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OPTIMAL ASSIGNMENT OF GENERATION TOLERANCES AND COST REDUCTION IN POWER SYSTEM EXPANSION PLANNING

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Abstract - A formulation of the generation expansion planning problem in the form of a tolerance assignment problem is presented. The general features of the approach are discussed. The principle of a combined tolerance/generation cost problem is stated. The inclusion of other aspects to achieve generality of the formulation is discussed. A new package called FLOPT5 is used to produce the numerical results.

INTRODUCTION

An important factor in the comparison of a number of alternative expansion plans is their respective capabilities in handling system uncertainties, unit outages, maintenance schedules, etc. When dealing with the generation system expansion plans, the designed values of the generating powers from the proposed stations are often subjected to uncertainties. The cost of reserve capacity and stand-by units as well as the required level of inspection and repair can be effectively reduced if the designed values of the nominal generations are well centered inside the region which defines the possible values of such generations so that no violations of the practical and natural restrictions take place. If the designed values of these generations are well centered inside this feasible region, this will be equivalent to having designed values of power generations subjected to larger associated allowable tolerances, which in turn indicates both higher reserve capacity (e.g., if the generated power from a particular station is required to be increased to cover some lack of overall generation during any contingency conditions) and lower cost of maintenance and associated inspections (e.g., allowing some unit outages while continuing to operate feasibly).

In practice, only lower bounds on the tolerances are required and other objectives have to be minimized. One of these objectives is obviously the generation cost. An overall generation cost figure for each station is used in formulating the corresponding cost criterion.

The simulation approaches have mainly been used in expansion planning [1,2]. In these approaches, a comparison between a large number of feasible expansion strategies is carried out with high computational effort involved in evaluating proposed alternatives.

Optimization techniques are also used in such problems. Quadratic programming [3], linear programming [4], and linear mixed integer programming [5] have been employed. A combination of both probabilistic simulation methods and dynamic programming has been presented [6].

In this paper, an approach for handling the combined problem is presented. A simplified system is first considered to clarify the analytical concepts. Then the solution of the pure tolerance assignment problem is presented, applicable to a simple 2-bus system where two possible objective functions are considered. The effect of weighting factors associated with the different tolerances is discussed. A special case arises in which it is possible to reach a nonunique optimum solution for the tolerance assignment problem (first problem); this may then be followed by the generation cost optimization (second problem). Finally, the general case of the combined problem is discussed and applied to a sample power system.

MAIN CONCEPTS

Consider the system in Fig. 1 which contains the 2-bus subsystem under consideration connected to the remaining system through the link W. Suppose first

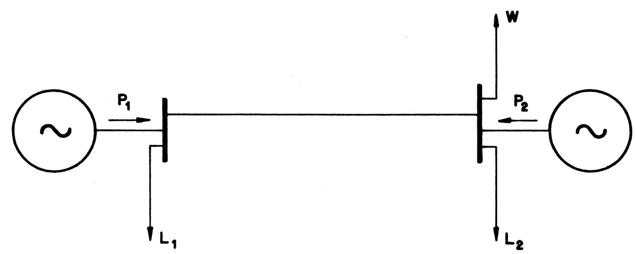

Fig. 1 Example of a 2-bus subsystem connected to the remaining system through link W.

that our goal is only to optimize the tolerances [7-12] associated with the designed nominal generated powers P_1^0 and P_2^0. In this case, our problem may be stated as follows: it is required to design the nominal powers P_1^0 and P_2^0, and the associated tolerances, ε_1 and ε_2, which minimize certain objective functions. Typically, the objective function may be in one of the following two forms:

$$C = \frac{w_1}{\varepsilon_1} + \frac{w_2}{\varepsilon_2} \tag{1}$$

or

$$C = w_1 \frac{P_1^0}{\varepsilon_1} + w_2 \frac{P_2^0}{\varepsilon_2} , \tag{2}$$

where w_1 and w_2 are weighting factors. The effect of the weighting factors is to emphasize the relative importance of each tolerance associated with the corresponding power.

