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FLOPT5 - A PROGRAM FOR MINIMAX OPTIMIZATION USING THE ACCELERATED LEAST PTH ALGORITHM

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Abstract

FLOPT5 is a package of subroutines primarily for solving least pth optimization problems. Its main features include Fletcher's quasi-Newton subroutine, a least pth objective formulation subroutine and the recent Charalambous least pth algorithm designed specifically for minimax problems. With appropriate utilization of these features, the program can solve a wide variety of optimization problems. These may range from unconstrained problems, problems subject to inequality or equality constraints to nonlinear minimax approximation problems. The program has been developed on a CDC 6400 computer. Some detailed examples of varying complexity are used to illustrate the versatility of the program. A FORTRAN IV listing is included. FLOPT5 may be regarded, from the user's point of view, as an upgraded FLOPT4, a previous package. Some results of performance comparison with FLOPT4 are also included.

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ACKNOWLEDGEMENTS

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The authors must also thank M.A. El-Kady and R.M. Biernacki for using a preliminary version of the program and making many helpful observations.

I. INTRODUCTION

FLOPT5 is a package of subroutines for solving least pth and minimax optimization problems. Its main features include a modification of the 1972 version of Fletcher's quasi-Newton subroutine [1], a least pth objective formulation subroutine and the recent Charalambous least pth algorithm with acceleration specifically for minimax problems [2]. With appropriate utilization of these features, the program can solve a wide variety of optimization problems. These may range from unconstrained problems, problems subject to inequality/equality constraints to minimax problems in general.

For solving constrained problems, the user may employ the Fiacco-McCormick method [3] or the Bandler-Charalambous minimax formulation [4] and least pth approximation.

The program FLOPT5 is an upgraded version of the program FLOPT4 developed by the authors [5]. Section IV deals with the specific improvements made in this program. The program has been developed on a CDC 6400 computer (with the NOS operating system) and is written in FORTRAN IV. Some examples of varying complexity have been included in this report to illustrate the versatility of this program. FLOPT5, in comparison with FLOPT4, should prove to be more convenient to use, faster in execution with improved printed output and a more informative gradient verification utility option.

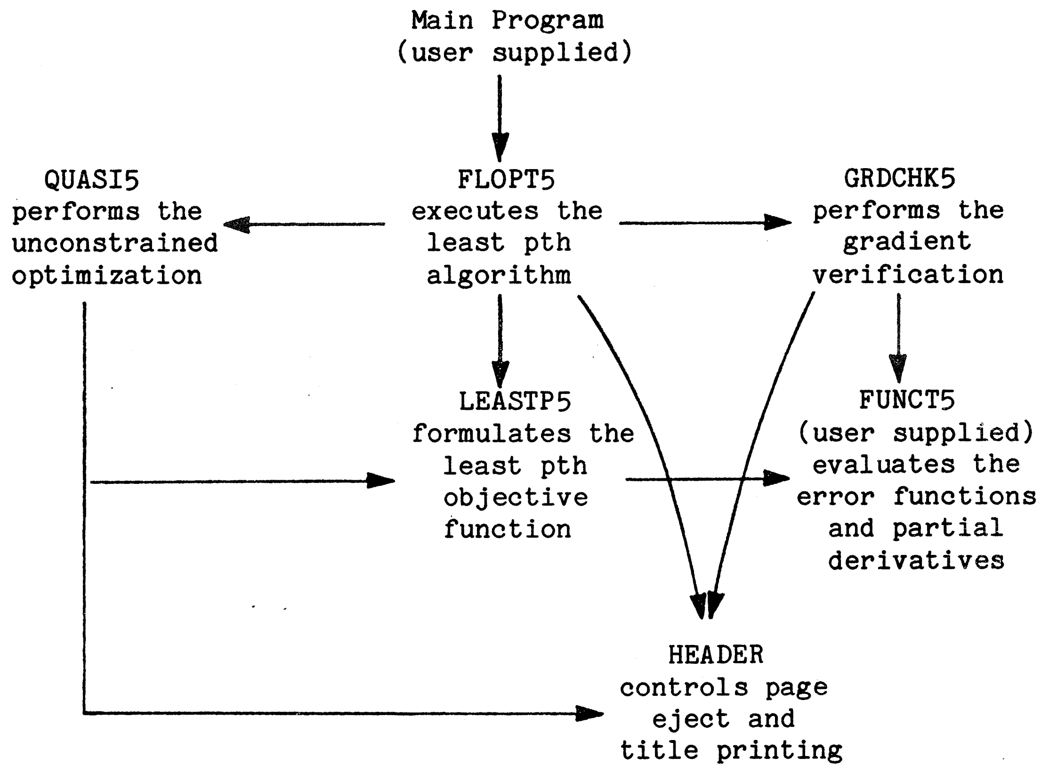

Fig. 1 Overall organization of the subroutines

Figure 1, with arrows emanating from calling subprograms and leading to called subroutines, highlights the overall organization of the program units.

In order to use the FLOPT5 package, the user has to provide the main program and a subroutine called FUNCT5. A discussion of the various subroutines, the variables, common blocks, etc., in the ensuing sections and some completely worked out examples will familiarize the user with the details necessary to use this package successfully.

II. VARIABLES

This section describes all the variables that could be of interest to the user. The essential information regarding the dimensions, initialization and default values is also provided in Table I in a condensed form. In addition, a more comprehensive explanation is included as comments in the program listing.

Logical Variables

FINISH — Set to TRUE by FLOPT5 when X, the solution point, has converged within EPS. It may be used in the main program as the termination criterion.

GRDCHK — When it is TRUE, the formulas for the partial derivatives supplied by the user in subroutine FUNCT5 are verified numerically by FLOPT5. Optimization is not performed. Hence, once the user is satisfied as to the accuracy of subroutine FUNCT5, GRDCHK should be initialized to FALSE in the main program. Gradient verification and optimization can never be performed simultaneously in the same call to FLOPT5. After the gradient verification, the control returns to the main program and it is up to the user to either stop the program execution or set GRDCHK to FALSE and then proceed with the optimization.

TABLE I ESSENTIAL INFORMATION ON DIMENSIONS, INITIALIZATION AND DEFAULT VALUES

VARIABLE NAME	INITIALIZED BY USER (1)	DIMENSIONS IN MAIN PROGRAM	DIMENSIONS IN FUNCT5	DEFAULT VALUE (2)
EPS	YES	N	-	-
ER	-	-	NR	-
EST	YES	-	-	0.
ETA	YES	-	-	0.
FACTOR	YES	-	-	10.
GRDCHK	YES	-	-	TRUE
G (or GE)	-	-	(N,NR)	-
IH	YES	-	-	1
IK	YES	-	-	1
IPT	YES	-	-	20
JD	-	NR	NR	-
MAX	YES	-	-	200
MXLINES	-	-	-	60
NR	YES	-	-	1
P	YES	-	-	10.
PRINTID	YES (3)	-	-	TRUE
X	YES	$N^2+7N+(N+4)NR+1$	N	-

(1) Variables may be initialized by any means other than a data statement in the main program.
(2) The user may take advantage of the default values to avoid initializing some variables.
(3) The only data card read by FLOPT5 may contain up to eighty characters. This string of characters appears on every page of the output and serves to identify the computer output. This card is not required, however, if the user is reading data before and/or after calls to FLOPT5 he must be prepared to include at least a blank card.

PRINTID A section of the printed output shows the values of many data items. This provides to the user a means of verifying his or her input to the program as well as the default values in effect. This printing can be turned off by initializing PRINTID to FALSE in the main program.

Integer Variables

IH Used as the index of a DO loop in the main program that calls FLOPT5 IK times.

IK The number of times FLOPT5 is called from the main program. Each call of FLOPT5 represents an iteration of the Charalambous algorithm.

IPT The results of the unconstrained minimization are printed for the first and last iterations of QUASI5 as well as after every IPT iterations within QUASI5. IPT=0 suppresses the entire printout.

JD An array identifying the active functions. A function with multiplier value larger than ETA is said to be active. Once a function becomes inactive, i.e. its multiplier drops below ETA, it is ignored in all subsequent optimizations by FLOPT5. Since the multipliers never assume negative values, by choosing ETA to be some negative value, the user could ensure that no functions ever become inactive.

MAX The maximum permissible number of function evaluations.

MODE For MODE = 1 an identity matrix is the initial estimate of the Hessian in Subroutine QUASI5. For MODE = 3 the initial estimate of the Hessian is a matrix which is in factored form generated by the last call to QUASI5. The user should be aware that MODE = 1 only for the first call to FLOPT5.

MXLINES The maximum permissible number of printed lines per page. This allows the user to alter the page length to fit a report.

N The number of variables in the problem. N must be greater than or equal to 2.

NA The number of active functions at any time. See also JD for further explanation.

NR The number of error functions in the problem.

Real Variables

EM Maximum of the error functions at a given point X. EM = maximum(ER(1), ..., ER(NR)).

EN A real array storing the normalized error function values. The relationship is: EN(I) = ER(I)/EM.

EPS A real array of N elements storing the "small" positive numbers which are used for testing the convergence of the minimax solution in FLOPT5 and the unconstrained solution in QUASI5.

ER A real array of NR elements storing the values of the error functions at a given point X.

ETA The parameter to select the active functions. See also JD for explanation.

FACTOR Multiplies P to update its value for a subsequent Charalambous iteration.

G or GE A real array of (N,NR) elements storing the partial derivatives of the error functions with respect to X(1), ..., X(N) at point X.

GU A real array of N elements storing the partial derivatives of U with respect to X(1), ..., X(N) at point X.

P The parameter of least pth approximation.

U The unconstrained least pth objective function at point X.

V A real array of NA elements storing the multipliers corresponding to the active functions.

X A real array of $N^2 + 7N + (N+4)NR+1$ elements. The first N elements store the starting point supplied by the user and at other times, the solution point. The remaining part of this array is used as scratch space by FLOPT5.

III. HOW TO USE THE PROGRAM

In order to solve a problem which can be formulated as a minimax problem, the user has to (i) prepare the main program, (ii) prepare subroutine FUNCT5 and (iii) provide a problem identification card.

The main program contains the necessary initialization and calls to subroutine FLOPT5. The number of times FLOPT5 is called depends on the number of desired iterations. For example, for a problem of unconstrained minimization involving only one function, it will be enough to call FLOPT5 once but for a minimax problem, generally, many iterations will be needed depending on the desired accuracy. FINISH may be used to exit the loop of iterations. Some COMMON cards should also be included in the main program in order to pass the initialized values to other subroutines. Arrays EPS, JD and X should be suitably dimensioned. Table I should be consulted for the purpose of dimensioning and initialization. Table II contains a comprehensive list of the COMMON blocks and should be consulted to select the relevant COMMON blocks for inclusion in the main program.

Subroutine FUNCT5 contains the user's definition of the problem to be solved. The input variables are NA, JD and X. The output variables are ER and G. It is rather easy, for any user, to make a mistake in the corresponding formulas. This program provides a means for discovering any possible inconsistencies. When GRDCHK is set TRUE in the main program, partial derivatives of the error functions are numerically approximated, based on the ER vector and compared with the user supplied values in array G. Both sets of values of the derivatives are printed

TABLE II AVAILABLE COMMON BLOCKS IN THE FLOPT5 PACKAGE

```
COMMON/FLOPT5A/FTIME,TTIME

COMMON/FLOPT5B/NLINES

COMMON/FLOPT5P/EM,EST,ETA,JV,MAX

COMMON/FLOPT5Q/DMIN,IEXIT,IFN,MODE

COMMON/FLOPT5X/FINISH,GRDCHK,PRINTID

COMMON/FLOPT5Y/IH,IK,IPT,N,NA,NR

COMMON/FLOPT5Z/FACTOR,P

COMMON/HEADER1/MXLINES
```

The type specification of the variables given above is as follows:

```
INTEGER   IEXIT,IFN,IH,IK,IPT,JV,MAX,MODE,MXLINES,N,NA,NLINES,NR

LOGICAL   FINISH,GRDCHK,PRINTID

REAL      DMIN,EM,EST,ETA,FACTOR,FTIME,P,TTIME
```

and if the discrepancy is greater than 10% they are also marked with an asterisk. The users are advised to perform this gradient verification before embarking on optimization.

The only data card read by FLOPT5, which may contain up to 80 characters, is assumed by the system to contain the problem identification supplied by the user which is a part of the title on each page. See comment 3 in Table I.

Suppose the user is solving several problems in one run by calling FLOPT5 several times. It must be emphasized that the variable MODE remains at 3, which causes the previous factored matrix H to be reused in the next problem. To reset the matrix to the unit matrix, the user must re-initialize MODE to 1 through the appropriate COMMON block shown in Table II.

Some examples have been included in this report for the purpose of illustration. A brief description of the subroutines is as follows.

FLOPT5 (EPS, JD, X)

Performs an iteration of the Charalambous algorithm. Calls QUASI5 for the unconstrained optimization. After an iteration is over, the multipliers corresponding to the error functions are calculated, which are used during the next iteration. Now, the value of P is updated for the next iteration and FINISH is set to TRUE if necessary.

FUNCT5 (ER, G, JD, X)

ER and G arrays are defined by the user in this subroutine. NA and array JD could be used to evaluate the error functions and their derivatives selectively. This technique is useful when the evaluation of error functions is expensive.

GRDCHK5 (ER1, ER2, G, JD, N, NR, W, X, Y, YP)

Evaluates the partial derivatives of the error functions by numerical perturbation and compares these values with the user supplied values in array G. All the values are printed out for each error function and those with greater than 10% discrepancy are marked with an asterisk. The control returns to the main program after the gradient verification, so a separate call to FLOPT5 is required for optimization. The user should preferably stop after the gradient verification.

HEADER

Performs the page eject whenever the number of printed lines on a page exceed MXLINES and prints a heading containing the date, time, page number and the user supplied problem identification.

LEASTP5 (EN, ER, ES, G, GU, JD, U, V, X)

Evaluates least pth objective U and its gradient vector.

QUASI5 (EN, EPS, ER, ES, G, GU, H, JD, U, V, W, X)

This subroutine is a modification of the 1972 version of Fletcher's unconstrained minimization method. The details of the original subroutine may be found in Fletcher's report [1]. The initial estimate of the Hessian for the first iteration of FLOPT5 is an identity matrix. For subsequent iterations of FLOPT5 the updated Hessian estimate is used. It should be noted that this matrix is always stored in factored form in array H and should not be confused wth the Hessian itself.

IV. FLOPT5 VERSUS FLOPT4

The FLOPT5 package solves the same type of problems as FLOPT4. The similarity, however, ends there. Some respects in which FLOPT5 is different from FLOPT4 are the following.

(1) The extrapolation option of FLOPT4 is not available in FLOPT5 [5].

(2) The least pth approximation algorithm of FLOPT4 has been replaced by the latest Charalambous algorithm in which multipliers are used to weight the different error functions in the evaluation of the unconstrained objective function. See [2] for details.

(3) The gradient verification utility in FLOPT5 numerically approximates the partial derivatives of each error function and reports these values along with the user-defined values as opposed to FLOPT4 in which only the partial derivatives of the unconstrained least pth objective function were calculated and reported. The present scheme is likely to make the error detection substantially easier for the user.

(4) Each page of the output has a two line heading. The first line contains the date, time and the page number and the second line displays a string, up to 80 characters long, which has been supplied by the user on the appropriate data card.

