is required.

The system (21) may be written in the perturbed form

$$\underset{\sim}{K}^S \delta\underset{\sim}{V}_M + \underset{\sim}{\overline{K}}^S \delta\underset{\sim}{V}_M^* = \delta\underset{\sim}{S}_M^* - \underset{\sim}{E}_M^* \delta\underset{\sim}{Y}_T \underset{\sim}{V}_M, \tag{22}$$

where $\delta\underset{\sim}{V}_M$, $\delta\underset{\sim}{V}_M^*$, $\delta\underset{\sim}{S}_M^*$ and $\delta\underset{\sim}{Y}_T$ represent first-order changes of $\underset{\sim}{V}_M$, $\underset{\sim}{V}_M^*$, $\underset{\sim}{S}_M^*$ and $\underset{\sim}{Y}_T$, respectively,

$$\underset{\sim}{K}^S \triangleq \underset{\sim}{E}_M^* \underset{\sim}{Y}_T \tag{23}$$

and $\underset{\sim}{\overline{K}}^S$ is a diagonal matrix of components of $\underset{\sim}{I}_M$, i.e.

$$\underset{\sim}{\overline{K}}^S \underset{\sim}{v} = \underset{\sim}{I}_M. \tag{24}$$

We write (22) in the form

$$\underset{\sim}{K}^S \delta\underset{\sim}{V}_M + \underset{\sim}{\overline{K}}^S \delta\underset{\sim}{V}_M^* = \underset{\sim}{d}^S, \tag{25}$$

where we have defined

$$\underset{\sim}{d}^S \triangleq \delta\underset{\sim}{S}_M^* - \underset{\sim}{E}_M^* \delta\underset{\sim}{Y}_T \underset{\sim}{V}_M. \tag{26}$$

Note that for constant $\underset{\sim}{Y}_T$, $\underset{\sim}{d}^S$ of (26) is simply $\delta\underset{\sim}{S}_M^*$ and (25) rigorously represents a set of linear equations to be solved in the well-known Newton-Raphson iterative method.

The form (25) must be adjusted for practical considerations. In practice, one bus is selected as a slack bus of specified voltage. Hence, the equation of (25) corresponding to the slack bus is replaced by

$$\underset{\sim}{k}_n^T \, \delta \underset{\sim}{V}_M + \bar{\underset{\sim}{k}}_n^T \, \delta \underset{\sim}{V}_M^* = \delta V_n^*, \tag{27}$$

where we have assigned the last bus, namely the nth bus, as a slack bus,

$$\underset{\sim}{k}_n = \underset{\sim}{0} \tag{28}$$

and

$$\bar{\underset{\sim}{k}}_n = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} . \tag{29}$$

Observe that in the load flow solution, the equation corresponding to the slack bus may be eliminated.

Moreover, a power system usually contains voltage-controlled buses or generator-type buses. Consider the equation of (25) corresponding to a generator bus g. Let

$$\tilde{S}_g \triangleq P_g + j \, |V_g| , \tag{30}$$

hence

$$\delta \tilde{S}_g^* = \delta P_g - j \, \delta |V_g| . \tag{31}$$

Since

$$2P_g = V_g \, I_g^* + V_g^* \, I_g , \tag{32}$$

then

$$2\delta P_g = V_g \, \delta I_g^* + I_g^* \, \delta V_g + V_g^* \, \delta I_g + I_g \, \delta V_g^* . \tag{33}$$

Using (13), we write I_g as

$$I_g = \underset{\sim}{y}_g^T \underset{\sim}{V}_M, \tag{34}$$

where $\underset{\sim}{y}_g^T$ represents the corresponding row of the bus admittance matrix $\underset{\sim}{Y}_T$, hence

$$\delta I_g = \underset{\sim}{y}_g^T \delta \underset{\sim}{V}_M + \underset{\sim}{V}_M^T \delta \underset{\sim}{y}_g. \tag{35}$$

Also,

$$\delta|V_g| = \delta(V_g V_g^*)^{1/2} = (V_g \delta V_g^* + V_g^* \delta V_g)/(2|V_g|). \tag{36}$$

