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SUFFICIENT TEST CONDITIONS FOR PARAMETER IDENTIFICATION OF ANALOG CIRCUITS BASED ON VOLTAGE MEASUREMENTS

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ABSTRACT

In this paper sufficient test conditions for identification of component values of linear analog circuits are investigated. Tests under consideration are assumed to be performed at a single frequency and consist of voltage measurements using different current excitations. Based on the fact that it is sufficient for this identification to perform nodal voltage measurements using all possible independent current excitations a systematic way to eliminate some unnecessary tests is proposed. A simple method for checking whether a reduced number of tests is sufficient for the identification is then formulated.

INTRODUCTION

Recently, there has been considerable interest in fault diagnosis for analog circuits (e.g., [1-9]). One of the possible approaches uses fault simulations and constructing a fault dictionary of mainly catastrophic faults (e.g., [9]). However, looking for so-called "soft faults" authors usually consider the identification of actual component values.

The solvability of the all parameter identification problem was initiated by Berkowitz [1]. His approach was mostly based on current measurements. The use of voltage measurements only was investigated by several other authors [3-5]. They formulated appropriate equations

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which can be used to calculate component values based on sufficient number of independent measurements. How to arrange for the least number of independent measurements, however, has not been known so far. This problem is studied in this paper in some detail.

Tests under consideration consist of voltage measurements using different current excitations. The tests are performed at a single frequency, so the most one can expect is to determine the values of passive admittances and control coefficients of controlled sources. Repeating the identification at different frequencies enables one to find the component values provided that there is a unique dependence of them on the frequency response. Thus, in this paper we are interested in determining the unknown nodal admittance matrix $\underset{\sim}{Y}_n$ of the equation

$$\underset{\sim}{Y}_n \underset{\sim}{V}_n = \underset{\sim}{I}^S, \tag{1}$$

where $\underset{\sim}{V}_n$ is the vector of nodal voltages measured and $\underset{\sim}{I}^S$ is the vector of applied nodal current excitations. We assume that the component admittance values can be found from $\underset{\sim}{Y}_n$, i.e., that there exists a unique solution to the system of equations

$$\underset{\sim}{B}_j \underset{\sim}{y} = \underset{\sim}{y}_j, \quad j = 1, 2, \ldots, n, \tag{2}$$

where $\underset{\sim}{y} = [y_1 \; y_2 \; \ldots \; y_p]^T$ is the vector of component admittances, $\underset{\sim}{y}_j$ is the jth column of $\underset{\sim}{Y}_n^T$ and $\underset{\sim}{B}_j$ is an appropriate matrix corresponding to the known network topology and consisting of 0, -1 and +1 values. The assumption means, for instance, that there are no parallel elements such as passive admittances or current sources controlled by the same voltage. As is well known, such elements are not solvable, so if they occur we usually treat them as a single component. Finally, note that the existance of the unique solution of (2) depends only upon the network topology.

GENERAL SUFFICIENT CONDITIONS

Consider the ith test as one which consists of nodal voltages $\underset{\sim}{V}_n^i$ measured when a known current vector $\underset{\sim}{I}^{Si}$ is applied. Equation (1) holds for every test, i.e., for vectors $\underset{\sim}{V}_n^i$ and $\underset{\sim}{I}^{Si}$, i = 1, 2, ..., each time with the same matrix $\underset{\sim}{Y}_n$. Taking n tests we can write a single matrix equation

$$\underset{\sim}{Y}_n \underset{\sim}{V}_t = \underset{\sim}{I}_t^S, \tag{3}$$

where $\underset{\sim}{V}_t = [\underset{\sim}{V}_n^1 \; \underset{\sim}{V}_n^2 \; \ldots \; \underset{\sim}{V}_n^n]$ is nxn matrix of voltages measured and $\underset{\sim}{I}_t^S = [\underset{\sim}{I}^{S1} \; \underset{\sim}{I}^{S2} \; \ldots \; \underset{\sim}{I}^{Sn}]$ is nxn matrix of consecutive test excitations. From (3) we find the unknown matrix $\underset{\sim}{Y}_n$ as

$$\underset{\sim}{Y}_n = \underset{\sim}{V}_t^{-1} \underset{\sim}{I}_t^S \tag{4}$$

provided that $\underset{\sim}{V}_t$ is nonsingular. As a consequence of equations (3) and (4) the following theorem gives general sufficient conditions for the identification.