The above minimization problem has to be performed subject to a set of constraints, for example,

$$g_1 = \varepsilon_1 \geq 0,\ g_2 = \varepsilon_2 \geq 0,\ g_3 = P_1 \geq 0,\ g_4 = P_2 \geq 0,$$

which ensure the nonnegativity of the actual fed powers and their respective tolerances, and

$$g_5 = \hat{P}_1 - P_1 \geq 0, \; g_6 = \hat{P}_2 - P_2 \geq 0, \; g_7 = P_1 + P_2 - L_t \geq 0,$$

which ensure that the fed powers are within the corresponding station predetermined maximum capability (constraints g_5 and g_6). The predetermined station capability represents practical limitations on the generating station layout and local conditions, the transmission part connecting this particular station with the remaining system taking into consideration the limited demand at this particular station-bus, etc. Constraint g_7 ensures sufficient generated power for the designed-area loads. In the above relations, L_t is the total load.

Additional constraints may be added according to system conditions.

In the previous formulas:

$$P_1 = P_1^0 + \epsilon_1 \mu_1, \; P_2 = P_2^0 + \epsilon_2 \mu_2.$$

In general, when dealing with k designed generated nominal powers, $\underset{\sim}{P}^0$, and their associated tolerances, $\underset{\sim}{\epsilon}$, we have [7-12]

$$\underset{\sim}{P} = \underset{\sim}{P}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu}, \tag{3}$$

where

$$\underset{\sim}{E} \triangleq \begin{bmatrix} \epsilon_1 & & & & \\ & \epsilon_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & & \epsilon_k \end{bmatrix}, \qquad \underset{\sim}{\mu} \triangleq \begin{bmatrix} \mu_1 \\ \mu_2 \\ \cdot \\ \cdot \\ \cdot \\ \mu_k \end{bmatrix}. \tag{4}$$

$\underset{\sim}{\mu} \in R_\mu$ is a random vector distributed according to a certain probability distribution. Considering the case of independent design parameters, we may define

$$R_\mu \triangleq \{\underset{\sim}{\mu} \mid -1 \leq \mu_i \leq 1, \; i = 1, 2, \ldots, k\}, \tag{5}$$

in which case the tolerance region, R_ϵ, is a convex regular polytope of k dimensions given by

$$R_\epsilon \triangleq \{\underset{\sim}{P} \mid \underset{\sim}{P} = \underset{\sim}{P}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu}, \; \underset{\sim}{\mu} \in R_\mu\}. \tag{6}$$

The extreme points of R_ϵ (which is centered at $\underset{\sim}{P}^0$) are its vertices. The number of these vertices is 2^k. They can be enumerated as follows. The rth vertex

$$\underset{\sim}{P}^r = \underset{\sim}{P}^0 + \underset{\sim}{E}\,\underset{\sim}{\mu}^r, \quad \mu_i^r \in \{-1, 1\}$$

has

$$r = 1 + \sum_{i=1}^{k} \left[\frac{\mu_i^r + 1}{2}\right] 2^{i-1} \tag{7}$$

The region of feasible points (the constraint region) is

$$R_c \triangleq \{\underset{\sim}{P} \mid \underset{\sim}{g}(\underset{\sim}{P}) \geq \underset{\sim}{0}\}, \tag{8}$$

where $\underset{\sim}{g}(\underset{\sim}{P})$ is the vector of constraints.

In worst-case design active vertices are important. Actually, only a few vertices are usually active at the solution, and those vertices, and at most a few additional ones, are usually predicted at the beginning so that the number of constraints actually used is much smaller than the maximum number possible. This number is basically at least equal to the product of the original number of constraints by the number of vertices (2^k).

Fig. 2 shows the solution of our simple example for particular values of $\hat{P}_1$, $\hat{P}_2$ and L_t. The active

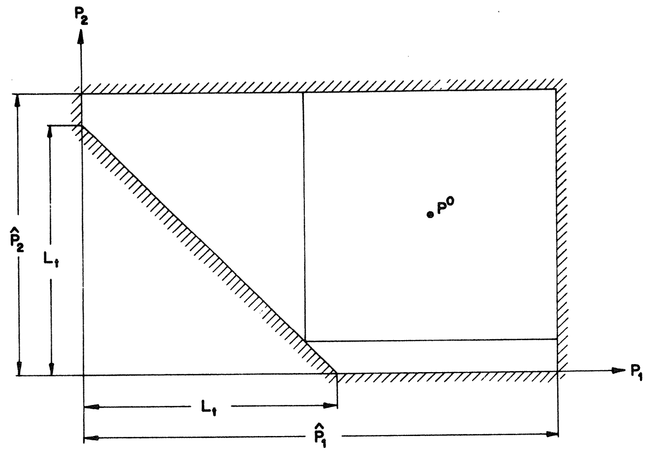

Fig. 2 Optimal solution of the 2-bus example for certain parameter values using objective (1) with uniform weighting of the tolerances.

constraints at the solution are g_5, g_6, and g_7. In this case, the solution is obtained with $w_1 = w_2$ for the absolute tolerance objective (1).

