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POSTPRODUCTION TUNING AND FAULT LOCATION OF ANALOG CIRCUITS

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Abstract

This report deals principally with two problems: postproduction identification of network parameters and fault detection for linear analog circuits. A number of different approaches are discussed and several methods proposed. The methods are based on measurements of voltage using mainly current excitations. The capabilities and limitations of these approaches are investigated and partially solved. Some unsolved problems are also indicated. Finally, some topics related to postproduction tuning are briefly discussed.

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1. INTRODUCTION

Computer-aided circuit design, which has become one of the most powerful tools in the design of analog electrical devices [2], enables us to deal with, for instance, manufacturing tolerance and tuning problems. Bandler, Liu and Tromp [3] formulated the design problem taking postproduction tuning into account. This work was extended by Polak and Sangiovanni-Vincentelli [17,18]. Although the algorithms proposed, for the time being, are computationally extravagant, there is hope for better and more efficient ones in the future. These methods are employed before a circuit is manufactured in order to assign the appropriate values of circuit parameters. A practical design, however, does not stop at that stage. The use of computer aids in further stages can also be helpful. Testing and tuning problems are of special interest.

Although there is a number of papers which deal with testing problems, only a few of them concern analog circuits [4-6, 9-11, 13-15, 19-25]. The main objective of testing is to check whether the circuit, which is already manufactured, meets the required specifications or not. If not, it should detect the source which causes the network to be wrong, principally, to indicate the element(s) which is (are) at fault. Then the elements or subnetworks which contain these elements can be replaced or repaired.

Another objective of testing is related to postproduction tuning. So-called deterministic tuning requires not only knowledge as to which elements have to be altered, but it is also necessary to know the actual values of network parameters in order to be able to calculate the amount of tuning to be carried out. This is the subject of the actual

parameter identification which is based on measurements of the network already manufactured. Most authors dealing with tuning problems assume that actual values of network parameters are available [1,7,8,12,16]. Nevertheless, since the elements can not usually be taken out of the network this can not be done directly. Therefore, appropriate methods of identification should exist.

The two objectives of testing are both subjects of fault analysis. However, it is felt that the term "fault analysis" is better suited to the situation when only a few elements are at fault and all remaining elements are correct. Then we want to locate the faulty elements. Thus, the situation when we are interested in actual values of all (or some) network elements can be better described by the term "parameter identification".

The solvability of the all parameter identification problem was first considered by Berkowitz [5]. He introduced the concept of accessible (and partly accessible) terminals where voltages and/or currents (or only voltages) can be applied and/or measured. From the theoretical point of view there is no difference as to which kind of excitation is used. However, from a practical point of view the use of current sources seems to be a little bit more reasonable. We will consider ideal current sources because for any nonideal source the source resistance can easily be treated as an additional element of the network.

We assume that no existing connection can be broken, hence current measurements are difficult to take. We may, however, consider that some ports can be shorted and the currents in these shorts measured. Therefore, voltage measurements are preferred over current measurements.

We will try to consider voltage measurements only and as few of them as possible.

The solvability of the all parameter identification problem was later investigated by several other authors [10,13,24]. Mayeda and Peponides [10] gave a topological characterization of the problem. Navid and Wilson [13], using symbollic network functions, formulated sufficient conditions for this solvabilty. Trick et al. [22-24] considered the problem of identification and showed how to formulate an appropriate system of equations using the adjoint network concept. They proved the very important result that, for linear networks, the problem can be solved by means of linear equations. Their approach, however, seems to be unnecessarily complicated, because many simulations of the adjoint have to be performed in order to formulate the equations. They formulate the equations using changes w.r.t. nominal values as unknowns. Of course we can assume that the nominal values of network parameters are known. This assumption is essential if we want to locate one or more faults assuming that the other elements are at their nominal values. For the purpose of identification this assumption is not essential, i.e., there is no need to know these values if we are interested in finding actual values of all parameters. The knowledge of the nominal values is only a matter of formulation of an appropriate system of equations (either actual values or actual changes can be used).

Although it is known how to check whether chosen tests are sufficient for identification no paper solves the problem of how to choose these tests to be independent (except the situation when we measure everything possible as in [13]). There are some other papers

which investigate this problem from the test point selection point of view [6,19,20].

Most papers on parameter identification assume tests to be performed at a single frequency [5,10,13,22-24]. This is quite a reasonable assumption since such identification provides the values of passive admittances and control coefficients of controlled sources. Repeating the identification at different frequencies enables us to identify the component values provided that there is a unique dependence of element values on the frequency response (as for canonical structures). Moreover, testing at a single frequency is essentially the same as that for resistive networks.

As is known [5], parallel elements are not solvable, so we assume that there are no direct parallel connections of elements or, alternatively, we have to be satisfied with the knowledge of the admittance of the whole connection. For instance, we can not determine individual values of two parallel resistors (obviously, even if we use measurements at different frequency points), so we have to satisfy ourselves with the composite resistor.

There are a few papers dealing with fault analysis in the foregoing sense, mostly to locate single faults. This can be done by constructing a fault dictionary using computer simulation of mainly single catastrophic faults [4,25]. Another approach uses certain analytical or geometrical invariants of element value changes [9,14,21]. The latter approach is worth developing since it enables us to deal not only with catastrophic faults and the computational effort required is much smaller than in the case of fault dictionaries.

This work deals with the two problems discussed above, namely, (a)

postproduction identification of network parameters and (b) fault detection. Analog linear and lumped networks are considered. Section 2 presents different approaches to the identification problem. Methods for selected elements as well as for the identification of all parameters are discussed. Section 3 includes methods for single- and multiple-fault detection. Finally, in Section 4, problems related to postproduction tuning are briefly discussed.

2. IDENTIFICATION

2.1 Matrix Parameter Identification

To begin with let us consider a very simple example of a two-port network. It is well known that any of the immittance, chain or hybrid matrices can be obtained from two appropriate sets of measurements. For instance, Fig. 2.1(a) shows the measurements giving us the chain matrix as

$$\underset{\sim}{A} = \begin{bmatrix} V_1^1/V_2^1 & V_1^2/I_2^2 \\ I_g^1/V_2^1 & I_g^2/I_2^2 \end{bmatrix}, \qquad (2.1.1)$$

where superscripts distinguish the two sets of measurements.

Using, preferably, current excitations and voltage measurements the two sets of measurements are as in Fig. 2.1(b). These measurements correspond directly to the impedance matrix, given by

$$\underset{\sim}{Z} = \begin{bmatrix} V_1^1/I_g^1 & V_1^2/I_g^2 \\ V_2^1/I_g^1 & V_2^2/I_g^2 \end{bmatrix}. \qquad (2.1.2)$$

Other matrices can be expressed using well-known relationships between them and $\underset{\sim}{Z}$, e.g., the chain matrix is

$$\underset{\sim}{A} = \begin{bmatrix} V_1^1/V_2^1 & (V_1^1V_2^2-V_2^1V_1^2)/I_g^2V_2^1 \\ I_g^1/V_2^1 & I_g^1V_2^2/I_g^2V_2^1 \end{bmatrix}. \qquad (2.1.3)$$

Now, consider the influence of a nonideal source and load environment (see Fig. 2.2) on our ability to make an identification. The chain matrix of the whole network $\underset{\sim}{A}_t$ is, obviously,

$$\underset{\sim}{A}_t = \begin{bmatrix} 1 & 0 \\ Y_S & 1 \end{bmatrix} \underset{\sim}{A} \begin{bmatrix} 1 & 0 \\ Y_L & 1 \end{bmatrix}. \qquad (2.1.4)$$

$\underset{\sim}{A}_t$ can be easily determined using (2.1.1) or (2.1.3). From (2.1.4) we have

$$\underset{\sim}{A} = \begin{bmatrix} 1 & 0 \\ -Y_S & 1 \end{bmatrix} \underset{\sim}{A}_t \begin{bmatrix} 1 & 0 \\ -Y_L & 1 \end{bmatrix}. \qquad (2.1.5)$$

We note that the matrix $\underset{\sim}{A}$ can be found if Y_S and Y_L are known. We also note that only A_{12} can be determined when Y_S and Y_L are unknown. Moreover, no more information is available even if we use any kind of port excitation and any kind of port measurement (including short circuit currents). This is simply because these excitations and measurements do not distinguish between parallel elements "within" the two-port represented by Y_{11} and Y_{22} and external parallel elements Y_S and Y_L, respectively. In other words shorting, for instance, the second port causes Y_L as well as Y_{22} to be shorted. If we consider another two-port network which replaces Y_L in Fig. 2.2 we note that, in general, we cannot identify the matrix elements of the two unknown subnetworks.

The above discussion gives us quite obvious, but important conclusions.

1. Two sets of measurements are required to identify all matrix

parameters of a two-port network. The existence of impedance matrix $\underset{\sim}{Z}$ is crucial for the preferable current excitations and open-circuit voltage measurements. This is, however, not a serious limitation form a practical point of view.

2. The identification of a subnetwork placed within a known environment is possible. In particular, we can use nonideal sources provided that we know the source admittances.

3. In general, the identification based on port excitations and measurements of a subnetwork placed within an unknown environment is impossible. In particular cases, however, it may be done.

Now, consider a general n-port network described by hybrid matrix $\underset{\sim}{H}$ and the equation

$$\begin{bmatrix} \underset{\sim}{V}_a \\ \underset{\sim}{I}_b \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{I}_a \\ \underset{\sim}{V}_b \end{bmatrix}, \tag{2.1.6}$$

where

$$\underset{\sim}{V}_a \triangleq [V_1 \; V_2 \; \dots \; V_k]^T,$$

$$\underset{\sim}{V}_b \triangleq [V_{k+1} \; V_{k+2} \; \dots \; V_n]^T,$$

$$\underset{\sim}{I}_a \triangleq [I_1 \; I_2 \; \dots \; I_k]^T,$$

$$\underset{\sim}{I}_b \triangleq [I_{k+1} \; I_{k+2} \; \dots \; I_n]^T.$$

Because n ports are available for measurements we can measure n voltages (and/or currents) at a time. So, in order to identify n^2 unknown elements of $\underset{\sim}{H}$ we have to arrange for n sets of independent measurements. The simplest way to do it is to apply one source at a time, sequentially to every port. Using the preferable current sources and voltage measurements the ith set of measurements can be described by (see Fig. 2.3)

$$\underset{\sim}{V}_a^i \triangleq [V_1^i \; V_2^i \; \ldots \; V_k^i]^T,$$

$$\underset{\sim}{V}_b^i \triangleq [V_{k+1}^i \; V_{k+2}^i \; \ldots \; V_n^i]^T, \tag{2.1.7}$$

$$I_j^i = 0 \quad \text{for} \quad j = 1, 2, \ldots, n; \quad j \neq i$$

with the excitation

$$I_i^i = I_g^i. \tag{2.1.8}$$

Values (2.1.7) and (2.1.8) satisfy equation (2.1.6). n systems of equations of the form (2.1.6) for i = 1, 2, ..., n can be written as a single matrix equation

$$\begin{bmatrix} \underset{\sim}{V}_{aa} & \underset{\sim}{V}_{ab} \\ \underset{\sim}{0} & \underset{\sim}{I}_{bb} \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{I}_{aa} & \underset{\sim}{0} \\ \underset{\sim}{V}_{ba} & \underset{\sim}{V}_{bb} \end{bmatrix}, \tag{2.1.9}$$

where

$$\underset{\sim}{V}_{aa} \triangleq [\underset{\sim}{V}_a^1 \; \underset{\sim}{V}_a^2 \; \ldots \; \underset{\sim}{V}_a^k],$$

$$\underset{\sim}{V}_{ab} \triangleq [\underset{\sim}{V}_a^{k+1} \; \underset{\sim}{V}_a^{k+2} \; \ldots \; \underset{\sim}{V}_a^n],$$

$$\underset{\sim}{V}_{ba} \triangleq [\underset{\sim}{V}_b^1 \; \underset{\sim}{V}_b^2 \; \ldots \; \underset{\sim}{V}_b^k],$$

$$\underset{\sim}{V}_{bb} \triangleq [\underset{\sim}{V}_b^{k+1} \; \underset{\sim}{V}_b^{k+2} \; \ldots \; \underset{\sim}{V}_b^n],$$

$$\underset{\sim}{I}_{aa} \triangleq \text{diag}(I_g^1 \; I_g^2 \; \ldots \; I_g^k),$$

$$\underset{\sim}{I}_{bb} \triangleq \text{diag}(I_g^{k+1} \; I_g^{k+2} \; \ldots \; I_g^n),$$

and $\underset{\sim}{0}$ denotes zero matrices of appropriate orders.

From (2.1.9) we find the solution

$$\underset{\sim}{H} = \begin{bmatrix} (\underset{\sim}{V}_{aa} - \underset{\sim}{V}_{ab}\underset{\sim}{V}_{bb}^{-1}\underset{\sim}{V}_{ba})\underset{\sim}{I}_{aa}^{-1} & \underset{\sim}{V}_{ab}\underset{\sim}{V}_{bb}^{-1} \\ -\underset{\sim}{I}_{bb}\underset{\sim}{V}_{bb}^{-1}\underset{\sim}{V}_{ba}\underset{\sim}{I}_{aa}^{-1} & \underset{\sim}{I}_{bb}\underset{\sim}{V}_{bb}^{-1} \end{bmatrix}. \qquad (2.1.10)$$

Note that existence of the inverse $\underset{\sim}{V}_{bb}^{-1}$ is essential for the solution. Of course, the inverse $\underset{\sim}{I}_{aa}^{-1}$ exists and is given by

$$\underset{\sim}{I}_{aa}^{-1} = \text{diag}\left(\frac{1}{I_g^1} \; \frac{1}{I_g^2} \; \ldots \; \frac{1}{I_g^k} \right).$$

Using exactly the same excitations, namely,

$$I_g^1 = I_g^2 = \ldots \triangleq I_g \qquad (2.1.11)$$

(2.1.10) can be simplified as

$$\underset{\sim}{H} = \begin{bmatrix} \frac{1}{I_g}(\underset{\sim}{V}_{aa} - \underset{\sim}{V}_{ab}\underset{\sim}{V}_{bb}^{-1}\underset{\sim}{V}_{ba}) & \underset{\sim}{V}_{ab}\underset{\sim}{V}_{bb}^{-1} \\ -\underset{\sim}{V}_{bb}^{-1}\underset{\sim}{V}_{ba} & I_g\underset{\sim}{V}_{bb}^{-1} \end{bmatrix}. \qquad (2.1.12)$$

Observe that, as before, the preferable current excitations and voltage measurements require the existence of the impedance matrix of the n-port. In the case when $\underset{\sim}{H}$ is to be impedance matrix (k=n) and under the assumption of (2.1.11) the solution is simply equal to the left hand side of the equation (2.1.9) divided by I_g.

A number of similar methods based on different excitations and measurements can be developed. But any one of them requires n independent sets of measurements unless additional information on the network is given (e.g., symmetry and reciprocity).

2.2 Identification of Selected Parameters

Consider the identification of a single two-terminal element within a known environment. The surrounding network can be replaced by the Thevenin equivalent as shown in Fig. 2.4. Assuming that the voltage V_x across the unknown element Z_x is known we find Z_x from

$$(V_{TH} - V_x)Z_x = V_x Z_{TH}. \tag{2.2.1}$$

Observe that the assumption

$$Z_{TH} \neq 0 \tag{2.2.2}$$

is crucial for the identification. We also note that the knowledge of a single voltage may be sufficient for identification of a single element.

The above approach can be generalized as follows. Consider n unknown elements of a network. The situation can be represented as an active n-port being terminated by unknown elements Y_1, Y_2, ..., Y_n (Fig.

2.5(a)). Assume that there exists a hybrid equivalent of the active n-port shown in Fig. 2.5(b). The equivalent is described as follows.

The vector of port voltage sources is

$$\underset{\sim}{V}_a^S \triangleq [V_1^S \; V_2^S \; \ldots \; V_k^S]^T, \tag{2.2.3}$$

the vector of port current sources is

$$\underset{\sim}{I}_b^S \triangleq [I_{k+1}^S \; I_{k+1}^S \; \ldots \; I_n^S]^T, \tag{2.2.4}$$

and the hybrid matrix $\underset{\sim}{H}'$ of the n-port without independent sources is defined by

$$\begin{bmatrix} \underset{\sim}{V}_a' \\ \underset{\sim}{I}_b' \end{bmatrix} = \underset{\sim}{H}' \begin{bmatrix} \underset{\sim}{I}_a \\ \underset{\sim}{V}_b \end{bmatrix}, \tag{2.2.5}$$

where

$$\underset{\sim}{H}' = \begin{bmatrix} \underset{\sim}{H}_{aa} & \underset{\sim}{H}_{ab} \\ \underset{\sim}{H}_{ba} & \underset{\sim}{H}_{bb} \end{bmatrix}. \tag{2.2.6}$$

According to Fig. 2.5(b) we have

$$\underset{\sim}{V}_a' = \underset{\sim}{V}_a - \underset{\sim}{V}_a^S,$$

$$\underset{\sim}{I}_b' = \underset{\sim}{I}_b^S - \underset{\sim}{Y}_{bb}\,\underset{\sim}{V}_b, \tag{2.2.7}$$

$$\underset{\sim}{I}_a = -\underset{\sim}{Y}_{aa}\,\underset{\sim}{V}_a,$$

where

$$\underset{\sim}{Y}_{aa} \triangleq \text{diag}(Y_1\ Y_2\ \ldots\ Y_k)$$

and

$$\underset{\sim}{Y}_{bb} \triangleq \text{diag}(Y_{k+1}\ Y_{k+2}\ \ldots\ Y_n).$$

Substituting (2.2.7) into (2.2.5), and after some manipulations we obtain

$$-\underset{\sim}{I} \triangleq \begin{bmatrix} \underset{\sim}{Y}_{aa}\underset{\sim}{V}_a \\ \\ \underset{\sim}{Y}_{bb}\underset{\sim}{V}_b \end{bmatrix} = \begin{bmatrix} \underset{\sim}{H}_{aa}^{-1} & \underset{\sim}{0} \\ \\ \underset{\sim}{H}_{ba}\underset{\sim}{H}_{aa}^{-1} & \underset{\sim}{1} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_a^S \\ \\ \underset{\sim}{I}_b^S \end{bmatrix} + \begin{bmatrix} \underset{\sim}{H}_{aa}^{-1} & -\underset{\sim}{H}_{aa}^{-1}\underset{\sim}{H}_{ab} \\ \\ \underset{\sim}{H}_{ba}\underset{\sim}{H}_{aa}^{-1} & \underset{\sim}{H}_{bb} - \underset{\sim}{H}_{ba}\underset{\sim}{H}_{aa}^{-1}\underset{\sim}{H}_{ab} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_a \\ \\ \underset{\sim}{V}_b \end{bmatrix}, \tag{2.2.8}$$

where $\underset{\sim}{1}$ is the identity matrix of an appropriate order and $\underset{\sim}{I}$ is the vector of currents through the unknown elements. From (2.2.8) it is seen that the existence of the inverse $\underset{\sim}{H}_{aa}^{-1}$ is necessary to obtain the solution. This is equivalent to the existence of the admittance matrix of the n-port (observe that the matrix in the last part of (2.2.8) is actually the admittance matrix). On the other hand we have considered another assumption, i.e., that the hybrid equivalent exists. It can be shown that the existence of a hybrid matrix is sufficient for the

existence of the corresponding hybrid equivalent. Therefore, if we assume that the admittance matrix exists we can consider a Norton equivalent and, according to the above discussion, we can find the solution. This leads to the following theorem.