Using (33), (34), (35) and (36), it is straightforward to show that $\delta\tilde{S}_g^*$ of (31) is given by

$$\delta\tilde{S}_g^* = \underset{\sim}{k}_g^T \delta \underset{\sim}{V}_M + \underset{\sim}{\bar{k}}_g^T \delta \underset{\sim}{V}_M^* + V_g^* \underset{\sim}{V}_M^T \delta \underset{\sim}{y}_g/2 + V_g \underset{\sim}{V}_M^{*T} \delta \underset{\sim}{y}_g^*/2, \tag{37}$$

where

$$\underset{\sim}{k}_g \overset{\Delta}{=} (V_g^*/2)\, \underset{\sim}{y}_g + [\underset{\sim}{y}_g^{*T} \underset{\sim}{V}_M^*/2 - jV_g^*/(2|V_g|)]\, \underset{\sim}{\mu}_g \tag{38}$$

and

$$\underset{\sim}{\bar{k}}_g \overset{\Delta}{=} (V_g/2)\, \underset{\sim}{y}_g^* + [\underset{\sim}{y}_g^T \underset{\sim}{V}_M/2 - j\, V_g/(2|V_g|)]\, \underset{\sim}{\mu}_g \tag{39}$$

and where $\underset{\sim}{\mu}_g$ is a column vector of unity gth element and zero other elements. Using (37), the equation of (25) corresponding to the gth bus is replaced by

$$\underset{\sim}{k}_g^T \delta \underset{\sim}{V}_M + \underset{\sim}{\bar{k}}_g^T \delta \underset{\sim}{V}_M^* = d_g, \tag{40}$$

where

$$d_g = \delta P_g - j\, \delta|V_g| - V_g^* \underset{\sim}{V}_M^T \delta \underset{\sim}{y}_g/2 - V_g \underset{\sim}{V}_M^{*T} \delta \underset{\sim}{y}_g^*/2. \tag{41}$$

We write (25), including (27) for slack bus and (41) for generator buses, in the form

$$\underset{\sim}{K}\, \delta \underset{\sim}{V}_M + \underset{\sim}{\bar{K}}\, \delta \underset{\sim}{V}_M^* = \underset{\sim}{d}. \tag{42}$$

Note that the elements of $\delta \underset{\sim}{V}_M$ and $\delta \underset{\sim}{V}_M^*$, namely, δV_i and δV_i^*, $i = 1,\ldots,n$ can be replaced by the relative quantities $\delta V_i/|V_i|$ and $\delta V_i^*/|V_i|$, respectively. In this case the elements k_{ij} and $\bar{k}_{ij}$ of the ith row of the coefficient matrices $\underset{\sim}{K}$ and $\underset{\sim}{\bar{K}}$ are replaced by $|V_j| k_{ij}$ and $|V_j|\, \bar{k}_{ij}$, respectively. Note also that we could equally well specify $|V_g|^2$ instead of $|V_g|$ for a generator bus. In this case $|V_g|^2$, replaces $|V_g|$ in (30) as a control variable and the required modifications for subsequent derivation can be performed in a straightforward manner.

IV. MODES OF FORMULATION

In the previous section, we have considered the complex formulation of power system equations. We shall exploit this formulation to derive compact forms of sensitivity expressions. In this section, we investigate, via suitable transformations, the relationship between the complex formulation and other formulations. This investigation provides the possibility of formulating the adjoint equations to be solved in the same mode as the original load flow problem. Hence, the available Jacobian of the load flow may be used in solving the adjoint system.

