INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-236

FAULT LOCATION OF ANALOG CIRCUITS

R.M. Biernacki and J.W. Bandler

September 1979

FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

SOC-236

FAULT LOCATION OF ANALOG CIRCUITS

R.M. Biernacki and J.W. Bandler

September 1979, No. of Pages: 31

Revised:

Key Words: Fault analysis, testing, circuit theory, computer aided design

Abstract: This paper deals with fault detection for linear analog circuits. The described methods are based on measurements of voltage using current excitations and have been developed for the location of single as well as for multiple faults. They utilize certain algebraic invariants of faulty elements. Computationally, they depend on checking the consistency or inconsistency of suitable sets of linear equations. The equations themselves are formulated via adjoint circuit simulations.

Description:

Related Work: SOC-233, SOC-235.

Price: $ 6.00.

FAULT LOCATION OF ANALOG CIRCUITS

Radoslaw M. Biernacki and John W. Bandler, Fellow, IEEE

Abstract

This paper deals with fault detection for linear analog circuits. The described methods are based on measurements of voltage using current excitations and have been developed for the location of single as well as for multiple faults. They utilize certain algebraic invariants of faulty elements. Computationally, they depend on checking the consistency or inconsistency of suitable sets of linear equations. The equations themselves are formulated via adjoint circuit simulations.

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant A7239.

R.M. Biernacki is on leave from the Institute of Electronics Fundamentals, Technical University of Warsaw, Warsaw, Poland. J.W. Bandler is with the Group on Simulation, Optimization and Control and Department of Electrical and Computer Engineering, McMaster University, Hamilton, Canada, L8S 4L7.

I. INTRODUCTION

Although there is a number of papers which deal with testing problems, only a few of them concern analog circuits [1-17]. The main objective of testing is to check whether the circuit, which is already manufactured, meets the required specifications or not. If not, it should detect the source which causes the network to be wrong, principally, to indicate the element(s) which is (are) at fault. Then the elements or subnetworks which contain these elements can be replaced or repaired.

By a fault we mean not only an unwanted short or open circuit but also, more generally, any large change in the value of an element w.r.t. its nominal value. Since the meaning of the term "large change" is not precise enough we will consider any change in element value as a fault. Of course, we assume that the network design, i.e., the topology as well as the nominal values of the parameters are known.

Fault analysis consists of two stages: fault detection and fault evaluation. Fault detection can be done by the method which identifies all element values [2-4, 6, 8, 11-12, 14-16] and then comparing the nominal and actual values. Thus, fault evaluation is being done simultaneously. This approach, however, can be too general. It may also be too difficult if, for instance, the network is not element-value solvable. Usually, we look for one, two or several faults and there is no need to identify everything as though we did not know anything about the network.

The fault detection should locate the faults, i.e., identify elements which are out of their nominal values. Once we know which elements are at fault, the fault evaluation is simply equivalent to the identification of selected parameters discussed in [4].

There are a few papers dealing with fault analysis without identifying all elements mostly to locate single faults. This can be done by constructing a fault dictionary using computer simulation of mainly single catastrophic faults [1,17]. Another approach uses certain analytical or geometrical invariants of element value changes [5, 9, 10, 13]. The latter approach is worth developing since it enables us to deal not only with catastrophic faults and the computational effort required is much smaller than in the case of fault dictionaries.

This paper presents a new approach to fault detection in the foregoing sense. Analog linear and lumped networks are considered. Methods for single as well as for multiple fault location are proposed. The methods are based on checking consistency or inconsistency of certain equations which are invariant on faulty elements. The measurement tests are assumed to be performed at a single frequency point. We consider mainly current excitations and voltage measurements. As is known [2], parallel elements are not solvable, so we also assume that there are no direct parallel elements.

II. SINGLE-FAULT DETECTION

Consider a network function f as a function of a single element Y. For many cases it can be expressed as a bilinear function

$$f = \frac{A + BY}{C + DY} \,. \tag{1}$$

The direct use of (1) for the single-fault detection is impossible since f can be changed either by a change of Y or by changes of the coefficients A, B, C, D which depend on values of other elements.

Now consider two different network functions f_1 and f_2 of the same element Y as

$$f_1 = \frac{A_1 + B_1 Y}{C_1 + D_1 Y} , \quad f_2 = \frac{A_2 + B_2 Y}{C_2 + D_2 Y} . \tag{2}$$

If the two functions essentially depend on Y, i.e., $A_i D_i - B_i C_i \neq 0$ for $i = 1, 2$, then each of them can be solved for Y and the solution is

$$Y = \frac{A_1 - C_1 f_1}{-B_1 + D_1 f_1} = \frac{A_2 - C_2 f_2}{-B_2 + D_2 f_2} . \tag{3}$$

From (3) we find the relation

$$(C_1 B_2 - D_1 A_2) f_1 + (A_1 D_2 - B_1 C_2) f_2 = (A_1 B_2 - B_1 A_2) + (C_1 D_2 - D_1 C_2) f_1 f_2 , \tag{4}$$

which holds for any value of Y provided that all other elements are fixed. If the two network functions are of the same type (e.g., trans-impedances) the denominators $C_1 + D_1 Y$ and $C_2 + D_2 Y$ are determined by the same characteristic polynomial of the network. Hence, they can differ only by a constant multiplier, so $C_1 D_2 - D_1 C_2 = 0$ and (4) becomes a linear relation

$$a f_1 + b f_2 = c \tag{5}$$

where $a \triangleq C_1 B_2 - D_1 A_2$, $b \triangleq A_1 D_2 - B_1 C_2$ and $c \triangleq A_1 B_2 - B_1 A_2$.

Equation (5) gives us the relationship between values of f_1 and f_2 when all network elements except Y are kept unchanged. In other words the coefficients a, b and c depend only on nominal values of all other elements. Similar relationships between f_1 and f_2 can be derived for all other elements Y_1, Y_2, ..., Y_p. This is done for nominal values of all elements. Therefore we obtain p equations

$$a^i f_1 + b^i f_2 = c^i, \quad i = 1, 2, \ldots, p, \tag{6}$$

each of them corresponding to a certain element of the network. Superscript i denotes the index of the element. We will use these equations for the single-fault detection.

Based on measurements, we find the actual values of f_1 and f_2. If there is a single fault within the network, i.e., one of the elements Y_1, Y_2, ..., Y_p is changed, then the corresponding equation of (6) is satisfied since all other elements are at their nominal values. All other equations are likely to be unsatisfied. To be able to identify uniquely the fault location it is required that

$$\det \begin{bmatrix} a^k & b^k \\ a^\ell & b^\ell \end{bmatrix} \neq 0, \tag{7}$$

for any k, ℓ, $k \neq \ell$. If these conditions are fulfilled then all of the equations in (6) are satisfied only by the nominal values f_1^0 and f_2^0 and no two equations can be satisfied by the same values f_1, f_2 different from f_1^0, f_2^0. The two-dimensional (e.g., DC network) geometrical interpretation of this is given in Fig. 1. The equations (6) describe straight lines in the two-dimensional space f_1, f_2. They all intersect at the point corresponding to the nominal values of all elements.