Theorem 1 If a given linear network can be described by the nodal equation (1) and the test excitations are chosen in such a way that $\underset{\sim}{I}_t^S$ is a nonsingular matrix then $\underset{\sim}{V}_t$ is also nonsingular and the solution (4) exists.

Proof of the theorem follows from equation (3) since $n = \text{rank}\, \underset{\sim}{I}_t^S \leq \text{rank}\, \underset{\sim}{V}_t \leq n$.

Thus, in order to identify all component values we may arrange for n independent current excitations, measure all nodal voltages and then apply equations (4) and (2). This approach, however, can be redundant. In the next section we shall discuss the problem of eliminating some unnecessary tests.

TEST GENERATION

One of possible choices of independent excitations is to apply a unit current consequtively to all n nodes, i.e., we may consider

$$\underset{\sim}{I}_t^S = \underset{\sim}{1} \, . \tag{5}$$

This together with (4) give us the very important relation

$$\underset{\sim}{V}_t = \underset{\sim}{Y}_n^{-1} \tag{6}$$

which imposes certain relations between elements of the matrix $\underset{\sim}{V}_t$. For instance, if $\underset{\sim}{Y}_n$ is symmetrical (as for reciprocal networks) then $\underset{\sim}{V}_t$ is so. In general, $\tilde{Y}_n$ has a particular (usually sparse) form corresponding to the known network topology, hence there are certain constraints to the elements of its inverse. There are three types of these constraints:

(1) $$y_{ij} = 0 \text{ (or } \Delta_{ji} = 0\text{)}, \tag{7}$$

for nonincident nodes (Δ_{ji} denotes the appropriate minor of $\tilde{V}_t$),

(2) $$y_{ij} = y_{ji} \text{ (or } \Delta_{ji} = \Delta_{ij}\text{)}, \tag{8}$$

for passive branches,

(3) $$\begin{aligned} &y_{ik} + y_{jk} = y_{ki} + y_{kj} \\ \text{and/or} \quad &y_{im} + y_{jm} = y_{mi} + y_{mj}, \end{aligned} \tag{9}$$

for voltage controlled current sources.

The constraints (7)-(9) are known, so we should take advantage of this in order to eliminate unnecessary measurements. Since the constraints are independent their total number is equal to the number of elements of $\tilde{V}_t$ which can uniquely be determined based on remaining elements. However, they cannot be chosen arbitrarily. From the other side what we intend to do in order to perform the least number of tests is to eliminate whole columns of $\tilde{V}_t$. In the following we propose a systematic way which enables us to indicate tests sufficient for component value identification. The mehtod is based on the assumption that all voltages measured as well as all components have nonzero values.

Equation (6) rewritten in a slightly different form

$$\tilde{V}_t^T \, \tilde{Y}_n^T = \tilde{1} \tag{10}$$

can be considered as n systems of equations

$$\tilde{V}_t^T \, \tilde{y}_j = \tilde{e}_j, \; j = 1, 2, \ldots, n, \tag{11}$$

where $\tilde{e}_j$ denotes the jth column of unit matrix. Note that coefficients of each equation in (11) are taken from only one measurement test and also that unknown elements of $\tilde{y}_j$ correspond to the jth nodal cut-set branch admittances.

Let us suppose that, for certain j, $\tilde{y}_j$ has a number, say $n-k_1$, elements of zero value. Since the solution of (11) is unique we do not need to know the elements of $n-k_1$ columns of $\underset{\sim}{V}_t^T$ corresponding to zero-value elements of $\underset{\sim}{y}_j$. Therefore, eliminating them we obtain a reduced system (11) having n equations and k_1 unknowns. Then we can also eliminate exactly $n-k_1$ equations (tests). The resulting system has to contain the jth equation for which the right hand side equals 1. Otherwise, if we did not choose the jth equation the resulting system would be homogenious and consequently, since the solution is different from zero, singular.