Transformation for Rectangular Formulation

We define the transformation matrix

$$\underset{\sim}{L}^q \triangleq \begin{bmatrix} \underset{\sim}{L}_1 & \underset{\sim}{L}_1^* \\ \underset{\sim}{L}_2 & \underset{\sim}{L}_2^* \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \underset{\sim}{1}^n & \underset{\sim}{1}^n \\ -\underset{\sim}{j}^n & \underset{\sim}{j}^n \end{bmatrix}, \tag{43}$$

hence

$$(\underset{\sim}{L}^q)^{-1} = \begin{bmatrix} \underset{\sim}{1}^n & \underset{\sim}{j}^n \\ \underset{\sim}{1}^n & -\underset{\sim}{j}^n \end{bmatrix}, \tag{44}$$

n denoting the number of buses in the power network. It follows, using (6) and (15), that

$$\begin{bmatrix} \underset{\sim}{V}_{M1} \\ \\ \underset{\sim}{V}_{M2} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{L}_1 & \underset{\sim}{L}_1^* \\ \\ \underset{\sim}{L}_2 & \underset{\sim}{L}_2^* \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_M \\ \\ \underset{\sim}{V}_M^* \end{bmatrix}, \tag{45}$$

hence

$$\begin{bmatrix} \delta\underset{\sim}{V}_{M1} \\ \\ \delta\underset{\sim}{V}_{M2} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{L}_1 & \underset{\sim}{L}_1^* \\ \\ \underset{\sim}{L}_2 & \underset{\sim}{L}_2^* \end{bmatrix} \begin{bmatrix} \delta\underset{\sim}{V}_M \\ \\ \delta\underset{\sim}{V}_M^* \end{bmatrix}. \tag{46}$$

Using the perturbed form (46), it is straightforward to show that (42) can be written in the form

$$\begin{bmatrix} (\underset{\sim}{K}_1 + \overline{\underset{\sim}{K}}_1) & (-\underset{\sim}{K}_2 + \overline{\underset{\sim}{K}}_2) \\ \\ -(\underset{\sim}{K}_2 + \overline{\underset{\sim}{K}}_2) & (-\underset{\sim}{K}_1 + \overline{\underset{\sim}{K}}_1) \end{bmatrix} \begin{bmatrix} \delta\underset{\sim}{V}_{M1} \\ \\ \delta\underset{\sim}{V}_{M2} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{d}_1 \\ \\ -\underset{\sim}{d}_2 \end{bmatrix}, \tag{47}$$

where we have set

$$\underset{\sim}{K} = \underset{\sim}{K}_1 + j\,\underset{\sim}{K}_2\,, \tag{48}$$

$$\overline{\underset{\sim}{K}} = \overline{\underset{\sim}{K}}_1 + j\,\overline{\underset{\sim}{K}}_2\,, \tag{49}$$

and

$$\underset{\sim}{d} = \underset{\sim}{d}_1 + j\,\underset{\sim}{d}_2\,. \tag{50}$$

The 2n x 2n matrix of coefficients in (47), denoted by $\underset{\sim}{K}^{crt}$, constitutes the well-known Jacobian matrix of the flow problem in rectangular form. Moreover, writing (42) in the form

$$[\underset{\sim}{K} \quad \overline{\underset{\sim}{K}}] \begin{bmatrix} \delta\underset{\sim}{V}_M \\ \\ \delta\underset{\sim}{V}_M^* \end{bmatrix} = \underset{\sim}{d}, \tag{51}$$

it follows that

$$[\underset{\sim}{K}\ \overline{\underset{\sim}{K}}] = [\underset{\sim}{K}^q\ \overline{\underset{\sim}{K}}^q]\begin{bmatrix} \underset{\sim}{L}_1 & \underset{\sim}{L}_1^* \\ & \\ \underset{\sim}{L}_2 & \underset{\sim}{L}_2^* \end{bmatrix}, \tag{52}$$

where $\underset{\sim}{K}^q$ and $\overline{\underset{\sim}{K}}^q$ are formed directly from the Jacobian of (47) as

$$\underset{\sim}{K}^q = (\underset{\sim}{K}_1 + \overline{\underset{\sim}{K}}_1) + j(\underset{\sim}{K}_2 + \overline{\underset{\sim}{K}}_2) \tag{53}$$

and

$$\overline{\underset{\sim}{K}}^q = (-\underset{\sim}{K}_2 + \overline{\underset{\sim}{K}}_2) - j(-\underset{\sim}{K}_1 + \overline{\underset{\sim}{K}}_1)\ . \tag{54}$$

Observe that (52) relates the Jacobian of the complex formulation (42) to the Jacobian of the rectangular formulation (47).