Since the nominal values satisfy equations (6) we can use the changes

$$\Delta f_j = f_j - f_j^0, \; j = 1, 2, \tag{8}$$

instead of f_1 and f_2. Thus, we have homogeneous equations

$$a^i \, \Delta f_1 + b^i \, \Delta f_2 = 0, \quad i = 1, 2, \ldots, p. \tag{9}$$

To use these equations we do not need to know the values c^i, $i = 1, 2, \ldots, p$, but we have to know f_1^0 and f_2^0.

The actual values of the network functions f_1 and f_2 are to be identified by measurements. Using, preferably, current excitation and voltage measurements the two network functions should be certain impedances or trans-impedances

$$f_j = V_j^m / I_{gj}, \quad j = 1, 2. \tag{10}$$

Thus, the equation (9) can be directly expressed using the measured voltages V_1^m and V_2^m instead of f_1 and f_2. If the excitation currents I_{g1} and I_{g2} are at different values then the coefficients a^i (or b^i) have to be rescaled. Otherwise, this is not necessary.

The two excitations I_{g1} and I_{g2} do not need to be applied to the same port, but if they are then the voltage measurements V_1^m and V_2^m can be taken simultaneously (i.e., at the same measurement test). We now derive a simple method which supplies the coefficients of equation

(9) for the latter case.

Consider the representation of the network shown in Fig. 2. Note that the 4-port network consists of elements which are at their nominal values, so it does not depend on any fault.

According to Fig. 2 we have

$$\underset{\sim}{V} = \begin{bmatrix} V_1^m \\ V_2^m \\ V_I \\ V_i \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} 0 \\ 0 \\ I_g \\ -V_i \Delta Y_i \end{bmatrix}, \tag{11}$$

where V_1^m and V_2^m are the voltages measured and I_g is the excitation (i.e., we consider the network functions $f_j = V_j^m/I_g$, $j = 1,2$). Since the left hand side of (11) can be expressed as

$$\underset{\sim}{V} = \underset{\sim}{V}^0 + \Delta\underset{\sim}{V}, \tag{12}$$

where $\underset{\sim}{V}^0$ is the nominal vector obtained for $\Delta Y_i = 0$, we find

$$\Delta\underset{\sim}{V} = \begin{bmatrix} \Delta V_1^m \\ \Delta V_2^m \\ \Delta V_I \\ \Delta V_i \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -V_i \Delta Y_i \end{bmatrix}. \tag{13}$$

Thus

$$\begin{bmatrix} \Delta V_1^m \\ \Delta V_2^m \end{bmatrix} = I_i \begin{bmatrix} Z_{14} \\ Z_{24} \end{bmatrix} . \tag{14}$$

Eliminating I_i from (14) we obtain

$$Z_{24}\ \Delta V_1^m - Z_{14}\Delta V_2^m = 0. \tag{15}$$

Note that in order to be able to eliminate I_i at least one of Z_{14} and Z_{24} has to be different from zero. The equation (15) is one of the equations (9). It corresponds to Y_i, so $a^i = Z_{24}$ and $b^i = -Z_{14}$.

In this way we can find all equations (9). But it would be inconvenient to consider as many different 4-port networks as the number of elements. We propose to use the adjoint network simulation for this purpose. The method is explained in Fig. 3. According to Fig. 3(a) we have

$$\begin{bmatrix} \cdot \\ \cdot \\ \vdots \\ \hat{V}_{i1} \end{bmatrix} = \underset{\sim}{Z}^T \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} \\ Z_{12} \\ Z_{13} \\ Z_{14} \end{bmatrix} \tag{16}$$

so that

$$\hat{V}_{i1} = Z_{14}. \tag{17}$$

Similarly, according to Fig. 3(b) we find

$$\hat{V}_{i2} = Z_{24}. \tag{18}$$

Finally, equation (15) can be rewritten in the form

$$\hat{V}_{i2}\ \Delta V_1^m - \hat{V}_{i1}\ \Delta V_2^m = 0. \tag{19}$$

It can be shown that the above discussion is valid for all elements of the network under a mild condition that the measured voltages V_1^m and V_2^m essentially depend on all elements. Moreover, it does not matter if the port of the element Y_i is the same as the port of excitation or a port of measurement. Similarly, the port of excitation can be one of the ports of measurement. Therefore, in order to obtain the coefficients of the equations (9) two simulations of the adjoint network are required. First, we apply a unit current to the first measurement port and calculate the voltages across all elements $\hat{V}_{11}$, $\hat{V}_{21}$, ..., $\hat{V}_{p1}$. Second, applying a unit current to the second measurement port we find $\hat{V}_{12}$, $\hat{V}_{22}$, ..., $\hat{V}_{p2}$. Finally, we formulate the equations (19) for i = 1, 2, ..., p and check the condition (7). In fact, only one simulation is required since in both cases we have to solve exactly the same system of equations with different right hand sides. It can be calculated simultaneously, or alternatively, using the same LU factorization.

Although a rather rare case, it is possible that not all determinants (7) are different from zero. If two equations in (9), say for i_1 and i_2, are found to be linearly dependent it means that V_1^m and V_2^m are influenced by Y_{i_1} and Y_{i_2} similarly. The corresponding straight lines in Fig. 1 are identical and we cannot distinguish a fault of Y_{i_1}

from a fault of Y_{i_2}. This situation can appear, for instance, if two elements are symmetrical to each other w.r.t. the voltages measured. To remedy this problem we can choose two other voltages in order to replace at least one of the two equations.

Finally, it is to be noted that the above method can be used to detect more general faults like shorts between nonincident nodes. We can simply consider nonexisting elements between such nodes as elements of nominal value Y = 0 and we can derive the equations of the form (9) for these elements.

III. MULTIPLE-FAULT DETECTION

We now generalize the foregoing approach in order to be able to deal with several simultaneous faults within the network. These faults are represented as external loads of (n+k)-port network shown in Fig. 4. We consider n ports of measurement with

$$\underset{\sim}{V}^m \triangleq [V_1^m \; V_2^m \; \dots \; V_n^m]^T \tag{20}$$

and

$$\underset{\sim}{I}^m \triangleq [I_1^m \; I_2^m \; \dots \; I_n^m]^T. \tag{21}$$

The ports of fault are described by

$$\underset{\sim}{V}^x = [V_1^x \; V_2^x \; \dots \; V_k^x]^T$$

and

$$\underset{\sim}{I}^{x} = [I_1^x \; I_2^x \; \ldots \; I_k^x]^T = -[V_1^x \Delta Y_1^x \; V_2^x \Delta Y_2^x \; \ldots \; V_k^x \Delta Y_k^x]^T, \tag{22}$$

where $k \leq n-1$.