Taking all possible values of $\hat{P}_1$, $\hat{P}_2$ and L_t leading to the active constraints of Fig. 2 it can be shown that the solution for each formulation is as follows.

Absolute Tolerance Formulation (1):

$$P_1^0 = \hat{P}_1 - \lambda/2 \left(1 + \sqrt{\frac{w_2}{w_1}}\right)$$

$$P_2^0 = \hat{P}_2 - \lambda/2 \left(1 + \sqrt{\frac{w_1}{w_2}}\right)$$

$$\epsilon_1 = \lambda/2 \left(1 + \sqrt{\frac{w_2}{w_1}}\right)$$

$$\epsilon_2 = \lambda/2 \left(1 + \sqrt{\frac{w_1}{w_2}}\right)$$

where

$$\lambda = \hat{P}_1 + \hat{P}_2 - L_t$$

Relative Tolerance Formulation (2):

$$P_1^0 = \hat{P}_1 - \lambda/2 \left(1 + \sqrt{\frac{w_2}{w_1}} \sqrt{\frac{\hat{P}_2}{\hat{P}_1}}\right)$$

$$P_2^0 = \hat{P}_2 - \lambda/2 \left(1 + \sqrt{\frac{w_1}{w_2}} \sqrt{\frac{\hat{P}_1}{\hat{P}_2}}\right)$$

$$\epsilon_1 = \lambda/2 \left(1 + \sqrt{\frac{w_2}{w_1}} \sqrt{\frac{\hat{P}_2}{\hat{P}_1}}\right)$$

$$\varepsilon_2 = \lambda/2\ \left(1 + \sqrt{\frac{w_1}{w_2}}\sqrt{\frac{\hat{P}_1}{\hat{P}_2}}\right)$$

We notice that the effect of increasing the weighting factor associated with a certain tolerance is to increase the value of that tolerance, i.e., the tolerance region tends to be elongated in the corresponding parameter direction.

Also, we notice that $\hat{P}_1$ and $\hat{P}_2$ appearing in the relative tolerance formulation of the objective function act essentially as weighting factors for the respective tolerances.

For extreme values of w_1/w_2 (or alternatively, extreme values of $\hat{P}_1/\hat{P}_2$) we obtain the situation shown in Fig. 3.

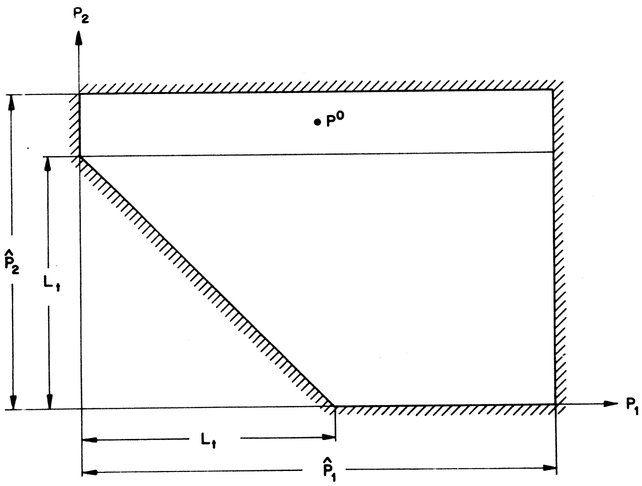

Fig. 3 Optimal solution of the 2-bus example using objective (1) with extreme nonuniform weighting $w_1 > w_2$.

GENERAL PROBLEM DESCRIPTION

We have considered basic concepts of the pure tolerance problem applied to a simplified example. In practice, however, the subsystem under planning expansion requires additional considerations. Nonlinear constraints are included as functions of generating powers involving, for example, environmental conditions (air pollution) [13,14], automatic generation control [15], and security [16].

In general, we must consider uncertainties in the generated powers and the deterministic criterion involving the generation economy, system losses, etc., combined together into one optimization problem. So, it is necessary to reformulate our problem stating our objective and describing constraints. We shall call the subsystem containing the design parameters "the working system" and the remaining system "the slack area".