(5) The number of printed lines on a page can be manipulated by the

user by varying MXLINES. This could be useful if one is trying to accommodate the output in a report. Also, the printing takes place in such a way that one logical block of printing material is never split between two pages.

(6) The processing time is reported in three ways.

(a) After each iteration of the Charalambous algorithm, the total CPU time for this iteration is reported.

(b) At the end of optimization, if it terminates normally, the CPU seconds for overall processing are reported as well as the CPU seconds exclusively for the function evaluations along with the number of function evaluations. This enables the user to judge the efficiency of subroutine FUNCT5, which has been prepared by him or her. The number of function evaluations required to solve a problem is a common measure of the efficiency of an algorithm. The CPU seconds for processing overheads, which may be defined as any operations other than a function evaluation, usually reflect how well a package has been programmed. It is felt that a comparison of two optimization packages solving the same problems can be easily made if these three properties of the packages are known. Hence, the task of comparing similar future packages with FLOPT5 should be easy.

V. EXAMPLES

Seven examples are presented in this section to illustrate the flexibility and power of FLOPT5. For each example, a complete listing of the main program, subroutine FUNCT5 and the output has been provided.

Example 1: Rosenbrock's function [6]

Minimize

$$U = 100\,(x_1^2 - x_2)^2 + (1 - x_1)^2.$$

The function has a minimum value of zero at $x_1 = x_2 = 1.0$. The starting point used was $x_1 = -1.2$, $x_2 = 1.0$.

```
      PROGRAM TST( INPUT,OUTPU1,TAPE5= INPUT,TAPE6=OUTPU1)
C
C     MAIN PROGRAM OF ROSENBROCK'S FUNCTION
C
C     THIS IS A PROBLEM OF UNCONSTRAINED MINIMIZATION INVOLVING ONLY ONE
C     FUNCTION. IT IS ENOUGH TO CALL FLOPT5 ONLY ONCE IN ORDER TO OBTAIN
C     THE DESIRED SOLUTION
C
      DIMENSION EPS(2), X(25)
C
      LOGICAL FINISH,GRDCHK,PRINTID
C
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
C
      DATA EPS/2*1.E-8/,X( 1) ,X( 2) /-1.2,1.0/
C
      GRDCHK=.FALSE.
      N=2
      NR= 1
      CALL FLOPT5 (EPS,JD,X)
      STOP
      END
```

```
      SUBROUTINE FUNCT5 (ER,GE,JD,X)
C
C     ROSENBROCK"S FUNCTION
C
C     THE ROLE OF THIS SUBROUTINE IS TO RETURN THE VALUES OF THE ERROR
C     FUNCTIONS AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO X(1),
C     X(2), ... ,X(N)
C
      DIMENSION ER(1), GE(1), X(1)
C
      A=X(1)*X(1)
      B=A-X(2)
      C=1.0-X(1)
C
C     THE OBJECTIVE FUNCTION IS DEFINED HERE
C
      ER(1)=100.*B*B+C*C
C
C     THE GRADIENT VECTOR IS DEFINED HERE
C
      GE(1)=400.*X(1)*(A-X(2))-C-C
      GE(2)=-200.*B
      RETURN
      END
```

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

INPUT DATA FOR FLOPT5

NUMBER OF ERROR FUNCTIONS .... NR = 1

PREDICTED FUNCTION LOWER BOUND .... EST = 0.

STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION CRITERION

	VARIABLE VECTOR X(I)		TEST VECTOR EPS(I)
1	-.12000000E+01	1	.10000000E-07
2	.10000000E+01	2	.10000000E-07

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 1 OF FLOPT5 WITH PARAMETER P= .10000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	1	.24200000E+02	1	-.12000000E+01	1	-.21560000E+03
			2	.10000000E+01	2	-.88000000E+02
20	27	.14762556E+00	1	.64388633E+00	1	.30030459E+01
			2	.40016442E+00	2	-.28850382E+01
37	47	.25874638E-24	1	.10000000E+01	1	-.76028073E-11
			2	.10000000E+01	2	.42632564E-11

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .094 SECONDS

SUMMARY OF PROCESSING TIME IN SECONDS

FOR 48 FUNCTION EVALUATIONS = 0.000
TOTAL OVERALL PROCESSING TIME = .094

Example 2: A minimax example [7]

Minimize the maximum of the following three functions

$$e_1 = x_1^2 + x_2^4$$

$$e_2 = (2 - x_1)^2 + (2 - x_2)^2$$

$$e_3 = 2 \exp(-x_1 + x_2).$$

This example is presented here to illustrate the output generated by FLOPT5 when the variable GRDCHK is set to TRUE in the main program. As the user may notice, gradient verification and optimization are not performed in the same run. Having verified the accuracy of subroutine FUNCT5, the user must set GRDCHK to FALSE in the main program in order to perform optimization.

```
      PROGRAM TST( INPUT,OUTPU2,TAPE5=INPUT,TAPE6=OUTPU2)
C
C     MAIN PROGRAM OF MINIMAX EXAMPLE
C
C     THIS IS A MINIMAX PROBLEM INVOLVING THREE FUNCTIONS. FLOPT5 WILL
C     HAVE TO BE CALLED AS MANY TIMES AS THE USER WISHES TO UPDATE
C     THE VALUE OF PARAMETER P
C
      DIMENSION EPS(2), JD(3), X(40)
C
      LOGICAL FINISH,GRDCHK,PRINTID
C
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
C
      DATA EPS/2*1.0E-8/,X(1),X(2)/2*2.0/
C
      N=2
      NR=3
      GRDCHK=.TRUE.
      IK=1
      P=4.
      FACTOR=4.
C
      DO 10 IH=1,IK
      CALL FLOPT5 (EPS,JD,X)
      IF (FINISH) STOP
10    CONTINUE
C
      STOP
      END
```

```
      SUBROUTINE FUNCT5 (ER,GE,JD,X)
C
C     A MINIMAX EXAMPLE
C
C     THE ROLE OF THIS SUBROUTINE IS TO RETURN THE VALUES OF THE ERROR
C     FUNCTIONS AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO X(1),
C     X(2), ... ,X(N)
C
      DIMENSION ER(1), GE(2,1), JD(1), X(1)
C
      Y1=X(1)*X(1)
      Y2=X(2)*X(2)
      Y3=X(1)+X(1)
      Y4=X(2)+X(2)
C
      DO 40 I=1,3
      K=JD(I)
      GO TO (10,20,30), K
10    ER(1)=Y1+Y2*Y2
      GE(1,1)=Y3
      GE(2,1)=(Y2+Y2)*Y4
      GO TO 40
20    ER(2)=8.-4.*(X(1)+X(2))+Y1+Y2
      GE(1,2)=-4.+Y3
      GE(2,2)=-4.+Y4
      GO TO 40
30    ER(3)=2.*EXP(-X(1)+X(2))
      GE(1,3)=-ER(3)
      GE(2,3)=ER(3)
40    CONTINUE
C
      RETURN
      END
```

```
USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

INPUT DATA FOR FLOPT5
---------------------

NUMBER OF ERROR FUNCTIONS ................ NR =    3

PREDICTED FUNCTION LOWER BOUND .......... EST = 0.

PARAMETER TO SELECT ACTIVE FUNCTIONS .... ETA = 0.

STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION CRITERION

                                      VARIABLE              TEST
                                     VECTOR X(I)        VECTOR EPS(I)

                                   1  .20000000E+01   1  .10000000E-07
                                   2  .20000000E+01   2  .10000000E-07

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT
--------------------------------------------------------

MAXIMUM OF THE ERROR FUNCTIONS ........... EM =   .20000000E+02

                                      WEIGHT            NORM. ERROR
                                     VECTOR V(I)        VECTOR EN(I)

                                   1  .10000000E+01   1  .10000000E+01
                                   2  .10000000E+01   2 0.
                                   3  .10000000E+01   3  .10000000E+00
```

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

VERIFICATION OF THE USER DEFINED PARTIAL DERIVATIVES IN SUBROUTINE FUNCT5

ERROR FUNC. NO.	USER DEFINED PARTIAL DERIVATIVE	NUMERICALLY APPROXIMATED DERIVATIVE	DIFFERENCE (PERCENTAGE W.R.T. NUMERICAL)
1	1 .40000000E+01	1 .40000000E+01	1 .45806700E-06
	2 .32000000E+02	2 .32000000E+02	2 .74840045E-07
2	1 0.	1 -.71054274E-08	1 .10000000E+03 *
	2 0.	2 -.35527137E-08	2 .10000000E+03 *
3	1 -.20000000E+01	1 -.20000000E+01	1 .10279564E-06
	2 .20000000E+01	2 .20000040E+01	2 .20012018E-03

FUNCTION CALLS MADE FOR THE ABOVE CALCULATIONS = 6

AVERAGE CPU SECONDS FOR EACH FUNCTION CALL = 0.0000

* INDICATES THAT THE DIFFERENCE EXCEEDS 10 PERCENT

USER DEFINED AND NUMERICAL DERIVATIVE VALUES IN THE INTERVAL (-.1E-19,+.1E-19)
ARE LOWERED OR RAISED TO THE NEAREST LIMIT TO CALCULATE PERCENTAGE DIFFERENCE

PERCENTAGE DIFFERENCE=100. ABS((USER DEFINED - NUMERICAL)/NUMERICAL VALUE)

END OF GRADIENT VERIFICATION PROGRAM

Example 3: A minimax example [7]

Minimize the maximum of the following three functions

$$e_1 = x_1^2 + x_2^4$$

$$e_2 = (2 - x_1)^2 + (2 - x_2)^2$$

$$e_3 = 2 \exp(-x_1 + x_2).$$

The minimax solution is defined by the functions e_1 and e_2 at the point $x_1 = 1.13904$, $x_2 = 0.89956$, where $e_1 = e_2 = 1.95222$ and $e_3 = 1.57408$. Using least pth approximation with p = 4, 16, 64, 256, 1024, an effort of 44 function evaluations yielded $x_1 = 1.1390377$, $x_2 = 0.8995599$.

```
      PROGRAM TST(INPUT,OUTPU3,TAPE5=INPUT,TAPE6=OUTPU3)
C
C     MAIN PROGRAM OF MINIMAX EXAMPLE
C
C     THIS IS A MINIMAX PROBLEM INVOLVING THREE FUNCTIONS. FLOPT5 WILL
C     HAVE TO BE CALLED AS MANY TIMES AS THE USER WISHES TO UPDATE
C     THE VALUE OF PARAMETER P
C
      DIMENSION EPS(2), JD(3), X(40)
C
      LOGICAL FINISH,GRDCHK,PRINTID
C
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /FLOPT5Z/ FACTOR,P
      DATA EPS/2*1.0E-8/,X(1),X(2)/2*2.0/
C
      NR=3
      GRDCHK=.FALSE.
      IK=5
      N=2
      P=4.
      FACTOR=4.
C
      DO 10 IH=1,IK
      CALL FLOPT5 (EPS,JD,X)
      IF (FINISH) STOP
10    CONTINUE
C
      STOP
      END
```

```
      SUBROUTINE FUNCT5 (ER,GE,JD,X)
C
C     A MINIMAX EXAMPLE
C
C     THE ROLE OF THIS SUBROUTINE IS TO RETURN THE VALUES OF THE ERROR
C     FUNCTIONS AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO X(1),
C     X(2), ... ,X(N)
C
      DIMENSION ER(1), GE(2,1), JD(1), X(1)
C
      Y1=X(1)*X(1)
      Y2=X(2)*X(2)
      Y3=X(1)+X(1)
      Y4=X(2)+X(2)
C
      DO 40 I=1,3
      K=JD(I)
      GO TO (10,20,30), K
10    ER(1)=Y1+Y2*Y2
      GE(1,1)=Y3
      GE(2,1)=(Y2+Y2)*Y4
      GO TO 40
20    ER(2)=8.-4.*(X(1)+X(2))+Y1+Y2
      GE(1,2)=-4.+Y3
      GE(2,2)=-4.+Y4
      GO TO 40
30    ER(3)=2.*EXP(-X(1)+X(2))
      GE(1,3)=-ER(3)
      GE(2,3)=ER(3)
40    CONTINUE
C
      RETURN
      END
```

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

INPUT DATA FOR FLOPT5

NUMBER OF ERROR FUNCTIONS .... NR = 3

INITIAL VALUE OF THE PARAMETER .... P = .40000000E+01

MULTIPLYING FACTOR FOR P .... FACTOR = .40000000E+01

PREDICTED FUNCTION LOWER BOUND .... EST = 0.

PARAMETER TO SELECT ACTIVE FUNCTIONS .... ETA = 0.

STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION CRITERION

	VARIABLE VECTOR X(I)		TEST VECTOR EPS(I)
1	.20000000E+01	1	.10000000E-07
2	.20000000E+01	2	.10000000E-07

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .20000000E+02

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.10000000E+01	1	.10000000E+01
2	.10000000E+01	2	0.
3	.10000000E+01	3	.10000000E+00

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 1 OF FLOPT5 WITH PARAMETER P= .40000000E+01

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	2	.20000500E+02	1	.20000000E+01	1	.39977002E+01
			2	.20000000E+01	2	.31999600E+02
10	14	.24033042E+01	1	.12008090E+01	1	-.24628529E-07
			2	.82623537E+00	2	.54009724E-07

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .20164297E+01

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.39141735E+00	1	.94621380E+00
2	.46203219E+00	2	.10000000E+01
3	.14655046E+00	3	.68198003E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .116 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 2 OF FLOPT5 WITH PARAMETER P= .16000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	16	.19578550E+01	1	.12008090E+01	1	-.51190004E+00
			2	.82623537E+00	2	-.10871581E+01
7	24	.19334275E+01	1	.11402478E+01	1	.28856595E-11
			2	.89589589E+00	2	.38711115E-11

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19582197E+01

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.42963553E+00	1	.99293203E+00
2	.56407849E+00	2	.10000000E+01
3	.62859774E-02	3	.79992245E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .067 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 3 OF FLOPT5 WITH PARAMETER P= .64000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	26	.19527811E+01	1	.11402478E+01	1	-.40896712E+00
			2	.89589589E+00	2	-.54234177E+00
6	33	.19520319E+01	1	.11390014E+01	1	-.24333663E-08
			2	.89954250E+00	2	-.54362084E-08

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19523253E+01

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.43049931E+00	1	.99983004E+00
2	.56950069E+00	2	.10000000E+01
3	.81452096E-08	3	.80627304E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .067 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 4 OF FLOPT5 WITH PARAMETER P= .25600000E+03

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	35	.19522254E+01	1	.11390014E+01	1	-.29935271E-01
			2	.89954250E+00	2	-.38260829E-01
3	39	.19522245E+01	1	.11390376E+01	1	.99088333E-11
			2	.89955990E+00	2	.12335414E-10

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19522247E+01

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.43048122E+00	1	.99999971E+00
2	.56951878E+00	2	.10000000E+01
3	.11681172E-31	3	.80629943E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .062 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 5 OF FLOPT5 WITH PARAMETER P= .10240000E+04

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	41	.19522245E+01	1	.11390376E+01	1	-.29032413E-03
			2	.89955990E+00	2	-.37107811E-03
2	44	.19522245E+01	1	.11390377E+01	1	-.26751936E-11
			2	.89955994E+00	2	-.30979666E-11

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19522245E+01

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.43048117E+00	1	.10000000E+01
2	.56951883E+00	2	.10000000E+01
3	.25899541-127	3	.80629950E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .059 SECONDS

SUMMARY OF PROCESSING TIME IN SECONDS

FOR 45 FUNCTION EVALUATIONS = 0.000
TOTAL OVERALL PROCESSING TIME = .371

Example 4: A microwave circuit example

The design of a three-section 100-percent relative bandwidth 10:1 transmission-line transformer [8] is considered. In this case, we let the error function e_i be the modulus of the reflection coefficient sampled at the 11 normalized frequencies (w.r.t. 1 GHz)

$$\{0.5, 0.6, 0.7, 0.77, 0.9, 1.0, 1.1, 1.23, 1.3, 1.4, 1.5\}.$$

Gradient vectors with respect to section lengths and characteristic impedances are obtained using the adjoint network method. Using p = 8, 48, 288, we get a reflection coefficient magnitude of 0.19729 (optimal to 5 figures) after 61 function evaluations. The necessary effort required is summarized in Table III. Note that the sample points are read from the main program and passed to subroutine FUNCT5 via a COMMON block named USER. At the end of each iteration of FLOPT5 the responses of the transformer at the local solution are printed. In subroutine FUNCT5 the error functions and their gradients are obtained from the subroutine NET which defines the reflection coefficient of the transformer.