We assume that the impedance matrix $\underset{\sim}{Z}$ of the (n+k)-port network exists. According to Fig. 4 we have

$$\begin{bmatrix} \underset{\sim}{V}^{m} \\ \underset{\sim}{V}^{x} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_{mm} & \underset{\sim}{Z}_{mx} \\ \underset{\sim}{Z}_{xm} & \underset{\sim}{Z}_{xx} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}^{m} \\ \underset{\sim}{I}^{x} \end{bmatrix}. \tag{23}$$

Assuming that the ports of measurement are open circuited or are excited by independent current sources we find that the nominal voltage vector is described by

$$\begin{bmatrix} \underset{\sim}{V}^{m0} \\ \underset{\sim}{V}^{x0} \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} \underset{\sim}{I}^{m} \\ \underset{\sim}{0} \end{bmatrix}. \tag{24}$$

Hence, the voltage change vector can be expressed as

$$\begin{bmatrix} \Delta\underset{\sim}{V}^{m} \\ \Delta\underset{\sim}{V}^{x} \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} \underset{\sim}{0} \\ \underset{\sim}{I}^{x} \end{bmatrix}, \tag{25}$$

and, in particular,

$$\Delta\underset{\sim}{V}^{m} = \underset{\sim}{Z}_{mx} \, \underset{\sim}{I}^{x}. \tag{26}$$

$\underset{\sim}{Z}_{mx}$ is a rectangular matrix having more rows than columns. Assuming that $\underset{\sim}{Z}_{mx}$ is a full column rank matrix we can find the solution of the equation (26) as

$$\underset{\sim}{I}^{x} = (\underset{\sim}{Z}_{mx}^{T} \underset{\sim}{Z}_{mx})^{-1} \underset{\sim}{Z}_{mx}^{T} \Delta\underset{\sim}{V}^{m}. \tag{27}$$

Therefore, eliminating $\underset{\sim}{I}^{x}$ from (26) and (27) we find the equation

$$[\underset{\sim}{Z}_{mx}(\underset{\sim}{Z}_{mx}^{T}\underset{\sim}{Z}_{mx})^{-1}\underset{\sim}{Z}_{mx}^{T} - \underset{\sim}{1}]\Delta\underset{\sim}{V}^{m} = \underset{\sim}{0}, \tag{28}$$

which is a generalization of equation (15). Using the notation

$$\overline{\underset{\sim}{A}} \triangleq \underset{\sim}{A}(\underset{\sim}{A}^{T}\underset{\sim}{A})^{-1}\underset{\sim}{A}^{T} \tag{29}$$

for a full column rank matrix $\underset{\sim}{A}$, the left hand side of (28) can be rewritten in the form

$$(\overline{\underset{\sim}{Z}}_{mx} - \underset{\sim}{1})\Delta\underset{\sim}{V}^{m}. \tag{30}$$

Given a vector of voltage changes $\Delta\underset{\sim}{V}^{m}$ we can calculate the expression (30). It is equal to $\underset{\sim}{0}$ regardless of the element changes ΔY_1, ΔY_2, ..., ΔY_k if all other elements are kept at their nominal values. In other words if (30) is different from zero it means that there is another element at fault besides the elements Y_1, ..., Y_k.

In order to be able to detect k simultaneous faults we need to know expressions similar to (30) for all possible combinations consisting of k elements.

As before, the matrix $\underset{\sim}{Z}_{mx}$ can be found by means of the adjoint network. For the adjoint network we have

$$\begin{bmatrix} \hat{\underset{\sim}{V}}^m \\ \hat{\underset{\sim}{V}}^x \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_{mm}^T & \underset{\sim}{Z}_{xm}^T \\ \underset{\sim}{Z}_{mx}^T & \underset{\sim}{Z}_{xx}^T \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{I}}^m \\ \hat{\underset{\sim}{I}}^x \end{bmatrix} . \tag{31}$$

Let $\hat{\underset{\sim}{I}}^x = \underset{\sim}{0}$. Then we obtain

$$\hat{\underset{\sim}{V}}^x = \underset{\sim}{Z}_{mx}^T \hat{\underset{\sim}{I}}^m , \tag{32}$$

where $\hat{\underset{\sim}{I}}^m$ is the vector of an adjoint network excitation. Taking n linearly independent excitations $\hat{\underset{\sim}{I}}^{m1}$, $\hat{\underset{\sim}{I}}^{m2}$, ..., $\hat{\underset{\sim}{I}}^{mn}$ we have the equation

$$[\hat{\underset{\sim}{V}}^{x1} \; \dots \; \hat{\underset{\sim}{V}}^{xn}] = \underset{\sim}{Z}_{mx}^T \, [\hat{\underset{\sim}{I}}^{m1} \; \dots \; \hat{\underset{\sim}{I}}^{mn}] , \tag{33}$$

which can be solved for $\underset{\sim}{Z}_{mx}^T$. The simplest solution can be obtained by applying a unit current, successively to all measurement ports (see Fig. 5). Then

$$[\hat{\underset{\sim}{I}}^{m1} \; \dots \; \hat{\underset{\sim}{I}}^{mn}] = \underset{\sim}{1} , \tag{34}$$

and

$$\underset{\sim}{Z}_{mx}^T = [\hat{\underset{\sim}{V}}^{x1} \; \dots \; \hat{\underset{\sim}{V}}^{xn}] . \tag{35}$$

Thus, we need n simulations of the adjoint network (with the same LU factorization) in order to obtain the coefficients of the expression

(30) for all possible combinations of k elements. We apply a unit source to the measurement ports and calculate voltages across all elements of the adjoint nominal network. Taking the values corresponding to a certain combination of elements we find the corresponding matrix $\underset{\sim}{Z}_{mx}$. In this way we obtain the matrices $\underset{\sim}{Z}^{j}_{mx}$ j = 1, 2, ..., $\binom{p}{k}$ for all possible combinations.

If there are k faults within the network we can detect them by checking the expressions (30) for all possible combinations of k elements. The expression which corresponds to the elements at fault is equal to zero while the other expressions are likely to be different from zero. This enables us to indicate the suitable combination. However, the approach is limited. Some problems which may arise are discussed in the following section.

IV. INTERPRETATION

We now discuss the assumptions and the capacity of the approach presented in this paper. In order to use it we have to formulate an appropriate set of p equations for single-faults, $\binom{p}{2}$ matrix equations for double-faults, $\binom{p}{3}$ matrix equations corresponding to three simultaneous faults etc. This can be done by practically one simulation of the adjoint nominal network (with n different excitations). Given measured voltages we calculate the voltage changes w.r.t. nominal values and check the equations. We start with equations corresponding to single faults. If all equations except one are not satisfied we can suppose that there is a single fault in the element which corresponds to the satisfied equation. (Although a rare case, it is possible that the situation is caused by two or more faults of other elements; this can be verified by other equations.) If all equations corresponding to single faults are not satisfied we have to go further and check the equations corresponding to double faults, etc.