Theorem 2.1

Identification of n elements based on voltages across those elements (Fig. 2.5(a)) is possible if and only if there exists the admittance matrix of the corresponding n-port (after shorting independent voltage sources and open-circuiting independent current sources).

The solution can also be obtained directly without looking for a hybrid equivalent. According to Theorem 2.1 and using a representation of the network of Fig. 2.5(a), which is shown in Fig. 2.6, there exists a matrix $\underset{\sim}{H}$ such that

$$\begin{bmatrix} \underset{\sim}{I} \\ \underset{\sim}{I}_V \\ \underset{\sim}{V}_I \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{V} \\ \underset{\sim}{V}_V \\ \underset{\sim}{I}_I \end{bmatrix}, \tag{2.2.9}$$

where $\underset{\sim}{H}_{11}$ is the admittance matrix of the n-port of Fig. 2.5(b). From (2.2.9) we have

$$\underset{\sim}{I} = \underset{\sim}{H}_{11} \underset{\sim}{V} + \underset{\sim}{H}_{12} \underset{\sim}{V}_V + \underset{\sim}{H}_{13} \underset{\sim}{I}_I. \tag{2.2.10}$$

Once we know $\underset{\sim}{I}$ then we can easily find the values of the unknown elements, since

$$Y_i = -I_i/V_i \quad \text{or} \quad Z_i = -V_i/I_i \tag{2.2.11}$$

for i = 1, 2, ..., n.

Now, consider that the measurement ports are different from the ports of the elements which are to be identified. Assume that there exists a hybrid equivalent as shown in Fig. 2.7. According to Fig. 2.7 we have (m and x identify the measurement and identification ports, respectively, and a, b, c, d identify ports of the same kind within the two groups)

$$\begin{bmatrix} \underset{\sim}{V}_a - \underset{\sim}{V}^S_a \\ \underset{\sim}{I}_b + \underset{\sim}{I}^S_b \\ \underset{\sim}{V}_c - \underset{\sim}{V}^S_c \\ \underset{\sim}{I}^S_d \end{bmatrix} = \underset{\sim}{H} \begin{bmatrix} \underset{\sim}{I}_a \\ \underset{\sim}{V}_b \\ \underset{\sim}{0} \\ \underset{\sim}{V}_d \end{bmatrix}, \tag{2.2.12}$$

where

$$\underset{\sim}{H} = \left[\begin{array}{cc:cc} \underset{\sim}{H}_{aa} & \underset{\sim}{H}_{ab} & \underset{\sim}{H}_{ac} & \underset{\sim}{H}_{ad} \\ \underset{\sim}{H}_{ba} & \underset{\sim}{H}_{bb} & \underset{\sim}{H}_{bc} & \underset{\sim}{H}_{bd} \\ \hdashline \underset{\sim}{H}_{ca} & \underset{\sim}{H}_{cb} & \underset{\sim}{H}_{cc} & \underset{\sim}{H}_{cd} \\ \underset{\sim}{H}_{da} & \underset{\sim}{H}_{db} & \underset{\sim}{H}_{dc} & \underset{\sim}{H}_{dd} \end{array}\right] = \begin{bmatrix} \underset{\sim}{H}_{xx} & \underset{\sim}{H}_{xm} \\ \underset{\sim}{H}_{mx} & \underset{\sim}{H}_{mm} \end{bmatrix} \tag{2.2.13}$$

and $\underset{\sim}{H}_{xx}$, $\underset{\sim}{H}_{xm}$, $\underset{\sim}{H}_{mx}$, $\underset{\sim}{H}_{mm}$ are n x n matrices. Observe that in order to solve the system (2.2.12) for unknown vectors $\underset{\sim}{I}_a$, $\underset{\sim}{V}_b$, $\underset{\sim}{V}_a$, $\underset{\sim}{I}_b$ we have to know the inverse $\underset{\sim}{H}^{-1}_{mx}$. This corresponds to the existence of the transmission matrix linking ports of identification as the input with ports of measurement as the output. Assuming that there exists a mixed

"transmission-hybrid" representation of the network described by (see Fig. 2.8)

$$\begin{bmatrix} \underset{\sim}{V}^x \\ \underset{\sim}{I}^x \\ \underset{\sim}{I}_V \\ \underset{\sim}{V}_I \end{bmatrix} = \underset{\sim}{A} \begin{bmatrix} \underset{\sim}{V}^m \\ \underset{\sim}{I}^m \\ \underset{\sim}{V}_V \\ \underset{\sim}{I}_I \end{bmatrix}, \qquad (2.2.14)$$

we find, for $\underset{\sim}{I}^m = \underset{\sim}{0}$,

$$\begin{bmatrix} \underset{\sim}{V}^x \\ \underset{\sim}{I}^x \end{bmatrix} = \begin{bmatrix} \underset{\sim}{A}_{11} \\ \underset{\sim}{A}_{21} \end{bmatrix} \underset{\sim}{V}^m + \begin{bmatrix} \underset{\sim}{A}_{13} & \underset{\sim}{A}_{14} \\ \underset{\sim}{A}_{23} & \underset{\sim}{A}_{24} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_V \\ \underset{\sim}{I}_I \end{bmatrix}. \qquad (2.2.15)$$

The above discussion gives us the following theorem.

Theorem 2.2

Existence of the transmission-type matrix defined by (2.2.14) is necessary and sufficient for identification of n unknown elements based upon n voltage measurements if ports of measurement are different from ports of identification.

The requirements of Theorem 2.1 can easily be verified. The admittance matrix exists if and only if no port can be shorted by shorting all the remaining ports. In contrast, verifying the conditions of Theorem 2.2 is more difficult. This is simply because the elements of a general transmission matrix of a 2n-port network (unlike a 2-port) cannot be defined as ratios of single input and single output in the presence of shorts and openings of other ports. Moreover, the existence

of the transmission matrix is not related to the existence of any particular hybrid matrix. Hence, Theorem 2.2 is not very useful in practice and one should look for another and simpler criterion. Nevertheless, we observe that in both cases there exists a limit to the number of elements which can be identified. Usually, Theorems 2.1 and 2.2 are satisfied as far as the identification of one or two elements is concerned. The more elements we want to consider the more unlikely it is to satisfy the corresponding theorem. The number of elements which can still be identified strongly depends upon topology and elements chosen. But in any case, for a particular network, there exists a maximum number of elements which can be identified by methods described in this section and this number is less than the total number of elements in the network. In the next section we deal with the problem of identification of all elements since this cannot be done by the above methods.

2.3 Identification of All Parameters

We now consider the situation when all network elements are unknown. We assume that voltages across all elements are available. Since Kirchhoff's voltage law is satisfied (i.e., we assume that measurements are accurate enough) we can consider nodal voltages only. Using the preferable current excitations we have a generalized branch shown in Fig. 2.9. As is well known, a network with p branches and r nodes can be described by the branch-node incidence matrix

$$\underset{\sim}{A} = [\lambda_{ik}], \tag{2.3.1}$$

where

$$\lambda_{ik} = \begin{cases} +1, & \text{for the kth branch directed towards the ith node,} \\ -1, & \text{for the kth branch directed away from the ith node,} \\ 0, & \text{for the kth branch not incident with the ith node,} \end{cases} \tag{2.3.2}$$

$i = 1, 2, \ldots, r-1$ and $k = 1, 2, \ldots, p$.

Following the typical nodal approach we introduce the vector of nodal current excitations as

$$\underset{\sim}{I}^S = \underset{\sim}{A} \begin{bmatrix} J_1 \\ J_2 \\ . \\ . \\ . \\ J_p \end{bmatrix}. \tag{2.3.3}$$

This enables us to write Kirchhoff's current law in the form

$$\underset{\sim}{A}\ \underset{\sim}{I} = -\underset{\sim}{I}^S, \tag{2.3.4}$$

where

$$\underset{\sim}{I} \triangleq [I_1\ I_2\ \ldots\ I_p]^T \tag{2.3.5}$$

is the vector of branch currents.

Using the notation

$$\underset{\sim}{Y} = [Y_1\ Y_2\ \ldots\ Y_p]^T \tag{2.3.6}$$

for the vector of branch admittances, and

$$\underset{\sim}{U} = \mathrm{diag}(U_1\ U_2\ \ldots\ U_p) \tag{2.3.7}$$

for the matrix of branch voltages, we can write Ohm's law for all branches of the network as

$$\underset{\sim}{I} = -\underset{\sim}{U}\ \underset{\sim}{Y}. \tag{2.3.8}$$

Since Kirchhoff's voltage law is satisfied automatically we note that equation (2.3.8) together with (2.3.4) are all the available equations for the network. The current vector $\underset{\sim}{I}$ is of no interest, so eliminating it from (2.3.8) and (2.3.4) we find

$$(\underset{\sim}{\Lambda}\ \underset{\sim}{U})\underset{\sim}{Y} = \underset{\sim}{I}^S. \tag{2.3.9}$$

This is simply the system of equations which has been sought. It contains r-1 equations with the p unknown values of Y_1, Y_2, ..., Y_p. Matrix $\underset{\sim}{\Lambda}$ consists of r-1 linearly independent rows, so if branch voltages are different from zero then the matrix $(\underset{\sim\sim}{\Lambda U})$ also consists of r-1 linearly independent rows. Note that p can be equal to r-1 only if the network graph is a tree. In this case all network elements can easily be determined if the excitations chosen are such that there is a nonzero current in every branch of the tree. This is a rather obvious result since, knowing the excitations, we know immediately all branch currents. In other cases we always have p > r-1 and we are not able to identify all elements Y_1, Y_2, ..., Y_p based only on the equation (2.3.9). If some of those elements are known (at least p-r+1 of them)

we can solve (2.3.9) for the remaining parameters provided that the resulting system contains an appropriate number of linearly independent equations. This is another approach to the problems considered in the foregoing section.

Now, we are interested in the identification of all parameters of the network. Since the number of equations in (2.3.9) is less than the number of unknowns we have to find additional equations based on other set(s) of measurements. According to (2.3.9) one set of measurements gives us at most r-1 independent equations. This means that we need at least m sets of measurements, where

$$m = \text{int}\left(\frac{p}{r-1} \right) \tag{2.3.10}$$

and int(x) denotes the smallest integer x_o such that $x \leq x_o$. Because the number of branches p is between r-1 (for a tree-network) and r(r-1)/2 (for a complete-graph network), i.e.,

$$r-1 \leq p \leq \frac{r(r-1)}{2}, \tag{2.3.11}$$

we find that

$$1 \leq m \leq \text{int}\left(\frac{r}{2} \right). \tag{2.3.12}$$

For typical networks m is expected to equal 2 or 3. Every set of measurements $\underset{\sim}{U}^i$ provides the appropriate system of equations (2.3.9) as

$$(\underset{\sim}{\Lambda}^i \; \underset{\sim}{U}^i)\underset{\sim}{Y} = \underset{\sim}{I}^{Si} \tag{2.3.13}$$

for $i = 1, 2, \ldots, M$ where $M \geq m$.

All of those systems give us the final matrix equation

$$\begin{bmatrix} \underset{\sim}{A}^1 \underset{\sim}{U}^1 \\ \underset{\sim}{A}^2 \underset{\sim}{U}^2 \\ \vdots \\ \underset{\sim}{A}^M \underset{\sim}{U}^M \end{bmatrix} \underset{\sim}{Y} = \begin{bmatrix} \underset{\sim}{I}^{S1} \\ \underset{\sim}{I}^{S2} \\ \vdots \\ \underset{\sim}{I}^{SM} \end{bmatrix}. \qquad (2.3.14)$$

Usually, different sets of measurements are obtained only under different excitations, while the network topology is not changed at all, so

$$\underset{\sim}{A}^1 = \underset{\sim}{A}^2 = \ldots = \underset{\sim}{A}^M. \qquad (2.3.15)$$

However, if we can short certain nodes, different ones for different measurements, then in general, we should consider different matrices $\underset{\sim}{A}^i$.

The system (2.3.14) is required to contain exactly p independent equations. Roughly speaking, the systems (2.3.13) should be "independent" of each other. In other words, we have to arrange for M "independent" measurements. How to arrange for these independent measurements, however, is not known so far. Nevertheless, several directions can be proposed.

It would seem to be optimal if the subsequent measurements provided equations which formed an independent system along with all previously obtained equations and, furthermore, if the final system was not ill-conditioned. In other words, we want the rows of matrices

$$\underset{\sim}{A}^1 \; \underset{\sim}{U}^1, \begin{bmatrix} \underset{\sim}{A}^1 & \underset{\sim}{U}^1 \\ & \\ \underset{\sim}{A}^2 & \underset{\sim}{U}^2 \end{bmatrix}, \begin{bmatrix} \underset{\sim}{A}^1 & \underset{\sim}{U}^1 \\ \underset{\sim}{A}^2 & \underset{\sim}{U}^2 \\ \underset{\sim}{A}^3 & \underset{\sim}{U}^3 \end{bmatrix}, \ldots \qquad (2.3.16)$$

to be linearly independent. Assuming (2.3.15) holds we have M = m. Of course if p/(r-1) is not an integer then the last system (for i = M) contains a few more equations and the system (2.3.14) is overdetermined. Alternatively, for the last set of measurements, we can make an appropriate number of shorts such that the matrix $\underset{\sim}{A}^M$ consists of p-(m-1)(r-1) rows and the system (2.3.14) has exactly p equations. For this approach, we would propose to use different locations for the excitations for the different measurements. These excitations should be as remote form one another as possible.

Now, as an important example, we apply the foregoing theory to ladder networks.

Methods for Ladder Networks

Consider the ladder network shown in Fig. 2.10. The branch-node incidence matrix $\underset{\sim}{A}$ consists of r-1 = n+1 rows and p = 2n+1 columns and its structure is

$$\underset{\sim}{A}^1 = \begin{bmatrix} 1 & 1 & & & & & & & \\ & -1 & 1 & 1 & & & & & \\ & & & -1 & 1 & 1 & & & \\ & & & & & \cdot & \cdot & \cdot & \cdot \\ & & & & & & & -1 & 1 & 1 \\ & & & & & & & & -1 & 1 \end{bmatrix} . \qquad (2.3.17)$$

Using only the input source I_1^S for the first set of voltage measurements, i.e.,

$$\underset{\sim}{I}^{S1} = [I_1^{S1} \quad 0 \quad 0 \quad \ldots \quad 0]^T, \qquad (2.3.18)$$

we obtain the first subsystem of equations (2.4.13) as

$$\begin{bmatrix} U_1^1 & U_2^1 & & & & & & \\ & -U_2^1 & U_3^1 & U_4^1 & & & & \\ & & & -U_4^1 & U_5^1 & U_6^1 & & \\ & & & & & \cdot & \cdot & \cdot \\ & & & & & & -U_{2n}^1 & U_{2n+1}^1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_{2n+1} \end{bmatrix} = \begin{bmatrix} I_1^{S1} \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} . \qquad (2.3.19)$$

According to (2.3.10) we find m = 2, so we have to arrange for another set of measurements. Since we need exactly n more equations we can make exactly 1 short. A number of different approaches can be suggested. We will discuss three of them.

1. We use the same excitations, i.e.,

$$\underset{\sim}{I}^{S2} = \underset{\sim}{I}^{S1} \tag{2.3.20}$$

and the output port is shorted. The appropriate matrix $\underset{\sim}{\Lambda}^2$ is obtained from (2.3.17) by dropping the last row.

2. We use only the output source I^S_{2n+1} for the second set of measurements, i.e.,

$$\underset{\sim}{I}^{S2} = [0 \quad 0 \quad \ldots \quad 0 \; I^{S2}_{2n+1}]^T, \tag{2.3.21}$$

and the input port is shorted. The appropriate matrix $\underset{\sim}{\Lambda}^2$ is obtained from (2.3.17) by dropping the first row.

