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A UNIFIED APPROACH TO POWER SYSTEM SENSITIVITY ANALYSIS AND PLANNING

PART III: CONSISTENT SELECTION OF ADJOINING COEFFICIENTS

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Abstract

A unified approach to power system sensitivity analysis and planning has been presented in Part I of the paper. The approach utilizes a generalized adjoint network concept with complex adjoining coefficients set to proper values which allow the required sensitivity evaluation. Here, we present a unified study for consistent selection of the adjoining coefficients where the restrictions imposed by the type of system and the particular function considered are investigated. The study, hence, justifies the use of the approach described in Part I as a general network approach.

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I. INTRODUCTION

Adjoint network approaches [1,2] have been successfully applied for efficient sensitivity analysis and gradient evaluation of electrical systems. In Part I of the paper [3], an approach to system sensitivity analysis based upon a generalized adjoint network concept has been presented. Although the approach has been applied to power system sensitivity analysis, other electrical and analogous systems can be handled as well.

For a particular system, the feasibility of obtaining the required sensitivities depends entirely on the number and type of different system elements, the function considered, and the selected adjoining coefficients. Good understanding of the restrictions imposed due to these factors allows proper formulation of the adjoint system.

In the first two sections, we explore some theoretical features of the conjugate notation introduced in Part I of the paper and utilize them to investigate consistent selection of the adjoining coefficients. A special technique which allows fast and easy selection of proper adjoining coefficients is presented in Section V. The more general case of functional adjoining coefficients is introduced in Section VI. More flexibility in controlling the adjoint system is afforded by exploiting the functional adjoining coefficients.

Two electrical systems, namely a typical linear electronic system and a power system [3] are used as examples throughout the paper.

II. PRELIMINARIES

In Part I of the paper, we have considered several forms of Tellegen terms summed via the complex coefficients α, $\bar{\alpha}$, β, $\bar{\beta}$, ξ, $\bar{\xi}$, ν and $\bar{\nu}$ together with a number of group terms adjoined to Tellegen sum via the complex coefficients Γ_k and $\bar{\Gamma}_k$, k denoting the kth group term. Sensitivity expressions for a particular system can be derived by suitable selection of these adjoining coefficients. Each system has, in general, a number of element types. For each element b, and according to its type, we have defined a set of element variables $\underset{\sim}{z}_b$ describing the practical state $\underset{\sim}{x}_b$ and control $\underset{\sim}{u}_b$ variables associated with it, $\underset{\sim}{x}_b$ and $\underset{\sim}{u}_b$ denoting two component column vectors, and

$$\underset{\sim}{z}_b = \begin{bmatrix} \underset{\sim}{x}_b \\ \underset{\sim}{u}_b \end{bmatrix} . \tag{1}$$

Also, we have denoted the basic variables associated with element b by the vector

$$\underset{\sim}{w}_b = \begin{bmatrix} \underset{\sim}{w}_{bv} \\ \\ \underset{\sim}{w}_{bi} \end{bmatrix} \triangleq \begin{bmatrix} V_b \\ V_b^* \\ -- \\ I_b \\ I_b^* \end{bmatrix} , \tag{2}$$

where V_b and I_b are the complex voltage and current associated with element b, respectively, and * denotes the complex conjugate.

The first-order changes of both $\underset{\sim}{z}_b$ and $\underset{\sim}{w}_b$ are related by

$$\delta \underset{\sim}{w}_b^T = \delta \underset{\sim}{z}_b^T \underset{\sim}{M}^b , \tag{3}$$

T denotes transposition and the matrix $\underset{\sim}{M}^b$ has the partitioned form

$$\underset{\sim}{M}^b = \begin{bmatrix} \underset{\sim}{M}^b_{11} & \underset{\sim}{M}^b_{12} \\ & \\ \underset{\sim}{M}^b_{21} & \underset{\sim}{M}^b_{22} \end{bmatrix} , \tag{4}$$

where $\underset{\sim}{M}^b_{11}$, $\underset{\sim}{M}^b_{12}$, $\underset{\sim}{M}^b_{21}$ and $\underset{\sim}{M}^b_{22}$ are 2x2 matrices.

The element b of the adjoint network is defined either by equation (31) of Part I, namely

$$\hat{\underset{\sim}{\eta}}_{bx} = \underset{\sim}{M}^b_{11} \hat{\underset{\sim}{f}}_{bi} + \underset{\sim}{M}^b_{12} \hat{\underset{\sim}{f}}_{bv} , \tag{5}$$

where $\hat{\underset{\sim}{f}}_{bi}$ and $\hat{\underset{\sim}{f}}_{bv}$ are given by (17) of Part I and $\hat{\underset{\sim}{\eta}}_{bx}$ is set according to (23) of Part I, or by equation (39) of Part I, namely

$$\bar{\underset{\sim}{\theta}}_{bi} \hat{\underset{\sim}{w}}_{bi} = \bar{\underset{\sim}{\theta}}_{bv} \hat{\underset{\sim}{w}}_{bv} + \underset{\sim}{\theta}_b , \tag{6}$$

where the 2x2 matrices $\bar{\underset{\sim}{\theta}}_{bi}$ and $\bar{\underset{\sim}{\theta}}_{bv}$ and the vector $\underset{\sim}{\theta}_b$ are given by (40) and (41) of Part I and the adjoint variables $\hat{\underset{\sim}{w}}_{bi}$ and $\hat{\underset{\sim}{w}}_{bv}$ are given by

$$\hat{\underset{\sim}{w}}_b = \begin{bmatrix} \hat{\underset{\sim}{w}}_{bv} \\ --- \\ \hat{\underset{\sim}{w}}_{bi} \end{bmatrix} \triangleq \begin{bmatrix} \hat{V}_b \\ \hat{V}^*_b \\ -- \\ \hat{I}_b \\ \hat{I}^*_b \end{bmatrix} . \tag{7}$$