TABLE III COMPUTATIONAL EFFORT FOR THE TRANSFORMER PROBLEM

Parameter p	Total function evaluations	Function evaluations	Number of error functions	Reflection coefficient
8	28	28	11	.21016566
48	45	17	11	.19825604
288	61	16	10	.19729076
1728	70	9	6	.19729063
10368	74	4	4	.19729063

```
      PROGRAM TST(INPUT,OUTPU5,TAPE5=INPUT,TAPE6=OUTPU5)
C
C     MAIN PROGRAM OF MICROWAVE CIRCUIT PROBLEM
C
C     THIS MINIMAX PROBLEM INVOLVES ELEVEN ERROR FUNCTIONS
C
      DIMENSION EPS(6), JD(11), X(190)
      DIMENSION GRAD(6), WN(11)
C
      LOGICAL FINISH,GRDCHK,PRINTID
C
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /FLOPT5Z/ FACTOR,P
      COMMON /USER/ WN
C
      DATA WN/.5,.6,.7,.77,.9,1.,1.1,1.23,1.3,1.4,1.5/
      DATA EPS/6*1.0E-8/,GRDCHK/.FALSE./
      DATA X(1),X(2),X(3),X(4),X(5),X(6)/.8,1.5,1.2,3.,.8,6./
C
      FACTOR=6.
      IK=5
      N=6
      NR=11
      P=8.
C
      DO 30 IH=1,IK
      CALL FLOPT5 (EPS,JD,X)
      WRITE (6,50)
C
      DO 20 I=1,NA
      K=JD(I)
      S=WN(K)
      CALL NET (X,S,ARHO,ATNG,GRAD,0)
      WRITE (6,40) K,S,K,ATNG
20    CONTINUE
C
      IF (FINISH) STOP
30    CONTINUE
C
      STOP
C
40    FORMAT (6X,2(I4,E15.8))
C
50    FORMAT (*-RESPONSES OF THE THREE SECTION TRANSMISSION-LINE TRANSFO
     1RMER*/1X,60(*-*)/1H0,30X,*REFLECTION*/12X,*FREQUENCY*,10X,
     2*COEFFICIENT*,8X/12X,*VECTOR S(I)*,8X,*VECTOR ATNG(I)*/)
C
      END
```

```
      SUBROUTINE FUNCT5 (ER,GE,JD,X)
C
C     A MICROWAVE CIRCUIT EXAMPLE
C
C     THE ROLE OF THIS SUBROUTINE IS TO RETURN THE VALUES OF THE ERROR
C     FUNCTIONS AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO X(1),
C     X(2), ... ,X(N)
C
      DIMENSION ER(1), GE(6,1), JD(1), X(1)
      DIMENSION GRAD(6)
C
      COMMON /USER/ WN(11)
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
C
      DO 10 I=1,NA
      K=JD(I)
      CALL NET (X,WN(K),ARHO,ATN,GRAD,1)
      ER(K)=ATN
      DO 10 J=1,N
10    GE(J,K)=GRAD(J)
C
      RETURN
      END
```

```
      SUBROUTINE NET (AX,S,ARHO,ATN,GRAD,IGRAD)
C
C     THREE SECTION 10 TO 1 TRANSFORMER
C
      COMPLEX A,B,C,D,VG,RHO,CJRHO,TVG,XIG
      COMPLEX XI(21),V(21),G(20)
C
      DIMENSION AX(1), GRAD(1), THETA(20), XL(20), Z(20)
C
      DATA XLQ,FACT/7.4948125,0.2095844728/
      BETA=FACT*S
      M=3
      MP1=M+1
C
      DO 10 I=1,M
      J=I+I
      XL(I)=XLQ*AX(J-1)
      Z(I)=AX(J)
10    CONTINUE
C
      RG=1.0
      RL=10.0
      XI(MP1)=CMPLX(1.0,0.0)
      V(MP1)=RL*XI(MP1)
C
      DO 20 J=1,M
      I=M+1-J
      IP1=I+1
      THETA(I)=BETA*XL(I)
      TH=THETA(I)
      CTH=COS(TH)
      STH=SIN(TH)
      A=CMPLX(CTH,0.)
      B=CMPLX(0.,(Z(I)*STH))
      C=CMPLX(0.,(STH/Z(I)))
      D=A
      V(I)=A*V(IP1)+B*XI(IP1)
      XI(I)=C*V(IP1)+D*XI(IP1)
20    CONTINUE
C
      XIG=-XI(1)
      VG=V(1)-XIG*RG
      RHO=1.+(RG+RG)*XIG/VG
      CJRHO=CONJG(RHO)
      AR=CJRHO*RHO
      ATN=SQRT(AR)
      IF (IGRAD.EQ.0) RETURN
      TVG=(RG+RG)/VG
C
      DO 30 I=1,M
      TH=THETA(I)
      J=I+I
      IP1=I+1
      G(J)=(V(I)*XI(I)-V(IP1)*XI(IP1))/(VG*Z(I))
      J1=J-1
      G(J1)=BETA*(V(I)*XI(IP1)-V(IP1)*XI(I))/(VG*SIN(TH))*XLQ
30    CONTINUE
C
      M2=M+M
C
      DO 40 I=1,M2
      GRAD(I)=REAL(TVG*CJRHO*G(I))/ATN
40    CONTINUE
C
      RETURN
      END
```

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INPUT DATA FOR FLOPT5

NUMBER OF ERROR FUNCTIONS .... NR = 11

INITIAL VALUE OF THE PARAMETER .... P = .80000000E+01

MULTIPLYING FACTOR FOR P .... FACTOR = .60000000E+01

PREDICTED FUNCTION LOWER BOUND .... EST = 0.

PARAMETER TO SELECT ACTIVE FUNCTIONS .... ETA = 0.

STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION CRITERION

	VARIABLE VECTOR X(I)		TEST VECTOR EPS(I)
1	.80000000E+00	1	.10000000E-07
2	.15000000E+01	2	.10000000E-07
3	.12000000E+01	3	.10000000E-07
4	.30000000E+01	4	.10000000E-07
5	.80000000E+00	5	.10000000E-07
6	.60000000E+01	6	.10000000E-07

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .38813233E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.10000000E+01	1	.59179736E+00
2	.10000000E+01	2	.17145885E+00
3	.10000000E+01	3	.67754936E+00
4	.10000000E+01	4	.88663403E+00
5	.10000000E+01	5	.10000000E+01
6	.10000000E+01	6	.90913294E+00
7	.10000000E+01	7	.72325952E+00
8	.10000000E+01	8	.46585921E+00
9	.10000000E+01	9	.38438299E+00
10	.10000000E+01	10	.40759033E+00
11	.10000000E+01	11	.62071597E+00

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ITERATION NO. 1 OF FLOPT5 WITH PARAMETER P= .80000000E+01

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	2	.42348353E+00	1	.80000000E+00	1	-.41877277E+00
			2	.15000000E+01	2	.10293966E+00
			3	.12000000E+01	3	.53785205E+00
			4	.30000000E+01	4	.27081936E-01
			5	.80000000E+00	5	-.46087917E+00
			6	.60000000E+01	6	-.52074793E-01
20	24	.23663055E+00	1	.98827693E+00	1	-.46778805E-07
			2	.16286836E+01	2	.36413600E-07
			3	.10000448E+01	3	-.34872074E-06
			4	.31622779E+01	4	.10649350E-06
			5	.98827691E+00	5	-.12989564E-06
			6	.61399279E+01	6	-.56382486E-07
23	28	.23663055E+00	1	.98827692E+00	1	-.19521885E-08
			2	.16286836E+01	2	.34568939E-08
			3	.10000448E+01	3	-.47667719E-08
			4	.31622777E+01	4	.22805595E-09
			5	.98827692E+00	5	-.50726703E-08
			6	.61399279E+01	6	.15500211E-09

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .21016566E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.35478268E+00	1	.10000000E+01
2	.29369814E-06	2	.13524902E+00
3	.71865105E-01	3	.79604299E+00
4	.23497497E+00	4	.94283789E+00
5	.16724124E-01	5	.64637269E+00
6	.15606601E-07	6	.88930230E-01
7	.25059015E-02	7	.49284972E+00
8	.13548396E+00	8	.87151540E+00
9	.63070045E-01	9	.78133494E+00
10	.63072715E-05	10	.20960966E+00
11	.12058660E+00	11	.85713276E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS 1.486 SECONDS

RESPONSES OF THE THREE SECTION TRANSMISSION-LINE TRANSFORMER

	FREQUENCY VECTOR S(I)		REFLECTION COEFFICIENT VECTOR ATNG(I)
1	.50000000E+00	1	.21016566E+00
2	.60000000E+00	2	.28424699E-01
3	.70000000E+00	3	.16730090E+00
4	.77000000E+00	4	.19815215E+00
5	.90000000E+00	5	.13584534E+00
6	.10000000E+01	6	.18690080E-01

7	.11000000E+01	7	.10358009E+00
8	.12300000E+01	8	.18316261E+00
9	.13000000E+01	9	.16420977E+00
10	.14000000E+01	10	.44052753E-01
11	.15000000E+01	11	.18013987E+00

ITERATION NO. 2 OF FLOPT5 WITH PARAMETER P= .48000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	30	.20584524E+00	1	.98827692E+00	1	-.31742227E+00
			2	.16286836E+01	2	-.38934937E+00
			3	.10000448E+01	3	-.62457044E+00
			4	.31622777E+01	4	.19762598E-08
			5	.98827692E+00	5	-.31742226E+00
			6	.61399279E+01	6	.10327922E+00
12	45	.19652620E+00	1	.10001261E+01	1	.49965129E-07
			2	.16356942E+01	2	.23465524E-07
			3	.99984960E+00	3	-.40643386E-07
			4	.31622777E+01	4	.44542934E-08
			5	.10001261E+01	5	.27429982E-08
			6	.61136120E+01	6	-.93040556E-08

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19825604E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.34978587E+00	1	.99089463E+00
2	.14990341E-38	2	.20374774E+00
3	.16904333E-03	3	.87143016E+00
4	.32794009E+00	4	.99824885E+00
5	.68637861E-11	5	.62580615E+00
6	.31229025-144	6	.12236987E-02
7	.12196653E-11	7	.62808077E+00
8	.20532242E+00	8	.10000000E+01
9	.16031088E-03	9	.87286836E+00
10	.40055943E-37	10	.20469738E+00
11	.11662228E+00	11	.99048900E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .850 SECONDS

RESPONSES OF THE THREE SECTION TRANSMISSION-LINE TRANSFORMER

	FREQUENCY VECTOR S(I)		REFLECTION COEFFICIENT VECTOR ATNG(I)
1	.50000000E+00	1	.19645084E+00
2	.60000000E+00	2	.40394220E-01
3	.70000000E+00	3	.17276629E+00
4	.77000000E+00	4	.19790886E+00
5	.90000000E+00	5	.12406985E+00
6	.10000000E+01	6	.24260566E-03
7	.11000000E+01	7	.12452080E+00
8	.12300000E+01	8	.19825604E+00
9	.13000000E+01	9	.17305142E+00
10	.14000000E+01	10	.40582490E-01
11	.15000000E+01	11	.19637042E+00

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ITERATION NO. 3 OF FLOPT5 WITH PARAMETER P= .28800000E+03

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	47	.19768514E+00	1	.10001261E+01	1	.69642582E-01
			2	.16356942E+01	2	.31878361E+00
			3	.99984960E+00	3	-.14766142E+00
			4	.31622777E+01	4	.11989155E-07
			5	.10001261E+01	5	.69642520E-01
			6	.61136120E+01	6	-.85290426E-01
10	61	.19729040E+00	1	.99999994E+00	1	-.99195178E-08
			2	.16347070E+01	2	-.60763507E-09
			3	.10000000E+01	3	-.33104515E-07
			4	.31622776E+01	4	-.30713958E-08
			5	.99999993E+00	5	-.17003464E-07
			6	.61173041E+01	6	.34915137E-09

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19729076E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.34997043E+00	1	.10000000E+01
2	.37834795-239	2	.20001002E+00
3	.12960300E-20	3	.87169333E+00
4	.32802430E+00	4	.99999906E+00
5	.69280156E-69	5	.62794659E+00
7	.12305494E-69	7	.62794565E+00
8	.20534851E+00	8	.99999860E+00
9	.12290316E-20	9	.87169321E+00
10	.10115779-237	10	.20001042E+00
11	.11665676E+00	11	.99999919E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .782 SECONDS

RESPONSES OF THE THREE SECTION TRANSMISSION-LINE TRANSFORMER

	FREQUENCY VECTOR S(I)		REFLECTION COEFFICIENT VECTOR ATNG(I)
1	.50000000E+00	1	.19729076E+00
2	.60000000E+00	2	.39460129E-01
3	.70000000E+00	3	.17197704E+00
4	.77000000E+00	4	.19729058E+00
5	.90000000E+00	5	.12388896E+00
7	.11000000E+01	7	.12388783E+00
8	.12300000E+01	8	.19729049E+00
9	.13000000E+01	9	.17197702E+00
10	.14000000E+01	10	.39460209E-01
11	.15000000E+01	11	.19729060E+00

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ITERATION NO. 4 OF FLOPT5 WITH PARAMETER P= .17280000E+04

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	63	.19729063E+00	1	.99999994E+00	1	-.22217448E-03
			2	.16347070E+01	2	-.32133525E-03
			3	.10000000E+01	3	-.21063204E-03
			4	.31622776E+01	4	-.30736698E-08
			5	.99999993E+00	5	-.22218157E-03
			6	.61173041E+01	6	.85869548E-04
4	70	.19729063E+00	1	.10000000E+01	1	-.43273034E-06
			2	.16347071E+01	2	.24855136E-06
			3	.10000000E+01	3	-.46226108E-06
			4	.31622776E+01	4	-.34146856E-08
			5	.10000000E+01	5	-.43990457E-06
			6	.61173037E+01	6	-.65699248E-07

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19729063E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.34997039E+00	1	.10000000E+01
3	.13233598-123	3	.87169439E+00
4	.32802466E+00	4	.10000000E+01
8	.20534848E+00	8	.10000000E+01
9	.12549483-123	9	.87169439E+00
11	.11665647E+00	11	.10000000E+01

