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POWER NETWORK SENSITIVITY ANALYSIS AND FORMULATION SIMPLIFIED

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Abstract

An approach for direct and compact derivation of sensitivity expressions in electrical power networks is presented. The approach utilizes a compact complex notation to facilitate the derivation and subsequent formulation. It employs only complex matrix manipulations and exploits the elements of the Jacobian matrix already available from the solution of the load flow problem.

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I. INTRODUCTION

This paper presents an approach for compact derivation of sensitivity expressions in electrical power networks. These expressions for first-order changes of system states and other functions of interest and their total derivatives w.r.t. practical control variables are required in a number of applications [1].

Several approaches to sensitivity calculations in power systems have been described [2-5]. Those approaches which utilize the Jacobian matrix already available from the solution of the load flow problem [2-4] employ the power flow equations, originally complex [6], in a real form.

Ease of derivation and compactness in formulation of the required sensitivity expressions can, however, be gained by preserving this basic complex form. Furthermore, the conjugate notation [5] provides a useful tool for formulating sensitivity expressions in terms of formal partial derivatives. In this paper we exploit the conjugate notation to derive, using only complex matrix manipulations, the required sensitivity expressions in the compact complex form.

The notation is described in Section II while the problem formulation in terms of complex state and control variables and the method proposed for sensitivity calculations are presented in Sections III and IV, respectively.

II. NOTATION

In the conjugate notation [5], a complex variable

$$\zeta_i = \zeta_{i1} + j\zeta_{i2} \tag{1}$$

and its complex conjugate ζ_i^* replace, as independent quantities, the real and imaginary parts of the variable. Hence, the first-order change of a continuous function f of a set of complex variables arranged in a column vector $\underset{\sim}{\zeta}$,

$$\underset{\sim}{\zeta} = \underset{\sim}{\zeta}_1 + j\underset{\sim}{\zeta}_2 \tag{2}$$

and their complex conjugate $\underset{\sim}{\zeta}^*$ can be expressed as

$$\delta f = \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}} \right)^T \delta\underset{\sim}{\zeta} + \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}^*} \right)^T \delta\underset{\sim}{\zeta}^*, \tag{3}$$

where δ denotes first-order change, T denotes transposition and $\partial f/\partial\underset{\sim}{\zeta}$ and $\partial f/\partial\underset{\sim}{\zeta}^*$ are column vectors representing the formal [5] partial derivatives of f w.r.t. $\underset{\sim}{\zeta}$ and $\underset{\sim}{\zeta}^*$, respectively.

It can be shown that, for a real function f, we can write

$$\frac{\partial f}{\partial \underset{\sim}{\zeta}^*} = \left(\frac{\partial f}{\partial \underset{\sim}{\zeta}} \right)^*. \tag{4}$$

III. PROBLEM FORMULATION

The power flow equations of an electrical power network are represented by a set of complex equality constraints in the form

$$\underset{\sim}{S}_M^* - \underset{\sim}{E}_M^* \underset{\sim}{Y}_T \underset{\sim}{V}_M = \underset{\sim}{0}, \tag{5}$$

where $\underset{\sim}{S}_M$ is a vector of the bus powers,

$$\underset{\sim}{S}_M = \underset{\sim}{P}_M + j\ \underset{\sim}{Q}_M, \tag{6}$$

$\underset{\sim}{V}_M$ is a vector of bus voltages, $\underset{\sim}{Y}_T$ is the bus admittance matrix of dimension nxn, n denoting number of buses in the power network and $\underset{\sim}{E}_M$ is a diagonal matrix of components of $\underset{\sim}{V}_M$ in corresponding order.