The solution of the adjoint system (74) of Part I provides the adjoint variables $\hat{\underset{\sim}{w}}_b$. The derivatives (formal derivatives) of the function f w.r.t. the control variables are calculated using (26) or (28) of Part I, namely

$$\frac{df}{d\underset{\sim}{u}_b} = \frac{\partial f}{\partial \underset{\sim}{u}_b} - \hat{\underset{\sim}{\eta}}_{bu} \tag{8}$$

or

$$\frac{df}{d\zeta_{bi}} = [(\frac{\partial f}{\partial \underset{\sim}{u}_b})^T - \hat{\underset{\sim}{\eta}}_{bu}^T] \frac{\partial \underset{\sim}{u}_b}{\partial \zeta_{bi}} , \tag{9}$$

where $\hat{\underset{\sim}{\eta}}_{bu}$ is given by (32) of Part I, namely

$$\hat{\underset{\sim}{\eta}}_{bu} = \underset{\sim}{M}_{21}^{b} \hat{\underset{\sim}{f}}_{bi} + \underset{\sim}{M}_{22}^{b} \hat{\underset{\sim}{f}}_{bv}. \tag{10}$$

III. REMARKS ON THE CONJUGATE NOTATION

We have introduced and utilized the conjugate notation in Parts I and II of the paper [3,4]. The use of conjugate notation has facilitated the derivations and subsequent formulation of the equations to be solved. In classical complex algebra, the variables of a system of complex linear equations are defined independently [5], e.g., x_1, x_2, etc., and this is the case in real algebra. Since the use of conjugate notation implies in some cases a set of complex variables and their complex conjugates to appear in the same linear equations, a special analysis is required to reveal the properties of such systems of linear equations regarding, for example, rank, consistency conditions, etc.

Through Parts I and II of the paper, the application of conjugate notation has been performed in a straightforward manner since the assumption of consistency of (6) was made when defining the adjoint elements. In this part, namely Part III, the consistency of (6) is discussed for suitable selection of the adjoining coefficients. In order to facilitate the consistency study performed in the following sections, we state here the following theorems.

Theorem 1

Let $\underset{\sim}{\theta}, \overline{\underset{\sim}{\theta}} \in C^{mxn}$, where

$$\underset{\sim}{\theta} = \underset{\sim}{\theta}_1 + j\, \underset{\sim}{\theta}_2 \tag{11}$$

and

$$\overline{\underset{\sim}{\theta}} = \overline{\underset{\sim}{\theta}}_1 + j\, \overline{\underset{\sim}{\theta}}_2, \tag{12}$$

$\underset{\sim}{\theta}_1, \underset{\sim}{\theta}_2, \overline{\underset{\sim}{\theta}}_1, \overline{\underset{\sim}{\theta}}_2 \in R^{mxn}$. Then the two matrices $\underset{\sim}{\theta}^c \in C^{2mx2n}$ and $\underset{\sim}{\theta}^r \in R^{2mx2n}$ defined as

$$\underset{\sim}{\theta}^c \triangleq \begin{bmatrix} \underset{\sim}{\theta} & \overline{\underset{\sim}{\theta}} \\ \overline{\underset{\sim}{\theta}}^* & \underset{\sim}{\theta}^* \end{bmatrix} \tag{13}$$

and

$$\underset{\sim}{\theta}^r \triangleq \begin{bmatrix} (\underset{\sim}{\theta}_1 + \overline{\underset{\sim}{\theta}}_1) & (\overline{\underset{\sim}{\theta}}_2 - \underset{\sim}{\theta}_2) \\ (\underset{\sim}{\theta}_2 + \overline{\underset{\sim}{\theta}}_2) & (\underset{\sim}{\theta}_1 - \overline{\underset{\sim}{\theta}}_1) \end{bmatrix} \tag{14}$$

have the same rank.

Proof

Let $\underset{\sim}{1}^{\ell}$ be the identity matrix of order ℓ,

$$\underset{\sim}{j}^{\ell} \triangleq j\, \underset{\sim}{1}^{\ell} \tag{15}$$

and define the two unitary matrices

$$\underset{\sim}{U}_L \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} \underset{\sim}{1}^m & \underset{\sim}{j}^m \\ \underset{\sim}{1}^m & -\underset{\sim}{j}^m \end{bmatrix} \tag{16}$$

and

$$\underset{\sim}{U}_R \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} \underset{\sim}{1}^n & \underset{\sim}{1}^n \\ -\underset{\sim}{j}^n & \underset{\sim}{j}^n \end{bmatrix}. \tag{17}$$

Since $\underset{\sim}{U}_L$ and $\underset{\sim}{U}_R$ are nonsingular, hence [5]

$$\text{rank}[\underset{\sim}{U}_L \, \underset{\sim}{\theta}^r \, \underset{\sim}{U}_R] = \text{rank}[\underset{\sim}{\theta}^r].$$

But

$$\underset{\sim}{U}_L \, \underset{\sim}{\theta}^r \, \underset{\sim}{U}_R = \underset{\sim}{\theta}^c,$$

hence

$$\text{rank}[\underset{\sim}{\theta}^r] = \text{rank}[\underset{\sim}{\theta}^c] \quad \blacksquare$$

<u>Theorem 2</u>

Let $\underset{\sim}{\theta}$, $\overline{\underset{\sim}{\theta}} \in C^{mxn}$ given by (11) and (12), and let $\underset{\sim}{\theta} \in C^m$ and $\underset{\sim}{w} \in C^n$, where

$$\underset{\sim}{\theta} = \underset{\sim}{\theta}_1 + j\,\underset{\sim}{\theta}_2 \tag{18}$$

and

$$\underset{\sim}{w} = \underset{\sim}{w}_1 + j\,\underset{\sim}{w}_2, \tag{19}$$

$\underset{\sim}{\theta}_1$, $\underset{\sim}{\theta}_2 \in R^m$ and $\underset{\sim}{w}_1$, $\underset{\sim}{w}_2 \in R^n$. Then the system of complex linear equations

$$\underset{\sim}{\theta}\,\underset{\sim}{w} + \overline{\underset{\sim}{\theta}}\,\underset{\sim}{w}^* = \underset{\sim}{\theta} \tag{20}$$

has a solution $\underset{\sim}{w}$ if and only if

$$\text{rank}[(\underset{\sim}{\theta}^c, \underset{\sim}{\theta}^c)] = \text{rank}[\underset{\sim}{\theta}^c],$$

where

$\underset{\sim}{\theta}^{c} \in C^{2m}$ is defined as

$$\underset{\sim}{\theta}^{c} \triangleq \begin{bmatrix} \underset{\sim}{\theta} \\ \\ \underset{\sim}{\theta}^{*} \end{bmatrix} \tag{21}$$

and $\underset{\sim}{\theta}^{c}$ is given by (13).