Transformation for Polar Formulation

For polar formulation, we set

$$V_i = |V_i| \angle \delta_i\ ,\ i = 1, \ldots, n, \tag{55}$$

where V_i are elements of $\underset{\sim}{V}_M$, and we define the vectors

$$|\underset{\sim}{V}| \triangleq \begin{bmatrix} |V_1| \\ \cdot \\ \cdot \\ \cdot \\ |V_n| \end{bmatrix} \tag{56}$$

and

$$\underset{\sim}{\delta} \triangleq \begin{bmatrix} \delta_1 \\ \cdot \\ \cdot \\ \cdot \\ \delta_n \end{bmatrix}. \tag{57}$$

Then, we define the transformation matrix

$$\underset{\sim}{L}^p \triangleq \begin{bmatrix} \underset{\sim}{L}_\delta & \underset{\sim}{L}_\delta^* \\ \underset{\sim}{L}_v & \underset{\sim}{L}_v^* \end{bmatrix}, \tag{58}$$

where $\underset{\sim}{L}_\delta$, $\underset{\sim}{L}_\delta^*$, $\underset{\sim}{L}_v$ and $\underset{\sim}{L}_v^*$ are diagonal matrices whose elements represent the formal partial derivatives $\partial\delta_i/\partial V_i$, $\partial\delta_i/\partial V_i^*$, $\partial|V_i|/\partial V_i$ and $\partial|V_i|/\partial V_i^*$, respectively, hence

$$\underset{\sim}{L}_\delta \triangleq \text{diag}\ \{L_{\delta i}\} \tag{59}$$

and

$$\underset{\sim}{L}_v \triangleq \text{diag}\ \{L_{vi}\}, \tag{60}$$

where

$$L_{\delta i} = -j/(2\ V_i) \tag{61}$$

and

$$L_{vi} = V_i^*/(2|V_i|)\ . \tag{62}$$

The inverse of $\underset{\sim}{L}^p$ is given by

$$(\underset{\sim}{L}^p)^{-1} = \begin{bmatrix} \tilde{\underset{\sim}{L}}_\delta & \tilde{\underset{\sim}{L}}_v \\ \tilde{\underset{\sim}{L}}_\delta^* & \tilde{\underset{\sim}{L}}_v^* \end{bmatrix}, \tag{63}$$

where $\tilde{\underset{\sim}{L}}_\delta$, $\tilde{\underset{\sim}{L}}_\delta^*$, $\tilde{\underset{\sim}{L}}_v$ and $\tilde{\underset{\sim}{L}}_v^*$ are diagonal matrices whose elements are the partial derivatives $\partial V_i/\partial\delta_i$, $\partial V_i^*/\partial\delta_i$, $\partial V_i/\partial|V_i|$ and $\partial V_i^*/\partial|V_i|$, respectively, hence

$$\tilde{\underset{\sim}{L}}_\delta \triangleq \text{diag}\ \{\tilde{L}_{\delta i}\} \tag{64}$$

and

$$\tilde{\underset{\sim}{L}}_v \triangleq \text{diag}\ \{\tilde{L}_{vi}\}\ , \tag{65}$$

where

$$\tilde{L}_{\delta i} = jV_i \tag{66}$$

and

$$\tilde{L}_{vi} = V_i/|V_i| \ . \qquad (67)$$

Similarly to (46), we may write

$$\begin{bmatrix} \delta\underset{\sim}{\delta} \\ \delta|\underset{\sim}{V}| \end{bmatrix} = \begin{bmatrix} \underset{\sim}{L}_\delta & \underset{\sim}{L}_\delta^* \\ \underset{\sim}{L}_v & \underset{\sim}{L}_v^* \end{bmatrix} \begin{bmatrix} \delta\underset{\sim}{V}_M \\ \delta\underset{\sim}{V}_M^* \end{bmatrix} . \qquad (68)$$