The variables in (5) are classified in practice as complex state and control variables. The complex state variables include the load-type bus voltages arranged in the vector

$$\underset{\sim}{\zeta}_x^L \triangleq \underset{\sim}{V}_L = \underset{\sim}{V}_{L1} + j\ \underset{\sim}{V}_{L2}, \tag{7}$$

the generator bus state variables defined as

$$\underset{\sim}{\zeta}_x^G \triangleq \underset{\sim}{Q}_G + j\ \underset{\sim}{\delta}_G, \tag{8}$$

where $\underset{\sim}{\delta}_G$ and $\underset{\sim}{Q}_G$ are vectors of phase angles of generator bus voltages and generator bus reactive powers, respectively, and the slack bus power

$$\zeta_x^n \triangleq S_n = P_n + j\ Q_n. \tag{9}$$

Also, the complex control variables include the load bus powers

$$\underset{\sim}{\zeta}_u^L \triangleq \underset{\sim}{S}_L = \underset{\sim}{P}_L + j\ \underset{\sim}{Q}_L, \tag{10}$$

the generator bus control variables defined as

$$\underset{\sim}{\zeta}_u^G \triangleq \underset{\sim}{P}_G + j\ |\underset{\sim}{V}_G|, \tag{11}$$

where $|\underset{\sim}{V}_G|$ is a vector of magnitudes of generator bus voltages and the slack bus control variable

$$\zeta_u^n \triangleq \delta_n + j\,|V_n|, \tag{12}$$

where $|V_n|$ and δ_n denote, respectively, the magnitude and phase angle of the slack bus voltage V_n. We may also consider the line admittances $\underset{\sim}{\zeta}_u^t$ contained in bus admittance matrix $\underset{\sim}{Y}_T$.

IV. SENSITIVITY CALCULATIONS

When solving (5) in the conventional load flow problem by the Newton-Raphson method [6] using cartesian coordinates, the state variables $\underset{\sim}{\zeta}_x^G$ of (8) and ζ_x^n of (9) are preferably considered, in formulating the equations, as

$$\underset{\sim}{\tilde{\zeta}}_x^G \triangleq \underset{\sim}{V}_G = \underset{\sim}{V}_{G1} + j\,\underset{\sim}{V}_{G2} \tag{13}$$

and

$$\tilde{\zeta}_x^n \triangleq V_n = V_{n1} + j\,V_{n2}. \tag{14}$$

Hence, we may write the power flow equations in the form

$$\underset{\sim}{h}(\underset{\sim}{\zeta}_x, \underset{\sim}{\zeta}_x^*, \underset{\sim}{\zeta}_u^t, \underset{\sim}{\zeta}_u^{t*}) = \underset{\sim}{\zeta}_u^{m*}, \tag{15}$$

where

$$\underset{\sim}{\zeta}_x = \underset{\sim}{\zeta}_{x1} + j\,\underset{\sim}{\zeta}_{x2} \tag{16}$$

is a vector of the state variables $\underset{\sim}{\zeta}_x^L$, $\underset{\sim}{\tilde{\zeta}}_x^G$ and $\tilde{\zeta}_x^n$ of (7), (13) and (14), respectively, and

$$\underset{\sim}{\zeta}_u^m = \underset{\sim}{\zeta}_{u1}^m + j\,\underset{\sim}{\zeta}_{u2}^m \qquad (17)$$

is a vector of the control variables $\underset{\sim}{\zeta}_u^L$, $\underset{\sim}{\zeta}_u^G$ and ζ_u^n of (10), (11) and (12), respectively.

We write (15) in the perturbed form

$$\underset{\sim}{K}\,\delta\underset{\sim}{\zeta}_x + \underset{\sim}{\overline{K}}\,\delta\underset{\sim}{\zeta}_x^* = \delta\underset{\sim}{\zeta}_u^{m*} - \underset{\sim}{H}_{\zeta u}^t\,\delta\underset{\sim}{\zeta}_u^t - \underset{\sim}{\overline{H}}_{\zeta u}^t\,\delta\underset{\sim}{\zeta}_u^{t*}, \qquad (18)$$

where $\underset{\sim}{K}$, $\underset{\sim}{\overline{K}}$, $\underset{\sim}{H}_{\zeta u}^t$ and $\underset{\sim}{\overline{H}}_{\zeta u}^t$ denote, respectively, the formal derivatives $(\partial\underset{\sim}{h}^T/\partial\underset{\sim}{\zeta}_x)^T$, $(\partial\underset{\sim}{h}^T/\partial\underset{\sim}{\zeta}_x^*)^T$, $(\partial\underset{\sim}{h}^T/\partial\underset{\sim}{\zeta}_u^t)^T$ and $(\partial\underset{\sim}{h}^T/\partial\underset{\sim}{\zeta}_u^{t*})^T$.