<u>Proof</u>

Separating (20) into real and imaginary parts using (11), (12), (18) and (19), we get

$$(\underset{\sim}{\theta}_1 + \overline{\underset{\sim}{\theta}}_1)\, \underset{\sim}{w}_1 + (\overline{\underset{\sim}{\theta}}_2 - \underset{\sim}{\theta}_2)\, \underset{\sim}{w}_2 = \underset{\sim}{\theta}_1 \tag{22a}$$

and

$$(\underset{\sim}{\theta}_2 + \overline{\underset{\sim}{\theta}}_2)\, \underset{\sim}{w}_1 + (\underset{\sim}{\theta}_1 - \underset{\sim}{\theta}_1)\, \underset{\sim}{w}_2 = \underset{\sim}{\theta}_2 \tag{22b}$$

or, using (14),

$$\underset{\sim}{\theta}^{r}\, \underset{\sim}{w}^{r} = \underset{\sim}{\theta}^{r}, \tag{23}$$

where

$$\underset{\sim}{w}^{r} \triangleq \begin{bmatrix} \underset{\sim}{w}_1 \\ \\ \underset{\sim}{w}_2 \end{bmatrix} \tag{24}$$

and

$$\underset{\sim}{\theta}^{r} \triangleq \begin{bmatrix} \underset{\sim}{\theta}_1 \\ \\ \underset{\sim}{\theta}_2 \end{bmatrix} . \tag{25}$$

We define the nonsingular matrix

$$\bar{U}_R \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} 1^n & 1^n & 0 \\ -j^n & j^n & 0 \\ 0 & 0 & 2 \end{bmatrix} , \tag{26}$$

hence

$$\text{rank}[U_L(\theta^r, \theta^r) \bar{U}_R] = \text{rank}[(\theta^r, \theta^r)],$$

where U_L is given by (16). But

$$U_L(\theta^r, \theta^r) \bar{U}_R = (\theta^c, \theta^c),$$

hence

$$\text{rank}[(\theta^r, \theta^r)] = \text{rank}[(\theta^c, \theta^c)]. \tag{27}$$

Now, the system of equations (23) has a solution if and only if [5]

$$\text{rank }[(\theta^r, \theta^r)] = \text{rank}[\theta^r],$$

hence the theorem is proved using (27) and Theorem 1 ■

IV. CRITERIA FOR ADJOINING COEFFICIENTS SELECTION

In this section, we derive the required conditions which (6) must satisfy for proper definition of the adjoint system. First, equation (6) must be consistent. The results of the previous section allow us to state the following corollary.

Corollary

Equation (6) is consistent if and only if

$$\operatorname{rank}[(\bar{\underset{\sim}{\theta}}_b, \underset{\sim}{\theta}_b^c)] = \operatorname{rank}[(\bar{\underset{\sim}{\theta}}_b)], \tag{28}$$

where

$$\bar{\underset{\sim}{\theta}}_b \triangleq \begin{bmatrix} -\bar{\underset{\sim}{\theta}}_{bv} & \bar{\underset{\sim}{\theta}}_{bi} \\ -\bar{\underset{\sim}{\theta}}^*_{bv}\bar{\underset{\sim}{1}} & \bar{\underset{\sim}{\theta}}^*_{bi}\bar{\underset{\sim}{1}} \end{bmatrix}, \tag{29}$$

$$\underset{\sim}{\theta}_b^c \triangleq \begin{bmatrix} \underset{\sim}{\theta}_b \\ \underset{\sim}{\theta}^*_b \end{bmatrix} \tag{30}$$

and

$$\bar{\underset{\sim}{1}} \triangleq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . \tag{31}$$

Proof

The conjugate of (6) is written as

$$\bar{\underset{\sim}{\theta}}^*_{bi}\, \hat{\underset{\sim}{w}}^*_{bi} = \bar{\underset{\sim}{\theta}}^*_{bv}\, \hat{\underset{\sim}{w}}^*_{bv} + \underset{\sim}{\theta}^*_b. \tag{32}$$

Since

$$\hat{\underset{\sim}{w}}^*_{bi} = \bar{\underset{\sim}{1}}\, \hat{\underset{\sim}{w}}_{bi} \tag{33a}$$

and

$$\hat{\underset{\sim}{w}}^*_{bv} = \bar{\underset{\sim}{1}}\, \hat{\underset{\sim}{w}}_{bv}, \tag{33b}$$

hence

$$\overline{\theta}^{*}_{\sim bi}\ \overline{1}_{\sim}\ \hat{w}_{\sim bi} = \overline{\theta}^{*}_{\sim bv}\ \overline{1}_{\sim}\ \hat{w}_{\sim bv} + \theta^{*}_{\sim b}. \tag{34}$$

Equations (6) and (34) are written together as

$$\overline{\theta}_{\sim b}\ \hat{w}_{\sim b} = \theta^{c}_{\sim b}. \tag{35}$$

When the variables $\hat{w}_{\sim b}$ and the corresponding columns of $\overline{\theta}_{\sim b}$ are rearranged such that (6) has the same form as (20), the rank of $\overline{\theta}_{\sim b}$ is preserved. Hence from Theorem 2, the corollary is proved ■

In order to uniquely define the adjoint elements with proper relations between adjoint variables, we also require [3], in addition to (28), that the system of four real equations (6) has rank 2. Hence, from Theorem 1 and the previous corollary, the matrix $\overline{\theta}_{\sim b}$ of (29) must be of rank 2. In summary, the conditions which (6) must satisfy are

$$\text{rank}[(\overline{\theta}_{\sim b},\ \theta^{c}_{\sim b})] = \text{rank}[(\overline{\theta}_{\sim b})] = 2. \tag{36}$$

Note that the elements of the matrices $M^{b}_{\sim 11}$ and $M^{b}_{\sim 12}$ and the complex adjoining coefficients form the matrix $\overline{\theta}_{\sim b}$. The matrix $M^{b}_{\sim}$ of (4) depends solely upon the element-type modelling. Moreover, the vector $\theta^{c}_{\sim b}$ contains the derivatives of the function f w.r.t. the states associated with element b. Thus we require a proper selection of the adjoining coefficients which satisfy (36) for a particular element-type modelling and for a given function f.