To be able to detect the suitable fault combination the equations (28) are required to be "independent" in a certain sense. More precisely, we do not want to face the situation when two or more equations (for the same k) are satisfied simultaneously for $\Delta \underset{\sim}{V}^m \neq \underset{\sim}{0}$. But this is not always possible. For instance, if only element Y_1 is at fault then, checking all equations for double faults, all equations corresponding to those combinations which contain Y_1 like Y_1Y_2, Y_1Y_3, ... are satisfied. In other words, it is possible that two equations of the form (28) are simultaneously satisfied for certain $\Delta \underset{\sim}{V}^m$, but generally such an implication does not exist. This is the case we are interested in. The concept of <u>block independent equations</u> will help us to state the problem.

Consider equation (26) in a slightly more general form

$$\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}, \tag{36}$$

where $\underset{\sim}{A}$ is an n x k-matrix, k < n. The rank of $\underset{\sim}{A}$ is assumed to be

$$\text{rank}\ \underset{\sim}{A} = k, \tag{37}$$

so the matrix $\underset{\sim}{A}^T\underset{\sim}{A}$ is nonsingular. System (36) is overdetermined. As is known, the solution of (36) exists if and only if (compare with (28))

$$\underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\,\underset{\sim}{A}^T\underset{\sim}{b} = \underset{\sim}{b}, \tag{38}$$

or

$$(\bar{\underset{\sim}{A}} - \underset{\sim}{1})\underset{\sim}{b} = \underset{\sim}{0}. \tag{39}$$

In other words, the left hand side of (39) is equal to zero if and only if the system (36) is consistent. Taking (28) and (26) into account, that is to say, the expression (30) is equal to zero if and only if there exists the solution $\underset{\sim}{I}^{x}$ of the system (26) for given $\Delta\underset{\sim}{V}^{m}$. Therefore, if we want systems (28) to be "independent" we actually do not want different systems (26) (for different combinations) to be simultaneously consistent (or inconsistent) for any $\Delta\underset{\sim}{V}^{m}$. Hence, we come to the following definition. Consider two overdetermined systems of equations

$$\underset{\sim}{A}_1\underset{\sim}{x}_1 = \underset{\sim}{b} \quad \text{and} \quad \underset{\sim}{A}_2\underset{\sim}{x}_2 = \underset{\sim}{b}, \tag{40}$$

and assume that n x k-matrices $\underset{\sim}{A}_1$ and $\underset{\sim}{A}_2$ are of full column rank.

<u>Definition 1</u>

Systems (40) are said to be <u>block dependent</u> if for any $\underset{\sim}{b}$ both are consistent or both are inconsistent.

If systems (40) are not block dependent then they are called <u>block independent</u>.

The conditions of Definition 1 are equivalent to the logical expression

$$\forall_{\underset{\sim}{x}_1} \exists_{\underset{\sim}{x}_2} \underset{\sim}{A}_1\underset{\sim}{x}_1 = \underset{\sim}{A}_2\underset{\sim}{x}_2 \text{ and } \forall_{\underset{\sim}{x}_2} \exists_{\underset{\sim}{x}_1} \underset{\sim}{A}_1\underset{\sim}{x}_1 = \underset{\sim}{A}_2\underset{\sim}{x}_2. \tag{41}$$

Consider, for example, the second part of (41). For any $\underset{\sim}{x}_2$ this is a consistent system of equations w.r.t. $\underset{\sim}{x}_1$, so according to the previous discussion (compare with (38))

$$\bar{\underset{\sim}{A}}_1\underset{\sim}{A}_2\underset{\sim}{x}_2 = \underset{\sim}{A}_2\underset{\sim}{x}_2 \tag{42}$$

or

$$(\bar{\underset{\sim}{A}}_1 - \underset{\sim}{1})\underset{\sim}{A}_2\, \underset{\sim}{x}_2 = \underset{\sim}{0}. \tag{43}$$

Since the above equation has to be valid for any $\underset{\sim}{x}_2$ we find the condition

$$\bar{\underset{\sim}{A}}_1\underset{\sim}{A}_2 = \underset{\sim}{A}_2. \tag{44}$$

Similarly, from the first part of (41) we find

$$\bar{\underset{\sim}{A}}_2\underset{\sim}{A}_1 = \underset{\sim}{A}_1. \tag{45}$$

The conditions (44) and (45) are necessary and sufficient for the systems (40) to be block dependent. In fact, only one of the two conditions has to be checked. To show it we introduce the notion of <u>block dependent matrices</u>. Assuming, as before, $\underset{\sim}{A}_1$ and $\underset{\sim}{A}_2$ to be n x k-matrices of full column rank we call them block dependent matrices if (44) holds. It will be denoted by $\underset{\sim}{A}_1 \sim \underset{\sim}{A}_2$. The relation has the following properties. It is reflexive since $\bar{\underset{\sim}{A}}\,\underset{\sim}{A} = \underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}(\underset{\sim}{A}^T\underset{\sim}{A}) = \underset{\sim}{A}$.

It is commutative because if $\bar{\mathbf{A}}\,\mathbf{B} = \mathbf{B}$ then

$$\bar{\mathbf{B}}\,\mathbf{A} = \mathbf{B}(\mathbf{B}^T\mathbf{B})^{-1}\,\mathbf{B}^T\mathbf{A} = \bar{\mathbf{A}}\,\mathbf{B}\,[\mathbf{B}^T\,\bar{\mathbf{A}}^T\,\bar{\mathbf{A}}\,\mathbf{B}]^{-1}\,\mathbf{B}^T\mathbf{A} =$$

$$= \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}\,[\mathbf{B}^T\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}]^{-1}\,\mathbf{B}^T\mathbf{A} =$$

$$= \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}(\mathbf{A}^T\mathbf{B})(\mathbf{A}^T\mathbf{B})^{-1}\,(\mathbf{A}^T\mathbf{A})(\mathbf{B}^T\mathbf{A})^{-1}\,(\mathbf{B}^T\mathbf{A}) =$$

$$= \mathbf{A}.$$

In the above derivation we utilized two properties: (1) the matrix $(\mathbf{A}^T\mathbf{A})$ is symmetrical, and (2) the matrix $\mathbf{A}^T\mathbf{B}$ as well as its transpose $\mathbf{B}^T\mathbf{A}$ are nonsingular. The latter property follows the assumption $\mathbf{B} = \bar{\mathbf{A}}\mathbf{B} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}(\mathbf{A}^T\mathbf{B})$ because $k = \text{rank}\ \mathbf{B} \leq \text{rank}(\mathbf{A}^T\mathbf{B}) \leq k$. Thus, if one of the expressions (44) and (45) holds then the second one holds also. The relation of block independent matrices is also transitive. Assuming $\mathbf{A} \sim \mathbf{B}$ and $\mathbf{B} \sim \mathbf{C}$ we have

$$\mathbf{C} = \mathbf{B}(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{C} =$$

$$= \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}\,[\mathbf{B}^T\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{B}]^{-1}\,\mathbf{B}^T\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C} =$$

$$= \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}(\mathbf{A}^T\mathbf{B})(\mathbf{A}^T\mathbf{B})^{-1}(\mathbf{A}^T\mathbf{A})(\mathbf{B}^T\mathbf{A})^{-1}(\mathbf{B}^T\mathbf{A})(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C} =$$

$$= \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{C} =$$

$$= \bar{\mathbf{A}}\mathbf{C}.$$

Therefore, the relation of block dependent matrices is an equivalence relation. This is a generalization of the linear dependence of vectors. Similarly, the condition (44) (or (45)) is a generalization of the condition (7) (with the equality symbol). Using the condition (44) we can find out which equations of the form (28) are dependent. In other words, we can determine the combinations, whose influence on the vector $\Delta \underset{\sim}{V}^m$ is similar, i.e., based on $\Delta \underset{\sim}{V}^m$ we cannot distinguish these combinations. Then, we should change measurement tests to be able to determine which combination actually occurs.