Now, suppose that we have chosen exactly k_1 equations, including the jth equation. In order to check whether this subsystem is solvable or not we can employ the fact that a subdeterminant of $\underset{\sim}{V}_t^T$ is different from zero if and only if the complementary subdeterminant (obtained by removing the rows and columns chosen) of the transpose to the inverse of $\underset{\sim}{V}_t^T$, i.e., of $\underset{\sim}{Y}_n$, is different from zero. Although the particular value of this determinant depends on particular values of unknown elements of $\underset{\sim}{Y}_n$ it can be "structurally" (i.e., dependently on zero elements) either of zero or nonzero value. Thus, using this criterion one can indicate the groups of k_1-1 tests which together with the jth test are not sufficient to determine the unknown elements of $\underset{\sim}{y}_j$. Any other group of k_1 tests (including the jth test) is topologically sufficient for this identification. In other words, except very particular values of $\underset{\sim}{Y}_n$ (e.g., some elements have zero values, although they are assumed to be different from zero) the corresponding submatrix of $\underset{\sim}{V}_t^T$ is nonsingular.

The simplest situation occurs when $\underset{\sim}{y}_j$ has only 2 nonzero elements. Then, assuming that all voltages measured are different from zero (e.g., the network is not weakly connected) any test (equation) can be chosen together with the jth equation.

One of possible choices of k_1 independent equations is that that we consider the tests corresponding to the excitations at all nodes j_1, j_2, ..., j_{k-1} which are incident with the jth node. This is simply the same result as that of Trick et al. in [4]. However, if we intended to use such tests at every cut-set we would eventually have to consider all nodal excitations as in Theorem 1. In fact, what we actually want to find are some other excitations which can replace all or part of the excitations at nodes j_1, j_2, ..., j_{k-1}.

In some cases the nonzero elements of $\underset{\sim}{y}_j$ can be dependent. For instance, this is the case when no admittance exists between the jth and the datum nodes. Then we can eliminate one more test out of k_1-1 tests chosen.

Based on one of proper groups of k_1 tests we are able to identify all admittances of the jth cut-set, so although we do not need, for the time being, to indicate which group has been chosen we can consider these elements as known. Then, consider another column $\underset{\sim}{y}_\ell$ such that it contains one of the elements of $\underset{\sim}{y}_j$. We now know not only all of $n-k_2$ zero elements of $\underset{\sim}{y}_\ell$ but also that nonzero admittance $y_{\ell m}$ which is common with $\underset{\sim}{y}_j$. As before after eliminating $n-k_2$ columns of $\underset{\sim}{V}_t^T$ corresponding to zero-elements we can properly select k_2 equations (including the ℓth equation). Since $y_{\ell m}$ is known and is different from zero it can be shown that after dropping the ℓth equation we obtain k_2-1 independent equations with k_2-1 remaining unknowns. This is quite an important result: knowing at least one of elements of a cut-set we do not have to use the direct excitation of this cut-set in order to solve for remaining elements. The circuit interpretation of this is the following. The value of $y_{\ell m}$ as well as the voltage across this element are known. Thus, after finding the current we can replace $y_{\ell m}$ by an independent current source and then express it as a combination of nodal current excitations. Doing so with all currents of the jth cut-set we eliminate the jth node. Repeating this for all k independent excitations at the jth node and all incident k-1 nodes we now express them as linear combinations of k-1 excitations. They are dependent from the remaining network point of view and, in particular, the ℓth excitation (test) can be determined as a linear combination of the jth excitation (test), which is already chosen, and some other ones.

As before, we do not want, for the time being, to indicate which group of k_2-1 equations has been chosen. However, it is desirable that these k_2-1 equations are included in one of group of k_1 equations considered in the first step. If this is not satisfied we have to augment them.

Using this approach we proceed with the subsequent columns of $\underset{\sim}{Y}_n^T$ and finally we can decide which equations (tests) may be chosen in order to identify all elements of $\underset{\sim}{Y}_n$.