Since the set of adjoining coefficients α, $\overline{\alpha}$, β, $\overline{\beta}$, ξ, $\overline{\xi}$, ν and $\overline{\nu}$ is common to all element types described in a particular system, we expect that the more element types in a system, the more restrictions, hence, the more difficulty there will be in selecting these adjoining coefficients to satisfy (36). On the other hand, the adjoining coefficients Γ_k and $\overline{\Gamma}_k$ are only common to those elements within certain group terms [3] which may include a few element types. Consequently, we expect more flexibility in adjusting these coefficients to satisfy (36) for certain elements.

Examples of element types of particular systems are shown in Tables I, II and III. Tables I and II represent typical linear electronic circuits. Two element types are described for each system, namely, the node elements (e.g., source elements) and the line elements. One representation of the power system [3] is shown in Table III in which four element types describing the loads, the generators, the slack generator and the transmission elements are considered.

A comparison between the electronic system of Table I or II and the power system of Table III is of particular interest [2]. The state and control variables associated with the source elements of an electronic system are simply the basic variables $\underset{\sim}{w}_b$ of (2) which classify them as either current sources (Table I) or voltage sources (Table II). In a power system the situation is different. The state and control variables associated with the source elements are nonlinear functions of the basic variables $\underset{\sim}{w}_b$ which results in nonlinear load flow equations, and also difficulty with respect to the consistent selection of the adjoining coefficients as we shall see later on.

At the end of this section we state some important forms of equation (6) which satisfy condition (36). As shown in Part I, equation (6) has the form

$$\tilde{\phi}_b^k \hat{I}_b + \bar{\phi}_b^k \hat{I}_b^* = \tilde{\psi}_b^k \hat{V}_b + \bar{\psi}_b^k \hat{V}_b^* + \hat{W}_b^{Sk}, \tag{37}$$

where k = 1, 2 denotes the first and second complex equations of (6), respectively. It can be shown that each of the following conditions is equivalent to (36).

Alternative Condition 1

$$\tilde{\phi}_b^1 = \tilde{\phi}_b^2,\ \bar{\phi}_b^1 = \bar{\phi}_b^2,\ \tilde{\psi}_b^1 = \tilde{\psi}_b^2,\ \bar{\psi}_b^1 = \bar{\psi}_b^2 \tag{38a}$$

and

$$\hat{W}_b^{S1} = \hat{W}_b^{S2}, \tag{38b}$$

in which the two complex equations of (37) are identical.

Alternative Condition 2

$$\tilde{\phi}_b^1 = \bar{\phi}_b^{2*},\ \bar{\phi}_b^1 = \tilde{\phi}_b^{2*},\ \tilde{\psi}_b^1 = \bar{\psi}_b^{2*},\ \bar{\psi}_b^1 = \tilde{\psi}_b^{2*} \tag{39a}$$

and

$$\hat{W}_b^{S1} = \hat{W}_b^{S2*}, \tag{39b}$$

in which the first complex equation of (37) is the conjugate of the second one.

Alternative Condition 3

$$\tilde{\phi}_b^1 = \bar{\phi}_b^{1*},\ \tilde{\psi}_b^1 = \bar{\psi}_b^{1*},\ \hat{W}_b^{S1} \text{ is real} \tag{40a}$$

or

$$\tilde{\phi}_b^1 = -\bar{\phi}_b^{1*},\ \tilde{\psi}_b^1 = -\bar{\psi}_b^{1*},\ \hat{W}_b^{S2} \text{ is imaginary} \tag{40b}$$

and

$$\tilde{\phi}_b^2 = \bar{\phi}_b^{2*},\ \tilde{\psi}_b^2 = \bar{\psi}_b^{2*},\ \hat{W}_b^{S2} \text{ is real} \tag{40c}$$

or

$$\tilde{\phi}_b^2 = -\bar{\phi}_b^{2*},\ \tilde{\psi}_b^2 = -\bar{\psi}_b^{2*} \text{ and } \hat{W}_b^{S2} \text{ is imaginary,} \tag{40d}$$

in which each of the two equations of (37) represents one real equation.

Observe, for example, that for a real function f and under condition (9) of Part I, Table I of the same part shows a proper system by conditions (40a) and (40c) for all elements while Table II of the same part shows a proper adjoint system by condition (39) for the load elements, slack generator, and transmission elements.

V. A SPECIAL CONSISTENCY CRITERION

In the previous section, we have derived the required conditions for proper definition of the adjoint system to be solved. Since we are searching for proper adjoining coefficients which satisfy condition (36) rather than checking the condition itself, the form (36) may not be adequate for direct use in selecting the various adjoining coefficients.

In this section, we state a special technique for selecting the adjoining coefficients. The technique presented is based on a few assumptions regarding the coefficients and hence it satisfies a somewhat more restricted criterion than (36). The technique, however, allows fast and easy selection of proper adjoining coefficients for different systems of different element types.