The following points have to be considered.

Demand

The estimated load curve of the overall working system demand is fixed in the sense that both the maximum and the minimum load points can be regarded as fixed points. The constraints involving load limitations are formulated accordingly. For example, the aforementioned constraint g_7 involves the maximum load point.

Alternatively, the estimated demand of the working system as well as its generation schemes are subjected to cyclic conditions, e.g., availability (or lack) of some generations at certain periods of the load curve which arise from some practical and local limitations. In this case, the estimated load curve of the overall working system demand is discretized into corresponding intervals during each of which the demand is considered constant and the optimization problem is solved based on a particular demand. The result of the successive solutions of the optimization problem corresponding to the different loads (different intervals of the load curve) is the optimum settings (schedule) of the designed generating powers. In this respect, another formulation may be obtained if the nominal powers in the ith interval of the load curve associated with the jth generating station are fixed. In this case, fixed nominal values have to be designed over the whole load curve period. However, the associated tolerances, are still dependent on the load curve leading, for example, to the objective functions

$$C(\underset{\sim}{\varepsilon}^i,\ i),\ i \in I_{\ell c}$$

or

$$C(\underset{\sim}{\varepsilon}^i,\ \underset{\sim}{P}^{0c},\ i),\ i \in I_{\ell c},$$

where $\underset{\sim}{\varepsilon}^i$ represents the tolerance vector in the ith interval, $\underset{\sim}{P}^{0c}$ is the common nominal power vector and $I_{\ell c}$ is an index set corresponding to the load curve intervals.

From now on we omit the superscript i as it is understood that our optimization problem is to be solved in this paper for a specified load configuration.

Transmission Losses

The losses of the transmission system are neglected. This is a reasonable assumption at this stage of planning rather than, for example, under operating conditions when the minimum loss problem is of equal interest to the economic dispatch problem. If the transmission system of the new expanded subsystem is at least roughly of known parameters, the losses as a function of the generating powers can be taken into consideration through any of the well-known formulas (e.g., the B-coefficient approach).

Reactive Power Compensation

Reactive power compensation is assumed to be completely available so that no problems due to bus voltage fluctuations or excessive VAr of generating units will arise. In fact, reactive power planning problems [17,18] attempt to cope with such situations.

Slack Area Loads

The slack area generators have a connected capacity which deals with the slack area loads and the initial load sharing with the working system. The excess generating power of the working system has to be allowed to share in supplying the slack area loads without redundancy. This represents an additional constraint which ensures the capability of the expanded generating stations to cover the estimated loads in the working system without causing additional burden on the slack area and at the same time allows the possibility of serving the operating economic dispatch and contingency conditions in the slack area. In the 2-bus example the corresponding additional constraint is

$$g_8 = -\ (P_1 + P_2) + L_s + L_t \geq 0,$$

where L_s is the total slack area load.

Dispatching Cost at Planning Stage

Economic dispatch considerations are to some extent different from those under operating conditions in the sense that under operating conditions the operator usually deals with existing dispatch curves of the different generating stations. On the other hand, the planner deals only with rough figures of the suitable or inevitable kinds of units at a particular location. The overall dispatch curve of a particular generating station is used as a part of the generation cost at this stage. The designed powers subjected to the corresponding tolerances are preferably biased towards regions of lower dispatching costs.

COST REDUCTION VIA OPTIMAL POWER CENTERING

The previous discussion may clarify the priority of optimizing tolerances which deal with the probabilistic situations arising in practical operations. Actually, only lower bounds on the tolerances are needed. This will create a feasible region for the second optimization problem in which we aim at minimizing the total generation cost. In this case we can directly deal with one optimization problem in the form

$$\min_{\underset{\sim}{P}^0} C_F (\underset{\sim}{P}^0) \tag{9}$$

subject to

$$\underset{\sim}{P}^0 \in R_f, \tag{10}$$

where

$$R_f \triangleq \{\underset{\sim}{P}^0 | g_i(\underset{\sim}{P}^0,\underset{\sim}{\varepsilon}) \geq 0,\ \underset{\sim}{\varepsilon} \geq \underset{\sim}{\delta},\ \underset{\sim}{\delta} \geq \underset{\sim}{0},\ i \in I_c\}, \tag{11}$$

- $\underset{\sim}{\varepsilon}$ is the vector of tolerances as described before,
- $\underset{\sim}{\delta}$ is a prescribed lower bound on the tolerances $\underset{\sim}{\varepsilon}$ and
- I_c is the index set defining the constraints of the problem.