Using the perturbed form (68), it is straightforward to show that (42) can also be written in the form

$$\begin{bmatrix} \underset{\sim}{K}_1^p & \overline{\underset{\sim}{K}}_1^p \\ -\underset{\sim}{K}_2^p & -\overline{\underset{\sim}{K}}_2^p \end{bmatrix} \begin{bmatrix} \delta\underset{\sim}{\delta} \\ \delta|\underset{\sim}{V}| \end{bmatrix} = \begin{bmatrix} \underset{\sim}{d}_1 \\ -\underset{\sim}{d}_2 \end{bmatrix} , \qquad (69)$$

where we have set

$$\underset{\sim}{K}^p = \underset{\sim}{K}_1^p + j\ \underset{\sim}{K}_2^p \qquad (70)$$

and

$$\overline{\underset{\sim}{K}}^p = \overline{\underset{\sim}{K}}_1^p + j\ \overline{\underset{\sim}{K}}_2^p , \qquad (71)$$

and where the matrices $\underset{\sim}{K}^p$ and $\overline{\underset{\sim}{K}}^p$ are related to $\underset{\sim}{K}$ and $\overline{\underset{\sim}{K}}$ through the relationship

$$[\underset{\sim}{K} \ \ \overline{\underset{\sim}{K}}] = [\underset{\sim}{K}^p \ \ \overline{\underset{\sim}{K}}^p] \begin{bmatrix} \underset{\sim}{L}_\delta & \underset{\sim}{L}_\delta^* \\ \underset{\sim}{L}_v & \underset{\sim}{L}_v^* \end{bmatrix} . \qquad (72)$$

The 2n x 2n matrix of coefficients in (69), denoted by $\underset{\sim}{K}^{plr}$, constitutes the well-known Jacobian matrix of the load flow problem in polar form. Observe that (72) relates the Jacobian of the complex formulation (42) to the Jacobian of the polar formulation (69), where $\underset{\sim}{K}^p$ and $\overline{\underset{\sim}{K}}^p$ are formed directly from the Jacobian of (69).

At the end of this section, we illustrate the foregoing concepts by two simple examples.

Example 1

Consider, first, the 2-bus sample power system of Fig. 1 which consists of a load bus and a slack bus. The solution of the load flow equations (21) is given by

$$V_1 = 0.7352 - j\ 0.2041$$

and

$$S_2 = 5.6705 + j\ 1.0706.$$

Note that S_2 is the injected power at bus 2. The matrices $\underset{\sim}{K}$ and $\overline{\underset{\sim}{K}}$ of (51) are given by

$$\underset{\sim}{K} = \begin{bmatrix} (8.0852 - j\ 12.0097) & (-8.4934 + j\ 13.4802) \\ 0 & 0 \end{bmatrix}$$

and

$$\overline{\underset{\sim}{K}} = \begin{bmatrix} (-5.2623 + j\ 5.5411) & 0 \\ 0 & 1 \end{bmatrix} .$$

Hence, using cartesian coordinates, the matrix of coefficients of (47) has, using (48) and (49), the form

$$\underset{\sim}{K}^{crt} = \begin{bmatrix} 2.8229 & -8.4934 & 17.5508 & -13.4802 \\ 0 & 1 & 0 & 0 \\ 6.4686 & -13.4802 & -13.3475 & 8.4934 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

which is the Jacobian of the load flow problem in cartesian coordinates when the slack bus equations are included.

For the polar formulation, the matrices $\tilde{\underset{\sim}{L}}_\delta$ and $\tilde{\underset{\sim}{L}}_V$ of (63) are given by

$$\tilde{\underset{\sim}{L}}_\delta = \begin{bmatrix} (0.2041 + j\ 0.7352) & 0 \\ 0 & j \end{bmatrix}$$

and

$$\underset{\sim}{\tilde{L}_v} = \begin{bmatrix} (0.9636 - j\,0.2675) & 0 \\ 0 & 1 \end{bmatrix}.$$