We write (17) of Part I in the form

$$\hat{\underset{\sim}{f}}_b = \begin{bmatrix} \hat{\underset{\sim}{f}}_{bi} \\ --- \\ \hat{\underset{\sim}{f}}_{bv} \end{bmatrix} = \begin{bmatrix} \hat{\mathcal{I}}_b \\ \hat{\bar{\mathcal{I}}}_b \\ -- \\ -\hat{U}_b \\ -\hat{\bar{U}}_b \end{bmatrix} \tag{41}$$

where we have defined

$$\hat{\mathcal{I}}_b \triangleq \alpha \hat{I}_b + \bar{\xi} \hat{I}^*_b + D_{ib}, \tag{42a}$$

$$\hat{\bar{\mathcal{I}}}_b \triangleq \bar{\alpha} \hat{I}^*_b + \xi \hat{I}_b + \bar{D}_{ib}, \tag{42b}$$

$$\hat{U}_b \triangleq \beta \hat{V}_b + \bar{\nu} \hat{V}^*_b + D_{vb}, \tag{43a}$$

$$\hat{\bar{U}}_b \triangleq \bar{\beta} \hat{V}^*_b + \nu \hat{V}_b + \bar{D}_{vb} \tag{43b}$$

and, using (38) of Part I,

$$D_{ib} \triangleq \sum_k \lambda_{bk} N^k_{ib}, \tag{44a}$$

$$\bar{D}_{ib} \triangleq \sum_k \lambda_{bk} \bar{N}^k_{ib}, \tag{44b}$$

$$D_{vb} \triangleq -\sum_k \lambda_{bk} N^k_{vb} \tag{45a}$$

and

$$\bar{D}_{vb} \triangleq -\sum_k \lambda_{bk} \bar{N}^k_{vb}, \tag{45b}$$

where λ_{bk} is given by (8) of Part I.

Under the assumptions

$$\bar{\xi}\,\bar{\xi}^* - \alpha\,\alpha^* \neq 0 \tag{46a}$$

and

$$\bar{\nu}\,\bar{\nu}^{*} - \beta\,\beta^{*} \neq 0, \tag{46b}$$

one may write

$$\hat{\bar{I}}_{b} = A_{i}\,\hat{I}_{b} + \bar{A}_{i}\,\hat{I}_{b}^{*} + \tilde{A}_{ib} \tag{47}$$

and

$$\hat{\bar{U}}_{b} = A_{v}\,\hat{U}_{b} + \bar{A}_{v}\,\hat{U}_{b}^{*} + \tilde{A}_{vb}, \tag{48}$$

where

$$A_{i} \triangleq \frac{\bar{\xi}^{*}\,\bar{\alpha} - \xi\,\alpha^{*}}{\bar{\xi}\,\bar{\xi}^{*} - \alpha\,\alpha^{*}}, \tag{49a}$$

$$\bar{A}_{i} \triangleq \frac{\xi\,\bar{\xi} - \alpha\,\bar{\alpha}}{\bar{\xi}\,\bar{\xi}^{*} - \alpha\,\alpha^{*}}, \tag{49b}$$

$$\tilde{A}_{ib} \triangleq \bar{D}_{ib} - A_{i}\,D_{ib} - \bar{A}_{i}\,D_{ib}^{*}, \tag{49c}$$

$$A_{v} \triangleq \frac{\bar{\nu}^{*}\,\bar{\beta} - \nu\,\beta^{*}}{\bar{\nu}\,\bar{\nu}^{*} - \beta\,\beta^{*}}, \tag{50a}$$

$$\bar{A}_{v} \triangleq \frac{\nu\,\bar{\nu} - \beta\,\bar{\beta}}{\bar{\nu}\,\bar{\nu}^{*} - \beta\,\beta^{*}} \tag{50b}$$

and

$$\tilde{A}_{vb} \triangleq \bar{D}_{vb} - A_{v}\,D_{vb} - \bar{A}_{v}\,D_{vb}^{*}. \tag{50c}$$

Now, according to the element types used in a particular system, the gross coefficients A_i, $\bar{A}_i$ and $\tilde{A}_{ib}$ of (49) and A_v, $\bar{A}_v$ and $\tilde{A}_{vb}$ of (50) have to satisfy certain relationships to fulfil condition (36). The more element types used to describe certain system, the more those relationships will be.

The impact of using the gross coefficients is obvious. Instead of dealing directly with the numerous coefficients α, $\bar{\alpha}$, etc., while searching for a proper adjoint system representation, we only investigate conditions on fewer gross coefficients. Moreover, in the absence of group terms, $\tilde{A}_{ib}$ and $\tilde{A}_{vb}$ are automatically zero and only A_i, $\overline{A}_i$, A_v and $\overline{A}_v$ are left for study.

We illustrate the use of the gross coefficients by the following examples.

Example 1

For the electronic system described in Table I, and using (41), (47) and (48), equation (6) is written, for node elements, as

$$\frac{\partial f}{\partial V_j} = \hat{I}_j \tag{51a}$$

and

$$\frac{\partial f}{\partial V_j^*} = \hat{\overline{I}}_j = A_i \hat{I}_j + \overline{A}_i \hat{I}_j^* + \tilde{A}_{ij} \tag{51b}$$

and, for line elements, as

$$Y_t \frac{\partial f}{\partial I_t} = \hat{I}_t - Y_t \hat{U}_t \tag{52a}$$

and

$$Y_t^* \frac{\partial f}{\partial I_t^*} = \hat{\overline{I}}_t - Y_t^* \hat{\overline{U}}_t = A_i \hat{I}_t + \overline{A}_i \hat{I}_t^* + \tilde{A}_{it} - Y_t^* (A_v \hat{U}_t + \overline{A}_v \hat{U}_t^* + \tilde{A}_{vt}). \tag{52b}$$

Now, for a real function f, we have

$$\frac{\partial f}{\partial V_j} = \left(\frac{\partial f}{\partial V_j^*} \right)^* \tag{53a}$$

and

$$\frac{\partial f}{\partial I_t} = \left(\frac{\partial f}{\partial I_t^*} \right)^* \tag{53b}$$

so that (51) is consistent if

$$A_i = 0,\ \overline{A}_i = 1 \text{ and } \tilde{A}_{ij} = 0. \tag{54}$$

Using (54), equation (52) is also consistent if

$$A_v = 0,\ \overline{A}_v = 1 \text{ and } \tilde{A}_{vt} = \tilde{A}_{it} = 0. \tag{55}$$

Under conditions (54) and (55), any of (51a) or (51b) can be used to define the adjoint node elements, also, any of (52a) or (52b) can be used to define the adjoint line elements.