The approach presented in this section is based on the assumption of the existence of the impedance matrix (Figs. 2 and 4) This assumption, however, is not essential since the impedance matrix exists for most practical networks. A more crucial assumption is the one which concerns the matrix $\underset{\sim}{Z}_{mx}$ in (26) to be of full column rank. The assumption means that there exist exactly k linearly independent rows of $\underset{\sim}{Z}_{mx}$. These rows correspond to those voltages which we can use to uniquely determine $\underset{\sim}{I}^x$ as well as $\underset{\sim}{V}^x$. This is simply the problem of the identification of elements ΔY_1^x, ΔY_2^x, ..., ΔY_k^x which was discussed in [4]. Under this assumption, the inverse of a full column submatrix of $\underset{\sim}{Z}_{mx}$ exists and, as a consequence, the conditions of Theorem 2 in [4] are satisfied. This is seen directly from (23) since, knowing $\underset{\sim}{I}^x$ as a solution of (26), we have

$$\underset{\sim}{V}^x = \underset{\sim}{Z}_{xm} \underset{\sim}{I}^m + \underset{\sim}{Z}_{xx} \underset{\sim}{I}^x, \quad (46)$$

where $\underset{\sim}{I}^m$ is a given vector of excitations. Then, according to [4] we find the element values ΔY_1^x, ΔY_2^x, ..., ΔY_k^x.

As mentioned in [4], the more unknown elements we want to consider the more unlikely it is to satisfy Theorem 2 (or Theorem 1). In other words, there is an upper bound of k for which we are able to construct the equation (28) and, as a consequence, to detect k simultaneous faults. If we want to consider more simultaneous faults we can use the method of identification of all elements described in, for instance, [4].

V. EXAMPLES

Example 1

Consider a simple resistive network shown in Fig. 6 with nominal values of elements $G_i^0 = 1$, $i = 1, \ldots, 5$. Assume that, for single-fault location, the network is excited at the port 11' and voltage measurements are taken at the ports 11' and 33'. For $I_g = 1A$ nominal responses are $V_{11'}^0 = 5/8$ and $V_{33'}^0 = 1/8$. We easily find equations (19) corresponding to subsequent elements of the network as

$$\frac{1}{8}\,\Delta V_{11'} - \frac{5}{8}\,\Delta V_{33'} = 0, \qquad (47a)$$

$$\frac{1}{8}\,\Delta V_{11'} + \frac{3}{8}\,\Delta V_{33'} = 0, \qquad (47b)$$

$$\frac{2}{8}\,\Delta V_{11'} - \frac{2}{8}\,\Delta V_{33'} = 0, \qquad (47c)$$

$$\frac{3}{8}\,\Delta V_{11'} + \frac{1}{8}\,\Delta V_{33'} = 0, \qquad (47d)$$

$$\frac{5}{8}\,\Delta V_{11'} - \frac{1}{8}\,\Delta V_{33'} = 0. \qquad (47e)$$

Now, measuring voltages $V_{11'} = 2/3$ and $V_{33'} = 1/6$ we have $\Delta V_{11'} = \Delta V_{33'} = 1/24$ and notice that equation (47c) is satisfied while all others are not. Assuming a single-fault within the network we find the element G_3 responsible for faulty responses. $G_3 = 0.5$ gives us the responses.

As was mentioned in Section II we can also consider a nonexisting element of value $G_{13}^0 = 0$ between nodes 1 and 3. The corresponding equation is

$$\frac{4}{8}\,\Delta V_{11'} + \frac{4}{8}\,\Delta V_{33'} = 0 \qquad (47f)$$

Measuring voltages $V_{11'} = V_{33'} = \frac{3}{8}$ we observe that only equation (47f) is satisfied. This situation corresponds to a short-circuit between the nodes 1 and 3.

Example 2

Consider the same network as in Example 1. For double-fault location we choose the port 11' as a port of excitation with $I_g = 1A$ and ports 11', 22' and 33' as ports of measurement with nominal voltages $V_{11'}^0 = 5/8$, $V_{22'}^0 = 2/8$ and $V_{33'}^0 = 1/8$. According to (35) we find the following matrices $\underset{\sim}{Z}_{mx}^{ij}$ corresponding to all combinations G_i, G_j of network elements. Since for any $\alpha \neq 0$ $\overline{\alpha \underset{\sim}{A}} = \overline{\underset{\sim}{A}}$ we multiply these matrices by 8 for the sake of simplicity. We have

$$8\underset{\sim}{Z}_{mx}^{12} = \begin{bmatrix} 5 & 3 \\ 2 & -2 \\ 1 & -1 \end{bmatrix}, \qquad 8\underset{\sim}{Z}_{mx}^{13} = \begin{bmatrix} 5 & 2 \\ 2 & 4 \\ 1 & 2 \end{bmatrix},$$

$$8\underset{\sim}{Z}_{mx}^{14} = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 1 & -3 \end{bmatrix}, \qquad 8\underset{\sim}{Z}_{mx}^{15} = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 1 & 5 \end{bmatrix},$$

$$8\underset{\sim}{Z}_{mx}^{23} = \begin{bmatrix} 3 & 2 \\ -2 & 4 \\ -1 & 2 \end{bmatrix}, \qquad 8\underset{\sim}{Z}_{mx}^{24} = \begin{bmatrix} 3 & 1 \\ -2 & 2 \\ -1 & -3 \end{bmatrix},$$

$$8\underset{\sim}{Z}_{mx}^{25} = \begin{bmatrix} 3 & 1 \\ -2 & 2 \\ -1 & 5 \end{bmatrix}, \qquad 8\underset{\sim}{Z}_{mx}^{34} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 2 & -3 \end{bmatrix},$$

$$8\underset{\sim}{Z}_{mx}^{35} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 2 & 5 \end{bmatrix}, \qquad 8\underset{\sim}{Z}_{mx}^{45} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ -3 & 5 \end{bmatrix},$$

Let the voltages measured be

$$\underset{\sim}{V}^{m} = [\ \tfrac{1}{2}\ ,\ \tfrac{3}{8}\ ,\ \tfrac{1}{8}\ ]^{T}.$$

Then $\Delta\underset{\sim}{V}^{m} = [\ -\frac{1}{8}\ ,\ \frac{1}{8}\ ,\ 0]^{T}$ and we can check that

$$(\overline{\underset{\sim}{Z}}_{mx}^{24} - \underset{\sim}{1})\Delta\underset{\sim}{V}^{m} = -\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1/8 \\ 1/8 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and the equation (28) is not satisfied for every other combination. Thus, the elements G_2 and G_4 are indicated as faulty elements. It can be checked that $G_2 = 4$ and $G_4 = 0.5$ cause such situation.