Hence, using (58), (63) and (72), the matrices $\underset{\sim}{K}^p$ and $\underset{\sim}{\overline{K}}^p$ are given by

$$\underset{\sim}{K}^p = \begin{bmatrix} (13.4802 + j\,8.4934) & (-13.4802 - j\,8.4934) \\ 0 & -j \end{bmatrix}$$

and

$$\underset{\sim}{\overline{K}}^p = \begin{bmatrix} (-1.9745 - j\,9.8031) & (-8.4934 + j\,13.4802) \\ 0 & 1 \end{bmatrix},$$

from which the matrix of coefficients of (69) has the form

$$\underset{\sim}{K}^{plr} = \begin{bmatrix} 13.4802 & -13.4802 & -1.9745 & -8.4934 \\ 0 & 0 & 0 & 1 \\ -8.4934 & 8.4934 & 9.8031 & -13.4802 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

which is the Jacobian of the load flow problem in polar coordinates when the slack bus equations are included.

Example 2

Now, consider the 2-bus sample power system of Fig. 2 which consists of a generator bus and a slack bus. The solution of the load flow equations (21) is given by

$$\delta_1 = -0.1995 \text{ rad},$$

$$Q_1 = 1.9929$$

and

$$S_2 = 4.2742 - j\,1.7131.$$

The matrices $\underset{\sim}{K}$ and $\underset{\sim}{\overline{K}}$ of (51) are given by

$$\underset{\sim}{K} = \begin{bmatrix} (2.3920 - j\ 9.4199) & (-4.4300 + j\ 8.2864) \\ 0 & 0 \end{bmatrix}$$

and

$$\underset{\sim}{\overline{K}} = \begin{bmatrix} (2.1938 + j\ 8.4398) & (-4.4300 - j\ 8.2864) \\ 0 & 1 \end{bmatrix}.$$

Hence, using cartesian coordinates, the matrix of coefficients of (47) has, using (48) and (49), the form

$$\underset{\sim}{K}^{crt} = \begin{bmatrix} 4.5858 & -8.8600 & 17.8597 & -16.5729 \\ 0 & 1 & 0 & 0 \\ 0.9802 & 0 & -0.1982 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

which is the Jacobian of the load flow problem in cartesian coordinates when the slack bus equations are included.

For the polar formulation, the matrices $\underset{\sim}{\tilde{L}}_{\delta}$ and $\underset{\sim}{\tilde{L}}_{v}$ of (63) are given by

$$\underset{\sim}{\tilde{L}}_{\delta} = \begin{bmatrix} (0.1784 + j\ 0.8822) & 0 \\ 0 & j \end{bmatrix}$$

and

$$\underset{\sim}{\tilde{L}}_{v} = \begin{bmatrix} (0.9802 - j\ 0.1982) & 0 \\ 0 & 1 \end{bmatrix}.$$

Hence, using (58), (63) and (72), the matrices $\underset{\sim}{K}^{p}$ and $\underset{\sim}{\overline{K}}^{p}$ are given by

$$\underset{\sim}{K}^{p} = \begin{bmatrix} 16.5729 & -16.5729 \\ 0 & -j \end{bmatrix}$$

and

$$\underset{\sim}{\overline{K}}^{p} = \begin{bmatrix} 0.9556 - j\ 1.0 & -8.8600 \\ 0 & 1 \end{bmatrix},$$

from which the matrix of coefficients of (69) has the form

$$\underset{\sim}{K}^{plr} = \begin{bmatrix} 16.5729 & -16.5729 & 0.9556 & -8.8600 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

which is the Jacobian of the load flow problem in polar coordinates when the slack bus equations are included.

V. CONCLUSIONS

We have utilized a special complex notation to effectively derive and compactly describe transformations relating different modes of formulation of power network equations. These transformations are necessary to derive generalized power network sensitivity expressions which are common to all modes of formulation. Hence, the Jacobian matrix of the load flow solution may be directly used. The derived transformations relate both cartesian and polar forms to the basic complex form and include all types of buses in practice. Flexibility of different definitions of practical variables in the equations has been discussed.