Different values of the specified lower bounds on the different generation tolerances reflect the relative importance of such tolerances.

A SPECIAL OPTIMIZATION PROBLEM

Generally, we consider the case of a k-bus working system with special attention to the 2-bus example (Appendix). Specifically, we consider that the k-bus working system contains k generating buses. However, for a working system which contains load buses, the symbol k will refer to the number of generating buses in the system.

Consider first the tolerance assignment problem. The absolute tolerance formulation is considered with objective function in the form

$$C = \sum_{i=1}^{k} \frac{w_i}{\varepsilon_i}. \tag{12}$$

Let $w_i = 1$, $i = 1, 2, \ldots, k$. The effect of weighting factors will be considered later. The main constraints for the k-bus working system are

$$g_i = \varepsilon_i \geq 0,\ g_{k+i} = P_i \geq 0,\ g_{2k+i} = \hat{P}_i - P_i \geq 0, \quad i = 1, 2, \ldots, k, \tag{13}$$

and

$$g_{3k+1} = \sum_{i=1}^{k} P_i - L_t \geq 0, \tag{14}$$

$$g_{3k+2} = -\sum_{i=1}^{k} P_i + L_s + L_t \geq 0. \tag{15}$$

These constraints are simply the general form of the 2-bus case previously described. Additional constraints arising from the practical limitations and local conditions may be formulated and appended.

Note that the constraints g_{3k+1} and g_{3k+2} are parallel, as illustrated in Fig. 4 for the 2-bus case.

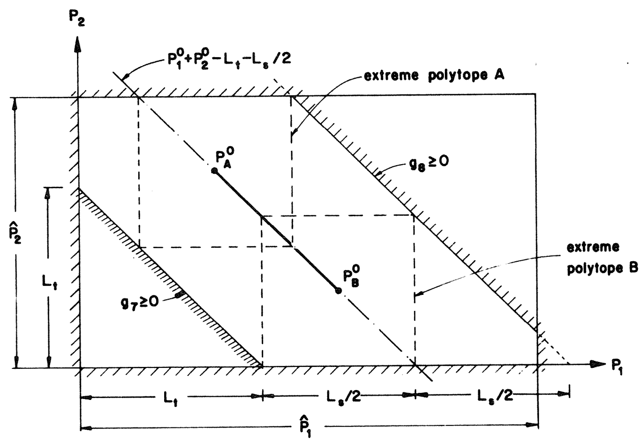

Fig. 4 Illustration of nonuniqueness of the optimal tolerance solution of the 2-bus example using uniform weighting.

Thus for the general k-bus case, and with these constraints active, it can be shown that the solution is given by

$$\varepsilon_i = \frac{L_s}{2k} \tag{16}$$

and

$$\sum_{i=1}^{k} P_i^0 = L_t + \frac{L_s}{2}. \tag{17}$$

We observe, as verified in Fig. 4, that the solution is not unique. In the general case (17) is a k-dimensional hyperplane and the essential feasible region for the second optimization problem is

$$R_f' \triangleq \{\underset{\sim}{P}^0 \mid \sum_{i=1}^{k} P_i^0 = L_t + \frac{L_s}{2},\ R_\varepsilon \subset R_c\}, \tag{18}$$

where R_ε and R_c are given by (6) and (8), respectively.

Now, we consider the second optimization problem. Suppose that the rough cost formulas of the different bus generations based on nominal powers are given by

$$C_{F_i} = A_i (P_i^0)^2 + B_i P_i^0 + C_i,\ i = 1, 2, \ldots, k, \tag{19}$$

where A_i, B_i, and C_i are cost coefficients. Then the total cost given by

$$C_F = \sum_{i=1}^{k} C_{F_i} \tag{20}$$

is required to be minimized subject to

$$\underset{\sim}{P}^0 \in R_f'. \tag{21}$$

Effect of Weighting Factors

Considering the general formula of the absolute tolerance objective (12), it can be shown that for the same previous constraints considered to be active the solution of the tolerance assignment problem is given by

$$\varepsilon_j = L_s \sqrt{w_j} \Big/ \Big(2 \sum_{i=1}^{k} \sqrt{w_i}\Big),\ j = 1, 2, \ldots, k \tag{22}$$

and

$$\sum_{i=1}^{k} P_i^0 = L_t + \frac{L_s}{2}. \tag{23}$$

Note that (23) is independent of the weighting factors and is hence identical to (17). The second optimization problem can therefore be carried out exactly as described before. However, the boundaries of the problem are altered. In the simple case of 2-bus system, the coordinates of the extreme points A and B are dependent on the ratio w_1/w_2. Fig. 5 shows a typical case in which $w_1 > w_2$.