Now, consider another situation when voltages measured are

$$\underset{\sim}{V}^{m} = [\ \tfrac{2}{3}\ ,\ \tfrac{1}{3}\ ,\ \tfrac{1}{6}\ ]^{T}.$$

We find that

$$(\overline{\underset{\sim}{Z}}_{mx}^{12} - \underset{\sim}{1})\Delta\underset{\sim}{V}^{m} = \frac{1}{5}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & -4 \end{bmatrix}\begin{bmatrix} 1/24 \\ 1/12 \\ 1/24 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

We also find that equation (28) is now satisfied for the matrices $\underset{\sim}{Z}_{mx}^{13}$ and $\underset{\sim}{Z}_{mx}^{23}$ because the two matrices together with $\underset{\sim}{Z}_{mx}^{12}$ are block dependent. Therefore we cannot distinguish which one of the three combinations actually appears. Nevertheless, for some reason we can be satisfied with the knowledge of the region where the faulty elements are located. This is indicated by the inconsistency of equation (28) for all other combinations.

VI. CONCLUSIONS

Fault analysis, which can be done using methods of identification, needs its own approaches especially in the case when only a few faults occur. Methods based on the bilinear dependence of network functions on a circuit parameter have been developed for single-fault detection. A particular approach utilizing a single current excitation and measurements of two voltages has been proposed. The adjoint network simulation has been found to be a convenient way for the necessary calculations. This approach has been successfully extended in order to deal with multiple-fault detection. However, there is a limit to the number of simultaneous faults which can be considered.

REFERENCES

[1] J.D. Bastian, "Fault isolation of analog circuits", Proc. 12th Asilomar Conf. Circuits, Systems and Computers, Western Periodicals, (North Hollywood, Nov. 1978).

[2] R.S. Berkowitz, "Conditions for network-element-value solvability", IRE Trans. Circuit Theory, vol. CT-9, 1962, pp. 24-29.

[3] R.M. Biernacki and J.W. Bandler, "Postproduction tuning and fault location of analog circuits", Faculty of Engineering, McMaster University, Hamilton, Canada, Report SOC-233, 1979.

[4] R.M. Biernacki and J.W. Bandler, "Postproduction parameter identification of analog circuits", IEEE Int. Symp. Circuits and Systems (Houston, TX, 1980), submitted.

[5] G.O. Martens and J.D. Dyck, "Fault identification in electronic circuits with the aid of bilinear transformations", IEEE Trans. Reliability, vol. R-21, 1972, pp. 99-104.

[6] W. Mayeda and G. Peponides, "Determination of component values in passive networks under limited measurements", Proc. 12th Asilomar Conf. on Circuits, Systems and Computers, Western Periodicals (North Hollywood, Nov. 1978), pp. 761-764.

[7] H.M. Merrill, "Failure diagnosis using quadratic programming", IEEE Trans. Reliability, vol. R-22, 1973, pp. 207-213.

[8] N. Navid and A.N. Wilson, Jr., "Fault diagnosis for resistive analog circuits", Proc. IEEE Int. Symp. Circuits and Systems (Tokyo, 1979), pp. 882-885.

[9] E.C. Neu, "A new n-port network theorem", Proc. 13th Midwest Symp. Circuit Theory (Minneapolis, 1970), pp. IV.5.1 - IV.5.10.

[10] E.C. Neu, "A general method for the identification of branch parameter changes", Proc. 14th Midwest Symp. on Circuit Theory (Denver, 1971), pp. 17.2-1 to 17.2-8.

[11] R. Saeks, S.P. Singh and R.W. Liu, "Fault isolation via components simulation", IEEE Trans. Circuit Theory, vol. CT-19, 1972, pp. 634-640.

[12] N. Sen and R. Saeks, "A measure of testability and its application to test point selection - theory", Proc. 20th Midwest Symp. Circuits and Systems (Lubbock, Texas, 1977), pp. 576-583.

[13] S. Seshu and R. Waxman, "Fault isolation in conventional linear systems - a feasibility study", IEEE Trans. Reliability, vol. R-15, pp. 11-16, May 1966.

[14] T.N. Trick and C.J. Aljajian, "Fault diagnosis of analog circuits", Proc. 20th Midwest Symp. Circuits and Systems (Lubbock, Texas, 1977), pp. 211-215.

[15] T.N. Trick and A.A. Sakla, "A new algorithm for the fault analysis and tuning of analog circuits", Proc. IEEE Int. Symp. Circuits and Systems (New York, NY, 1978), pp. 156-160.

[16] T.N. Trick, W. Mayeda and A.A. Sakla, "Determination of component values from node voltage measurements", Proc. IEEE Int. Symp. Circuits and Systems (Tokyo, 1979), pp. 878-881.

[17] R.E. Tucker and L.P. McNamee, "Computer-aided design application to fault detection and isolation techniques", Proc. IEEE Int. Symp. Circuits and Systems (Phoenix, AZ, 1977), pp. 684-687.

FIGURE CAPTIONS

Fig. 1 Geometrical interpretation of single-fault detection based on checking of equations (6). The actual values f_1 and f_2 corresponding to a change of Y_2 are also indicated.

Fig. 2 Representation of a network with a single fault as a 4-port network with the external load ΔY_i. The nominal value Y_i^0 is included in the 4-port. The port of excitation and ports of measurements are also indicated.

Fig. 3 Adjoint network simulations leading to the coefficients of equation (9).

Fig. 4 Network with k simultaneous faults represented by (n+k)-port with n ports of measurement. The impedance matrix $\tilde{Z}$ depends only on nominal values of network elements.

Fig. 5 Adjoint network simulations leading to the coefficients of equation (28).

Fig. 6 A simple resistive network example.