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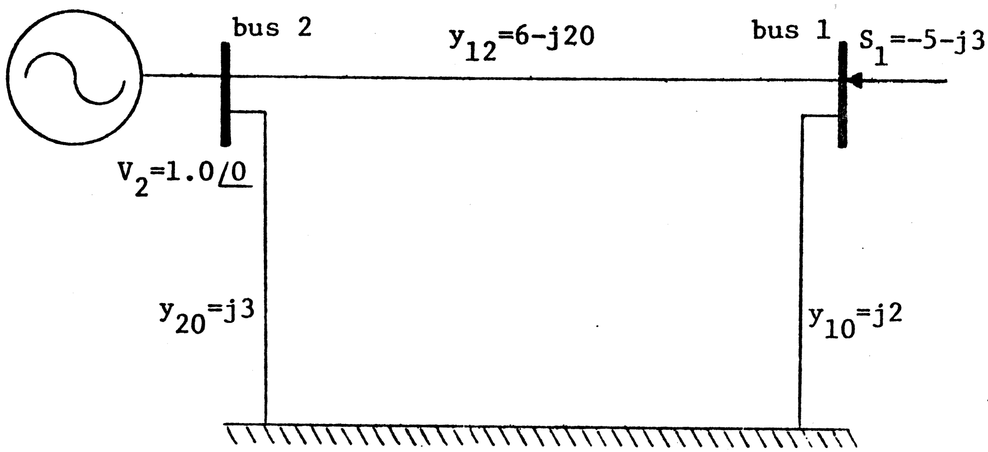

Fig. 1 2-bus load-slack sample power system

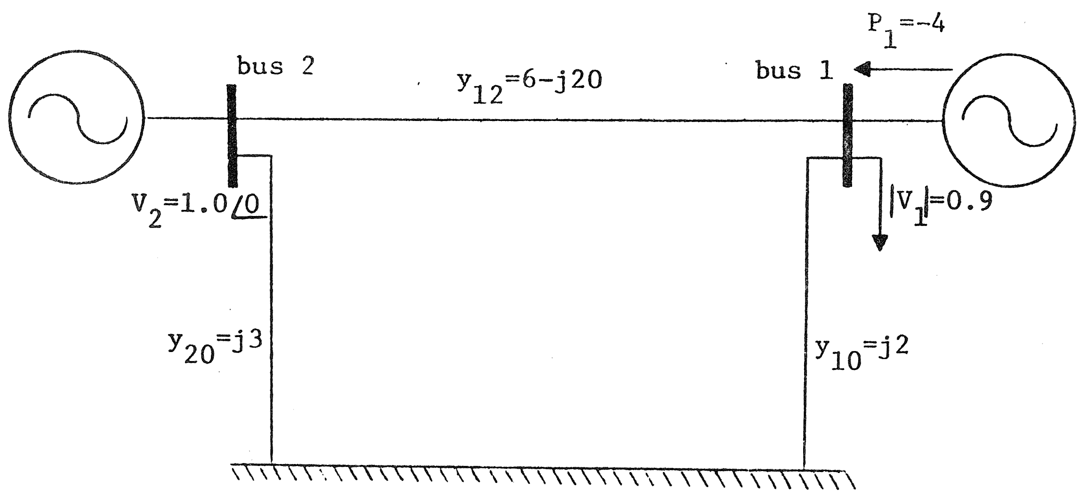

Fig. 2 2-bus generator-slack sample power system

SOC-248

GENERALIZED POWER NETWORK SENSITIVITIES
PART I: MODES OF FORMULATION

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Abstract: A unified study of the class of adjoint network approaches to power system sensitivity analysis which exploits the Jacobian matrix of the load flow solution is presented. Generalized sensitivity expressions which are easily derived, compactly described and effectively used for calculating first-order changes and gradients of functions of interest are obtained. These generalized sensitivity expressions are common to all modes of formulation, e.g., polar and cartesian. A first step towards deriving these generalized sensitivity expressions is performed here, in Part I, where we utilize a special complex notation to compactly describe the transformations relating different ways of formulating power network equations. This special notation and the derived transformations are used in Part II to effectively derive the required sensitivity expressions only by matrix manipulations.

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