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .423 SECONDS

RESPONSES OF THE THREE SECTION TRANSMISSION-LINE TRANSFORMER

	FREQUENCY VECTOR S(I)		REFLECTION COEFFICIENT VECTOR ATNG(I)
1	.50000000E+00	1	.19729063E+00
3	.70000000E+00	3	.17197713E+00
4	.77000000E+00	4	.19729063E+00
8	.12300000E+01	8	.19729063E+00
9	.13000000E+01	9	.17197713E+00
11	.15000000E+01	11	.19729063E+00

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 5 OF FLOPT5 WITH PARAMETER P= .10368000E+05

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	72	.19729063E+00	1	.10000000E+01	1	-.26700609E-05
			2	.16347071E+01	2	.18742431E-05
			3	.10000000E+01	3	-.27616463E-05
			4	.31622776E+01	4	-.34146888E-08
			5	.10000000E+01	5	-.26772351E-05
			6	.61173037E+01	6	-.50012755E-06
1	74	.19729063E+00	1	.10000000E+01	1	-.26700609E-05
			2	.16347071E+01	2	.18742431E-05
			3	.10000000E+01	3	-.27616463E-05
			4	.31622776E+01	4	-.34146888E-08
			5	.10000000E+01	5	-.26772351E-05
			6	.61173037E+01	6	-.50012755E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .19729063E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.34997016E+00	1	.10000000E+01
4	.32802680E+00	4	.10000000E+01
8	.20534833E+00	8	.10000000E+01
11	.11665470E+00	11	.10000000E+01

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .156 SECONDS

SUMMARY OF PROCESSING TIME IN SECONDS

FOR 75 FUNCTION EVALUATIONS = 2.772
TOTAL OVERALL PROCESSING TIME = 3.697

RESPONSES OF THE THREE SECTION TRANSMISSION-LINE TRANSFORMER

	FREQUENCY VECTOR S(I)		REFLECTION COEFFICIENT VECTOR ATNG(I)
1	.50000000E+00	1	.19729063E+00
4	.77000000E+00	4	.19729063E+00
8	.12300000E+01	8	.19729063E+00
11	.15000000E+01	11	.19729063E+00

Example 5: Beale constrained function [9]

Minimize

$$f = 9 - 8x_1 - 6x_2 - 4x_3 + 2x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 2x_1x_3$$

subject to

$$x_i \geq 0, \quad i = 1, 2, 3$$

$$3 - x_1 - x_2 - 2x_3 \geq 0.$$

The function has a minimum $f = 1/9$ at $\underset{\sim}{x} = [4/3 \ 7/9 \ 4/9]^T$.

The conversion of the constrained problem into an unconstrained minimax problem has been carried out using the Bandler-Charalambous technique [4].

This problem has also been solved by DISOPT3 [10], which utilizes the Charalambous algorithm [11].

```
      PROGRAM TST( INPUT,OUTPU6,TAPE5= INPUT,TAPE6=OUTPU6)
C
C     MAIN PROGRAM FOR BEALE CONSTRAINED FUNCTION
C
      DIMENSION EPS(3), JD(5), X(70)
C
      LOGICAL FINISH,GRDCHK,PRINTID
C
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /FLOPT5Z/ FACTOR,P
C
      DATA EPS/3*.1E-6/,X(1),X(2),X(3)/3*.5/,GRDCHK/.FALSE./
C
      FACTOR=2.
      IK=5
      N=3
      NR=5
      P=10.
C
      DO 10 IH=1,IK
      CALL FLOPT5 (EPS,JD,X)
      IF (FINISH) STOP
10    CONTINUE
C
      STOP
      END
```

```
      SUBROUTINE FUNCT5 (ER,GE,JD,X)
C
C     THE BEALE PROBLEM
C
      DIMENSION CONS(5), GCONS(3,5)
      DIMENSION ER(1), GE(3,1), JD(1), X(1)
C
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
C
      DATA AL/1./
C
10    ER(1)=9.-8.*X(1)-6.*X(2)-4.*X(3)+2.*(X(1)**2+X(2)**2)+X(3)**2+2.*X
     1(1)*(X(2)+X(3))
      GE(1,1)=-8.+4.*X(1)+2.*(X(2)+X(3))
      GE(2,1)=-6.+4.*X(2)+2.*X(1)
      GE(3,1)=-4.+2.*X(3)+2.*X(1)
C
      DO 100 I=1,NA
      J=JD(I)
      GO TO (100,20,40,60,80,100), J
C
20    CONS(2)=X(1)
      GCONS(1,2)=1.
      GCONS(2,2)=0.
      GCONS(3,2)=0.
      ER(2)=ER(1)-AL*CONS(2)
      DO 30 IJ=1,3
      GE(IJ,2)=GE(IJ,1)-AL*GCONS(IJ,2)
30    CONTINUE
      GO TO 100
C
40    CONS(3)=X(2)
      GCONS(1,3)=0.
      GCONS(2,3)=1.
      GCONS(3,3)=0.
      ER(3)=ER(1)-AL*CONS(3)
      DO 50 IJ=1,3
      GE(IJ,3)=GE(IJ,1)-AL*GCONS(IJ,3)
50    CONTINUE
      GO TO 100
C
60    CONS(4)=X(3)
      GCONS(1,4)=0.
      GCONS(2,4)=0.
      GCONS(3,4)=1.
      ER(4)=ER(1)-AL*CONS(4)
      DO 70 IJ=1,3
      GE(IJ,4)=GE(IJ,1)-AL*GCONS(IJ,4)
70    CONTINUE
      GO TO 100
C
80    CONS(5)=3.-X(1)-X(2)-2.*X(3)
      GCONS(1,5)=-1.
      GCONS(2,5)=-1.
      GCONS(3,5)=-2.
      ER(5)=ER(1)-AL*CONS(5)
      DO 90 IJ=1,3
      GE(IJ,5)=GE(IJ,1)-AL*GCONS(IJ,5)
90    CONTINUE
C
100   CONTINUE
C
      RETURN
      END
```

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

INPUT DATA FOR FLOPT5

NUMBER OF ERROR FUNCTIONS .... NR = 5

INITIAL VALUE OF THE PARAMETER .... P = .10000000E+02

MULTIPLYING FACTOR FOR P .... FACTOR = .20000000E+01

PREDICTED FUNCTION LOWER BOUND .... EST = 0.

PARAMETER TO SELECT ACTIVE FUNCTIONS .... ETA = 0.

STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION CRITERION

	VARIABLE VECTOR X(I)		TEST VECTOR EPS(I)
1	.50000000E+00	1	.10000000E-06
2	.50000000E+00	2	.10000000E-06
3	.50000000E+00	3	.10000000E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .22500000E+01

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.10000000E+01	1	.10000000E+01
2	.10000000E+01	2	.77777778E+00
3	.10000000E+01	3	.77777778E+00
4	.10000000E+01	4	.77777778E+00
5	.10000000E+01	5	.55555556E+00

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 1 OF FLOPT5 WITH PARAMETER P= .10000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	2	.23000048E+01	1	.50000000E+00	1	-.44054831E+01
			2	.50000000E+00	2	-.33244442E+01
			3	.50000000E+00	3	-.22392689E+01
11	15	.11700904E+00	1	.13382190E+01	1	.76120471E-08
			2	.77452070E+00	2	-.10786977E-07
			3	.43630175E+00	3	.17279277E-07

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .11439206E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.77452070E+00	1	.10000000E+01
5	.22547930E+00	5	.87187180E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .120 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 2 OF FLOPT5 WITH PARAMETER P= .20000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	17	.11304491E+00	1	.13382190E+01	1	-.20255983E+00
			2	.77452070E+00	2	-.20255985E+00
			3	.43630175E+00	3	-.40511966E+00
5	23	.11111093E+00	1	.13333696E+01	1	.15962921E-06
			2	.77775360E+00	2	.15308335E-06
			3	.44438401E+00	3	.32075511E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .11113529E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.77775345E+00	1	.10000000E+01
5	.22224655E+00	5	.99902121E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .061 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 3 OF FLOPT5 WITH PARAMETER P= .40000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	25	.11111147E+00	1	.13333696E+01	1	-.65312843E-02
			2	.77775360E+00	2	-.65312909E-02
			3	.44438401E+00	3	-.13062567E-01
3	29	.11111111E+00	1	.13333335E+01	1	.81577290E-10
			2	.77777769E+00	2	.31485130E-10
			3	.44444422E+00	3	-.47037619E-10

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .11111120E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.77777769E+00	1	.10000000E+01
5	.22222231E+00	5	.99999640E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .061 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 4 OF FLOPT5 WITH PARAMETER P= .80000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	31	.11111111E+00	1	.13333335E+01	1	-.49102278E-04
			2	.77777769E+00	2	-.49102328E-04
			3	.44444422E+00	3	-.98204766E-04
2	34	.11111111E+00	1	.13333333E+01	1	-.41473366E-07
			2	.77777778E+00	2	-.41995984E-07
			3	.44444444E+00	3	-.83007322E-07

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .11111111E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.77777782E+00	1	.10000000E+01
5	.22222218E+00	5	.99999999E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .049 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 5 OF FLOPT5 WITH PARAMETER P= .16000000E+03

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	36	.11111111E+00	1	.13333333E+01	1	-.30340934E-06
			2	.77777778E+00	2	-.30393197E-06
			3	.44444444E+00	3	-.60687924E-06
1	37	.11111111E+00	1	.13333333E+01	1	-.30340934E-06
			2	.77777778E+00	2	-.30393197E-06
			3	.44444444E+00	3	-.60687924E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .11111111E+00

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.77777808E+00	1	.10000000E+01
5	.22222192E+00	5	.99999999E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .056 SECONDS

SUMMARY OF PROCESSING TIME IN SECONDS

FOR 38 FUNCTION EVALUATIONS = .001
TOTAL OVERALL PROCESSING TIME = .347

Example 6: The Wong problem 1 [10, 11]

Minimize

$$f = (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2$$
$$+ 10\, x_5^6 + 7\, x_6^2 + x_7^4 - 4\, x_6\, x_7 - 10\, x_6 - 8\, x_7$$

subject to

$$-2\, x_1^2 - 3\, x_2^4 - x_3 - 4\, x_4^2 - 5\, x_5 + 127 \geq 0$$
$$-7\, x_1 - 3\, x_2 - 10\, x_3^2 - x_4 + x_5 + 282 \geq 0$$
$$-23\, x_1 - x_2^2 - 6\, x_6^2 + 8\, x_7 + 196 \geq 0$$
$$-4\, x_1^2 - x_2^2 + 3\, x_1\, x_2 - 2\, x_3^2 - 5\, x_6 + 11\, x_7 \geq 0$$

The optimal solution found in 89 function evaluations is

$f = 680.63006$

$x_1 = 2.3304994$	$x_2 = 1.9513724$
$x_3 = -0.47754140$	$x_4 = 4.3657262$
$x_5 = -0.62448697$	$x_6 = 1.0381310$
$x_7 = 1.5942267$	

This problem has also been solved by the program DISOPT3 [10] using the Charalambous algorithm [11].

```
      PROGRAM TST(INPUT,OUTPU7,TAPE5=INPUT,TAPE6=OUTPU7)
C
C     MAIN PROGRAM FOR WONG PROBLEM 1
C
      DIMENSION EPS(7), JD(5), X(155)
C
      LOGICAL FINISH,GRDCHK,PRINTID
C
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /FLOPT5Z/ FACTOR,P
C
      DATA EPS/7*.1E-6/,GRDCHK/.FALSE./
      DATA X/1.,2.,0.,4.,0.,1.,1.,148*0./
C
      FACTOR=2.
      IK=15
      N=7
      NR=5
      P=10.
C
      DO 10 IH=1,IK
      CALL FLOPT5 (EPS,JD,X)
      IF (FINISH) STOP
10    CONTINUE
C
      STOP
      END
```

```
      SUBROUTINE FUNCT5 (ER,GE,JD,X)
C
C     THE FIRST WONG PROBLEM
C
      DIMENSION CONS(5), GCONS(7,5)
      DIMENSION ER(1), GE(7,1), JD(1), X(1)
C
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
C
      DATA AL/10./
C
10    ER(1)=(X(1)-10.)**2+5.*(X(2)-12.)**2+X(3)**4+3.*(X(4)-11.)**2+10.*
     1X(5)**6+7.*X(6)**2+X(7)**4-4.*X(6)*X(7)-10.*X(6)-8.*X(7)
      GE(1,1)=2.*(X(1)-10.)
      GE(2,1)=10.*(X(2)-12.)
      GE(3,1)=4.*X(3)**3
      GE(4,1)=6.*(X(4)-11.)
      GE(5,1)=60.*X(5)**5
      GE(6,1)=14.*X(6)-4.*X(7)-10.
      GE(7,1)=4.*X(7)**3-4.*X(6)-8.
C
      DO 100 I=1,NA
      J=JD(I)
      GO TO (100,20,40,60,80,100), J
C
20    CONS(2)=-2.*X(1)**2-3.*X(2)**4-X(3)-4.*X(4)**2-5.*X(5)+127.
      GCONS(1,2)=-4.*X(1)
      GCONS(2,2)=-12.*X(2)**3
      GCONS(3,2)=-1.
      GCONS(4,2)=-8.*X(4)
      GCONS(5,2)=-5.
      GCONS(6,2)=0.
      GCONS(7,2)=0.
      ER(2)=ER(1)-AL*CONS(2)
      DO 30 IJ=1,7
      GE(IJ,2)=GE(IJ,1)-AL*GCONS(IJ,2)
30    CONTINUE
      GO TO 100
C
40    CONS(3)=-7.*X(1)-3.*X(2)-10.*X(3)**2-X(4)+X(5)+282.
      GCONS(1,3)=-7.
      GCONS(2,3)=-3.
      GCONS(3,3)=-20.*X(3)
      GCONS(4,3)=-1.
      GCONS(5,3)=1.
      GCONS(6,3)=0.
      GCONS(7,3)=0.
      ER(3)=ER(1)-AL*CONS(3)
      DO 50 IJ=1,7
      GE(IJ,3)=GE(IJ,1)-AL*GCONS(IJ,3)
50    CONTINUE
      GO TO 100
C
60    CONS(4)=-23.*X(1)-X(2)**2-6.*X(6)**2+8.*X(7)+196.
      GCONS(1,4)=-23.
      GCONS(2,4)=-2.*X(2)
      GCONS(3,4)=0.
      GCONS(4,4)=0.
      GCONS(5,4)=0.
      GCONS(6,4)=-12.*X(6)
      GCONS(7,4)=8.
      ER(4)=ER(1)-AL*CONS(4)
      DO 70 IJ=1,7
      GE(IJ,4)=GE(IJ,1)-AL*GCONS(IJ,4)
70    CONTINUE
      GO TO 100
C
80    CONS(5)=-4.*X(1)**2-X(2)**2+3.*X(1)*X(2)-2.*X(3)**2-5.*X(6)+11.*X(
     17)
      GCONS(1,5)=-8.*X(1)+3.*X(2)
      GCONS(2,5)=-2.*X(2)+3.*X(1)
      GCONS(3,5)=-4.*X(3)
      GCONS(4,5)=0.
```

```
      GCONS(5,5)=0.
      GCONS(6,5)=-5.
      GCONS(7,5)=11.
      ER(5)=ER(1)-AL*CONS(5)
      DO 90 IJ=1,7
      GE(IJ,5)=GE(IJ,1)-AL*GCONS(IJ,5)
90    CONTINUE
C
100   CONTINUE
C
      RETURN
      END
```

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

INPUT DATA FOR FLOPT5

NUMBER OF ERROR FUNCTIONS .... NR = 5

INITIAL VALUE OF THE PARAMETER .... P = .10000000E+02

MULTIPLYING FACTOR FOR P .... FACTOR = .20000000E+01

PREDICTED FUNCTION LOWER BOUND .... EST = 0.