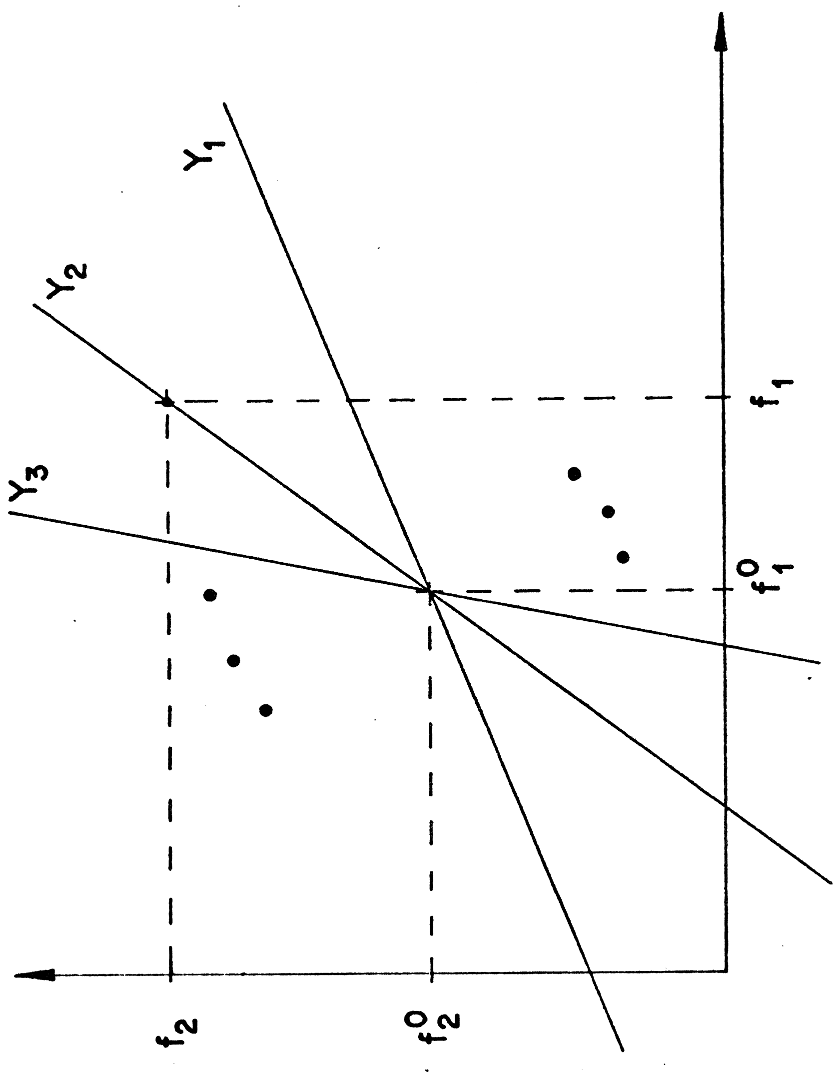


Fig. 1

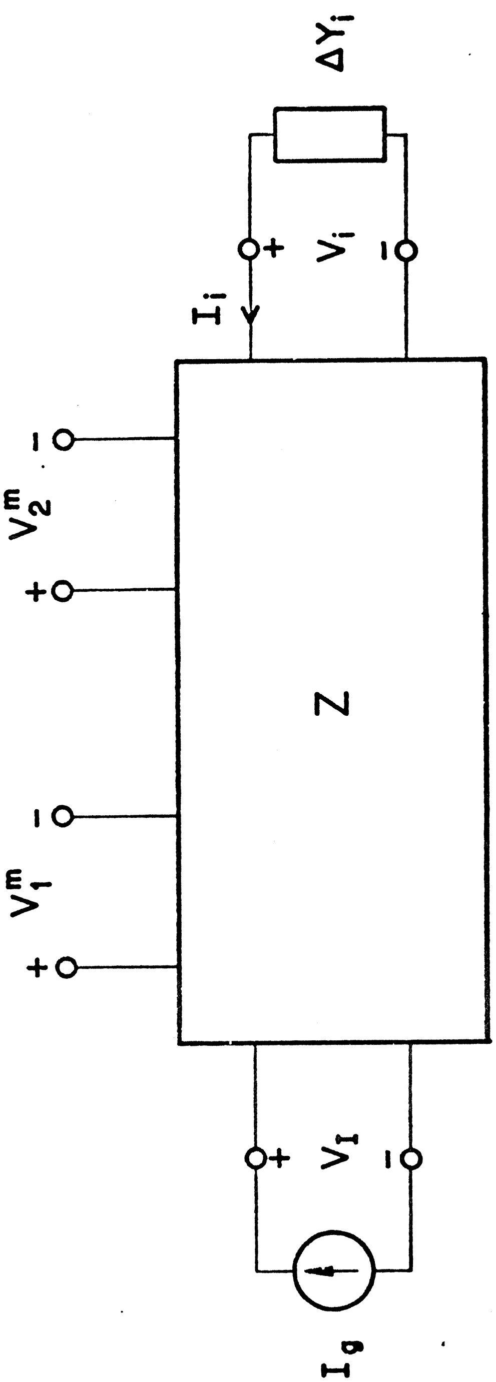
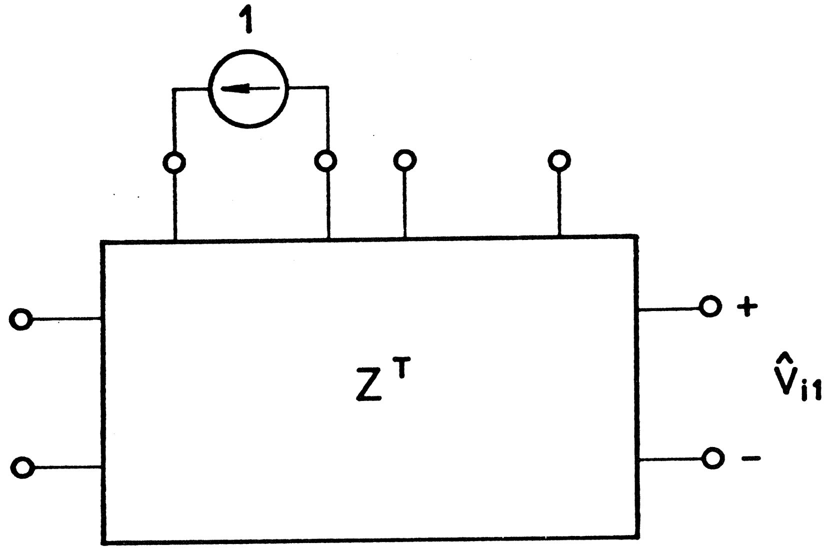
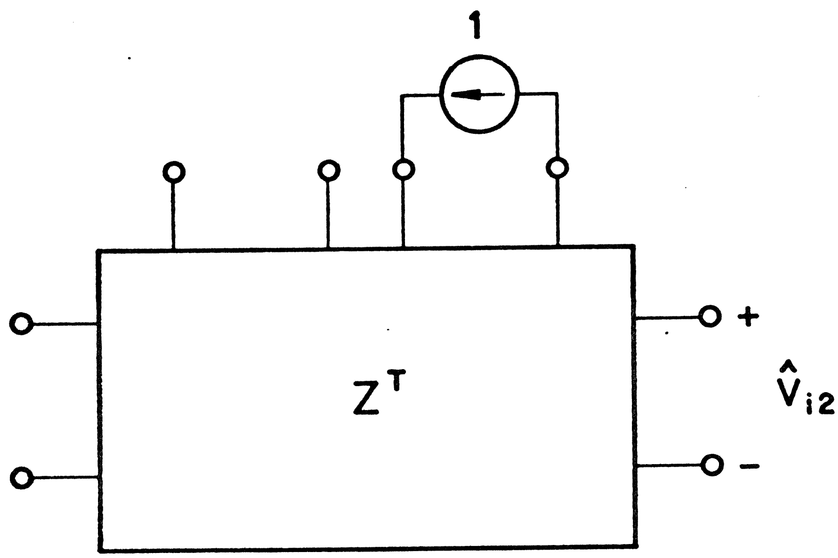