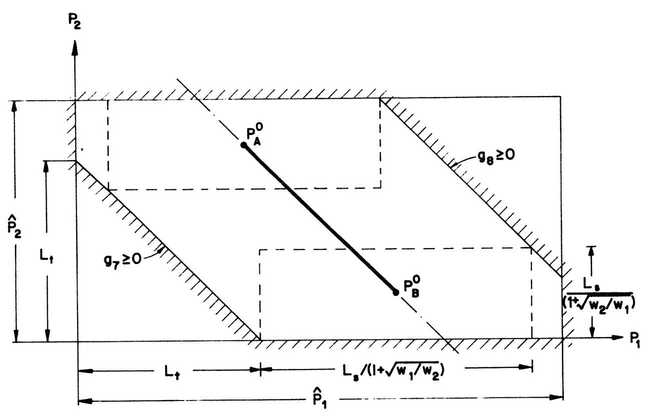

Fig. 5 Nonunique solutions of the 2-bus example for $w_1 > w_2$. The coordinates of P_A^0 and P_B^0 are, respectively, $(L_t + L_s - \hat{P}_2 - \varepsilon_1,\ \hat{P}_2 - \varepsilon_2)$ and $(L_t + \varepsilon_1,\ \varepsilon_2)$, where $2\varepsilon_1$ and $2\varepsilon_2$ are given in the figure.

3-BUS EXAMPLE

Our approach is applied to a sample power system containing a 3-bus working system and the slack area. The sample system is shown in Fig. 6. Table I represents the maximum powers and the loads expected for the expansion planning. An additional constraint is formed to ensure the possibility of supplying the loads of the working subsystem which contains buses 1 and 2 from its own generations.

The pure tolerance problem is first considered. The objective function of (12) is minimized subject to (13), (14) and (15) with the additional constraint

$$P_1 + P_2 - 1.6 \geq 0.$$

The minimization was performed using a new package called FLOPT5 [19] on a CDC 6400 digital computer. The package uses a recent highly efficient and robust least pth method due to Charalambous [20]. (It may be remarked that this paper is the first to present numerical results with FLOPT5, which supercedes all previous versions of the well-known FLOPT series available to the authors.) It gave the results shown in Table II for a number of different weighting factors. The effect of the relative size of the slack area is shown by considering two values of L_s.

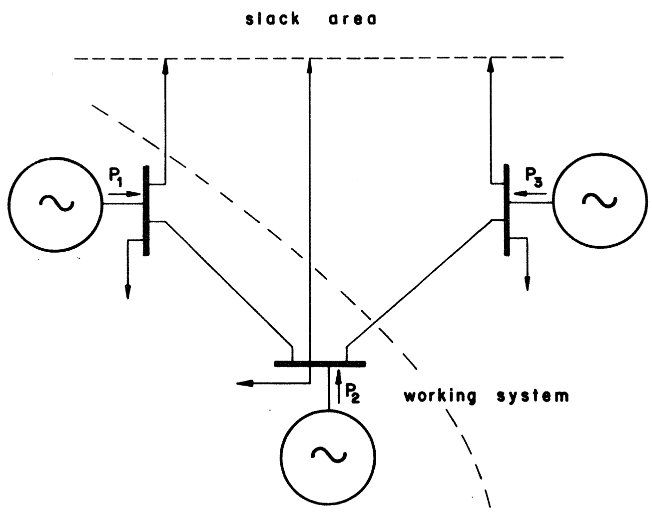

Fig. 6 Three-bus working area with 2-bus subsystem connected to the slack area.

Table I
Data of the Sample Power System

Bus Code i	Max. Generation (pu) $\hat{P}_i$	Load (pu)
1	1.7	0.7
2	1.2	0.9
3	1.4	0.85
Total load		2.45

The generation cost (9) is next minimized subject to specified lower bounds on the generation tolerances as given by (10). The additional constraint is considered again. Two generation cost formulas are used, the coefficients of which are shown in Table III. The data of Table I is also used with $L_s = 1$ pu.