3. We do not make any shorts, i.e., we use the same branch-node matrix

$$\underset{\sim}{\Lambda}^2 = \underset{\sim}{\Lambda}^1, \tag{2.3.22}$$

and the resulting system of equations (2.3.14) will be overdetermined. According to the previous discussion we apply the output source and the vector $\underset{\sim}{I}^{S2}$ is in the form of (2.3.21).

Note that regardless of the method chosen, for any row of $\underset{\sim}{\Lambda}^2$ we can find an identical row within the matrix $\underset{\sim}{\Lambda}^1$. Hence the linear independence or linear dependence of the final system (2.3.14) consists in the particular values of voltages $U^1_1, U^1_2, \ldots, U^1_p$ in comparison with $U^2_1, U^2_2, \ldots, U^2_p$. Because of this the first method is likely to be ill-

conditioned. It can be caused by relatively insensitive behaviour of voltages across the elements located close to the input w.r.t. a change of the output load. Using the same excitation for the two measurements we can meet the situation that the corresponding equations in both subsystems are "nearly" the same. From this point of view it is obvious that we are looking for quite a different excitation for the second set of measurements. The second and the third methods satisfy this requirement. These two methods are similar and we can discuss both simultaneously. The only difference is that the second method provides one less equation and that the values of voltages are a little different (in particular, $U_1^2 = 0$ and $U_2^2 = -U_3^2$). Therefore, we will discuss the third method and most of the following results will be applicable to the second method. Now, the second subsystem of equations (2.3.13) is similar to (2.3.19). The only difference is that superscripts "1" are replaced by superscripts "2" and the right hand side of (2.3.19) is replaced by $\underset{\sim}{I}^{S2}$ given by (2.3.21). The resulting system of equations (2.3.14), after reordering, can be expressed in the form

$$\underset{\sim}{A}\ \underset{\sim}{Y} = \underset{\sim}{B} \tag{2.3.23}$$

where

$$\underset{\sim}{B} = \begin{bmatrix} \underset{\sim}{I}^{S1} \\ \underset{\sim}{I}^{S2} \end{bmatrix} = [I_1^{S1}\ 0\ 0\ \ldots\ 0\ I_{2n+1}^{S2}]^T \tag{2.3.24}$$

and

$$\underset{\sim}{A} = \begin{bmatrix} U_1^1 & U_2^1 & & & & & & & \\ U_1^2 & U_2^2 & & & & & & & \\ & -U_2^1 & U_3^1 & U_4^1 & & & & & \\ & -U_2^2 & U_3^2 & U_4^2 & & & & & \\ & & & -U_4^1 & U_5^1 & U_6^1 & & & \\ & & & -U_4^2 & U_5^2 & U_6^2 & & & \\ & & & & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & & & & -U_{2n}^1 & U_{2n+1}^1 & \\ & & & & & & -U_{2n}^2 & U_{2n+1}^2 & \end{bmatrix} . \qquad (2.3.25)$$

The particular sparse form of the matrix $\underset{\sim}{A}$ enables us to find explicit formulae for the solution of (2.3.23). It is more convenient to use nodal voltages $V_1^i, \ldots, V_{n+1}^i$, $i = 1, 2$, such that

$$U_{2k-1}^i = V_k^i \quad \text{and} \quad U_{2k}^i = V_k^i - V_{k+1}^i \qquad (2.3.26)$$

for $i = 1, 2$ and $k = 1, 2, \ldots$. From the last two equations we obtain

$$Y_{2n+1} = \frac{I_{2n+1}^{S2} \, (V_n^1 - V_{n+1}^1)}{\Delta_n} \qquad (2.3.27)$$

and

$$Y_{2n} = \frac{I_{2n+1}^{S2} \quad V_{n+1}^1}{\Delta_n} , \qquad (2.3.28)$$

where

$$\Delta_n = V_n^1 V_{n+1}^2 - V_n^2 V_{n+1}^1. \tag{2.3.29}$$

Substituting Y_{2n} into the preceding two equations we can determine Y_{2n-1} and Y_{2n-2}. In this way we find the recurrent formulae

$$Y_{2k+1} = \frac{\Delta_k'}{\Delta_k} Y_{2k+2}, \tag{2.3.30}$$

$$Y_{2k} = \frac{\Delta_{k+1}}{\Delta_k} Y_{2k+2}, \tag{2.3.31}$$

where

$$\Delta_k = \det \begin{bmatrix} V_k^1 & V_{k+1}^1 \\ V_k^2 & V_{k+1}^2 \end{bmatrix} \tag{2.3.32}$$

and

$$\Delta_k' = \det \begin{bmatrix} V_k^1 - V_{k+1}^1 & V_{k+1}^1 - V_{k+2}^1 \\ V_{k+1}^2 - V_k^2 & V_{k+2}^2 - V_{k+1}^2 \end{bmatrix}. \tag{2.3.33}$$

From (2.3.31) and (2.3.28) we notice that

$$Y_{2k} \Delta_k = I_{2n+1}^{S2} V_{n+1}^1 \tag{2.3.34}$$

or

$$Y_{2k} = \frac{I_{2n+1}^{S2} \quad V_{n+1}^{1}}{\Delta_k} \tag{2.3.35}$$

for k = 1, 2, ..., n.

Using (2.3.35), (2.3.30) and (2.3.27), we find Y_2, Y_3, ..., Y_{2n+1}. Finally, from the first equation of the system (2.3.23) we have

$$Y_1 = \frac{I_1^{S1} - (V_1^1 - V_2^1)Y_2}{V_1^1} . \tag{2.3.36}$$

Alternatively, since the system is overdetermined, we obtain from the second equation

$$Y_1 = \frac{V_2^2 - V_1^2}{V_1^2} Y_2 \tag{2.3.37}$$

and both solutions should be identical. Of course, this second equation does not appear in the second method (because of shorting the input port). The above solution may be described by the term <u>backward solution</u>.

Similarly, starting from the first two equations we can derive the <u>forward solution</u> as

$$Y_1 = \frac{I_1^{S1}(V_2^2 - V_1^2)}{\Delta_1} , \tag{2.3.38}$$

$$Y_{2k} = \frac{I_1^{S1}\, V_1^2}{\Delta_k}, \quad k = 1, 2, \ldots, n, \tag{2.3.39}$$

$$Y_{2k+1} = \frac{\Delta_k'}{\Delta_{k+1}} Y_{2k}, \quad k = 1, 2, \ldots, n-1, \tag{2.3.40}$$

and

$$Y_{2n+1} = \frac{I_{2n+1}^{S2} - (V_{n+1}^2 - V_n^2)Y_{2n}}{V_{n+1}^2}, \tag{2.3.41}$$

or

$$Y_{2n+1} = \frac{V_n^1 - V_{n+1}^1}{V_{n+1}^1} Y_{2n}. \tag{2.3.42}$$

For the second method, only the backward solution exists and Y_1 is expressed by (2.3.36). For the third method we can use the backward as well as the forward solution and the two solutions should be identical. They can be different from each other if the measurements are inaccurate. Then the question arises of how to take advantage of the fact that the system (2.3.23) is overdetermined.

The solvability of the problem depends on the determinants (2.3.32). They have to be different from zero. In other words any two successive nodal voltages for the two tests cannot be linearly dependent. Also V_{n+1}^2 and V_1^1 should have nonzero values. From a physical point of view we see that none of the nodal voltages (except V_{n+1}^1 and V_1^2) can be equal to zero. This is because if $V_k^1 = 0$ or $V_k^2 = 0$

then $V^1_{k+1} = V^1_{k+2} = \ldots = 0$ or $V^2_{k-1} = V^2_{k-2} = \ldots = 0$, respectively. Then also $\Delta_k = \Delta_{k+1} = \ldots = 0$ or $\Delta_k = \Delta_{k-1} = \ldots = 0$ and the solution does not exist. This corresponds, for instance, to the situation when the frequency of excitation is the resonant frequency of a shunt element. To remedy the situation we can change the frequency of excitation and/or arrange for other measurements, i.e., use other ports of excitations. Otherwise, if all nodal voltages are different from zero the solution is likely to exist. For instance, for a resistive ladder network the voltages V^1_1, V^1_2, ... and V^2_{n+1}, V^2_n, V^2_{n-1}, ... are consecutively smaller and, as a consequence, the determinants (2.3.32) are different from zero. The only exception occurs when two successive voltages are identical, i.e., a series element is a short circuit. In this case, although we can identify $Y_{2k} = \infty$, we cannot identify Y_{2k-1} and Y_{2k+1} separately. Only the composite parallel connection of Y_{2k-1} and Y_{2k+1} can be determined. This corresponds to the assumption that the network does not contain parallel connections of elements which are to be identified.

The above example of the ladder network parameter identification gives us some guidance as to how to arrange for independent tests of measurements as well as some problems which can arise. However, these are not satisfactory enough and more general and precise methods and properties should be sought. In particular, methods for active networks are of great importance. We deal with this problem in the following subsection.

Active Networks

We now consider a network which consists of passive as well as active lumped elements. Control sources are taken into account as models of active elements. We will consider only voltage controlled current sources (VCCS) which are typical for the nodal approach. It is sufficiently general for many practical cases.

The general formulation discussed at the beginning of this section can easily be extended to identify unknown control coefficients besides all other passive admittances. Of course, if the control coefficient of a VCCS is known, we can treat this source as independent since the controlling voltage is also known.

Consider a network with passive branches and s voltage controlled current sources. The VCCS elements are described by the equation

$$J_k^c = Y_k^c \, U_k^c, \tag{2.3.43}$$

for k = 1, 2, ..., s.

For our purposes we have to treat the controlled branches as different from those which contain passive elements and/or independent sources even if they are parallel. Hence, the branch-node incidence matrix for the network can be expressed as

$$\underset{\sim}{A} = [\underset{\sim}{A}_p \quad -\underset{\sim}{A}_a] \tag{2.3.44}$$

where $\underset{\sim}{A}_p$ is the (r-1) x p matrix described by (2.3.1) and $\underset{\sim}{A}_a$ is an (r-1) x s matrix constructed for all controlled branches in the same way as $\underset{\sim}{A}_p$. Now, Kirchhoff's current low can be written in the form

$$\underset{\sim}{\Lambda}_p \underset{\sim}{I} + \underset{\sim}{\Lambda}_a \underset{\sim}{J}^c = -\underset{\sim}{I}^S, \tag{2.3.45}$$

where

$$\underset{\sim}{J}^c = [J_1^c \; J_2^c \; \ldots \; J_s^c]^T. \tag{2.3.46}$$

Using the notation

$$\underset{\sim}{U} = \mathrm{diag}(U_1 \; U_2 \; \ldots \; U_p \; U_1^c \; U_2^c \; \ldots \; U_s^c) \tag{2.3.47}$$

we finally find the equation

$$(\underset{\sim}{\Lambda} \; \underset{\sim}{U})\underset{\sim}{Y} = \underset{\sim}{I}^S, \tag{2.3.48}$$

where $\underset{\sim}{\Lambda}$ is given by (2.3.44), $\underset{\sim}{I}^S$ is described by (2.3.3) and $\underset{\sim}{Y}$ is the vector of unknown parameters

$$\underset{\sim}{Y} = [Y_1 \; Y_2 \; \ldots \; Y_p \; Y_1^c \; Y_2^c \; \ldots \; Y_s^c]^T. \tag{2.3.49}$$

The system (2.3.48) contains r-1 equations with p+s unknowns. As before, in order to obtain an appropriate number of independent equations we have to arrange for other tests. The number of tests which we need is at least

$$m = \mathrm{int}\left(\frac{p+s}{r-1}\right). \tag{2.3.50}$$

The same approaches are valid as for the choice of independent measurements.

3. FAULT ANALYSIS

3.1 Problem Formulation

Fault analysis is strongly related to the problem of identification, which was discussed in Section 2. By a fault we mean not only an unwanted short or open circuit but also, more generally, any large change in the value of an element w.r.t. its nominal value. Since the meaning of the term "large change" is not precise enough we will consider any change in element value as a fault. Of course, we assume that the network design, i.e., the topology as well as the nominal values of the parameters are known.

Fault analysis consists of two stages: fault detection and fault evaluation. Fault detection can be done by the method which identifies all element values and then comparing the nominal and actual values. Thus, fault evaluation is being done simultaneously. This approach, however, can be too general. It may also be too difficult if, for instance, the network is not element-value solvable. Usually, we look for one, two or several faults and there is no need to identify everything as though we did not know anything about the network.

The fault detection should locate the faults, i.e., identify elements which are out of their nominal values. Once we know which elements are at fault, the fault evaluation is simply equivalent to the identification of selected parameters discussed in subsection 2.3. Therefore, in this Section, we will mainly deal with problems of fault detection.

3.2 Single-fault Detection

Consider a network function f as a function of a single element Y. For many cases it can be expressed as a bilinear function

$$f = \frac{A + BY}{C + DY} . \tag{3.2.1}$$

The direct use of (3.2.1) for the single-fault detection is impossible since f can be changed either by a change of Y or by changes of the coefficients A, B, C, D which depend on values of other elements.

Now consider two different network functions f_1 and f_2 of the same element Y as

$$f_1 = \frac{A_1 + B_1Y}{C_1 + D_1Y} , \quad f_2 = \frac{A_2 + B_2Y}{C_2 + D_2Y} . \tag{3.2.2}$$

If the two functions essentially depend on Y, i.e., $A_iD_i - B_iC_i \neq 0$ for $i = 1, 2$, then each of them can be solved for Y and the solution is

$$Y = \frac{A_1 - C_1f_1}{-B_1 + D_1f_1} = \frac{A_2 - C_2f_2}{-B_2 + D_2f_2} . \tag{3.2.3}$$

From (3.2.3) we find the relation

$$(C_1B_2 - D_1A_2)f_1 + (A_1D_2 - B_1C_2)f_2 = (A_1B_2 - B_1A_2) + (C_1D_2 - D_1C_2)f_1f_2, \tag{3.2.4}$$

which holds for any value of Y provided that all other elements are fixed. If the two network functions are of the same type (e.g., trans-impedances) the denominators $C_1 + D_1Y$ and $C_2 + D_2Y$ are determined by the same characteristic polynomial of the network. Hence, they can differ only by a constant multiplier, so $C_1D_2 - D_1C_2 = 0$ and (3.2.4) becomes a linear relation

$$af_1 + bf_2 = c \tag{3.2.5}$$

where $a \triangleq C_1B_2 - D_1A_2$, $b \triangleq A_1D_2 - B_1C_2$ and $c \triangleq A_1B_2 - B_1A_2$.

Equation (3.2.5) gives us the relationship between values of f_1 and f_2 when all network elements except Y are kept unchanged. In other words the coefficients a, b and c depend only on nominal values of all other elements. Similar relationships between f_1 and f_2 can be derived for all other elements Y_1, Y_2, ..., Y_p. This is done for nominal values of all elements. Therefore we obtain p equations

$$a^if_1 + b^if_2 = c^i, \quad i = 1, 2, \ldots, p, \tag{3.2.6}$$

each of them corresponding to a certain element of the network. Superscript i denotes the index of the element. We will use these equations for the single-fault detection.

Based on measurements, we find the actual values of f_1 and f_2. If there is a single fault within the network, i.e., one of the elements Y_1, Y_2, ..., Y_p is changed, then the corresponding equation of (3.2.6) is satisfied since all other elements are at their nominal values. All other equations are likely to be unsatisfied. To be able to identify

uniquely the fault location it is required that

$$\det \begin{bmatrix} a^k & b^k \\ a^\ell & b^\ell \end{bmatrix} \neq 0, \qquad (3.2.7)$$

for any k, ℓ, k ≠ ℓ. If these conditions are fulfilled then all of the equations in (3.2.6) are satisfied only by the nominal values f_1^0 and f_2^0 and no two equations can be satisfied by the same values f_1, f_2 different from f_1^0, f_2^0. The two-dimensional (e.g., DC network) geometrical interpretation of this is given in Fig. 3.1. The equations (3.2.6) describe straight lines in the two-dimensional space f_1, f_2. They all intersect at the point corresponding to the nominal values of all elements.

Since the nominal values satisfy equations (3.2.6) we can use the changes

$$\Delta f_j = f_j - f_j^0, \; j = 1, 2, \qquad (3.2.8)$$

instead of f_1 and f_2. Thus, we have homogeneous equations

$$a^i \, \Delta f_1 + b^i \, \Delta f_2 = 0, \quad i = 1, 2, \ldots, p. \qquad (3.2.9)$$

To use these equations we do not need to know the values c^i, i = 1, 2, ..., p, but we have to know f_1^0 and f_2^0.

The actual values of the network functions f_1 and f_2 are to be identified by measurements. Using, preferably, current excitation and voltage measurements the two network functions should be certain impedances or trans-impedances

$$f_j = V_j^m / I_{gj}, \quad j = 1, 2. \tag{3.2.10}$$

Thus, the equation (3.2.9) can be directly expressed using the measured voltages V_1^m and V_2^m instead of f_1 and f_2. If the excitation currents I_{g1} and I_{g2} are at different values then the coefficients a^i (or b^i) have to be rescaled. Otherwise, this is not necessary.

The two excitations I_{g1} and I_{g2} do not need to be applied to the same port, but if they are then the voltage measurements V_1^m and V_2^m can be taken simultaneously (i.e., at the same measurement test). We now derive a simple method which supplies the coefficients of equation (3.2.9) for the latter case.

Consider the representation of the network shown in Fig. 3.2. Note that the 4-port network consists of elements which are at their nominal values, so it does not depend on any fault.