In terms of the adjoining coefficients α, $\overline{\alpha}$, etc., it is obvious from (49) and (50) that

$$\overline{\alpha} = \alpha^*,\ \overline{\beta} = \beta^*,\ \overline{\xi} = \xi^* \text{ and } \overline{\nu} = \nu^*, \tag{56a}$$

$$\overline{D}_{ib} = D^*_{ib} \text{ and } \overline{D}_{vb} = D^*_{vb} \tag{56b}$$

is sufficient to satisfy (36).

Note that (56b) is an alternative condition to (9e) of Part I, namely,

$$\overline{\Gamma}_k = \Gamma^*_k \text{ for all k.} \tag{56c}$$

<u>Example 2</u>

For the power system described in Table III, and following a similar procedure to that of Example 1 for the different element types of the system, it is a straightforward to show that, for a real function f, condition (56) is also sufficient [4] to satisfy (36).

Example 3

Consider, again, the electronic system of Table I. Let

$$f = V_k, \tag{57}$$

where V_k is a certain complex node voltage. With no group terms and using (41), (47) and (48), equation (6) is written, for the node element $j \neq k$, as

$$0 = \hat{I}_j \tag{58a}$$

and

$$0 = \hat{\bar{I}}_j = A_i \hat{I}_j + \bar{A}_i \hat{I}_j^*, \tag{58b}$$

which requires no restrictions on A_i or $\bar{A}_i$, and for the node element k, as

$$1 = \hat{I}_k \tag{59a}$$

and

$$0 = \hat{\bar{I}}_k = A_i \hat{I}_k + \bar{A}_i \hat{I}_k^*, \tag{59b}$$

which requires

$$A_i = -\bar{A}_i. \tag{60}$$

Also, for line elements, we write

$$0 = \hat{I}_t - Y_t \hat{U}_t \tag{61a}$$

and

$$0 = A_i \hat{I}_t + \bar{A}_i \hat{I}_t^* - Y_t^*(A_v \hat{U}_t + \bar{A}_v \hat{U}_t^*), \tag{61b}$$

which requires

$$A_i Y_t = A_v Y_t^* \text{ for all } t. \tag{62}$$

Hence

$$A_i = A_v = 0 \tag{63}$$

and

$$\overline{A}_i = \overline{A}_v. \tag{64}$$

Conditions (60), (63) and (64) are simply

$$A_i = A_v = \overline{A}_i = \overline{A}_v = 0, \tag{65}$$

or, from (49) and (50),

$$\xi\ \overline{\xi} = \alpha\ \overline{\alpha}, \tag{66a}$$

$$\nu\ \overline{\nu} = \beta\ \overline{\beta}, \tag{66b}$$

$$\overline{\xi}^*\ \overline{\alpha} = \xi\ \alpha^* \tag{66c}$$

and

$$\overline{\nu}^*\ \overline{\beta} = \nu\ \beta^*. \tag{66d}$$

Observe that any member of the family of adjoining coefficients satisfying (66) can lead to the required sensitivities of V_k. In particular, the member

$$\alpha = \beta = 1 \tag{67a}$$

and

$$\overline{\alpha} = \overline{\beta} = \xi = \overline{\xi} = \nu = \overline{\nu} = 0, \tag{67b}$$

or

$$\overline{\xi} = \overline{\nu} = 1 \tag{68a}$$

and

$$\alpha = \bar{\alpha} = \beta = \bar{\beta} = \xi = \nu = 0, \tag{68b}$$

may be used. For the member (67), the adjoint system is defined, using (58a), (59a) and (61a), as

$$\hat{I}_j = 0 \text{ for } j \neq k, \tag{69a}$$

$$\hat{I}_k = 1 \tag{69b}$$

and

$$\hat{I}_t = Y_t \hat{V}_t, \tag{69c}$$

while, for the member (68), the adjoint system is defined as

$$\hat{I}_j = 0 \text{ for } j \neq k, \tag{70a}$$

$$\hat{I}_k = 1 \tag{70b}$$

and

$$\hat{I}_t = Y_t^* \hat{V}_t. \tag{70c}$$

As illustrated by the above examples, the technique described in this section allows direct selection of proper adjoining coefficients for a given system and for a certain function. In this respect, sensitivities of some of the complex functions of practical interest can be obtained directly by appropriate adjustment of these coefficients.

On the other hand, the adjoining coefficients play an important role in the adjoint network formulation. The freedom acquired by defining a family of possible adjoining coefficients can be utilized to alter the modelling of the adjoint elements. In some cases, it is possible to achieve certain modelling for a particular adjoint element.

In the next section, more freedom in selecting the adjoining coefficients is afforded by considering the more general case of functional adjoining coefficients.

VI. FUNCTIONAL ADJOINING COEFFICIENTS

In the analysis so far, we have considered the case of constant adjoining coefficients in (7) of Part I. Since these adjoining coefficients are basically multipliers of zero quantities, the restriction of constant adjoining coefficients can be relaxed. In fact, the adjoining coefficients can be functions of the basic variables $\underset{\sim}{w}_b$ of (2). Moreover, since

$$\tau(\underset{\sim}{\overline{w}})\ \delta h(\underset{\sim}{\overline{w}}) = h(\underset{\sim}{\overline{w}})\ \delta\tau(\underset{\sim}{\overline{w}}) = 0, \tag{71}$$

where $\tau(\underset{\sim}{\overline{w}})$ stands for any of the adjoint coefficients, $\underset{\sim}{\overline{w}}$ denotes a vector of the basic variables $\underset{\sim}{w}_b$, and

$$h(\underset{\sim}{\overline{w}}) = 0 \tag{72}$$

represents any of the terms (5) or (6) of Part I, the adjoining coefficients are not required to be perturbed in (10) of Part I. Hence, the sensitivity expressions derived so far are still valid even when the adjoining coefficients are functions of the basic variables.

On the other hand, the adjoining coefficients can be also functions of the adjoint variables $\underset{\sim}{w}_b$ of (7). The case when the set of adjoining coefficients α, $\overline{\alpha}$, β, $\overline{\beta}$, ξ, $\overline{\xi}$, ν and $\overline{\nu}$ are functions of the adjoint variables usually results in nonlinear adjoint equations to be solved. The case when the adjoining coefficients Γ_k and $\overline{\Gamma}_k$ are linear functions of the adjoint variables is indeed of particular interest.