PARAMETER TO SELECT ACTIVE FUNCTIONS .... ETA = 0.

STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION CRITERION

	VARIABLE VECTOR X(I)		TEST VECTOR EPS(I)
1	.10000000E+01	1	.10000000E-06
2	.20000000E+01	2	.10000000E-06
3	0.	3	.10000000E-06
4	.40000000E+01	4	.10000000E-06
5	0.	5	.10000000E-06
6	.10000000E+01	6	.10000000E-06
7	.10000000E+01	7	.10000000E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .71400000E+03

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.10000000E+01	1	.10000000E+01
2	.10000000E+01	2	.81792717E+00
3	.10000000E+01	3	-.27114846E+01
4	.10000000E+01	4	-.13949580E+01
5	.10000000E+01	5	.94397759E+00

```
USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO.    1 OF FLOPT5 WITH PARAMETER P=   .10000000E+02
--------------------------------------------------------------

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD
-----------------------------------------------------------------

ITER. FUNC. OBJECTIVE             VARIABLE              GRADIENT
 NO.  EVAL. FUNCTION            VECTOR X( I)          VECTOR GU( I)

  0     2  .75272643E+03    1   .10000000E+01    1 -.82089809E+01
                            2   .20000000E+01    2 -.78706625E+01
                            3 0.                 3  .10185471E+01
                            4   .40000000E+01    4 -.13334000E+02
                            5 0.                 5  .50927356E+01
                            6   .10000000E+01    6  .18500121E+02
                            7   .10000000E+01    7 -.49448364E+02

 17    26  .72279067E+03    1   .15736285E+01    1  .89821959E-06
                            2   .19207655E+01    2  .36282777E-05
                            3 -.21294119E+00     3  .13252928E-06
                            4   .42321180E+01    4  .15674206E-05
                            5 -.63089200E+00     5 -.90355470E-06
                            6   .76103565E+00    6  .13187909E-06
                            7   .18670823E+01    7 -.93016456E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT
--------------------------------------------------------

MAXIMUM OF THE ERROR FUNCTIONS ........... EM =  .70497960E+03

                                  WEIGHT              NORM. ERROR
                                VECTOR V( I)         VECTOR EN( I)

                            1   .74378554E+00    1  .10000000E+01
                            2   .11993786E+00    2  .81647971E+00
                            5   .13627660E+00    5  .82814841E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS   .244  SECONDS
```

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 6 OF FLOPT5 WITH PARAMETER P= .32000000E+03

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	75	.68063007E+03	1	.23303227E+01	1	-.61653113E-01
			2	.19513796E+01	2	.50900474E-01
			3	-.47747532E+00	3	.10149956E-01
			4	.43657515E+01	4	.13758457E-01
			5	-.62448577E+00	5	.19697085E-02
			6	.10380941E+01	6	-.25540697E-01
			7	.15942723E+01	7	.56189474E-01
4	80	.68063006E+03	1	.23304947E+01	1	-.54581357E-05
			2	.19513726E+01	2	-.50129374E-04
			3	-.47753964E+00	3	-.59879959E-06
			4	.43657269E+01	4	-.21444421E-04
			5	-.62448695E+00	5	-.36443046E-05
			6	.10381301E+01	6	.47826968E-06
			7	.15942279E+01	7	-.45273656E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .68063009E+03

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.84916627E+00	1	.10000000E+01
2	.11397191E+00	2	.99999998E+00
5	.36861824E-01	5	.99999880E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .087 SECONDS

ITERATION NO. 7 OF FLOPT5 WITH PARAMETER P= .64000000E+03

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	82	.68063006E+03	1	.23304947E+01	1	-.33058498E-02
			2	.19513726E+01	2	.25860892E-02
			3	-.47753964E+00	3	.54044150E-03
			4	.43657269E+01	4	.68120573E-03
			5	-.62448695E+00	5	.96947500E-04
			6	.10381301E+01	6	-.13630786E-02
			7	.15942279E+01	7	.29993723E-02
2	85	.68063006E+03	1	.23304993E+01	1	-.37590892E-05
			2	.19513724E+01	2	-.38433303E-04
			3	-.47754137E+00	3	-.53469042E-06
			4	.43657262E+01	4	-.16620020E-04
			5	-.62448698E+00	5	-.29092823E-05
			6	.10381310E+01	6	.53415400E-06
			7	.15942267E+01	7	-.58431323E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .68063006E+03

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.84916659E+00	1	.10000000E+01
2	.11397195E+00	2	.10000000E+01
5	.36861461E-01	5	.99999998E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .076 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 8 OF FLOPT5 WITH PARAMETER P= .12800000E+04

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	87	.68063006E+03	1	.23304993E+01	1	-.88792708E-04
			2	.19513724E+01	2	.59222106E-04
			3	-.47754137E+00	3	.14183149E-04
			4	.43657262E+01	4	.12843309E-04
			5	-.62448698E+00	5	.13087060E-05
			6	.10381310E+01	6	-.35782705E-04
			7	.15942267E+01	7	.79312776E-04
1	89	.68063006E+03	1	.23304994E+01	1	.48726444E-04
			2	.19513724E+01	2	.66394546E-05
			3	-.47754140E+00	3	-.68067589E-05
			4	.43657262E+01	4	.67759040E-05
			5	-.62448697E+00	5	.88287267E-06
			6	.10381310E+01	6	.18379099E-04
			7	.15942267E+01	7	-.40333935E-04

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .68063006E+03

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.84916617E+00	1	.99999999E+00
2	.11397202E+00	2	.99999999E+00
5	.36861816E-01	5	.10000000E+01

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .073 SECONDS

Example 7: The Wong problem 2 [10, 11]

Minimize

$$f = x_1^2 + x_2^2 + x_1 x_2 - 14 x_1 - 16 x_2 + (x_3 - 10)^2 + 4(x_4 - 5)^2 + (x_5 - 3)^2 + 2(x_6 - 1)^2 + 5 x_7^2 + 7(x_8 - 11)^2 + 2(x_9 - 10)^2 + (x_{10} - 7)^2 + 45$$

subject to

$$-3(x_1 - 2)^2 - 4(x_2 - 3)^2 - 2 x_3^2 + 7x_4 + 120 \geq 0$$

$$-5x_1^2 - 8x_2 - (x_3 - 6)^2 + 2x_4 + 40 \geq 0$$

$$-0.5(x_1 - 8)^2 - 2(x_2 - 4)^2 - 3x_5^2 + x_6 + 30 \geq 0$$

$$-x_1^2 - 2(x_2 - 2)^2 + 2x_1 x_2 - 14x_5 + 6x_6 \geq 0$$

$$-4x_1 - 5x_2 + 3x_7 - 9x_8 + 105 \geq 0$$

$$-10x_1 + 8x_2 + 17x_7 - 2x_8 \geq 0$$

$$3x_1 - 6x_2 - 12(x_9 - 8)^2 + 7x_{10} \geq 0$$

$$8x_1 - 2x_2 - 5x_9 + 2x_{10} + 12 \geq 0$$

The optimal solution found in 96 function evaluations is

$f = 24.30621$

$x_1 = 2.1719964$	$x_2 = 2.3636830$
$x_3 = 8.7739257$	$x_4 = 5.0959845$
$x_5 = 0.99065477$	$x_6 = 1.4305740$
$x_7 = 1.3216442$	$x_8 = 9.8287258$
$x_9 = 8.2800916$	$x_{10} = 8.3759267$

This problem has also been solved by the program DISOPT3 [10], using the Charalambous algorithm [11].

```
      PROGRAM TST(INPUT,OUTPU8,TAPE5=INPUT,TAPE6=OUTPU8)
C
C     MAIN PROGRAM FOR WONG PROBLEM 2
C
      DIMENSION EPS(10), JD(9), X(300)
C
      LOGICAL FINISH,GRDCHK,PRINTID
C
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /FLOPT5Z/ FACTOR,P
C
      DATA X/2.,3.,5.,5.,1.,2.,7.,3.,6.,10.,290*0./
      DATA EPS/10*.1E-6/,GRDCHK/.FALSE./
C
      IK=15
      N=10
      FACTOR=2.
      NR=9
      P=10.
C
      DO 10 IH=1,IK
      CALL FLOPT5 (EPS,JD,X)
      IF (FINISH) STOP
10    CONTINUE
C
      STOP
      END
```

```
      SUBROUTINE FUNCT5 (ER,GE,JD,X)
C
C     THE SECOND WONG PROBLEM
C
      DIMENSION CONS(9), GCONS(10,9)
      DIMENSION ER(1), GE(10,1), JD(1), X(1)
C
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
C
      DATA AL/10./
C
10    ER(1)=X(1)**2+X(2)**2+X(1)*X(2)-14.*X(1)-16.*X(2)+(X(3)-10.)**2+4.
     1*(X(4)-5.)**2+(X(5)-3.)**2+2.*(X(6)-1.)**2+5.*X(7)**2+7.*(X(8)-11.
     2)**2+2.*(X(9)-10.)**2+(X(10)-7.)**2+45.
      GE(1,1)=2.*X(1)+X(2)-14.
      GE(2,1)=2.*X(2)+X(1)-16.
      GE(3,1)=2.*(X(3)-10.)
      GE(4,1)=8.*(X(4)-5.)
      GE(5,1)=2.*(X(5)-3.)
      GE(6,1)=4.*(X(6)-1.)
      GE(7,1)=10.*X(7)
      GE(8,1)=14.*(X(8)-11.)
      GE(9,1)=4.*(X(9)-10.)
      GE(10,1)=2.*(X(10)-7.)
C
      DO 180 I=1,NA
      J=JD(I)
      GO TO (180,20,40,60,80,100,120,140,160,180), J
C
20    CONS(2)=-3.*(X(1)-2.)**2-4.*(X(2)-3.)**2-2.*X(3)**2+7.*X(4)+120.
      GCONS(1,2)=-6.*(X(1)-2.)
      GCONS(2,2)=-8.*(X(2)-3.)
      GCONS(3,2)=-4.*X(3)
      GCONS(4,2)=7.
      GCONS(5,2)=0.
      GCONS(6,2)=0.
      GCONS(7,2)=0.
      GCONS(8,2)=0.
      GCONS(9,2)=0.
      GCONS(10,2)=0.
      ER(2)=ER(1)-AL*CONS(2)
      DO 30 IJ=1,10
      GE(IJ,2)=GE(IJ,1)-AL*GCONS(IJ,2)
30    CONTINUE
      GO TO 180
C
40    CONS(3)=-5.*X(1)**2-8.*X(2)-(X(3)-6.)**2+2.*X(4)+40.
      GCONS(1,3)=-10.*X(1)
      GCONS(2,3)=-8.
      GCONS(3,3)=-2.*(X(3)-6.)
      GCONS(4,3)=2.
      GCONS(5,3)=0.
      GCONS(6,3)=0.
      GCONS(7,3)=0.
      GCONS(8,3)=0.
      GCONS(9,3)=0.
      GCONS(10,3)=0.
      ER(3)=ER(1)-AL*CONS(3)
      DO 50 IJ=1,10
      GE(IJ,3)=GE(IJ,1)-AL*GCONS(IJ,3)
50    CONTINUE
      GO TO 180
C
60    CONS(4)=-.5*(X(1)-8.)**2-2.*(X(2)-4.)**2-3.*X(5)**2+X(6)+30.
      GCONS(1,4)=8.-X(1)
      GCONS(2,4)=-4.*(X(2)-4.)
      GCONS(3,4)=0.
      GCONS(4,4)=0.
      GCONS(5,4)=-6.*X(5)
      GCONS(6,4)=1.
      GCONS(7,4)=0.
      GCONS(8,4)=0.
      GCONS(7,4)=0.
```

```
      GCONS(8,4)=0.
      GCONS(9,4)=0.
      GCONS(10,4)=0.
      ER(4)=ER(1)-AL*CONS(4)
      DO 70 IJ=1,10
      GE(IJ,4)=GE(IJ,1)-AL*GCONS(IJ,4)
70    CONTINUE
      GO TO 180
C
80    CONS(5)=-X(1)**2-2.*(X(2)-2.)**2+2.*X(1)*X(2)-14.*X(5)+6.*X(6)
      GCONS(1,5)=-2.*X(1)+2.*X(2)
      GCONS(2,5)=-4.*(X(2)-2.)+2.*X(1)
      GCONS(3,5)=0.
      GCONS(4,5)=0.
      GCONS(5,5)=-14.
      GCONS(6,5)=6.
      GCONS(7,5)=0.
      GCONS(8,5)=0.
      GCONS(9,5)=0.
      GCONS(10,5)=0.
      ER(5)=ER(1)-AL*CONS(5)
      DO 90 IJ=1,10
      GE(IJ,5)=GE(IJ,1)-AL*GCONS(IJ,5)
90    CONTINUE
      GO TO 180
C
100   CONS(6)=-4.*X(1)-5.*X(2)+3.*X(7)-9.*X(8)+105.
      GCONS(1,6)=-4.
      GCONS(2,6)=-5.
      GCONS(3,6)=0.
      GCONS(4,6)=0.
      GCONS(5,6)=0.
      GCONS(6,6)=0.
      GCONS(7,6)=3.
      GCONS(8,6)=-9.
      GCONS(9,6)=0.
      GCONS(10,6)=0.
      ER(6)=ER(1)-AL*CONS(6)
      DO 110 IJ=1,10
      GE(IJ,6)=GE(IJ,1)-AL*GCONS(IJ,6)
110   CONTINUE
      GO TO 180
C
120   CONS(7)=-10.*X(1)+8.*X(2)+17.*X(7)-2.*X(8)
      GCONS(1,7)=-10.
      GCONS(2,7)=8.
      GCONS(3,7)=0.
      GCONS(4,7)=0.
      GCONS(5,7)=0.
      GCONS(6,7)=0.
      GCONS(7,7)=17.
      GCONS(8,7)=-2.
      GCONS(9,7)=0.
      GCONS(10,7)=0.
      ER(7)=ER(1)-AL*CONS(7)
      DO 130 IJ=1,10
      GE(IJ,7)=GE(IJ,1)-AL*GCONS(IJ,7)
130   CONTINUE
      GO TO 180
C
140   CONS(8)=3.*X(1)-6.*X(2)-12.*(X(9)-8.)**2+7.*X(10)
      GCONS(1,8)=3.
      GCONS(2,8)=-6.
      GCONS(3,8)=0.
      GCONS(4,8)=0.
      GCONS(5,8)=0.
      GCONS(6,8)=0.
      GCONS(7,8)=0.
      GCONS(8,8)=0.
      GCONS(9,8)=-24.*(X(9)-8.)
      GCONS(10,8)=7.
      ER(8)=ER(1)-AL*CONS(8)
      DO 150 IJ=1,10
```

```
      GE(IJ,8)=GE(IJ,1)-AL*GCONS(IJ,8)
150   CONTINUE
      GO TO 180
C
160   CONS(9)=8.*X(1)-2.*X(2)-5.*X(9)+2.*X(10)+12.
      GCONS(1,9)=8.
      GCONS(2,9)=-2.
      GCONS(3,9)=0.
      GCONS(4,9)=0.
      GCONS(5,9)=0.
      GCONS(6,9)=0.
      GCONS(7,9)=0.
      GCONS(8,9)=0.
      GCONS(9,9)=-5.
      GCONS(10,9)=2.
      ER(9)=ER(1)-AL*CONS(9)
      DO 170 IJ=1,10
      GE(IJ,9)=GE(IJ,1)-AL*GCONS(IJ,9)
170   CONTINUE
C
180   CONTINUE
C
      RETURN
      END
```

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

INPUT DATA FOR FLOPT5

NUMBER OF ERROR FUNCTIONS .... NR = 9

INITIAL VALUE OF THE PARAMETER .... P = .10000000E+02

MULTIPLYING FACTOR FOR P .... FACTOR = .20000000E+01

PREDICTED FUNCTION LOWER BOUND .... EST = 0.