According to Fig. 3.2 we have

$$\underset{\sim}{V} = \begin{bmatrix} V_1^m \\ V_2^m \\ V_I \\ V_i \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} 0 \\ 0 \\ I_g \\ -V_i \Delta Y_i \end{bmatrix}, \tag{3.2.11}$$

where V_1^m and V_2^m are the voltages measured and I_g is the excitation (i.e., we consider the network functions $f_j = V_j^m / I_g$, $j = 1,2$). Since the left hand side of (3.2.11) can be expressed as

$$\underset{\sim}{V} = \underset{\sim}{V}^0 + \Delta\underset{\sim}{V}, \tag{3.2.12}$$

where $\underset{\sim}{V}^0$ is the nominal vector obtained for $\Delta Y_i = 0$, we find

$$\Delta\underset{\sim}{V} = \begin{bmatrix} \Delta V_1^m \\ \Delta V_2^m \\ \Delta V_I \\ \Delta V_i \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -V_i \Delta Y_i \end{bmatrix} . \tag{3.2.13}$$

Thus

$$\begin{bmatrix} \Delta V_1^m \\ \Delta V_2^m \end{bmatrix} = I_i \begin{bmatrix} Z_{14} \\ Z_{24} \end{bmatrix} . \tag{3.2.14}$$

Eliminating I_i from (3.2.14) we obtain

$$Z_{24}\ \Delta V_1^m - Z_{14} \Delta V_2^m = 0. \tag{3.2.15}$$

Note that in order to be able to eliminate I_i at least one of Z_{14} and Z_{24} has to be different from zero. The equation (3.2.15) is one of the equations (3.2.9). It corresponds to Y_i, so $a^i = Z_{24}$ and $b^i = -Z_{14}$.

In this way we can find all equations (3.2.9). But it would be inconvenient to consider as many different 4-port networks as the number of elements. We propose to use the adjoint network simulation for this purpose. The method is explained in Fig. 3.3. According to Fig. 3.3(a) we have

$$\begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \hat{V}_{i1} \end{bmatrix} = \underset{\sim}{Z}^T \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} \\ Z_{12} \\ Z_{13} \\ Z_{14} \end{bmatrix} \tag{3.2.16}$$

so that

$$\hat{V}_{i1} = Z_{14}. \tag{3.2.17}$$

Similarly, according to Fig. 3.3(b) we find

$$\hat{V}_{i2} = Z_{24}. \tag{3.2.18}$$

Finally, equation (3.2.15) can be rewritten in the form

$$\hat{V}_{i2}\ \Delta V_1^m - \hat{V}_{i1}\ \Delta V_2^m = 0. \tag{3.2.19}$$

It can be shown that the above discussion is valid for all elements of the network under a mild condition that the measured voltages V_1^m and V_2^m essentially depend on all elements. Moreover, it does not matter if the port of the element Y_i is the same as the port of excitation or a port of measurement. Similarly, the port of excitation can be one of the ports of measurement. Therefore, in order to obtain the coefficients of the equations (3.2.9) two simulations of the adjoint network are required. First, we apply a unit current to the first measurement port and calculate the voltages across all elements $\hat{V}_{11}$, $\hat{V}_{21}$, ..., $\hat{V}_{p1}$. Second, applying a unit current to the second measurement port we find $\hat{V}_{12}$, $\hat{V}_{22}$, ..., $\hat{V}_{p2}$. Finally, we formulate the equations (3.2.19) for i = 1, 2, ..., p and check the condition (3.2.7). In fact, only one simulation is required since in both cases we have to

solve exactly the same system of equations with different right hand sides. It can be calculated simultaneously, or alternatively, using the same LU factorization.

Although a rather rare case, it is possible that not all determinants (3.2.7) are different from zero. If two equations in (3.2.9), say for i_1 and i_2, are found to be linearly dependent it means that V_1^m and V_2^m are influenced by Y_{i_1} and Y_{i_2} similarly. The corresponding straight lines in Fig. 3.1 are identical and we cannot distinguish a fault of Y_{i_1} from a fault of Y_{i_2}. This situation can appear, for instance, if two elements are symmetrical to each other w.r.t. the voltages measured. To remedy this problem we can choose two other voltages in order to replace at least one of the two equations.

Finally, it is to be noted that the above method can be used to detect more general faults like shorts between nonincident nodes. We can simply consider nonexisting elements between such nodes as elements of nominal value Y = 0 and we can derive the equations of the form (3.2.9) for those elements.

3.3 Multiple-fault Detection

We now generalize the foregoing approach in order to be able to deal with several simultaneous faults within the network. These faults are represented as external loads of (n+k)-port network shown in Fig. 3.4. We consider n ports of measurement with

$$\underset{\sim}{V}^m \triangleq [V_1^m \; V_2^m \; \ldots \; V_n^m]^T \qquad (3.3.1)$$

and

$$\underset{\sim}{I}^m \triangleq [I_1^m \; I_2^m \; \ldots \; I_n^m]^T. \tag{3.3.2}$$

The ports of fault are described by

$$\underset{\sim}{V}^x = [V_1^x \; V_2^x \; \ldots \; V_k^x]^T$$

and

$$\underset{\sim}{I}^x = [I_1^x \; I_2^x \; \ldots \; I_k^x]^T = -[V_1^x \Delta Y_1^x \; V_2^x \Delta Y_2^x \; \ldots \; V_k^x \Delta Y_k^x]^T, \tag{3.3.3}$$

where $k \leq n-1$.

We assume that the impedance matrix $\underset{\sim}{Z}$ of the (n+k)-port network exists. According to Fig. 3.4 we have

$$\begin{bmatrix} \underset{\sim}{V}^m \\ \underset{\sim}{V}^x \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}_{mm} & \underset{\sim}{Z}_{mx} \\ \underset{\sim}{Z}_{xm} & \underset{\sim}{Z}_{xx} \end{bmatrix} \begin{bmatrix} \underset{\sim}{I}^m \\ \underset{\sim}{I}^x \end{bmatrix}. \tag{3.3.4}$$

Assuming that the ports of measurement are open circuited or are excited by independent current sources we find that the nominal voltage vector is described by

$$\begin{bmatrix} \underset{\sim}{V}^{m0} \\ \underset{\sim}{V}^{x0} \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} \underset{\sim}{I}^m \\ \underset{\sim}{0} \end{bmatrix}. \tag{3.3.5}$$

Hence, the voltage change vector can be expressed as

$$\begin{bmatrix} \Delta V^m \\ \sim \\ \Delta V^x \\ \sim \end{bmatrix} = \underset{\sim}{Z} \begin{bmatrix} 0 \\ \sim \\ I^x \\ \sim \end{bmatrix}, \tag{3.3.6}$$

and, in particular,

$$\Delta \underset{\sim}{V}^m = \underset{\sim mx}{Z} \; \underset{\sim}{I}^x. \tag{3.3.7}$$

$\underset{\sim mx}{Z}$ is a rectangular matrix having more rows than columns. Assuming that $\underset{\sim mx}{Z}$ is a full column rank matrix we can find the solution of the equation (3.3.7) as

$$\underset{\sim}{I}^x = (\underset{\sim mx}{Z}^T \; \underset{\sim mx}{Z})^{-1} \; \underset{\sim mx}{Z}^T \; \Delta \underset{\sim}{V}^m. \tag{3.3.8}$$

Therefore, eliminating $\underset{\sim}{I}^x$ from (3.3.7) and (3.3.8) we find the equation

$$[\underset{\sim mx}{Z}(\underset{\sim mx}{Z}^T \underset{\sim mx}{Z})^{-1} \underset{\sim mx}{Z}^T - \underset{\sim}{1}] \Delta \underset{\sim}{V}^m = \underset{\sim}{0}, \tag{3.3.9}$$

which is a generalization of equation (3.2.15). Using the notation

$$\overline{\underset{\sim}{A}} \triangleq \underset{\sim}{A}(\underset{\sim}{A}^T \underset{\sim}{A})^{-1} \underset{\sim}{A}^T \tag{3.3.10}$$

for a full column rank matrix $\underset{\sim}{A}$, the left hand side of (3.3.9) can be rewritten in the form

$$(\overline{\underset{\sim mx}{Z}} - \underset{\sim}{1}) \Delta \underset{\sim}{V}^m. \tag{3.3.11}$$

Given a vector of voltage changes $\Delta\underset{\sim}{V}^m$ we can calculate the expression (3.3.11). It is equal to $\underset{\sim}{0}$ regardless of the element changes ΔY_1, ΔY_2, ..., ΔY_k if all other elements are kept at their nominal values. In other words if (3.3.11) is different from zero it means that there is another element at fault beside the elements Y_1, ..., Y_k.

In order to be able to detect k simultaneous faults we need to know expressions similar to (3.3.11) for all possible combinations consisting of k elements.

As before, the matrix $\underset{\sim}{Z}_{mx}$ can be found by means of the adjoint network. For the adjoint network we have

$$\begin{bmatrix} \hat{\underset{\sim}{V}}^m \\ \hat{\underset{\sim}{V}}^x \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Z}^T_{mm} & \underset{\sim}{Z}^T_{xm} \\ \underset{\sim}{Z}^T_{mx} & \underset{\sim}{Z}^T_{xx} \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{I}}^m \\ \hat{\underset{\sim}{I}}^x \end{bmatrix} . \qquad (3.3.12)$$

Let $\hat{\underset{\sim}{I}}^x = \underset{\sim}{0}$. Then we obtain

$$\hat{\underset{\sim}{V}}^x = \underset{\sim}{Z}^T_{mx}\ \hat{\underset{\sim}{I}}^m, \qquad (3.3.13)$$

where $\hat{\underset{\sim}{I}}^m$ is the vector of an adjoint network excitation. Taking n linearly independent excitations $\hat{\underset{\sim}{I}}^{m1}$, $\hat{\underset{\sim}{I}}^{m2}$, ..., $\hat{\underset{\sim}{I}}^{mn}$ we have the equation

$$[\hat{\underset{\sim}{V}}^{x1} \ \ldots\ \hat{\underset{\sim}{V}}^{xn}] = \underset{\sim}{Z}^T_{mx}\ [\hat{\underset{\sim}{I}}^{m1} \ \ldots\ \hat{\underset{\sim}{I}}^{mn}], \qquad (3.3.14)$$

which can be solved for $\underset{\sim}{Z}^T_{mx}$. The simplest solution can be obtained by applying a unit current, successively to all measurement ports (see Fig. 3.5). Then

$$[\hat{\underset{\sim}{I}}^{m1} \ \ldots \ \hat{\underset{\sim}{I}}^{mn}] = \underset{\sim}{1}, \tag{3.3.15}$$

and

$$\underset{\sim mx}{Z}^{T} = [\hat{\underset{\sim}{V}}^{x1} \ \ldots \ \hat{\underset{\sim}{V}}^{xn}]. \tag{3.3.16}$$

Thus, we need n simulations of the adjoint network (with the same LU factorization) in order to obtain the coefficients of the expression (3.3.11) for all possible combinations of k elements. We apply a unit source to the measurement ports and calculate voltages across all elements of the adjoint nominal network. Taking the values corresponding to a certain combination of elements we find the corresponding matrix $\underset{\sim mx}{Z}$. In this way we obtain the matrices $\underset{\sim mx}{Z}^{j}$ $j = 1, 2, \ldots, \binom{p}{k}$ for all possible combinations.

If there are k faults within the network we can detect them by checking the expressions (3.3.11) for all possible combinations of k elements. The expression which corresponds to the elements at fault is equal to zero while the other expressions are likely to be different from zero. This enables us to indicate the suitable combination. However, the approach is limited. Some problems which may arise are discussed in the following subsection.

3.4 Interpretation

We now discuss the assumptions and the capacity of the approach presented in this chapter. In order to use it we have to formulate an appropriate set of p equations for single-faults, $\binom{p}{2}$ matrix equations for double-faults, $\binom{p}{3}$ matrix equations corresponding to three

simultaneous faults etc. This can be done by practically one simulation of the adjoint nominal network (with n different excitations). Given measured voltages we calculate the voltage changes w.r.t. nominal values and check the equations. We start with equations corresponding to single faults. If all equations except one are not satisfied we can suppose that there is a single fault in the element which corresponds to the satisfied equation. (Although a rare case, it is possible that the situation is caused by two or more faults of other elements; this can be verified by other equations.) If all equations corresponding to single faults are not satisfied we have to go further and check the equations corresponding to double faults, etc.

To be able to detect the suitable fault combination the equations (3.3.9) are required to be "independent" in a certain sense. More precisely, we do not want to face the situation when two or more equations (for the same k) are satisfied simultaneously for $\Delta\underset{\sim}{V}^m \neq \underset{\sim}{0}$. But this is not always possible. For instance, if only element Y_1 is at fault then, checking all equations for double faults, all equations corresponding to those combinations which contain Y_1 like Y_1Y_2, Y_1Y_3, ... are satisfied. In other words, it is possible that two equations of the form (3.3.9) are simultaneously satisfied for certain $\Delta\underset{\sim}{V}^m$, but generally such an implication does not exist. This is the case we are interested in. The concept of block independent equations will help us to state the problem.

Consider equation (3.3.7) in a slightly more general form

$$\underset{\sim}{A}\ \underset{\sim}{x} = \underset{\sim}{b}, \qquad (3.4.1)$$

where $\underset{\sim}{A}$ is an n x k-matrix, k < n. The rank of $\underset{\sim}{A}$ is assumed to be

$$\text{rank } \underset{\sim}{A} = k, \tag{3.4.2}$$

so the matrix $\underset{\sim}{A}^T\underset{\sim}{A}$ is nonsingular. System (3.4.1) is overdetermined. As is known, the solution of (3.4.1) exists if and only if (compare with (3.3.9))

$$\underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\ \underset{\sim}{A}^T\underset{\sim}{b} = \underset{\sim}{b}, \tag{3.4.3}$$

or

$$(\overline{\underset{\sim}{A}} - \underset{\sim}{1})\underset{\sim}{b} = \underset{\sim}{0}. \tag{3.4.4}$$

In other words, the left hand side of (3.4.4) is equal to zero if and only if the system (3.4.1) is consistent. Taking (3.3.9) and (3.3.7) into account, that is to say, the expression (3.3.11) is equal to zero if and only if there exists the solution $\underset{\sim}{I}^x$ of the system (3.3.7) for given $\Delta\underset{\sim}{V}^m$. Therefore, if we want systems (3.3.9) to be "independent" we actually do not want different systems (3.3.7) (for different combinations) to be simultaneously consistent (or inconsistent) for any $\Delta\underset{\sim}{V}^m$. Hence, we come to the following definition. Consider two overdetermined systems of equations

$$\underset{\sim}{A}_1\underset{\sim}{x}_1 = \underset{\sim}{b} \quad \text{and} \quad \underset{\sim}{A}_2\underset{\sim}{x}_2 = \underset{\sim}{b}, \tag{3.4.5}$$

and assume that n x k-matrices $\underset{\sim}{A}_1$ and $\underset{\sim}{A}_2$ are of full column rank.

Definition 3.1

Systems (3.4.5) are said to be block dependent if for any $\underset{\sim}{b}$ both are consistent or both are inconsistent.

If systems (3.4.5) are not block dependent then they are called block independent.

The conditions of Definition 3.1 are equivalent to the logical expression

$$\underset{\underset{\sim}{x}_1}{\forall}\ \underset{\underset{\sim}{x}_2}{\exists}\ \underset{\sim}{A}_1\underset{\sim}{x}_1 = \underset{\sim}{A}_2\underset{\sim}{x}_2 \quad \text{and} \quad \underset{\underset{\sim}{x}_2}{\forall}\ \underset{\underset{\sim}{x}_1}{\exists}\ \underset{\sim}{A}_1\underset{\sim}{x}_1 = \underset{\sim}{A}_2\underset{\sim}{x}_2. \tag{3.4.6}$$

Consider, for example, the second part of (3.4.6). For any $\underset{\sim}{x}_2$ this is a consistent system of equations w.r.t. $\underset{\sim}{x}_1$, so according to the previous discussion (compare with (3.4.3))

$$\underset{\sim}{\bar{A}}_1\underset{\sim}{A}_2\underset{\sim}{x}_2 = \underset{\sim}{A}_2\underset{\sim}{x}_2 \tag{3.4.7}$$

or

$$(\underset{\sim}{\bar{A}}_1 - \underset{\sim}{1})\underset{\sim}{A}_2\ \underset{\sim}{x}_2 = \underset{\sim}{0}. \tag{3.4.8}$$

Since the above equation has to be valid for any $\underset{\sim}{x}_2$ we find the condition

$$\underset{\sim}{\bar{A}}_1\underset{\sim}{A}_2 = \underset{\sim}{A}_2. \tag{3.4.9}$$

Similarly, from the first part of (3.4.6) we find

$$\bar{A}_2 A_1 = A_1. \qquad (3.4.10)$$

The conditions (3.4.9) and (3.4.10) are necessary and sufficient for the systems (3.4.5) to be block dependent. In fact, only one of the two conditions has to be checked. To show it we introduce the notion of block dependent matrices. Assuming, as before, A_1 and A_2 to be n x k-matrices of full column rank we call them block dependent matrices if (3.4.9) holds. It will be denoted by $A_1 \sim A_2$. The relation has the following properties. It is reflexive since $\bar{A}\,A = A(A^TA)^{-1}(A^TA) = A$. It is commutative because if $\bar{A}\,B = B$ then

$$\bar{B}\,A = B(B^TB)^{-1}\,B^TA = \bar{A}\,B\,[B^T\bar{A}^T\bar{A}\,B]^{-1}\,B^TA =$$

$$= A(A^TA)^{-1}A^TB\,[B^TA(A^TA)^{-1}A^TA(A^TA)^{-1}A^TB]^{-1}\,B^TA =$$

$$= A(A^TA)^{-1}(A^TB)(A^TB)^{-1}\,(A^TA)(B^TA)^{-1}\,(B^TA) =$$

$$= A.$$

In the above derivation we utilized two properties: (1) the matrix (A^TA) is symmetrical, and (2) the matrix A^TB as well as its transpose B^TA are nonsingular. The latter property follows the assumption $B = \bar{A}B = A(A^TA)^{-1}(A^TB)$ because $k = \text{rank}\,B \leq \text{rank}(A^TB) \leq k$. Thus, if one of the expressions (3.4.9) and (3.4.10) holds then the second one holds also. The relation of block independent matrices is also transitive. Assuming $A \sim B$ and $B \sim C$ we have

$$\underset{\sim}{C} = \underset{\sim}{B}(\underset{\sim}{B}^T\underset{\sim}{B})^{-1}\underset{\sim}{B}^T\underset{\sim}{C} =$$

$$= \underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\underset{\sim}{A}^T\underset{\sim}{B}\,[\underset{\sim}{B}^T\underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\underset{\sim}{A}^T\underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\underset{\sim}{A}^T\underset{\sim}{B}]^{-1}\;\underset{\sim}{B}^T\underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\underset{\sim}{A}^T\underset{\sim}{C} =$$

$$= \underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}(\underset{\sim}{A}^T\underset{\sim}{B})(\underset{\sim}{A}^T\underset{\sim}{B})^{-1}(\underset{\sim}{A}^T\underset{\sim}{A})(\underset{\sim}{B}^T\underset{\sim}{A})^{-1}(\underset{\sim}{B}^T\underset{\sim}{A})(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\underset{\sim}{A}^T\underset{\sim}{C} =$$

$$= \underset{\sim}{A}(\underset{\sim}{A}^T\underset{\sim}{A})^{-1}\underset{\sim}{A}^T\underset{\sim}{C} =$$

$$= \underset{\sim\sim}{\bar{A}}\underset{\sim}{C}.$$

Therefore, the relation of block dependent matrices is an equivalence relation. This is a generalization of the linear dependence of vectors. Similarly, the condition (3.4.9) (or (3.4.10)) is a generalization of the condition (3.2.7) (with the equality symbol). Using the condition (3.4.9) we can find out which equations of the form (3.3.9) are dependent. In other words, we can determine the combinations, whose influence on the vector $\Delta\underset{\sim}{V}^m$ is similar, i.e., based on $\Delta\underset{\sim}{V}^m$ we cannot distinguish these combinations. Then, we should change measurement tests to be able to determine which combination actually occurs.