Fig. 2



(a)



(b)

Fig. 3

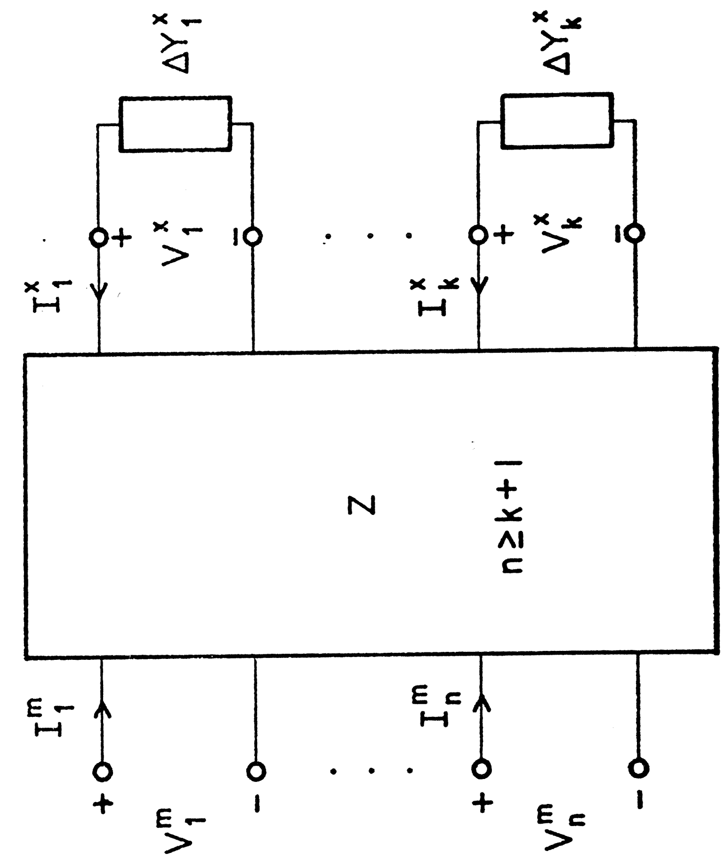


Fig. 4

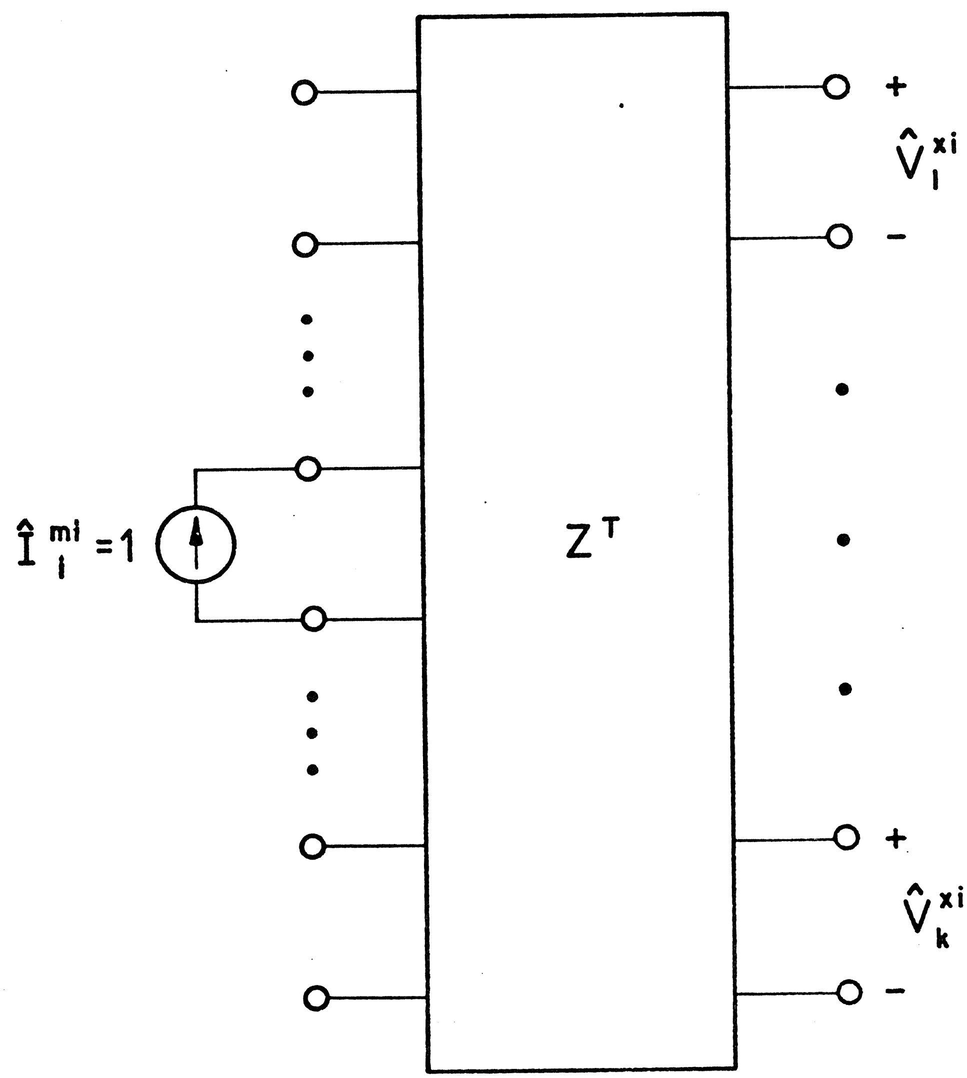


Fig. 5

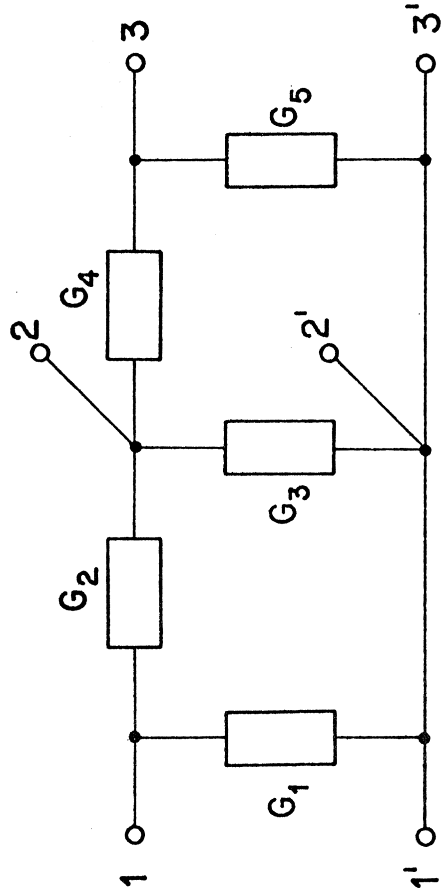


Fig. 6