We consider the case in which the adjoining coefficients Γ_k and $\overline{\Gamma}_k$ are linear functions of the adjoint variables $\hat{\underset{\sim}{w}}_b$ contained in the kth group term in the forms

$$\Gamma_k = \gamma_{k0} + \sum_{b\in B_k} (\gamma^b_{kv} \hat{V}_b + \gamma^{b\bullet}_{kv} \hat{V}^*_b + \gamma^b_{ki} \hat{I}_b + \gamma^{b\bullet}_{ki} \hat{I}^*_b) \qquad (73a)$$

and

$$\overline{\Gamma}_k = \overline{\gamma}_{k0} + \sum_{b\in B_k} (\overline{\gamma}^b_{kv} \hat{V}_b + \overline{\gamma}^{b\bullet}_{kv} \hat{V}^*_b + \overline{\gamma}^b_{ki} \hat{I}_b + \overline{\gamma}^{b\bullet}_{ki} \hat{I}^*_b), \qquad (73a)$$

where B_k is the set of elements forming the kth group term, and as indicated before, the coefficients γ_{k0}, γ^b_{kv}, etc., are in general functions of the basic variables $\underset{\sim}{w}_b$.

It is straightforward to show that the forms (73) still lead to a linear, although less sparse, adjoint system to be solved. In the resulting form of adjoint system, the diagonal matrices $\underset{\sim}{\Phi}_{ij}$ and $\underset{\sim}{\Psi}_{ij}$, i,j = 1, 2 of (64) and (74) of Part I are replaced by the equivalent matrices $\underset{\sim}{\Phi}^e_{ij}$ and $\underset{\sim}{\Psi}^e_{ij}$, respectively.

In general, the matrices $\underset{\sim}{\Phi}^e_{ij}$ and $\underset{\sim}{\Psi}^e_{ij}$ are no longer diagonal matrices. The more adjoint variables appearing in (73), the more will be the off diagonal elements of $\underset{\sim}{\Phi}^e_{ij}$ and $\underset{\sim}{\Psi}^e_{ij}$.

Since the number and type of group terms to be considered in a particular problem are entirely dictated by the type of the system and the function f, we shall not proceed towards general derivations for different systems and different classes of functions. Instead, we illustrate by a simple example the concepts stated in this section.

Example 4

Consider the simple 2-bus system of Fig. 1. The system consists of a load, a slack generator and three transmission elements. Required data in pu for the problem is shown in Fig. 1. Table IV shows the currents and voltages of different elements resulting from the a.c. load flow solution.

Suppose we are interested in the sensitivities of the complex load bus voltage. The adjoining coefficients may be set according to the special case described in Part II of the paper in which we may define the two real functions

$$f_1 = \text{Re}\,\{V_1\} = (V_1 + V_1^*)/2 \qquad (74a)$$

and

$$f_2 = \text{Im}\,\{V_1\} = j(V_1^* - V_1)/2. \qquad (74b)$$

The sensitivities of f_1 and f_2 are obtained in the same way described in Part II. The adjoint matrix of coefficients and the RHS vectors for both f_1 and f_2 are shown in Table V.

Alternatively, we may utilize the functional adjoining coefficients to obtain the sensitivities of the complex function

$$f = V_1, \qquad (75)$$

directly, while altering the modelling of the adjoint system.

For simplicity, we let

$$\xi = \bar{\xi} = \nu = \bar{\nu} = 0 \qquad (76a)$$

and

$$\Gamma_k = \overline{\Gamma}_k = 0 \quad \text{for all } k \neq 1, \tag{76b}$$

where we have considered the group terms

$$V_1 - V_2 - V_5 = 0 \tag{77a}$$

and

$$V_1^* - V_2^* - V_5^* = 0, \tag{77b}$$

adjoined via coefficients Γ_1 and $\overline{\Gamma}_1$, respectively.

With various adjoint elements modelled according to (5), it is a straightforward to show that the consistent selection of the adjoining coefficients requires, for example,

$$\alpha = \beta = 1,$$

$$\overline{\alpha} = \overline{\beta} = I_1/V_1^*(Y_3 + Y_5),$$

$$\Gamma_1 = -\hat{I}_1 - 1/(\overline{\beta\beta}^* - 1)$$

and

$$\overline{\Gamma}_1 = -\overline{\beta}\,\hat{I}_1^* - \overline{\beta}/(\overline{\beta\beta}^* - 1).$$

Observe that the above selection of the functional adjoining coefficients leads to modelling the load element in the adjoint system as a voltage source in the form

$$\hat{V}_1 = 1/[(\overline{\beta\beta}^* - 1)(Y_3 + Y_5)].$$

The derivatives of the complex load bus voltage shown in Table VI are calculated from (8) using the solution of the resulting simple adjoint network.

VII. CONCLUSIONS

We have presented a useful theoretical study which allows proper selection of the adjoining coefficients described in the generalized adjoint network concept presented in Part I of the paper. The freedom acquired by exploiting both constant and functional adjoining coefficients has been investigated so that complex function sensitivities for different systems of different element types may be evaluated via proper definition of the adjoint system.

The theoretical foundation of consistent modelling of different adjoint elements has been established by deriving suitable consistency criteria. These consistency criteria may be used to handle the more general branch modelling of power networks as distinct from that of typical electronic circuits.

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TABLE I

A TYPICAL LINEAR ELECTRONIC CIRCUIT WITH CURRENT SOURCES

Element Type	Symbol	$\underset{\sim}{x}_b$	$\underset{\sim}{u}_b$	$\underset{\sim}{M}^b_{11}$	$\underset{\sim}{M}^b_{12}$
Node Elements	j	$\begin{bmatrix} V_j \\ V^*_j \end{bmatrix}$	$\begin{bmatrix} I_j \\ I^*_j \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
Line Elements	t	$\begin{bmatrix} I_t \\ I^*_t \end{bmatrix}$	$\begin{bmatrix} Y_t \\ Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1/Y_t & 0 \\ 0 & 1/Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$Y_t = I_t/V_t$ is the admittance of line t.