The approach presented in this chapter is based on the assumption of the existence of the impedance matrix (Figs. 3.2 and 3.4) This assumption, however, is not essential since the impedance matrix exists for most practical networks. A more crucial assumption is the one which concerns the matrix $\underset{\sim mx}{Z}$ in (3.3.7) to be of full column rank. The assumption means that there exist exactly k linearly independent rows of $\underset{\sim mx}{Z}$. These rows correspond to those voltages which we can use to uniquely determine $\underset{\sim}{I}^x$ as well as $\underset{\sim}{V}^x$. This is simply the problem of the identification of elements ΔY^x_1, ΔY^x_2, ..., ΔY^x_k which was discussed in subsection 2.3. Under this assumption, the inverse of a full column

submatrix of $\underset{\sim}{Z}_{mx}$ exists and, as a consequence, the conditions of Theorem 2.2 are satisfied. This is seen directly from (3.3.4) since, knowing $\underset{\sim}{I}^x$ as a solution of (3.3.7), we have

$$\underset{\sim}{V}^x = \underset{\sim}{Z}_{xm}\, \underset{\sim}{I}^m + \underset{\sim}{Z}_{xx}\, \underset{\sim}{I}^x, \tag{3.4.11}$$

where $\underset{\sim}{I}^m$ is a given vector of excitations. Then, according to (2.3.11) we find the element values ΔY_1^x, ΔY_2^x, ..., ΔY_k^x.

As mentioned in subsection 2.3, the more unknown elements we want to consider the more unlikely it is to satisfy Theorem 2.2 (or Theorem 2.1). In other words, there is an upper bound of k for which we are able to construct the equation (3.3.9) and, as a consequence, to detect k simultaneous faults. If we want to consider more simultaneous faults we can use the method of identification of all elements described in subsection 2.4.

4. POSTPRODUCTION TUNING

4.1 Problem Formulation

According to [3], a design consists of the nominal vector

$$\underset{\sim}{\phi}^0 \triangleq [\phi_1^0 \ \phi_2^0 \ \dots \ \phi_p^0]^T \tag{4.1.1}$$

of the network parameters $\phi_1, \dots, \phi_p$, the tolerance matrix

$$\underset{\sim}{E} \triangleq \text{diag}(\epsilon_1 \ \epsilon_2 \ \dots \ \epsilon_p), \tag{4.1.2}$$

and the tuning matrix

$$\underset{\sim}{T} \triangleq \text{diag}(t_1 \ t_2 \ \dots \ t_p). \tag{4.1.3}$$

Usually only some, say k, elements are tunable, so the tuning matrix can be represented as

$$\underset{\sim}{T} = \text{diag}(t_1 \ \dots \ t_k \ 0 \ \dots \ 0). \tag{4.1.4}$$

An outcome of the manufacturing process implies a point

$$\underset{\sim}{\phi}^\mu \triangleq \underset{\sim}{\phi}^0 + \underset{\sim}{E} \ \underset{\sim}{\mu}, \tag{4.1.5}$$

where $\underset{\sim}{\mu} \in R_\mu$ and R_μ is usually considered as

$$R_\mu \triangleq \{\underset{\sim}{\mu} \in \mathbb{R}^p \mid -1 \leq \mu_i \leq 1, \ i = 1, 2, \dots, p\}. \tag{4.1.6}$$

An outcome $\underset{\sim}{\phi}^\mu$ meets the required specifications if it belongs to the constraint region R_c, defined as

$$R_c \triangleq \{\underset{\sim}{\phi} \mid \underset{\sim}{g}(\underset{\sim}{\phi}) \geq \underset{\sim}{0}\}, \tag{4.1.7}$$

where $\underset{\sim}{g}$ is the vector of constraint functions.

After tuning, an outcome $\underset{\sim}{\phi}^{\mu\rho}$ can be expressed by

$$\underset{\sim}{\phi}^{\mu\rho} = \underset{\sim}{\phi}^{\mu} + \underset{\sim}{T}\,\underset{\sim}{\rho}, \tag{4.1.8}$$

where $\underset{\sim}{\rho}$ is a scaling vector corresponding to the actual amount of tuning. Since the tuning range is defined by the matrix $\underset{\sim}{T}$ we consider $\underset{\sim}{\rho}$ as an element of the set

$$R_{\rho} = \{\underset{\sim}{\rho} \in \mathbb{R}^{p} \mid a_i \leq \rho_i \leq b_i,\ -1 \leq a_i \leq 0,\ 0 \leq b_i \leq 1\}. \tag{4.1.9}$$

A worst-case design centering, tolerancing and tuning algorithm [3,11,12] supplies a design such that

$$\forall_{\underset{\sim}{\mu} \in R_{\mu}} \exists_{\underset{\sim}{\rho} \in R_{\rho}} \text{ such that } \underset{\sim}{\phi}^{\mu\rho} \in R_c. \tag{4.1.10}$$

Postproduction tuning consists of two stages: (a) tuning assignment and (b) carrying out this assignment. If only one element is to be tuned then the tuning can often be carried out directly by experiment without tuning assignment. This approach, however, can hardly be used if many elements are to be tuned or if the tuning process is irreversible.

The objective of the tuning assignment problem is to find an appropriate vector $\underset{\sim}{\rho}$ such that

$$\underset{\sim}{\phi}^{a\rho} \triangleq \underset{\sim}{\phi}^{a} + \underset{\sim}{T}\,\underset{\sim}{\rho} \in R_c, \tag{4.1.11}$$

where $\underset{\sim}{\phi}^a$ is the actual parameter vector

$$\underset{\sim}{\phi}^a \triangleq [\phi_1^a \ \phi_2^a \ \cdots \ \phi_k^a \ \cdots \ \phi_p^a]^T. \tag{4.1.12}$$

The vector $\underset{\sim}{\phi}^a$ has to be identified by means of any method based on postproduction measurements.

In terms of optimal design problems the tuning assignment is quite similar to design centering. To show this let us consider the k-dimensional subspace of tunable elements $\underset{\sim}{\phi}_t$ such that

$$\underset{\sim}{\phi} \triangleq \begin{bmatrix} \underset{\sim}{\phi}_t \\ \underset{\sim}{\phi}_r \end{bmatrix}, \tag{4.1.13}$$

where

$$\underset{\sim}{\phi}_t \triangleq [\phi_1 \ \ \phi_2 \ \cdots \ \phi_k]^T$$

and

$$\underset{\sim}{\phi}_r \triangleq [\phi_{k+1} \ \phi_{k+2} \ \cdots \ \phi_p]^T.$$

Let $\underset{\sim}{\phi}_r^a$ denote the actual values of the parameters which are not tunable. Then

$$R_{ct} \triangleq R_c(\underset{\sim}{\phi}_r^a) \triangleq \{\underset{\sim}{\phi}_t \in \mathbb{R}^k \mid \underset{\sim}{g}(\underset{\sim}{\phi}) \geq \underset{\sim}{0} \text{ and } \underset{\sim}{\phi}_r = \underset{\sim}{\phi}_r^a\} \tag{4.1.14}$$

is the constraint region for the tuning assignment problem. The additional conditions which appear in (4.1.14) can be treated as additional equality constraints

$$\underset{\sim}{h}(\underset{\sim}{\phi}) = \underset{\sim}{\phi}_r - \underset{\sim}{\phi}_r^a = \underset{\sim}{0} \tag{4.1.15}$$

used explicitly in the optimization process. However, it is felt that the better approach makes use of these constraints during the analysis of the network. In other words, we consider constraint functions

$$\underset{\sim}{g}_a(\underset{\sim}{\phi}_t) \triangleq \underset{\sim}{g}(\underset{\sim}{\phi})\Big|_{\underset{\sim}{\phi}_r=\underset{\sim}{\phi}_r^a}, \tag{4.1.16}$$

where $\underset{\sim}{\phi}$ is given by (4.1.13). Then the optimization (i.e., tuning assignment) can be done directly in the subspace $\underset{\sim}{\phi}_t$ with the constraint region

$$R_{ct} = \{\underset{\sim}{\phi}_t \mid \underset{\sim}{g}_a(\underset{\sim}{\phi}_t) \geq \underset{\sim}{0}\}. \tag{4.1.17}$$

A three-dimensional interpretation of this is shown in Fig. 4.1.

Unlike optimal design centering we now have other constraints since the set R_ρ is limited, i.e., only certain amounts of tuning are available. These constraints can be represented in the form

$$\begin{aligned} &\underset{\sim}{T}_t^{-1}(\underset{\sim}{\phi}_t - \underset{\sim}{\phi}_t^a) - \underset{\sim}{a} \geq \underset{\sim}{0} \\ \text{and} \quad &\underset{\sim}{b} - \underset{\sim}{T}_t^{-1}(\underset{\sim}{\phi}_t - \underset{\sim}{\phi}_t^a) \geq 0, \end{aligned} \tag{4.1.18}$$

where $\underset{\sim}{T}_t$ consists of first k columns and rows of $\underset{\sim}{T}$ and components of the vectors a and b are given by (4.1.9) for i = 1, ..., k. We use the notation

$$\underset{\sim}{g}_\rho(\underset{\sim}{\phi}_t) \geq \underset{\sim}{0} \tag{4.1.19}$$

for the constraints (4.1.18).

A number of different approaches which have been developed for optimal design centering can now be adapted in tuning assignment. We will discuss some of them [2]. We start with minimax tuning assignment.

An optimal tuning assignment in the minimax sense can be formulated as the following constrained minimax problem

$$\underset{\underset{\sim}{\phi}_t}{\text{minimize}} \quad \max_i(-g_a^i(\underset{\sim}{\phi}_t)) \tag{4.1.20}$$

subject to (4.1.19), where $g_a^i(\underset{\sim}{\phi}_t)$ denotes the ith component of $\underset{\sim}{g}_a(\underset{\sim}{\phi}_t)$.

This is an exact approach. If we take into account an uncertainty in carrying out the tuning assignment we should consider the tolerances on the tuning. We discuss this problem in the following sections.

4.2 Fixed Tuning Tolerance Problem

Two different approaches can be taken into account. First, we deal with the absolute tuning tolerances, i.e.,

$$\underset{\sim}{\phi}_t^{a\rho} = \underset{\sim}{\phi}_t^a + \underset{\sim}{T}_t\underset{\sim}{\rho}_t^0 + \underset{\sim}{D}\delta, \tag{4.2.1}$$

where $\underset{\sim}{\rho}_t^0$ is the nominal tuning,

$$\underset{\sim}{D} = \text{diag}(d_1\ d_2\ \ldots\ d_k) \tag{4.2.2}$$

is the tuning tolerance matrix, $\underset{\sim}{\delta} \in R_\delta$ and R_δ can be considered as

$$R_\delta \triangleq \{\underset{\sim}{\delta} \in \mathbb{R}^k \mid -1 \leq \delta_i \leq 1,\ i = 1, 2, \ldots k\}. \tag{4.2.3}$$

Given the actual parameter vector $\underset{\sim}{\phi}_t^a$ and the nominal tuning assignment $\underset{\sim}{\rho}_t^0$ we see that all the outcomes fall into the box, whose size is defined by the tolerances $\underset{\sim}{D}$, as is shown in Fig. 4.2(a). This

inaccurate tuning can be treated as the accurate tuning $\underset{\sim}{\rho}^0$ but starting from an inaccurate point $\underset{\sim}{\phi}_t^{a\delta}$, described by

$$\underset{\sim}{\phi}_t^{a\delta} = \underset{\sim}{\phi}_t^{a} + \underset{\sim\sim}{D}\underset{\sim}{\delta}, \tag{4.2.4}$$

as is shown in Fig. 4.2(b).

Assuming that the appropriate set of candidates for a worst case tuning (often vertices of the tuning tolerance region) $\underset{\sim}{\delta}_1$, $\underset{\sim}{\delta}_2$, ..., $\underset{\sim}{\delta}_N$ is available then the fixed tuning tolerance problem can be solved using (4.1.20), where $\underset{\sim}{g}_a(\underset{\sim}{\phi}_t)$ is extended over all worst-case candidates $\underset{\sim}{g}_{a\delta_1}(\underset{\sim}{\phi}_t)$, $\underset{\sim}{g}_{a\delta_2}(\underset{\sim}{\phi}_t)$, ..., $\underset{\sim}{g}_{a\delta_N}(\underset{\sim}{\phi}_t)$.

The fixed absolute tuning tolerance approach seems to be a little unrealistic for small amounts of tuning, especially if some of the tunable elements do not need to be trimmed at all. Alternatively, we can consider relative tuning tolerances to be fixed. This can be expressed by

$$\underset{\sim}{\phi}_t^{a\rho} = \underset{\sim}{\phi}_t^{a} + \underset{\sim t\sim}{T}_t^{a\,0}\underset{\sim}{\rho}_t + \underset{\sim\sim\sim}{D\Delta}\underset{\sim}{\rho}_t^0, \tag{4.2.5}$$

where $\underset{\sim}{D} = \mathrm{diag}(d_1\ d_2\ \ldots\ d_k)$ is now the matrix of relative tuning tolerances and $\underset{\sim}{\Delta} \triangleq \mathrm{diag}(\underset{\sim}{\delta})$, $\underset{\sim}{\delta} \in R_\delta$, represents a random effect of tuning.

The preceding approach cannot be applied directly to this problem since the size of the box varies with the nominal tuning amount $\underset{\sim}{\rho}_t^0$. Nevertheless, assuming that the candidates for a worst case tuning $\underset{\sim}{\delta}_1$, $\underset{\sim}{\delta}_2$, ..., $\underset{\sim}{\delta}_N$ are available we can consider the matrices

$$\underset{\sim}{T}_t^a + \underset{\sim\sim}{D}\Delta_i, \quad i = 1, 2, \ldots, N, \tag{4.2.6}$$

as corresponding worst case tuning ranges. Hence, (4.1.20) extended over all worst case tuning ranges is an appropriate minimax formulation of the fixed relative tuning tolerance problem.

4.3 Variable Tuning Tolerance Problem

For better centering of the tuning assignment the variable tolerance approach can be used. This might enable us to carry out the tuning as inaccurately as possible. Different optimization techniques are utilized for the optimal design centering [2]. The one which is based on a nonlinear programming formulation minimizing a suitable cost function [3] seems to be the most useful approach to the variable tuning tolerance problem.

Now, besides the nominal tuning $\underset{\sim}{\rho}^0$ (we now omit the subscript t for the sake of simplicity) the component tuning tolerances of (4.2.2) are the variables of the problem. The suitable cost function

$$C(\underset{\sim}{\rho}^0, \underset{\sim}{d}), \tag{4.3.1}$$

where $\underset{\sim}{d} = [d_1 \; d_2 \; \ldots \; d_k]^T$ should possess the well-known properties

$$C(\underset{\sim}{\rho}^0, \underset{\sim}{d}) \to \text{constant} \quad \text{as } \underset{\sim}{d} \to \infty \tag{4.3.2}$$

and

$$C(\underset{\sim}{\rho}^0, \underset{\sim}{d}) \to \infty \quad \text{as } d_i \to 0 \tag{4.3.3}$$

which can be satisfied by the function $C_1(\underset{\sim}{\rho}^0, \underset{\sim}{d})$ of the form

$$C_1(\underset{\sim}{\rho}^0, \underset{\sim}{d}) = \sum_{i=1}^{k} \alpha_i \frac{|\rho_i^0|}{d_i}, \tag{4.3.4}$$

where α_i are constant weights.