PARAMETER TO SELECT ACTIVE FUNCTIONS .... ETA = 0.

STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION CRITERION

	VARIABLE VECTOR X(I)		TEST VECTOR EPS(I)
1	.20000000E+01	1	.10000000E-06
2	.30000000E+01	2	.10000000E-06
3	.50000000E+01	3	.10000000E-06
4	.50000000E+01	4	.10000000E-06
5	.10000000E+01	5	.10000000E-06
6	.20000000E+01	6	.10000000E-06
7	.70000000E+01	7	.10000000E-06
8	.30000000E+01	8	.10000000E-06
9	.60000000E+01	9	.10000000E-06
10	.10000000E+02	10	.10000000E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .75300000E+03

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.10000000E+01	1	.10000000E+01
2	.10000000E+01	2	-.39442231E+00
3	.10000000E+01	3	.93359894E+00
4	.10000000E+01	4	.88047809E+00
5	.10000000E+01	5	.94687915E+00
6	.10000000E+01	6	-.92961487E-02
7	.10000000E+01	7	-.55378486E+00
8	.10000000E+01	8	.86719788E+00
9	.10000000E+01	9	.84063745E+00

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 1 OF FLOPT5 WITH PARAMETER P= .10000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION	I	VARIABLE VECTOR X(I)	I	GRADIENT VECTOR GU(I)
0	2	.83403970E+03	1	.20000000E+01	1	.12220232E+02
			2	.30000000E+01	2	.10989327E+02
			3	.50000000E+01	3	-.16074643E+02
			4	.50000000E+01	4	-.42948443E+01
			5	.10000000E+01	5	.37032160E+02
			6	.20000000E+01	6	-.11186646E+02
			7	.70000000E+01	7	.82458588E+02
			8	.30000000E+01	8	-.13193374E+03
			9	.60000000E+01	9	-.67731597E+02
			10	.10000000E+02	10	-.23412785E+01
20	42	.27501208E+02	1	.21663393E+01	1	.63071491E+01
			2	.23255575E+01	2	.19953444E+01
			3	.87418195E+01	3	.46146565E+01
			4	.50944225E+01	4	-.11614058E+01
			5	.94714744E+00	5	-.93512743E+00
			6	.14760845E+01	6	.56574693E+00
			7	.13729658E+01	7	-.49190885E+00
			8	.98375795E+01	8	.19805577E+01
			9	.82038165E+01	9	-.52371205E+00
			10	.83565327E+01	10	.26182125E-01
33	56	.27460677E+02	1	.21560750E+01	1	-.33325720E-04
			2	.23437964E+01	2	-.44758896E-05
			3	.87376438E+01	3	-.63261020E-06
			4	.51001572E+01	4	.20002682E-05
			5	.93844476E+00	5	-.13800442E-06
			6	.14417621E+01	6	.11614172E-05
			7	.13575275E+01	7	.10061624E-04
			8	.98225645E+01	8	-.51995822E-05
			9	.82103567E+01	9	.32999759E-05
			10	.84317146E+01	10	-.13368754E-05

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .26157974E+02

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.57092360E+00	1	.10000000E+01
2	.20976647E-02	2	.53636695E+00
3	.32720946E-01	3	.72782596E+00
5	.29450788E-01	5	.71936043E+00
6	.17216261E+00	6	.87528847E+00
7	.49472872E-01	7	.76203790E+00
9	.14317152E+00	9	.85753762E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .744 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 4 OF FLOPT5 WITH PARAMETER P= .80000000E+02

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	86	.24306209E+02	1	.21719928E+01	1	-.28088919E-02
			2	.23636803E+01	2	-.21233966E-02
			3	.87739286E+01	3	-.96232076E-03
			4	.50959848E+01	4	.81013242E-03
			5	.99065229E+00	5	-.17162769E-02
			6	.14305745E+01	6	.73534416E-03
			7	.13216471E+01	7	.92835060E-02
			8	.98287291E+01	8	.32429676E-02
			9	.82800787E+01	9	-.10270578E-01
			10	.83759368E+01	10	.41078029E-02
4	92	.24306209E+02	1	.21719963E+01	1	-.628086 15E-05
			2	.23636829E+01	2	-.12760877E-04
			3	.87739258E+01	3	.10551200E-04
			4	.50959845E+01	4	-.22717833E-05
			5	.99065475E+00	5	-.15011504E-05
			6	.14305740E+01	6	.64826320E-06
			7	.13216442E+01	7	.55922196E-05
			8	.98287258E+01	8	-.23238948E-04
			9	.82800916E+01	9	.16390917E-05
			10	.83759267E+01	10	-.68039049E-06

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .24306228E+02

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.58133984E+00	1	.99999927E+00
2	.20545810E-02	2	.10000000E+01
3	.31202877E-01	3	.99999906E+00
5	.28704921E-01	5	.99999923E+00
6	.17165305E+00	6	.99999925E+00
7	.47452031E-01	7	.99999916E+00
9	.13759270E+00	9	.99999915E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .153 SECONDS

USER: THIS IS YOUR SPACE. USE IT AS YOU WISH.

ITERATION NO. 5 OF FLOPT5 WITH PARAMETER P= .16000000E+03

UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLETCHERS METHOD

ITER. NO.	FUNC. EVAL.	OBJECTIVE FUNCTION		VARIABLE VECTOR X(I)		GRADIENT VECTOR GU(I)
0	94	.24306209E+02	1	.21719963E+01	1	-.78579153E-04
			2	.23636829E+01	2	-.65069472E-04
			3	.87739258E+01	3	.49963541E-04
			4	.50959845E+01	4	-.24147132E-05
			5	.99065475E+00	5	-.75374786E-05
			6	.14305740E+01	6	.32352610E-05
			7	.13216442E+01	7	.97533575E-04
			8	.98287258E+01	8	-.10284618E-05
			9	.82800916E+01	9	-.97198409E-04
			10	.83759267E+01	10	.38854610E-04
1	96	.24306209E+02	1	.21719964E+01	1	.32001961E-05
			2	.23636830E+01	2	.39224626E-04
			3	.87739257E+01	3	-.36266857E-04
			4	.50959845E+01	4	.11933656E-04
			5	.99065477E+00	5	.72956038E-05
			6	.14305740E+01	6	-.31255980E-05
			7	.13216442E+01	7	.10746210E-04
			8	.98287258E+01	8	.13219422E-03
			9	.82800916E+01	9	-.55834407E-04
			10	.83759267E+01	10	.22370321E-04

WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT

MAXIMUM OF THE ERROR FUNCTIONS .... EM = .24306211E+02

	WEIGHT VECTOR V(I)		NORM. ERROR VECTOR EN(I)
1	.58134007E+00	1	.99999994E+00
2	.20545349E-02	2	.99999979E+00
3	.31202328E-01	3	.99999982E+00
5	.28704984E-01	5	.99999995E+00
6	.17165486E+00	6	.10000000E+01
7	.47451681E-01	7	.99999989E+00
9	.13759155E+00	9	.99999988E+00

EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS .107 SECONDS

VI. COMPARISON OF RESULTS

Table IV compares the number of function evaluations presented in Charalambous' report required to solve each problem, with those required using FLOPT4 and FLOPT5. The corresponding starting points were used.

The superiority of the new package over FLOPT4 and sufficient consistency of the results with those of Charalambous are evident to conclude that the algorithm appears to have been correctly implemented.

TABLE IV COMPARISON OF THE RESULTS OF CHARALAMBOUS, FLOPT4 AND FLOPT5

Problem		Charalambous' Report [2]	FLOPT4 [5]	FLOPT5
Rosenbrock	a	-	2.	10.
	b	-	2.	10.
	c	-	47	47
	d	-	$2.6x10^{-25}$	$2.6x10^{-25}$
Minimax with 3 functions	a	-	4.	4.
	b	-	4.	4.
	c	-	52	44
	d	-	1.9522246	1.9522245
Microwave Circuit	a	10.	8.	8.
	b	10.	6.	6.
	c	78	89	70
	d	.1972906	.19729063	.19729063
Beale	a	-	10.	10.
	b	-	2.	2.
	c	-	39	34
	d	-	.11111111	.11111111
Wong Problem 1	a	10.	10.	10.
	b	10.	2.	2.
	c	107	110	89
	d	680.6301	680.63006	680.63006
Wong Problem 2	a	10.	10.	10.
	b	10.	2.	2.
	c	120	101	96
	d	24.30621	24.30621	24.30621

a: Value of parameter p
b: Value of FACTOR
c: The number of function evaluations
d: The optimal function value