Minimization was again performed using FLOPT5 giving the results shown in Table IV for different values of specified lower bounds on the tolerances. We note that all these bounds are exactly active.

CONCLUSIONS

An approach for solving the design problem in power system generation expansion planning in the form of a combined probabilistic (tolerance assignment) and generation cost problem was presented. Minimization of the tolerance objective leads to increasing the effective reserve capacity of the respective stations and to reducing the required level of maintenance and

Table II
Results for Tolerance Optimization

Parameters				Solution					
w_1	w_2	w_3	L_s	P_1^0	P_2^0	P_3^0	ϵ_1	ϵ_2	ϵ_3
1	1	1	1	1.3213	0.8229	0.8058	0.1667	0.1667	0.1667
1	1	1	10	1.3917	0.8917	1.0917	0.3083	0.3083	0.3083
2	1	1	1	1.4756	0.9020	0.5724	0.2071	0.1464	0.1464
3	2	1	1	1.4593	0.9336	0.5571	0.2089	0.1705	0.1206

Table III
Coefficients of Generation Cost Formulas

Case	i	A_i	B_i	C_i
1	1	100	400	25
	2	130	150	50
	3	240	110	20
2	1	10	170	60
	2	20	200	75
	3	25	150	55

Table IV
Results for Generation Cost Reduction

Case	Parameters			Solution			
	δ_1	δ_2	δ_3	P_1^0	P_2^0	P_3^0	Cost
1	0.05	0.05	0.05	0.5978	1.1500	0.8522	982.3
	0.10	0.10	0.10	0.7395	1.1000	0.9105	1066.9
	0.14	0.12	0.10	0.7979	1.0800	0.9321	1102.5
	0.15	0.15	0.15	0.8814	1.0500	0.9687	1157.8
2	0.05	0.05	0.05	1.6242	0.0758	0.9000	663.0
	0.10	0.10	0.10	1.6000	0.2000	0.9500	693.5
	0.14	0.12	0.10	1.5600	0.3000	0.9500	706.4
	0.15	0.15	0.15	1.5500	0.3500	1.0000	725.0

repair. The concept of dividing the whole interconnected power system into a working system and a slack area was utilized. Under certain conditions we showed that the nonunique optimal solution of the pure tolerance problem permitted subsequent optimization of generation cost with fixed load sharing between the working system and the slack area, without degradation of the tolerance solution.

APPENDIX

Consider the 2-bus example. To solve this problem analytically, we take the region R_μ as the infinite set of discrete points $\underset{\sim}{\mu}(i)$, i = 1, 2, 3, ..., where

$$-1 \leq \mu_1(i) \leq 1,$$
$$-1 \leq \mu_2(i) \leq 1,$$

The optimality conditions for objective function (1) lead to

$$\begin{bmatrix} -\frac{w_1}{\varepsilon_1^2} \\ -\frac{w_2}{\varepsilon_2^2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ 0 \\ 0 \end{bmatrix} + \sum_i u_3(i) \begin{bmatrix} \mu_1(i) \\ 0 \\ 1 \\ 0 \end{bmatrix} + \sum_i u_4(i) \begin{bmatrix} 0 \\ \mu_2(i) \\ 0 \\ 1 \end{bmatrix} +$$

$$\sum_i u_5(i) \begin{bmatrix} -\mu_1(i) \\ 0 \\ -1 \\ 0 \end{bmatrix} + \sum_i u_6(i) \begin{bmatrix} 0 \\ -\mu_2(i) \\ 0 \\ -1 \end{bmatrix} + \sum_i u_7(i) \begin{bmatrix} \mu_1(i) \\ \mu_2(i) \\ 1 \\ 1 \end{bmatrix} +$$

$$+ \sum_i u_8(i) \begin{bmatrix} -\mu_1(i) \\ -\mu_2(i) \\ -1 \\ -1 \end{bmatrix}$$

and

$$u_1 g_1 = u_2 g_2 = u_3(i) g_3(i) = \dots = u_8(i) g_8(i) = 0, \quad i = 1, 2, \dots$$

where u_1, u_2, $u_3(i)$, ..., $u_8(i) \geq 0$ are the multipliers.