We remark that the elements of the complex matrices $\underset{\sim}{K}$ and $\underset{\sim}{\overline{K}}$ constitute the well-known Jacobian matrix of the load flow problem in the rectangular form.

We now write (18) in the consistent form

$$\begin{bmatrix} \underset{\sim}{K} & \underset{\sim}{\overline{K}} \\ \underset{\sim}{\overline{K}}^* & \underset{\sim}{K}^* \end{bmatrix} \begin{bmatrix} \delta\underset{\sim}{\zeta}_x \\ \delta\underset{\sim}{\zeta}_x^* \end{bmatrix} = \begin{bmatrix} \delta\underset{\sim}{\zeta}_u^{m*} \\ \delta\underset{\sim}{\zeta}_u^m \end{bmatrix} - \begin{bmatrix} \underset{\sim}{H}_{\zeta u}^t & \underset{\sim}{\overline{H}}_{\zeta u}^t \\ \underset{\sim}{\overline{H}}_{\zeta u}^{t*} & \underset{\sim}{H}_{\zeta u}^{t*} \end{bmatrix} \begin{bmatrix} \delta\underset{\sim}{\zeta}_u^t \\ \delta\underset{\sim}{\zeta}_u^{t*} \end{bmatrix}. \qquad (19)$$

For a real function f of $\underset{\sim}{\zeta}_x$, $\underset{\sim}{\zeta}_x^*$, $\underset{\sim}{\zeta}_u^m$, $\underset{\sim}{\zeta}_u^{m*}$, $\underset{\sim}{\zeta}_u^t$ and $\underset{\sim}{\zeta}_u^{t*}$, we may, using (4), write

$$\delta f = \underset{\sim}{f}_{\zeta x}^T\,\delta\underset{\sim}{\zeta}_x + \underset{\sim}{f}_{\zeta x}^{*T}\,\delta\underset{\sim}{\zeta}_x^* + \underset{\sim}{f}_{\zeta u}^{mT}\,\delta\underset{\sim}{\zeta}_u^m + (\underset{\sim}{f}_{\zeta u}^{m*})^T\,\delta\underset{\sim}{\zeta}_u^{m*} + \underset{\sim}{f}_{\zeta u}^{tT}\,\delta\underset{\sim}{\zeta}_u^t + (\underset{\sim}{f}_{\zeta u}^{t*})^T\,\delta\underset{\sim}{\zeta}_u^{t*}, \qquad (20)$$

where $\underset{\sim}{f}_{\zeta x}$, $\underset{\sim}{f}_{\zeta u}^m$ and $\underset{\sim}{f}_{\zeta u}^t$ denote, respectively, the formal partial derivatives of f w.r.t. $\underset{\sim}{\zeta}_x$, $\underset{\sim}{\zeta}_u^m$ and $\underset{\sim}{\zeta}_u^t$.

Using (19) and (20), it can be shown that

$$\delta f = [\hat{\underset{\sim}{V}}^T \quad \hat{\underset{\sim}{V}}^{*T}] \left\{ \begin{bmatrix} \delta \underset{\sim}{\zeta}_u^{m*} \\ \\ \delta \underset{\sim}{\zeta}_u^{m} \end{bmatrix} - \begin{bmatrix} \underset{\sim}{H}_{\zeta u}^{t} & \underset{\sim}{\bar{H}}_{\zeta u}^{t} \\ \\ \underset{\sim}{\bar{H}}_{\zeta u}^{t*} & \underset{\sim}{H}_{\zeta u}^{t*} \end{bmatrix} \begin{bmatrix} \delta \underset{\sim}{\zeta}_u^{t} \\ \\ \delta \underset{\sim}{\zeta}_u^{t*} \end{bmatrix} \right\}$$