However, for our purposes the property (4.3.3) should be modified since for $\rho_i^0 = 0$ we do not need to consider any tolerance at all, i.e., d_i should be equal to zero. Then, obviously, there is no cost of such untuned "tuning". In other words, (4.3.4) should be rewritten in the form

$$C_1(\underset{\sim}{\rho}^0, \underset{\sim}{d}) = \sum_{i\in J} \alpha_i \frac{|\rho_i^0|}{d_i}, \tag{4.3.5}$$

where $J \triangleq \{i \mid \rho_i^0 \neq 0\}$. Moreover, we would prefer to tune only as few elements as possible. This can be expressed as another property of the cost function, namely, that it is a decreasing function of an absolute value of component tuning $|\rho_i^0|$ for the relative tolerance being fixed, i.e., $\rho_i^0/d_i = \text{const}$. On the other hand, we deal with additional constraints (4.1.19). Both, constraints (4.1.19) and forcing the "zero-tuning", if possible, can be applied simultaneously by considering an additional cost function, which is actually an interior barrier function, of the form

$$C_2(\underset{\sim}{\rho}^0) = -\sum_{i=1}^{k} \beta_i \frac{(\rho_i^0)^2}{(\rho_i^0-a_i)(\rho_i^0-b_i)}, \tag{4.3.6}$$

where β_i are constant weight coefficients.

Finally, the variable tuning tolerance problem can be formulated as

$$\underset{\underset{\sim}{\rho}^0,\ \underset{\sim}{d}}{\text{minimize}}\ [C_1(\underset{\sim}{\rho}^0,\ \underset{\sim}{d}) + C_2(\underset{\sim}{\rho}^0)] \tag{4.3.7}$$

subject to

$$\underset{\sim}{\phi}_t^{a\rho} \in R_{ct} \text{ for all } \underset{\sim}{\delta} \in R_\delta, \tag{4.3.8}$$

where $\underset{\sim}{\phi}_t^{a\rho}$ is given by (4.2.1).

Any algorithm for the above problem should avoid some numerical difficulties which may arise because the cost function is not differentiable at points $\rho_i^0 = 0$, $d_i = 0$. Again, the solution of (4.3.7) can be obtained assuming that the set of worst case candidates is available.

4.4 Uncertainty of Identification

As a consequence of inaccurate measurements we should consider the uncertainty of identification of element values. In other words, we should take into account a region R_t^a of all possible points $\underset{\sim}{\phi}_t^a$ instead of a single point $\underset{\sim}{\phi}_t^a$. Since the actual parameter vector $\underset{\sim}{\phi}^a$ depends on the vector of measurements $\underset{\sim}{V} = [V_1\ V_2\ \ldots\ V_M]^T$ as

$$\underset{\sim}{\phi}^a = \underset{\sim}{\phi}^a(\underset{\sim}{V}) \tag{4.4.1}$$

and we measure the values $\underset{\sim}{V}^0 = [V_1^0\ V_2^0\ \ldots\ V_M^0]^T$ then

$$R_t^a = \{\underset{\sim}{\phi}_t^a \mid \underset{\sim}{\phi}^a \in R^a\}, \tag{4.4.2}$$

where

$$R^a = \{\underset{\sim}{\phi}^a \mid \underset{\sim}{\phi}^a = \underset{\sim}{\phi}^a(\underset{\sim}{V}^0 + \underset{\sim}{F}\ \underset{\sim}{\xi}) \text{ for all } \underset{\sim}{\xi} \in R_{\underset{\sim}{\xi}}\}, \tag{4.4.3}$$

where $\underset{\sim}{F} \triangleq \text{diag}(f_1\ f_2\ \ldots\ f_M)$ corresponds to the inaccuracy of the measurements and $R_{\underset{\sim}{\xi}} \triangleq \{\underset{\sim}{\xi} \mid -1 \leq \xi_i \leq 1\}$. The general properties of the dependence of the region R^a on the vector $\underset{\sim}{\xi}$ are still not known and should be investigated. A more serious problem, however, is associated with finding the constraint region (4.1.14). Now, it can be defined as

$$R_{ct} = \bigcap_{\underset{\sim}{\phi}_r^a \in R_r^a} R_c(\underset{\sim}{\phi}_r^a), \tag{4.4.4}$$

where $R_r^a = \{\underset{\sim}{\phi}_r^a \mid \underset{\sim}{\phi}^a \in R^a\}$.

Finding the constraint region (4.4.4) may be extremely difficult and some approximation methods might be useful.

Finally, the tuning assignment problem consists of finding the appropriate vector $\underset{\sim}{\rho}^0$ such that the whole region R_a^t would be placed within the constraint region (4.4.4). For exact tuning the situation would be like that of Fig. 4.2(b). Again, tuning tolerances can be taken into account.

5. CONCLUSIONS

A wide range of topics in the field of postproduction parameter identification, tuning and fault detection have been discussed. Methods of identification and fault detection presented here are oriented to linear analog electrical networks.

In the field of parameter identification we have discussed the methods for finding: (a) general hybrid matrix parameters, (b) values of selected elements and (c) values of all elements. The methods are based mainly on voltage measurements of the network, which is excited by current source(s). The limitations for the selected element identification have been derived and formulated in Theorems 2.1 and 2.2.

For identification of all elements a simple approach based on nodal analysis has been proposed. As a very important example we present a method for ladder networks. The method is much simpler than that of Trick and Sakla [14] and, because of a particular sparse form of the equations, we obtain explicit recurrent formulae for the solution. For arbitrary network topologies, however, there are still many open questions and unsolved problems.

Fault analysis, which can be done using methods of identification, needs its own approaches especially in the case when only a few faults occur. Methods based on the bilinear dependence of network functions on a circuit parameter have been developed for single-fault detection. A particular approach utilizing a single current excitation and measurements of two voltages has been proposed. The adjoint network simulation has been found to be a convenient way for the necessary calculations. This approach has been successfully extended in order to

deal with multiple-fault detection. However, there is a limit to the number of simultaneous faults which can be considered.

Finally, a variety of approaches to the problem of postproduction tuning have been discussed. Starting with the simplest minimax formulation based on the assumption that the identification and tuning are accurate enough we have gradually introduced more complicated problems taking tolerances of tuning and inaccurate measurements into account. Some of these problems are extremely difficult and there is a great need to develop them in the future.

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FIGURE CAPTIONS

Fig. 2.1 Two-port measurements.

Fig. 2.2 Two-port network with source and load.

Fig. 2.3 n-port network. The current excitation corresponding to the ith set of measurements is shown.

Fig. 2.4 Single element identification.

Fig. 2.5 Active n-port and its hybrid equivalent.

Fig. 2.6 Representation of active n-port of Fig. 2.5(a) as an (n+m)-port with m external excitations.

Fig. 2.7 Hybrid equivalent of a 2n-port.

Fig. 2.8 (2n+m)-port with m external excitations.

Fig. 2.9 Generalized branch.

Fig. 2.10 Ladder network.

Fig. 3.1 Geometrical interpretation of single-fault detection based on checking of equations (3.2.6). The actual values f_1 and f_2 corresponding to a change of Y_2 are also indicated.

Fig. 3.2 Representation of a network with a single fault as a 4-port network with the external load ΔY_k. The nominal value Y_k^0 is included in the 4-port. The port of excitation J and ports of measurements are also indicated.

Fig. 3.3 Adjoint network simulations giving coefficients of equation (3.2.9).

Fig. 3.4 Network with k simultaneous faults represented by (n+k)-port with n ports of measurement. The impedance matrix $\underset{\sim}{Z}$ depends only on nominal values of network elements.

Fig. 3.5 Adjoint network simulations giving coefficients of equation (3.3.9).

Fig. 4.1 Constraint region R_{ct} in the tunable element subspace.

Fig. 4.2 An illustration of the fixed tuning tolerance problem indicating (a) the tuning tolerance region and (b) corresponding "starting" box.

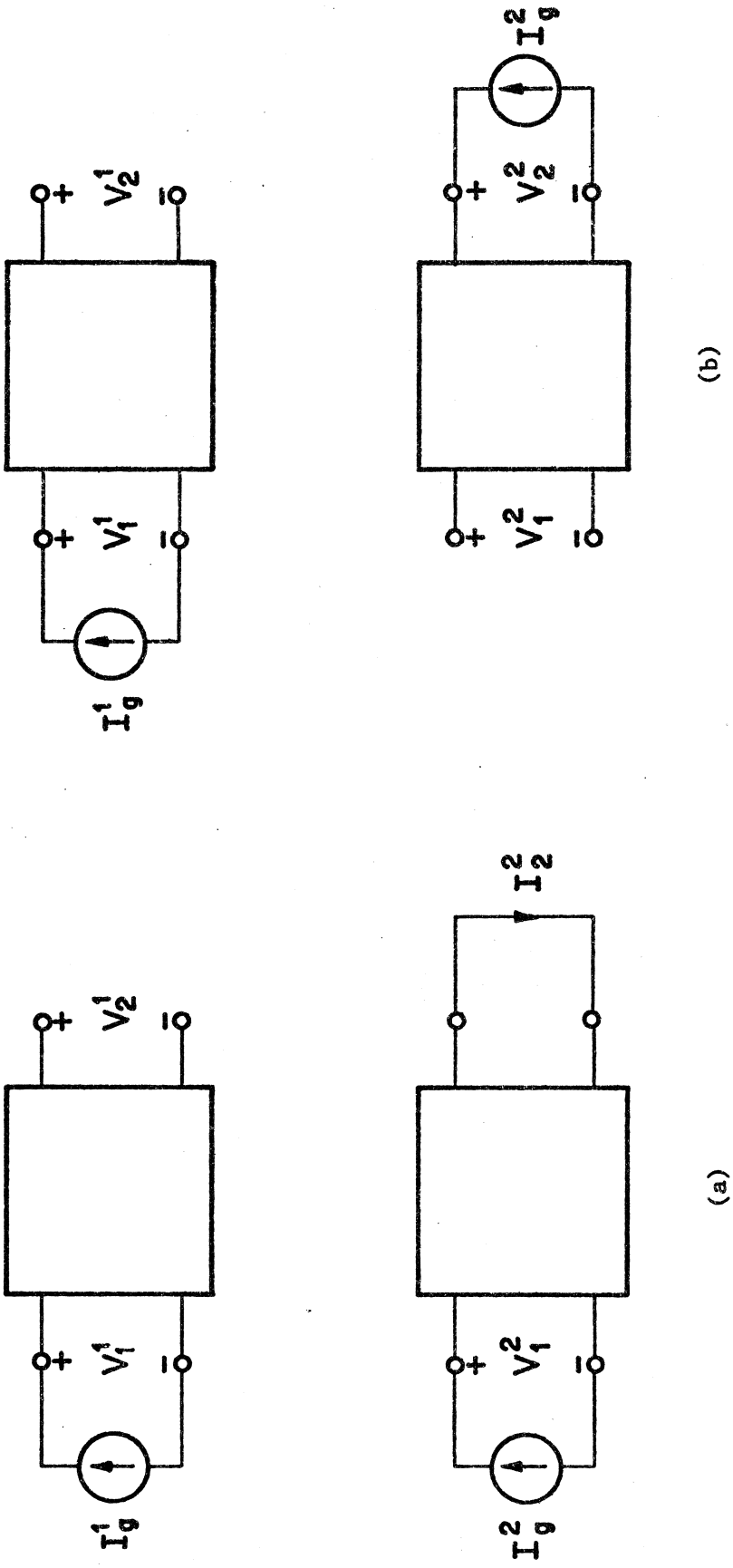
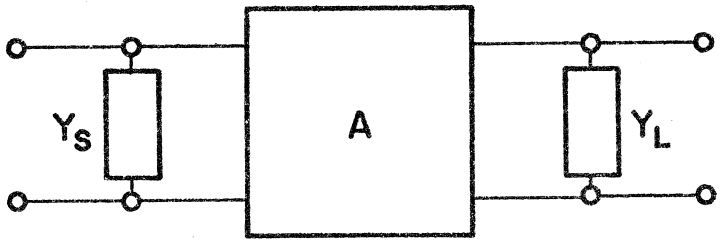


Fig. 2.1



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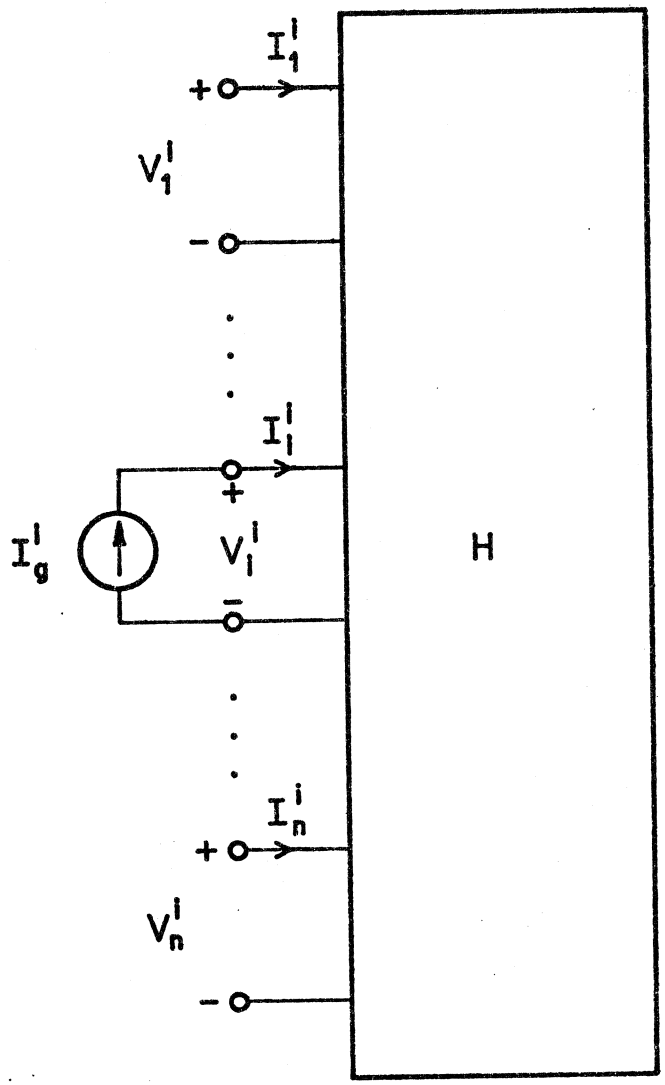


Fig. 2.3

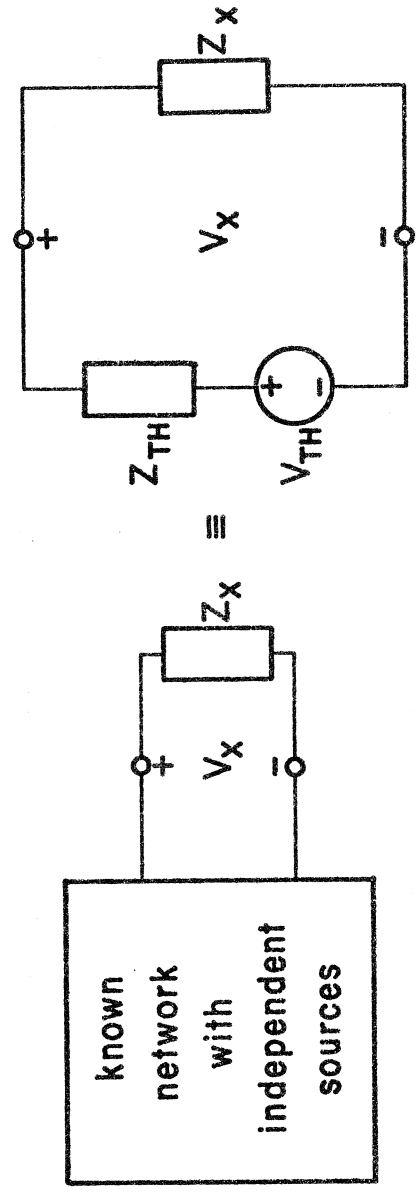


Fig. 2.4

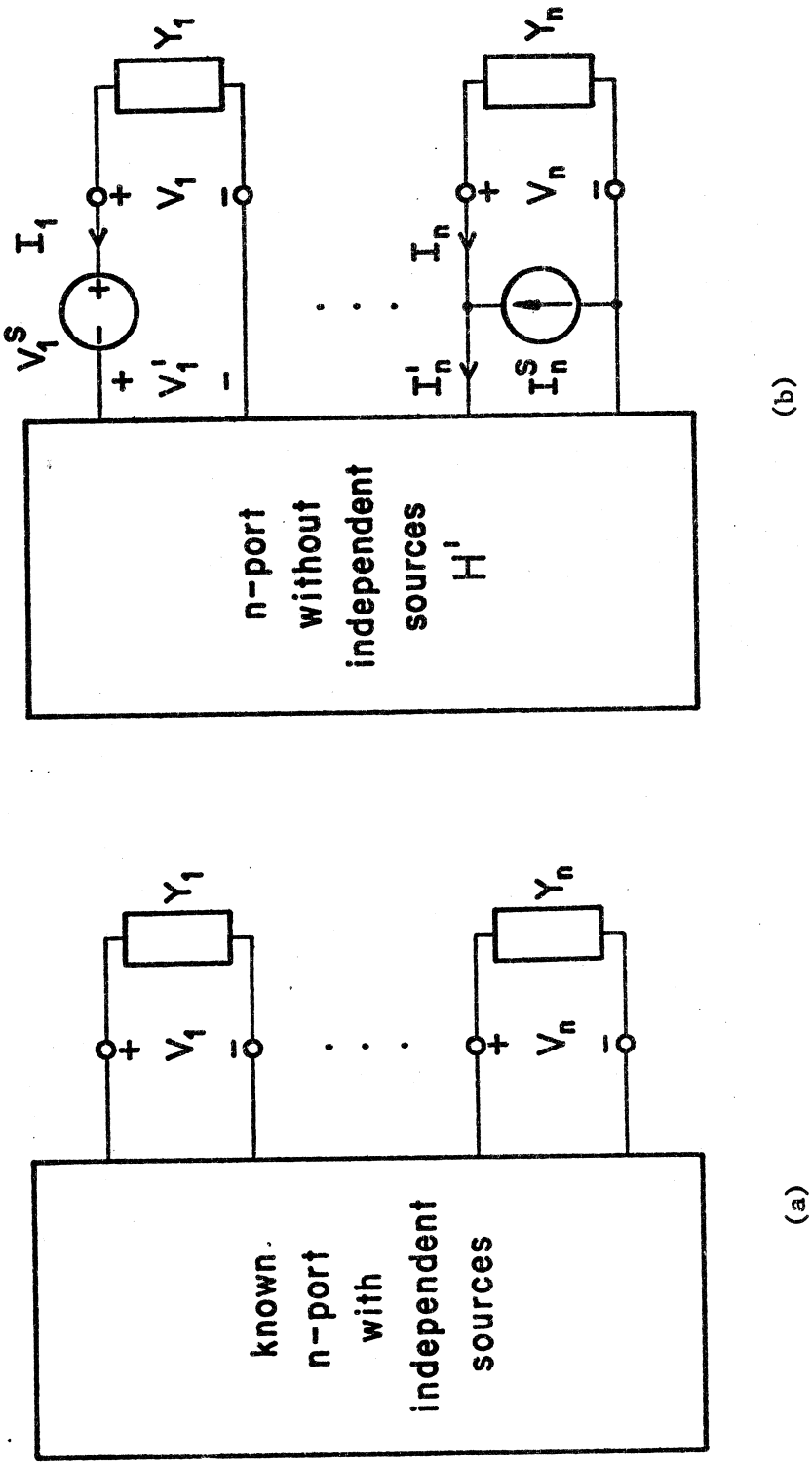

Y1
Yn
I1
V1
Vn
In
V1S
V1'
I'n
InS
n-port
without
independent
sources
H'
known
n-port
with
independent
sources
(b)
(a)