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LISTINGS OF SUBROUTINES

FLOPT5, GRDCHK5, HEADER, LEASTP5 and QUASI5

```
      SUBROUTINE FLOPT5 (EPS,JD,X)
C
C     THIS IS THE CHIEF SUBROUTINE OF FLOPT5 PACKAGE. IT EXECUTES THE
C     CHARALAMBOUS ALGORITHM PUBLISHED IN REPORT 28-0-280677 OF THE
C     DEPARTMENT OF SYSTEMS DESIGN, UNIVERSITY OF WATERLOO, CANADA
C
      INTEGER IEXIT,IFN,IH,IK,IPT,JD(1),JV,M,MODE,N,NA,NR
      LOGICAL FINISH,GRDCHK,PRINTID,QTEMP
      REAL DMIN,EM,EPS(1),EST,FACTOR,P,X(1)
C
      COMMON /FLOPT5A/ FTIME,TTIME
      COMMON /FLOPT5B/ NLINES
      COMMON /FLOPT5P/ EM,EST,ETA,JV,MAX
      COMMON /FLOPT5Q/ DMIN,IEXIT,IFN,MODE
      COMMON /FLOPT5X/ FINISH,GRDCHK,PRINTID
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /FLOPT5Z/ FACTOR,P
      COMMON /HEADER1/ MXLINES
C
C     THE DEFAULT VALUES OF THE VARIABLES LISTED IN TABLE 1 OF REPORT
C     SOC-218 ARE ASSIGNED BY THE FOLLOWING DATA STATEMENTS
C
      DATA FINISH,GRDCHK,PRINTID/.FALSE.,.TRUE.,.TRUE./
      DATA EST,ETA,IFN,JV/4*0/,IH,IK,MODE,NR/4*1/,IPT/20/,MAX/200/
      DATA FACTOR,P/2*10./,FTIME,TTIME/2*0./
C
C     A DESCRIPTION OF ALL THE VARIABLES USED BY THIS AS WELL AS OTHER
C     SUBROUTINES NOW FOLLOWS
C
C     ************************ INTEGER VARIABLES ***********************
C
C     IEXIT  A FLAG USED BY QUASI5 TO STOP THE PROGRAM EXECUTION AND
C            PRINT A MESSAGE IF THE CHOSEN VALUE OF EPS IS TOO SMALL
C            (IEXIT=2) OR, IF MAX HAS BEEN EXCEEDED (IEXIT=3). IEXIT=1
C            INDICATES A NORMAL EXIT AND NO MESSAGE IS PRINTED
C
C     IFN    COUNTS THE FUNCTION EVALUATIONS
C
C     IH     USED AS THE INDEX OF A DO LOOP IN THE MAIN PROGRAM THAT
C            CALLS FLOPT5 IK TIMES
C
C     IK     THE NUMBER OF TIMES FLOPT5 IS CALLED FROM THE MAIN PROGRAM.
C            EACH CALL OF FLOPT5 REPRESENTS AN ITERATION IN THE CHARA-
C            LAMBOUS ALGORITHM
C
C     IPT    THE RESULTS OF THE UNCONSTRAINED MINIMIZATION ARE PRINTED
C            FOR THE FIRST AND THE LAST ITERATIONS OF QUASI5 AS WELL AS
C            AFTER EVERY IPT ITERATIONS WITHIN QUASI5. IT MUST BE NOTED
C            THAT IPT=0 SUPPRESSES THE ENTIRE PRINTOUT
C
C     JD     AN ARRAY WHICH IDENTIFIES THE ACTIVE FUNCTIONS
C
C     JV     IF NONZERO, MULTIPLIERS ARE ALSO CALCULATED IN LEASTP5
C
C     MAX    MAXIMUM PERMISSIBLE NUMBER OF FUNCTION EVALUATIONS.
C            EXECUTION STOPS IF MAX IS EXCEEDED
C
C     MXLINES THE MAXIMUM PERMISSIBLE NUMBER OF LINES ON A PAGE
C
C     MODE   FOR MODE=1 AN IDENTITY MATRIX IS THE INITIAL ESTIMATE OF
C            THE HESSIAN IN SUBROUTINE QUASI5. FOR MODE=3 THE INITIAL
C            ESTIMATE OF THE HESSIAN IS A MATRIX WHICH IS IN THE
C            DECOMPOSED FORM LDL(TRANSPOSE) AND HAS BEEN GENERATED
C            BY THE LAST CALL TO QUASI5
C
C     N      THE NUMBER OF VARIABLES IN THE PROBLEM. N.GE.2
C
C     NA     THE NUMBER OF ACTIVE FUNCTIONS. IF THE REDUCTION SCHEME IS
C            USED, A FUNCTION WHOSE MULTIPLIER V DOES NOT EQUAL OR
C            EXCEED ETA AT THE STARTING POINT OF AN OPTIMIZATION(EXCEPT
C            THE FIRST) IS CONSIDERED INACTIVE AND DROPPED FROM FUTURE
C            CONSIDERATION. WHEN THE REDUCTION SCHEME IS NOT USED, NA
C            IS SET EQUAL TO NR BY FLOPT5
```

```
C
C   NR      THE NUMBER OF ERROR FUNCTIONS IN THE PROBLEM
C
C   ************************ LOGICAL VARIABLES ***********************
C
C   FINISH SET TO TRUE WHEN X HAS CONVERGED WITHIN EPS
C
C   GRDCHK SHOULD BE SET TO FALSE BY THE USER WHEN GRADIENT CHECK IS
C          NOT REQUIRED
C
C   PRINTID SHOULD BE SET TO FALSE BY THE USER WHEN INPUT DATA IS NOT
C          REQUIRED TO BE PRINTED
C
C   ************************   REAL VARIABLES   **********************
C
C   EM     EQUALS THE MAXIMUM OF THE ERROR FUNCTIONS
C
C   EPS    THIS ARRAY OF N ELEMENTS IS USED FOR TESTING THE CONVER-
C          GENCE OF THE SOLUTION OF THE UNCONSTRAINED OPTIMIZATION AS
C          WELL AS THE MINIMAX SOLUTION
C
C   ER     AN ARRAY OF NR ELEMENTS CONTAINING THE VALUES OF THE ERROR
C          FUNCTIONS. ARRAY EN CONTAINS THE NORMALIZED VALUES AND
C          ARRAY ES CONTAINS THE NORMALIZED VALUES RAISED TO POWER P
C
C   EST    USERS GUESS OF THE OPTIMAL OBJECTIVE FUNCTION VALUE
C
C   ETA    USED BY THE REDUCTION SCHEME TO SELECT ACTIVE FUNCTIONS,
C          I.E., THOSE FUNCTIONS WITH MULTIPLIER VALUES .GE. ETA
C
C   FACTOR MULTIPLIES P TO UPDATE ITS VALUE FOR A SUBSEQUENT ITERATION
C
C   G OR   ARRAY OF (N,NR) ELEMENTS STORING THE PARTIAL DERIVATIVES OF
C   GE     THE ERROR FUNCTIONS
C
C   GU     ARRAY OF (N) ELEMENTS STORING THE GRADIENT VECTOR OF U
C
C   H      THIS ARRAY OF N*(N+1)/2 ELEMENTS STORES THE CURRENT
C          ESTIMATE OF THE HESSIAN MATRIX AT X
C
C   P      THE PARAMETER OF LEAST PTH APPROXIMATION
C
C   U      VALUE OF THE UNCONSTRAINED OBJECTIVE FUNCTION
C
C   V      ARRAY OF (NA) ELEMENTS STORING THE MULTIPLIERS
C
C   W      AN ARRAY OF 4*N ELEMENTS USED AS WORKING SPACE
C
C   X      AN ARRAY OF N**2+7*N+(N+4)*NR+1 ELEMENTS USED FOR
C          SCRATCH WORK BY FLOPT5. THE FIRST N ELEMENTS ALWAYS
C          STORE THE BEST AVAILABLE SOLUTION AND MUST BE INITIAL-
C          IZED WITH THE STARTING POINT
C
      CALL SECOND (T1)
      IF (IH.NE.1) GO TO 60
      CALL HEADER
      MXLINES=MAX0(MXLINES,20,12+NR,13+N)
      NA=NR
      DO 10 I=1,NA
10    JD(I)=I
      IFN=0
      IF (.NOT.PRINTID) GO TO 30
      WRITE (6,150) NR
      NLINES=NLINES+5
      IF (IK.GT.1) WRITE (6,170) P,FACTOR
      IF (IK.GT.1) NLINES=NLINES+4
      WRITE (6,190) EST
      NLINES=NLINES+2
      IF (NR.GT.1) WRITE (6,180) ETA
      IF (NR.GT.1) NLINES=NLINES+2
      IF (NLINES+N+6.GT.MXLINES) CALL HEADER
      WRITE (6,200) (I,X(I),I,EPS(I),I=1,N)
      NLINES=NLINES+N+6
```

```
      QTEMP=IPT.NE.0.AND.NR.GT.1
      IF (.NOT.QTEMP) GO TO 30
      LEN=N+1
      LER=LEN+NR
      LES=LER+NR
      LG=LES+NR
      LGU=LG+N*NR
      LV=LGU+N
C
      DO 20 I=1,NA
      X(LV-1+I)=1.
20    CONTINUE
C
      CALL LEASTP5 (X(LEN),X(LER),X(LES),X(LG),X(LGU),JD,U,X(LV),X)
      IF (NLINES+9+NA.GT.MXLINES) CALL HEADER
      WRITE (6,130) EM,(I,X(LV-1+I),I,X(LEN-1+I),I=1,NA)
      NLINES=NLINES+9+NA
30    IF (.NOT.GRDCHK) GO TO 40
      NNR=N*NR
      LER1=N+1
      LER2=LER1+NR
      LG=LER2+NR
      LW=LG+NNR
      LY=LW+NNR
      LYP=LY+NNR
      CALL GRDCHK5 (X(LER1),X(LER2),X(LG),JD,N,NR,X(LW),X,X(LY),X(LYP))
      GO TO 120
40    CONTINUE
      LEN=2*N+1
      LER=LEN+NR
      LES=LER+NR
      LG=LES+NR
      LGU=LG+N*NR
      LH=LGU+N
      LV=LH+N*N
      LW=LV+NR
C
      DO 50 I=1,NA
      X(LV+I-1)=1.
50    CONTINUE
C
C
C     THE UNCONSTRAINED OPTIMIZATION IS NOW PERFORMED BY CALLING QUASI5
C
60    CONTINUE
C
      DO 70 I=1,N
      X(N+I)=X(I)
70    CONTINUE
C
      IF (IPT.EQ.0) GO TO 80
      CALL HEADER
      WRITE (6,140) IH,P
      NLINES=NLINES+8
80    CALL QUASI5 (X(LEN),EPS,X(LER),X(LES),X(LG),X(LGU),X(LH),JD,U,X(LV
     1),X(LW),X)
      JV=1
      CALL LEASTP5 (X(LEN),X(LER),X(LES),X(LG),X(LGU),JD,U,X(LV),X)
      JV=0
      IF (IPT.EQ.0) GO TO 90
      IF (NA.NE.1.AND.NLINES+9+NA.GT.MXLINES) CALL HEADER
      IF (NA.NE.1) WRITE (6,130) EM,(JD(I),X(LV-1+JD(I)),JD(I),X(LEN-1+J
     1D(I)),I=1,NA)
      IF (NA.NE.1) NLINES=NLINES+9+NA
90    MODE=3
      P=P*FACTOR
C
      DO 100 I=1,N
      IF (ABS(X(I)-X(N+I)).GT.EPS(I)) GO TO 110
100   CONTINUE
C
      FINISH=.TRUE.
110   CONTINUE
```

```
120   IF (GRDCHK) RETURN
      IF (IPT.EQ.0) RETURN
      CALL SECOND (T2)
      TTIME=TTIME+T2-T1
      IF (NLINES+2.GT.MXLINES) CALL HEADER
      WRITE (6,160) T2-T1
      NLINES=NLINES+2
      IF (IH.NE.IK) RETURN
      IF (NLINES+6.GT.MXLINES) CALL HEADER
      WRITE (6,210) IFN,FTIME,TTIME
      NLINES=NLINES+6
      RETURN
C
130   FORMAT (*0WEIGHTS AND NORMALIZED ERRORS AT THE NEXT STARTING POINT
     1*/1H ,56(*-*)/*0MAXIMUM OF THE ERROR FUNCTIONS *,11(*.*),* EM =*,E
     215.8/1H0,31X,*WEIGHT*,13X,*NORM. ERROR*/31X,*VECTOR V(I)*,8X,*VECT
     3OR EN(I)*/1H0,24X,99(I4,E15.8,I4,E15.8/25X))
C
140   FORMAT (* ITERATION NO. *,I3,* OF FLOPT5 WITH PARAMETER P=*,E15.8/
     11H ,61(*-*)/*0UNCONSTRAINED OPTIMIZATION USING 1972 VERSION OF FLE
     2TCHERS METHOD */1H ,65(*-*)/*0ITER. FUNC. OBJECTIVE*,10X,*VARIABLE
     3*,11X,*GRADIENT*/2X,*NO.  EVAL. FUNCTION*,10X,*VECTOR X(I)*,8X,*VE
     4CTOR GU(I)*)
C
150   FORMAT (* INPUT DATA FOR FLOPT5*/1H ,21(*-*)/*-NUMBER OF ERROR FUN
     1CTIONS *,16(*.*),* NR =*,I5)
C
160   FORMAT (*0EXECUTION TIME FOR THE ABOVE OPTIMIZATION IS *,F6.3,*  S
     1ECONDS*)
C
170   FORMAT (*0INITIAL VALUE OF THE PARAMETER *,12(*.*),* P =*,E15.8/*0
     1MULTIPLYING FACTOR FOR P *,13(*.*),* FACTOR =*,E15.8)
C
180   FORMAT (*0PARAMETER TO SELECT ACTIVE FUNCTIONS .... ETA =*,E15.8)
C
190   FORMAT (*0PREDICTED FUNCTION LOWER BOUND *,10(*.*),* EST =*,E15.8)
C
200   FORMAT (*0STARTING POINT AND TEST QUANTITIES FOR THE TERMINATION C
     1RITERION*/1H0,31X,*VARIABLE*,14X,*TEST*/31X,*VECTOR X(I)*,8X,*VECT
     2OR EPS(I)*/1H0,24X,99(I4,E15.8,I4,E15.8/25X))
C
210   FORMAT (*0SUMMARY OF PROCESSING TIME IN SECONDS*/1H ,37(*-*)//* FO
     1R*,I5,* FUNCTION EVALUATIONS =*,F10.3/* TOTAL OVERALL PROCESSING T
     2IME =*,F10.3)
      END
```

```
      SUBROUTINE GRDCHK5 (ER1,ER2,G,JD,N,NR,W,X,Y,YP)
C
C     VERIFIES THE USER DEFINED PARTIAL DERIVATIVES IN SUBROUTINE FUNCT5
C
      INTEGER JD(1)
      REAL ER1(1),ER2(1),G(N,1),W(N,1),X(1),Y(N,1),YP(N,1)
C
      COMMON /FLOPT5A/ FTIME,TTIME
      COMMON /FLOPT5B/ NLINES
      COMMON /HEADER1/ MXLINES
      COMMON /FLOPT5Q/ DMIN,IEXIT,IFN,MODE
C
      DO 10 I=1,NR
      JD(I)=I
10    CONTINUE
C
      CALL HEADER
      WRITE (6,100)
      NLINES=NLINES+7
      CALL SECOND (T1)
      CALL FUNCT5 (ER1,G,JD,X)
      CALL SECOND (T2)
      IFN=IFN+1
      FTIME=FTIME+T2-T1
C
      DO 30 I=1,N
      Z=X(I)
      DX=1.E-6*Z
      IF (ABS(DX).LT.1.E-10) DX=1.E-10
      DX2=DX*2.
      X(I)=Z+DX
      CALL SECOND (T1)
      CALL FUNCT5 (ER1,W,JD,X)
      CALL SECOND (T2)
      IFN=IFN+1
      FTIME=FTIME+T2-T1
      X(I)=Z-DX
      CALL SECOND (T1)
      CALL FUNCT5 (ER2,W,JD,X)
      CALL SECOND (T2)
      FTIME=FTIME+T2-T1
      IFN=IFN+1
C
      DO 20 J=1,NR
      YDUM=(ER1(J)-ER2(J))/DX2
      IF (ABS(YDUM).LT.1.E-20) YDUM=SIGN(1.E-20,YDUM)
      GDUM=G(I,J)
      IF (ABS(GDUM).LT.1.E-20) GDUM=SIGN(1.E-20,GDUM)
      YP(I,J)=ABS((YDUM-GDUM)/YDUM)*100.
      Y(I,J)=YDUM
20    CONTINUE
30    CONTINUE
C
      DO 70 J=1,NR
      I=1
      IF (NLINES+1.GT.MXLINES) CALL HEADER
      IF (NLINES.EQ.3) WRITE (6,100)
      IF (NLINES.EQ.3) NLINES=NLINES+7
      IF (YP(1,J).LE.10.) WRITE (6,80) J,I,G(1,J),I,Y(1,J),I,YP(1,J)
      IF (YP(1,J).GT.10.) WRITE (6,90) J,I,G(1,J),I,Y(1,J),I,YP(1,J)
      NLINES=NLINES+1
C
      DO 50 I=2,N
      IF (NLINES+1.GT.MXLINES) CALL HEADER
      IF (YP(I,J).GT.10.) GO TO 40
      WRITE (6,110) I,G(I,J),I,Y(I,J),I,YP(I,J)
      GO TO 50
40    WRITE (6,120) I,G(I,J),I,Y(I,J),I,YP(I,J)
50    NLINES=NLINES+1
      IF (NLINES+1.GT.MXLINES) GO TO 70
      WRITE (6,60)
60    FORMAT (1H )
      NLINES=NLINES+1
```

```
70    CONTINUE
      IF (NLINES+15.GT.MXLINES) CALL HEADER
      WRITE (6,130) IFN,FTIME/IFN
      NLINES=NLINES+15
      RETURN
C
80    FORMAT (1X,I3,2X,3(I4,E15.8))
C
90    FORMAT (1X,I3,2X,3(I4,E15.8),2H *)
C
100   FORMAT (74H VERIFICATION OF THE USER DEFINED PARTIAL DERIVATIVES I
     1N SUBROUTINE FUNCT5/1X,73(1H-)//6H ERROR,5X,12HUSER DEFINED,7X,11H
     2NUMERICALLY,8X,10HDIFFERENCE/6H FUNC.,5X,9HPARTIAL   ,10X,12HAPPROX