As shown in Fig. 4, based on the active vertices indicated, we have

$$u_1 = u_2 = u_3(i) = u_4(i) = u_5(i) = u_6(i) = 0, \quad i = 1, 2, \dots$$

and the minimization of the constraints $g_7(i)$ and $g_8(i)$ w.r.t. $\underset{\sim}{\mu}(i)$ lead to

$$u_7(1) \neq 0, \; \mu_1(1) = -1, \; \mu_2(1) = -1$$

$$u_7(j) = 0, \; j = 2, 3, \dots$$

also

$$u_8(2) \neq 0, \; \mu_1(2) = 1, \text{ and } \mu_2(2) = 1$$

$$u_8(j) = 0, \; j = 1, 3, 4, \dots$$

Finally we get a set of equations the solution of which is shown in Fig. 4.

Suppose that the cost formulas for the two generating stations based on nominal powers are

$C_{F_1} = A_1 \, (P_1^0)^2 + B_1 \, P_1^0 + C_1$ for the first station, and

$C_{F_2} = A_2 \, (P_2^0)^2 + B_2 \, P_2^0 + C_2$ for the second station.

The total cost given by

$$C_F = C_{F_1} + C_{F_2}$$

is required to be minimized subject to the constraints which define the feasible portion AB of Fig. 4, that is the equality constraint

$$h(P_1^0, P_2^0) = P_1^0 + P_2^0 - L_t - \frac{L_s}{2} = 0$$

and the inequality constraints, namely, either $P_1^0 \geq P_{1A}$, and $P_1^0 \leq P_{1B}$, or $P_1^0 \leq P_{2B}$, and $P_2^0 \geq P_{2A}$.

The solution of this optimization problem can be easily obtained as follows. Substituting

$$P_2^0 = L_t + \frac{L_s}{2} - P_1^0 = \alpha - P_1^0,$$

where

$$\alpha = L_t + \frac{L_s}{2}$$

we get

$$C_F = a \, (P_1^0)^2 + b \, P_1^0 + c,$$

where

$$a = A_1 + A_2,$$

$$b = B_1 - 2\alpha A_2 - B_2,$$

$$c = C_1 + A_2 \alpha^2 + B_2 \alpha + C_2,$$

to be minimized subject to the first set of inequality constraints. Normally, both a and c are positive. Depending on the state of b we have the following two cases.

<u>Case of b ≥ 0</u>

Here, the optimal solution is at point A and is given by

$$\underset{\sim}{P}_A^0 = \begin{bmatrix} L_t - \hat{P}_2 + \frac{3}{4} L_s \\ \hat{P}_2 - \frac{1}{4} L_s \end{bmatrix},$$

and

$$\varepsilon_1 = \varepsilon_2 = \frac{L_s}{4} .$$

<u>Case of b < 0</u>

Here, we have three possible cases for $\check{P}_1^0 = - b/2a$.

Case of $P_{1A} \leq \check{P}_1^0 \leq P_{1B}$

The optimal solution is

$$\underset{\sim}{P}^0 = \begin{bmatrix} - \frac{[B_1 - 2\alpha A_2 - B_2]}{2[A_1 + A_2]} \\ L_t + \frac{L_s}{2} + \frac{[B_1 - 2\alpha A_2 - B_2]}{2[A_1 + A_2]} \end{bmatrix}$$

and

$$\varepsilon_1 = \varepsilon_2 = \frac{L_s}{4} .$$

Case of $P_{1A} \geq \check{P}_1^0$

The solution corresponds to the case of b ≥ 0.

Case of $\check{P}_1^0 \geq P_{1B}$

The optimal solution is at point B and is given by

$$\underset{\sim}{P}_B^0 = \begin{bmatrix} L_t + \frac{L_s}{4} \\ \\ \frac{L_s}{4} \end{bmatrix},$$

and

$$\epsilon_1 = \epsilon_2 = \frac{L_s}{4} .$$

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SOC-219

OPTIMAL ASSIGNMENT OF GENERATION TOLERANCES AND COST REDUCTION IN POWER SYSTEM EXPANSION PLANNING

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Abstract: A formulation of the generation expansion planning problem in the form of a tolerance assignment problem is presented. The general features of the approach are discussed. The principle of a combined tolerance/generation cost problem is stated. The inclusion of other aspects to achieve generality of the formulation is discussed. A new package called FLOPT5 is used to produce the numerical results.

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