Fig. 2.5

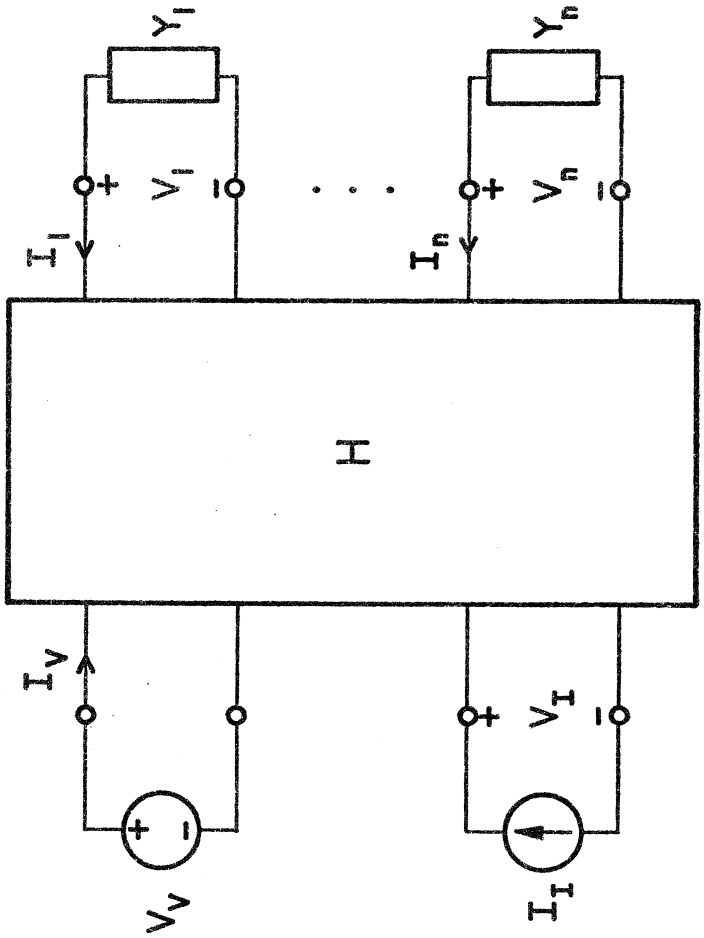


Fig. 2.6

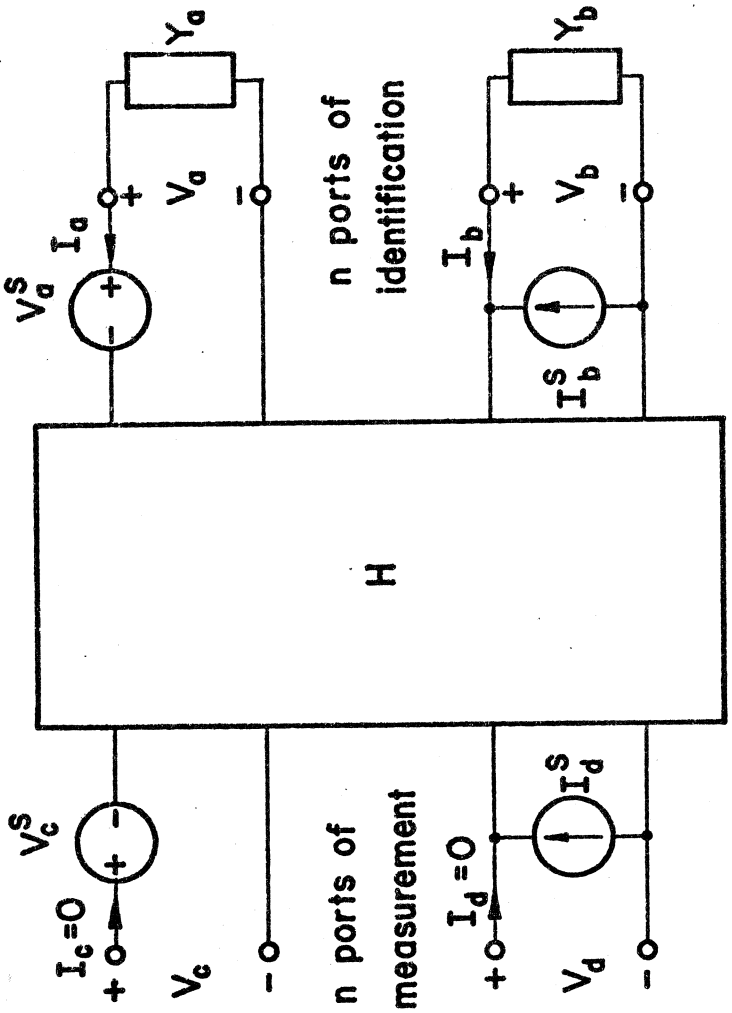

Ya
Va
+
-
Vs a
Ia
n ports of identification
Yb
Vb
+
-
Ib
Is b
H
Vs c
Ic=0
+
Vc
-
n ports of measurement
Id=0
+
Vd
-
Is d

Fig. 2.7

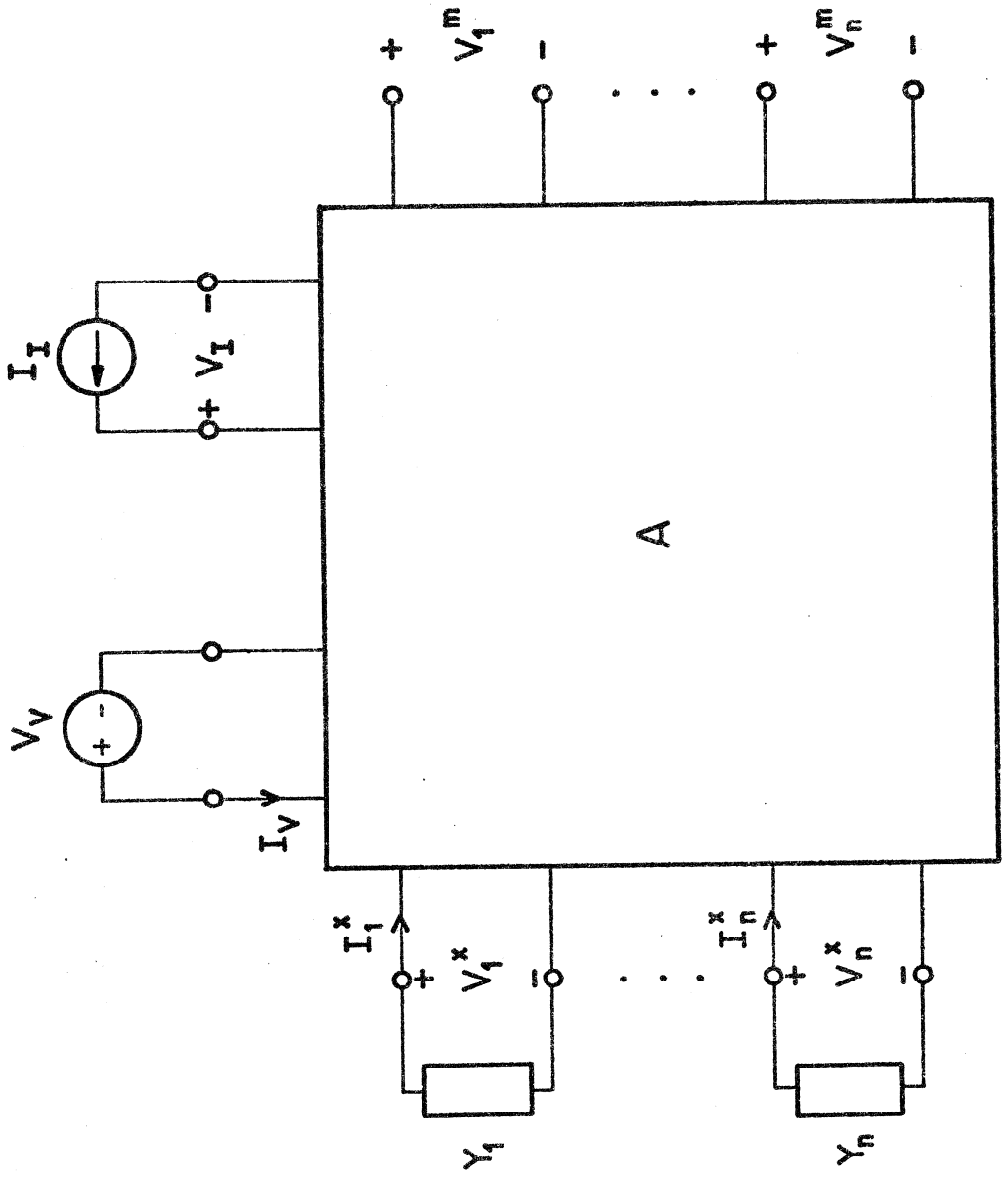


Fig. 2.8

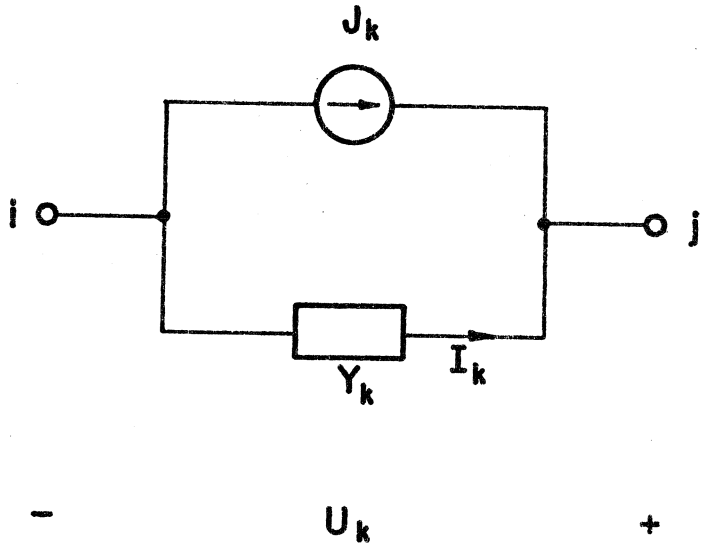


Fig. 2.9

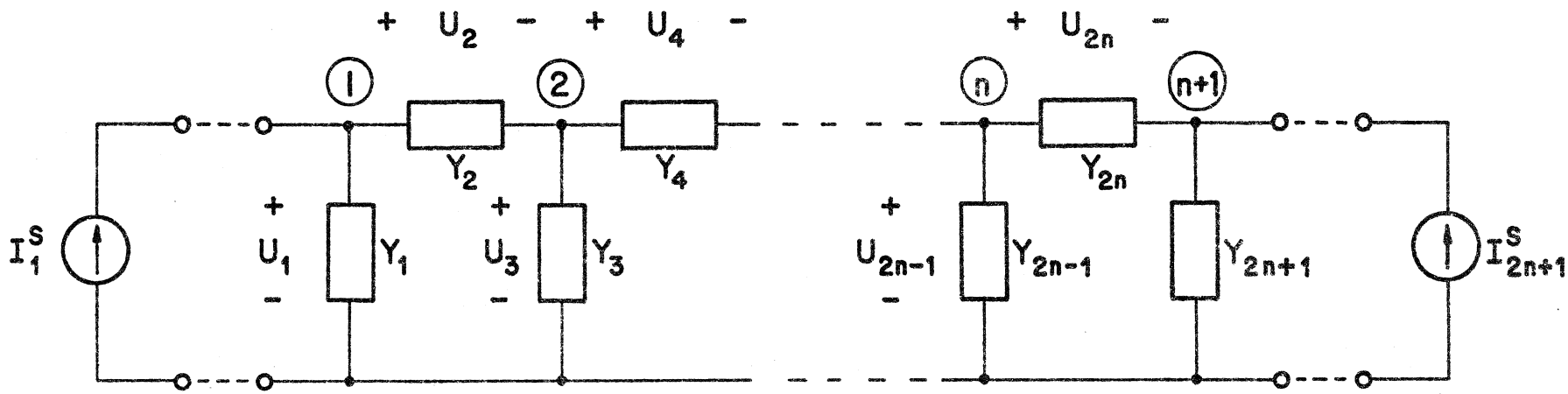
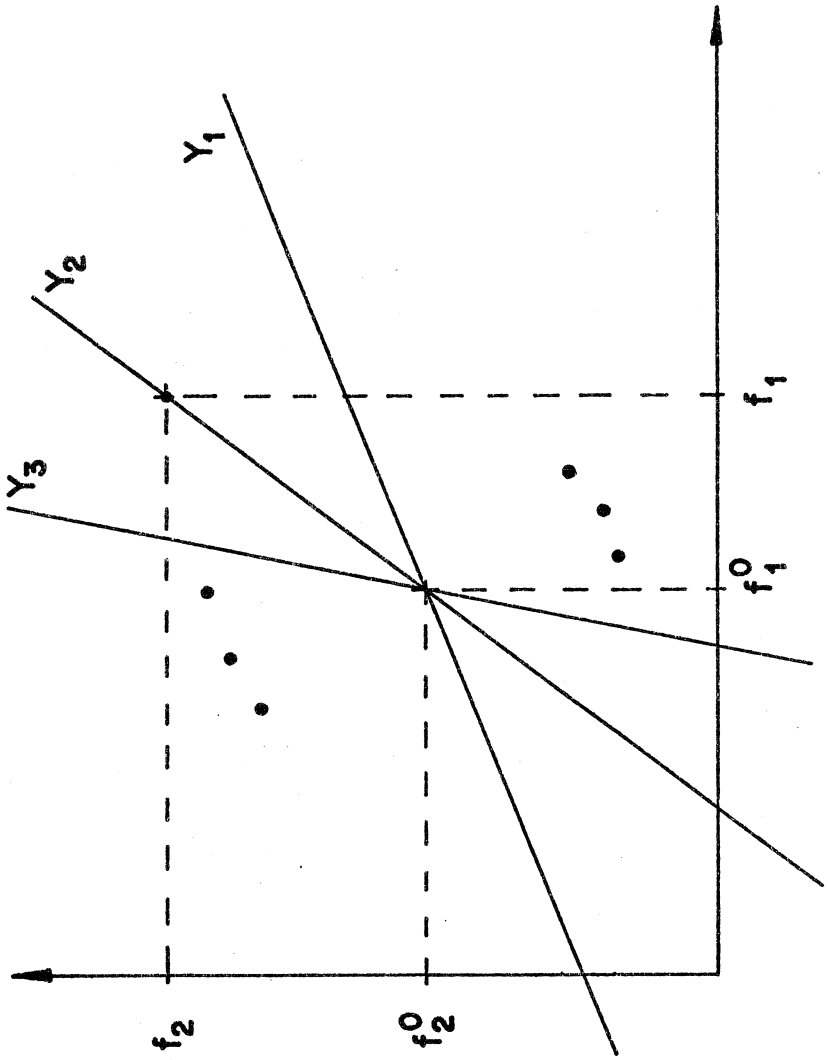