     3IMATED,7X,11H(PERCENTAGE/4H NO.,7X,10HDERIVATIVE,9X,10HDERIVATIVE,
     49X,17HW.R.T. NUMERICAL)/)
C
110   FORMAT (6X,3(I4,E15.8))
C
120   FORMAT (6X,3(I4,E15.8),2H *)
C
130   FORMAT (//49H FUNCTION CALLS MADE FOR THE ABOVE CALCULATIONS =,I3/
     1/49H AVERAGE CPU SECONDS FOR EACH FUNCTION CALL      =,F8.4//51H *
     2INDICATES THAT THE DIFFERENCE EXCEEDS 10 PERCENT//* USER DEFINED A
     3ND NUMERICAL DERIVATIVE VALUES IN THE INTERVAL (-.1E-19,+.1E-19)*/
     4* ARE LOWERED OR RAISED TO THE NEAREST LIMIT TO CALCULATE PERCENTA
     5GE DIFFERENCE*//* PERCENTAGE DIFFERENCE=100. ABS((USER DEFINED - N
     6UMERICAL)/NUMERICAL VALUE)*////* END OF GRADIENT VERIFICATION PROG
     7RAM*)
C
      END
```

```
      SUBROUTINE HEADER
C
C     PRINTS HEADING ON A NEW PAGE. FIRST LINE CONTAINS THE DATE, TIME
C     AND PAGE NUMBER. THE SECOND LINE CONTAINS THE PROBLEM IDENTIFICA-
C     TION PROVIDED BY THE USER WHICH CAN BE UPTO 80 CHARACTERS LONG. IT
C     IS ONLY CALLED WHEN A PAGE EJECT IS NECESSARY
C
      REAL BUF(7),PROB(8)
      LOGICAL QFIRST
      INTEGER MXLINES,NLINES,PAGE
C
      COMMON /HEADER1/ MXLINES
      COMMON /HEADER2/ BUF,PAGE,PROB,QFIRST
      COMMON /FLOPT5B/ NLINES
C
      DATA BUF/6*10H          ,10H   PAGE   /,QFIRST/.TRUE./
      DATA MXLINES/60/,NLINES/0/,PAGE/0/,PROB/8*10H          /
C
      IF (.NOT.QFIRST) GO TO 20
      QFIRST=.FALSE.
      CALL DATE (BUF(2))
      CALL TIME (BUF(4))
      READ (5,10) PROB
10    FORMAT (8A10)
      IF (EOF(5)) 20,20
C
20    PAGE=PAGE+1
      WRITE (6,30) BUF,PAGE,PROB
30    FORMAT (1H1,7A10,I3/1H ,8A10/)
      NLINES=3
      RETURN
      END
```

```
      SUBROUTINE LEASTP5 (EN,ER,ES,G,GU,JD,U,V,X)
C
C     FORMULATES THE LEAST PTH OBJECTIVE FUNCTION AND ITS GRADIENT VEC-
C     TOR. INPUT - JD,JV,N,NA,P AND X. OUTPUT - EM,EN,ER,ES,G,GU,U AND V
C
      INTEGER JD(1)
      REAL EN(1),ER(1),ES(1),G(1),GU(1),U,V(1),X(1)
C
      COMMON /FLOPT5A/ FTIME,TTIME
      COMMON /FLOPT5P/ EM,EST,ETA,JV,MAX
      COMMON /FLOPT5Q/ DMIN,IEXIT,IFN,MODE
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /FLOPT5Z/ FACTOR,P
C
      CALL SECOND (T1)
      CALL FUNCT5 (ER,G,JD,X)
      CALL SECOND (T2)
      FTIME=FTIME+T2-T1
      IFN=IFN+1
C
C     THE MAXIMUM OF THE ERROR FUNCTIONS IS BEING DETERMINED, AND IF
C     FOUND TO BE ZERO IT WILL BE SET EQUAL TO A SMALL NEGATIVE NUMBER
C
      EM=ER(JD(1))
C
      DO 10 I=1,NA
      EM=AMAX1(EM,ER(JD(I)))
10    CONTINUE
C
      IF (EM.NE.0.) GO TO 30
C
      DO 20 I=1,NA
      J=JD(I)
      ER(J)=ER(J)-1.E-10
20    CONTINUE
C
      EM=EM-1.E-10
C
C     LEAST PTH OBJECTIVE FUNCTION U IS BEING FORMULATED HERE
C
30    Q=SIGN(P,EM)
      S1=0.
C
C     WEIGHTS ARE ONLY CALCULATED WHEN JV IS NOT EQUAL TO 0
C
      IF (JV.EQ.0) GO TO 80
      SDUM=0.
C
      DO 60 J=1,NA
      I=JD(J)
      EN(I)=ER(I)/EM
      IF (EM.LT.0.) GO TO 40
      IF (ER(I).LE.0.) GO TO 50
40    ES(I)=V(I)*(EN(I)**(Q-1.))
      SDUM=SDUM+ES(I)
      GO TO 60
50    ES(I)=0.
60    CONTINUE
C
      K=0
      DO 70 J=1,NA
      I=JD(J)
      V(I)=ES(I)/SDUM
      IF (V(I).LE.ETA) GO TO 70
      K=K+1
      JD(K)=I
70    CONTINUE
      NA=K
80    CONTINUE
      DO 100 I=1,NA
      J=JD(I)
      EN(J)=ER(J)/EM
      IF (EM.LT.0.) GO TO 90
```

```
      IF (ER(J).LE.0.) GO TO 100
90    ES(J)=EN(J)**Q
      S1=S1+ES(J)*V(J)
100   CONTINUE
C
      ST=S1**(1./Q)
      U=EM*ST
C
C     GRADIENTS ARE CALCULATED BY THE FOLLOWING PROGRAM SEGMENT
C
      ST=ST/S1
C
      DO 130 I=1,N
      S2=0.
      DO 120 J=1,NA
      K=JD(J)
      IF (EM.LT.0.) GO TO 110
      IF (ER(K).LE.0.) GO TO 120
110   S2=S2+ES(K)*G((K-1)*N+I)*V(K)/EN(K)
120   CONTINUE
      GU(I)=ST*S2
130   CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE QUASI5 (EN,EPS,ER,ES,G,GU,H,JD,U,V,W,X)
C
C     THIS SUBROUTINE IS BASED ON THE 1972 VERSION OF FLETCHERS METHOD
C     OF UNCONSTRAINED OPTIMIZATION. REFER TO REPORT AERE-R7125 FOR THE
C     THEORETICAL BACKGROUND AND THE ORIGINAL PROGRAM. ALTHOUGH ESSEN-
C     TIALLY THE SAME AS THE ORIGINAL FLETCHERS PROGRAM, SOME MINOR
C     CHANGES HAVE BEEN MADE, FOR EXAMPLE, 1) THE PORTION OF THE ORIGIN-
C     AL PROGRAM WHICH DECOMPOSES H INTO LDL(TRANSPOSE) HAS BEEN RE-
C     MOVED, 2) THE DO LOOP THAT INITIALLY DETERMINES THE SMALLEST ELE-
C     MENT ALONG THE DIAGONAL OF D HAS BEEN REMOVED AND 3) SOME CODE
C     FOR CONTROLLING THE PRINTING HAS BEEN ADDED
C
      INTEGER JD(1)
      REAL EN(1),EPS(1),ER(1),ES(1),G(1),GU(1),H(1),U,V(1),W(1),X(1)
C
      COMMON /FLOPT5B/ NLINES
      COMMON /FLOPT5P/ EM,EST,ETA,JV,MAX
      COMMON /FLOPT5Q/ DMIN,IEXIT,IFN,MODE
      COMMON /FLOPT5Y/ IH,IK,IPT,N,NA,NR
      COMMON /HEADER1/ MXLINES
C
C     INITIALIZATION
C
      NP=N+1
      N1=N-1
      NN=N*NP/2
      IS=N
      IU=N
      IV=N+N
      IB=IV+N
      IEXIT=0
      IF (MODE.EQ.3) GO TO 30
C
C     THE INITIAL ESTIMATE OF H, AN IDENTITY MATRIX, IS GENERATED HERE
C
      IJ=NN+1
C
      DO 20 I=1,N
      DO 10 J=1,I
      IJ=IJ-1
      H(IJ)=0.
   10 CONTINUE
      H(IJ)=1.
   20 CONTINUE
C
      DMIN=1.
C
C     INITIAL PRINTING AND INITIALIZATION
C
   30 Z=EST
      ITN=0
      CALL LEASTP5 (EN,ER,ES,G,GU,JD,U,V,X)
      DF=U-EST
      IF (DF.LE.0.0) DF=1.0
   40 IF (IPT.EQ.0.OR.MOD(ITN,IPT).NE.0) GO TO 50
      IF (NLINES+N+2.GT.MXLINES) CALL HEADER
      WRITE (6,370) ITN,IFN,U,(I,X(I),I,GU(I),I=1,N)
      NLINES=NLINES+2+N
C
C     AN ITERATION OF QUASI5 BEGINS. IT INVOLVES SELECTION OF ALPHA,
C     THE LINE SEARCH PARAMETER, AND UPDATING OF H FOR THE NEXT ITERA-
C     TION OF QUASI5
C
   50 ITN=ITN+1
C
C     THE DIRECTION OF SEARCH, WHICH IS THE PRODUCT OF THE INVERSE OF
C     THE HESSIAN H WITH THE GRADIENT VECTOR GU, IS FOUND HERE. THE ELE-
C     MENTS OF THIS VECTOR ARE W(N+1), W(N+2),   ......, W(2N)
C
      W(1)=-GU(1)
C
      DO 70 I=2,N
      IJ=I
```

```
      I1=I-1
      Z=-GU(I)
      DO 60 J=1,I1
      Z=Z-H(IJ)*W(J)
      IJ=IJ+N-J
60    CONTINUE
      W(I)=Z
70    CONTINUE
C
      W(IS+N)=W(N)/H(NN)
      IJ=NN
C
      DO 90 I=1,N1
      IJ=IJ-1
      Z=0.
      DO 80 J=1,I
      Z=Z+H(IJ)*W(IS+NP-J)
      IJ=IJ-1
80    CONTINUE
      W(IS+N-I)=W(N-I)/H(IJ)-Z
90    CONTINUE
C
C     THE SCALAR PRODUCT OF GU WITH THE DIRECTION OF SEARCH IS NOW FOUND.
C     IT MUST BE NEGATIVE OR ELSE THE FUNCTION CAN NOT BE MINIMIZED ANY
C     FURTHER. GS IS TESTED TO ENSURE THIS
C
      GS=0.
C
      DO 100 I=1,N
      GS=GS+W(IS+I)*GU(I)
100   CONTINUE
C
      IEXIT=2
      IF (GS.GE.0.) GO TO 320
C
C     ALPHA, THE LINE SEARCH PARAMETER, WILL NOW BE CALCULATED USING
C     EITHER THE QUADRATIC FIT, THE CUBIC INTERPOLATION, OR THE LINEAR
C     EXTRAPOLATION. AN INEXACT LINE SEARCH IS MADE HERE
C
      GS0=GS
      ALPHA=-2.*DF/GS
      IF (ALPHA.GT.1.) ALPHA=1.
      DF=U
      TOT=0.
110   IEXIT=3
      IF (IFN.EQ.MAX) GO TO 320
      ICON=0
      IEXIT=1
C
      DO 120 I=1,N
      Z=ALPHA*W(IS+I)
      IF (ABS(Z).GE.EPS(I)) ICON=1
      X(I)=X(I)+Z
120   CONTINUE
C
      CALL LEASTP5 (EN,ER,ES,G,W,JD,FY,V,X)
C
C     ELEMENTS W(1),W(2), .... ,W(N) NOW CONTAIN THE GRADIENT VECTOR.
C     GYS, IN THE FOLLOWING SECTION, IS THE SCALAR PRODUCT OF THE GRAD-
C     IENT AT THE NEXT POINT WITH THE PRESENT DIRECTION OF SEARCH
C
      GYS=0.
C
      DO 130 I=1,N
      GYS=GYS+W(I)*W(IS+I)
130   CONTINUE
C
      IF (FY.GE.U) GO TO 140
      IF (ABS(GYS/GS0).LE..9) GO TO 160
      IF (GYS.GT.0.) GO TO 140
C
C     LINEAR EXTRAPOLATION FOR ALPHA IS PERFORMED HERE
C
```

```
      TOT=TOT+ALPHA
      Z=10.
      IF (GS.LT.GYS) Z=GYS/(GS-GYS)
      IF (Z.GT.10.) Z=10.
      ALPHA=ALPHA*Z
      U=FY
      GS=GYS
      GO TO 110
C
C     CUBIC INTERPOLATION TO FIND ALPHA IS PERFORMED HERE
C
140   DO 150 I=1,N
      X(I)=X(I)-ALPHA*W(IS+I)
150   CONTINUE
C
      IF (ICON.EQ.0) GO TO 320
      Z=3.*(U-FY)/ALPHA+GYS+GS
      ZZ=SQRT(Z*Z-GS*GYS)
      GZ=GYS+ZZ
      Z=1.-(GZ-Z)/(ZZ+GZ-GS)
      ALPHA=ALPHA*Z
      GO TO 110
C
C     THE LINE SEARCH HAS BEEN COMPLETED AND A NEW POINT HAS BEEN OB-
C     TAINED. H MUST BE UPDATED NOW
C
160   ALPHA=TOT+ALPHA
      U=FY
      IF (ICON.EQ.0) GO TO 300
      DF=DF-U
      DGS=GYS-GS0
      LINK=1
C
C     IF THE FOLLOWING TEST IS TRUE, THE DFP FORMULA WILL BE USED FOR
C     UPDATING H, OTHERWISE, THE COMPLEMENTARY DFP FORMULA WILL BE USED
C
      IF (DGS+ALPHA*GS0.GT.0.) GO TO 180
C
      DO 170 I=1,N
      W(IU+I)=W(I)-GU(I)
170   CONTINUE
C
      SIG=1./(ALPHA*DGS)
      GO TO 250
180   ZZ=ALPHA/(DGS-ALPHA*GS0)
      Z=DGS*ZZ-1.
C
      DO 190 I=1,N
      W(IU+I)=Z*GU(I)+W(I)
190   CONTINUE
C
      SIG=1./(ZZ*DGS*DGS)
      GO TO 250
200   LINK=2
C
      DO 210 I=1,N
      W(IU+I)=GU(I)
210   CONTINUE
C
      IF (DGS+ALPHA*GS0.GT.0.) GO TO 220
      SIG=1./GS0
      GO TO 250
220   SIG=-ZZ
      GO TO 250
C
230   DO 240 I=1,N
      GU(I)=W(I)
240   CONTINUE
C
      GO TO 40
250   W(IV+1)=W(IU+1)
C
      DO 270 I=2,N
```

```
      IJ=I
      I1=I-1
      Z=W(IU+I)
      DO 260 J=1,I1
      Z=Z-H(IJ)*W(IV+J)
      IJ=IJ+N-J
260   CONTINUE
      W(IV+I)=Z
270   CONTINUE
C
      IJ=1
C
      DO 280 I=1,N
      IVI=IV+I
      IBI=IB+I
      Z=H(IJ)+SIG*W(IVI)*W(IVI)
      IF (Z.LE.0.) Z=DMIN
      IF (Z.LT.DMIN) DMIN=Z
      H(IJ)=Z
      W(IBI)=W(IVI)*SIG/Z
      SIG=SIG-W(IBI)*W(IBI)*Z
      IJ=IJ+NP-I
280   CONTINUE
C
      IJ=1
C
      DO 290 I=1,N1
      IJ=IJ+1
      I1=I+1
      DO 290 J=I1,N
      W(IU+J)=W(IU+J)-H(IJ)*W(IV+I)
      H(IJ)=H(IJ)+W(IB+I)*W(IU+J)
290   IJ=IJ+1
C
      IF (LINK-2) 200,230,230
C
C     THE UPDATING OF H IS NOW COMPLETE AND THE NEXT ITERATION BEGINS
C
300   DO 310 I=1,N
      GU(I)=W(I)
310   CONTINUE
C
320   IF (IPT.EQ.0) GO TO 330
      IF (NLINES+N+2.GT.MXLINES) CALL HEADER
      WRITE (6,370) ITN,IFN,U,(I,X(I),I,GU(I),I=1,N)
      NLINES=NLINES+2+N
330   IF (IEXIT-2) 360,340,350
340   WRITE (6,380) IEXIT
      CALL EXIT
350   WRITE (6,390) IEXIT
      CALL EXIT
360   RETURN
C
370   FORMAT (1H0,I3,2X,I4,E15.8,99(I4,E15.8,I4,E15.8/25X))
C
380   FORMAT (1H1,*IEXIT =*,I2/1H0,*EPS CHOSEN IS TOO SMALL*)
C
390   FORMAT (1H1,*IEXIT =*,I2/1H0,*MAXIMUM NUMBER OF ALLOWABLE
     1FUNCTION EVALUATIONS HAVE BEEN PERFORMED*)
C
      END
```

SOC-218

FLOPT5 - A PROGRAM FOR MINIMAX OPTIMIZATION USING THE ACCELERATED LEAST PTH ALGORITHM

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Key Words: Unconstrained optimization, gradient minimization methods, penalty function methods, least pth optimization, minimax optimization, multiplier methods

Abstract: FLOPT5 is a package of subroutines primarily for solving least pth optimization problems. Its main features include Fletcher's quasi-Newton subroutine, a least pth objective formulation subroutine and the recent Charalambous least pth algorithm designed specifically for minimax problems. With appropriate utilization of these features, the program can solve a wide variety of optimization problems. These may range from unconstrained problems, problems subject to inequality or equality constraints to nonlinear minimax approximation problems. The program has been developed on a CDC 6400 computer. Some detailed examples of varying complexity are used to illustrate the versatility of the program. A FORTRAN IV listing is included. FLOPT5 may be regarded, from the user's point of view, as an upgraded FLOPT4, a previous package. Some results of performance comparison with FLOPT4 are also included.

Description: Contains Fortran listing, user's manual.
Source deck available for $100.00.
The listing contains 777 cards of which 280 are comments.

Related Work: SOC-151, SOC-174.

Price: $80.00.