The above discussion is also valid in any case when equation (1) is based on a system of independent cut-sets instead of nodal cut-sets. In particular, it holds for any fundamental system of cut-sets (generated by a certain tree). Given a tree, $\underset{\sim}{V}_n$ and $\underset{\sim}{V}_t$ represent voltages measured across the tree branches and the unit matrix of (5) corresponds to unit excitation at terminals of tree branches, each at a time. The vector $\underset{\sim}{y}_j$ corresponds to jth cut-set admittances. As a conclusion from our considerations we can formulate the following theorem.

Let us choose a sequence of independent cut-sets $\gamma_1, \gamma_2, \ldots, \gamma_n$. Let $\hat{\gamma}_1, \hat{\gamma}_2, \ldots, \hat{\gamma}_n$ be reduced cut-sets defined as

$$\hat{\gamma}_1 = \gamma_1 \quad \text{and} \quad \hat{\gamma}_i = \gamma_i - \bigcup_{k=1}^{i-1} \gamma_k .$$

Let β_i denotes the set of indices of cut-sets determined by independent voltages (e.g., tree voltages) corresponding to all elements of $\hat{\gamma}_i$. Finally, consider a set of test indices α such that card $\alpha \geq$ max (card β_i).

<u>Theorem 2</u> Tests α are sufficient for the identification of component admittances if there exist $\alpha_i \subset \alpha$, card α_i = card β_i, such that det $\underset{\sim}{Y}_n (\alpha_i | \beta_i) \neq 0$ for $i = 1, 2, \ldots, n$, where $\underset{\sim}{Y}_n (\alpha_i | \beta_i)$ denotes the submatrix of $\underset{\sim}{Y}_n$ obtained by removing α_i rows and β_i columns.

It is to be noted that the choice of independent cut-sets as well as their sequence is crucial for better selection of tests. Now, we give an example of implementation of Theorem 2.

EXAMPLE

Consider a simple resistive active circuit shown in Fig. 1.

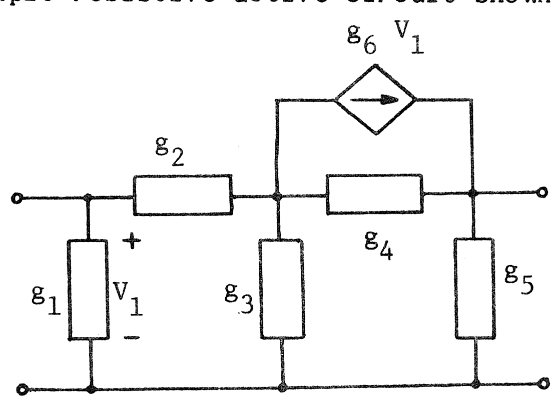

Fig. 1 An active circuit example

Let us choose the sequence of cut-sets $\gamma_1 = \{1,2\}$, $\gamma_2 = \{2,3,5\}$, $\gamma_3 = \{4,5,6\}$ based on the tree $\{1,3,4\}$. The reduced sequence is $\hat{\gamma}_1 = \{1,2\}$, $\hat{\gamma}_2 = \{3,5\}$, $\hat{\gamma}_3 = \{4,6\}$, so we find $\beta_1 = \{1,2\}$, $\beta_2 = \{2,3\}$ and $\beta_3 = \{1,3\}$. Based on the matrix

$$\underset{\sim}{Y}_n = \begin{bmatrix} g_1+g_2 & -g_2 & 0 \\ -g_2 & g_2+g_3-g_5 & g_5 \\ g_6 & -g_5 & g_4+g_5 \end{bmatrix}$$

we find that tests $\alpha = \{1,2\}$ are sufficient for the identification provided that $g_5 \neq 0$ and $g_6 \neq 0$. Simiarly we can check that tests $\alpha = \{1,3\}$ are sufficient provided that $g_2 \neq 0$ and $g_5 \neq 0$.

CONCLUSIONS

Sufficient test conditions for identification of all component values are investigated in this paper. It is shown that measuring nodal voltages using all possible independent current excitation is sufficient for this identification. Then a systematic way to eliminate some unnecessary tests is proposed. However, the choice of a sequence of cut-sets is crucial for this elimination, i.e., the solution may be or may not be optimal. The results are summarized in Theorem 2 which gives a simple method of checking whether a reduced number of tests is sufficient for the identification. This is illustrated by a simple active circuit example.

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