Fig. 2.10



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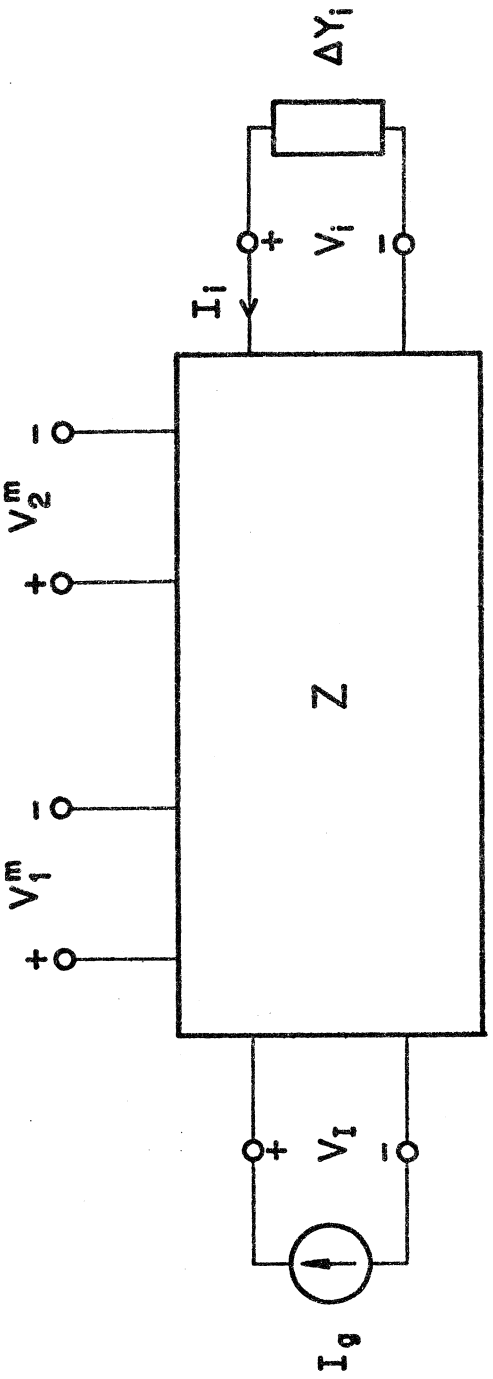
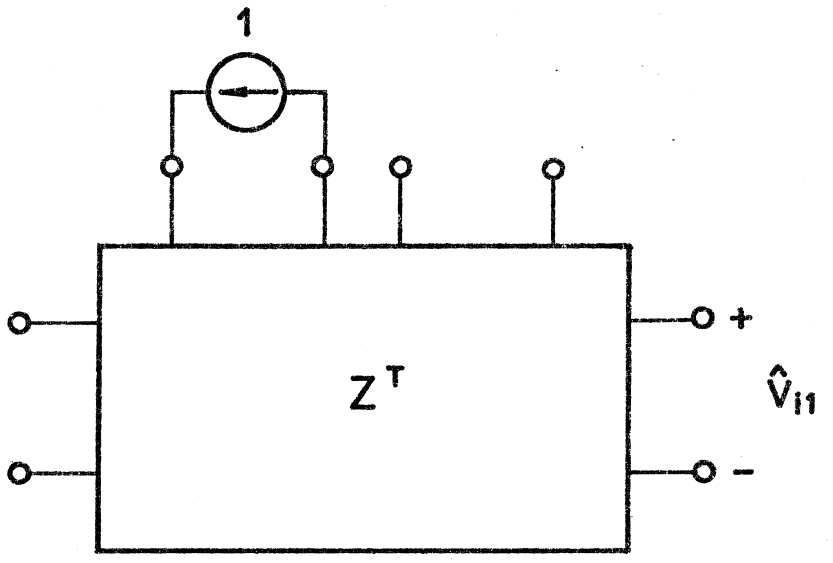
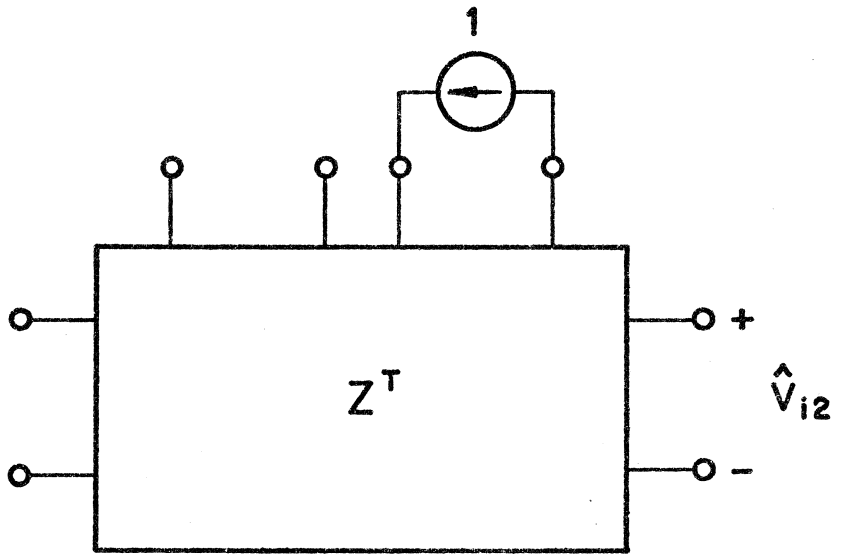


Fig. 3.2



(a)



(b)

Fig. 3.3

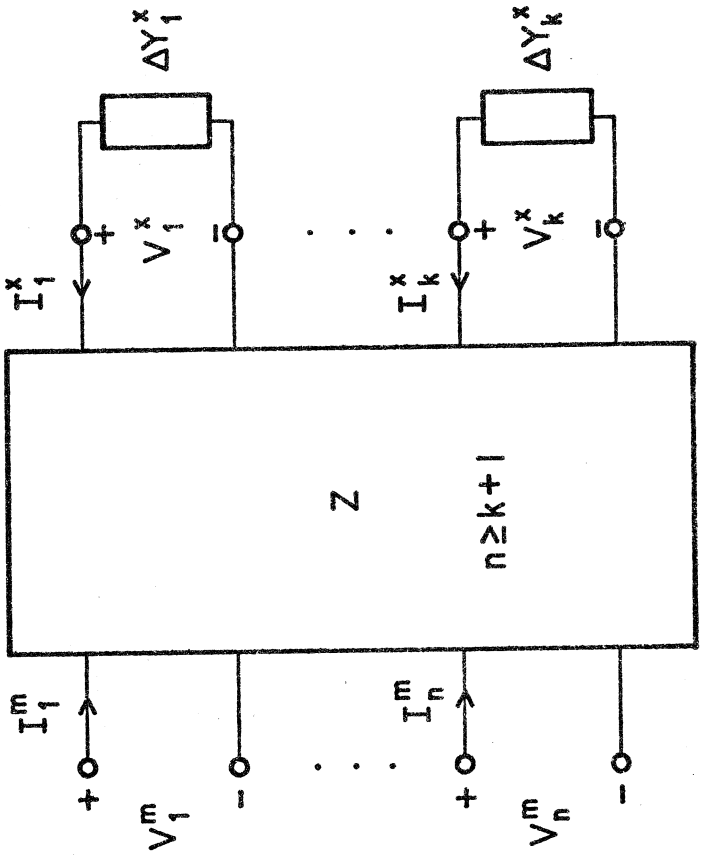


Fig. 3.4

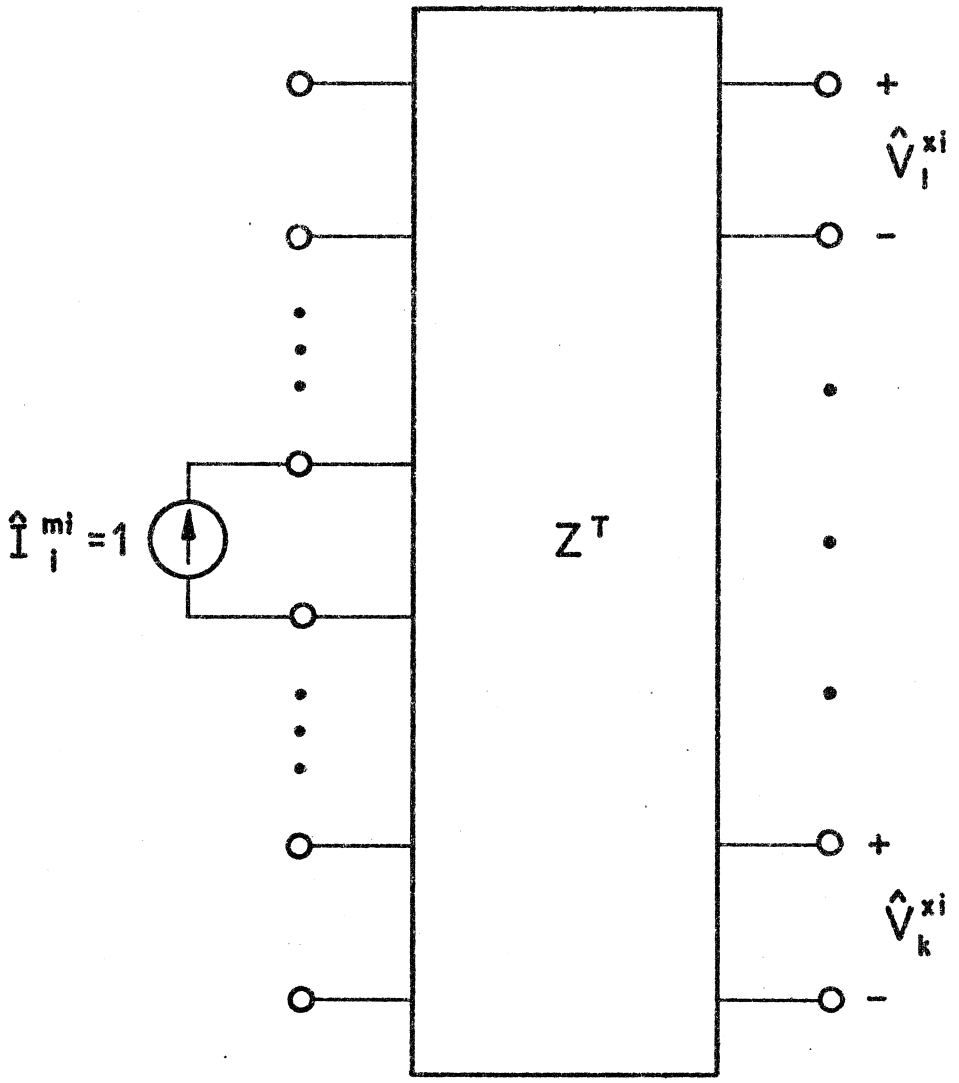


Fig. 3.5

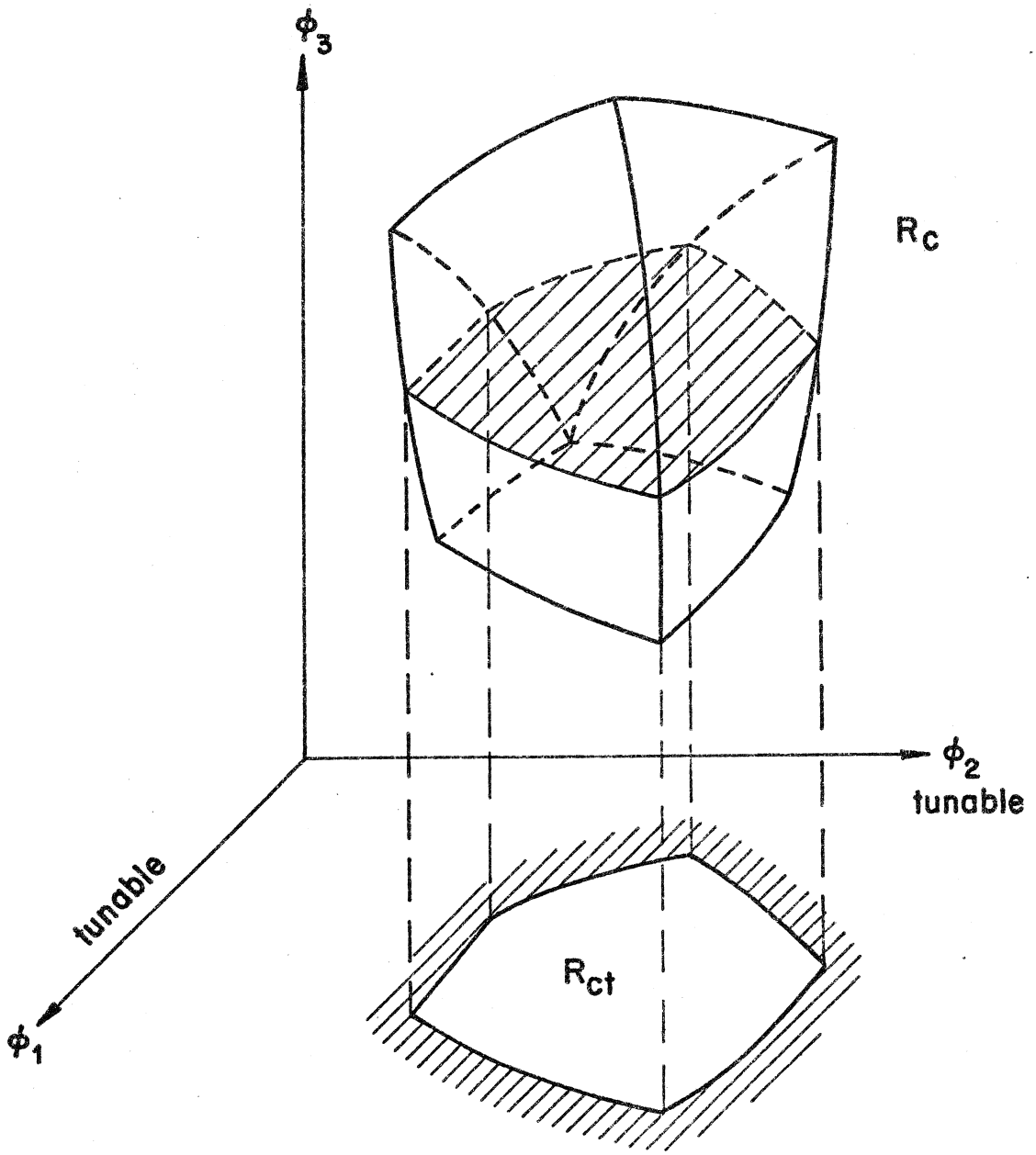


Fig. 4.1

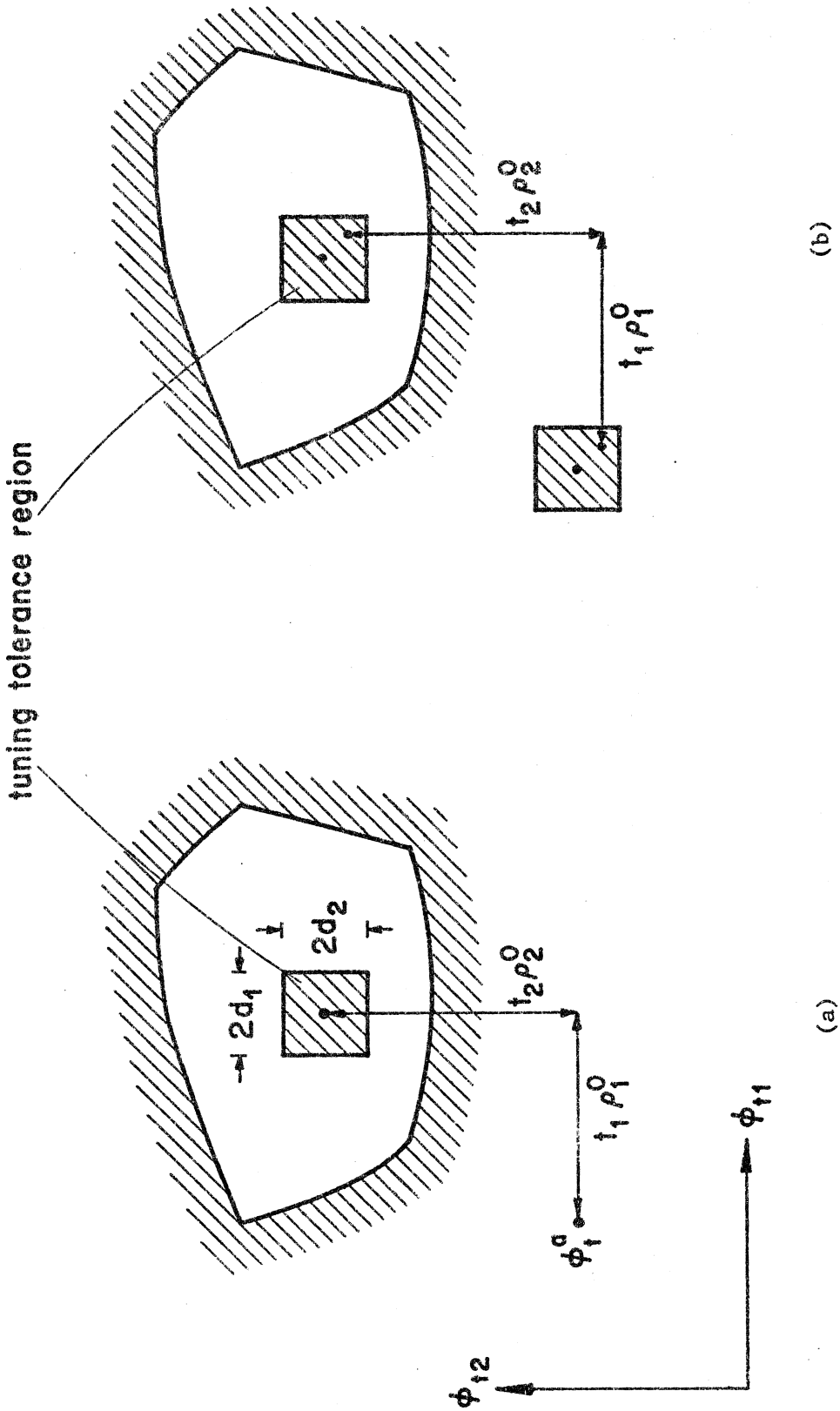

tuning tolerance region
$2d_1$
$2d_2$
$t_2\rho_2^0$
$t_1\rho_1^0$
ϕ_1^a
ϕ_{t1}
ϕ_{t2}
(a)
(b)

Fig. 4.2