$$+ \underset{\sim}{f}_{\zeta u}^{mT} \delta \underset{\sim}{\zeta}_u^{m} + (\underset{\sim}{f}_{\zeta u}^{m*})^T \delta \underset{\sim}{\zeta}_u^{m*} + \underset{\sim}{f}_{\zeta u}^{tT} \delta \underset{\sim}{\zeta}_u^{t} + (\underset{\sim}{f}_{\zeta u}^{t*})^T \delta \underset{\sim}{\zeta}_u^{t*}, \qquad (21)$$

where $\hat{\underset{\sim}{V}}$ are complex adjoint variables obtained from solving the adjoint equations

$$[\underset{\sim}{K}^T \quad \underset{\sim}{\bar{K}}^{*T}] \begin{bmatrix} \hat{\underset{\sim}{V}} \\ \hat{\underset{\sim}{V}}^{*} \end{bmatrix} = \underset{\sim}{f}_{\zeta x}. \qquad (22)$$

The required total formal derivatives of f w.r.t. $\underset{\sim}{\zeta}_u^m$ and $\underset{\sim}{\zeta}_u^t$ are obtained directly from (21) as follows

$$\frac{df}{d\underset{\sim}{\zeta}_u^m} = \hat{\underset{\sim}{V}}^{*} + \underset{\sim}{f}_{\zeta u}^{m} \qquad (23)$$

and

$$\frac{df}{d\underset{\sim}{\zeta}_u^{tT}} = \underset{\sim}{f}_{\zeta u}^{tT} - \hat{\underset{\sim}{V}}^T \underset{\sim}{H}_{\zeta u}^{t} - \hat{\underset{\sim}{V}}^{*T} \underset{\sim}{\bar{H}}_{\zeta u}^{t*} . \qquad (24)$$

In practice, gradients w.r.t. real and imaginary parts of the defined control variables are of direct interest. These gradients are simply obtained from

$$\frac{df}{d\underset{\sim}{\zeta}_1} = 2 \text{ Re}\{ \frac{df}{d\underset{\sim}{\zeta}} \} \qquad (25)$$

and

$$\frac{df}{d\underset{\sim}{\zeta}_2} = -\ 2\ \mathrm{Im}\{\ \frac{df}{d\underset{\sim}{\zeta}}\ \}, \tag{26}$$

where $\underset{\sim}{\zeta}$ can be $\underset{\sim}{\zeta}_u^m$ or $\underset{\sim}{\zeta}_u^t$.

For a given real function f, the adjoint equations (22) are formulated using the elements of the Jacobian of the load flow problem at the solution point. The solution of (22) is then substituted into (23) - (26) to obtain the required total derivatives of f w.r.t. control variables.

V. CONCLUSIONS

A compact complex notation has been utilized to describe an approach for compact derivation of sensitivity expressions required in power system studies. The approach has been described using only complex matrix manipulations. It exploits the Jacobian matrix of the original load flow problem. The approach has been illustrated using the cartesian coordinates. The approach is also applicable to the polar form through suitable transformations.

REFERENCES

[1] A.M. Sasson and H.M. Merrill, "Some applications of optimization techniques to power system problems", Proc. IEEE, vol. 62, 1974, pp. 959-972.

[2] H.W. Dommel and W.F. Tinney, "Optimal power flow solutions", IEEE Trans. Power Apparatus and Systems, vol. PAS-87, 1968, pp. 1866-1876.

[3] S.W. Director and R.L. Sullivan, "A tableau approach to power system analysis and design", Proc. IEEE Int. Symp. Circuits and Systems (New York, 1978), pp. 605-609.

[4] R. Fischl and R.G. Wasely, "Efficient computation of optimal load flow sensitivities", IEEE Canadian Communications and Power Conf., Cat. No. 78 CH1373-0 REG7, 1978, pp. 401-404.

[5] J.W. Bandler and M.A. El-Kady, "A unified approach to power system sensitivity analysis and planning, Part I: family of adjoint systems", Proc. IEEE Int. Symp. Circuits and Systems (Houston, TX, 1980, pp. 681-687.

[6] J.E. Van Ness and J.H. Griffin, "Elimination methods for load-flow studies", Proc. AIEE Power Apparatus and Systems, vol. 80, 1961, pp. 299-304.

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