TABLE II

A TYPICAL LINEAR ELECTRONIC CIRCUIT WITH VOLTAGE SOURCES

Element Type	Symbol	$\underset{\sim}{x}_b$	$\underset{\sim}{u}_b$	$\underset{\sim}{M}^b_{11}$	$\underset{\sim}{M}^b_{12}$
Node Elements	j	$\begin{bmatrix} I_j \\ I^*_j \end{bmatrix}$	$\begin{bmatrix} V_j \\ V^*_j \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Line Elements	t	$\begin{bmatrix} I_t \\ I^*_t \end{bmatrix}$	$\begin{bmatrix} Y_t \\ Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1/Y_t & 0 \\ 0 & 1/Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

TABLE III

A REPRESENTATION OF A POWER SYSTEM

Element Type	Symbol	$\underset{\sim}{x}_b$	$\underset{\sim}{u}_b$	$\underset{\sim}{M}^b_{11}$	$\underset{\sim}{M}^b_{12}$
Load Elements	ℓ	$\begin{bmatrix} V_\ell \\ V^*_\ell \end{bmatrix}$	$\begin{bmatrix} S_\ell \\ S^*_\ell \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & -I^*_\ell/V_\ell \\ -I_\ell/V^*_\ell & 0 \end{bmatrix}$
Elements Generator	g	$\begin{bmatrix} V_g \\ I_g \end{bmatrix}$	$\begin{bmatrix} \|V_g\|^2 \\ 2P_g \end{bmatrix}$	$\begin{bmatrix} 1 & -V^*_g/V_g \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -j2Q_g/V^2_g \\ 1 & -V^*_g/V_g \end{bmatrix}$
Slack Generator	n	$\begin{bmatrix} I_n \\ I^*_n \end{bmatrix}$	$\begin{bmatrix} V_n \\ V^*_n \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Transmission Elements	t	$\begin{bmatrix} I_t \\ I^*_t \end{bmatrix}$	$\begin{bmatrix} Y_t \\ Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1/Y_t & 0 \\ 0 & 1/Y^*_t \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$S_m = P_m + jQ_m$ is the power of element m, m can be ℓ, g or n.

TABLE IV

SOLUTION OF EXAMPLE 4

b	I_b	V_b
1	5.2623-j5.5411	0.7352-j0.2041
2	-5.6705+j1.0706	1.0+j0.0
3	0.4082+j1.4705	0.7352-j.2041
4	0.0+j3.0	1.0+j0.0
5	-5.6705+j4.0706	-0.2648-j0.2041

TABLE V

ADJOINT SYSTEM OF EXAMPLE 4 WITH CONSTANT COEFFICIENTS

Adjoint Matrix of Coefficients	RHS Vector $f = \text{Re}\{V_1\}$	RHS Vector $f = \text{Im}\{V_1\}$
$\begin{bmatrix} 1.2972 & -6.0 & 9.1581 & -20.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ -26.8419 & 20.0 & 10.7283 & -6.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$	$\begin{bmatrix} -0.5 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$	$\begin{bmatrix} 0.0 \\ 0.0 \\ 0.5 \\ 0.0 \end{bmatrix}$

TABLE VI

DERIVATIVES OF V_1 OF EXAMPLE 4

b	Associated Derivatives	
1	$\frac{df}{dS_1} = -0.0348+j0.0366$	$\frac{df}{dS_1^*} = -0.0535-j0.0794$
2	$\frac{df}{dV_2} = 1.5248-j0.0462$	$\frac{df}{dV_2^*} = 0.7896+j0.1579$
3	$\frac{df}{dY_3} = -0.0311-j0.0462$	$\frac{df}{dY_3^*} = -0.0203+j0.0213$
4	$\frac{df}{dY_4} = 0.0$	$\frac{df}{dY_4^*} = 0.0$
5	$\frac{df}{dY_5} = -0.0080+j0.0231$	$\frac{df}{dY_5^*} = -0.0022-j0.0127$

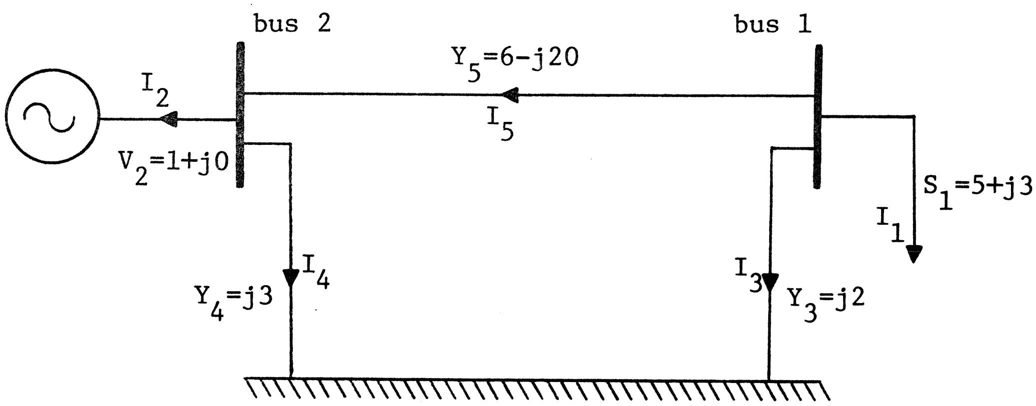

Fig. 1 2-bus system of Example 4

SOC-241

A UNIFIED APPROACH TO POWER SYSTEM SENSITIVITY ANALYSIS AND PLANNING PART III: CONSISTENT SELECTION OF ADJOINING COEFFICIENTS

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Key Words: Power system analysis, adjoint networks, sensitivity analysis, conjugate notation, modelling, consistency criteria

Abstract: A unified approach to power system sensitivity analysis and planning has been presented in Part I of the paper. The approach utilizes a generalized adjoint network concept with complex adjoining coefficients set to proper values which allow the required sensitivity evaluation. Here, we present a unified study for consistent selection of the adjoining coefficients where the restrictions imposed by the type of system and the particular function considered are investigated. The study, hence, justifies the use of the approach described in Part I as a general network approach.

Description:

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